ELECTRICAL ENGINEERING

IN HONOR OF WILLIAM CONRAD ROENTGEN WHOSE DISCOVERY OF X-RAY CHALLENGED THE UNKNOWN FORCES OF NATURE ~ IN HONOR ALSO OF THOSE WHOSE VISION, DESIRE FOR KNOWLEDGE, DEVOTION AND PERSEVERANCE EXPANDED HIS FINDINGS TO FORGE NEW TOOLS FOR DIAGNOSIS AND THERAPY AS WELL AS FOR EXPLORING THE STRUCTURE OF MATTER

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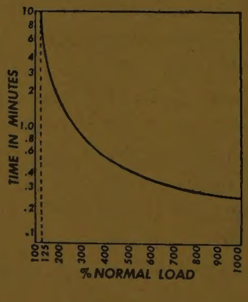

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Some Experiences With the X Ray . . . . W. D. Coolidge . . . 423
X-Ray History and Development . . . . W. D. Coolidge, E. E. Charlton . . . 427
Industrial X-Ray Developments . . . . C. D. Moriarty . . . 433
X Rays an Early Institute Topic . . . 435
Scientific Importance of X Rays . . . . L. Henry Garland . . . 437
50 Years of X-Ray Progress in Europe . . . 444
Statistical Methods in Quality Control—VII . . . . AIEE Subcommittee . . . 448
Miscellaneous Short Items:
  X Ray in Industrial Inspection, 426
  Betatron for 100,000,000-Volt X Rays, 450
Institute Activities . . . 451
Of Current Interest . . . 461

TRANSACTIONS SECTION

(Follows EE page 470; a preprint of pages 807–72 of the 1945 volume. Any discussion of these papers will appear in the December 1945 "Supplement to Electrical Engineering—Transactions Section" and in the 1945 volume.)

Resonant-Circuit Constant-Current Regulators . . . . G. M. Kevern . . . 807
Parallel Operation of 400-Cycle Aircraft Generators . . . . L. G. Levoy, Jr. . . . 811
Electric Power in a Steel Plant . . . . R. W. Graham . . . 816
Maintenance of Good Brush Performance . . . . W. C. Kalb . . . 819
Carrier-Current Relaying . . . . A. J. McConnell, T. A. Cramer, H. T. Seeley . . . 825
Resistivity of Wood and Plastics . . . . Richard C. Weatherwax, Alfred J. Stamm . . . 833
Per-Unit Impedances of Synchronous Machines—II . . . . A. W. Rankin . . . 839
Quality Control of Motor Windings and Cores . . . . P. H. Trickey . . . 842
Power Systems for Auxiliary Drives . . . . J. B. McClure, S. I. Whittlesey . . . 844
Study of Costs of Power-Transmission Systems . . . . John G. Holm . . . 852
Equivalent Circuits of the Synchronous Machine . . . . A. W. Rankin . . . 861
Modern Practice in Power-Plant Auxiliary Equipment . . H. N. Muller, Jr . . . 868

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ELECTRICAL ENGINEERING
VOLUME 64, NUMBER 12
DECEMBER 1945

Some Experiences With the X Ray

W. D. COOLIDGE
MEMBER AIEE

November 1945 commemorates the 50th anniversary of the discovery of the penetrating power of the X ray by William Conrad Roentgen. Here another pioneer discusses some of his early experiences with the tube, its gradual evolution, and its current importance.

THE 50-YEAR PERIOD which this month brings to a close began during my senior year as an electrical-engineering student at the Massachusetts Institute of Technology. That year marked the announcement of Roentgen's discovery of the X ray which soon was followed by intensive work by several members of the teaching staff at the institute on various forms of high-voltage generators to operate the tube. This work was done in collaboration with Doctor Francis Williams of Boston, one of the earliest pioneers in the application of X rays in the medical field, and it was my exciting privilege to witness many of these developments.

In addition to this I spent many evenings with a friend who had a large static machine from which various X-ray tubes were operated. Later I built a static machine of my own for the same purpose.

I became acquainted very early with the destructive effect of X rays upon living tissue. In fact it was only a few months after the announcement of Roentgen's discovery that I required medical care for an X-ray burn covering most of the back of one hand.

MEETING WITH ROENTGEN

In 1898, while a student in the University of Leipsic, Germany, I was greatly thrilled one day when Professor Roentgen entered the room in which I was working on my thesis. Geheimrath Gustav Wiedemann had died recently and Roentgen had been called in to take his place as head of the physics department. The impression which I received was an unforgettable one. It was heightened by the fact that I had previously pictured Roentgen as a frail and sickly looking individual rather than as the robust person before me who was so tall that he could just enter the door without stooping.

The new physics institute in Leipsic had just been completed and the living quarters of the Roentgens would have been on the top floor. However the front windows overlooked the cemetery and I heard that this view proved unacceptable to Mrs. Roentgen. Be that as it may, the call to Leipsic was not accepted.

ROENTGEN TUBE AS VACUUM FURNACE

In 1905, when I left Massachusetts Institute of Technology and joined the staff of the research laboratory of the General Electric Company in Schenectady, N. Y., I found there an urgent need for a vacuum furnace for melting refractory metals, including tungsten, which were being studied in the hope of finding something better than carbon for the filament of the incandescent lamp. My early experience with X-ray tubes naturally suggested that any material placed at the focus of the cathode rays in such a tube either could be melted or vaporized and, because of the vacuum, without contamination from the atmosphere. With this in mind I returned to the institute and for several weeks, with the help of a large Heinz induction coil and other equipment kindly lent me by the physics department, I had the experience of working on the development of X-ray tubes in which, for the first time perhaps, one wanted the targets to melt. For these experiments I did not get beyond the stage of using graphite targets. This was perhaps fortunate for me as graphite, with its low atomic number, is a very inefficient source of X rays and I was using much energy in the tubes and did not have adequate means of protection. In fact, when Doctor Williams visited me in order to acquaint himself with the X-ray output of one of the tubes, he said, upon leaving, that in all his experience he had never before been so much exposed to the rays.

TUNGSTEN-TARGET DEVELOPMENT

In Schenectady, when, after several years' work on tungsten, we had succeeded in making that originally brittle metal ductile and had learned how to make good lamp filaments from it, we looked for other applications. One of the first of these was as a substitute for platinum in the target of the X-ray tube.

Experiments were needed not only to determine design features but also to learn the best metallurgical characteristics of the tungsten metal for the target face. To facilitate the experiments the most powerful X-ray generator of that time, a 10-kw Snook transformer machine, was acquired. As the purpose of the study was the development of as robust a target as possible, the experiments involved serious overloading of the tubes. This overloading developed weaknesses in this type of tube which manifested themselves in the form of punctures in the glass envelopes and in cracks due to local overheating. These troubles subsequently were reduced by operating the tubes immersed in oil but new limitations then presented themselves. With the heavier loads which could be used with oil immersion the aluminum cathodes melted. In addition there was the ever-present difficulty of controlling the gas pressure in the tubes.

THE HOT-CATHODE HIGH-VACUUM TUBE

To avoid the melting of the aluminum cathodes, we tried replacing them with tungsten. This resulted in a tube which was hopelessly unstable ("cranky" in the language of the time).

In another tube the target and cathode both were made of massive tungsten and were identical in form. This tube also was very unstable. The cathode showed essentially the same heating as the target, and the positive ions which bombarded it were about as sharply focused as were the electrons striking the target. It was at first impossible to operate it for more than an instant at a time as, with the application of high voltage, the gas immediately would clean up. It later developed, however, that if the gas pressure then were increased quickly by means of the regulator, the tube again could be operated for an instant, and that if this procedure were repeated several times in quick succession the cathode would become very hot and, in this condition, the tube could be operated continuously. This seemed to

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An article prepared for the *American Journal of Roentgenology and Radium Therapy*, December 1945, volume 54, and published in *Electrical Engineering* in collaboration with that publication in joint recognition of Roentgen's contribution to science.

W. D. Coolidge is X-ray consultant, Research Laboratory, General Electric Company, Schenectady, N. Y.

bespeak interesting possibilities for a tube in which the cathode could readily be heated at will. The idea of getting electrons from a hot body was not new. Thomas Edison had observed it in the incandescent lamp and O. W. Richardson had investigated the relation between electron emission and temperature. There was, however, much skepticism among physicists at the time as to whether electron emission would continue in case the gas were completely removed from the hot body.

At just the time, however, when we had become conscious of the fact that most of the limitations of the original type of Roentgen tube were due to the gas content, without which it could not operate, and were wishing for a stable source of electrons in a high vacuum, Doctor Irving Langmuir, of our laboratory, was studying electron emission from hot tungsten filaments. He found that even in the highest vacuum the emission was stable and reproducible and that it was even favored in amount by freeing the cathode filament of its original gas content.

These circumstances led me to the construction of experimental high-vacuum tubes with a heated tungsten filament as cathode and a tungsten disk as anode. Until these tubes had been continuously pumped for many hours by an exhaust system which was good at the time, but which now would be considered very slow, they showed some of the green fluorescence of the glass which always had attended the operation of a Roentgen tube. However, as the electrodes became freed of gas, this fluorescence became less and finally disappeared completely. The tube was then stable and controllable, and we were able to satisfy ourselves that its behavior was the same as it would be even if it had a perfect vacuum. The positive ions, essential to the operation of the earlier tubes, were no longer needed and the limitations, most of which were due to the presence of those positive ions, were gone with them.

Doctor Lewis Gregory Cole in New York was the first roentgenologist to be equipped with the new type of tube. To introduce it and me to the medical profession, he gave a dinner in New York on December 27, 1913, to which he had invited many prominent roentgenologists. A powerful high-voltage generator had been installed in the dining room by Doctor Harry Waite, of the firm of Waite and Bartlett, and with it I was privileged to demonstrate the new tube. Upon recalling the affair now, it seems surprising that the audience stayed through the demonstration. Up until that time the capacity of high-voltage X-ray generators had been considerably in excess of what the tubes could stand for any length of time, but with one of the new tubes having a sufficiently large focal spot this was no longer true. I opened the machine up wide and, with the limited amount of protection which the open lead-glass bowl of that time afforded, the audience must have received many more X rays than they were accustomed to experience.

Doctors Cole, James Case, Walter Dodd, Eugene Caldwell, George Pfahler, Preston M. Hickey, and many other roentgenologists were very helpful to us in those early days in adapting the new tube to the various medical applications.

Courtesy *Radiology*

Dr. W. C. Röntgen

MYSTERY OF ROENTGENOGRAPHY

The gas-filled X-ray tube had not been easy to control and its successful use had called for both patience and experience. It was probably as a result of the idiosyncrasies of that type of tube that it was not uncommon to hear an expert roentgenologist say that once upon a time he had made a wonderful roentgenograph whose quality he had never since been able to equal. There seemed to be some mystery connected with this—possibly some particular X-ray spectrum was most desirable.

A factor which had contributed to the difficulty of clearing up the mystery was the runaway tendency of the discharge through the gas-filled tube. This had made it necessary that the high-voltage source employed should not have good regulation; that is, that for any given setting of the controls, its voltage should fall rapidly with the milliamperage drawn by the tube.

As a result, it had always been very difficult to know what voltage had been effective in producing a given roentgenograph. As ordinarily used, the parallel spark gap indicated the voltage required to initiate the discharge but this was usually quite different from, and appreciably higher than, the voltage across the tube when it was later carrying current and so producing X rays.

With the new stable tube, in which the starting and running voltages were the same, we found it desirable to use a high-voltage source which did not have a falling voltage characteristic but instead had good regulation. It was then possible to know in advance what both the milliamperage and the voltage were going to be.

Another factor which must have played some role in creating the mystery was the size of focal spot. In a gas tube this was not constant but could vary appreciably with the pressure even during an exposure. Not only this, but as the pressure changed, the position of the focal spot could change also.

There was at the time but little knowledge concerning the matter of focal-spot size. It was quite common for the manufacturer to receive an order for a tube with "pin-point focus." Such a tube, had it been delivered, would of course have been unsatisfactory because of the energy limitation which this would have imposed. This limited knowledge concerning the size of focal spot was such a handicap in the art that for some time after beginning the sale of the hot-cathode tube we supplied with each one a natural-size X-ray pinhole camera picture of its focal spot.

With the hot-cathode tube, in which current and voltage were under independent and accurate control, and the focal spot was of known and unvarying size and fixed location, we were in a position to attempt the solution of the mystery concerning that occasional outstanding roentgenograph that could not be duplicated. As a subject was needed for the experiment, I called on one of my medical friends for assistance. (I had temporarily and unintentionally sacrificed my own back hair to previous experiments and did not like to practice on other living subjects.) My friend provided me with material which was very useful over a period of months. I wish that I might make fitting acknowledgment at this time to the willing and complacent subject of those experiments. (I cannot think of it even today without smelling formaline.)

Later experiments with the detached leg of another passive subject led to a serious misunderstanding. As it happened, upon completion of these experiments, we wrapped the member in question in a fabric which was easily available to us, varnished cambric, and took it to the works' incinerator. The operator in charge removed the cover and we inserted the package, making no attempt at explanation, as the attendant did not understand English very well. It seems that, after we left, he raised the cover of the incinerator, and, looking into the furnace, saw our varnished cambric unwrap itself, revealing what seemed to be evidence of a ghastly crime. We later received a visit from our works' detective who was in a very serious mood. We explained and explained but have never been sure that he has ceased to regard us with suspicion.

The conclusion drawn from these experiments was that there was no mystery—that, other things being equal, contrast was determined solely by the voltage used and that definition was a function of the size and fixity of position of focal spot. We concluded that if one had his choice he would not use the mixture of rays which the tube gives, but would take monochromatic radiation of a wave length dependent on the thickness of the part to be radiographed; but that, not having this choice, he must be content with that mixture of rays coming from the tube when operated with the voltage which gives the desired contrast. Increasing the breadth of the spectrum used, by making a part of the exposure with low voltage and the rest with high, did not help.

FIELD CURRENTS

It appeared early in the course of our work that if cathode and target were brought very close together, or if there were a sharp point on the cathode, a high-voltage discharge could take place even from a cold cathode and in the highest attainable vacuum.

In poorer vacua a similar effect had been observed and described by H. A. Rowland, N. R. Carmichael, and L. J. Briggs, and by R. W. Wood.

Upon learning that electrons could be pulled out of a cold cathode, even in the highest vacuum, by means of a high potential gradient, the question arose as to whether a tube based upon this principle might not, because of the greater simplicity involved in the control equipment, be more attractive than a hot-cathode tube.

Extensive efforts which we made at the time, however, as well as all subsequent experience, have borne out the conclusion then reached that such a discharge is not sufficiently stable. Nor does it offer the great flexibility of the hot-cathode tube which permits independent control of milliamperage and kilovoltage.

Not only did we decide not to make use of field currents but it soon developed that we must take steps to avoid them, as they otherwise set a limit to the voltage which a tube could support. For operation at high voltage we found it helpful to avoid sharp edges and corners on the electrodes and to space them well apart. Even with these and other precautions, however, it proved difficult to get above a few hundred thousand volts until we hit upon the expedient of grading the potential by means of auxiliary electrodes. The resulting cascade tube made it possible to avoid field-current limitations and so to build tubes for any desired voltage.

THE RADIATOR TUBE

World War I brought interesting experiences in connection with the development of X-ray equipment. For use near the front it was clearly desirable to have a simple self-contained and dependable generating outfit of adequate power and maximum portability. It was obviously desirable to eliminate, if possible, the mechanical high-voltage rectifying switch which was in general use at the time. This was before the days of the kenotron but was still possible if we could develop a hot-cathode Roentgen tube which could be depended upon to rectify its own current. To meet this requirement it was imperative that no part of the focal spot ever should reach the temperature at which appreciable thermionic emission would take place. This would lead to electron bombardment of the cathode filament thus raising its temperature and causing run-away with consequent destruction of the tube. One factor was very favorable; the amount of available energy was going to be definitely limited by the capacity of the gasoline-electric generator. The "universal" tube with its solid tungsten target would have been safe for occasional use, starting with a cold target, but not for the very frequent use which war service required. These considerations led to the development of the "radiator" tube with its composite target, heavy copper stem, and external radiator for rapid heat removal. The target itself was that which had been developed earlier for the gas tube but much experimentation was required before we were able to free the large mass of copper sufficiently from gas. For X-ray protection, a heavy two-piece lead-glass shield was developed to surround the tube completely. This tube and shield served for both the United States Army portable and bedside outfits.

I knew of no lightweight gasoline-electric set suitable for the power supply of the portable outfit. It seemed possible that a motorcycle could be used to advantage as a highly portable power supply and belted to an electric generator at the point where X rays were required. Through the courtesy of the Indian Motorcycle Company I was provided with a motorcycle on which I took my first motorcycle ride. One experience was enough to satisfy me that this was not the way to treat the roentgenologist.

Through the application of tungsten contacts in the automobile-ignition system, I had become acquainted with Charles F. Kettering, then of the Delco Company, and this connection suggested the 1-kw house lighting set of that company. This was a lightweight 32-volt gas-engine-driven d-c generator. We needed an a-c generator and, for ready adaptation to a high-voltage transformer which we had found already in production by the Victor X-Ray Company, we required 115 volts. The Delco Company obligingly made changes required in the generator, but the engine did not respond well to having the full X-ray load thrown suddenly upon it. I found it necessary to ask for Doctor Kettering's personal assistance. He spent a whole week with us in Schenectady and so modified that engine that it was able to accept the suddenly applied load promptly.

Doctor John S. Shearer, then professor of physics at Cornell University, had assumed the responsibility for the design of the "portable" table and he co-operated closely with us in harmonizing table and X-ray source.

Shipment of the tube caused us much worry at first. Our experience was all in favor of a lightweight open-work crate which permitted the handler to see the glass, and so made him constantly aware of its fragility. But army shipping instructions called for a solid crate suspended from the corners of another solid crate and we were told that the tube so packed must stand dropping from a dray onto a stone pavement. We found that our tubes would stand this treatment but, in the first lot shipped abroad in this way, 40 per cent were broken. We then were permitted to use our fragile open-work crates and, when shipped in this way to the war zone, only one per cent were broken.

OIL-IMMERSED EQUIPMENT

The fact that the hot-cathode tube could be made very small, especially for oil immersion, could rectify its own current, had a long life, and did not need to be seen during operation, made it seem very attractive to install it in the same metal container with the transformer serving as the source of high voltage. In this way it was easy to secure excellent X-ray protection, as well as protection from electric shock. Many small outfits of this type have been used for dental work and other purposes, while larger ones have been used for therapy.

In connection with our work on what might be called a glorified Lenard tube, we had tried to develop the induction coil for very high voltage. Using a mercury-turbine interrupter for the primary current, we had succeeded in attaining 900,000 volts. The difficulties, however, increased rapidly with voltage and it appeared very difficult to go much higher by this method.

In 1932, when I became director of the laboratory, Doctor E. E. Charlton was made head of our X-ray section and, ably assisted by W. F. Westendorp and others, still holds that position. Since then I have been able to devote much less time to this field, but have had the privilege of being a very interested observer.

THE RESONANCE TRANSFORMER

My earlier experience with the induction coil and other high-voltage sources has made me very appreciative of the low-frequency resonance transformer developed by Mr. Westendorp. This required no iron in the center, thus permitting the Roentgen tube to be placed there where it would be electrostatically shielded by the transformer and where the connections between tube and transformer would be very short and also completely shielded.

It has been very interesting to watch the development in Doctor Charlton's section of the 1,000,000- and 2,000,000-volt Roentgen outfits employing the resonance transformer and compressed-gas insulation. By this method it looks practicable to go still higher to perhaps 4,000,000 or 5,000,000 volts and with several milliamperes of current.

It also has been a pleasure and a privilege to watch the development of the betatron. After Doctor Kerst, working at the University of Illinois, had built a successful small machine which gave 2,300,000-volt X rays, he procured a leave of absence from the university and joined our research staff. He was with us for more than a year. During that time, with the help of Doctor Charlton's group, a larger machine embodying the same principles was designed and built. This operated satisfactorily at 20,000,000 volts. Since then we have built a still larger machine of the same type which gives X rays of any voltage from 2,000,000 or so up to 100,000,000. The fact that it has been possible to proceed so directly from Doctor Kerst's first machine to one giving over 40 times the voltage would augur well for future further advances in this direction.

SIGNIFICANCE OF ROENTGEN'S DISCOVERY

During the past 50 years the X rays indirectly have extended our vision greatly. Our eyes respond directly to a region of the spectrum which is only one octave in length. The rays were first produced by Roentgen at perhaps 25,000 volts. Through the years we gradually have worked up to a few million volts and now, with the betatron, to 100,000,000 volts. In the other direction we have gone down to as low as 1,000 volts. The X-ray spectrum available today then covers a range of more than 16 octaves and indirectly extends our vision to this extent.

The X rays have shown us the relation between the various chemical elements. They serve as a means of chemical analysis and they enable us to measure the distance between the atoms in a crystal and so to determine crystalline structure.

They also perform an invaluable service in medical diagnosis and are a powerful therapeutic agent.

The X rays occupy a rapidly expanding place in industry. They are used to show cavities and foreign inclusions in structural and other materials, to indicate whether vital internal parts of a structure are present and in proper position, and recently as a thickness gauge which does not require contact with the part to be measured.

The very-low-voltage X rays are used in microroentgenography as a means of studying the composition of very thin metallic specimens and to reveal structure in delicate botanical and zoological tissues. They may be used to produce genetic changes in plants and animals.

Lastly, and to the physicist most important, the X rays have, perhaps more than any other single agency, advanced our knowledge of the structure of matter.

At the close of these reminiscences I take the opportunity to acknowledge the help and the pleasure which I have derived from my close association through the years with the many doctors, dentists, physicists, industrialists, and others interested in the various uses of X rays.

X Ray in Industrial Inspection

X-ray equipment, manufactured by the North American Philips Company, Inc., used for radiography or fluoroscopy in the inspection of industrial parts for flaws. Operator is shown placing a trayful of small metal castings into specimen chamber. The lead-lined door then is closed, after which the unit may be energized. An eyepiece below the tray permits visual inspection. For radiography the film is placed under tray in chamber

X-Ray History and Development

W. D. COOLIDGE   E. E. CHARLTON
MEMBER AIEE

Development of X-ray equipment from the early gas tube used 50 years ago by William Conrad Roentgen to the present-day induction electron accelerator is traced. Topics discussed include the various types of tubes involved in X-ray history, the limitations of each, and the attainment of an invaluable industrial and scientific tool.

SINCE its birth, the X-ray tube has undergone many radical changes. The general method of producing X rays is, however, still the same, namely by accelerating electrons to a high velocity and then suddenly stopping them by collision with a solid body, the so-called target.

Depending upon the method used in generating the electrons, Roentgen tubes may be classified into two general groups, gas tubes and high-vacuum tubes. In the first group, the electrons are freed from a cold cathode by positive ion bombardment, thus necessitating a certain gas pressure. In the second group, the vacuum is made as good as possible and the electrons are freed from the cathode either by heat, by bombardment by other electrons, or by the use of a potential gradient high enough to remove them electrostatically.

THE GAS TUBE

The first X-ray tube was of a form previously employed by Crookes in his experiments on electrical discharges through rarefied gas. The electrons liberated by positive ion bombardment from the flat aluminum cathode were emitted in a direction perpendicular to its surface and, under the impressed-voltage gradient, traveled in straight lines to the glass wall of the tube where they generated X rays.

This first X-ray source soon was greatly improved by Campbell-Swinton through the introduction of a platinum-foil target and by Professor H. Jackson through the substitution of a concave cathode for the original flat one. A later step of great importance was the addition of a device for regulating the vacuum. The early tubes were small and easily ruined as so little energy was necessary to melt the thin electrodes and to overheat portions of the glass envelope.

The power of the tubes was increased greatly by making them larger and with more massive cathodes and targets. The development of the target in particular received much attention. It resulted in a change from the use of a simple piece of sheet platinum to a platinum-faced disk of nickel brazed to a massive block of copper. This greatly increased the rate of heat flow away from the focal spot and also increased the heat-storage capacity.

A further substantial increase in tube power was obtained later by the development of a tungsten-faced copper target which consisted of a disk of wrought tungsten onto which a large mass of oxygenfree copper had been cast in a vacuum. The principal properties desired in the target facing are high atomic number for maximum X-ray efficiency, a high melting point, high thermal conductivity to allow maximum energies for a given size of focal spot, and a low vapor pressure to reduce the amount of target metal vaporized. Of all the chemical elements, tungsten combines these properties best.

The electrical characteristics of the gas tube were determined mainly by the gas pressure existing when the tube was carrying current. Owing to two opposing effects this pressure could be either higher or lower than it was before the tube was energized. Heat development resulting from positive ion and electron bombardment tended to liberate adsorbed gas from the glass and from the electrodes and so tended to raise the gas pressure. On the other hand, there was an electrical pumping action during operation which tended to reduce the pressure.

Even though the pressure might remain constant, the electrical discharge through the gas tube was of a runaway character. To combat this and to stabilize the discharge, it was necessary that the high-voltage source should have very poor regulation, that is, that its voltage should decrease rapidly as the current drawn by the tube increased.

If the pressure in the tube were too high, the voltage at all settings of the control would be too low. In the ordinary gas tube the only recourse then was to electrical cleanup. By operating the tube with current low enough to avoid appreciable heating, the gas pressure gradually would decrease. If the pressure were too low, it could be increased by admitting gas from the regulator.*

The initial pressure required for satisfactory operation was strongly dependent on the past history of the tube. The useful life was limited by the permanent electrical cleanup of gas and especially by the removal of adsorbed gas from the cathode. This last effect manifested itself by marked instability which finally became so bad that it was useless to add gas from the regulator as, on the application of high voltage, it would be immediately cleaned up. The tube then could be returned to its original condition only by rebuilding it with a fresh aluminum cathode, and it was necessary that this aluminum should contain hydrogen. It was apparently the gradual loss of this hydrogen from the cathode that was most responsible for instability. This was, at least in effect, recognized by the manufacturers of such tubes who carefully refrained from operating them any longer than necessary while connected to the pump.

As a result of the tube and circuit characteristics, it was difficult to know in advance what the voltage across the tube terminals during an exposure would be, and even difficult to know afterwards what the X-ray producing voltage had been. The tube voltage was measured by means of a spark gap connected in parallel with the tube but, as customarily used, this indicated the starting or breakdown voltage which was often much higher than the running voltage.

The size of focal spot was not constant but depended on the gap pressure and could vary appreciably even during an exposure. Not only this, but, as the pressure changed, the location of the focal spot could change also.

In the early hot-cathode tube of J. E. Lilienfeld[1] the main electrodes were the same as in the ordinary gas tube. By means of current flow between a pair of auxiliary electrodes, the cathode of which was heated, the gas of the tube was ionized. By varying the discharge current passing between the auxiliary electrodes, the conductivity between the main electrodes could be controlled. The tube operated with a gas pressure somewhat lower than that of the ordinary gas tube. It was still a gas tube, however, and was not operable if the pressure became too low.

THE HIGH-VACUUM TUBE

Most of the troubles experienced with gas X-ray tubes could be associated with the gas itself and the positive ion bombardment that took place when that gas was present. It was very desirable to get rid of the gas but this made it imperative to have some other mechanism for getting electrons out of the cathode.

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* This assumed various forms, often consisting of a side tube containing a chemical which upon being heated, as by the passage of a spark, gave off gas.

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W. D. Coolidge is X-ray consultant and E. E. Charlton is a research physicist, both with the General Electric Company, Schenectady, N. Y.

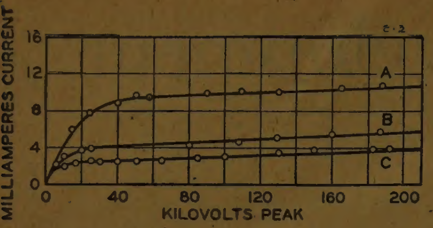

Figure 1. Curves showing the relation of current to voltage in the hot-cathode high-vacuum tube

Filament amperes: A—3.6; B—3.5; C—3.4

Edison, in his work on the incandescent lamp, had shown that in the vacuum of the lamp current could be made to flow from the hot filament to an anode. Much additional light had been shed on this phenomenon by the work of O. W. Richardson[2] and others. They connected electron emission with the temperature of the hot body. The general belief had come to be, however, that the whole hot-cathode effect resulted from gas contained in the cathode itself and that no current would flow from a hot cathode which had been freed completely from gas. Irving Langmuir's studies[3] of electron emission from hot tungsten lamp filaments demonstrated, however, that electron emission not only persisted in high vacuum but was favored by getting rid of the last traces of gas in the filament and other parts of the tube. Thus he was able to realize conditions which were stable and reproducible.

Coolidge,[4] encouraged by the work of Doctor Langmuir, developed an X-ray tube with a hot tungsten filamentary cathode and a solid tungsten target. He found that, even with the relatively high voltages and large masses of metal involved, it was possible to get and to maintain a vacuum sufficiently high to permit stable and reproducible operation.*

The relation of the current to the impressed voltage in a tube of this type is shown in Figure 1. The different curves are for different filament temperatures and show that, over the operating range of X-ray voltages, the discharge current is practically independent of voltage.

In one of the first Coolidge tubes (Figure 2A) the cathode consisted of a spiral tungsten filament mounted behind a centrally perforated tungsten or molybdenum focusing disk. Both filament and disk were set in the cathode side arm and the anode consisted of a circular tungsten disk attached to the end of a tungsten support rod. From the earliest form the design soon changed to that shown in Figure 2B, with its different cathode construction and its more massive target.

J. E. Lilienfeld[6] later developed a type of hot-cathode tube in which the primary electrons were produced from a hot filament in a side tube, the so-called ignition chamber. These primary electrons were caused to bombard the inside of the perforated main cathode where they liberated secondary electrons. The electrons passing through the perforated cathode were accelerated, by the high electrostatic field, toward the target with the production of X rays. The tube was more complicated than the simple hot-cathode type and apparently had no advantages over the latter.

In the gas tube the emission of electrons from the cathode was produced by positive ion bombardment, and in the hot-cathode tube by thermionic effect.

Experimental phenomena involving the pulling out of electrons from cold metals by high potential gradients have been observed and studied by many investigators.*[7]

H. A. Rowland, N. R. Carmichael, and L. J. Briggs[8] made early experimental tubes in which the cathode and anode consisted of aluminum wires mounted with their co-operating ends only a millimeter apart. With the best vacuum which they could produce, they found that current would pass through such a tube between the ends of the electrodes, generating X rays at a minute point on the end of the electrode which functioned as anode.

R. W. Wood[9] found that he could pass discharges from an induction coil between platinum spheres 1.5 millimeters in diameter placed 1 to 5 millimeters apart in a highly evacuated chamber over mercury, and that very penetrating X rays were produced at the anode sphere. The experiments of both Rowland and Wood were examples of field-current discharge.

Lilienfeld[10] developed an X-ray tube of this type in which the cathode was a wire with a pointed tip placed a few millimeters from the target and facing a depression in the target which served as the focal area. The function of this depressed area was to localize the focal spot which otherwise wandered about.

C. M. Slack and L. F. Ehrke[11] have developed a field-current tube for very rapid roentgenology which, for example, will depict a bullet in flight in its passage through an obstacle.

The electrical characteristics of the field-current tube resemble those of the gas tube in that the current and voltage are not independent of each other. Like the gas tube, therefore, it does not have the flexibility of the hot-cathode tube nor does it have its stability.

The main advantages of the hot-cathode high-vacuum tube over the gas tube are:

1. Flexibility; voltage and current may be varied independently at will.
2. It can be designed for much higher voltage.
3. Electrical stability permits more accurate reproducibility of results.
4. Nonvarying size and position of focal spot.
5. For a given energy input the tube can be made much smaller, thus facilitating X-ray protection and giving increased latitude in the design of auxiliary equipment.
6. Design of tube permits greater flexibility in use of hollow-anode construction to facilitate adequate X-ray protection, to reduce scattered and stray radiation, and to generate X rays at any desired location in the vacuum envelope.
7. As visual inspection during operation is not required, it can be enclosed entirely and out of sight, thus facilitating both electrical and X-ray protection.
8. Operation can be directly from a transformer without auxiliary rectifying device, thus making possible a very simple outfit which can be used for many purposes.
9. Long life.

Many forms of hot-cathode tubes of the high-vacuum type now have been de-

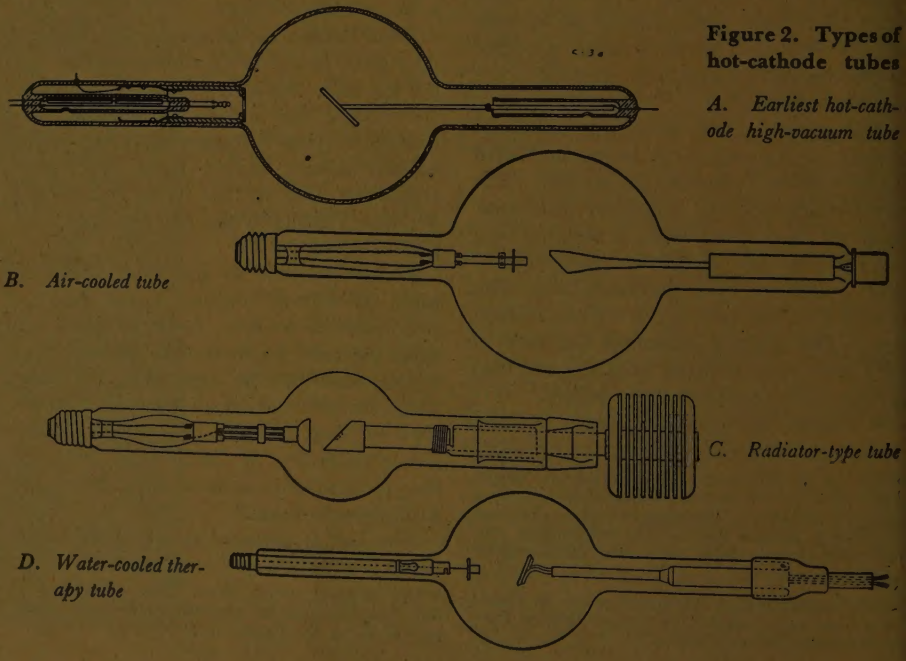

Figure 2. Types of hot-cathode tubes

A. Earliest hot-cathode high-vacuum tube

B. Air-cooled tube

C. Radiator-type tube

D. Water-cooled therapy tube

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* The idea of using a hot cathode in an X-ray tube was not new at this time, but the principle had never been successfully applied in a vacuum so good that positive ions did not play either an essential or harmful role. The hot cathode of A. Wehnelt and W. Trenkle[5] employed a lime-coated hot cathode as a main electrode, but was not operable at useful X-ray voltages

* K. T. Compton and I. Langmuir have reviewed the results of this work on field currents.

veloped to cover a wide range of usefulness. They vary in size from that of an oil-immersed dental tube with a bulb diameter of $1\frac{1}{2}$ and a length of 4 inches, up to that of the 1,400,000-volt tube at the National Bureau of Standards, which is 12 inches in diameter and 24 feet long.

The "universal" tube with a solid tungsten target is a form that has been used for both diagnostic and therapeutic purposes for many years. As its target may get very hot, it is intended to be operated only with rectified current.

Another typical design, for diagnostic work only, is a radiator-type tube, shown in Figure 2C, built with a copper-backed tungsten target. It operates over a wide range of energy ratings and is capable of rectifying its own current provided the energy used is not sufficient to heat the focal spot to a temperature at which appreciable electron emission would take place.

Another typical type, with a high energy rating for continuous operation, is a water-cooled tube (Figure 2D) especially developed for therapeutic use while for internal body-cavity work[12] a special form of therapy tube has been developed.

For use at very low voltages (approximately 1,000 to 10,000 volts) thin beryllium windows may be utilized.

The electrons are emitted from a hot tungsten filament, and the tube can be so designed that the current flowing from cathode to anode is either emission limited or space-charge limited. In the former case, over the operating range of voltage, all of the available electrons from the filament are used and the milliamperage is, therefore, dependent only on filament temperature. In the latter case, the filament always is operated at a temperature in excess of that required to emit the desired number of electrons and the milliamperage is limited by the negative space charge due to the electrons. The amount of this space charge is determined by the electron velocity and so by voltage used.

The tube used for the data contained in Figure 1, when operating with a filament current as high as 3.6 amperes, was evidently emission limited from about 50 kilovolts upward, while for lower voltages it was space charge limited.

For maximum flexibility, it is desirable to design the tube to be emission limited in order that any desired current can be used with any desired voltage. With a space-charge-limited tube, the lower the anode voltage, the less current can be made to flow through the tube.

With an emission-limited tube the current changes very rapidly with filament temperature as shown in Figure 1. This makes it desirable to have a constant source of voltage to heat the filament. Storage batteries were used at first, but since have been replaced generally by transformers for filament excitation. The effect of fluctuations in line voltage is minimized by some form of stabilizer such as the Kearsley[13] or constant-current transformer type.[14]

The hot-cathode tube, unlike its predecessor, permits the independent control of the quality and intensity of X-ray output. The quality is determined primarily by the applied voltage and secondarily by wave form and target material. It is also somewhat dependent on the angle, referred to as the tube axis, at which the X rays are emitted. These conditions all being fixed, the intensity is simply proportional to the current.

The X rays produced are in general of two kinds; those characteristic of the target material, and those which are independent of target material and, like white light, include a considerable range of wave lengths. Of the latter, the shortest wave lengths, λ_0, bear the following simple relation to the voltage:

$$\lambda_0 = \frac{12,340}{\text{voltage}}$$

in which λ_0 is expressed in angstrom units (1 angstrom $= 10^{-8}$ centimeters). For practical purposes, the *effective* wave length in general can be considered to be about twice the minimum value.

In comparison with visible radiation, which lies between 4,000 and 7,000 angstroms, X rays in use today range in effective wave length from about 0.00025, corresponding to 100,000,000 volts, to 25.0 angstroms, corresponding to about 1,000 volts. Table I gives the voltage range and the corresponding effective wave lengths of radiation for various medical purposes.

In the hot-cathode high-vacuum tube, even with perfect focusing of the primary electrons, there is in general a considerable X-ray production from the surface of the target outside of the focal area.[15] This is due to high-velocity secondary electrons emitted from the focal spot. They cannot go, as in the gas tube, straight to the glass walls, as these are, in the high-vacuum tube, relatively close to cathode potential. They must therefore return to the anode where their impact gives rise to the Roentgen radiation which may be observed, by means of a pinhole camera, as coming from other than the focal area. For diagnostic purposes this parasitic radiation would be troublesome if it were much more intense than it is. Its effect, however, could be reduced if necessary by special design.

Table I

Application	Voltage	Effective Wave Length (Angstroms)
Dental roentgenography	50,000–70,000	0.5 to 0.4
General roentgenography	30,000–100,000	0.8 to 0.24
Therapy	1,000–1,000,000	24.7 to 0.025

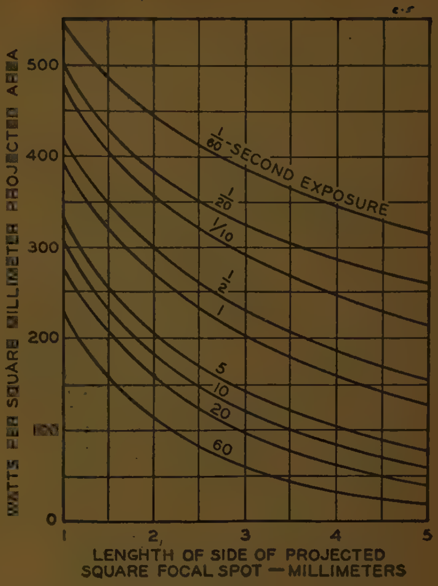

Figure 3. Tungsten-target focal-spot loading data

Single phase, full wave, 60 cycles; line focus 20-degree target angle

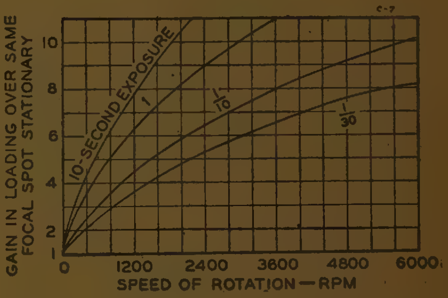

Figure 4. Gain from rotation of target

The intensity of the rays is dependent upon their direction with respect to that of the cathode rays which produced them and the relation of these two quantities is affected strongly by the voltage used. At voltages much below 1,000,000 it is customary, for most purposes, to use X rays emanating from the face of the target. It is convenient to designate these rays as the "reflected rays." With higher voltages the rays passing through the target ("transmitted rays") are employed more often. In the case of tubes intended for use in body cavities and employing relatively low voltages, transmitted rays ordinarily are used. Penetrating power also is affected by direction of emission but not to as great an extent as intensity.

Measurement of the intensity of the reflected rays from a universal tube showed[16] a maximum in close to the direction of the generally used "central ray," decreasing to half this value at an angle (measured around the girdle of the tube) of about 75 degrees on either side. This was with a voltage of 127,000. Petrauskas, Van Atta, and Myers,[17] using the unfiltered "transmitted" beam from a target placed at right angles to the cathode rays, found at 2,350,000 volts maximum X-ray intensity close to the central ray dropping to half

Table II

Voltage (Millions)	Efficiency (Measured, Per Cent)	Efficiency (From Theory, Per Cent)
2.35	10.4	8.3
1.63	5.8	5.6
0.90	3.0	3.4

value at about 37 degrees from this direction. Doctor Kerst,[18] working with the induction electron accelerator at 20,000,000 volts, found the intensity dropping from maximum to half value in about four degrees. With the 100,000,000-volt induction electron accelerator operating at full voltage, the corresponding angle is only one degree.[19]

EFFICIENCY OF X-RAY PRODUCTION

The efficiency is directly proportional to the atomic number of the target metal and increases rapidly with voltage. Rutherford and Barnes,[20] operating a hot-cathode tungsten-target tube from a static machine, obtained at 96,000 volts an efficiency of transformation of cathode-ray energy into X-ray energy of 0.2 per cent.

In Table II recent measurements made by Petrauskas, Van Atta, and Myers, with the help of the large Van de Graaff static machine at the Massachusetts Institute of Technology, show the dependence of efficiency upon voltage. The rays were taken from a gold target and in the forward direction (that of the cathode rays) and corrected for absorption in the target.

Under normal conditions, current can pass through the tube in only one direction. If, however, any portion of the focal spot is allowed to become heated to the temperature of the cathode filament, on a-c excitation, current will flow through the tube in both directions. The resulting electron bombardment of the cathode will raise its temperature, thus increasing the target bombardment, and so cause runaway. While, then, the tube may be operated satisfactorily from alternating current, serving as its own rectifier, with any given design the capacity of the tube is always greater when operating from a unidirectional source.

Even with rectifier current, the energy input must not be so high as to cause appreciable vaporization of the tungsten from the focal spot because the tungsten vapor, by becoming ionized, may cause instability, signalized by rapid rise of current.

TUBES FOR ROENTGENOGRAPHY

For the production of radiographs of the highest technical quality the tube should be so designed and constructed as to permit the production of the greatest possible X-ray intensity, consistent with satisfactory tube life, from a given size of focal spot. The target in most general use is the composite one developed earlier for the gas tube which has a tungsten facing of a certain thickness in good thermal contact with a large mass of copper. The copper with its high heat conductivity serves to take the heat away rapidly from the focal spot, distributing it to its large mass which serves as a reservoir, from which it can escape later by radiation and conduction before the next operation of the tube. The thickness of the tungsten facing will be chosen just sufficient to prevent the melting of the copper at its hottest point just behind the focal spot.

In this same connection, cathode design is also important as it determines not only the size but also the distribution of energy over the focal spot. This last must be as uniform as possible because, for a given area, the limit to allowable energy input is set by the hottest part of the focal spot.

For the maximum possible X-ray intensity per unit of focal area, direct current at constant potential would be employed to operate the tube. As single-phase a-c high-voltage apparatus is much simpler, however, it is used ordinarily. With a-c high-voltage supply, the maximum available intensity per unit area is appreciably greater if an auxiliary rectifying device, such as full-wave kenotron rectifier, is employed. Here again, however, a simpler system, in which the X-ray tube rectifies its own current, often is used.

In the line-focus tube[21,22] a rectangular electron beam is used to provide a projected square focal spot in the useful roentgenographic direction. The gain obtainable by this method[23] is determined by the minimum angle of the target face with the direction of the useful beam that will provide adequate film coverage for the largest film at the minimum distance to be used. In general, a 20-degree angle has been found to be the most practical for roentgenographic uses. For this, the gain over a 45-degree angle may be as much as threefold. In some special tubes 15 degrees are used and even 10-degree target angles have been employed. The gain for 15 degrees may be about fourfold and for 10 degrees as much as fivefold. The coverage at 10 degrees is, however, so small that a minimum six-feet target-to-film distance should be used. There is, furthermore, the disadvantage in the use of so small an angle as 10 degrees that the development, with use, of any considerable roughness of the focal area may cause serious loss of X-ray intensity in the useful direction.

Various subjects for roentgenography may require widely different amounts of radiation, as well as different limits in time of exposure. To facilitate the revelation of as much detail as possible in all cases, tubes are made with a variety of focal-spot sizes ranging from about one millimeter to nine millimeters. For focal spots of these sizes the allowable loading of a tungsten target three millimeters thick, cast in copper, varies from about 50 watts to 600 watts per square millimeter, depending on size of focal spot and time of exposure. Figure 3 gives safe values for tubes operating on single-phase full-wave rectified 60-cycle current. The smaller focal spots will stand higher specific loadings because the surrounding tungsten is more effective in heat removal from them than in the case of larger focal areas.

To simplify the technique of roentgenography of the various parts of the human body, double-focus tubes often are employed. In these tubes, by means of a double cathode, either of two radically different sizes of focal spot may be used. The smaller is employed to give fine detail in body extremities and the larger for thicker parts requiring more energy.

The loadings of a stationary target cannot exceed certain definite values safely. By rapid rotation of the target, however, relatively cold metal can be advanced constantly to take the electron bombardment, and so the permissible loadings can be increased greatly. Such rotation was suggested by R. W. Wood in 1897, and later by Rollins and Elihu Thomson. In 1915 Coolidge reported on experimental work with a tube in which the rotating target was supported by ball bearings and ran at 750 rpm, yielding a gain of twofold or threefold in the amount of energy which could be carried on a given focal area.

The target is rotated best by means of an induction motor whose stator is without and whose rotor is within the tube and carries the target. A commercial tube with a plain sleeve bearing which embodied this principle was described by Bouwers[24] in 1929.

Ball bearings, to be used successfully, must be made of metal which is hard at the temperatures to which they are subjected in this application. This requirement is fulfilled by certain precipitation hardening alloy steels.

In our early ball-bearing rotating-target tubes the drastic heat treatment required for the exhaust removed the last trace of lubricant from the bearings. As a result the friction was high and, to operate at all well, it was necessary that the bearings have appreciably more than the customary radial clearance. This made them very noisy. It since has been found that this difficulty can be overcome by coating the bearings either with a vaporized thin film of barium[26] or vacuum-coated silver.[25]

In the rotating-target tube rotation is combined with the use of the line focus. The gain to be derived by rotation is shown in Figure 4 for various speeds.[27]

In rotating-target tubes on the market today a speed of 3,000 to 3,500 rpm is

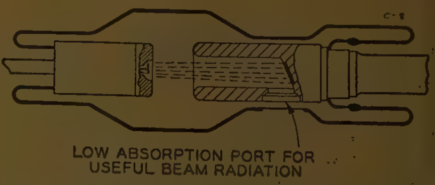

Figure 5. Self-rectifying therapy tube with hood anode construction

employed. The gain in loading as compared to a stationary-target tube varies with the exposure time and is about as the square root of the speed.

TUBES FOR FLUOROSCOPY AND THERAPY

In the medical field the same tubes used for roentgenography are, in general, suitable for fluoroscopy as the requirements of the latter service have been considered and met in the design of roentgenographic tubes. Most medical fluoroscopy is done with a current of two milliamperes to five milliamperes at voltages from 40 kv peak to 85 kv peak, depending upon the technique employed and the part of the body being studied. In this service the tube may be energized for a few seconds or for several minutes at a time.

For industrial fluoroscopic work special tubes may be required, as currents of as high as 15 to 30 milliamperes at voltages of 85 to 250 kv peak may be needed.

Because the detail recognizable in fluoroscopy is not as great as in roentgenography, exact size of focal spot is not as important as in the latter service. For the requisite long time of operation, the total heat developed is considerable, and the anode must be capable of handling it.

In therapy the main requisite is to provide a sufficiently large beam of the desired kind of radiation having essentially uniform intensity over its entire cross section.

The skin-focus distance, dictated in general by depth-dosage considerations, ranges from about 25 centimeters to 100 centimeters, and the largest field to be covered is about 20 centimeters by 20 centimeters.

The size of focal spot is relatively unimportant. Usually round, it varies from about one-quarter inch to one inch in diameter.

Therapy-treatment periods range from several minutes up to as much as an hour in special cases, so that the duty of the tube for all practical purposes must be considered as continuous. This means that the main design problem in tubes for therapy concerns the removal of heat from the anode.

This may be accomplished either by using a solid tungsten target and allowing this to heat up to a high temperature where it can radiate the requisite amount of energy, or a composite tungsten–copper target may be cooled by rapidly flowing water or oil. In the case of oil-immersed tubes a heavy anode stem may be employed to conduct heat out to the oil where it may be dissipated by natural convection.

The voltage employed in superficial therapy may be as low as 5,000 or 10,000, in which case the radiation would not be transmitted to a useful extent through the ordinary glass envelope. For such work a thin window of beryllium may be used.

Voltages from 200 to 400 kv peak make possible the treatment of the deepest seated tumors found in the body. Tubes produced today for these voltages have a thick-walled pyrex envelope to avoid puncture. They usually are cooled by circulating oil in the back of the target and at 200 kv well may carry as much as 10 or even 30 milliamperes.

They may be operated from either a rectified- or unrectified-current source. The performance of such tubes, when operating directly on alternating current, is improved considerably by the use of a hooded target, as shown in Figure 5. The presence of the hood reduces the number of secondary electrons emanating from the target and reaching the glass envelope where otherwise their presence in sufficient number might lead to puncture.

For 400,000 volts, tube design is much the same as for 200,000 volts except that all physical dimensions have to be increased. At this voltage, less current is required in general and at present five milliamperes is in common use.

During the last few years much interest has been shown in the therapeutic use of voltages of 1,000,000 or more. For such voltages, and even for much lower ones, the tube can be sectionalized to advantage[28] and provided with a multiplicity of hollow accelerating electrodes. This gives a more uniform gradient along the length of the tube and reduces dielectric stresses in the glass envelope. At the same time it serves to prevent the formation of troublesome field currents which otherwise may take place from the cathode due to the stronger field which exists there in case only a single pair of electrodes is used. The accelerating electrodes usually are connected to suitable taps in the high-voltage source. With such high voltages the target may be so designed as to make possible the use of either the radiation coming through the target or that given off from the face. At these voltages the intensity is greater in the "transmitted" than in the "reflected" beam and the effective wave length is shorter.

Such tubes may be operated either while undergoing continuous evacuation from a suitable pumping system or sealed off.

This was followed by commercial transformer installations in which the tube consisted of a multiplicity of cylindrical glass sections cemented or sealed together and provided with hollow cylindrical accelerating electrodes.

With the advent of the low-frequency resonance transformer,[29] it became possible to put the tube inside of the transformer. This location facilitates the connection of the various accelerating electrodes to the transformer and at the same time provides electrostatic shielding for both the tube and the connecting leads.

An early 1,000,000-volt example of this type of equipment was installed in 1939 at the Memorial Hospital in New York.[29] It was stationary and arranged for therapeutic use with either the reflected or transmitted beam. It was a 12-section tube and was pumped continuously.

A later similar 1,000,000-volt outfit,[30] developed originally for industrial roentgenography, is smaller, has a sealed-off tube, and can be operated in any position. A similar outfit,[31] also with sealed-off tube and operable in any position, has been developed for 2,000,000 volts. It seems entirely feasible by this method to go still higher to perhaps as much as 4,000,000 or 5,000,000 volts.

For X-ray diffraction work it is desirable to have radiation which, as nearly as possible, is monochromatic. This is obtained by using, not the general Roentgen radiation, but that characteristic of the target material. As for different purposes different wave lengths are required, this necessitates the use of tubes with different target metals, such as copper and molybdenum. The voltages used for these two metals are usually 30 to 35 kv for copper and 35 to 45 kv for molybdenum and the characteristic wave lengths are 1.539 angstroms for copper and 0.709 angstrom for molybdenum. These radiations usually are taken out through thin beryllium windows.[33] To increase the capabilities of the diffraction tube, it may be provided with a number of windows, thus making possible the simultaneous study of a corresponding number of specimens.

THE PROTECTION PROBLEM

Protection must be provided from both the X rays and the high voltage. This is accomplished much more readily with the modern type of tube than with its predecessor, and for two reasons. First, the modern tube, for the same service, can be much smaller than the gas tube, and, second, it does not need to be seen during operation.

As X rays are emitted in all directions from the front of the focal spot, lead, or its equivalent, is used either in the walls of the tube or in the tube enclosure to absorb all but the useful beam. The electrodes of the tube usually are sufficiently massive to guard against most of the radiation which otherwise would escape from the ends.

X-ray protection was secured readily in the "Metalix" tube[32] of the Philips Company, with its grounded metal envelope surrounding the central portion, Figure 6. The same result also was secured by the construction in which the radiator-type tube was provided with a two-piece thick-walled shield of glass having a high lead content.

The small tube developed for medical diagnostic use with a portable outfit, derives its X-ray protection from its very thick

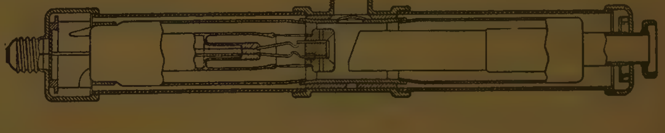

Figure 6. "Metalix" tube

envelope of glass which has a high lead content. The useful beam of rays was taken out through a leadfree glass window.

In tubes in which the target is located in a metal extension chamber attached to the tube envelope and, more particularly, in one and two million-volt tubes in which the X rays are transmitted through the target as well, X-ray protection is facilitated, to a great measure, by surrounding this chamber with an adequate wall of lead.

Full electrical protection is obtained by enclosing the entire high-voltage circuit in grounded metal. Two different methods are used for obtaining this result.

One of these is a small Roentgen tube, together with filament transformer and high-voltage transformer, all in the same oil-filled metal case. Such a system is used extensively for dental work, therapy, and other applications. In some cases, compressed gas is used in place of oil for the high-voltage insulation.

The other method of securing electrical protection consists of housing the tube in a grounded oil-filled metal enclosure, and connecting it to the high-voltage source by means of heavily insulated flexible metalclad cable. For rapid medical roentgenography this method is preferable to the former, as it permits the use of a powerful high-voltage source of rectified current while retaining relatively light weight in the part which has to be moved, namely, the tube and its enclosure.

With both systems lead, or its equivalent, will be used around the tube and the metal enclosure can be made not only to provide electrical protection but also to increase the X-ray protection.

For the production of X rays corresponding to voltages in excess of a few million, the induction electron accelerator is today the most attractive looking device. It has proved successful in the 20,000,000-[18] and the 100,000,000-[19] volt sizes and presumably can be used for higher voltages.

The tube is a hot-cathode high-vacuum device consisting of a hollow toroid of glass or other insulating material about 18 inches in diameter for the 20,000,000- and six feet in diameter for the 100,000,000-volt sizes. The electrons from a hot filament are focused electrostatically and are accelerated with some 20,000 volts or more in a direction tangential to the axis of the toroid. By means of a time-varying magnetic field they are accelerated further and focused and constrained to follow a circular path within the tube. In the case of the 100,000,000-volt machine they encircle the field 250,000 times in 1/240 seconds, receiving on an average a 400-volt push each time around. After traveling in this circular path for about 800 miles and receiving the energy which they would have had if they had passed between two electrodes having a potential difference of 00,000,000 volts, they are caused to leave their circular orbit and impinge upon a tungsten target where they produce X rays. Or they may be caused to leave the circular orbit at any desired earlier time in the magnetic cycle, thus producing X rays corresponding to any voltage up to 100,000,000.

The electron current in the beam is very low compared with that which ordinarily has been used in Roentgen tubes and of the order of a microampere, depending to a great extent on the frequency employed for the time-varying magnetic field.

The tubes may be built either as unitary structures or in sections cemented together and continuously pumped. The 100,000,000-volt tube consists of 16 pie-shaped sections of heat treated pyrex glass having an elliptical cross-section of 8 inches by $4^{7}/_{8}$ inches. The ends of the section are ground flat and smooth and to the correct angle, and the joints, coated on the outside with glyptal paint, are vacuum tight.

The induction electron accelerator, as a multimillion-volt X-ray source, will be used extensively in scientific research and also should find therapeutic and industrial applications.

SUMMARY

During the 50 years since Roentgen's great discovery, the X-ray tube, from a very uncertain and relatively feeble source of radiation, has been developed into a powerful precision tool of great stability, flexibility, and ease of control, permitting accurate reproducibility of results and capable of operation over a very wide range of current and voltage. In the diagnostic field, definition, for a given speed, has been improved greatly through the years. For therapeutic work the high-voltage rays now obtainable make possible the treatment of deepseated lesions. The various industrial applications make use of the entire range of wave lengths which can be derived from the tube, at least corresponding to voltages up to a few million. The gradual increase which has taken place in the allowable voltage has been attended by a corresponding extension in the range of usefulness of the rays in industry. Much X-ray protection now can be built into the tube itself, and the modern tube lends itself readily to the attainment of complete electrical protection. From the point where only an expert, with years of experience, could obtain the best results, the X-ray tube has developed into an instrument which is as easy to operate as an incandescent lamp.

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Industrial X-Ray Developments

C. D. MORIARTY

Advances in the production of X-ray materials, techniques in the recording of X-ray results, and the opportunities for the electrical engineer in the field of industrial X-ray equipment are discussed in this review of an application of the rays whose powers were first observed by William Conrad Roentgen a half century ago.

IN a consideration of industrial X-ray developments it is necessary to go back 30 years, and to stress particularly the influence of World War I on medical X-ray equipment and technique. At the outset it can be said that portability of equipment was a prime requirement, and with patience, perseverance, and diligence the objective was achieved.

Now, another world catastrophe has influenced the field of the X ray, and the termination of World War II finds many advances and improvements in the production of X-ray materials and the techniques of application. Not the least among these are:

1. Raising the operating potential of X-ray equipment first to 1,000,000 volts and then to 2,000,000 volts.

2. Production of new X-ray film to extend the use of this high voltage.

3. Broadened application of the X-ray technique of visual inspection of industrial products.

4. Application of photoelectric equipment to industrial units.

RECORDING X-RAY RESULTS

Fundamentally, there are three techniques of indicating or recording industrial X-ray results. They are:

1. Radiographic method. This is the chemical development of a resultant latent image on an X-ray film. The method combines high sensitivity with ease of interpretation of the results, and provides a permanent record. It is, however, relatively slow and costly.

2. Fluoroscopic method. A visual examination is made of a chemically coated screen which gives off visible light in proportion to the quantity of X rays impinging on it. This method combines ease of interpretation of results with speed and low cost. It is, however, of low sensitivity.

3. Electrical method. This is the varying of electric current in proportion to X-ray radiation by means of a phototube or by ionization. The newest of methods, it combines highest sensitivity with speed and low cost. The results, a summation of several possible variables, may be difficult to interpret.

RADIOGRAPHIC EXAMINATIONS

The time is not far distant when radiography of small parts will resemble present blueprinting methods, using a low-voltage high-current X-ray source. Instead of employing the slow stop-and-go technique, the objects being radiographed will be in constant motion. Such a procedure already is being used. Figure 1 shows the application of the method using 1,000,000-volt X rays.

Advantages of continuous motion are readily appreciated. The speed of the conveyor governs the time of exposure for a given thickness of material; the useful life of the X-ray unit is extended since the on–off cycles are eliminated or considerably reduced, and the variable in time of bringing up the voltage is eliminated. This makes for uniformity of results. Such methods of radiography require a powerful source of X rays, either in voltage or current, to counteract the relative slowness of X-ray film if normal conveyor speeds are to be maintained.

FLUOROSCOPIC METHODS

When records are desired, or careful analysis and interpretation needed, radiographic examinations will be required. However, the fluoroscopic method approximates the radiographic method except that it leaves no record. It is less sensitive, but is limited in speed only by that of the human eye. Within the limits of present fluorescent screens there are many places where fluoroscopy is applicable, even though more power is required than for radiography, and even though the resultant detail is not so good. It is ideal for rapid X-ray examination of small parts prior to machining, since it will show whether internal defects will interfere with machined, drilled, or tapped areas. It is a low-cost method sorting out grossly defective material, and avoids a waste of time on subsequent costly examinations such as radiography.

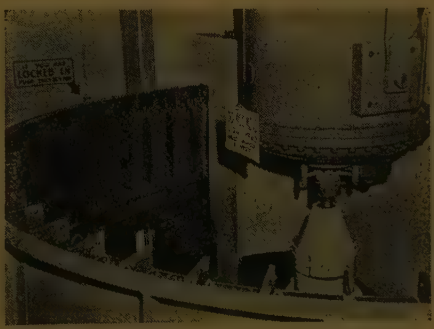

Figure 1. One-million-volt X-ray equipment for inspecting 155-millimeter shells at Ravenna ordnance plant, Apco, Ohio

Just as in the case of radiography, the trend in fluoroscopic examination is for continuous motion, as shown in Figure 3. The X-ray source in such installations is usually a low-voltage high-current unit. The supporting structure for fluoroscopic examination consists of plastic trays on a race-track type of conveyor. The objects, resting on turntables mounted on these trays, rotate while they travel between the X-ray source and the fluorescent screen. Over the fluorescent screen is an inch-thick plate of lead glass which prevents X rays from reaching the operator but permits efficient transmission of visible light from the screen.

Hundreds of thousands of objects have been examined fluoroscopically with the unit illustrated in Figure 2. Where interpretation or the nature of nonhomogeneous masses is the paramount question, the unit is used as a visual inspection tool. The eyes of the operator then are the limiting feature in speed and sensitivity. When considering the possibility of increasing the effectiveness of fluoroscopy, it should be remembered that there is always the hope of finding some fluorescent substance that will give off more visible light when exposed to X rays.

ELECTRICAL METHODS

In the field of electronic recording of X rays the future is particularly promising. One reason is because the field is relatively new.

The ability of X rays to ionize gases is well known, and has been applied advantageously in ionization-chamber methods of detecting and measuring X-ray strengths. Practical considerations encountered in industrial plants have, however, held back application of the ionization method to industrial inspection. Since 1912, with the advent of the Coolidge tube, X rays have been created in a high vacuum. Now it seems reasonable to expect that they will also be detected in high vacuum, just as visible light is detected today with a high-vacuum phototube.

Among recent new electron tubes is the multiplier phototube, combining an amplifier and a phototube in one unit. Itself a highly sensitive detector of visible light, it

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may be used indirectly for X rays. By the addition of a fluorescent substance on the transparent glass envelope of the multiplier phototube, this tube, opaque to external visible light, becomes a sensitive detector of X rays. The effects of temperature and humidity on the fluorescent substance under X-ray conditions need thorough investigation before results can be evaluated in absolute terms; but already the device has been put to several practical applications.

The first X-ray inspection job to which the multiplier phototube was applied served to illustrate its potentialities. Hand-grenade fuses were carried on a conveyor so as to intercept an X-ray beam aimed at the phototube (Figure 3). The phototube was given one-fifth second in which to determine:

1. If the fuse contained the proper powder charge.
2. To record the information on a photoelectric recorder.
3. To activate a thyratron (if there were insufficient powder), which in turn controlled a stamping device to mark any defective fuses.

Three quarters of a million fuses were thus tested in very short order. The rate of testing—4,000 per hour—was not limited by electrical but rather by mechanical characteristics of the particular setup.

On the basis of such a method, many new applications suggest themselves. Some of the present-day fluoroscopic examinations, such as the ones used on citrus fruits and canned goods may be converted advantangeously to this automatic method. Like most electronic developments, the new method introduces speed and sensitivity into a field heretofore paced by time exposures in radiography or endurance of the human eye in fluoroscopy.

Another interesting application of the phototube method was the inspection of a large turbine rotor (Figure 4). The steel forging was 13 inches in diameter, with a hole 2 inches in diameter made lengthwise through the center and "dead-ended" inside. The problem was to check for no homogeneous masses, and to measure an eccentricity of the two-inch bore. Th thickness was too great for radiography o fluoroscopy, but previous tests of the photo tube with 1,000,000-volt X rays had show good results on 12 inches of steel.

The phototube was inserted in the bor and the 1,000,000-volt X ray was pointe at it; the rays had to penetrate a radi thickness of about six inches to reach th tube. The rotor was turned at consta speed during examination, the phototu and X-ray source remaining fixed in pos tions. The output of the phototube wa supplied to a photoelectric recorder whic charted any change in radial thickne and hence any eccentricity of the bore.

Photoelectric circuits may be used i industrial X-ray work for control purpos as well as for recording. They can aut matically control the amount of exposu an X-ray film is to receive (Figure 5) or th brilliancy of a fluorescent screen. Suc control applications should take muc of the uncertainty out of industrial X-ra

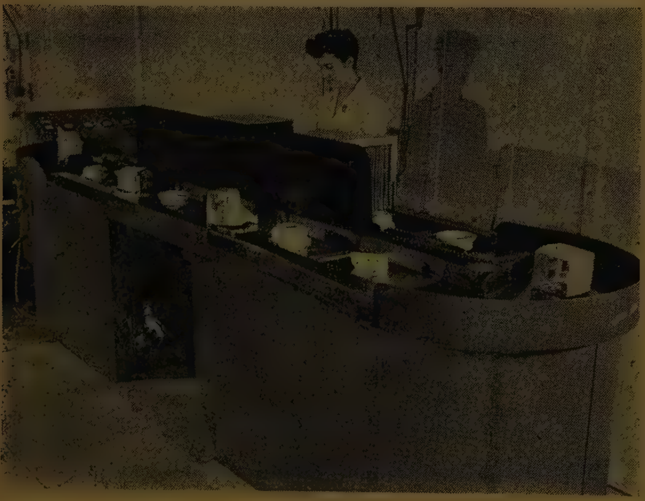

Figure 2. X-ray fluoroscopic unit for examination of light-alloy castings

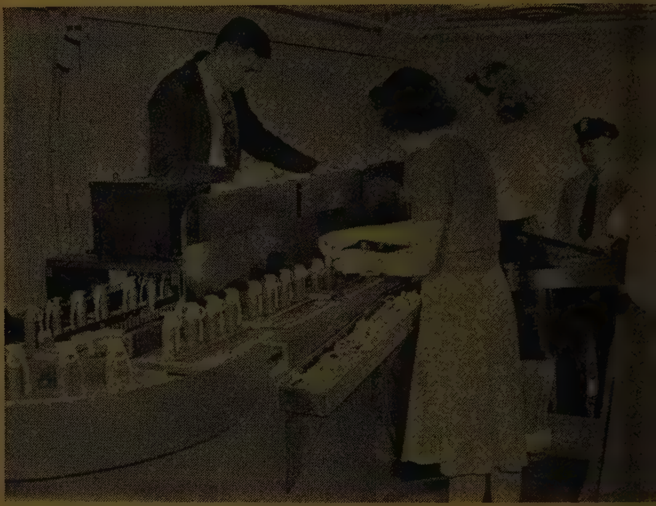

Figure 3. Phototube method of high-speed inspection of han grenade fuses

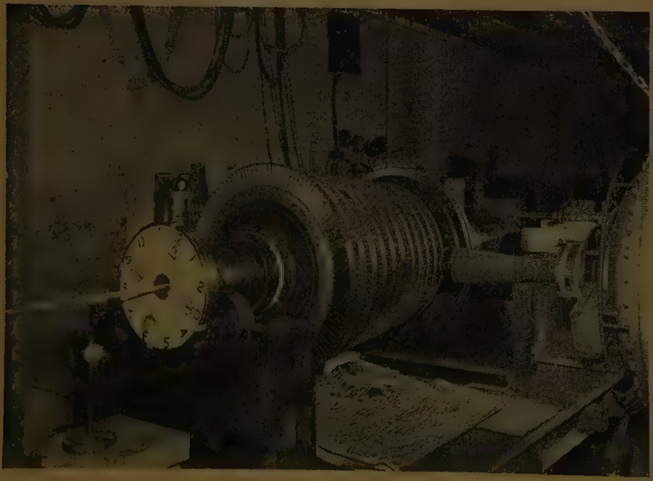

Figure 4. Phototube method of inspecting turbine rotor for eccentricity of bore (phototube unit on end of rod)

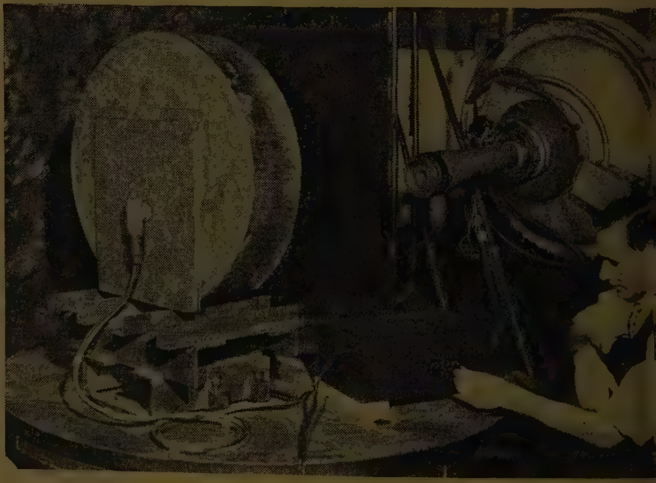

Figure 5. Proper X-ray film exposure is assured by the phot tube shown against the back of the film holder

laboratories. Variations in X-ray voltages, current, distance, thickness, or the kind of material will be taken into consideration automatically by phototube control.

COMBINATION METHODS

Most industrial X-ray examinations are made to answer either or both of two questions: "Is the material homogeneous?" and "If not, what is the nature of the nonhomogeneity?" It is the answer to the second question that is time-consuming and therefore a costly item in X-ray examination. By the same token, the value of an X-ray examination over other nondestructive methods is its ability to answer this second question. Either radiography or fluoroscopy will supply the answer to this question; the electrical method of recording X rays provides the best means for answering the first question.

Why not, therefore, combine the different methods? Let there first be an exploration with the electrical method, with interpretation where necessary with radiography or fluoroscopy. It can be done; in fact, Figure 3 shows a performance of both electric and fluoroscopic examinations. Such a combination of methods to be successful must have each method working at highest speed consistent with efficiency. Here again is where continuous motion of the object under examination comes into play.

Motion and photoelectric circuits in industrial X-ray work are among the major advances to be expected in the near future. Their possibilities are indicated by some of the jobs already done, and additional applications depend only on the ingenuity of the electrical engineer.

X Rays an Early Institute Topic

One of the leading subjects presented at Institute meetings a half century ago was the newly discovered X ray. These excerpts from the words of men whose names became recognized as being among the leaders in electrical engineering emphasize the advances in scientific knowledge that have been made during the ensuing years.

IN 1895 William Conrad Roentgen observed an invisible radiation which penetrated substances opaque to all other known radiations. In AIEE *Transactions* for the following year are recorded some of the first technical discussions of the subject. These now assume renewed interest in relation to the accumulation of knowledge since that time.

A meeting of western members of the Institute was held at Chicago, Ill., on March 25, 1896, for the purpose of exchanging information on the new ray. Charles E. Scribner presented a historical sketch of the discovery, and described experiments made by him and the apparatus used. Doctor James Burry also spoke on "The Surgical Value of the Roentgen Ray." Wilbur M. Stine* presented details of his experiments, discussing the point of formation of the rays, penumbral effects, polarization of the rays, sensitivity of photographic plates, and other matters. He concluded:

"Much stress has also been placed on fluorescent glass bulbs as the source of the ray. To test this a cube of uranium glass was powerfully excited by a focused arc light, but only negative results were obtained in repeated and lengthy exposures. The surface of the cube was also kept charged by a Holtz machine, but no results were obtained. The fluorescence of the bulb of the Crookes tube seems rather an accidental than a causative phenomenon. The kinematics of the Roentgen ray was carefully and exhaustively studied. In short, no evidences were found of diffraction, refraction, reflection, or interference. There are many appearances of such character, which have evidently misled some rather untrained experimenters, yet when carefully studied are found of negative value."

At the 111th meeting of the AIEE on December 16, 1896, at New York, N. Y., there was a topical discussion "The Roentgen Ray, and Its Relation to Physics." In opening the discussion, Henry A. Rowland† said:

"A gentleman asked me a few moments ago if I knew anything about the X rays. I told him no; that what I was going to tell tonight was what I did not know about the X rays. I do not suppose anybody can do much more than that, because all of us know so very little about them. We were very much surprised, something like a year ago, by this very great discovery. But I cannot say that we know very much more about it now than we did then. The whole world seems to have been working on it for all this time without having discovered a great deal with respect to it."

His discussion foreshadowed subsequent applications:

"Now, one important point with respect to these rays is as to whether they are homogeneous. Are they like light which can be divided up into a large number of different wave lengths, or are they homogeneous? There seems to be a great deal of evidence that they are not all the same; that one ought to get a spectrum of them in some way . . . there seems to be little reason to doubt that a number of rays really do exist; that whatever they are that come from the object, they are not all the same; some of them penetrate bodies better than others, and very likely someone will get up some sort of filter that will filter them out, and allow us to use them and to find if they have different properties. At the present we are rather in the dark with regard to this point."

He also gave credit to an earlier scientist:

"I must say that Lenard, when he first experimented upon this subject—and I regard his experiments as quite as valuable as Roentgen's, probably—, he got several kinds of rays coming out through an aluminum window. He got rays which were deflected by the magnet, as well as others. He had not separated them however. When the Lenard paper came to the laboratory I remarked to my students: 'That is the best discovery that has been made in many a day.' I immediately set somebody to work experimenting. He tried to get some results and would probably have discovered the Roentgen rays at that time if it had not been that the University of Chicago called him off, and Johns Hopkins University was very poor and could not call him back, and he had to stop in the midst of his work."

Describing means of excitation for the tubes, Elihu Thomson‡ said:

"But the most effective method of excitation which we have yet been able to use has been one of which the public has heard nothing so far and I mean to speak of it now. It was arranged by Mr. Hermann Lemp (whose name is familiar to many of you) in this way: He simply took a 12-inch inductorium, a coil giving a 12-inch spark ordinarily, and excited the primary with alternating currents at 125 cycles. This gives in the secondary, as in any step-up transformer, a great increase of potential, such that we may find that the spark darts five or six inches between terminals, and, of course, after it is started, there is a continuous arcing somewhat difficult to stop unless you blow it out or shut off the current, which latter is probably the easiest way. Now this high potential discharge of the secondary is, of course, an alternating current—an alternating current somewhat of the same wave form as the impressed primary wave. But if we rotate, by a little synchronous motor, a break piece which picks out one direction only of the secondary discharge and leaves that in the other direction open-circuited, then you see we have an admirable source of uni-directional discharges of great power. Of so great power indeed are they that you would not dare to put them upon your Crookes tube without modification. You must put up a high resistance, such as a water resistance—a long glass tube filled with water—to cut down the flow which would destroy almost any tube you might try. The commutating device made properly simply consists, for example, of two insulated terminals in series with the discharge, and a connecting wire revolving synchronously between them. With a spark gap between the wire and fixed terminals, of course we do not need friction or any contact. You are then chopping off the tops of the waves or are taking the very highest potential of each and every wave all in one direction, and you are also giving them a spark gap between the fixed terminals and revolving wire which is favorable to the generation of the rays. The spark gap regulates itself in a measure, because just as soon as the potential is such that it can easily jump a gap then the commutator wire anticipates the discharge and the current leaps the spark gap. If there should be a weaker discharge, the wire comes up nearer to the terminals before the gap is jumped, so that in this case the discharges are wonderfully uniform and you get them at the rate of 125 per second, which is a rapid rate, and excites tubes wonderfully well."

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† Professor of physics, Johns Hopkins University, Baltimore, Md.

‡ Electrician, General Electric, and Thomson Electric Welding Companies, Lynn, Mass., and AIEE president, 1889–90.

He indicated the difficulties which beset the early experimenters:

"We often come across very curious things in this work with different forms of tubes under different conditions, and I think there is hardly a more fascinating field than working with these vacuum tube arrangements. There are no two alike. You can hardly produce the same exact effects twice. I may mention a curious thing which I noticed the other day. We had a spherical bulb with a wedge of platinum, the cathodes opposite each other. We made one of these actual cathodes, and the platinum the anode, with commutated current excitation. The vacuum in the tube was a little low, and one would suppose that the side of the platinum nearest the real cathode would get hot, but I was astonished to find that there was a bright spot on the other side opposite the idle cathode and none on the side opposite the real cathode. The latter side was not even hot, but the apparently idle cup on the other side of the tube had sent out something or other which produced a red hot spot opposite to it and gave rays. That is only one of the curious things we find, which among the many others serve to mix us up and carry us perhaps farther away from the real thing that we all are looking for."

His experiences also showed the risks to which these pioneers were exposed:

"I was interested some time ago in regard to the effect of Roentgen rays on the tissues. I had read a few times that certain people had been burned by Roentgen rays. I did not believe it. These rays went through tissue so easily that their action could not amount to anything, but it was certainly worthwhile investigating so as to know. So I used a tube, which happened to be a heavy blue glass tube with a clear glass window. The blue glass did not allow the rays to get out, and they were absorbed except through the clear glass portion, where I wanted them for use. I put my finger up to the clear glass window and kept the other fingers pretty well shielded by the blue glass of other parts of the tube. I exposed the finger for half an hour to the rays, a Holtz machine being the source of electricity. I put the finger up pretty close to the tube, and after half an hour I thought that perhaps it was not long enough; perhaps it was not half enough. But if there were to be any effect it would be equivalent to a few hours distance, and as I got tired I went no farther. I shut down the tube and went away. Five, six, seven, eight days passed and nothing happened, and I felt that people had been mistaken about the effect of the rays. But on the ninth day the finger began to redden; on the twelfth day there was a blister, and a very sore blister. On the thirteenth or fourteenth day after exposure, the blister had included all the skin down to the part not exposed and had gone around the finger almost to the other side. The whole of the epidermis came away and left an ulcer without any possibility of recovering its own epidermis except from the edges, and I had to go through that painful process of having a raw sore there and the epidermis growing in from the side and gradually closing up. Only three days ago was the sore actually closed, and the skin is yet very tender, and nature does not appear to have found out how to make a good skin over that finger."

Michael I. Pupin* described some of his experiments and discussed the various views on X-ray theory, particularly a theory of Helmholtz which called for bodily movement of the ether. A. E. Kennelly† then said:

"Just one word about the theory of the subject. Whatever may be the nature of the Roentgen ray, it must be either a bodily movement of matter, or a movement of a disturbance in matter, or a bodily movement of ether; or, finally, a movement of a disturbance in ether. As regards the first supposition, that X rays are streams of matter in motion, or of projected molecules, there is an experiment which seems to negative it. A Crookes radiometer vane does not, as far as I have been able to discover, recede from an excited Roentgen ray tube. One would suppose that it would be powerfully repelled, if X rays were bombarding streams of particles . . . . As regards the three remaining hypotheses, it is to be hoped that future researches will show that X rays are ultra-ultra-violet light rays. Not only would this be the simplest conception that we can at present frame, since it would only call for further extension of the spectrum, but it would also keep X rays within the limits of electro-magnetic waves and in the immediate province of electricians."

A communication from Wilbur M. Stine and included in the published record told of some of the techniques used in extending the life of the tubes:

"After a time a period of decline sets in. The vacuum becomes extremely high, the fluorescence is scarcely perceptible, and the ray is weakly emitted, but its penetrating power has become so great that even the bony skeleton of the body becomes quite transparent. It is now customary at this stage to heat the tube with the flame of an alcohol lamp. As some erroneous statements have been made in this connection, the phenomena will be treated in detail. When the flame is first applied to the tube it darkens, being doubtless short-circuited by the aqueous vapor condensed on its surface. From time to time occasional flashes occur in the tube, but at length, with surprising suddenness, the entire bulb fills with the usual glow accompanying a highly active state, and the ray is emitted with great power. If at this juncture, the heat is not very cautiously applied, the vacuum will be quickly lowered to the point of breaking-down; the impact plane will grow red hot, and the penetrating power of the ray is feeble. By deftly reversing the tube and manipulating the heating, the vacuum can be controlled within wide limits."

After adjournment of the meeting, a communication was received from Charles T. Rittenhouse§ for inclusion in the *Transactions*. It included these opinions:

"Every endeavor should be made to co-ordinate the results of experiments with Roentgen rays with the generally accepted theories of light, rather than to propose new theories, concocted without mature deliberation, likely to be upset at any moment by the announcement of some new phenomenon. It seems to me that it would be preferable to withhold opinions regarding the exact nature of Roentgen rays until a sufficient number of facts have been collected to form a basis for the advancement of a new theory. . . . In conclusion it may be said that when it is considered that Roentgen rays most likely consist of vibrations of such small wave-lengths as to be comparable with the distances between the molecules of the most dense substances, it can scarcely be expected that regular reflection will take place to any large extent, but that dense substances having a most highly polished surface will still be as rough to these rays as a badly polished surface to ordinary light. In other words, regular reflection and diffusion are relative terms only, so that a given surface subjected to one mode of vibration behaves entirely different when receiving vibrations of a higher or lower order."

No further material on the subject was recorded during the few remaining years of that century.

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Scientific Importance of X Rays

L. HENRY GARLAND

A practicing radiologist outlines the scientific and medical importance of the X ray first noted 50 years ago by William Conrad Roentgen and summarizes, briefly, a physician's view of its value to science in general.

"Come, come, and sit you down; you shall not budge;
You go not, till I set you up a glass
Where you may see the inmost part of you."
(*Hamlet, act III, scene 4, lines 23–25*)

THE DISCERNING English dramatist wrote these amazingly prophetic lines some 300 years before a modest German physicist announced the discovery of a "new type of rays." Yet, had he known, Shakespeare hardly could have penned a more apt description of a fluoroscopic examination—aided, it is granted in this instance, by a mirror, in which the good Queen of Denmark might "see her inmost part." To continue the remarkable coincidence, the Queen, like many an apprehensive patient, replies to Hamlet:

"What wilt thou do? Thou wilt not murder me?"

And Polonius cries forth:

"What ho! help, help, help!"

Before the days of shockproof X-ray equipment, such a scene might have taken place in a radiologist's office; today it should be a rarity (unless Polonius were confronted with the statement for an unusually prolonged and complicated series of examinations).

In the half century which has elapsed since Roentgen's announcement there have been many developments in medical science which, at first glance, might seem to dwarf the tremendous importance of the discovery of X ray. These developments include the perfection of antitoxic sera, of remarkable antibiotic agents (the sulfa drugs, penicillin, and similar molds), of stored blood or blood derivatives, and finally of planned atomic disintegration. Yet, we believe it is safe to hazard the guess that in another 50 years we still shall look upon the X ray as one of the developments of major scientific import of all time, as well as one of the most valuable weapons for medical care. We shall look back and thank the pioneers in physics, electricity, radiology, and general medical pedagogy who made equipment practicable and who encouraged physicians to specialize in the field of X-ray diagnosis and treatment.

GENERAL CONSIDERATIONS

To appreciate the importance of X rays it is desirable that certain basic facts be kept in mind. These rays are penetrating radiations corresponding to light rays but having much shorter wave lengths. They may be used in a manner similar to light rays for the inspection of some materials and, by virtue of their peculiar properties, for the analysis of others. Gross inspection methods include simple roentgenography (or X-ray "photography") and roentgenoscopy (or X-ray fluoroscopy); detailed inspection methods include various other means such as microradiography, Roentgen-spectroscopy, Roentgen-diffraction, and so forth. In practice, X rays usually are generated by allowing a stream of high-speed electrons to impinge upon a metal target. They have wave lengths of from 10.0 to 0.01 angstroms, and affect a sensitized film in a manner similar to that of light rays. They also cause certain substances to fluoresce. The materials through which they pass are ionized and give rise to scattered X rays. Because of this ionization, the rays are of use in the treatment of certain medical conditions. X rays can be reflected, refracted, and polarized by special means (including the use of crystals).

The principal use of X rays lies in their ability to penetrate opaque objects. Such objects arrest the rays approximately in direct proportion to their densities. Substances of low atomic weight such as cotton, gauze, and aluminum, are traversed readily by the rays, while other substances such as bone and heavy metals are opaque, and cast a dense shadow on a sensitized film. Some materials are transparent to light rays but opaque to X rays (for example, a plate of lead glass).

In the radiographic examination of materials (both animate and inanimate) it is important to remember that the delineation of an object depends on its differing in density from its surroundings. Unless such difference exists, the outline of the object cannot be shown on ordinary roentgenograms. A simple example of this is the demonstrability of the heart in X rays of the chest. Surrounded as it is by air-filled lungs, the heart is readily visible in chest films. However, if nature had placed it in the middle of the liver, no distinct shadow would be cast as the liver is of approximately the same density.

Under ordinary circumstances, X rays are not directly visible and require a specially prepared surface for their detection (a fluoroscopic screen or sensitized film). We use the adjective "ordinary" because you actually can see X rays with a little training and practice. Unfortunately, if you look at them sufficiently long, you will lose your eyelashes, and probably your eyebrows, if not most of your hair. This is because of the destructive effect of large amounts of X rays on the hair follicles and other living tissues. However, it is safe to gaze at the rays for a few minutes, using a small beam (perhaps one-inch diameter). One must sit in total darkness for some 30 minutes before one can appreciate the faint fluorescence produced by the rays on the human retina; one is then able to see small lead objects placed between the X-ray tube and the human eye, and, if one has an opaque lens (cataract) or cornea, one can ascertain whether or not the optic nerve is intact. The method is used occasionally by radiologists, at the request of eye specialists, to assure that the patient has an intact retina and nerve before surgical procedures on the cornea or lens are performed. It is to be noted that the objects will seem upside down, since the focussing power of the human lens will be ineffective with X rays.

The action of X rays on photographic film is similar to that of light rays. Ordinary X-ray films are coated on both sides with a silver halide emulsion, especially sensitive to the violet range of the spectrum, because they are used mostly with double intensifying screens which glow with that color. These screens consist of thin pieces of cardboard coated with calcium tungstate and a protective transparent film. They are placed in a special bakelite or aluminum-fronted frame, and for satisfactory results must make perfect contact with the film. Special nonscreen films may be used for improved detail, but are from five to ten times slower than screen films. The modern film has a cellulose-acetate base (with no greater fire hazard than paper). The finished roentgenogram is a negative image and is studied in a flashed opal glass illuminator.

The fluoroscopic screen usually is made with zinc sulphide because it must carry an image to be viewed directly and this chemical fluoresces in the color range to which the eye is quite sensitive (yellow green). Roentgen's original screen was made of barium platinocyanide, which was much less efficient than the present type. All fluoroscopic screens must be covered with lead glass to protect the operator from exposure to X rays. The operator must "adapt" his eyes prior to examina-

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tion by remaining in a darkened room for several minutes or by wearing special goggles before beginning the work.

A beam of X rays consists of electromagnetic radiations of various wave lengths. If of very short wave length they are of high penetrating power; if of very long wave length they may penetrate only a few millimeters of tissue. The quality of a beam of rays may be measured by various methods, the most convenient one in ordinary practice being the determination of their absorption by some material such as copper or aluminum. A spectrometor can be employed, but is too time-consuming and troublesome for routine use. The quality is expressed as the thickness of an absorber which reduces the intensity of a given beam to one half its initial value, and the resulting figure is known as the half-value layer for that beam. For low-voltage rays (up to 20 kv) cellophane may be used as the absorber; between 20 and 120 kv, aluminum; between 120 and 400 kv, copper; and for more than 400 kv, lead or tin. A refinement of the method is to record a second half-value layer, the ratio of the second to the first being used as an index of homogeneity. It is to be noted that the expression half-value layer is only part of the description of the quality of a given beam; for scientific purposes one also must specify the nature of the target material, the tube wall, and the generator wave form.

The quantity of X rays in a given beam at a given point may be measured by recording the ionization in a fixed volume of air or gas at that point. The "roentgen" is the unit of X-ray quantity and is defined as "that quantity of X or gamma radiation such that the associated corpuscular emission per 0.001293 gram of air produces, in air, ions carrying one electrostatic unit of electricity of either sign." The mass of one cubic centimeter of air at 0 centigrade and 760 millimeters of mercury is 0.001293 gram; in biological work, one roentgen equals 83 ergs per gram of tissue. The rays usually are measured by an electroscope attached to a small thimble chamber composed of a special light plastic material. It is of interest to note that the almost universally used instrument at the present time is a "condenser Roentgen meter" designed and manufactured in Cleveland, Ohio.

X RAYS IN SCIENCE

The importance of X rays to science in general is so great that a volume would be necessary to describe it. However, a few of the more interesting applications of X rays will be mentioned in this article and an attempt made to give a clue to their true value. For the sake of brevity the sciences will be grouped into a few general categories.

Anatomy and Physiology. The importance of X rays to anatomy, comparative anatomy, paleontology, and associated sciences is now well appreciated. However, it is not realized generally that the modern method of teaching anatomy to medical and other students involves the use of roentgenological demonstration of the skeleton and its various associated soft parts, both in the cadaver and in the living subject. In this manner the appearance and behavior of bony and other structures in the living is demonstrated in a way never before possible. The study of the skeletal development in vertebrate embryos is facilitated enormously by Roentgen methods. The anatomy of small and large animals is revealed in zoological work. The status and often the diseases of mummies may be determined without opening the wrappings or even the sarcophagi. Genetics has been furthered by studying the behavior of many species, notably drosophila melanogaster (the common fruit fly) following exposure to specific quantities of X rays (with the production of subsequent mutations of certain types).

Physiology is greatly indebted to roentgenologic methods, notably in connection with the study of the alimentary tract, the circulation of the blood, and the functioning of moving parts, especially joints. X-ray "movies" as well as fluoroscopy and "still" films have been used extensively in this field. As an example of their value in one small department the X-ray (kymographic) determination of cardiac output can be cited.[2]

An X-ray film is made of the subject's chest, in the erect position, at a measured distance (say five or six feet), using a lead grid between the patient and the film. The exposure takes about two seconds' time and the film is moved slowly downwards a distance of 12 millimeters during the exposure. When the film is dried, the outline of the heart appears as a serrated border, the "peaks" representing the shadow in maximum expansion and the "valleys" in maximum contraction. The peaks are joined by one line, the valleys by another. The rest of the heart shadow is completed as shown in Figure 1. The area in these two phases then is measured with a planimeter and the figures corrected for distortion. The corrected figures then are converted into volumes according to a table established from experimental and cadaver work. The difference in volume between expansion and contraction gives the output per beat. In many normal subjects the average stroke output per ventricle is 60 cubic centimeters; a person with a rapid pulse tends to have a smaller and one with a slow pulse a correspondingly larger ouput. Therefore, it is more informative to speak of the output per minute than the output per stroke; this in turn varies with body size, which may be expressed fairly simply in terms of total body surface. The number of liters of blood pumped per minute per square meter of body surface is termed the cardiac index. This index thus may be found by using this particular X-ray method. (Needless to say, there are other methods of determining this index, but the roentgenkymographic one is a simple and reliable one when correctly used.)

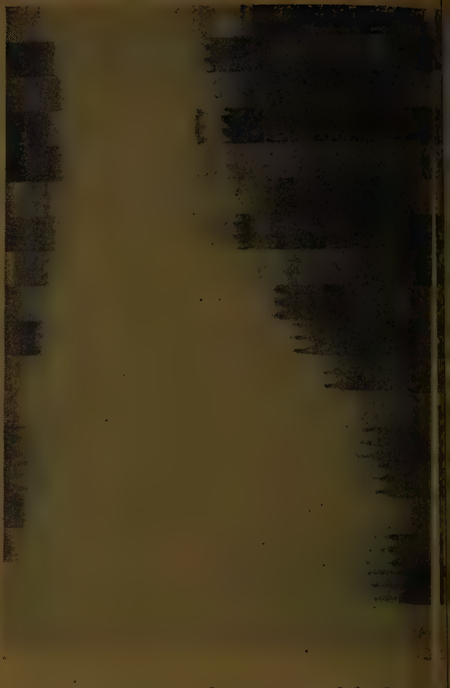


Figure 1. Roentgenkymogram of the heart of an adult male, exposure time two seconds

Three contractions of the heart are visible in each of the frames with the tip or peak of each wave representing maximum expansion and the base or valley maximum contraction of the heart (diastole and systole, respectively). The contractions are of normal amplitude and shape

Our knowledge of the functions and behavior of the alimentary tract in health and disease is largely dependent on the use of Roentgen methods of examination. Some of the earliest investigators in this field have been American physiologists who did their original work on cats (to whom they fed bismuth in milk, and so forth). Food also may be used, provided it is opaque to the rays. On an early morning walk near San Francisco several years ago, I came across a plump horned toad. Having a little time to spare that day, I brought the toad into my fluoroscopic room and placed it on the table. I then captured a common house fly and dusted it with barium sulphate. After setting the tasty meal beside the toad I turned on the X rays and watched the inevitable and fascinating sequence of events. The barium cast just as good a shadow in the toad's stomach as it does in the human, and the progress of the meal could be studied easily.

Physics and Chemistry. The use of X rays in the sciences of physics and chemistry and all their innumerable ramifications is an ever-expanding chapter in X-ray history. The gross analysis of many materials may be performed in part by roentgenoscopy or roentgenography; and the detailed analysis by methods such as Roentgen diffraction, crystallography, or electron microscopy. The diffraction method of measuring small objects is well established. The principle depends on the fact that a beam of light or X rays which has traversed a collection of small objects is seen surrounded by a series of rings or diffraction spectra, from the diameter or pattern of which the size of the object can be calculated. A familiar example is the ring visible around lights on a misty night, the diameter of which is determined by the mean diameter of the mist particles. The interpretation of X-ray diffraction patterns in terms of the ultimate structure of crystals and solids has its foundations in crystallography. By such methods it is found that rubbers, plastics, and fibers, although superficially different, are intrinsically similar materials. X-ray studies also have assisted greatly in analyzing the structure of the higher polymers. The essential features of such diffraction apparatus are:

1. A source of X rays.
2. A device to limit the rays to a beam of minimum divergence.
3. A holder to support the test specimen in the beam.
4. A means for recording the X rays diffracted from the sample of critical angles (determined by the crystal structure of the samples).

A recent issue of *Electrical Engineering* included a description of a method of X-ray analysis of unknown chemical substances by employing a new photoelectric Roentgen intensimeter.[1] The meter is said to be so delicate that if the X rays are passed through a pile of 100 sheets of paper, the difference in absorption caused by adding or subtracting a single sheet can be recorded.

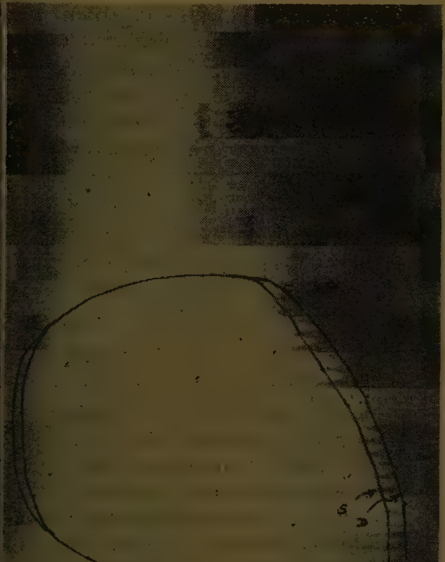


Figure 2. Same case as shown in Figure 1 illustrating method of tracing the outline of the heart shadow in contraction (S) and expansion (D).

In this way the output of the heart is ascertained

As a direct result of investigations in physics and chemistry there are numerous industrial and commercial X-ray developments which include the following:

1. The examination of various complicated appliances, such as radio tubes, without the necessity of breaking open the tubes.
2. The examination of castings and welds.
3. The examination of packages and personnel for concealed material.
4. The examination of edible materials for defects or impurities (such as candy bars and oranges).
5. The behavior of solids and liquids under projectile bombardment (such as high-speed roentgenography of bullets).

The roentgenographic examination of metal parts in connection with the airplane industry is said to have used up more film per month during the last few months of the recent war than was used in all medical procedures in the United States during the same period. The importance of skilled interpretation of these roentgenograms and of adequate protection for the employees operating the X-ray equipment obviously is great.

Since the early days of X-ray development, packages, clothing, and even personnel have been inspected for contraband and other illegally possessed materials. Packages and similar objects may be inspected without harm, provided they do not contain unprocessed film or other sensitized material. Individuals may be examined only under stringent conditions as exposure of large amounts of the body to X rays, especially if repeated, is fraught with ultimate danger to the individual; possibly resulting later in skin damage, anemia, or infertility.

It is to be recollected that metals and other materials may be radiographed either with X rays or radium rays. It is therefore appropriate to consider a few aspects of radioactive substances as many of them have properties analogous to those of X rays. Radium rays are of three general types:

1. Alpha rays, which are positively charged particles of very low penetrating power, being stopped by a sheet of ordinary paper.
2. Beta rays, which are negative electrons, of moderate penetrating power, stopped by a thin metal filter (two millimeters of brass or 0.5 millimeter platinum or their equivalent).
3. Gamma rays, which are electromagnetic radiations (photons) of considerable penetrating power, the hardest ones traversing several centimeters of lead.

Because of their different charges, these three types of rays can be separated in a magnetic field. The alpha rays are deflected slightly in one direction, the beta more strongly in the opposite direction, and the gamma not at all. Radium disintegrates slowly, its half period being 1,590 years. For medical purposes it usually is kept in small containers in the form of a radium salt. Many of these containers can be placed together and the group then used like an X-ray tube.

Since the work of Rutherford, in 1919, we have been able to produce artificially radioactive substances. One method of doing this is to use the cyclotron, by means of which the nuclei of atoms are transformed into new unstable substances. High-speed protons, deuterons, and neutrons have been used to transform most elements. The resultant radiations are of many types (alpha, beta, gamma, neutron, and positron). Many biologically useful radio elements are produced by deuteron bombardment and include radiophosphorus, radiosodium, radioiron, radioiodine, and radiostrontium. These substances can be administered to patients and their exact method of localization and storage in the human body studied by means of Geiger counters and similar apparatus. In this manner the metabolism of certain living tissues, in both health and disease, can be studied more completely than ever before. For example, totally new information concerning the need for and method of use of iron in the human body, in conditions like anemia, has been obtained.

An indirect development of high-voltage tubes in physics is the electron microscope which permits much higher magnifications of minute objects than are feasible with optical microscopes. With this instrument the image of the object is viewed either on a fluorescent screen or is recorded on a photographic film. Recent developments include modifications by which the instrument may be used either as a diffraction camera or a microscope. Stereoscopic electron micrographs can be made. The usual electron microscope operates at about 60 kv, but there is a small table model operating at 30 kv. The structure and behavior of viruses has been studied for the first time, and the detailed structure of bacteria has been revealed. Chromosomes, the tiny rodlike particles that bear the major responsibility for inherited characteristics, may be examined, and by means of ingenious methods the actual location of certain specific genes in a number of chromosomes has been determined.

Biology. X rays have been used in most of the various biological sciences, notably in botany and zoology. The architecture of many forms of plant life have been studied by macro as well as microradiography. Mutations have been produced by bombardment of seedlings and rootlings with X ray, and new hybrids successfully developed. In fact, one physician insists that a special type of begonia he grows is a direct result of irradiation of a former plant. He gives no credit to Mendel's cosmic ray but, unfortunately, I am not sufficiently familiar with botany to know whether or not he is correct.

In zoology, several forms of animal life have been studied with X rays. With the larger type of animal one naturally has to make segmental studies. About a generation ago an ailing elephant at the London zoo was subjected to X-ray examination. The tired pachyderm lay on her side and had her torso marked with chalk into a series of rectangles, each a little less than 14 by 17 inches in size. These then were numbered in sequence and a series of roentgenograms made. We cannot vouch for the quality of the films made through the thicker parts, but those of the extremities which we saw were quite good. Race horses, greyhounds, parrots, and other domestic pets frequently are subjected to examination in connection with injuries, foreign bodies, and certain diseases. Before the development of shockproof apparatus, cats were particularly difficult to X ray as they have a strong dislike for the hair-raising qualities of the corona from exposed high-voltage wires.

Miscellaneous. One of the lesser known but more valuable uses of X rays is in connection with art. Many pigments contain lead and other radiopaque salts and the roentgenogram of a canvas often reveals shadows different from those on the visible painting. Areas of overpainting, alterations, and erasures can be detected. The authenticity of some old masters has been proved and of others disproved by such means. A recent refinement includes sectional radiography of the canvas in which, by keeping the tube and film in motion, a very fine layer of the painting can be registered to the exclusion of other layers.

X RAYS IN MEDICINE

The importance of roentgenologic methods in medicine, dentistry, and allied sciences is increasing continually. They permit the examination of parts of the body hitherto inaccessible to study and the detection of disease at a stage when it may be cured readily. They extend the physician's eye to an incalculable distance. They are also of use in the prevention of disease and in its treatment. (A small precancerous skin nodule, keratosis, may be cured before it develops into a malignancy; established, localized, accessible cancer can be destroyed.) Our knowledge of many diseases such as stomach ulcer, lung tuberculosis, and bone cancer has been entirely revolutionized and reoriented since the discovery of X rays. They have provided an impetus to investigation and research in preventive medicine such as no other weapon ever placed at the disposal of the doctor.

Table I. The Electromagnetic Spectrum

Type of Radiation	Range of Wave Lengths	Range of Frequencies
Electrical waves	1,000,000 to 20 kilometers	10^{-1} to 10^4
Radio waves	20 to 0.01 meters	10^4 to 10^{11}
Infrared rays	120,000 to 7,700 angstroms*	10^{11} to 10^{15}
Visible rays	7,700 to 3,900 angstroms	
Ultraviolet rays	3,900 to 1,800 angstroms	
X rays	10 to 0.01 angstroms	
Gamma rays	0.1 to 0.02 angstrom	
Cosmic rays	0.00001 ± angstrom	

* 1 angstrom = 10^{-8} centimeters; 100,000,000 angstroms = 1 centimeter

One might arrange this section according to the various medical specialities, but I think it will be of more interest to nonmedical readers to consider the importance of the rays according to each of the various systems of the human body. However, it is timely at this point to acknowledge the debt of roentgenology to members of the specialties outside of its own field, physicians who have contributed greatly to the refinements of many techniques in diagnosis as well as treatment. The reader should know that there are now "national qualifying boards for specialists." These boards examine eligible graduate physicians and certify those who pass successfully. At the present time there are boards in the following branches of medicine:

1. Anaesthesiology.
2. Dermatology and syphilology.
3. Internal medicine.
4. Neurological surgery.
5. Obstetrics and gynecology.
6. Ophthalmology.
7. Orthopedic surgery.
8. Otolaryngology.
9. Pathology.
10. Pediatrics.
11. Plastic surgery.
12. Psychiatry and neurology.
13. Radiology.
14. Surgery.
15. Neurology.

X-ray diagnostic methods involve first and foremost intelligent use: one case may be diagnosed quickest without X rays at all; the next may need only fluoroscopy; the third, fluoroscopy, roentgenograms, and special-section techniques; the fourth may require repeated roentgenograms. Only a trained physician knows the correct answer to these questions: only a trained physician can be economic in the exposure of your tissues to a radiation known to be noxious. The wise layman selects his physician–radiologist according to his experience or ability, and not according to the newness or extent of his equipment. In assessing the value of a roentgenogram it is important to remember that it is just a shadowgraph and not a true photograph and as such is subject to erroneous interpretation. Furthermore, the roentgenogram is a projection on a flat surface of everything on every plane between the X-ray tube and the film. Thus it is desirable that the interpreter familiarize himself with the projected appearance of normal structures of various shapes. Multiple views, preferably at right angles, usually are essential; stereoscopic roentgenograms frequently are necessary. The examination of moving parts such as the stomach, heart, or diaphragm frequently requires fluoroscopy as well as multiple films.

There are many shadows in roentgenograms of healthy persons which cause errors in interpetation. These include the normal epiphyseal or growth line in bones (often mistaken for a fracture), the overlapping of a bony margin, or the slender canal of the artery to the bone. The existence of a congenital fissure or cleft also may be mistaken for a fracture. Calcium deposits are fairly common in various tissues; they are normal findings in rib cartilages, laryngeal cartilages, and certain other areas, but occasionally they are mistaken for tuberculous lesions, foreign bodies, and so on. This is one of the reasons why a consultant roentgenologist may require stereoscopic or other additional projections even though you present yourself with a perfectly good film made elsewhere only a few days before. A small wart on the back may be projected on a film in a manner identical with that of a kidney stone or gall stone. Visual examination of the undressed patient or stereoscopic projections should prevent an error being made. In general, it is advisable for persons to be undressed when having X rays taken, since objects such as ear-rings have been mistaken for misplaced tooth-root fragments, buttons for gallstones, and the edge of folded clothing for fractures. Faulty darkroom technique sometimes results in undeveloped or fogged areas of films which can suggest (in chest films) pneumonia or (in films of the limbs) diseased bone, even to the initiated.

The use of X-ray methods in medical diagnosis may be outlined in relation to the various systems of the body.

Alimentary System. X-ray examination of the alimentary system permits the early diagnosis of a vast number of common disease conditions, ranging from adhesions to volvulus. Both fluoroscopic and film examinations are usually essential. The fluoroscopic examination discloses facts regarding the mobility and function of organs which cannot be obtained from films alone; the films in turn reveal detail of structure which cannot be appreciated on the fluoroscopic screen. The value of either method depends on the skill, patience, and experience of the examiner. When it is a question of a condition such as early stomach cancer, the examination may have to be repeated two or three times before a reliable opinion can be rendered. For the upper part of the system (the gullet, stomach, and small intestine) it is necessary for the patient to come fasting. He is given a drink of barium sulphate suspended in water and the appearance of these portions of the tract is studied under the fluoroscope. Patients often complain at the chalky taste of the bariumized water. The reason that flavoring agents are not used is that in some persons they stimulate large amounts of gastric secretion which dilutes the test meal undesirably. In the early days of roentgenology the barium or bismuth used to be given in flavored preparations. However, if these contained much milk or cream, stomach emptying would be (physiologically) delayed. If they contained much sugar, stomach emptying would be hastened. These conditions often resulted in erroneous conclusions as to the presence of gastric stasis. In the average case four ounces of barium sulphate (by weight) are given in about eight ounces of water. This eventually mixes with the other contents of the intestinal tract and does not require laxation for its natural ejection.

The lower portion of the alimentary system normally is examined by barium enema. Sixteen ounces of barium sulphate are suspended in two quarts of warm water, with the addition of some medium such as acacia solution to maintain suspension during the procedure. After fluoroscopic and roentgenographic examinations the patient evacuates the suspension.

Conditions diagnosable by X ray include varicose veins in the gullet, ulcers, and tumors in the stomach or duodenum. Before the days of X ray, duodenal ulcer was considered a rarity; a person with severe and prolonged indigestion was usually diagnosed stomach ulcer. Since the development of Roentgen methods it has been found that most of such cases are actually due to duodenal ulcer, the latter being over 20 times as common as gastric ulcer. This was one of numerous revolutionary findings during the first two decades of this century.

Gastric cancer is one of the most common and serious malignant tumors in man; unfortunately it tends to be asymptomatic in its early stages and therefore its presence is not manifested until it is pretty well advanced. Persistent disturbance of digestion in men over 40 renders X-ray examination of the stomach advisable. The growth shows as a small intrusion on the barium shadow or as a small zone of immobility of the stomach wall.

The normal appendix usually is visible at some time during X-ray examination of the intestinal tract. It may be seen to fill and empty but if it does remain filled for some days this finding alone is not of grave importance. The X-ray evidence of acute disease in the appendix consists of marked local tenderness, absence of or incomplete filling, delay in the passage of the barium in the adjacent terminal inches of small bowel, and, occasionally, a local inflammatory mass. However, the diagnosis of acute appendicitis is more reliable when made by methods other than radiological ones.

X-ray examination is invaluable in the study and elucidation of the various diseases of the colon.

The liver and spleen are normally visible in most abdominal roentgenograms. Their visibility may be enhanced by intravenous injection of thorium dioxide sol (thorotrast), a drug which has the property of depositing itself in the small cells lining certain portions of the liver and spleen (the reticuloendothelial cells). The thorium is opaque and aids in the diagnosis of certain tumorous and cystic diseases of these organs. After seven years, thorium degenerates into a mildly radioactive product, but the amount necessary for ordinary examination has been shown to be so small that no harmful late radioactive changes develop. Another area of the alimentary tract which may be examined by selective methods is the gall bladder. If you ingest a suitable iodine preparation, the material will be excreted in your bile and will be concentrated in the gall bladder; provided the duct between the liver and gall bladder is not blocked and you remain fasting. This test is a valuable method for the detection of nonopaque cholesterin stones in the gall bladder; indeed, it is the only method by which such may be diagnosed preoperatively.

Cardio-vascular System. The accurate determination of heart size in the living human being can be made only by roentgenological methods. It is to be noted, in passing, that this is not always a very important factor, since cardiac function rather than size governs most healthy lives, and function can be assessed by many means much more accurate than roentgenologic ones. However, the problem of heart size does come up in some individuals and Roentgen methods are then invaluable. Correction should always be made for the amount of magnification present, the phase of respiration in which the patient happens to be during the exposure, and similar items. As in the case of the alimentary tract, complete examination of the cardiovascular system involves both fluoroscopy and roentgenography. In recent years the use of opaque substances has permitted the outlining of the individual heart chambers in the roentgenogram and the study of the circulation in a manner one could scarcely have even dreamed of 50 years ago. Besides giving information concerning the size, shape, position, and mobility of the heart, Roentgen methods also may disclose the presence or absence of coronary disease and its sequelae. Records of the beating heart may be obtained on a single film by means of kymographic apparatus and deductions as to the presence of localized areas of heart-muscle disease deducted therefrom. The condition of the arteries and veins in the extremities can be studied by means of both plain films and films made with contrast media and the presence of varicose veins in the deep venous circulation of the leg can be detected and unnecessary operations avoided.

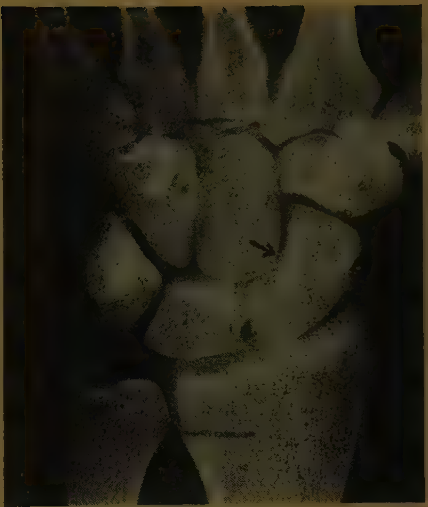
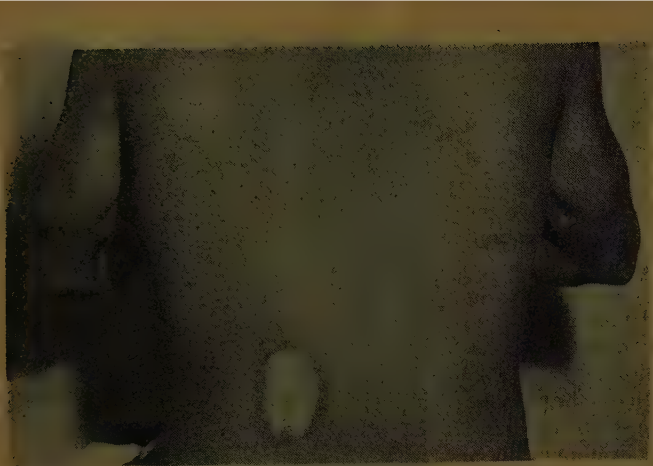


Figure 3. Left wrist of adult male showing faint fracture line in one of the small bones (the scaphoid)

Fracture is indicated by arrow

Central Nervous System. Medical advances in the diagnosis and treatment of diseases of the brain, spinal cord, and peripheral nerves is one of the most interesting chapters in modern science. The normal brain casts no distinguishing shadow in the routine roentgenogram. However, by very simple methods, its outlines can be revealed. This consists of performing a spinal puncture (in the lower portion of the back), withdrawing from 75 to 150 cubic centimeters of cerebrospinal fluid, and injecting in its place a corresponding amount of air. Upon placement of the patient in an erect position this gas will ascend into the cerebrospinal fluid pathways around, between, and in the various lobes of the brain. Several plain or stereoscopic roentgenograms then are made from various angles, with the patient both erect and horizontal. The air absorbs during the next few days and is replaced by normal fluid. By this method various types of brain injury, brain disease, and tumors may be detected. Considerable experience is desirable in interpreting these pneumo-encephalograms.

The spinal cord may be studied by similar methods, or by the injection of radio-opaque contrast media into various portions of the spinal canal. The media most commonly used at the present time are preparations containing iodine (iodized poppy-seed oil and ethyl iodophenylundecylate). About three cubic centimeters of the opaque oil are injected in the lower lumbar area and the patient is fluoroscoped



Oval area of pallor surrounded by brown pigmentation is the effect of a second-degree burn suffered some months before when the patient underwent X-ray examination of his stomach by an untrained worker

Figure 4. Localized bleaching of the skin of the lower back of an adult male as a result of excessive Roentgen irradiation (healed X-ray burn)

on a special table. By tilting the body up and down, the lake of opaque oil can be made to traverse the spinal canal. Indentations and certain other alterations of its shadow will disclose the presence of ruptured disks, tumors, and so forth.

Genito-urinary System. It is now possible to examine the outlines of virtually every portion of the genito-urinary system. One can determine the function of each kidney following intravenous injection of an iodine salt selectively excreted by the kidneys. The seminal vesicles and ducts may be outlined by contrast media. Similarly, the cavity of the uterus and the slender canals of the fallopian tubes can be shown. In this manner an extraordinary variety of conditions, normal as well as abnormal, may be detected. The number, size, and approximate age of infants in utero may be told. The exact alterations in the shape of the fetal bones during the actual process of birth have been studied by serial roentgenograms. The remarkable molding and elongation of the fetal skull have been shown, as well as the disposition of the fetal limbs and placenta. The latter has been studied by arteriographic injection of opaque media and valuable information as to the uniovular or multiovular nature of twins ascertained. Abnormal gestations have been diagnosed correctly in time to save the mother an unnecessarily prolonged pregnancy or labor. These cases are some of the most tragic incidents that a radiologist encounters. The normal fetal skeleton does not contain sufficient calcium to cast a clear shadow in routine roentgenograms before a gestation period of 12 weeks. Within increasing accuracy after that time the fetal parts may be shown. Attempts have been made to determine the sex of the fetus by X-ray methods, but no practical method has yet been found. Extensive studies have been made on the influence of the shape as well as the size of the female pelvis on spontaneous delivery. Female pelves are classified into four general types, based on their shape. By careful Roentgen examination, the probability of easy or difficult labor can be prognosed with considerable accuracy in selected cases.

Osseous System (Bones and Joints). The first use of X-rays was in the examination of cases of suspected fracture. From that day its uses have been extended to include the study of bone growth, bone tumors, joints, tendons, bursae, and adjacent structures. In the correct diagnosis of all bone conditions, and in the treatment of many of them, X rays are essential.

The detection of a gross fracture in an ordinary roentgenogram is a simple matter; the detection of fine or fissure fractures is often extremely difficult. An important example of this is in injuries involving the wrist, probably the most commonly injured area in the body (see Figure 3). One type of "sprain injury" results in fracture of the scaphoid or navicular bone, a small bone in the wrist joint. This bone has a very critical blood supply. If injured, healing requires immediate and complete immobilization for many weeks. Fractures of this bone are often difficult to detect except in films of the highest technical quality; three or four views may be necessary before the crack can be confirmed or excluded. If overlooked, and the wrist is not immobilized, most of these cases result in nonunion and chronic arthritis in the wrist joint, a serious source of disablement in laborers. Attempts have been made to restore the function of the joint by removing the two broken pieces of scaphoid and replacing them with a synthetic bone (made of a biologically tolerable metal such as vitallium) but these results have not been conspicuously successful. The only method of assuring safety is early X-ray diagnosis and complete immobilization.

Fractures of the ribs are quite common and usually unimportant injuries and the vast majority of them heal without any particular treatment. However, compensation and legal considerations often require an answer as to whether or not a fracture is present in a given case of alleged chest injury. In at least ten per cent of actual fracture cases, the fracture line is not immediately demonstrable by ordinary X-ray methods. It is concealed by its obliquity or by overlapping parts. Therefore, your physician may tell you in such a case that there is "no X-ray evidence of fracture" rather than "there is no fracture." If it should be of legal importance to confirm a suspected fracture in such a case, re-examination at the end of four-weeks time usually will provide the answer. By that time a little "fuzz" of new bone will be present in the fracture site and will be visible in the roentgenograms.

Roentgen methods are also invaluable in the detection of various types of bone disease due to infection, tumor, and so forth. However, the shadows cast often are not characteristic of one particular infection. For example, a bone that has been disused for several weeks (perhaps in the foot, when a patient is wearing an extensive plaster splint for fracture of the upper leg) may cast a shadow identical with that of one extensively diseased. Therefore, it is necessary for the roentgenologist to have some of the clinical facts or history of a given case before rendering an interpretation of a film. The film is not misleading; our deductions, in the absence of clinical data, may be misleading.

Some diseases cause fairly characteristic changes in bone, but these are in the minority. The more experience the observer has the more he realizes that a host of different conditions can produce identical pathological and, therefore, identical radiological changes. A good example of this is leprosy. Many years ago an author reported "characteristic" atrophy of the terminal phalanges as a fairly early sign of this disease. On studying cases of nerve disease of other types, and of obliterative or spastic vascular diseases of various types, it was found that quite identical changes occurred in many of them. Phosphorus poisoning causes changes in the jaw bone similar to those of infection and of radium poisoning. Thus, it is important to maintain reserve in accepting reports of new or "characteristic" findings in disease.

One interesting finding in roentgenograms of children's extremities is that seen in lead poisoning. Children who have licked the paint off of their toys or pens sometimes develop signs of joint disease or leg weakness. If they have been following this dietary indiscretion for some time, the growing ends of long bones will show dense lines due to actual deposit of lead salts therein.

The roentgenologist is required to have a general knowledge of the development, anatomy, and pathology of the teeth. Satisfactory examination of the teeth requires careful technique and even more careful interpretation. Without the latter, early abscesses at the roots of the teeth, early areas of caries or decay in the crowns, and similar processes may be overlooked.

The value of a consultant specialist is almost nowhere better seen that in certain cases of dental radiography. The dentist or family doctor is apt to look at such films with his thoughts concentrated purely on the dental structures and innocently may neglect a malignant growth in the adjacent bony mandible, a lesion which an expert in the field of X-ray interpretation would be apt to detect readily. It is not suggested that ability in such interpretation is confined exclusively to the roentgenologist. Any person, professional or otherwise, can learn how to interpret films of certain parts of the body after a fairly short period of training. However, the human tendency is to concentrate on the matters in which one is most interested and to overlook other data, even though such are quite apparent on retrospect.

Normal joint cartilage casts no distinguishing shadow in roentgenograms so routine Roentgen methods are not of much value in the early diagnosis of many types of joint disease. However, "soft-tissue" films of joints, and films made following intra-articular injection of air do provide valuable diagnostic information in many cases. Roentgenograms are of considerable value in the differential diagnosis of established cases of joint disease (chronic rheumatism, gout, specific infection). Bleeding in and about joint areas, such as occurs in hemophiliacs, presents fairly characteristic changes.

Respiratory System. If roentgenology had made no other contribution to medicine than the ability to study the shadows cast by the lungs in living individuals it would have performed a tremendous boon. But before considering this portion of the system, let us commence at the upper portion of the respiratory tract, namely the nose and nasal accessory sinuses. The cartilages, bones, and even the skin of the nose can be radiographed with simplicity, and various injuries and other conditions accurately diagnosed. The nasal accessory sinuses, nasal passages, and adjacent areas can be recorded and various conditions ranging from sinusitis to cancer detected. The air passages leading from the nose to the lungs can be portrayed and the true and false vocal cords may be studied both fluoroscopically and radiographically. A special technique known as body-section roentgenography (laminography or omography) permits the obtaining of films of such areas as the cords relatively free from under- or overlying shadows. In this method the tube and film are moved synchronously but in opposite directions about a fulcrum, the location of which depends on the height above the X-ray table of the area to be studied.

All types and varieties of diseases of the lungs and pleura are amenable to X-ray diagnosis, and many require such examination for their elucidation. Communicable diseases such as active lung tuberculosis provide an excellent example. In recent years the training of increased numbers of physicians in radiology plus the development of special X-ray apparatus has permitted mass surveys of hundreds of thousands of individuals. The best method involves the use of standard (14- by 17-inch) films. A slightly less expensive and currently popular method is the photofluorographic one, in which the fluoroscopic image is photographed on small or roll film (35-millimeter, 70-millimeter or 100-millimeter widths, depending on the type of equipment). The film comes in rolls of from 35 to 100 frames, permitting a like number of exposures. Some of the newer units have built-in photoelectric cells by which the X-ray exposures are timed automatically. This strip film is processed in special developer, then dried and viewed either in a magnifying transilluminator or by means of a projection unit. The detail in the films is naturally not as great as in the conventional 14- by 17-inch film but it is sufficiently good to permit screening of lungs for significant lesions. The prime object in the method is to detect cases of open pulmonary tuberculosis, that is, patients with cavities or other lesions from which they cough bacilli and so innocently infect their fellow citizens. Miniature films permit the detection of most such lesions. About one per cent of apparently healthy adults are found to have some significant lung condition on such surveys, but fortunately only one in 400 has evidence of "open" disease.

The X-ray unit has not made the stethoscope obsolete but it has given the physician a weapon with which he may detect numerous diseases of the chest at a stage long before they could otherwise be discovered. In this manner they have been a major factor in saving countless lives and in preventing much advanced disease.

LOCALIZATION OF FOREIGN BODIES

X rays are essential for the detection and accurate localization of most foreign bodies in the tissues. These bodies may be divided into two general types, nonopaque to the X rays and opaque. Surprising as it seems, nonopaque foreign bodies frequently may be localized, with considerable accuracy, by careful examination. For example, a peanut lodged in one of the bronchial tubes will itself cast no shadow, however, it will produce partial or complete obstruction of the bronchus. As a result the involved lobe of the lung will show either persistent distention with air on expiration, or gradual collapse. Nonopaque foreign bodies in the alimentary tract often may be located by giving the patient small barium-soaked cotton pledgets or swallows of barium cream. Nonopaque foreign bodies in other parts of the body sometimes are located by injecting radiopaque liquids into draining sinus tracts.

Opaque foreign bodies, common in war time, may be found and localized by various methods. The simplest is by roentgenoscopic and roentgenographic examination in two planes at right angles to each other. Other methods include parallax and triangulation. The detection and localization of small metallic bodies in the eye may be performed with the aid of special apparatus. That most generally employed involves examining the patient's eye in accurate relationship to two fixed objects placed at a known distance from the cornea. The data obtained from two films made at different angles are transferred to a special ruled chart, and the position of the body indicated in three different planes with an accuracy of less than one millimeter.

Protection against unnecessary or excessive exposure to X or radium rays is of the utmost importance for patient, operating personnel, and radiologist. Protection against excessive exposure from the direct beam is now well-established in responsible offices and departments by methods which include careful calibration of the apparatus, adequate distance between tube and patient's skin, use of lead-protected shockproof tubes, suitable diaphragms, and filters.

Protection against radiation scattered from the patient or the X-ray table is often more difficult but unfortunately, sometimes is overlooked. The best way to achieve this protection is to use as small a beam as possible in examining patients, to work as expeditiously as is consistent with thoroughness, and to stay at the maximum distance or behind the safest barrier available. Protective barriers usually are made of lead or concrete. If workers are not safeguarded properly they may develop injury to the blood or reproductive system with consequent dangers of anemia, leukemia, or sterility. The tolerance dose is the total X-ray energy that a person may receive continuously without suffering damage to the blood or reproductive organs. For most workers it is set at 0.2 roentgen per day (a dosage rate not exceeding 10^{-5} per second). Detailed rules for protection are available in the National Bureau of Standards Handbooks 20 (*X-Ray Protection*) and 23 (*Radium Protection*).

X RAYS IN TREATMENT

X rays are of considerable significance in the treatment of a large number of diseases, ranging from simple infections such as ringworm of the scalp to serious processes such as cancer. The physician performing Roentgen therapy must strive to be as careful in the calibration and handling of his apparatus as a physicist, and as accurate in the application of his rays as a surgeon is when applying his knife. The therapist has at his disposal a wide variety of equipment supplying low-voltage beams for superficial treatment (40 to 100 kv); intermediate voltage for more deeply seated lesions (120 to 150 kv), high voltage for deep-seated lesions (180 to 220 kv) and, finally, extra-high voltage for a few selected conditions or for biological research (400 to 1,000 kv). He must select a filter suited to the procedure desired, varying from less than one millimeter aluminum up to as much as five millimeters of copper. He calculates his dose in roentgens, and delivers small doses to most inflammatory or benign conditions, and very high doses to certain localized malignant lesions. A small dose ranges from 10 to 100 roentgens and may need to be repeated at intervals for weeks or even months. For example, generalized acne vulgaris may require weekly doses of 100 roentgens for three months. On the other hand a small localized cancer may require 6,000 roentgens delivered either at one session or, depending on the amount of associated infection and similar complications, in several sessions. The essential effect of X and gamma rays on the tissues is a destructive one. Cancer cells are slightly more sensitive to such radiations than are normal cells, and for this reason it is possible to destroy some cancers without permanent injury to the normal surrounding tissues.

The following is a partial list of conditions in which X or gamma rays are of value: Inflammatory diseases of the skin and adjacent tissues; certain thickenings of the skin (plantar warts, keratoses); disturbed function of certain glands such as salivary and thyroid (hyperthyroidism); many benign and malignant tumor conditions.

Just as the mere possession of a knife does not make you a surgeon, so the possession of X-ray apparatus or radium does not make you a radiation therapist. The safe and efficient application of radiation methods in disease requires as much skill and even more training than many types of surgery. It also requires recognition of the fact that there are many conditions far better treated by nonradiological methods and a few which actually are rendered worse by Roentgen treatment.

The rays may be applied to human tissues by a variety of methods including external application of the beam, internal (intracavitary) application, interstitial application (that is, direct insertion of radium or radon seeds into the tissues), and various combinations of these methods, all designed to deliver the involved tissues a specific planned radiation dose.

FUTURE DEVELOPMENTS

The world has seen tremendous advances in the X-ray field during the past 50 years but there still are numerous developments, both in apparatus and technique, to which physicists and physicians are looking forward. These include the following:

1. More efficient recording media, including improved film emulsions, processing equipment, and fluorescent screens.

2. Improved simplified exposure meters and automatic timers.

3. Finer focal-spot diagnostic tubes.

4. Greatly improved fluoroscopes, perhaps an electron fluoroscope, and, as a result, better safer X-ray motion pictures.

5. More widespread use of X rays in teaching and in preventive medicine (including the installation of X-ray units in morgues to aid in research and routine autopsy work).

6. Improved methods of calculating the size and depth of tumors, so that radiation beams may be still more accurately aimed.

7. Improved selection of patients for both diagnosis and treatment (to reduce unnecessary expense and unnecessary exposure of human tissues—especially the reproductive organs—to X or gamma rays).

8. Finally, and most important of all, the training of more and better radiologists; medical physicians specializing in X-ray diagnosis and treatment.

The value of X rays to science in general and to medicine in particular is immense and ever-increasing. They have provided us with a weapon by which we may search out the structure of matter, as well as the hidden components of the body. The diagnosis of innumerable disease conditions is dependent largely or entirely on X-ray examinations and the treatment of several types of disease likewise requires Roentgen irradiation. The debt of mankind to Roentgen and his fellow workers in the field of physics and engineering cannot be repaid easily.

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50 Years of X-Ray Progress in Europe

The penetrating power of the X ray was discovered in Europe in 1895 and since then European scientists have made many contributions to X-ray development. This is a presentation of some of the important aspects of their research as conducted in one company.

A HALF-CENTURY ago the penetrating power of X rays, or Roentgen rays as they are known in most European countries, was discovered by William Conrad Roentgen, a 50-year old professor of physics at Wurzburg, Germany. For this discovery Roentgen, who was born in Lennep, Germany, in 1845, was awarded the Nobel Prize in 1901.

Roentgen's momentous discovery was made while experimenting with a Crookes tube, a highly evacuated glass container into which two electrodes, an anode and a cathode, were fused. At that time physicists were interested in cathode rays which had been discovered in 1876 but Roentgen was the first to obtain fluorescent action when he allowed the discharge from a powerful Ruhmkorff induction coil to pass through his tube and the rays emitted struck a screen covered with a barium platino-cyanide accidentally left lying in the vicinity. When the slightness of the absorption of X rays by all kinds of nontransparent substances, such as wood and cardboard, as well as the formation of sharp shadow images by metallic objects was determined, Roentgen realized that his rays could be employed for medical, diagnostical and technical purposes. Their value to medical therapy also was recognized quickly and as early as 1905 a special tube had been constructed for the so-called "cavity therapy." Unfortunately, in those days there was no question of protection against the X ray, whose action on the human organism still was unknown, nor of protection against high voltages, so that several scientific pioneers lost their lives as a result of excessive irradiation.

Only one of the most important properties of the X ray, its electromagnetic character, was not demonstrated by Roentgen himself and even that he attempted to prove. It was not until 1905 that Haga and Wind, with primitive means, succeeded in demonstrating diffraction phenomena with X rays. In 1912 Laue, Friedrich, and Knipping published their diffraction spectra, obtained by the scat-

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Based on "50 Years of Roentgen Rays" by J. H. van der Tuuk, in charge of X-ray tubes and rectifiers, X-ray Department, N. V. Philips Gloeilampenfabricken, Eindhoven, Holland.

tering of X rays by natural crystals, a discovery which formed the basis of modern crystal-structure analysis by means of X rays and which also opened the way to X-ray spectroscopy.

DEVELOPMENT OF THE X-RAY TUBE

Following the first X-ray tube, the so-called gas tube was introduced (see Figure 2). This glass tube was filled with a gas (for example, air) at a pressure of about 0.01 to 0.001 millimeter of mercury. Under the influence of high voltage, 30,000 to 50,000 volts, the positive ions always present in the gas moved towards the negative electrode (cathode) and upon collision with it they freed negative electrons or cathode rays. These negative electrons, because of their small mass, moved in the field at a great speed toward the positive electrode (anode) and upon striking it or the glass wall excited electromagnetic vibrations, the X rays. Roentgen quickly discovered that the penetrating power of the ray increased rapidly with an increase of the voltage of the tube, and that an increase in intensity was evident with an increase in current.

Roentgen also discovered that the efficiency of the tube was enhanced by introducing opposite the cathode a third electrode, the so-called anticathode, which is usually in direct electrical connection with the anode. By means of this improvement practically all of the electrons were captured by the metal of the anticathode, instead of part of them striking the glass wall. It was not long before platinum was in general use as material for the anticathode since, under otherwise similar conditions, the efficiency of the process of excitation of X rays was found to be approximately proportional to the atomic weight of the anticathode substance.

EARLY IMPROVEMENTS

During the first 15 or 20 years of X-ray history many important technical improvements were introduced. The surface of the cathode (an aluminum cathode proved most efficient) was made concave for the more perfect focusing of the cathode-ray beam on the anode. The anticathode was water-cooled in order to obtain a higher continuous loading of the tube. Finally, an automatic regulation of the gas pressure corresponding to a certain length of spark was instituted in order to insure the regular functioning of the tube. In this way, a development period of almost two decades saw a capricious laboratory instrument evolve into a commercial product with which the experienced X-ray technologist could carry out very useful work.

Meanwhile, in the United States, W. D. Coolidge directed research that perfected a ductile drawn-tungsten wire which made possible a tungsten filament as well as a tungsten anticathode in X-ray tubes. Investigations by Edison and Richardson had discovered the "thermionic emission" of incandescent metals and, in 1913, Coolidge constructed a "powerful Roentgen-ray tube with a pure electron discharge." This was an essential step in the process of rendering the X-ray tube an electrotechnically reproducible instrument. The improved vacuum technique (Langmuir) had made it possible to evacuate the glass tubes with metal electrodes thoroughly so that the electric current, which in tubes with tungsten filaments passed from the cathode to the anticathode, consisted of a completely controllable current of electrons. With this innovation the current through the tube as well as the voltage could be chosen practically independent of each other, an impossibility with the gas tubes. Regeneration arrangements were no longer necessary, the tube was evacuated to the extreme attainable vacuum, and the mysterious third electrode was no longer necessary. The same piece of metal served as target and as conductor for removing the electrons. It was also very important that these electron tubes, in contrast to the old gas tubes, were self-rectifying in principle, so that good-quality transformers could be used to feed them instead of the old-fashioned induction coil with complicated interrupters. High-vacuum rectifiers then appeared as a further step toward reliable X-ray apparatus.

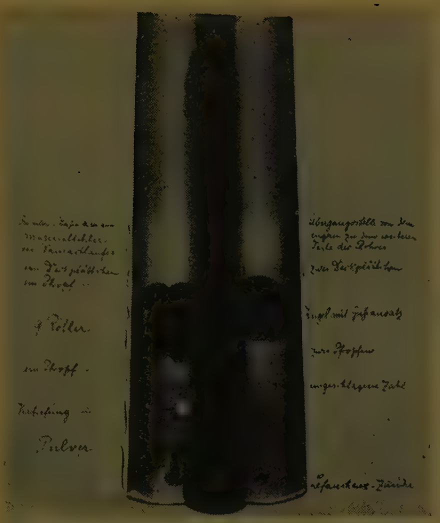

Figure 1. Historical X-ray photograph of an old sporting gun taken by Professor Roentgen

German handwriting is Roentgen's and refers to certain details of the picture

As previously mentioned, Coolidge introduced the tungsten anticathode which offered several advantages over the platinum used almost exclusively until then. While the atomic number differed only a little, the melting point of tungsten (3,400 degrees centigrade) is considerably higher than that of platinum (1,770 degrees centigrade), its heat conductivity is about twice as good, and, at higher temperatures, tungsten has a much lower vapor pressure than platinum; properties which make tungsten the ideal material for anticathodes. Both types of anticathode used by Coolidge, the wrought-tungsten stem for therapy tubes and the copper-backed tungsten plate for diagnostic tubes, still are used in modern X-ray tubes.

IMPETUS OF WORLD WAR I IN NETHERLANDS

In the Netherlands World War I was indirectly responsible for a new activity in the X-ray field. The country, as a neutral in that war, practically was cut off from the United States, England, and Germany, and Dutch roentgenologists were forced to go to native industry for their equipment. In Eindhoven, for example, the Philips Company was obliged to establish a special department for the repair of defective tubes which, in turn, led to the later creation of a separate research and development group.

By 1923 a method of welding chrome-iron to glass had been discovered and this was applied in the manufacture of X-ray tubes (the "Metalix" tube of the Philips Company). The main characteristic of these tubes was that the essential part, the discharge chamber proper, consisted of metal instead of glass. This made it possible to decrease the diameter of the tube considerably as the chance of breakdown of the remaining glass wall was reduced to a minimum. Moreover, the opening in the metal for the window could be chosen so that only a cone of useful X rays left the tube. By means of a lead covering and with the help of a tungsten plate in the cathode, direct primary rays were prevented from escaping from the tube except through the glass window, which removed the danger from X rays

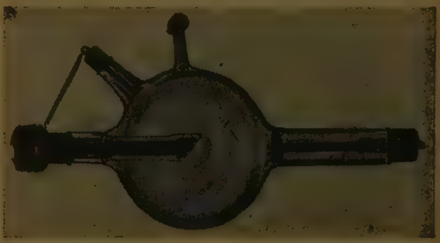

Figure 2. Early-type gas-filled X-ray tube

It is provided with a radiator cooling system and a regeneration device

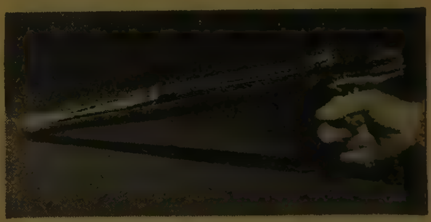

Figure 3. Shielded tube for cavity-therapy apparatus

Total length is 50 centimeters, total weight is approximately $5^1/_2$ pounds

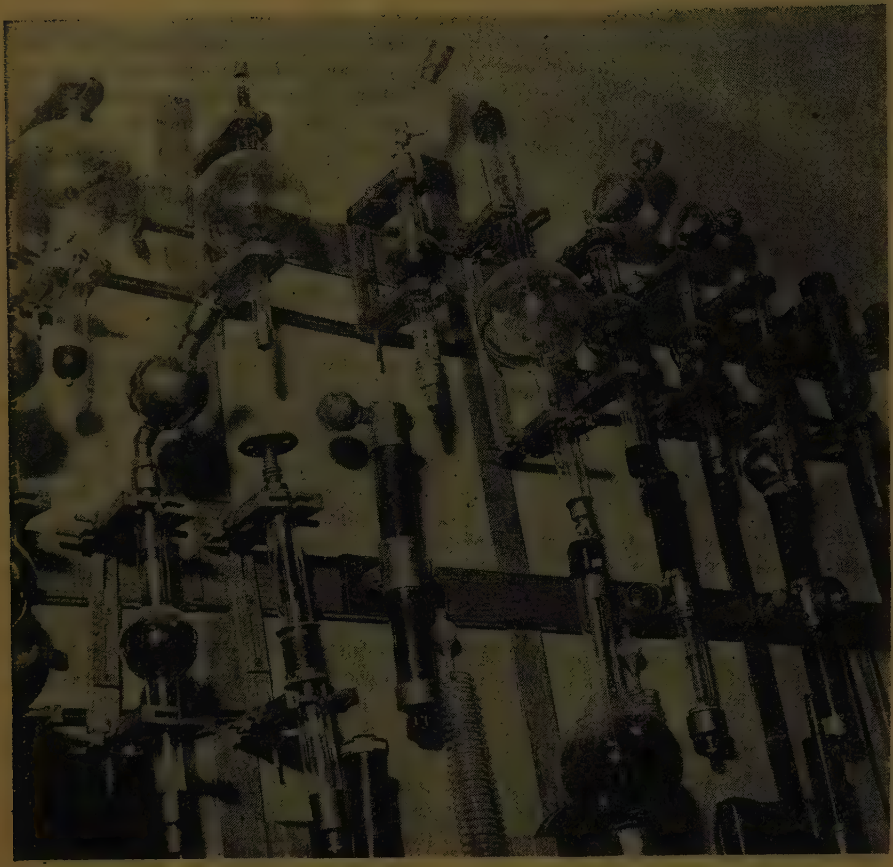

Figure 4. Historical types of X-ray tubes

when the tube was used with judgment and by experts. In another tube developed by that company the anode was synonomous with the rotor of an electric motor and was made to rotate by a rotating field shortly before and during the time the tube was loaded. In this way a sharper image of the rapidly moving organs of the human body (the heart, lungs, and stomach) could be obtained. Another development was the construction of a portable diagnostic X-ray apparatus with complete protection from high voltage and X rays in 1928.

A whole series of X-ray diagnostic apparatus for various general and special uses was developed within a short time and appeared on the market. This apparatus embodied the principle of complete automatic control which meant that upon the choice of voltage and exposure time the tube was charged automatically with approximately the maximum permissible load.

TUBES FOR THERAPY

Therapy tubes must operate at much higher voltages than diagnostic tubes and, as a result, the secondary electrons reflected from the focus have an important influence on the electrical behavior of the tube. With the introduction of a metal partition plate by which the discharge space between anode and cathode is divided into two parts so that only the primary electrons from the filament can pass through a narrow opening in the grounded partition plate, it was found possible to manufacture sealed rayproof tubes having a lifetime of several thousand hours for 200-kv, or even 400-kv, pulsating or constant direct current. (At the Cancer Hospital, Louvain, Belgium, four 400-kv tubes have been in daily use since 1935.) The use of a barium getter, which absorbs the last traces of active gases in the tube, has played an important part in these tubes as it has in the construction of a sealed 800,000-volt cascade tube. The cascade tube consists of three sections in series connected by metal tubes. In the first section there is a tungsten filament, in the third an anode. The electrons emitted by the filament are accelerated in the first section, pass through the first connecting tube, and again are accelerated in the second section. They then pass through the second connecting tube and undergo a third acceleration in the last section before they finally strike the anode. Each of the three sections carries one third of the voltage. Such a tube has been in use at the Cancer Institute, Amsterdam, since 1937.

ARTIFICIAL RADIOACTIVITY

Another interesting field in connection with the X ray is that of artificial radioactivity. Since Curie and Joliot first succeeded, in 1934, in making radioactive substances by artificial means, physicists all over the world have been exploring this new field of nuclear physics with great enthusiasm. The most suitable projectile for penetrating the atomic nuclei, and thus initiating various nuclear reactions, was found to be the neutron. The neutron is a particle of the weight of a hydrogen atom, but with no electrical charge, and is capable of easily entering the positively charged nucleus. The production of neutrons, which is carried out most effectively by the bombardment of lithium by heavy hydrogen ions, requires discharge tubes and generators for very high voltages as the efficiency of the reaction increases rapidly with the velocity of the ions of heavy hydrogen. In this connection special high-voltage apparatus of the cascade type, up to 2,000,000 volts, was built consisting of one transformer and several tubes and capacitors, each for 200 kv (see Figure 6) and neutron tubes were produced.

Apart from medical X-ray apparatus, special X-ray outfits for industrial purposes and crystal analysis have been made available. Although the efficiency of the microscopic testing of materials with the help of X rays had long been known, it was not until about 1935 that this X-ray testing, stimulated by several cases of official requirement of this testing for the quality of electrically welded seams, was applied on such a large scale for this purpose. A tube was perfected which has a specially constructed grounded anode, can be cooled with water from the mains, and may be loaded continuously with 2,000 watts on a focus of four millimeters to a maximum of 150 kv peak. With this instrument it is possible to photograph the most commonly occurring welded joints up to a thickness of four centimeters of steel, while fluoroscopy can handle a thickness of two centimeters. For the fluoroscopic X-ray testing of heavy light-metal castings special tubes with a thin glass window are preferable.

During World War II X-ray equipment was developed, consisting of a transformer and a tube with rotating anticathode, which was provided with a focus of the order of 0.5 to 0.7 millimeter. A tube with this extremely small focus is continuously loadable up to 100 kv peak alternating current and 600 watts continuous current. The point focus makes it possible to bring the object to be tested to within a few centimeters from the focus and to examine the object in its natural size (screen directly behind the object) or to examine an enlarged image (distance between screen and object twice or three times as long as the distance from focus to middle of the object). As the focus is practically a point, the enlarged image on the screen does not lose its definition immediately. On the contrary, upon enlargement the details become visible much more quickly and much more easily, because the shadow image and its detailed structure become relatively larger and sharper compared with the lack of sharpness of the grain of the fluoroscopic screen. With a normal focus of a tube with a stationary anode of two millimeters, this enlarging technique cannot be applied because in that case the geometrical lack of sharpness of the shadow image caused by the dimensions of the focus spoils the fluoroscopic image. It is even impossible to place an object at approximately the same distance from the focus as the thickness of the object because of the geometrical lack of sharpness.

CRYSTAL-STRUCTURE ANALYSIS

In 1930 transportable, entirely protected X-ray equipment was made available for crystal-structure analysis. This apparatus functions at about 45-kv alternating current and about five milliamperes. Tubes with anticathodes of copper, cobalt, iron, molybdenum, or tungsten and a focus of 1.5 millimeters, depending on the sample to be investigated, can be interchanged easily. Fan-cooled anodes are used. Later the unipolar grounded tube for microstructure was developed in which a water-cooled anode is used so that a loading of 50 to 60 kv peak at 20 milliamperes continuous current can be given on a line focus of 1.2 square millimeters. In addition to tubes with copper, tungsten, and molybdenum anodes, tubes with cobalt, iron, and chromium anodes also are produced. The tubes are provided with four or two windows of low-absorbent Lindemann glass. For these tubes a transportable high-voltage and radiationproof unit has been developed and various special cameras can be furnished as well. Particulary in the manufacture of artificial silk and quartz crystals for transmitters, industrial microstructure examination is employed on a large scale and the testing of the product with X rays forms a necessary link in the manufacturing process.

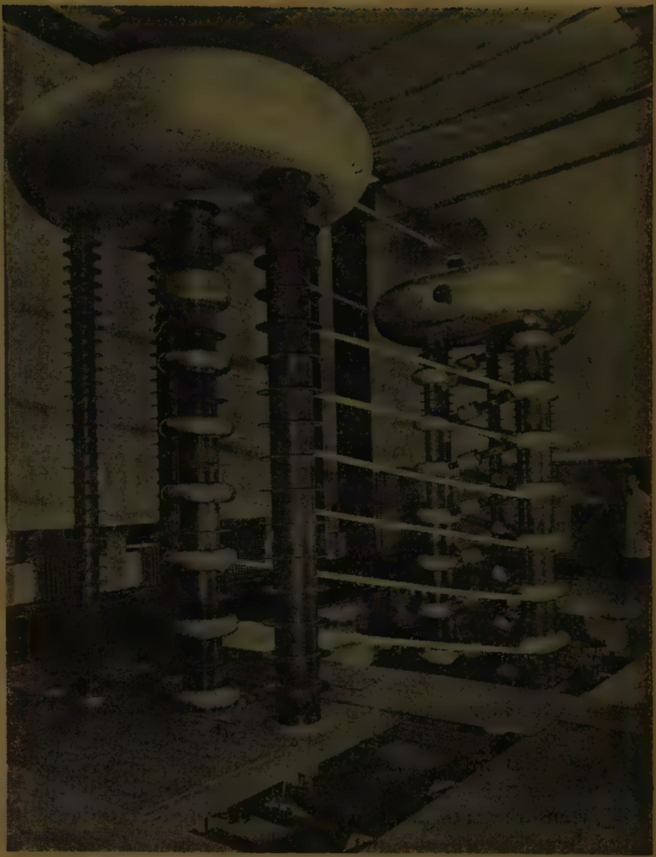

Figure 6. Neutron generator for 1,250,000 volts (Eindhoven, Holland)

In the foreground is the 13-foot acceleration tube with three insulating pillars for the shielding cap. Below is laboratory in which transmutations take place. To rear is high-voltage generator whose successive stages are connected by resistance (approximately 13 feet long) to corresponding sections of acceleration tube

While industrial X-ray technology was progressing there also was progress to report in the medical field. For stomach examinations "serial casette changers" were developed by means of which the film can be slid behind the screen with a single motion of the hand and the apparatus and tube were arranged for automatic and rapid changing over from fluoroscopy to photography. The increasing employment of X-ray examination for tuberculosis led not only to series fluoroscopy but also to mass chest radiography by taking miniature pictures of the image on the fluorescent screen. For the operating room an apparatus was developed which facilitates the removal of bullets and shell splinters. By means of X rays two beams of light are so adjusted that their point of intersection falls exactly on the bullet. The surgeon thus sees two light spots on the skin, can make his incision there, and can be guided further to the bullet by these beams of light. In addition, the "heart-phase switch" which enables the doctor to take an X-ray photograph at a certain phase of the heartbeat and an "exposure automat" which automatically switches off the tube when a certain density is reached have been realized experimentally.

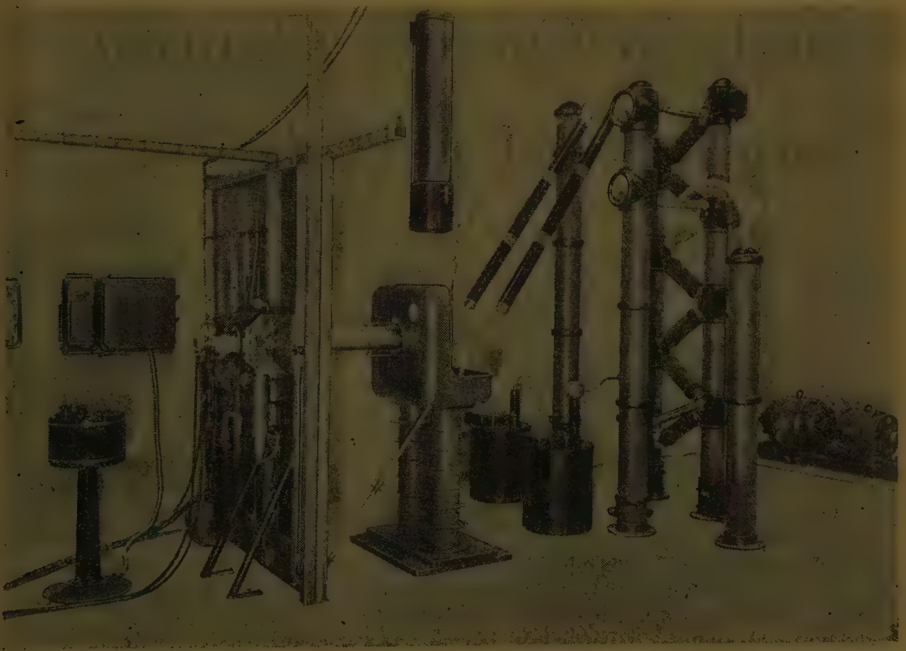

Figure 5. Complete neutron generator for 300 kv (Paris, France)

In the high-voltage room (right side of screen) are the canal-ray tube and the high-voltage generator. At extreme right is a 50–500-cycle converter used for feeding the positive-ion source. The isolated cooling pump is above the tube, one end of which protrudes through the screen into the workroom. Here neutrons are produced on a target which is connected to ground

X-RAY PROGRESS DURING WORLD WAR II

When the Germans invaded the Netherlands in 1940 the manufacture of new types of X-ray equipment in Eindhoven was brought to an end. The object was to manufacture as little as possible which might be of help to the German war effort. However, development proceeded. During the occupation plans were prepared for the manufacture of special hard glasses for discharge tubes at high voltage and for suitable alloys for hard glass–metal seals. Since 1940 X-ray and high-vacuum tubes of hard glass have been produced. In 1941 a small diagnostic tube with a stationary anode was put on the market which weighed only about 20 pounds. The hard-glass tube is placed inside a grounded rayproof and shockproof metal shield filled with oil. It is normally suitable for a power of more than 70 watts continuous at alternating and direct voltages up to 100 kv peak but, if desired, the continuous power can be increased by extra cooling of the jacket. A similar tube is suitable for a continuous power of 150 watts without any additional cooling of the shield to working voltages of 100 kv peak. These tubes are the double-focus type.

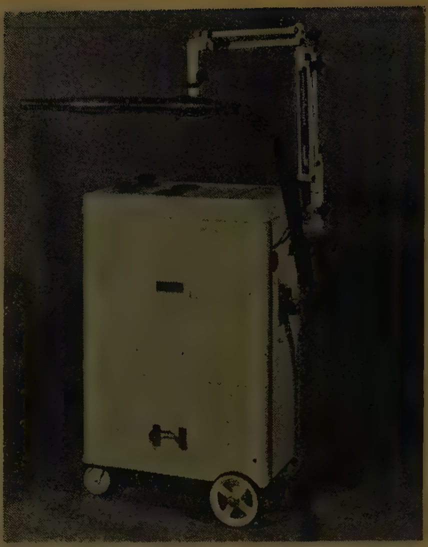

Figure 7. "Metalix" therapy apparatus suitable for short-distance and body-cavity irradiation

Mounted on wheels, the unit also incorporates a stand for the tube

RECTIFIERS

Typical rectifier tubes for X-ray purposes are the high-vacuum and gas-filled types. The high-vacuum tubes have a metal discharge chamber, the metal wall of which is at the same time the anode, surrounding the cathode filament. By this means the saturation emission at a given temperature of the filament is reached at a much lower and more reproducible anode voltage than in glass tubes. In the latter the cathode is not surrounded by the anode and the glass envelope usually takes on a negative charge and so functions as a grid. The gas-filled tubes contain mercury vapor and the discharge phenomenon in the transmitting phase is quite different from that in a vacuum tube. The strong negative space charge is lacking; on the contrary an arc is formed between the filament and the anode. With a momentary emission of 1,500 to 2,000 milliamperes the voltage difference between anode and cathode amounts to only about 25 to 30 volts. The advantages of these tubes are practically no voltage drop and a very low filament wattage. However, at very low room temperatures they are difficult to start and at high temperatures there is a chance of breakdown in the blocking phase.

A third tube combines the advantages of the high-vacuum and gas-filled tubes but does not have the disadvantages of the latter. This type is a high-vacuum rectifier with a thoriated-tungsten filament. A diagnostic tube of this type at an emission of 2,000 milliamperes has a voltage drop of about 2,000 volts and a filament power of about 35 watts. These tubes have a long life and require a high vacuum.

This is just a brief résumé of some of the developments in the Netherlands that have helped to make the X ray indispensable to modern life. Its importance in both the pure and applied sciences has long been recognized in Europe, where the penetrating power of the ray was discovered, as well as in the United States. European scientists have contributed a great deal to X-ray technology and, with the resumption of peace, undoubtedly will continue to contribute much more in the future.

Statistical Methods in Quality Control

VII. Control Charts for Analysis of Variables

Control charts and their use of analysis to determine possible factors in the process which might need correction are discussed in this article.

ENGINEERS recognize that control and interpretation of variations in processes and operations constitute a major problem. This is true whether the aim is to meet specifications consistently, to reduce variations for design improvement, or to interpret the reliability of experimental results. Obviously variations in results come from variations in the factors underlying the creative process. Each factor in turn is governed by a host of others. In short, the character of a result arises from a large number of causal factors. When a system contains a large mass of relatively small causal factors, indistinguishable in their individual influences, the resulting variations are said to be due to chance. When a predominant and therefore identifiable factor changes, we speak of an assignable cause or an assignable variation.

The statistical quality-control chart is sometimes considered primarily a tool for attaining better control during repetitive production operations. Nevertheless, its scope of application includes analysis of data. This article illustrates the latter.

As previously discussed in this series, an essential of control-chart procedure is proper classification of observations—rational subgrouping*—on a basis of possibly associated factors, such as machine, operator, supplier, and so forth, or upon time, that is, production sequence. The chart therefore can be used to test homogeneity, that is, for assignable peculiarities associated with a particular classification, as well as for removable instabilities and capabilities of processes in time. In fact, such analysis precedes the setting of operating control standards for corrective action during continuous production.

In illustration, visualize a multiple-head machine, essentially a battery of individually operated processes, molding a plastic base around a projecting pin. The pin projection is critical in subsequent assembly, and has specification limits of 0.036 inch and 0.065 inch. Process variables or factors influencing control of pin

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One of a series of articles prepared in the AIEE subcommittee on educational activities and sponsored by the AIEE subcommittee on statistical methods.

Personnel of the AIEE subcommittee on educational activities: J. Manuele, H. F. Dodge, A. I. Peterson, and R. E. Wareham.

* Rational subgrouping has been discussed previously in this series, and in reference 1, page 48, and reference 2, page 9.

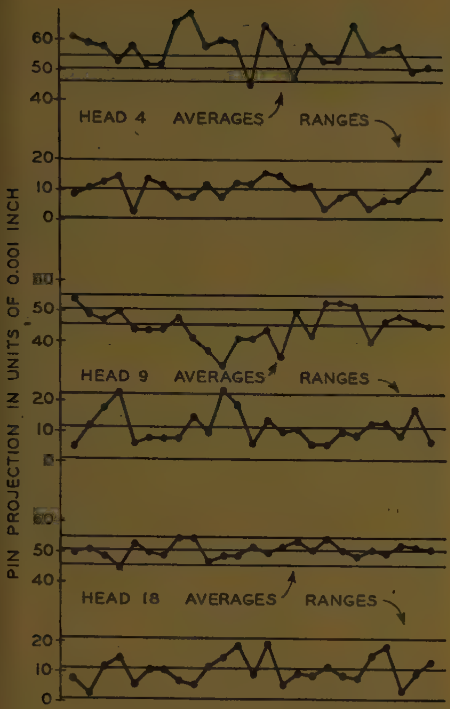

Figure 1. Control charts for heads number 4, number 9, and number 18

projection include supplied over-all pin lengths and plastic-material quality, where variability tends to influence all heads or machine units simultaneously, and individual head adjustments and characteristics substantially independent of each other.

In planning the rational subgrouping of test samples for initial analysis, it is desirable to reflect possible influences from principal over-all factors, as well as from those peculiar to individual heads. For reasons beyond this discussion, the sample size for any one head and time was set at seven in this case, although four or five usually suffice. Such samples per hour from each head provided the necessary data.

The average and range were calculated for each sample of seven. Table I illustrates the results for the first 25 hours for three of the heads, in units of 0.001 inch. The next step was the setting-up of control limits for analyzing the results. In each case, the mean value of ranges within samples provided an approximation of the chance variability or dispersion, without inflation from the instabilities in time and from the assignable differences between heads. To assist in comparing head performances, the central line in each case was taken as the midpoint of specification limits, that is, 0.0505 inch. Using the procedure outlined in reference 2, the control-limit calculations* for head number 18 are illustrated by

For averages:

$\overline{X}' \pm A_2\overline{R} = 0.0505 \pm (0.419)\ (0.0112)$
$= 0.0505 \pm 0.0047$
$= 0.0458$ and 0.0551

For ranges:

Central line $= \overline{R} = 0.01124$
Limits $= D_3\overline{R}$ and $D_4\overline{R}$
$= 0.076\ (0.0112)$ and $1.924\ (0.0112)$
$= 0.0009$ and 0.0215

The corresponding limits for the other two heads are

	Number 4	Number 9
For averages	0.0461 and 0.0549	0.0457 and 0.0553
For ranges	0.0008 and 0.0200	0.0009 and 0.0222

In this case, comparing heads which are supposed to be similar in performance and which in the data appear substantially homogeneous, it is permissible to average all sample ranges from all heads, which for the three units illustrated gives us

$\overline{R} = 0.01107$
Limits for averages $= 0.0505 \pm 0.419\ (0.01107)$
$= 0.0505 \pm 0.0046$
$= 0.0459$ and 0.0551
Limits for ranges $= 0.076\ (0.01107)$ and $1.924\ (0.01107)$
$= 0.0008$ and 0.0213

In examining the control charts, many interesting inferences were suggested. First, it was apparent that head number 4 operated at too high a level, as well as being out of control. Also, head number 9 performed at too low a level, as well as being out of control. Head number 18 operated in control and at the proper level. The machine setter's earlier claim that the

Table I. Control Sample Averages and Ranges

(Sample Size is Seven)

Date	Hour	Head Number 4		Head Number 9		Head Number 18	
		$\overline{X}$	R	$\overline{X}$	R	$\overline{X}$	R
-3	1	61	9	54	5	50	8
	2	59	11	49	12	51	3
	3	58	13	47	18	49	12
	4	53	15	50	23	45	15
	5	58	3	44	6	53	6
	6	52	14	44	8	50	11
	7	52	12	44	8	49	11
	8	66	8	48	8	55	7
9-4	1	69	8	41	15	55	6
	2	58	12	37	10	47	12
	3	60	8	32	24	49	15
	4	59	13	41	19	49	19
	5	45	12	41	6	52	9
	6	65	16	44	14	50	20
	7	59	15	35	10	52	6
	8	47	11	50	11	54	10
9-5	1	58	12	42	6	51	9
	2	53	4	53	6	55	12
	3	53	8	53	11	51	9
	4	65	10	52	9	49	8
	5	55	4	40	13	51	16
	6	57	7	47	13	50	19
	7	58	7	49	9	53	4
	8	50	11	47	18	52	10
	9	51	17	45	7	51	14
Sum		1,421	260	1,129	289	1,273	281
Number of items		25	25	25	25	25	25
Average		56.84	10.40	45.16	11.56	50.92	11.24

Table II. Head Relative Frequency Distributions

(Units of 0.001 Inch)

Cell Limits	Number 4 (Per Cent)	Number 9 (Per Cent)	Number 18 (Per Cent)	Composite (Per Cent)	
20.5–23.5		0.6		0.2	2.7
23.5–26.5		0.0		0.0	
26.5–29.5		1.1		0.4	
29.5–32.5		1.7		0.6	
32.5–35.5		4.6		1.5	
35.5–38.5		4.6	1.1	1.9	93.6
38.5–41.5	1.7	18.3	2.9	7.6	
41.5–44.5	1.1	13.7	6.3	7.1	
44.5–47.5	6.9	18.8	13.7	13.1	
47.5–50.5	9.1	12.6	21.8	14.6	
50.5–53.5	10.3	11.4	24.0	15.3	
53.5–56.5	18.9	9.1	17.7	15.3	
56.5–59.5	20.6	2.3	9.1	10.8	
59.5–62.5	11.4	0.6	3.4	4.9	
62.5–65.5	9.1	0.6		3.0	
65.5–68.5	5.1			1.7	3.7
68.5–71.5	2.9			1.0	
71.5–74.5	2.9			1.0	
	100.0	100.0	100.0	100.0	

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* For technique and factors for calculation see reference 1, pages 49–50.

trouble originated with the poor control of pin lengths supplied—an over-all effect—obviously was refuted, as the same material was used in head number 18 as in the others. Adjustment of the head levels was a relatively simple job as soon as the faulty units were identified, although there still remained some lack of control or stability about the average levels. Here again, it was difficult to defend the claim that over-all influences such as incoming material were to blame, since trends and short-run violations of limits did not correlate among the heads; in fact did not appear in head number 18 at all. Investigation guided by this information subsequently proved that fouling and play in positioning devices were the causes; that is, insufficient maintenance.

A second aspect of analysis, particularly with reference to the setting up of control standards for subsequent corrective supervision, was the approximation of attainable control, in the sense of dispersion with respect to specification limits and scrap. Since the mean ranges tended to indicate the degrees of simple chance variability alone, attainable for the product over-all if the assignable causes of instability were kept out of the process, it was possible to check both the rationality of the specification limits and the justice of control limits to be established. Using the known relationship of a mean range, which is substantially stable in these charts, and the controlled long-run dispersion of single observations, we calculated the limits on each side of specification bogey, or midpoint, that could include practically all of the results under controlled conditions.* For head number 18:

$$\bar{X}' \pm 3\frac{\bar{R}}{d_2} = 0.0505 \pm 3\ (1.0112/2.704)$$
$$= 0.0505 \pm 0.0124$$
$$= 0.0381 \text{ and } 0.0629$$

Corresponding calculations for the other heads provide

Number 4: 0.0390 and 0.0620
Number 9: 0.0380 and 0.0630

Such calculations for all head ranges together provide limits of 0.0382 and 0.0628.

Since there was practical compatability of these possible specification limits with the established ones of 0.036 and 0.065, the typical claim of "robbing the shop" did not stand up.

To demonstrate the effect of lack of control, that is, differences between heads as well as instability in time, the initial over-all relative frequency distributions of individual values from each head are shown in Table II, together with the composite result representing the product flowing to the next operation. Obviously, if the heads were all kept the same and controlled at proper levels, the total product could meet the performance here demonstrated by head number 18.

A last point should be stressed. Analysis for heterogeneity of two or more supposedly similar sources or groups, in this case perhaps 15 different heads, can be made with one sample from each, using the same chart procedure, except that there would be one point per group on a single chart. The mean range would provide control limits on each side of the grand average or the specification limits midpoint.

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* See discussion of inclusive limits in the second article of this series in the July 1945 issue of *Electrical Engineering*.

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2. Control Chart Method of Controlling Quality During Production. American War Standard ASA Z1.3, American Standards Association, New York, N. Y., 1942.

3. A First Guide to Quality Control for Engineers. Ministry of Supply, London, England, 1943.

Betatron Developed for Production of 100,000,000-Volt X Rays

With the removal of wartime restrictions, details concerning the construction of a 100,000,000-volt betatron or electron accelerator have been released by the research laboratory of the General Electric Company, Schenectady, N. Y. The new machine emits a 100,000,000-volt electron stream that produces X rays of the same energy and, it is said, can generate radiations in the laboratory which formerly were available only in the cosmic rays.

Located in a special building with concrete walls three feet thick as a protection against the dangerous rays emitted, the principal part of the betatron is a 130-ton electromagnet of laminated silicon steel and its heart is a toroid or doughnut-shaped vacuum tube of glass, 74 inches in diameter, which lies in the center between the two poles of the magnet.

The betatron operates on ordinary 60-cycle alternating current and when the machine is operated at full power, a beam of X rays two degrees in diameter is emitted. The betatron also may be operated at lower power but the resulting production is not very strong.

The 100,000,000-volt electron accelerator evolved from a 2,300,000-volt table-size model constructed in 1939 and a 20,000,000-volt betatron completed in 1941.

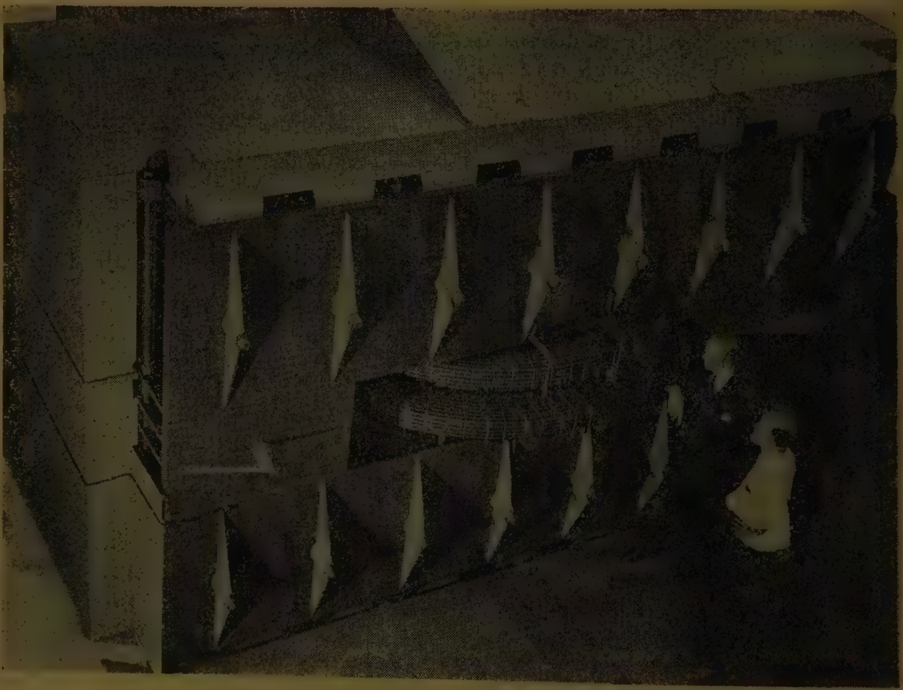


Figure 1. View of 100,000,000-volt induction electron accelerator

In the foreground are E. E. Charlton (left) and W. F. Westendorp (M '43) who were responsible for the design and construction of the machine

INSTITUTE ACTIVITIES

Winter Convention Program Will Explore New Frontiers of Electrical Engineering

Initiating the Institute's postwar series of conventions, a program replete with both technical and social activities has been arranged for the winter convention to be held in New York, N. Y., January 21–25, 1946. All prewar features have been restored, including the dinner-dance, smoker, and inspection trips. The challenging possibilities of the immediate future will be discussed at the general session by a prominent electrophysicist. Similarly reflecting this forward-looking keynote, many of the sessions and conferences comprising the technical program will explore the new frontiers of electrical engineering in their respective specialties. Convention headquarters will be in the Engineering Societies' Building, 33 West 39th Street. Early registration is urged.

Registration will be enforced and nonmember fees of $2 will be required. Only persons registered may attend sessions.

GENERAL SESSION

The Institute is fortunate in having J. R. Dunning, a prominent electrophysicist of broad vision and extensive research experience in the field of nuclear physics, to address the meeting. Active in the investigation of atomic fission for many years prior to his recent connection with the atomic-bomb project, Doctor Dunning is in an exceptional position to place before the electrical engineering profession the great possibilities of the future. His address will be inspirational, serving as a keynote for the forward outlook of the entire technical program, from which may come new and improved ideas in specialized fields of activity.

Two of the highest engineering honors, the Edison Medal and the Hoover Medal, will be presented at this session. Details of the awards will be announced later.

CONFERENCE ON INSTITUTE ACTIVITIES

The Institute is confronted by problems of growing urgency both in its relations to the engineering profession at large and in its relationship to highly specialized organizations and interests within its own general field. Early in this administrative year, the board of directors referred these problems to the committee on planning and co-ordination, who have appointed two special subcommittees to explore the professional and technical activities of the Institute in relation to those of other engineering and scientific societies and more particularly in relation to the desires and aspirations of the members of the Institute.

The relationship to other organizations is the easier part of the study. The desires and aspirations of the rank and file of the membership are more difficult of determination. This conference is intended as one of the first steps in sampling membership opinion on what the Institute should be doing as compared with what it is doing.

Members active in Institute affairs should take this opportunity to assist the subcommittees by giving thought to the problems in advance of the conference, by attending the meeting and stating their considered opinions and specific suggestions. It is hoped that every District and every Section will be represented by at least one member but the meeting is open to all members and their views on these possible new frontiers of the Institute will be welcomed.

TECHNICAL SESSIONS AND CONFERENCES

Technical sessions and conferences range through the whole realm of technical-committee activites. The sessions have been arranged to avoid conflict insofar as possible. Applied principles of electronics occur not only in the communications papers but also may be found in various papers throughout the program. Similarily, measurement technique may be found not only in the instruments and measurements sessions but also threads through many papers in other sessions. Abstracts of first available papers are given elsewhere in this issue.

CENTRAL STATIONS

In the field of power transmission one of the papers will analyze the lightning performance and construction of 220-kv lines for the past ten years. Another paper deals with formulas for conductor size according to the cost of resistance loss.

In the field of power generation one session will treat hydroelectric systems and another will be on excitation systems. The hydroelectric installations in this country and in Canada have played an important part in winning the war. In the past, however, very little has been published on the problems that are encountered in the design, operation, and maintenance of the hydroelectric installations, as compared with the steam plants. This series of papers is intended to give a representative cross section of the problems and experience relating to the hydroelectric systems.

The task of providing and controlling the excitation of large turbogenerators of the power system is being approached from a new angle. Instead of the conventional type of design, the motor-driven exciter, the electronic exciter, and the electronic voltage regulator are offered as a new solution of the problem.

In general, wartime requirements for protective devices did not bring forth any startlingly new developments. However, there have been some new and interesting applications of basically known devices which will be discussed in detail at the several protective-devices and relay sessions. These subjects will cover a range of protective relays, network protectors, new devices for the protection of series capacitors, pilot wire protection, and a number of interesting new circuit breakers. This program will be keyed in with the theme of "New Frontiers," as the discussions develop the new applications and the probable effect of new devices on future practice.

There will also be one conference session devoted to an informal discussion of transformer protection. In the past these conference sessions have been extremely helpful as a means of exchanging experiences.

As a means of helping to shape future practice, the users of protective devices are being encouraged to contribute papers recounting their experiences. It is expected that at least one such paper will be included in this program.

COMMUNICATION AND ELECTRONICS

In the field of communication three technical sessions and a conference on communication relays are in prospect. In addition there will be a joint session with the Institute of Radio Engineers, which will be addressed by a prominent speaker in the radio field. One of the sessions will be general with papers on transmission-line problems, telegraphy, infantry combat communications, and a turntable rejection filter. Another will be a symposium on sound recording. Radar will be the subject of the third session provided necessary clearances can be obtained. This session is sponsored jointly with the committee on electronics.

INDUSTRIAL POWER APPLICATIONS AND ELECTRIC MACHINERY

On the new frontiers of industry the committee on electronics has arranged a conference on "New Industrial Uses of Electronics Resulting From Wartime Developments." Each speaker will describe briefly some new application of electronics for peacetime use that has resulted directly or as a by-product from a wartime development. Some of the speakers will utilize demonstrations or sample devices in their presentations. Only about half the allotted time will be devoted to scheduled presentations. The remainder of the period will be available for questions or discussions of the ideas presented and, if time permits, for the suggestion or description of still other new applications of electronics by those in attendance.

Another session and conference will consider the voltage requirements and the possible use of higher voltages to provide more power for the industrial distribution systems of the future. An interim report will be presented, which may lay the groundwork for a final report. Other conference presentations will treat neutral grounding and the grounding of delta-connected transformers. One or two papers on induction heating also will be presented.

Machine tool and process drives will be the subject of another conference. New and more powerful drives for industry will be

Tentative Schedule of Events

Monday, January 21

9:30 a.m.—Protective devices
Communication
Conference on new industrial uses of electronics resulting from wartime developments

2:00 p.m.—Symposium on sound recording
Power system relays
Electric welding

Evening—Radiobroadcasts, theaters, etc.

Tuesday, January 22

9:30 a.m.—Protective devices
Conference on communication relays
Conference on industrial voltage requirements

2:00 p.m.—Radar
Conference on power relays
Industrial power applications

6:30 p.m.—Smoker at Hotel Commodore

Wednesday, January 23

10:00 a.m.—General session
Edison Medal presentation
Hoover Medal presentation
Address: Professor J. R. Dunning

2:00 p.m.—Conference on Institute activities
Conference on standardization of aircraft electric equipment
Conference on machine tool and process drives

8:00 p.m.—Joint meeting with Institute of Radio Engineers

Thursday, January 24

9:30 a.m.—Power transmission
Electric machinery
Instruments and measurements

2:00 p.m.—Electric machinery
Instruments and measurements
Conference on statistical methods applied to quality control

7:00 p.m.—Dinner-dance at Hotel Plaza

Friday, January 25

9:30 a.m.—Hydroelectric systems
Air transportation
Conference on lighting
Basic sciences

2:00 p.m.—Excitation systems
Air transportation
Conference on education

6:00–8:00 p.m.—Institute of Radio Engineers' party at Hotel Astor

discussed and there will be presentations on co-ordinated drives in the rubber industry and industrial application of Rototrol regulators.

In the field of electric welding, papers will be presented which deal with the characteristics of the electric arc, spot welding, and capacitor stabilization of arc-welding transformers as well as the design and measurements of capacitor-discharge transformers.

Two sessions on electric machinery have been scheduled at which problems of design, performance characteristics, and rating of electric motors will be treated.

AIR TRANSPORTATION

Engineers attacking electrical problems in aircraft are always on "New Frontiers." In one of the sessions the vital problem of rating electric apparatus will be presented as aviation's frontier, "service ceiling," advances higher and higher. Another paper will contribute a newly released study of aircraft-ignition problems. Still another will blaze new trails with an authoritative treatise on the new a-c system for very large airplanes and a companion paper will describe a precedent-setting circuit breaker for these airplanes. Other pioneering achievements to be presented include a system of electric gun charging and the application of carbon piles to aircraft motor speed-regulating devices.

In the conference on standardization of aircraft electric equipment the principal business will be the progress of subcommittees engaged in standardization activities. Anyone interested is invited to attend. Prepared memoranda from committee members and nonmembers will be welcomed and time will be made available for discussion from the floor by visitors.

HOTEL ACCOMMODATIONS

The hotel situation in the metropolitan area is acute and quite likely will remain so for some time. Members, therefore, are urged to make their plans early and arrange for accommodations directly with the hotel preferred. The Institute has no connections with any of the hotels and cannot undertake to make reservations.

NONMEMBER REGISTRATION REQUIRED

Arrangements are being made to enforce the rule that nonmembers attending the convention must register. Admission to all technical sessions will be strictly limited to those wearing the registration badges obtainable at the registration desk by nonmembers of the Institute on payment of the $2.00 nonmember registration fee. To avoid embarrassment and misunderstandings, members are urged to call this fact to the attention of their associates who are not members of the AIEE, but who may be planning to attend some of the sessions.

Popular Entertainment Features Restored for Convention

Smoker: The smoker, an event which fortunately we have been able to stage regularly throughout the war years, will be scheduled again this year on Tuesday evening, January 22, 1946, at the Hotel Commodore. Arrangements are being made so that you can get together with your friends for cocktails preceding dinner and show. Tickets for dinner and show this year are $6.50 each and tables can be reserved for six, eight, and ten by remittance in advance. It is absolutely necessary that you make your reservations now.

Dinner-Dance: Your convention committee had decided that the popular interest always evidenced in our dinner-dances of past years again warrants the revival of that affair as one of the features of the 1946 winter convention. The Plaza Hotel Grand Ballroom has been reserved for 7 p.m. on the evening of Thursday, January 24, 1946. Tickets for dinner and dance will be $8.00 per person. Tables can be reserved for eight and ten. For those who find it impossible to attend the dinner a very limited number of tickets for the dance only will be available at $3.00 per person. Please send in your reservation and remittance at earliest possible date so that the committee can make necessary arrangements.

Inspection Trips: An inspection-trips committee is engaged in setting up an interesting group of inspections. While definite commitments cannot be made at the present time as to places or installations to be visited, the committee hopes to be able to schedule a trip to the Bell Laboratories at Summit, N. J., an inspection of LaGuardia Airport, with an air trip over New York and vicinity; a complete inspection of an aircraft carrier, a large power station, and trips to other points of interest. Details will be available on the opening registration day at committee headquarters.

Special Trips for Women: A Women's Committee is now being organized to provide features of special interest to the ladies attending the convention. No details can be listed at this time, but an interesting program for the women is assured.

Theater and Broadcast Tickets: As a service to out-of-town members a limited number of theater tickets are being purchased in advance for Wednesday and Thursday evenings, January 23 and 24, 1946. Members from outside of the New York territory who wish to attend any of the shows listed below should send in the reservations with remittances at once. Please indicate first, second, and third choices, if possible.

Oklahoma	$4.80
Carousel	6.00
Bloomer Girl	5.40
Song of Norway	6.00
Follow the Girls	4.80
On the Town	4.80
Up in Central Park	6.00
I Remember Mama	4.20
Harvey	4.20
Life With Father	3.60

There will also be available tickets for a number of broadcast presentations. These tickets will be obtainable on application at the registration desk, but no definite listing of the broadcasts can be made at this time.

Future AIEE Meetings

Winter Convention
New York, N. Y., January 21–25, 1946

South West District Meeting
San Antonio, Tex., April 16–18, 1946

North Eastern District Meeting
Buffalo, N. Y., April 24–25, 1946

Southern District Meeting
May 13–16, 1946

Summer Convention
Detroit, Mich., June 24–28, 1946

Pacific Coast Convention
Seattle, Wash., August 26–30, 1946

Great Lakes District Meeting
Fort Wayne, Ind., late summer or early fall, 1946

AIEE Planning Committees Begin Their Searching Task

The two new subcommittees of the AIEE committee on planning and co-ordination are actively at work on their assignments to review the organization, methods of operation, and scope of the technical and the professional activities, respectively, of the Institute and of other engineering and scientific societies. This survey is intended to lead to specific recommendations of ways and means by which the Institute more effectively can pursue its stated objectives and thus better serve its membership. Activation of these new groups was announced in the November 1945 issue of *Electrical Engineering*, page 409.

Active assistance of the technical-program committee has been enlisted by the technical-activities subcommittee. Through individual technical committee chairmen information on the several national societies working in specialized fields related to those of the Institute's technical committees is being assembled. The technical-program committee also is continuing its efforts to achieve more efficient internal organization within and between the existing technical committees. As one chairman expressed it—"Let's put our present house in order, then see if we can plan a better one." M. D. Hooven (F '44), chairman of the technical-activities subcommittee, is also chairman of the technical-program committee and a member of the committee on planning and co-ordination. Other members of the subcommittee are F. A. Cowan (M '29), S. G. Hibben (M '45), H. M. Turner (F '44), and R. J. Wiseman (F '27), all of whom have been active over a period of many years in AIEE and industrial engineering fields.

The professional-activities subcommittee, under the chairmanship of T. G. LeClair (F '40) a member of the committee on planning and co-ordination, includes M. S. Coover (F '42), W. S. Hill (M '30), B. D. Hull (F '39), and F. E. Sanford (M '34), all of whom long have been active in Institute and professional affairs. This group is not as fortunate as their colleagues who are studying technical activities, in that they do not have a ready-made group of active members to call upon for support. Nor is their problem as clearly defined, in spite of the fact that much has been said and written about the proper place of the engineer in civic and professional affairs, his relation to collective bargaining, his economic status generally, and the bearing of his nontechnical interests on the matter of greater unity in the engineering profession. Thus they will need, and are desirous of receiving, all the help which members of the Institute can give them.

Both groups are holding regular meetings planning their work and conducting preliminary investigations principally of an introspective character. The stated intention of these groups is to examine critically and constructively what the Institute is doing—and what it is not doing—to the end that the Institute's future program and policy may serve most effectively the actual needs and desires of the membership and of the profession. The working groups believe that worthwhile improvements in policy or procedure can be effected only if based upon competent and thorough-going diagnosis of existing conditions. They are seeking information as to symptoms and will welcome it from any quarter. AIEE vice-presidents already have been called upon as consultants, thus establishing one set of direct channels to and from the Districts. Others will be called upon as the studies progress, but spontaneous comments and suggestions direct from individual members or Section groups are especially invited.

An open clinic or conference scheduled to be held during the forthcoming winter convention will provide opportunity for the subcommittees to report progress and to state certain problems as they see them, and for full and frank discussion by any who may wish to participate. Meanwhile, members are urged to send in their ideas, their criticisms, their suggestions as to what they think needs to be fixed, to be changed, to be done. Communications may be addressed to any of the members of either group, to any of the vice-presidents, or to headquarters.

AIEE Board of Directors Meeting Held in Cleveland

The regular meeting of the board of directors of the American Institute of Electrical Engineers was held in Cleveland, Ohio, October 18, 1945. The policy was adopted of holding the May and October meetings outside New York, N. Y., within a certain area, consideration to be given when choosing the time and place, to holding the meeting in conjunction with a District or Section meeting.

Members of the board of directors were elected to serve on the AIEE nominating committee as follows: C. B. Carpenter, C. M. Laffoon, F. L. Lawton, J. R. North, and H. B. Wolf. J. F. Fairman and W. B. Morton were designated alternates.

H. H. Henline, secretary, AIEE, was reappointed AIEE representative on the board of directors of the Engineering Societies Personnel Service, Inc., for the year 1946.

Sidney Withington was appointed AIEE representative on the main accrediting committee of the Engineers' Council for Professional Development, succeeding E. L. Moreland.

After a review of AIEE conventions and District meetings previously authorized tentatively, and consideration of new requests for District meetings, the following schedule was approved for the year 1946: *Conventions*—Winter, New York, N. Y., January 21–25; Summer, Detroit, Mich., June 24–28; Pacific Coast, Seattle, Wash., during week of August 26–30. *District meetings*—South West, San Antonio, Tex., April 16–18; North Eastern, Buffalo, N. Y., April 24–25; Southern, in North or South Carolina, May 13–16, and Great Lakes, Fort Wayne, Ind., late summer or early fall.

The board endorsed resolutions, drafted at a conference of representatives of engineering organizations and the American Association for the Advancement of Science held in Washington, D. C., urging provision in any extension of the Selective Service Law for assigning quotas of selected young men to scientific and technical schools for training in lieu of active service with the Army and Navy—the purpose being to set in motion a procedure for overcoming the alarming deficit in trained technical personnel, so necessary for national welfare and defense, which has resulted from the applications of the Selective Service regulations.

Upon recommendation of the Welding Research Council, the board approved the idea of inviting the other Founder Societies to join in the sponsorship of the work of the Welding Research Council, for which the AIEE and the American Welding Society have been the sponsors in the past.

The board approved a change in name of the joint conference committee (consisting of the presidents, immediate past presidents, secretaries of the Founder Societies and the American Institute of Chemical Engineers) to "Engineers Joint Council." A suggested apportionment of the expenses of the Engineers Joint Council among the constituent societies according to their income from dues also was approved. The board approved a recommendation of the Engineers Joint Council that the National Society of Professional Engineers be invited to join the Joint Committee on Economic Status of the Engineer as a full participating member.

A budget for the appropriation year beginning October 1, 1945, recommended by the finance committee, was adopted amounting to $482,000. The budget included an appropriation for the committee on planning and co-ordination to carry out a project assigned to it by the board of directors, namely, that of reviewing the technical activities of the Institute and the Institute's relations with other organizations, and recommending any changes in organization or methods that may seem desirable.

Other actions taken included the following: Minutes of the meeting of the board of directors held August 1, 1945, and of the executive committee held September 14, 1945, were approved. Recommendations adopted by the board of examiners at its meeting September 20, 1945, were reported and approved. Upon recommendation of that board, the following actions were taken: 13 applicants were transferred and one was elected to the grade of Fellow; 68 applicants were transferred and 47 were elected to the grade of Member; one Member was reinstated; 200 applicants were elected to the grade of Associate, and 289 Students were enrolled.

Those present were: *President*—William E. Wickenden, Cleveland, Ohio. *Past president*—N. E. Funk, Philadelphia, Pa. *Vice-presidents*—C. B. Carpenter, Portland, Oreg., M. S. Coover, Ames, Iowa, J. F. Fairman, New York, N. Y., E. S. Fields, Cincinnati, Ohio, R. T. Henry, Buffalo, N. Y., L. M. Robertson, Denver, Colo., H. B. Wolf, Charlotte, N. C. *Directors*—P. L. Alger, Schenectady, N. Y., J. M. Flanigen, Atlanta, Ga., C. W. Mier, Dallas, Tex., S. H. Mortensen, Milwaukee, Wis., W. B. Morton, Bethlehem, Pa., J. R. North, Jackson, Mich., D. A. Quarles, New York, N. Y., W. C. Smith, San Francisco, Calif., W. R. Smith, Newark, N. J. *Treasurer*—W. I. Slichter, New York, N. Y. *Secretary*—H. H. Henline, New York, N. Y.

Other matters were discussed but not acted upon at this meeting.

1945 December Supplement Contents Announced

Papers which appeared in the AIEE technical-paper program for 1945 and were not published in the monthly *Transactions* section of *Electrical Engineering* or in the June supplement will appear in the December 1945 "Supplement to Electrical Engineering—Transactions Section." The December supplement will contain five technical papers, discussions of those papers, and discussions of the papers already published in the July–December monthly sections. Issuance of the December supplement will complete publication of papers and discussions received in 1945.

Copies of the supplement will not be available until some time after the first of the year, at which time published copies will be mailed to those who entered advance orders. Others then may obtain copies at 50 cents each from the AIEE order department, 33 West 39th Street, New York 18, N. Y.

Papers appearing in the December 1945 supplement, abstracts of which have been published in *Electrical Engineering* in advance of the meetings, are:

Electric Machinery

45-93—A Generalized Circle Diagram for a Four-Terminal Network and Its Application to the Capacitor Single-Phase Motor; *J. G. Tarboux (F '43).* Abstracted in the April 1945 issue, pages 156–7.

Communication

45-145—Report of Conference on Radio-Frequency Cables; *AIEE committee on communication.* Abstracted in the September 1945 issue, page 331.

45-146—A Study of Wave Shapes for Radio-Noise-Meter Calibration; *C. W. Frick (A '19).* Abstracted in the September 1945 issue, page 331.

45-162—Fundamentals of the Amplidyne Generators; *J. L. Bower (M '45).* Abstracted in the September 1945 issue, page 331.

Power Transmission and Distribution

45-141—The Frequency of Occurrence and the Distribution of Lightning Flashes to Transmission Lines; *R. H. Golde (A '42).* Abstracted in the September 1945 issue, page 332.

Welding Forum in Detroit

A resistance welding forum, sponsored by the industrial power group of the AIEE Michigan Section in collaboration with the Industrial Electrical Engineering Society, an affiliate of the Engineering Society of Detroit, and the Saginaw Valley Division of the American Welding Society was held at the Engineering Society of Detroit on November 9, 1945. A feature of the meeting was a lobby display of welded parts by the Resistance Welder Manufacturers Association. More than 200 attended the afternoon and evening sessions at which J. D. Gordon of Progressive Welder Company presided.

At the afternoon session the papers presented were: "Application of Capacitors to Flash Welders," by J. F. Deffenbaugh (A '42), Federal Machine and Welder Company, Warren, Ohio; "Resistance Welders," by J. E. Ponkow, Federal Machine and Welder Company; "Design and Measurements of Condenser-Discharge Welding Transformers," by T. W. Dietz, Taylor Winfield Company, Warren, and G. M. Stein (M '38), Westinghouse Electric Corporation, Sharon, Pa., and "A New High-Speed Control for Gun Welders," by Frank Roby (M '41), Square D Company, Milwaukee, Wis.

The evening session was devoted to two papers. They were: "Standard Types of Electronic Welding Control," by E. H. Vedder (M '35), Westinghouse Electric Corporation, East Pittsburgh, Pa., and "Off-Standard Resistance Welding Control," by G. W. Garman (M '43), General Electric Company, Schenectady, N. Y.

ABSTRACTS • • • •

TECHNICAL PAPERS previewed in this section will be presented at the AIEE winter convention, New York, N. Y., January 21–25, 1946, and will be distributed in advance pamphlet form as soon as they become available. Copies may be obtained by mail from the AIEE order department, 33 West 39th Street, New York 18, N. Y., at prices indicated with the abstract; or at five cents less per copy if purchased at AIEE headquarters or at the convention registration desk.

Mail orders will be filled
AS PAMPHLETS BECOME AVAILABLE

Air Transportation

46-13—Pressure-Arc-Interruption Circuit Breakers for 400-Cycle Aircraft Electric-Power System; *B. O. Austin (M '43). 15 cents.* This paper describes a lightweight circuit breaker for use on 208Y–120-volt three-phase 400-cycle a-c system for large aircraft. The circuit breaker is operated electrically remotely controlled with latched-in contacts. The design is suitable for paralleling generators and busses, circuit interruption, and manual or automatic synchronization, and interlocks are provided for use in various control circuits. Operating solenoids are designed for low-current requirements and for wide ranges of voltages and ambient temperatures. The circuit breaker performs under extreme variations in environment and performs satisfactorily under all conditions of operation. The arc-interruption mechanism is of the self-pressurizing type, having two breaks per phase. Speed of closing the main contacts of the circuit breaker, and speed of opening are very rapid, and the mechanism is adaptable to the required speeds for proper circuit protection co-ordination.

46-14—Electrical Measurements on Aircraft-Engine Ignition Circuits; *W. E. Berkey (M '39). 15 cents.* A method of

Those present at the Southern District executive committee meeting held at Charlotte, N. C., October 27, 1945, were: W. J. Miller (F '43) and C. F. Sittloh (A '39)—Alabama Section; C. P. Almon, Jr. (A '37), W. O. Leffell (M '44), S. E. Lyons (A '37), and R. M. Alspaugh (M '41)—East Tennessee Section; C. W. Evans (M '40), J. M. Flanigen (M '25), G. F. Price (M '44), and R. O. Loomis (A '39)—Georgia Section; S. H. Gates (A '39), L. W. Anderson (A '32), and Brinkley Barnett (A '20)—Louisville Section; W. E. Lindemann (A '43), E. L. Biship (A '42), and R. C. Fuller (A '38)—Muscle Shoals Section; W. S. Leake (A '34), S. L. Kennedy (A '40), and A. D. Hargroder (A '43)—New Orleans Section; Otto Meier, Jr. (A '35), J. O. Kimbrey (A '44), W. J. Seeley (M '28), G. F. Bason (A '42), E. R. Davis (A '41), and H. W. Oetinger (M '35)—North Carolina Section; F. T. Tingley (M '31), J. L. Weeks (M '40), J. F. Rader (M '40), and D. H. Davis (A '40)—South Carolina Section; J. A. Rawls (M '27), C. H. Smoke (M '42), and J. H. Miller, Jr. (A '22)—Virginia Section; H. B. Wolf (M '37) vice-president, and C. B. Galphin (M '39) secretary, Southern District (4)

measuring ignition voltages with a cathode-ray oscillograph is described. Magneto and spark-plug voltage oscillograms covering several consecutive firing pulses were taken on a complete aircraft magneto ignition system under several engine operating conditions. Oscillograms taken with spark plugs firing in air show differences when compared with engine tests. An improved electronic oscillograph has been developed which is well adapted for ignition measurements.

46-1—All-Electric Gun Charging; *W. C. Rohn. 15 cents.* Power-operated charging means have been found desirable on remote automatic weapon installations for military aircraft. Chargers are used primarily to restore operation of a gun which has malfunctioned, but also may be used to set the weapon at safety or to stop a runaway gun. With the provision of suitable controls a charger can be made to function entirely automatically. Chargers have been predominantly fluid operated, whereas automatic or semiautomatic controls require electric power. An all-electric charger is considered which avoids the use of fluids and their associated installation and maintenance problems. The mechanism is mounted off the gun and connected by means of a flexible steel cable. This avoids transmission of the shock in firing to the charger unit and provides a maximum flexibility of installation. The design described will handle one weapon of 50-caliber or 20-millimeter size with fully automatic operation at a total apparatus weight of nine pounds. Two similar weapons may be served semiautomatically at a total weight of 11$^1/_2$ pounds.

46-3—Design Principles for A-C Electric Power Systems in Large Military Aircraft; *C. K. Chappuis (A '41), L. M. Olmstead (M '39). 15 cents.* This paper discusses the development of aircraft electric systems from the early days of aircraft development to superaircraft now in the design stage and defines the electric systems now in use and proposed. The primary discussion covers the basic design problems of 400-cycle three-phase electric systems for aircraft. Voltage control, load requirements, operational characteristics, types of power supply, equipment, protection, regulation, and d-c requirements are considered. A brief résumé of future trends for electric-power requirements in aircraft is included with figures as to power requirements of some aircraft now in design.

46-4—A Carbon-Pile Speed Governor; *C. T. Button (A '26). 15 cents.* The advantages of a centrifugally operated carbon pile for control of the speed of a d-c motor are discussed, and comparison is made with the problem of automatic voltage control of a generator. Though it is applicable for speed regulation of various types of d-c motors, the discussion centers around the use of such a device for controlling speed and hence output frequency of aircraft inverters, where an 8,000-rpm d-c motor drives a 400-cycle generator. The features of design of the governor are described in some detail, and the operating characteristics are covered in a general way.

46-15—Effect of Altitude on Ventilation and Rating of Aircraft Electric Machines; *C. G. Veinott (M '34). 15 cents.* Fundamental problems involved in the cooling and rating of aircraft electric machinery at medium and high altitudes are discussed in this paper, primarily to assist users in the application of such equipment and to assist builders in designing equipment to meet the requirements. In addition, flight tests are reported which confirm in part the blast-tube theory developed in a previous paper. For blast-cooled generators an equation has been developed to show the maximum permissible output in terms of entering-air temperature and density, ramming-head pressure, and winding temperature. For motors curves of total winding temperature at rated load have been plotted as a function of altitude; these curves were plotted for selected values of sea-level temperature rise. Specific conclusions are drawn as to:

1. Permissible generator output as a function of altitude and sea-level rise.
2. Limiting sea-level temperature rises for motors on the basis of constant or reduced life at altitude.

This paper presents a qualitative method of analysis rather than a definitive solution of the problems involved.

Communication

46-7—Electronic Regeneration of Teleprinter Signals; *H. F. Wilder (M '45). 15 cents.* The paper describes a teleprinter signal regenerator using an electric delay network to control the signal-scanning process, which with associated circuits for the start-and-stop functions provides a regenerator without mechanism. The composition of an illustrative teleprinter character and its susceptibility to interfering line currents are discussed. The development and the electrical characteristics of the delay network are described, and the regeneration of the signal character is explained.

46-10—The Solution of Transmission-Line Problems in the Case of Attenuating Transmission Line; *George Glinski (A '43). 15 cents.* The mathematical and graphical method of finding the receiving-end standing-wave ratio q_R from the measurement on the measuring section located far from the receiving end of attenuating line is presented. It is shown that the magnitude of receiving-end voltage-reflection coefficient K_R is a function of the total attenuation of the line between measuring section and receiving end and of the standing-wave ratio q_2 at the point where measuring section is inserted. Graphical method of finding the receiving-end impedance in the case of attenuating line is shown to be essentially the same as in the case of the lossless line, if a correction for the total attenuation is properly introduced.

Electric Machinery

46-5—Transients in A-C Motors and Generators; *George S. Smith (M '26). 15 cents.* This paper presents a method for obtaining a physical conception of the variations in currents, voltages, magnetic fields, and so forth, which occur in rotating a-c machinery during changes in operation. Its purpose is not to present an accurate method of determining such values but to offer a method for obtaining a clear conception of such variations, which should be of value to the operator of such machinery or to the engineer or student who must make use of the more accurate mathematical methods available. The method of arriving at the results is by properly combining the determinable components known to be present. In certain cases these components are identified in the mathematical results of previous workers. As examples of the use of this analysis, certain operations of induction motors, three-phase alternators, and single-phase alternators are analyzed. Oscillograms taken in the laboratory on similar machines are presented for comparison.

46-6—Temperature Rise of Water-Cooled Power Transformers; *J. R. Meador (M '40). 15 cents.* This paper contains an analysis of the oil temperature rise of water-cooled power transformers. It is shown that by factoring the effect of viscosity oil temperatures can be calculated more accurately for a variety of conditions. Equations and curves are given with supporting test data for determining the temperature rise for different loads, ingoing water temperatures, and rates of water flow. The variations in temperature rise with load, water rate, and water temperature are appreciably different from published data on the subject. A discussion of winding hot-spot temperature, factoring the effect of oil viscosity, is included, although no new test data are given.

Electric Welding

46-9—Measurement and Effect of Contact Resistance in Spot Welding; *R. A. Wyant. 15 cents.* Spot welding is the simplest of the electric resistance-welding processes which depend upon the Joule effect, $H=I^2RT$, for the generation of heat. The total resistance, R, consists of the body resistance of the metal between the electrodes plus the sum of the sheet-to-sheet and electrode-to-sheet contact resistances. Circuits are described for the measurement of these contact resistances before and during the formation of the weld. These measurements have been most useful in improving methods for the removal of oxide films from the surfaces of aluminum-alloy and magnesium-alloy sheet prior to spot welding. The welds are more uniform, and the electrodes tend to require less frequent cleaning, when the initial contact resistance is consistently low. Little is known about contact resistance and its effects in spot-welding other metals. The need for further research in this field is emphasized.

46-12—Investigation of Arc-Starting Characteristics of D-C Welders With Reference to Electrode Freezing; *J. M. Tyrner (A '41). 15 cents.* The starting of the arc in d-c welders was investigated. It was found that the starting performance is determined by the length of the time the operator may leave the electrode and work in contact. Generators should permit at least 40 milliseconds contact duration. Five generators of different designs were tested. Oscillograms of the current surge during the contact period were taken. It was found that the striking performance depends on the shape of the current surge.

Instruments and Measurements

46-2—The Measurement of Large Varying Currents; *A. C. Johnson (A '42). 15 cents.* In oscillographic studies where it is necessary to measure current in the range of from 5 to 100 kiloamperes, one often must resort to the use of shunts because:

(*a*). Addition of a current transformer may change materially the impedance of the circuit.

(*b*). Current transformers for large currents are unwieldy.

(*c*). The current may be pulsating direct current, which would saturate a current transformer and render it useless.

It is extremely important that the shunts used be carefully designed and so connected into the circuit that the measuring-circuit current is proportional to and inphase with the shunt current. The paper includes an analysis and a basis for the design of shunts for measuring these currents, so that there will be no voltage induced in the measuring circuit for any of the frequencies of harmonic components normally encountered in the measuring of current of these magnitudes. Consequently, the measuring-circuit current will be a function only of a shunt current and the ratio of the resistance of the shunt to that of the measuring circuit.

Power Transmission and Distribution

46-8—Formulas for Conductor Size According to Cost of Resistance Loss; *H. B. Dwight (F '26). 15 cents.* In designing electric conductors, especially for overhead lines, it is often advisable to choose the size of conductor according to the cost of resistance loss. When the cost of reducing the loss at peak load by increasing the cross section of the conductor becomes equal to the cost of producing additional power, the point at which this occurs is given by a fixed value of current density. Formulas for determining the most economical conductor size in the foregoing way are given in this paper. These are designed to make the determination more convenient and easy. Formulas are given for aluminum conductors and for conductors of varying cost per pound, such as insulated conductors. The formulas indicate the advisability of taking account of the cost of power in choosing a conductor size. The marked difference between the cases where the conductors are owned by the company which generates the power and the cases where energy is purchased by the kilowatt-hour is brought out. It is shown that this application of Kelvin's law, for that is what the method really amounts to, does not involve uncertainty in most cases, but is a simple definite engineering problem based on data that are usually known. To specify heavier conductors, at improved economy, by these formulas does not mean a larger investment by power companies which have plant extensions in prospect. The matter is a choice in engineering design, whether a certain fraction of the investment should be made in conductors or in generator-station capacity.

Protective Devices

46-11—A New Submersible Network Protector of Higher Rating; *G. G. Grissinger (M '38), F. D. Johnson (A '41). 15 cents.* Although network protectors having a continuous ampere rating of 3,000 amperes 60 cycles have been available for several years, they are of the open type for use in dry vaults and therefore not suitable for submersible service. This paper outlines the problems involved and the methods used in developing a 3,000-ampere submersible network protector. Since a submersible network protector is practically airtight, the watts loss in the protector must be kept to a minimum, and special features must be incorporated to dissipate the generated heat if temperatures are to be limited to satisfactory operating values. There being no known means available for making reasonably accurate measurements of watts loss at very low voltages, a special test circuit was devised. The principle of operation of this circuit is described. Once how much heat energy has to be dissipated is known and the temperature differential available, it becomes a matter of radiation, convection, and conduction. The methods adopted for making use of these factors in a practical way are discussed.

PERSONAL • • • • •

G. Ross Henninger (A '22, F '43) AIEE editor-on-leave who has been serving as a lieutenant colonel in the Army Air Forces, has been released from active duty and has returned to the staff of *Electrical Engineering.* A member of AIEE headquarters staff since 1930, and editor since 1933, he was appointed a lieutenant colonel in the Army Specialist Corps in 1942 to serve as a liaison officer on the headquarters staff of the Corps at Washington, D. C., assisting the Signal Corps and the Air Corps with their technical-personnel requirements. In the latter part of that year he was commissioned a lieutenant colonel in the Army of the United States and transferred to the Army Air Forces with immediate assignment to the then newly established Air Service Command Headquarters at Patterson Field, Dayton, Ohio. As chief of the reproduction branch he initially was responsible for the procurement and development of printing and other facilities required for the reproduction of technical data for the Army Air Forces, but in June 1943 he was reassigned as assistant chief of the Maintenance Data Section serving as executive officer in the administration of the section's nine branches. In October 1944 he became chief of the section and a member of the staff of the chief of the Maintenance Division of what had by that time become the Headquarters, Air Technical Service Command, through consolidation of the original Air Service and Matériel Commands. His expanded duties included direct responsibility for procurement, preparation, publication and dissemination of the technical information required for the operation, maintenance, and repair of all equipment used by the Army Air Forces. Prior to entering into active military service in 1942, Colonel Henninger had devoted a substantial amount of his time for several months to the United States Navy Department in Washington in the organization of an editorial and review section in the office of the Secretary of the Navy for the co-ordination of Navy publications. **W. R. MacDonald, Jr.** (A '33, M '44) AIEE assistant editor who has been serving recently as acting editor (*EE, Apr '45, p 160*) has been appointed associate editor.

G. R. Henninger

C. E. Ide (A '16, M '38) formerly president, East Tennessee Light and Power Company, Bristol, has been elected executive vice-president and general manager of the Toledo (Ohio) Edison Company. Mr. Ide was born in Brayton, Iowa, and was graduated from Iowa State College in 1914 with the degree of bachelor of science in electrical engineering. He started in with the Cities Service system working first with the Denver Gas and Electric Light Company, and later with the Toledo Railways and Light Company, in the New York, N. Y., office of Henry L. Doherty and Company and the Meridian (Miss.) Light and Railway Company. During World War I he was a member of the science and research division of the Signal Corps of the United States Army, returning to the Doherty company and remaining for the next eight years. In 1927 he became vice-president and general manager of the East Tennessee Light and Power Company, and in 1940 became president. **H. H. Kerr** (A '17, M '26) formerly general superintendent, engineering and operation, Toledo Edison Company, becomes vice-president in charge of operations. Mr. Kerr was born in Fort Worth, Tex., and was graduated from the University of Colorado in 1914 with the degree of bachelor of science in electrical engineering. After graduation he spent one year in the test course of the General Electric Company, Schenectady, N. Y. Then after 14 years service with the Public Service Company of Colorado he transferred to Toledo as superintendent of the electrical operating department of the Toledo Edison Company. He was named general superintendent of engineering operation, maintenance, and construction in 1943.

A. L. Pollard (M '40) formerly general superintendent of light and power, Puget Sound Power and Light Company, Seattle, Wash., has been appointed general superintendent of operations. This assignment together with his duties under the previous assignment will add the task of general supervision of the generating and transmission system. Mr. Pollard was first employed by Stone and Webster as student engineer with the Houghton County Electric Light and Power Company, Houghton, Mich. After service with the United States Army in World War I he returned to Stone and Webster working in Pawtucket, R. I., Tampa, Fla., El Paso, Tex., and for two years previous to his arrival in Seattle was power engineer for Stone and Webster Engineering Corporation, Boston, Mass. Mr. Pollard went to the Puget Sound Power and Light Company in May 1930 as superintendent of steam generation. In June 1934 in addition to his previous duties he was made superintendent of light and power of the central division, and two years later became general superintendent of light and power for the entire company. He continued in this capacity until his recent appointment.

William Henry Harrison (A '20, M '30 F '31) has been elected to receive the Hoover Medal for 1945 (*EE Nov '45, p 421*). General Harrison recently was elected vice-president in charge of operation and engineering, American Telephone and Telegraph Company, New York, N. Y., a position he held prior to 1940 when he went on leave of absence to head the production division of the War Production Board. He left the latter organization in 1942 when commissioned a colonel in the United States Army. Very soon after entering active duty he was promoted to brigadier-general and was given various assignments incident to Army production requirements. General Harrison was born in Brooklyn, N. Y., and was graduated in electrical engineering from Pratt Institute. He entered the employ of the New York Telephone Company in 1909. His work included apparatus inspection, assembling, and wiring. From 1915 until 1919 he was engaged in engineering department of the Western Electric Company, and in the latter year became a member of the engineering staff of the American Telephone and Telegraph Company. Five years later he was made equipment and building engineer. In 1929 General Harrison was appointed plant engineer with broadened responsibilities covering the general direction of the engineering, design, and layout of all phases of the Bell system plant. In 1933 he was elected vice-president in charge of operations of the Bell Telephone Company of Pennsylvania and the Diamond State Telephone Company of Delaware with headquarters in Philadelphia, Pa. In 1937 he was appointed assistant vice-president, department of operation and engineering, American Telephone and Telegraph Company. General Harrison is a past president (1937–38) of the Institute, was a member of the board of directors (1935–40), and has served on many AIEE committees including the following: technical program, 1929–36, chairman, 1931–33; publication, 1931–33; communication, 1935–36; award of Institute prizes, 1931–34, chairman 1931–33; co-ordination of Institute activities, 1931–33, and executive, 1936–37.

W. H. Harrison

George W. Garman (A '38, M '43) control division, General Electric Company, Schenectady, N. Y., has been appointed 1945–46 chairman of the AIEE committee on electric welding. Mr. Garman was born in South Bend, Ind., in 1903, and was graduated from the University of Washington with a bachelor-of-science degree in electrical engineering in 1926, and a master-of-science degree from Union College in 1933. He started in the test department of the General Electric Company and has been with the company since that year. For an interval prior to 1940 he spent part of his time in the electronics department working on the design and development of various electronic control equipment, and with the voltage-regulator engineering division, on the design and development of various electronic voltage regulators. Subsequently, he was transferred to the industrial control engineering division where he was engaged in the design and development of electronic motor controls and resistance-welding controls. He holds 24 patents covering equipments of this type. Since 1940 he has been in the control division and has been responsible for the sale and application of electronic resistance-welding controls. Mr. Garman is a member of the American Welding Society, and a member of several of its committees. He was a member of the American Standards Association war standard committee *C*-52, and chairman of the American Standards Association war standard subcommittee *C*-52.4 and *C*-52.5.

James De Kiep (M '43) is now chief engineer in charge of electrical and mechanical design and development of the Electric Machinery Manufacturing Company, Minneapolis, Minn. As stated previously (*EE, Nov '45, p 415*) he was formerly manager of the a-c motor engineering department of the Westinghouse Electric Corporation, East Pittsburgh, Pa.

J. A. Potts (A '25, M '36) general engineer, executive department, Wisconsin Electric Power Company, Milwaukee, has been elected 1945–46 president of the Engineers' Society of Milwaukee. Mr. Potts was graduated from the University of Wisconsin with a bachelor-of-science degree in electrical engineering. He was associated with Wisconsin utilities for many years and was chairman, Milwaukee Section, AIEE, 1936–37, and that Section's secretary, 1933–34. **V. R. Tate** (A '28, M '38) vice-president, secretary and attorney, Perfex Corporation, Milwaukee, was elected a director. He was graduated from the Milwaukee School of Engineering with the degree of bachelor of science in electrical engineering in 1927. After a period of service with Milwaukee Engineering and Manufacturing Company, Radio Corporation of America, New York, N. Y., Time-O-Stat Controls Company Elkhart, Ind., and Minneapolis-Honeywell Regulator Company, Minneapolis, Minn. He became associated with the Perfex Corporation in 1934.

G. W. Garman

W. C. Kalb (A '12, F '40) formerly commercial engineer, sales department, National Carbon Company, Cleveland, Ohio, is now manager, brush-service engineering department of the same company. Mr. Kalb was born in Ohio, and was graduated from Ohio State University with the degree of mechanical engineer in electrical engineering. He entered the employ of the National Carbon Company as electrical engineer in 1907, and returned to it in 1911 after a short period with Dunlap Engineering Company, Columbus, Ohio, as general superintendent. In 1913 he was made sales engineer of the National Carbon Company. Six years later he joined the Corliss Carbon Company, Bradford, Pa., as managing engineer and when the company was purchased by the National Carbon Company, Mr. Kalb remained as superintendent of the Corliss plant. He has been commercial engineer since 1940. He holds various patents on brushes and has written many technical articles. Mr. Kalb has been very active in AIEE affairs and has held membership on many committees. They include the following: industrial and domestic power, 1918–22; electrochemistry and electrometallurgy, 1931–34 (chairman 1932–34) and 1944–45; technical program, 1932–34, 1943–45, and Standards, 1943–45.

L. C. F. Horle (A '20, F '35) radio consulting engineer, New York, N. Y., has been made chief engineer, engineering department Radio Manufacturers Association, Washington, D. C. Mr. Horle is a graduate of the Stevens Institute of Technology, Hoboken, N. J., and from 1914 until 1916 was a member of the institute faculty. From 1916 to 1917 he was employed by the Public Service Corporation of New Jersey, Newark. Later positions were with the radio laboratory of the Navy Yard, Washington, D. C.; the DeForest Radio Telephone and Telegraph Company, New York, N. Y.; and the National Bureau of Standards, Washington. Mr. Horle became a consulting engineer in 1932.

H. J. Reich (A '32, M '43) formerly special research associate, radio-research laboratory, Harvard University, Cambridge, Mass., has been appointed professor of electrical engineering, Yale University, New Haven, Conn., effective January 1, 1946. Doctor Reich was born on Staten Island, N. Y., October 25, 1900, and received a mechanical-engineer degree from Cornell University in 1924, and a doctor-of-philosophy degree in physics in 1929. After serving as an instructor of physics 1924–29 at Cornell University, he became assistant professor of electrical engineering at the University of Illinois, Urbana, and later professor of electrical engineering. On January 1, 1944, he was granted a leave of absence to join the staff of the radio-research laboratory at Harvard University. He is the author of several textbooks and some 40 technical papers covering electron tubes, tube circuits, and related subjects. He is past chairman (1936–37) of the Urbana Section, AIEE. Doctor Reich is also a senior member of the Institute of Radio Engineers, a member of American Physical Society, American Association for the Advancement of Science, and Society for the Promotion of Engineering Education.

E. U. Condon (M '44) formerly associate director of research, Westinghouse Electric Corporation, East Pittsburgh, Pa., has been appointed in the capacity of a director of the National Bureau of Standards, Washington, D. C. Doctor Condon was graduated from the University of California with a bachelor-of-arts degree in 1924 and a doctor-of-philosophy degree in 1926. From 1928 to 1937 he was associate professor of physics Princeton (N. J.) University and while there wrote a number of papers and two books on quantum mechanics and the theory of atomic spectra. In 1937 he became affiliated with the research laboratories of the Westinghouse company, as associate director. In this capacity he has had charge of fundamental research including the microwave radio and radar program. From 1940 to 1943 Doctor Condon was chairman of a company committee on radar which had the responsibility of building up the research and development program of the company.

B. F. DeLanty (A '21, M '34) former general manager and chief engineer, Pasadena (Calif.) Municipal Light and Power Department, has retired. Mr. DeLanty was born in New Ungeness, Wash., and was educated at Whitworth College. In 1899 he became identified with the Prescott (Ariz.) Electric Company remaining until 1907 when he accepted a position with the City of Pasadena as superintendent of construction for the Light and Power Department. He subsequently became general manager and chief electrical engineer. **E. L. Bettannier** (A '11, M '26) formerly chief electrical engineer, and assistant general manager has been named general manager and chief engineer. Mr. Bettannier was born in Corning, Ia., and was educated at Throop College, now the California Institute of Technology, and the Van Der Naillen School of Engineering. In 1909 he joined the Pacific Light and Power Company, Los Angeles, Calif., but a short time later was employed by the Municipal Light and Power Department, City of Pasadena. Subsequently, he became junior electrical engineer.

P. H. Crago (M '43) railroad representative, general commercial department, Federal Telephone and Radio Corporation, Newark, N. J., has been made supervisor of the New York, N. Y., branch of Wilmotte Laboratory, Inc., Washington, D. C., which provides industrialists with engineering services. Mr. Crago was graduated from the Carnegie Institute of Technology with a bachelor-of-science degree in physics in 1922. For three years he was employed by the Nachod Signal Company, Louisville, Ky., and in 1925 became associated with the Union Switch and Signal Company, Swissvale, Pa. **G. C. Engel** (A '41) formerly production manager of General Electronics, Inc., and manager of the laboratory of General Time Instruments Corporation, New York, N. Y., will be associated with Mr. Crago. Mr. Engel was graduated from Stevens Institute of Technology in 1927 as mechanical engineer. In 1927 he joined the Bell Telephone Laboratories, New York, N. Y., and later was employed by Thomas A. Edison, Inc., West Orange, N. J., and the Sylvania Corporation, Clifton, N. J.

Frederick Krug (A '17, F '36) general manager and director, Montreal (Quebec, Canada) Engineering Company, Ltd., was recently elected in addition to his present office a director of the International Power Company, Ltd., and vice-president and director of the Monterey Railway, Light and Power Company, southern properties of the Montreal Engineering Company. Mr. Krug studied at Cooper Union and the New Mexico School of Mines. He has been identified with several companies and at one time was assistant superintendent, electrical department, New York and Honduras Rosario Mining Company, San Juancito, Honduras, Central America, and superintendent of power production for the Puerto Rico Railway, Light and Power Company, San Juan.

R. D. Birge, Jr. (A '39) formerly application engineer, The Louis Allis Company, Philadelphia, Pa., has been appointed Detroit, Mich., representative, storage-battery division, Philco Corporation. Mr. Birge was graduated from Purdue University in electrical engineering in 1938. He has been associated with the Louis Allis Company in Detroit and Philadelphia for four years.

E. B. Curdts (A '25, M '29) superintendent, substation maintenance, Narragansett Electric Company, Providence, R. I., has joined the staff of James G. Biddle Company, Philadelphia, Pa., as application engineer. Mr. Curdts became associated with the Providence company 16 years ago. Prior to that he served the Virginia Railway and Power Company, Petersburg, and its successor, the Virginia Electric Power Company.

C. O. Bickelhaupt (M '22, F '28) vice-president, American Telephone and Telegraph Company, New York, N. Y., and recently brigadier general, Army of the United States, has been elected to the additional office of secretary of the afore-mentioned organization. General Bickelhaupt became associated with the American Telephone and Telegraph Company in 1911 when he was employed as toll-traffic engineer and commercial engineer. In 1930, after an interval with the Cumberland Telephone Company and the Southern Bell Telephone and Telegraph Company, Atlanta, Ga., he returned to the American Telephone and Telegraph Company as assistant vice-president and later was named vice-president. In 1941 he was called to active duty with the Army and in 1944 was promoted to the rank of brigadier general.

C. L. Davis (A '29, M '37) formerly manager, Washington, D. C., office, patent department, Radio Corporation of America, now is engaged in the general practice of law in Washington and will continue to specialize in matters relating to patents and research. Mr. Davis attended the Iowa State College, George Washington University, and National University. He holds the degrees of bachelor and master of laws. From 1923 to 1925 he served in the Signal Corps, United States Army, as radio engineer. During the next three years he was radio engineer and part owner of radiobroadcasting station WJAF, Detroit, Mich. Before organizing the Washington office for the Radio Corporation of America he was instructor in radio at the School of Engineering, Milwaukee, Wis.

J. B. Coleman (A '25) chief engineer, special apparatus engineering division, Victor division, Radio Corporation of America, has been appointed assistant director of engineering for the Victor division with offices at Camden, N. J. Mr. Coleman was educated at the Carnegie Institute of Technology. He was employed by that institute as radio instructor, and later became associated with the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa. He joined the Radio Corporation of America in 1930.

OBITUARY • • • • •

Leslie LaMonde Dyer (A '10, M '37) for 37 years prior to his retirement in 1939, affiliated with the Southern California Edison Company, Ltd., and its predecessors, died August 24, 1945. Born November 22, 1878, in Abbot, Ia., Mr. Dyer first became affiliated with the San Gabriel Electric Company, Los Angeles, Calif., as assistant dynamo tender in September 1902 and later became switchboard operator at the company's Third Street substation. Subsequently he became chief operator at the Redondo Steam Plant, and in 1914 was made superintendent of the company's southern division. When in 1917 the systems of the Pacific Light and Power Company and the then Edison Electric Company were merged, Mr. Dyer was appointed assistant superintendent of transmission of the consolidated organization which was known as the Southern California Edison Company. Five years later he became superintendent of substation operation for the entire Edison system, a position which he held until his retirement in 1939. A Navy Spanish-American war veteran, Mr. Dyer was unable to restrain himself in retirement under the pressing manpower demands of World War II, and went on active duty in July 1941 as a civilian electrical inspector for the United States Navy at a Terminal Island (Calif.) shipyard, where he remained until July 1944, when failing health overtook him.

Jesse Robert Lovejoy (A '91, M '94, F '13) honorary vice-president, General Electric Company, Schenectady, N. Y., died October 31, 1945. Mr. Lovejoy was born November 10, 1863, at Columbus, Ohio, and was graduated from Ohio State University in 1884 with the degree of bachelor of science. He began work in the Thomson-Houston Electric Company at Lynn, Mass., in 1886 and the following year was appointed construction superintendent. With the formation of the General Electric Company in 1892 Mr. Lovejoy was appointed manager of the supply department. In 1900 he was placed in charge of the railway and lighting department, and seven years later became general sales manager when he was elected a vice-president. As vice-president in charge of sales he was active in the development of business throughout the company's foreign department. Subsequently, he was elected president of the Mexican General Electric Company, the South African General Electric Company, the Australian General Electric Company and the South American General Electric Company. In 1928 he was elected an honorary vice-president of General Electric Company.

Bertrand Perry Rowe (A '03, M '05, F '13) retired, Westinghouse Electric Corporation, East Pittsburgh, Pa., died September 21, 1945. He was born in Ithaca, N. Y., September 2, 1868, and was graduated from Cornell University in 1892 with the degree of mechanical engineer in electrical engineering. Early in his career he worked for the Short Electric Railways Company, Cleveland, Ohio, and for a short period was with the General Electric Company, New York, N. Y. Later he was associated with the Creston Colorada Mining and Milling Company, Sonora, Mexico, and then accepted a position on the engineering staff of the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa. He then served as general engineer and acted as a consulting engineer. During World War I he was assigned to remain at the Westinghouse works to prepare plans, specifications, estimates and projects and for the Government plants in Navy yards and munition plants. For five years before his retirement at the age of 65 he served as consulting engineer for the Los Angeles, (Calif.) district under the engineering department of that district. Mr. Rowe was a pioneer in the designing and installation of power plants, among them the Ontario Power Company, Niagara Falls, N. Y.

William Russell Cone Corson (A '93) chairman, board of directors, Hartford (Conn.) Steam Boiler Inspection and Insurance Company, died October 2, 1945. Mr. Corson was born in New York, N. Y., in 1870 and was graduated from Yale University with the degree of bachelor of arts in 1891. Shortly after graduation he entered the shops of the Eddy Electrical Manufacturing Company, Windsor, Conn., later becoming assistant electrical engineer, superintendent, and secretary. In 1902 he established a consulting office in Hartford, and five years later joined the Hartford Steam Boiler Inspection and Insurance Company as assistant engineer. In 1909 he was appointed assistant secretary, and secretary and treasurer in 1916. In 1921 he was made vice-president and treasurer and in 1927 he was elected president. He was elected chairman of the board in 1942.

Arthur A. Bolsterli (A '23, M '32) research engineer, Moloney Electric Company, St. Louis, Mo., died September 25, 1945. Mr. Bolsterli was born in Lucerne, Switzerland, in 1892, and was graduated in 1918 from the Zurick Polytechnic Institute with degrees in electrical and mechanical engineering. He had been associated with Oerlikon Company in England; Brown Boveri, Halfely Company; A. G. and Micafil in Switzerland; American Brown-Boveri Western Electric Company, Safety Car Heating and Lighting Company, Roller Smith Company and Moloney Electric Company in the United States. Mr. Bolsterli was the author of several technical papers. He was a member of the Institution of Electrical Engineers, England, and the Schweizerischer Elektrotechnischer Verein.

George Ellis Hutchinson (A '11, M '19) formerly chief engineer in charge of safety work, Utilities Mutual Insurance Company, New York, N. Y., died October 20, 1945 in Rochester, N. Y. Mr. Hutchinson was born in New York June 25, 1887, and was graduated from Columbia University in 1909 with the degree of electrical engineer. He was first associated with the Crocker Wheeler Company, Ampere, N. J. as draftsman, later becoming assistant engineer of the a-c design department. Later he was with the New York Edison Company. Mr. Hutchinson was with the National Carbon Company, Cleveland, Ohio, from 1920 to 1922, and in the latter year went with the Utilities Mutual Insurance Company as a safety engineer. During World War I he served as a lieutenant in the United States Navy.

MEMBERSHIP • • •

Recommended for Transfer

The board of examiners, at its meeting of November 15, 1945, recommended the following members for transfer to the grade of membership indicated. Any objections to these transfers should be filed at once with the secretary of the Institute.

To Grade of Fellow

Eastham, M., chief engr., General Radio Co., Cambridge, Mass.
Hamdi, A. F., director general, dept. of elec. research, Ministry of Public Works, Ankara, Turkey.
Lassen, E. U., asst. chief engr., Cutler-Hammer, Inc., Milwaukee, Wis.
Ryan, F. M., radio engr., American Tel. & Tel. Co., New York, N. Y.
Scholz, H. J., supervisory engr., Commonwealth & Southern Corp., Birmingham, Ala.
Shanklin, G. B., engr. in charge, cable div., General Electric Co., Schenectady, N. Y.
Stevenson, J. R., director, elec. div., Memphis Lt., Gas & Water Div., Memphis, Tenn.
7 to grade of Fellow

To Grade of Member

Blomqvist, E. A., asst., administration applied physics lab., Johns Hopkins Univ., Silver Spring, Md.
Bork, M. J., Jr., engr., Anaconda Wire & Cable Co., Hastings-on-Hudson, N. Y.
Brown, E. E., engr., Philadelphia Electric Co., Philadelphia, Pa.
Davis, E. R., industrial pr. engr., Duke Power Co., Charlotte, N. C.
Davis, S. W., div. engr., Southwestern Public Service Co., Amarillo, Tex.
Dexter, H. W., Jr., director, pr. & steam utilization div., Duquesne Lt. & Allegheny County Steam Heating Cos., Pittsburgh, Pa.
Elmer, W. B., section engr., Westinghouse Electric Corp., Cleveland, O.
Ellestad, I. M., trans. engr., Northwestern Bell Tel. Co., Omaha, Nebr.
Frakes, J. H., elec. engr., Westinghouse Electric Corp.. E. Pittsburgh, Pa.
Franks, S. T., elec. engr., Canadian & General Finance Co., Toronto, Ont., Can.
Grosser, G. E., switchgear specialist, Westinghouse Electric Corp., Philadelphia, Pa.
Keeler, O. E., switchgear engr., Westinghouse Electric Corp., San Francisco, Calif.
MacCallum, A. F., maint. engr., Westinghouse Electric Corp., Houston, Tex.
McDonough, M. E., ship elect. foreman, Federal Shipbuilding & DryDock Co., Port Newark, N. J.
Michener, H. P., Jr., asst. elec. engr., Underwriters' Laboratories, Inc., New York, N. Y.
Peatfield, R. R., supt., elec. engg. dept., Clinton Engineer Works, Tennessee Eastman Corp., Knoxville, Tenn.
Rankin, M. B., supervisory engr., Ohio Edison Co., Akron, O.
Schaefer, L. P., elec. engr., Naval Ordnance Lab., Navy Yard, Washington, D. C.
Smith, M. J., instructor in elec. engg., Rice Institute, Houston, Tex.
Trekell, H. E., asst. des. engr., meter div., General Electric Co., West Lynn, Mass.
Van Kleeck, R. E., sr. engr., Philadelphia Electric Co., Philadelphia, Pa.
York, J. W., design engr., motor div., Westinghouse Electric Corp., E. Pittsburgh, Pa.
22 to grade of Member

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. Any member objecting to the election of any of these candidates should so inform the secretary before January 15, 1946, or March 15, 1946, if the applicant resides outside of the United States or Canada.

To Grade of Member

Adams, E. F. (Re-election), Weston Elect. Inst. Corp., Newark, N. J.
Brooks, H. (Re-election), Westinghouse Elec. Corp., East Pittsburgh, Pa.
Brown, H. H., Toulmin & Toulmin, Dayton, Ohio
Buchanan, J. C., Tennessee Valley Authority, Knoxville
Burney, C. W., The Austin Co., Cleveland, Ohio
Carter, W. R., Electric Machinery Mfg. Co., Minneapolis, Minn.
Cheng, C. Y., National Resources Comm. of China, Chungking, China
Crego, V., Republic Steel Corp., Northern Ore Mines, Duluth, Minn.
Cutbill, H. W. (Re-election), Consolidated Edison Co. of N. Y., Inc., New York, N. Y.
Debes, C. N., Charles N. Debes & Asso., Rockford, Ill.
DeGuise, Y., Quebec Hydro, Montreal, Que., Can.
Elam, E. R., Kiefer Electric Co., Peoria, Ill.
Fingerman, S., Jr. (Re-election), U. S. Bureau of Reclamation, Denver, Colo.
Forostovsky, O. J., Western Electric Co., Kearny, N. J.
Fuchs, N. Y., Westinghouse Elec. Int'l., New York, N. Y.
Gilmore, H. S., Laramore & Douglass, Inc., Chicago, Ill.
Hanson, A. N., Western Electric Co., Chicago, Ill.
Hotchkiss, G. (Re-election), Western Union Telegraph Co., New York, N. Y.
Irvine, T. F. (Re-election), Federal Tel. & Radio Corp., Newark, N. J.
Joslin, W. M., Public Service Co. of Northern Ill., Maywood, Ill.
Kern, B. B., Penna. Power & Light Co., Shenandoah, Pa.
Madgett, J. P., Jr., Central Nebr. Pub. Pr. & Irrig. Dist., Hastings, Nebr.
Mathews, L. E. (Re-election), Bureau of Reclamation, Salt Lake City, Utah
Mead, R. S., Transcontinental & Western Air, Inc., Kansas City, Mo.
Naylor, J. T., Associated Telephone Co., Ltd., Santa Monica, Calif.
Nelsen, M. G., Guardian Elec. Mfg. Co., Chicago, Ill.
Nichols, C. R., Lt. Col., U. S. Military Academy, West Point, N. Y.
Nickerson, C. W. (Re-election), Broadway Maintenance Corp., Long Island City, N. Y.
Parker, J. P., McLellan & Partners, London, England
Payne, J. B., H. N. Roberts Consulting Engr., Lubbock, Tex.
Peters, A. W., Shawinigan Water & Power Co., Three Rivers, Que., Can.
Peterson, E. S., Automatic Elec. Co., Chicago, Ill.
Polk, O. H., Lt. Cdr., USNR (Re-election), U. S. Naval Academy, Annapolis, Md.
Price, E. P., Price & Hawk, Lubbock, Tex.
Reynolds, J. N., Crescent Insulated Wire & Cable Co., Trenton, N. J.
Ryan, J. E., General Electric Co., Schenectady, N. Y.
Salmon, J. T., Commonwealth Edison Co., Chicago, Ill.

Schairer, E. G., Armstrong Cork Co., Lancaster, Pa.
Shevelenko, D., G. M. Laboratories, Inc., Chicago, Ill.
Stafford, J. K. (Re-election), General Electric Co., Pittsfield, Mass.
Tuttle, A. S., International Nickel Co. of Canada, Ltd., Toronto, Ont., Canada
Van Nostrand, E. S., Southwestern Public Service Co., Amarillo, Tex.
Waggoner, W. R., The Commonwealth & Southern Corp., Birmingham, Ala.
Weiss, O., Charles M. Terry Pty. Ltd., Sydney, N.S.W., Australia

44 to grade of Member

To Grade of Associate

United States and Canada

1. North Eastern

Bacon, R. E., Holyoke Water Pr. Co., Holyoke, Mass.
Coupal, E. T., Central N. Y. Power Corp., Syracuse, N. Y.
Dana, D. W., General Electric Co., Lynn, Mass.
Fisk, D. B., General Electric Co., West Lynn, Mass.
Fiske, R. T., Westinghouse Electric Corp., Boston, Mass.
Hrabchak, E. W., General Electric Co., New Haven, Conn.
Johnson, C. L. (Re-election), Hart Mfg. Co., Hartford, Conn.
Kihlstrom, H. C., Westinghouse Elec. Corp., Buffalo, N. Y.
Lemelin, J. W., General Elec. Co., New Haven, Conn.
Lewis, J. B., Jr., General Elec. Co., Bridgeport, Conn.
Murdock, H. W., General Elec. Co., Boston, Mass.
Perkins, S. D., Westinghouse Electric Corp., Providence, R. I.
Peterson, W. C., General Electric Co., Pittsfield, Mass.
Pierce, G. A., Hartford Steam Boiler Inspection & Insurance Co., Boston, Mass.
Quinn, Q. Q., Conn. Light & Power Co., Waterbury, Conn.
Reynolds, H. E., Eastman Kodak Co., Rochester, N. Y.
Russell, J. S., Jr., General Electric Co., Schenectady, N. Y.
Smith, J. C., General Elec. Co., Buffalo, N. Y.
Smith, M. E., Oakes Electrical Companies, Holyoke, Mass.
Speed, D. K., Mass. Inst. of Tech., Cambridge, Mass.
Swenson, C. A., Curtiss-Wright Research Lab., Buffalo, N. Y.
Tanner, H. A. J., Submarine Signal Co., Boston, Mass.
Wells, W. H. (Re-election), The Connecticut Light & Power Co., Waterbury, Conn.
Williams, M., Rome Cable Corp., Rome, N. Y.
Wilson, P. H., General Elec. Co., Lynn, Mass.
Yoffe, B., U. S. Naval Drydock, South Boston, Mass.

2. Middle Eastern

Adams, W. D., USAAF, Wright Field, Dayton, Ohio
Batchelor, A. J. (Re-election), Toledo Edison Co., Toledo, Ohio
Behm, H. J., Delco Products Div., GMC, Dayton, Ohio
Center, C. E., Westinghouse Elec. Corp., East Pittsburgh, Pa.
Douds, R. A., Dravo Corp., Pittsburgh, Pa.
Durboraw, I. N., Jr., Atlas Powder Co., Wilmington, Del.
Fox, F. M., Reliance Elec. & Engr. Co., Cleveland, Ohio
Fuller, L. (Re-election), Westinghouse Elec. Corp., East Pittsburgh, Pa.
Gemmell, W. H., Philadelphia Electric Co., Philadelphia, Pa.
Gilbert, H. R., Cates & Shepard, Philadelphia, Pa.
Griffin, H. C., Pa. Power & Light Co., Harwood Mines, Pa.
Hanna, E. E., Imperial Electric Co., Akron, Ohio
Hensler, J. F., The Electric Storage Battery Co., Pittsburgh, Pa.
Hess, F. P., Pennsylvania Electric Co., Philadelphia, Pa.
Holdman, M., Ens., USNR, Naval Research Lab., Washington, D. C.
Kleinbach, W. S., Philadelphia Electric Co., Philadelphia, Pa.
Kuhar, W. A., Roller-Smith Co., Allentown, Pa.
McIntyre, R., General Electric Co., Erie, Pa.
Niedurny, W. J. (Re-election), Philadelphia Elec. Co., Philadelphia, Pa.
Perry, P. B., Cleveland Electric Illuminating Co., Cleveland, Ohio
Rickman, A. A., E. C. Ernst, Inc., Washington, D. C.
Rile, W. S., Bell Telephone Co. of Pa., Philadelphia, Pa.
Ryan, E. J., Philadelphia Elec. Co., Philadelphia, Pa.
Schommer, A. H., H. Fletcher Brown Vocational High School, Wilmington, Del.
Sckerl, A. M., U. S. Engineer Office, Cincinnati, Ohio
Shaprow, W. H., Standard Gas Equip. Corp., Baltimore, Md.
Shugars, J. G., Mutual Boiler Insurance Co. of Boston, Philadelphia, Pa.
Small, R. W., Philadelphia Electric Co., Philadelphia, Pa.
Thomas, C. C. (Re-election), Cincinnati Gas & Elec. Co., Cincinnati, Ohio
Wagner, H. A., Reliance Elec. Engineering, Cleveland, Ohio
Weaver, D. F., Jr., Potomac Electric Power Co., Washington, D. C.
Williams, J. R., Naval Research Lab., Washington, D. C.
Witte, F. R., U. S. Coast Guard Hqs., Washington, D. C.
Yerger, J. F., Lehigh Valley Railroad, Bethlehem, Pa.
Zimmerman, E. W., Reliance Electric & Engineering, Cleveland, Ohio

3. New York City

Arany, D. M., Federal Telephone & Radio Corp., Newark, N. J.
Ayer, G. R., Western Electric Co., Inc., New York, N. Y.
Baird, R. M., Jr., Westinghouse Electric Corp., Newark, N. J.
Bonthron, R. C., Westinghouse Elec. Int'l. Co., New York, N. Y.
Boyce, E. W. (Re-election), Smith Hinchman & Grylls, Inc., New York, N. Y.
Brennan, T. J., Western Electric Co., New York, N. Y.
Denike, A. R., J. G. White Engineering Corp., New York, N. Y.
Digirolanio, J. V. M., The Lionel Corp., Irvington, N. J.
Faber, S. S., Liquidometer Co., New York, N. Y.
Greene, R. B., American Transformer Co., Newark, N. J.
Kelly, J. T., War Dept., Transportation Corps, New York, N. Y.
Kirsch, M. J. (Re-election), Federal Engineering Co., New York, N. Y.
Kleinberger, R. C. (Re-election), Radio Receptor Co., New York, N. Y.
Magno, H. T., Federal Tel. & Radio Corp., New York, N. Y.
Mushnitzky, D., 42 North Broadway, Long Branch, N. J.
Pansius, J. L., Underwriters' Laboratories, Inc., New York, N. Y.
Pearsall, F. M., Jr., Bell Tel. Labs., Inc., New York, N. Y.
Reed, F. C., Jr., New York Telephone Co., Brooklyn, N. Y.
Steenbergen, E. G., Todd Shipyards Corp., New York, N. Y.
Stern, R. M., Bol, Ltd., New York, N. Y.
Swingle, D. M., Lt., Evans Signal Lab., SCEL, Belmar, N. J.
Ulm, E. H., Sylvania Electric Products, Inc., New York, N. Y.
Varza, G., U. S. Navy Yard, Brooklyn, N. Y.

4. Southern

Barron, O. E., Jr., Robbins & Myers, Inc., New Orleans, La.
Craven, E. H., Evans Electrical Construction Co., Pascagoula, Miss.
Crossley, W. W., Cities Service Refinery, Lake Charles, La.
Dewender, H. J., The Kellex Corp., Clinton Engineer Works, Knoxville, Tenn.
Dishner, W. D., Tennessee Eastman Corp., Kingsport, Tenn.
Early, C. H., Alabama Power Co., Birmingham, Ala.
Erickson, A. J., The Kellex Corp., Clinton Engineer Works, Knoxville, Tenn.
Ford, R. A., U. S. Waterways Experiment Station, Vicksburg, Miss.
Hardy, S. H., Shaw Hardy, Miami, Fla.
Hill, G. H., Cities Service Refinery, Lake Charles, La.
McCord, C., Tennessee Valley Authority, Knoxville, Tenn.
Molette, W. P., General Elec. Co., Richmond, Va.
Oldham, G. B., Godat & Heft, New Orleans, La.
Stone, W. T., Norfolk Navy Yard, Portsmouth, Va.
Trimble, C. R. (Re-election), C. R. Trimble Co., Charlotte, N. C.

5. Great Lakes

Blytheman, G. C., Fisher Body Ternstedt Div., Detroit, Mich.
Cragg, R. C., Gould Storage Battery Corp., Chicago, Ill.
Crist, J. U., Indiana & Michigan Electric Co., Elkhart, Ind.
Enyart, R. B., Goodman Mfg. Co., Chicago, Ill.
Froland, O., Kyle Corp., South Milwaukee, Wis.
Greenwood, G. E., The Detroit Edison Co., Detroit, Mich.
Hermelin, S. A. E., Automatic Elec. Co., Chicago, Ill.
Hinds, S. C., A. C. Spark Plug, Flint, Mich.
Hulbert, F. L. (Re-election), Power Equipment Co., Detroit, Mich.
Larsen, L. O., Western Electric Co., Chicago, Ill.
Larson, R. W., Wyman Gordon Co., Harvey, Ill.
Lenard, I. G., 1640 Greenleaf Ave., Chicago 26, Ill.
Livingston, E. E., Commonwealth Edison Co., Chicago, Ill.
Long, A. P., Wyman Gordon Co., Harvey, Ill.
McHaffie, M. (Miss), Harper J. Ransburg Co., Clayton, Ind.
Pavlenko, V. Z., Caterpillar Tractor Co., Peoria, Ill.
Rembusch, J. E., Public Service Co. of Northern Ill., Chicago, Ill.
Rohde, C. H., Defoe Shipbuilding Co., Bay City, Mich.
Rutishauser, M. H., Clymer Engineering & Sales, Inc., Milwaukee, Wis.
Salatin, J. F., Delco-Remy Div., General Motors, Anderson, Ind.
Scobell, R. H., Guardian Elec. Mfg. Co., Chicago, Ill.
Slaybaugh, R. D., Consumers Power Co., Jackson, Mich.
Steele, A. D., Minnesota Mining & Mfg. Co., St. Paul, Minn.
Wartzok, D. F., Indiana Service Corp., Fort Wayne, Ind.
White, T. J., Kyle Corporation, So. Milwaukee, Wis.
Wiesehahn, H. F., U. S. Rubber Co., Detroit, Mich.

6. North Central

Vollmer, M. F., Bureau of Reclamation, Denver, Colo.

7. South West

Battern, R. W., Pillsbury Mills, Inc., Enid, Okla.
Chilton, J. M., Texas A & M College, College Station, Tex.
Chunn, C. B., Southwestern Public Service Co., Amarillo, Texas
Claughton, J. C., Jr., Southwestern Public Service Co., Amarillo, Texas
Claybaugh, H. W., U. S. Engineer Office, Little Rock, Ark.
Dansby, W. W., Sr., Texas Electric Service Co., Ft. Worth, Texas
Depew, T. S., Standard Measuring & Equipment Co., Enid, Okla.
Frank, S. B., James R. Kearney Corp., St. Louis, Mo.
Groeteke, E. A., B-L Electric Mfg. Co., St. Louis, Mo.
Keller, T. T., Kansas Gas & Electric Co., Wichita, Kan.
Loughridge, R. F., Elevator Service Co., Ft. Worth, Texas
Macune, C. W., Consolidated Vultee Aircraft Corp., Ft. Worth, Texas
Miller, J. A., Southwestern Public Service Co., Plainview, Tex.
Northup, J. R., Oklahoma Gas & Electric Co., Enid, Okla.
Roberts, H. N., 2415—20th St., Lubbock, Texas
Smith, H. J., Southern Methodist University, Dallas, Tex.
Sterrett, P. S., Southwestern Public Service Co., Clovis, N. M.
Wallar, O. W., Fagan Electric Co., Little Rock, Ark.
Walvoord, R. H., Allis-Chalmers Mfg. Co., Amarillo, Texas
Waters, W. F., Southwestern Bell Telephone Co., Dallas, Tex.
Wayland, J. R., Southwestern Public Service Co., Plainview, Texas
Williams, W. L. C., General Electric Co., Oklahoma City, Okla.

8. Pacific

Bean, R. N., Lt., USNR, Degaussing Station, San Pedro, Calif.
Bettersworth, T. A., Pacific Gas & Elec. Co., San Francisco, Calif.
Bowser, W. A., Dept. of Water & Power, Los Angeles, Calif.
Denton, W. T., Dept. of Water & Power, Los Angeles, Calif.
Fraser, R. J., 539 44th Ave., San Francisco, Calif.
Holden, A. F., Pacific Gas & Elec. Co., San Francisco, Calif.
Hollingsworth, H. P., Naval Air Station, San Diego, Calif.
Humphrey, W. E., Dept. of Water & Power, Los Angeles, Calif.
Ingalls, R. E., C. E. Ingalls, San Francisco, Calif.
Kendall, F. I. S., Southern Calif. Tel. Co., Los Angeles, Calif.
Kostainsek, E. L., General Electric Co., San Francisco, Calif.
McCarvel, J. M., Westinghouse Elec. Corp., Los Angeles, Calif.
Moore, R. M., Southern Calif. Tel. Co., Los Angeles, Calif.
Retherford, H. N., Firestone Tire & Rubber Co., Los Angeles, Calif.
Shinn, E. S., Johannessen & Girand, Phoenix, Ariz.
Stirnus, W. F. J., Moore Dry Dock Co., Oakland, Calif.
Welch, T. R., United Aircraft Products, Inc., Los Angeles, Calif.
Younger, U. E., U. S. Navy, Research & Development Section, Pasadena, Calif.

9. North West

Anslow, L. E., Sr., Portland General Elec. Co., Portland, Oreg.
Eagan, F. E., R. W. Beck & Associates, Seattle, Wash.
Gill, B. J., Tinling & Powell, Spokane, Wash.

10. Canada

Ayers, R. E., Elec. Sub.Lieut., RCNVR, H.M.C.S. *Scotian* Halifax, N. S., Can.
Haskett, G. P., Westdale Electric, Toronto, Ont., Can.
Hausch, R. C., B. C. Electric Railway Co., Vancouver, B. C., Can.
Little, H. W., Northern Elec. Co., Lt., Montreal, P. Q., Can.
Maxwell, H. H., Bell Telephone Co. of Canada, Toronto, Ont., Can.
Peacock, R. F., Canadian General Electric Co., Toronto, Ont., Can.
Rose, P. E., Canadian General Elec. Co., Ltd., Montreal, Que., Can.
Taylor, D. E., Elec. Lt., R.C.N., Naval Service Headquarters, Ottawa, Ont., Can.
Wiklund, C. A., Amalgamated Electric Corp., Ltd., Toronto, Ont., Can.
Williams, J. W., Northern Elec. Co., Ltd., Montreal, Que., Can.

Elsewhere

Chaturvedi, A. C., P. W. D., Lucknow, U. P., India
Dayal, D., Govt. Ordnance Factory, Khamaria, Jubbulpore, C. P., India
Gault, F. E., U. S. Engineers, CPBC. Fort Armstrong, Honolulu, T. H.
Hamm, A., New Consolidated Goldfields, Johannesburg, South Africa
Ingalsbe, W. E., Construction Service, CPBC, Fort Armstrong, Honolulu, T. H.
Kamel, G., Edfu Power Station, Atwani, Egypt
Leech, W. P., The English Elec. Co., Ltd., Stafford, Eng.
Lissenden, P. H., c/o Constructora Nacional de Maquinaria Electrica, Madrid, Spain
Narayana Rao, K. L. R., Sri Jayachamarajendra Occupational Institute, Bangalore, S. India
Ranum, H. (Re-election), Elektricitets-Aktieselskabet A E G, Oslo, Norway
Row, V. L. N., E. I. Rly Technical School, Jamalpur, India
Siberry, L. A., Bridgend Urban Dist. Council Electricity Dept., Bridgend, Glam, S. Wales
Srivastava, A. S., Balrampur Estate Electric Supply Dept., Balrampur, Distt. Gonda U. P., India

Total to grade of Associate

United States and Canada, 179
Elsewhere, 13

OF CURRENT INTEREST

ECPD Plans Re-examination of Engineering Schools

Plans for the re-examination of the curricula of engineering schools previously accredited by the Engineers Council for Professional Development were announced at the ECPD annual meeting held in New York, N. Y., October 19–20, 1945, by the committee on engineering schools. It is contemplated that the proposed re-examination will be undertaken progressively as the schools recover from the effects of their wartime activity. Examination of the curricula of schools that already have applied to ECPD for accrediting will continue meanwhile.

Special attention is expected to be given by the committee, in its re-examination work, to those engineering schools, the graduates of which have consistently failed or made minimum grades in license examinations. Also, the accrediting of technical institutes, authorized last year, is expected to be undertaken soon.

SHORTAGE OF ENGINEERS

The apparent lack of knowledge evidenced by educational counselors at military camps and high schools concerning the serious shortage of graduate engineers, was an item of special concern to the committee on student selection and guidance, and consideration is being given as to ways and means of helping to correct that situation. With reference to this point, the committee reported that the demand for engineers appears to have reached a maximum whereas the supply is approaching a minimum, largely as the result of wartime interruption to engineering education and the failure of military policy adequately to recognize the essentiality of scientific training. The committee has written to local engineering societies requesting local activity in interesting returning veterans in engineering work or in further study of engineering subjects.

CIVIL SERVICE REQUIREMENTS

Concern was expressed by the committee on engineering schools and by the committee on professional recognition that Civil Service requirements for engineering positions in government service might be lowered under the Veterans Preference Act of 1944. Thus stimulated, ECPD adopted a resolution that "ECPD will be pleased to support the Civil Service Commission in its efforts to maintain professional standards of performance in science and engineering and to co-operate in the appointment of an advisory group to consider this problem in relation to the physical sciences, pure and applied."

UNIFORM MEMBERSHIP GRADES

The committee on professional recognition repeated a previous recommendation that the major engineering societies establish uniform membership requirements and uniform membership-grade nomenclature, and that these be limited to the following:

1. *Fellow*—an engineer of distinction.

2. *Member*—an engineer who has achieved education and experience equal to the "minimum definition" proposed by ECPD in 1933:

(*a*). Graduation from an approved course in engineering of four years or more in an approved school or college; a specific record of an additional four years or more of active practice in engineering work of a character satisfactory to the examining body (the examining body, in its discretion, may give credit for graduate study in counting years of active practice); and the successful passing of a written and oral examination covering technical, economic, and cultural subjects, and designed to establish the applicant's ability to be placed in responsible charge of engineering work and to render him a valuable member of society; or alternatively

(*b*). Eight years or more of active practice in engineering work of a character satisfactory to the examining body, and the passing of written and oral examination designed to show knowledge and skill approximating that attained through graduation from an approved engineering course, and also examinations written and oral covering technical, economic, and cultural subjects designed to establish the applicant's ability to be placed in responsible charge of engineering work and to render him a valuable member of society.

3. *Junior Member*—graduate of an approved school, or four years or more of satisfactory experience.

4. *Student Member*—a student pursuing the study of engineering in a recognized school under accredited curricula.

RECOMMENDATION NOT NEW

This recommendation harks back some 12 years to the committee's recommendation in the first annual report of ECPD in 1933, with the interesting contrast that the committee now recommends the fellow grade in addition to the three basic grades, and that the society membership be *limited* to these grades whereas the 1933 recommendation adopted by ECPD for submission to its constituent bodies specifically stipulated that the suggested basic grades of membership were "not intended to exclude the affiliation of other persons with any constituent society in a capacity other than the grades of membership indicated."

COMMITTEE'S 1933 PROPOSAL NOT SATISFACTORY

This proposal for standard grades of membership and common nomenclature for them has been variously studied by AIEE and other ECPD constituent bodies during the intervening 12 years, but the committee has been unable to resolve the administrative and other difficulties that would be incurred by at least some of the societies in attempting to apply the committee's proposal. For example, the great body of AIEE Associates is not at all satisfactorily accommodated by the committee's 1933 proposal, and much less so by the committee's less flexible current proposal.

In view of these and other aspects of the problem, the proposal was referred back to the committee by ECPD Council.

PROFESSIONAL ATTRIBUTES

In discussing the qualities which are believed to distinguish a professional from a nonprofessional, the committee on professional recognition recalled attention to the following list of attributes of a profession and its practitioners:

Attributes of a Profession

1. It must satisfy an indispensable and beneficial social need.

2. Its work must require the exercise of discretion and judgment and not be subject to standardization.

3. It is a type of activity conducted from a high intellectual plane.

(*a*). Its knowledge and skills are not common possessions of the general public; they are the result of tested research and experience, and are acquired through a special discipline of education and practice.

(*b*). Engineering requires a body of distinctive knowledge (science) and art (skill).

4. It must have group consciousness for the promotion of technical knowledge and professional ideals, and for rendering social services.

5. It should have legal status and must require well-formulated standards of admission.

Attributes of Professional Practitioners

1. They must have a service motive, sharing their advances in knowledge, guarding their professional integrity and ideals, and rendering gratuitous public service in addition to that engaged by clients.

2. They must recognize their obligations to society and to practitioners by living up to established and accepted codes of conduct.

3. They must assume relations of confidence and accept individual responsibility.

4. They should be members of professional groups, and they should carry their part of the responsibility of advancing professional knowledge, ideals, and practice.

The committee on professional recognition also recommended that ECPD constituent organizations study engineering salary schedules, develop a uniform code of ethics for engineers; that ECPD "urge the Civil Service Commission to make definite engineering educational requirements for entrance into engineering employment with the Federal Government."

CHAIRMAN LEE EMPHASIZES POTENTIALITIES

In re-emphasizing the opportunities which ECPD offers to the individual members of its constituent societies for social and professional contributions to the solidarity and the constructive usefulness of the engineering profession in a professional way as distinguished from specific techniques and sciences, Chairman Lee restated and drew especial attention to ECPD objectives:

"To co-ordinate and promote efforts to attain higher standards of education and practice, greater solidarity of the engineering profession, and greater effectiveness in dealing with technical, economic, and social problems. An immediate objective, now apparently practicable of attainment, is the development of a system whereby the progress of the young engineer toward professional standing can be recognized by the public, by the profession, and by the man himself, through the development of technical and other qualifications which will enable him to meet minimum professional standards.

"Through the standing (ECPD) committees, the boy who would become an engineer is found in the high school (committee on student selection and guidance), is carried through his college life (committee on engineering schools, committee on professional training), and into his junior engineering life, say up to ten years after graduation from college (committee on professional training), and thereafter throughout his engineering life (committee on professional recognition), to form a logical sequence of participation.

"The committee on student selection and guidance shall report to ECPD the means for the educational and vocational opportunities of engineers, in order that only those may seek entrance to the profession who have the high quality, aptitude, and capacity which are required of its members. To this end, the committee reports this year the continuance of the research work under Doctor W. K. Vaughn of the Carnegie Foundation, on the pre-engineering inventory whereby students entering the freshman year of our engineering colleges can be tested as to their fitness for the engineering courses. The past year has added 4,889 students tested to give a total of 13,982 tested in 25 engineering schools. This is a most important phase of our engineering activity . . . . The general characteristics of the engineering students presented in this report have significance. Preliminary study of test results indicates that many of these factors bear directly upon the success of individual students in the colleges of engineering. As we learn more about these factors and are able to relate them to student ability and accomplishment, examination techniques and guidance functions can be greatly strengthened and improved, thereby improving the whole process of engineering education. This is an opportunity open to every engineer in his local community who will give the time to work with the high schools in his territory to bring a better appreciation of the qualifications of the boy who would become an engineer.

". . . . Agencies for helping and guiding the Servicemen have been set up in localities throughout our land and local engineering groups can find plenty to do in aiding these agencies in the special guidance appropriate to the engineering field . . . and do not think, fellow engineers, that there is a surplus of engineers. There is, on the contrary, a mighty deficit. The committee estimates that the need for engineers is such that while in normal years 25,000 freshmen would enter engineering colleges, to meet present needs 65,000 freshmen would have to enter engineering colleges this year. The need for selection and guidance under these conditions is greater than ever.

ENGINEERING SCHOOLS

"The committee on engineering schools reports to ECPD the means for bringing about co-operation between the engineering profession and the engineering schools. As an intermediate step, the committee reports to the Council criteria for colleges of engineering which will insure to their graduates a sound educational foundation for the practice of engineering. In this committee is centered the accrediting activity of ECPD to give an accredited list of engineering courses recognized as standard . . . during the war years these inspections have been held to a minimum, as the committee has believed that engineering colleges could not be judged fairly while operating under war conditions. Even so, during the past three years, 26 curricula at 13 institutions were investigated, and special consideration given to 25 curricula at other institutions. With the resumption of normal activities, the normal accrediting program will be resumed.

"This year has seen the active setting up of a complete regional committee personnel, covering the entire United States . . . for the accrediting of technical-institute curricula. This includes a personnel representation of 125 engineers and educators . . . . The days to come will see the beginning of technical institute accrediting under the same guiding principles that have made collegiate accrediting an outstanding contribution of ECPD. . . . There is still much to be done and participation of members of local engineering societies interested in the work is open to those who will contribute of their time and ability to it." Negotiations with junior colleges now are under way and the prospect is that a working agreement soon will be arrived at with such colleges.

COMMITTEE ON PROFESSIONAL TRAINING

"The committee on professional training reports to ECPD plans for the further personal and professional development of young engineering graduates, and also of those without formal scholastic training. Here is a great opportunity for work with the young engineer as he leaves the engineering school and enters his chosen life work. A 'Manual for Junior Engineers' is in preparation under the guidance of this committee. A continuation of the upkeep of the 'Reading List for Junior Engineers' is an important objective of this committee. . . .

COMMITTEE ON PROFESSIONAL RECOGNITION

"The committee on professional recognition reports to ECPD methods whereby those engineers who have met suitable standards may receive corresponding professional recognition. This work is necessarily among the engineers who have advanced in the profession, and great responsibility rests with the committee in bringing into being those concepts of the engineering profession which they seek and to which their engineering spirit gives visualization. There is need here for a greater response. . . . . It is to be hoped that there will be recognition from the engineers of the local engineering societies and councils of the opportunities for bringing professional recognition into being, based upon accomplishment and service . . . this will require much individual and collective time of engineers, well spent."

PARTICIPATION IN PUBLIC AFFAIRS

Commenting upon ECPD participation in the Consultative Committee on Engineering to the War Manpower Commission during the war, Mr. Lee pointed out that "we have demonstrated that where engineers are willing to give of the limitless time demanded for the progression of those activities of which they would be a part, they are granted representation and are recognized. . . . So, again, I repeat this thought: That engineers can have those representations for which they ask and those recognitions for which they aspire, but these come only as the result of hard work."

Officers elected and committee chairmen appointed at the 1945 annual meeting of the ECPD were as follows:

Chairman—Everett S. Lee, engineer of the general engineering and consulting laboratory of the General Electric Company, Schenectady, N. Y. (re-election).

Vice-chairman—James W. Parker, president, general manager, and director of The Detroit Edison Company, Detroit, Mich.

Secretary—William N. Carey, secretary and executive officer of the American Society of Civil Engineers, New York, N. Y.

Assistant secretary—H. H. Henline, secretary, American Institute of Electrical Engineers, New York, N. Y.

Chairman committee on student selection and guidance—Carl J. Eckhardt, Jr., professor of mechanical engineering and superintendent of utilities, University of Texas, Austin, Tex.

Chairman committee on engineering schools—D. B. Prentice, president, Rose Polytechnic Institute, Terre Haute, Ind.

Chairman committee on professional training—C. A Pohl, consulting engineer, New York, N. Y.

Chairman committee on professional recognition—N. W. Dougherty, dean of engineering, University of Tennessee, Knoxville, Tenn.

Representatives to ECPD were appointed by the constituent bodies as follows:

American Society of Civil Engineers—Scott Lilly.

American Institute of Mining and Metallurgical Engineers—A. F. Greaves-Walker (reappointment).

American Society of Mechanical Engineers—John E. Younger.

American Institute of Electrical Engineers—M. S. Coover.

Society for the Promotion of Engineering Education—D. B. Prentice (reappointment).

Engineering Institute of Canada—deGaspé Beaubien.

American Institute of Chemical Engineers—W. T. Nichols.

National Council of State Boards of Engineering Examiners—George M. Shephard.

The ECPD executive committee for the ensuing year will include R. E. Bakenhus, ASCE; W. B. Plank, AIME; R. L. Goetzenberger, ASME; M. S. Coover, AIEE; S. D. Kirkpatrick, AIChE; H. S. Rogers, SPEE; C. R. Young, EIC; and C. C. Knipmeyer, NCSBEE, in addition to Chairman Lee, Vice-Chairman Parker, Secretary Carey, and Assistant Secretary Henline.

Microwave Radio to Augment Wire Services

A microwave radio-relay communications system with two-way voice transmission over radio links totalling 1,400 miles was demonstrated by the Bell Telephone Laboratories, New York, N. Y., October 31, 1945, over circuits between the West Street building of the New York Telephone Company, and Neshanic, N. J. Reputed to be the first microwave radio-relay system that has been used for long-distance telephony, this system employs pulse-position modulation—a form of frequency modulation which provides unusual stability of operation over great distances. All units of the system used by the armed services in the European theater of operations were supplied by the Western Electric Company, the manufacturing branch of the Bell system.

Operating in the microwave region, this system uses radio frequencies of nearly 5,000,000,000 cycles per second corresponding to a wavelength of less than half the length of an ordinary pencil. The waves can be concentrated into a sharp beam and require a clear unobstructed straight-line path for their transmission. They do not follow the earth's curvature but shoot off into space. Each system can carry eight high-grade telephone channels, any of which can be used for signaling and dialing. If taken out of telephone service, any or all channels will carry telegraph, facsimile, or pictures.

In the demonstration six simultaneous conversations were carried between New York and Neshanic, 40 miles away. At the same time a seventh telephone channel was transmitting the day's weather map by the Army's facsimile system, and over an eighth channel were passing 18 teletypewriter messages, one of which was originated by amateur typists.

Two more pairs of radio sets were then looped in at each end to provide five radio links giving two-way transmission over a total distance of 200 miles. Then, seven channels were themselves looped together to give 1,400 miles of two-way voice transmission by radio. Finally, by changing from two-way to one-way transmission, a 2,800-mile airline circuit was provided with both talker and listener being in New York. This is one of several radio-relay systems now being considered for application by the Bell system.

Pulse Time Modulation of Multiplex Telephone

The first unrestricted demonstration of pulse time modulation of multiplex telephone, telegraph, radiobroadcast, and television transmission was given by the International Telephone and Telegraph Corporation at New York, N. Y., September 27, 1945. The equipment demonstrated can carry 24 or more two-way telephone conversations, or many telegrams, or 12 or more different radiobroadcasting programs at the same time from the same radiobroadcasting center over one radio channel.

Pulse time modulation operates by emitting short pulses of high-frequency radio energy. These pulses are of constant amplitude, and are all of the same carrier frequency. Modulation is effected by transmitting a synchronizing, or "marker" pulse at fixed times, and following this pulse with the "signal" pulse at a variable time interval, the time interval being varied at a rate corresponding to the audio frequency of the signal being broadcast. In the present arrangement a single marker pulse is followed by the 24 signal pulses for the 24 channels.

From the top floor of the 32-story International Telephone Building the pulse time modulation channel goes in a sharp beam to a repeater station near Hazlet, N. J., then to another repeater station at the Federal Telecommunication Laboratories at Nutley, N. J., and from there back to the same room in the International Telephone Building, so that the circuit starts and finishes there, but is in reality an 80-mile long-distance telephone circuit. This long-distance telephone channel carries 24 or more conversations at the same time. Engineers point out that the repeater stations which boost the strength of the circuits' microwave energy are required approximately every 30 miles. These repeater stations operate automatically without any human attendance.

Pulse time modulation which has been referred to previously (*EE, Jan '45, p 43*) was developed by the laboratories division of the Federal Telephone and Radio Corporation.

UET Elects Officers

J. P. H. Perry, vice-president of the Turner Construction Company, New York, N. Y., was elected president of United Engineering Trustees, Inc., at a meeting on October 25, 1945. In this capacity he succeeds F. M. Farmer (A '02, F '13) vice-president and consulting engineer, Electrical Testing Laboratories, Inc., New York, N. Y.

Other officers elected were: R. M. Roosevelt, mining engineer, vice-president; Albert Roberts, secretary-treasurer, Minerals Separation North America Corporation, re-elected treasurer; C. R. Jones (A '16, M '30) eastern transportation manager, transportation division, Westinghouse Electric Corporation, New York, N. Y., re-elected assistant treasurer; John H. R. Arms, secretary of the Engineering Foundation, also re-elected secretary.

The United Engineering Trustees is a corporation which promotes the advancement of the engineering arts and sciences through the Engineering Foundation, and the Engineering Societies Library.

Mica Substitutes Developed to Meet Wartime Shortages

German wartime industry appears to have developed a substitute for mica as an electrical-insulating material that appears likely to remain in the peacetime market as a mica competitor. As disclosed in a report by the Publications Board of the United States Department of Commerce, a resin-impregnated glass fabric known as "Glushartgewebe" was developed by the Robert Bosch Company of Stuttgart for the insulation of commutator segments of its motors, generators, and other such machinery. Governing specifications for the basic glass fabric limited the individual threads to a diameter range of 0.015 to 0.022 millimeter, and a weight of not less than 125 nor more than 135 grams per square meter. The thread counts ranged from 19 to 21 per centimeter in the weave, and from 15 to 17 per centimeter in the warp. A special heat test revealed any excess oil that might have a tendency to remain in the fabric from the sheen process, as such oil would interfere with satisfactory impregnation.

The impregnating solution was a homogeneous mixture of 53.30 per cent (by weight) of partially polymerized resinol, 23.35 per cent alcohol, and 23.35 per cent of osmose-caoline. Machines similar to those used in the United States in the manufacture of varnished cambric were used in the manufacture of the impregnated glass fabric. Adjustable pressure rolls controlled the amount of solution used, at the rate of approximately 42 kilograms of the solution for each 100 meters of fabric. The glass

Nerve Center for Western Wind Tunnel

Westinghouse photo

Master control room of wind tunnel at Pasadena, Calif., where tomorrow's airplanes are being developed, from which all operations are directed and controlled and to which all scientific data are transmitted. This is operated by the California Institute of Technology and owned jointly by Consolidated Vultee Aircraft Corporation, Douglas Aircraft Company, Inc., Lockheed Aircraft Corporation, and North American Aviation, Inc.

fabric entered an oven at room temperature, reached a maximum of 120 degrees centigrade at the top of the oven and cooled to room temperature by the time it left the oven. The travel distance in the oven was about 14 meters.

After impregnation, the fabric was cut into strips 1,500 by 95 mm, folded into three thicknesses lengthwise, and then formed under heat and pressure into a molded piece 500 by 95 mm, the largest possible within the close thickness limits required for commutator insulating strips. Molded under a pressure of 350 kilograms per square centimeters at a temperature of 160 degrees centigrade for a period of five minutes, the material subsequently was given an oven baking at a temperature of 210 degrees centigrade to remove all the solvent.

The physical characteristics of this mica substitute, as reported by the German makers to American investigators, were as follows:

1. Withstood a temperature of 250 degrees centigrade continuously; 320 degrees centigrade for short periods.
2. Nonhygroscopic, showed no change in weight or dimension when exposed to relative humidity of 100 per cent and temperature of 21 degrees centigrade for 24 hours.
3. Withstood a commercial-manufacturing breakdown test of 1,000 volts, but is described as being able to withstand many times that voltage.

No information was indicated concerning the insulation resistance of the material or concerning the effect of humidity on such resistances. Chemists of the Bosch Company have indicated that the substitute material would continue in use even after natural mica again becomes available.

Regulations for Federal Technical Committees

The Federal Specifications Board, composed of representatives from ten Government agencies, has drafted regulations governing the operations of the 74 technical committees under its supervision. The Board was established by C. E. Mack, director of the Treasury's procurement division. In co-operation with the standards branch of the procurement division, it is responsible for the preparation and revision of Federal specifications for supplies used by Federal agencies.

Federal specifications are drafted by the technical committees for promulgation by the director of procurement. Each committee is composed of technical Government personnel and is responsible for drafting specifications on certain commodities.

Serving on these technical committees on a part-time basis are about 1,300 experts from the Government service. According to regulations drafted by the Federal Specifications Board, technical committees are composed of representatives from Government agencies responsible for quality and performance of materials and equipment. Drafts of proposed specifications will be submitted for comment and criticism to Government agencies, representative manufacturers, and recognized technical and professional societies. Among the latter are American Standards Association, American Society for Testing Materials, Society of Automotive Engineers, American Society of Civil Engineers, American Society of Mechanical Engineers, National Institute of Governmental Purchasing, and others. The technical committees will then incorporate any accepted suggestions in the final draft.

Engineers Testify at Senate Hearings on Science Legislation

Appearing in October before the Senate Committee hearings on proposed science legislation (*S* 1285, *S* 1297, and related bills) a panel of five prominent engineers presented engineering viewpoints and recommendations. This panel was appointed by the Engineers Joint Council (formerly known as Joint Conference Committee) and consisted of the following: Boris A. Bakhmeteff, chairman, honorary member, American Society of Civil Engineers; Doctor Harvey S. Mudd, president, American Institute of Mining and Metallurgical Engineers; Professor A. G. Christie, past president, American Society of Mechanical Engineers; F. Malcolm Farmer, Fellow and past president, American Institute of Electrical Engineers; Doctor George Grainger Brown, past president, American Institute of Chemical Engineers.

This panel recommended that:

1. The development of basic scientific research on a scale commensurate with the dominant position of the U.S.A. is a problem of pressing national necessity. The magnitude and urgency of the task make indispensable government support of such research through a National Research Foundation.

2. Federal funds, administered by the Foundation, should be allocated for purposes where government support is indispensable, and should not be diverted to fields where research may rely on other sources. The proposed National Research Foundation should promote basic scientific research only, leaving applications to industrial and technologic practices to the appropriate private, industrial, and public agencies.

3. Basic scientific research should include fundamental research in engineering sciences.

4. Social studies should be the object of a separate agency.

5. The preferable form of organization is to have control of the Foundation vested in a Board appointed solely on the basis of scientific competence and outside of any partisan or political consideration. The Director of the Foundation should be selected by the Board and be responsible to the latter.

6. The Foundation should be given the broadest authority to enact its own rules and regulations in all matters concerning basic scientific research, in subsidizing the training of future research personnel, and in matters of publication.

7. Engineering science should be recognized in the forming of the Board and the divisional substructure.

8. The Legislative Act establishing the Foundation should be confined to the sole purpose of advancing basic science and should not include controversial legislation dealing with patents.

The full text of the official testimony of this panel follows:

ENGINEERS INTERESTED IN RESEARCH

"Engineers are vitally interested in basic scientific research, for such research is the foundation of modern engineering. In fact, the position and role of the engineer in the human community is that of an active link between basic scientific research and technology. It is the engineer who makes use of the fruit of scientific progress and turns it to the practical service of man. Applied research is actually planned and carried out by the engineer. That is his recognized field. However, the engineer is directly concerned with, and actively engaged in, basic scientific research. Indeed, recent progress of technology has grown out of an unprecedented development of engineering science, meaning a fundamental knowledge of the laws of nature which permit the mastery of the resources and powers of nature. The significance of engineering science in present society is best illustrated by the example of Germany, which was the first country to recognize the vital importance of basic engineering research. The result was the miraculous technical achievement of which the world has been the recent witness, and of which humanity came so near becoming a victim.

"In many ways the practicing engineer bears the same relationship to fundamental research in the science of engineering as does the practicing physician to the basic investigations of the scholarly doctor in the medical and biological fields. The practicing engineer applies the basic principles discovered by engineering science to technological problems, just as the practicing physician uses scientific discoveries for healing the sick.

ENGINEERING VIEWPOINT

"In presenting the viewpoint of the engineering profession, which may rightfully consider itself as particularly expert in appraising the value and portent of scientific research, the undersigned panel unreservedly endorses the broad objects of the proposed legislation in regard to basic scientific research. The Engineering Profession stands undivided back of the words of the President, that

> 'Progress in scientific research and development is an indispensable condition to the future welfare and security of the nation.'

"Furthermore the circumstances under which this country is facing the problem of promoting basic scientific research, are unprecedented and are marked by pressing urgency. By the force of events growing out of the War, the United States has been thrust into a position of pre-eminent leadership in world affairs. It is incumbent on us to continue to preserve and maintain this leading part from this time forward. We must be prepared for any military eventuality. War has become a battle of scientists. Also this country must lead in science to assure national health, prosperity, and welfare. As the President stated:

> 'No nation can maintain a position of leadership in the world of today, unless it develops to the full its scientific and technological resources.'

"The American people have been foremost in technical ingenuity and industrial organization, and in research of 'applied' character. It is a well-acknowledged fact, on the other hand, that in the realm of basic sciences and basic scientific research the United States did not keep pace with the principal nations of the Old World. Indeed, to a large extent practical applied research in the United States relied on basic scientific material coming from overseas. The war has violently upset this balance. Europe is in eclipse. For years to come, in the intellectual and scientific realm the United States will have to depend on its own resources. This brings this country face to face with a problem of utmost gravity. Under the threat of losing its primacy, the United States 'must' speedily fill the void left open by the ravages of Europe, and within the shortest allowable period bring up its own scientific research to a level which, in scope and quality, will measure up to the requirements of this country's new world position.

"It is self-evident that the size and the urgency of the problem are such that scientific research in this country no longer can be allowed to depend on the course of natural development that prevailed in the past, and to rely upon the diminishing funds of private philanthropy. A systematic and generous yearly appropriation of government funds becomes a necessity. Under such circumstances the Engineering Panel joins its voice to the universally endorsed proposal of a special National Foundation for promoting and developing basic scientific research.

"The situation indeed bears a resemblance to that at the beginning of the war, when the country was called upon to build overnight a war industry capable of meeting the most formidable threat of all time. However, the conversion of our peacetime industry to war was largely a problem of material reorganization, while the present problem of bringing to life and stimulating creative, scientific endeavor largely lies in the spiritual and intellectual realm. Indeed, the delicacy of the problem requires the most careful and considerate approach. Methods must be chosen which would assure an optimum and most speedy development. It is equally imperative to abstain from measures which could impair or stultify the sought-for objective.

RECOMMENDATIONS

"In formulating the following opinions, the undersigned Engineering Panel is motivated by the desire to find the best possible solution for a problem of highest national importance:

(*a*) The primary purpose of the proposed Foundation should be basic scientific research. It is in this realm that the United States has been lagging behind. As a general principle, the Foundation should not spend government funds for research in fields which have been obtaining, and will continue to obtain, financial support from other sources. Accordingly, there is no need for the Foundation to support 'applied, research. Indeed, experience has shown that adequate funds and means were readily found in the past for research of applied practical character. Also there are many research men and organizations, in the field of technology and applied sciences, that are well supported by industry and partly by special public agencies for the purpose of developing new products and processes for business concerns of all sizes. Federal aid, on the other hand, is sorely needed and should be generously provided to enhance and support basic scientific research. The latter obviously is the foundation for practical applied advancement. But basic research in itself bears the distinction of being undertaken without any immediate idea of profit. The results are to be of service to humanity at large.

(*b*) It is the view of this panel that basic scientific research necessarily implies fundamental research in the engineering sciences. Although none of the proposed legislative proposals has so far deigned to mention engineering research by name, we feel that it is unnecessary as well as impractical to enumerate the different ramifications of science in the proposed Legislative Act. Research in the sciences should mean that the Foundation will promote basic scientific work on all possible lines, recognizing fundamental engineering research as one of its major objectives.

(*c*) In the opinion of this panel no useful purpose will be served by extending the scope of the Foundation to embrace 'social sciences.' The engineers in their wide contacts with 'men,' keenly appreciate the value and significance of better social understanding. The character of the problems, however, is essentially distinct from those dealing with the physical world. Social studies should be the object of a separate agency composed of an altogether different type of man. Placing social sciences under the same roof with natural sciences will help neither and impede both.

(*d*) In discerning the ways and means by which optimum progress in basic scientific research can be achieved, the undersigned panel wholeheartedly ranges itself back of the words of the President, that:

'Science can be co-ordinated and encouraged, it cannot be dictated to or regimented. Science cannot progress unless founded on the free intelligence of the Scientist . . . the Federal Research Agency . . . should in no way impede that freedom.'

In deciding upon the preferred form of organization and on the modes of functioning of the Foundation, this panel is guided by the conviction that progress in science is essentially a matter of free and uninhibited display of creative scientific endeavor. Accordingly, any plan intended to call to life and promote basic scientific research must devolve from the aim of providing a propitious atmosphere, in which creative human talent will assert itself to supreme advantage. Reduced to practice, the problem is to select scientists endowed with creative capacity and to place them in an environment where, with proper material support, scientific talent will thrive and bear fruit.

With regard to the form of organization, the essential feature is to place the Foundation in the hands of men, competent and experienced in scientific research and removed from all possible partisan or commercial influences. The type of organization, proved by the example of the large universities, the National Advisory Board for Aeronautics, and the numerous privately endowed non-profit institutions, is the American democratic method of group control as distinct from centralized authority exercised by a single official. The undersigned panel decidedly expresses its preference, therefore, for the type of structure reflected in the Magnuson Bill *S* 1285, which vests supreme control of the Foundation's affairs in a board, the members of which are chosen without regard to political or partisan affiliations and solely on the basis of their demonstrated interest, experience, and competence in matters of research.

Research, by its very nature, requires a 'climate' different from the technical formalities attendant on customary government routine. The Foundation should be given the widest authority to prescribe its own specific rules and regulations and to administer affairs in forms appropriate to the purpose of advancing basic sciences, and outside the usual bureaucratic routine.

(*e*) A Foundation of the size and scope contemplated will obviously require a strong and efficient executive structure. Accordingly, the Director should possess the broadest powers to insure promptness and efficiency in operations. But in his capacity as Chief Executive the Director should function under the general control of the board and should be responsible to the latter.

The board should consist of men, representing different ramifications of science and should be selected from panels submitted by the leading national scientific and professional associations. The board should necessarily include members representative of engineering. Engineering scientists should also be appointed to the different committees, which are to govern the work of the divisional sub-structure. The board should be left free to seek recommendations for committee appointments from appropriate scientific and professional associations. No single body should be privileged by legislative acts to offer such recommendations.

(*f*) The most crucial problem is that of men. There is need for scholars capable of creative leadership and for adequate staffs of scientifically trained personnel. The supply of such personnel has been diminishing in recent years and shows no prospect of immediate renewal. There is no way to provide for such personnel in the future except by generously appointing promising candidates to fellowships and scholarships. The training of such future scientific personnel must be raised to the highest possible level, commensurate with the requirements of the day. To achieve this purpose, and to procure in the shortest time an adequate host of properly trained men, may require policies and procedures in the way of fellowships and stipends which might substantially depart from previous practices and would boldly cut across routine. The Foundation should be given the broadest possible freedom of action in this respect.

The natural seat for the training of personnel is the universities. Except for a few privately endowed non-profit institutions and certain Government laboratories, the universities will also be the natural center for basic scientific research. By contrast to the Old World prototypes, where for centuries the universities flowered as centers of creative scientific activity, the American university in the past principally served the purpose of mass instruction. The duties connected with teaching left no time and opportunity to the academic personnel for scientific research and advancement. Under the stress of the new national requirements the climate of university life will necessarily have to change. The Foundation in its policy of contracts and subventions should be free to exercise such powers as will allow University Research to be located in surroundings where the scientifically minded staff will be able to devote the necessary time and effort to scientific pursuits free from the consuming burden of academic routine.

(*g*) In laying emphasis on basic scientific research as the prime objective of the proposed Foundation, this panel fully recognizes the fact that it is not always possible to draw the delimiting line between basic and applied pursuits. It is obvious, on the other hand, that all such activities as experimental researches looking to the development of new or the improvement of existing processes and devices, or the preparation of plans, specifications, standards, economic and industrial studies, or the experimental operation of pilot plants should not be included in the function of the Foundation. These most necessary and useful activities are the proper function of private industry, industrial research laboratories, engineering organizations, and the appropriate federal, state, or municipal agencies. A National Research Foundation should not only refrain from duplicating such activities, but should not utilize its facilities or energies for the immediate commercial advantage of any group of citizens. On the other hand, the work of the Foundation will ultimately aid all practical endeavor by extending the limits of basic knowledge and by increasing the supply of men trained for research.

(*h*) An essential condition to the success of a Foundation dedicated to the advancement of basic sciences, is to divest such Foundation from all duties and functions which are essentially alien to the spirit of free scientific pursuit. A most important instance of this character is the question of patents. The subject of patents is highly controversial. Patents have been qualified by some as 'the life of research.' Others are inclined to consider the very idea of patent protection as 'an embodiment of monopoly.' This panel understands that the whole subject of future national patent policies is in the process of consideration by special legislative agencies. In view of this fact and pending the forthcoming patent legislation, a Foundation dedicated to basic scientific research should be held free from any connections with predetermined patent policies. Moreover, patents have to do primarily with applied or industrial research, as distinct from the basic scientific research with which the Foundation should be concerned. If the Foundation is properly set up for the object of advancing basic sciences, the question of patents will not be serious, and in rare, exceptional instances, could be properly handled under the provisions of the general patent law through appropriate contractual relationships determined by the Board."

New System Requires One Frequency for Sound and Vision

A new system of television transmission embodying the transmission of sound by pulses similar to the small bursts of radio waves lasting but a few millionths of a second used in radar was announced October 31, 1945, by Pye, Ltd., a British firm. This system effects a televised program with sound by a single transmitting unit. Hitherto, a transmitting station had to have two single-purpose transmitting units, one to transmit the picture, and the other to broadcast sound. The public had to have a vision receiver and a sound receiver, either on separate sets or housed in one cabinet.

Use of the new system is said to reduce considerably the cost of television both for the broadcasting station and for home users. Again, the transmitting station will be able to reduce the equipment, as well as to cut down running costs. For the public the elimination of the sound receiver would mean a valuable saving on tubes and other components. When the television transmitter is in operation, there are short intervals of time during which no signal is being broadcast. These intervals occur during the time that the spot which traces the picture is returning to its starting point preparatory to making another line, and each represents approximately one-tenth of the total transmission time. Each of these idle periods lasts for ten-millionths of a second, and there are 10,000 of them during each second. It would now be possible to use the television transmitter during these idle periods to transmit the sound program. This would be done by taking a "sound snapshot" of the sound part of the program whenever the transmitter is not transmitting the vision part of the program. Using television of the present definition, 10,125 of these "sound snapshots" are transmitted per second.

The process is best compared with that of taking motion pictures. Thus, if the sound snapshots passed into a suitable receiver at the rate of 10,125 per second, the original sound program could be recovered. The sound snapshot is really a pulse similar to a radar pulse which is sent out by the picture transmitter, and the width of which is made to vary according to the sound that has been transmitted. With the system in operation, this modification to the transmitted wave form would enable both vision and sound to be transmitted on a single carrier by a single transmitter with a single aerial system. This pulse would be separated from the picture program in the television receiver and the variation of its width would be made to operate a loudspeaker.

The new system embodies other far-reaching improvements such as, eliminating interference in the television receiver between sound and vision.

Products Exposition in Chicago

The first of the "Products of Tomorrow Exposition," an industrial fair that will be presented annually in Chicago, Ill., will be held beginning April 27, 1946, and tentative plans call for a 24-day run. A nonprofit organization composed of American business men is behind the project. An announcement has been made that the exposition is being designed as a springboard for world industry to assist in the introduction of new designs, new inventions, and other developments to the general purchasing public as well as to the trade world as a whole. It will not be confined to any one phase of the industrial world or any one country and will be international in scope. Already England, Switzerland, Sweden, and Holland have made inquiries revealing interest in similar exhibits in those countries. Canada and South America are also interested in such expositions.

Both consumer and industrial products will be featured at the Chicago exposition, where a novel one-way traffic plan will ensure guidance to every exhibit and expedite the thousands of visitors expected. A nominal charge for admission will be in effect. No restrictions will be placed on the design of the displays although they will have to conform to the over-all picture of modern interior design.

The planning will be under the direction of Marcus Hinson, electronics engineer and physicist.

New Television Tube Assures 24-Hour Coverage

A new television camera tube more than 100 times as sensitive as any previously used in the studios which picked up scenes with infrared rays in a blacked-out room was demonstrated in a studio of the National Broadcasting Company, New York, N. Y., October 25, 1945. Evidence of the superiority of this new electronic "eye"—a wartime secret—known as RCA "Image Orthicon" came in the transmission of scenes from a rodeo being held in an indoor arena which showed its advantages over the conventional television pickup tubes in providing greater depth of perception and clearer views under shifting light conditions.

The tube was designed and developed by engineers of the Radio Corporation of America at Princeton and Camden, N. J. Credit for its creation was given largely to Doctors Albert Rise, Paul Weiner, and Harold Law of the laboratory staff and to V. K. Zworykin (M '22) associate director.

Especially suited for remote use out of doors under conditions of lighting "that would make the use of ordinary means impossible," the tube is 15 inches long, 3 inches in diameter, of light weight and is easy to use and operate according to the engineers. Its "seeing ability" is one-tenth that of the human eye and is about as sensitive as ordinary high-speed film.

Engineers listed specific advantages in performance of the new device. They are:

1. It extends the range of operations to practically all scenes of visual interest particularly those under low-lighting conditions.

2. Its improved sensitivity, permits greater depth of field and inclusion of background that might otherwise be blurred.

3. Its improved stability protects images from interferences due to exploding photo flashlight bulbs and other sudden bursts of brilliant light.

4. Its smaller size facilitates use of telephoto lens.

5. Its light weight is useful for portable television cameras.

6. Improved control provides unvarying transmission, despite wide fluctuations of light and shadow.

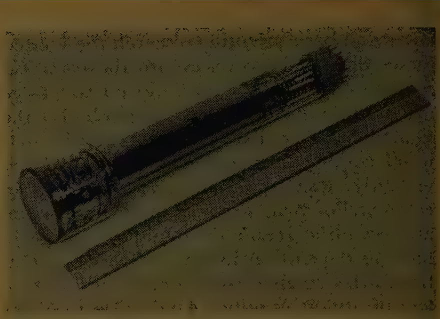


Figure 1. Television-camera "eye" known as the "Image Orthicon"

E. W. Engstrom, research director of the corporation's laboratories, explained that early models were built before World War II in an effort to improve the quality of television transmission. When war came the Armed Forces found need for television applications, and research and development continued at an accelerated pace in response to military requirements. The Image Orthicon tube emerged in its present form much sooner than would normally have been possible. No longer a military secret the tube makes use of 20 years of research not only in television pick-up tubes but also in electron optics, photo-emission, processes, electron multipliers, and special materials. According to Mr. Engstrom "we are today mere amateurs in its ultimate application."

J. F. Royal, vice-president in charge of television, National Broadcasting Company, declared that . . . "assures television of 24-hour coverage, in daylight, twilight, or moonlight—in good weather and in bad."

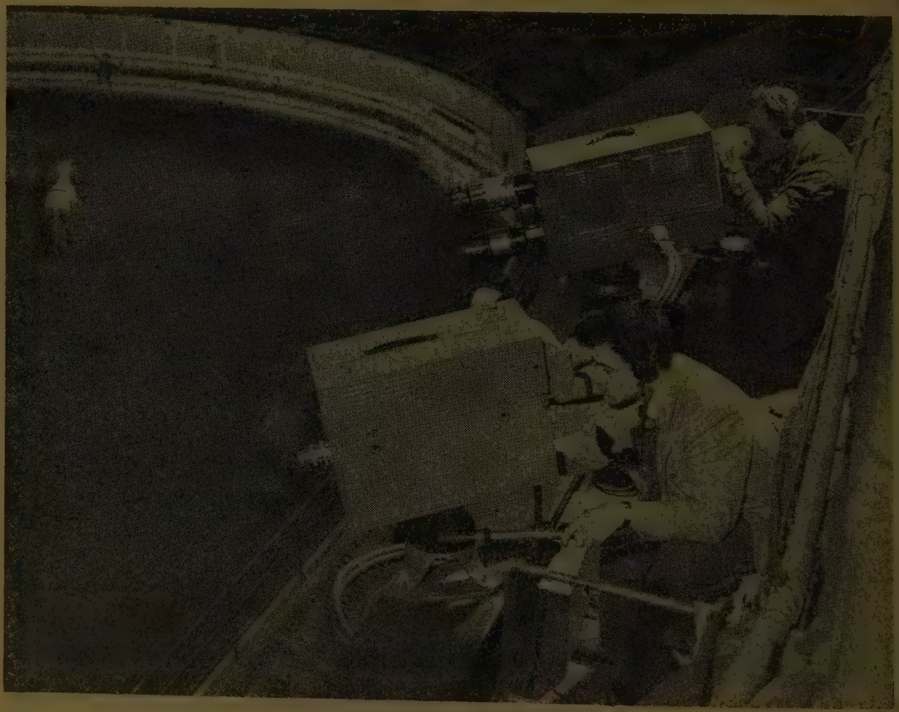


Figure 2. Scenes from the recent rodeo at Madison Square Garden, New York, N. Y. are picked up by the new television camera (foreground) and by a conventional camera

HONORS • • • • • •

J. M. R. Fairbairn to Receive Sir John Kennedy Medal

The Sir John Kennedy Medal for 1945 will be given to J. M. R. Fairbairn, retired chief engineer of the Canadian Pacific Railway. A graduate in civil engineering from the University of Toronto in 1893, Doctor Fairbairn's career with the Canadian Pacific Railway began in 1901 when he became principal assistant to the district engineer at Montreal, Quebec. His promotions were rapid until in 1918 he was appointed chief engineer of the system, a position which he filled continuously for more than 20 years until his retirement January 1, 1939. In 1941 Doctor Fairbairn accepted the position of director of works and buildings in the naval service of the department of national defense at Ottawa, Ontario, from which he resigned in 1942.

The medal, which is awarded as a recognition of outstanding merit in the profession, commemorates the great services rendered to the development of Canada, to engineering science, and to the profession by the late Sir John Kennedy, past president of the Engineering Institute of Canada.

OTHER SOCIETIES •

Material Handling Society Formed

A new society known as the Material Handling Society was formed in Pittsburgh, Pa., October 8, 1945. The suggested objectives are as follows:

1. To further the application of good material handling practices.
2. To encourage an interchange of ideas among the members.
3. To promote education and training in the science and practice of practical co-ordinated material handling.
4. To arrange for the collection and dissemination of information relating to all phases of material handling.

Temporary officers elected were: T. O. English, Aluminum Company of America, chairman, and Richard Rimbach, Materials Publishing Company, secretary. Several committees were organized, such as the program, membership, and nominating committees.

Further information may be obtained from Richard Rimbach, 1117 Wolfendale Street, Pittsburgh 12, Pa.

Canadian Group Aids Community Planning. Assistance in creating a national organization to foster community planning has been offered by the Engineering Institute of Canada, Montreal, Quebec, Canada, in a letter to all provincial representatives at the Dominion-Provincial Conference on Reconstruction. The institute believes that some such organization can be developed into an instrument of great significance and usefulness. This support from the institute is in line with the resolution passed at the annual meeting of the institute at Winnipeg, Canada, wherein such an organization was proposed.

Future Meetings of Other Societies

American Association for the Advancement of Science. March 27–29, 1946, St. Louis, Mo.

American Society for Testing Materials. Forty-ninth annual meeting. June 24–28, 1946, Buffalo, N. Y.

American Society of Heating and Ventilating Engineers. 52d annual meeting, January 28–30, 1946, New York, N. Y.

Illuminating Engineering Society. National convention, September 18–21, 1946, Quebec, Quebec, Canada.

Institute of Aeronautical Sciences. Wright Brothers Lecture. December 17, 1945, Washington, D. C.; 14th annual meeting. January 1946, New York, N. Y.

Institute of Radio Engineers. Winter technical meeting. January 23–26, 1946, New York, N. Y.

Louisiana Engineering Society. Annual Meeting. January 10–12, 1946. New Orleans, La.

National Association of Corrosion Engineers. Annual meeting and convention. May 7–9, 1946, Kansas City, Mo.

EDUCATION • • • •

Hoboken Cities Exchange Greetings. In commemoration of the 75th anniversary of the founding of Stevens Institute of Technology, Hoboken, N. J., the mayor of Hoboken, Antwerp, Belgium, has sent greetings to Hoboken, its sister city, in the United States. The mayor's letter, which congratulated the institute for its accomplishments during three quarters of a century, was prompted by greetings sent to him from Stevens Institute of Technology via the Belgian Information Center in May 1945.

More Eastman Kodak Fellowships. The establishment of three fellowships has been announced by the Tennessee Eastman Corporation, Kingsport, a subsidiary of the Eastman Kodak Company, Rochester, N. Y. These include one postgraduate-doctorate fellowship in textile chemistry, and two postgraduate master-degree fellowships—one in textile engineering, and one in textile chemistry. The latter two fellowships will be awarded to different institutions each year. The fellowships granted by the parent company were referred to previously (*EE, Oct '45, p. 377*).

Professional Teamwork at Stevens. While problems of national economy and education confront the country, the severest tests will come "in the every-day experiences of human relationships, of personal contacts," according to J. B. Blandford, Jr., administrator, National Housing Agency, speaking at the recent commencement exercises of Stevens Institute of Technology, Hoboken, N. J. Mr. Blandford asserted that even more important than technology, resources, or science, are the factors of collective bargaining, international relations, co-operation between professions, teamwork between Congress and the administrative branch of Government, and the relations between Government, individuals, and private enterprise—especially in attitudes between a citizen and his government.

Mathematics for Servicemen. The services of a mathematical consultant and a noncredit refresher course in high-school algebra and geometry are included in a program adopted by the mathematics department, University of Illinois, Urbana. Funds for the program, which is to aid veterans to adjust themselves to the study of mathematics, are provided by the university's division of special service for former Servicemen.

Housing for Veterans at Michigan. To relieve the housing shortage created by the admission of married veterans, the University of Michigan, Ann Arbor, has leased 39 side-by-side two-family portable dwellings from the abandoned Willow Run housing project at the Willow Run Bomber Plant outside Ann Arbor. The units, which will be transported to a site on the edge of the campus, will accommodate 78 veterans' families, will cost $25 per month, and will be completely furnished.

Homes for Married Veterans at Yale. A bureau to handle the housing problems of married discharged Servicemen who have resumed their education at Yale University, New Haven, Conn., has been established. Property owners and real estate agents co-operate by supplying information about vacancies and suitable accommodations. Several houses owned by the university, which before the war were used as dormitories for unmarried students and during the war were occupied by soldiers attached to the Army training school units at the University, will be made available.

Gift to Charlottesville for Peace. A fund of $300,000 has been given by J. H. Jones, former United States secretary of commerce, to create a Woodrow Wilson school of international affairs at the University of Virginia, Charlottesville. The campus where public affairs and world peace will be discussed was recently the site for training United States officers for military government in foreign lands.

Guidance for Veterans at Stevens. A Veterans Administration Guidance Center has been established at Stevens Institute of Technology, Hoboken, N. J. The center will be equipped to serve 150 veterans per week including men and women. Its purpose is to give advice or consultation along either vocational or educational lines to veterans eligible for such service under the "G.I. Bill of Rights."

New Library for Princeton Students. A construction contract has been let for the Harvey S. Firestone Memorial Library at Princeton University, Princeton, N. J. At present Princeton University has 1,000,000 volumes. About 800,000 will be placed in the new library which will accommodate more than 1,700,000 volumes.

INDUSTRY • • • • •

Training Opportunities for Veterans. A procedure for approving institutions in New York State offering on-the-job training to veterans under the "G.I. Bill of Rights" has been put into operation. The system, worked out by the State Labor Department and division of veterans' affairs, is aimed at guaranteeing to the veteran training which will equip him to earn a decent living in the field of his choice. All institutions and firms offering training programs are thoroughly investigated before approval. An advisory group composed of representatives of management, workers, and veterans works closely with the assistant commissioners in passing upon the merits of each proposed program. Industrial Commissioner Edward Corsi has announced that training opportunities for as many as 400 veterans have been approved and made available in a single day's operations.

Wilmington Company to Develop Plastics. Plans for a new plastics laboratory at its Parlin, N. J., plant has been announced by the Hercules Powder Company, Wilmington, Del. The laboratory will serve as headquarters for development and research on plastics, and tests will be conducted there on three plastics—ethyl cellulose, cellulose acetate, and nitrocellulose. New mixing equipment used in making plastics for experimentation and a new drawing press for laminating plastics are included in the laboratory project. Construction will start early in 1946.

Lighting Exposition Planned. The first international lighting exposition and conference will be opened in Chicago, Ill., April 25, 1946. It is being sponsored by the National Electrical Manufacturers Association, New York, N. Y. Consulting engineers, architects, building managers, electrical contractors, and executives of industrial and commercial establishments will have an opportunity to obtain first-hand information about the better and more efficient lighting units for industrial plants and offices. As a part of the exposition a series of lighting conferences will be conducted at which recognized authorities will discuss the latest trends in lighting practice.

"Forticel" New Plastic Announced. Development of a new plastic, "Forticel," was recently announced by Celanese Plastics Corporation, a division of Celanese Corporation of America, New York, N. Y. Made from cellulose and propionic acid it was a laboratory plastic for years until the Celanese company succeeded in developing a new process for producing propionic acid from natural gases in commercial quantities and at reasonable prices. Forticel is the result of years of intensive research devoted to the study of scores of chemical combinations developed by introducing organic acid or alcohol radicals into the cellulose molecular chain. As a thermoplastic, the most economical process for making Forticel into finished articles is by injection and extrusion molding according to the company. The new plastic has a low specific gravity, and has excellent electrical properties. Because of low-moisture absorption, its electrical properties are said to be little affected by changes in atmospheric conditions. It is believed that the properties of this plastic will enlarge the field of applications of the widely used cellulosic plastics.

Notebook for Engineers. The *Neoprene Notebook*, a quarterly "sourcebook for engineers and designers" containing facts about Neoprene, has resumed publication as of September 1945. Established in 1938, it was issued regularly until the summer of 1942. It is published at Wilmington, Del., by the rubber chemicals division of the E. I. du pont de Nemours and Company, Inc.

Edison Centennial Committee Formed. Formation of the Thomas A. Edison Centennial Committee which will conduct the international observance of the 100th anniversary of the birth of Thomas A. Edison on February 11, 1947, has been announced. C. F. Kettering (A '04, F '14) president of the American Association for the Advancement of Science, and vice-president in charge of research, General Motors Corporation, Dayton, Ohio, will be chairman of the committee.

Westinghouse Laboratory Opens in West. A new laboratory to which western industry can bring for study its problems of peacetime production by making use of scientific methods of high-frequency heating was opened recently by the Westinghouse Electric Corporation in Los Angeles, Calif. Doctor Russell A. Nielsen (M '45), formerly of the company's research laboratories at East Pittsburgh, Pa., will be in charge of the new laboratory.

Chemical Industries Exhibit. The 20th Exposition of Chemical Industries will be held in Grand Central Palace, New York, N. Y., February 25 to March 2, 1946. The announcement stated that the exposition, coming at a time when industries are in the throes of reconversion, will serve as an accelerator, providing opportunities for personal contact and business conferences among principals, technical staffs, manufacturers, and professional consultants.

Bell System to Construct Electronic Plant. As a part of the Bell Telephone System's postwar construction program, Western Electric Company has purchased property and will construct a new electronic-components plant at Allentown, Pa. In the new plant the company plans to manufacture vacuum tubes, quartz crystals, temperature-sensitive devices, and other precision products for use in the System's communications facilities and in other commercial applications.

LETTERS TO THE EDITOR

INSTITUTE members and subscribers are invited to contribute to these columns expressions of opinion dealing with published articles, technical papers, or other subjects of general professional interest. While endeavoring to publish as many letters as possible, Electrical Engineering reserves the right to publish them in whole or in part or to reject them entirely. Statements in letters are expressly understood to be made by the writers. Publication here in no wise constitutes endorsement or recognition by the AIEE. All letters submitted for publication should be typewritten, double-spaced, not carbon copies. Any illustrations should be submitted in duplicate, one copy an inked drawing without lettering, the other lettered. Captions should be supplied for all illustrations.

Voltage and Current Relations for Controlled Rectification With Inductive and Generative Loads

To the Editor:

In his interesting investigation about certain rectifier problems[1] K. P. Puchlowski remarks very correctly that analytical methods to deal with some rather basic rectifier relations are insufficiently treated in technical literature. I found that a simple analytical solution of some rectifier problems is available, if as an approximation the sine wave $y=A\cos\omega t$ is substituted by a parabola

$$y=A\left(1-\frac{\omega^2t^2}{2}\right)$$

This approximation gives very accurate results for a rectifier working period near the peak of the sine wave. I published this method to attack some rectifier problems in connection with calculations of half-wave battery charger in *Electrical Engineer and Merchandiser* (Melbourne, Australia), July 15, 1944. A similar substitution of a sine wave

$y=A\sin\omega t$

near its change of polarity would be

$$y=A\left(\omega t-\frac{\omega^3t^3}{6}\right)$$

BENNO GERSTMANN (A '41)

(Electrical engineer, Australian Block and Chain Company, Melbourne)

REFERENCE

1. Voltage and Current Relations for Controlled Rectification With Inductive and Generative Loads, K. P. Puchlowski. AIEE *Transactions*, volume 64, 1945, May section, pages 255–60.

Nomogram for Frequency Formula

To the Editor:

The familiar equation

$f=\frac{1}{2\pi\sqrt{LC}}$, in which

L = inductance in henries
C = capacitance in farads
f = frequency in cycles per second

is awkward to compute, but it lends itself to the three-parallel-line nomogram of the utmost simplicity and usefulness, and this is solved by a single secant cutting the three scales. To visualize it, assume three central sliders from any ten-inch slide rule. Place these upright near each outer edge of a paper, with the bottoms having unit graduation against a horizontal base line so that the graduations read upward. These will be the L and C scales. Place the third midway between the two, but upside down, so that the graduations increase downward and so aligned vertically that the base line intersects it at the graduation $\frac{1}{2\pi}=0.159$. This will be the f scale and will require an extension. Use the same modulus (length from one to ten) for all three scales. Then, any straight line (computing secant) across all three, cuts out values that satisfy the equation. For $L=1$, $C=1$, $f=0.159$ as computed from the equation.

Actually, the graduations may be ticked off from the ten-inch modulus of the slider.

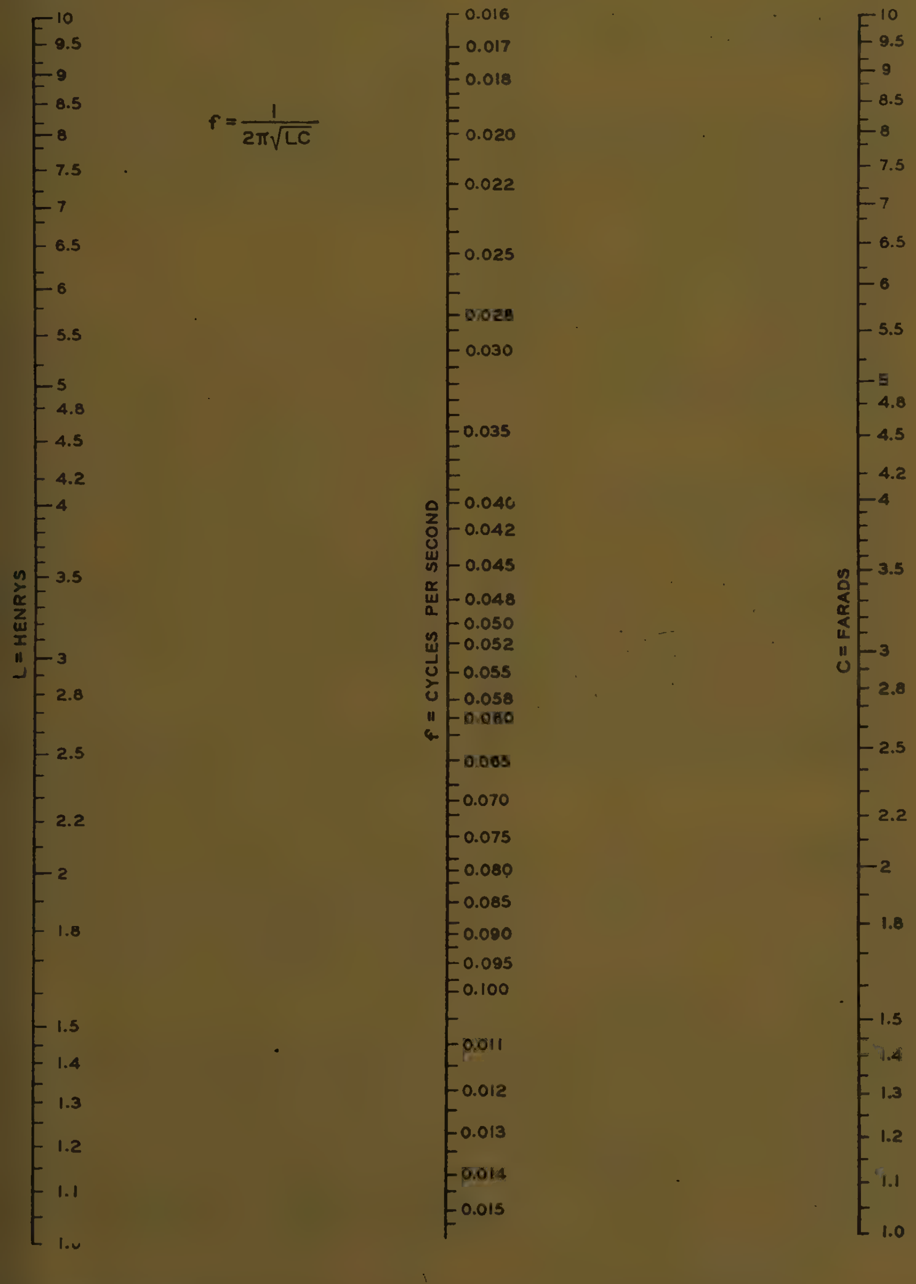
To increase its range the following rules are quite evident:

If *either* L or C is divided by 10^2, 10^4, and so forth, then f will be multiplied by 10, 10^2, . . .

If *either* L or C is multiplied by 10^2, 10^4, and so forth, then f will be divided by 10, 10^2, . . .

If *both* L and C are divided by 10, 10^2, 10^3, and so forth, then f will be multiplied by 10, 10^2, 10^3, . . .

If *both* L and C are multiplied by 10, 10^2, 10^3, and so forth, then f will be divided by 10, 10^2, 10^3, . . .



If inductance is in millihenries and capacitance in microfarads, the practical units, the inductance scale is to be multiplied by 10^3 and the capacitance scale by 10^6.

CARL P. NACHOD (A '07)

(Vice-president, Nachod and United States Signal Company, Inc., Louisville, Ky.)

NEW BOOKS . . . .

The following new books are among those recently received from the publishers. Books designated ESL are available at the Engineering Societies Library; these and thousands of other technical books may be borrowed from the library by mail by AIEE members. The Institute assumes no responsibility for statements made in the following summaries, information for which is taken from the prefaces of the books. All inquiries relating to the purchase of any book reviewed in these columns should be addressed to the publisher of the book in question.

Controllers for Electric Motors. By H. D. James and L. E. Markle. McGraw-Hill Book Company, New York, N. Y., 1945. 324 pages, illustrated, diagrams, charts, tables, 9 by 5³/₄ inches, cloth, $3.50. (ESL.)

The construction, performance, and operation of all types of commercial motor controls in general use are described. Protective devices are explained in detail, and brief instructions for installation and maintenance are included. Special control systems, such as for elevators, steel mills, and so forth, are not considered. The book is intended for technical students as well as for operating men and application engineers, and the treatment is such that only a limited knowledge of electric motors is required.

Electrical Power Uses in Marine Service. (Marine Electricians' Library, volume 3.) By J. M. Dodds. McGraw-Hill Book Company, New York, N. Y., and London, England, 1945. 444 pages, illustrated, diagrams, charts, tables, 8¹/₂ by 5¹/₄ inches, cloth, $4. (ESL.)

This volume presents the fundamentals of the ordinary incandescent lamp, as well as new applications of electrical principles in modern lamp sources. It also discusses the various new lamps for shore-plant and marine use, such as mercury-vapor lamps, marine searchlights, and the fluorescent lamp in illumination. The electrical propulsion of ships is discussed under two divisions: direct current for the relatively small power plant, and alternating current for the larger ship-propulsion plant.

Electromagnetic Engineering. Volume 1, Fundamentals. By R. W. P. King. McGraw-Hill Book Company, New York, N. Y., 1945. 580 pages, diagrams, charts, tables, 8¹/₂ by 5¹/₄ inches, cloth, $6. (ESL.)

This first volume of a three-volume series offers a systematic introduction to basic concepts of electromagnetism which are fundamental in the study of electromagnetic waves, antennas, electromagnetic horns, wave guides ultrahigh frequency, and microwave circuits. The physical and mathematical essentials of electrodynamics are developed logically and discussed critically for the purpose of application to engineering problems.

Procedure Handbook of Arc Welding Design and Practice. Eighth Edition. The Lincoln Electric Company, Cleveland 1, Ohio, 1945. 1,312 pages, illustrated, 6 by 9 inches, simulated leather, $1.50 in United States, $2.00 elsewhere.

Includes details of the most recent welding methods and techniques. Many new illustrations cover significant welding applications developed in various phases of war production. In addition to standard data on welding symbols, speeds and costs, characteristics of metals, preheating, and stress relieving, the revised handbook includes 16 new subjects, such as underwater cutting, shop ventilation, methods of testing, mathematical calculations for new weld-designed structures, and filler-metal specifications for arc-welding electrodes.

Elementary Electric-Circuit Theory. By R. H. Frazier. McGraw-Hill Book Company, New York, N. Y., 1945. 434 pages, $8^1/_4$ by $5^1/_4$ inches, cloth, $4.

Designed as a complete elementary exposition of electric-circuit theory requisite in the technical foundation of all students of electrical engineering regardless of their expected branches of specialization—electric power, communications, or electronics. A knowledge of physics that usually is acquired by the end of the sophomore year is adequate. Provides a basis for the more advanced study of power-system networks, communications networks, electron-tube circuits, and associated apparatus. By the use of problems a broad view of the frequency spectrum is given. The book is amply provided with references to historical sources, that show the origin of the various laws, principles, and theorems that are presented.

Thomas' Register of American Manufacturers. 35th edition, 1945. Thomas Publishing Company, New York, N. Y. No pagination, illustrated, $14^1/_4$ by 9 inches, cloth, $15. (ESL.)

This directory, over 6,000 pages in size, is a guide to the manufacturers and importers of practically every commercial product. The products are listed alphabetically in the main list, with the names and addresses of manufacturers and their capital ratings. An alphabetical list of manufacturers shows their branch offices. An extensive list of brand names enables manufacturers of brand goods to be found.

Piping Handbook. 4th edition. By S. Crocker. McGraw-Hill Book Company, New York, N. Y., and London, England, 1945. 1,376 pages, illustrated $7^1/_4$ by $4^1/_2$ inches, cloth, $7.00. (ESL.)

This standard work makes available under one cover a compilation of data for the effective design and use of piping in all its engineering and industrial applications. The book authoritatively covers scientific fundamentals, materials, design and installation practice, and many useful construction details. Separate chapters are devoted to closely allied topics, such as pumps and corrosion.

Plastics in Practice, a Handbook of Product Applications. By J. Sasso and M. A. Brown, Jr. McGraw-Hill Book Company, New York, N. Y., and London, England, 1945. 185 pages, illustrated, diagrams, charts, tables, 11 by $7^1/_2$ inches, cloth, $4. (ESL.)

This book brings within one cover a practical review of plastics in all their successful commercial applications, providing a handy key to the essential facts on how and why plastics are used. The treatment is organized around 103 actual uses of plastics, reviewing the problems encountered in developing them and outlining the reasons for the solutions adopted.

Power-System Stability, Volume 1, Steady-State Stability. By S. B. Crary. John Wiley and Sons, New York, N. Y.; Chapman and Hall, London, England, 1945. 291 pages, illustrated, diagrams, charts, tables, $8^3/_4$ by $5^1/_4$ inches, cloth, $4. (ESL.)

The fundamental theory underlying maintenance of continuous electric power under normal and abnormal operating conditions is to be provided by a two-volume set of which this is the first. The present volume devotes the first five chapters to the theory required for a study of the steady-state-stability characteristics of a system; the last seven chapters discuss the applications of the theory to system design. The theory of transient stability will be covered in the second volume.

Theory of Functions. Part one, Elements of the General Theory of Analytic Functions. By K. Knopp, translated by F. Bagemihl. Dover Publications, New York, N. Y., 1945. 146 pages, diagrams, tables, 7 by 4 inches, cloth, $1.25. (ESL.)

This is the first volume of a translation of Doctor Knopp's well-known monograph on the theory of functions. It is devoted to the fundamentals of the theory of functions of a complex variable: basic concepts, integral theorems, series and the expansion of analytic functions in series, singularities. There is a brief bibliography.

Introduction to Microwaves. By S. Ramo. McGraw-Hill Book Company, New York and London, 1945. 138 pages, $8^1/_2$ by $5^1/_4$ inches, cloth, $1.75. (ESL.)

The author presents a simple, nonmathematical discussion of microwaves, electricity with a frequency of alternation in billions of cycles per second. Physical pictures in the form of line diagrams are extensively used to illustrate the concepts discussed. The book is intended for both the beginner in the field and the nonspecialist who wants a brief treatment of the subject.

UHF Radio Simplified. By M. S. Kiver. D. Van Nostrand Co., New York, N. Y., 1945. 238 pages, illustrated, diagrams, chart, tables, $8^1/_2$ by $5^1/_2$ inches, cloth, $3.25. (ESL.)

The concepts of ultrahigh-frequency radio are presented as logical outgrowths of the more familiar low-frequency equipment. The opening chapter provides this transition, while the following chapters successively describe the important pieces of equipment—magnetic oscillators, wave guides, cavity resonators, antennas, measuring instruments, and so forth—and explain simply the principles of the production, transmission, and measuring of ultrahigh-frequency currents and waves.

Electronics Dictionary. By N. M. Cooke and John Markus. McGraw-Hill Book Company, Inc., New York, N. Y., 1945. 433 pages, illustrated, $5^1/_2$ by $8^3/_4$ inches, $5.

This is an up-to-date glossary containing definitions of more than 6,000 terms used in radio, television, industrial electronics, communications, facsimile, and sound recording. The book is written primarily at the level of the student, but the language of electronics is explained also for the engineers who must develop applications for electronic equipment and for those who must use and maintain such equipment. The authors have tried to set up a complete and consistent policy for abbreviating electronic terms based upon the groundwork laid by American Standards Association.

Electroplating, a Survey of Modern Practice, including the Analysis of Solutions. By S. Field and A. D. Weill. 5th edition. Pitman Publishing Corporation, New York, N. Y., and Chicago, Ill., 1945. 483 pages, illustrated, 7 by 5 inches, cloth, $5. (ESL.)

The object of this book is to provide the practical plater with a review of the technical advances in his field. The first ten chapters are devoted to fundamental principles of chemistry, metallurgy, electricity and electrodeposition, and to the mechanical or chemical cleaning of metal surfaces. The following ten chapters give practical information for the deposition of specific metals and alloys. Metal coloring is briefly discussed in the final chapter.

PAMPHLETS • • • •

The following recently issued pamphlets may be of interest to readers of "Electrical Engineering." All inquries should be addressed to the issuers.

Index of Microfilms (Series A—Lots 1–1737). Library of Congress, Photograph Section. Washington, D. C., 1945, 26 pages.

X-Ray Diffraction. Basic Theory, Principles and Applications. North American Philips Company, Inc., New York, N. Y. Text prepared by F. G. Firth. 16 pages. (Reprinted from April, May, June 1945 issues of *Petroleum Refiner*.)

Engineering-Design Development of X-Ray Spectrometer. J. S. Buhler. North American Philips Company, Inc., New York 17, N. Y., 12 pages. (Reprinted from June 1945 *Electrical Manufacturing*.)

Use of Low-Angle X-Ray Scattering in the Study of Catalysts, Viruses, and Other Materials. F. G. Firth. North American Philips Company, Inc., New York, N. Y., 4 pages. (Reprinted from *Rubber Age*, August 1945).

Government Guaranteed Production and Termination Loans. By R. A. Foulke. Dun and Bradstreet, Inc., 35 pages. (Reprint of three articles which appeared in *Dun's Review*.)

Scientists of Tomorrow. (Essays and report of the fourth annual science-talent search for the Westinghouse scholarships.) Science Service, Washington 6, D. C., 136 pages.

TRANSACTIONS SECTION

Preprint of Corresponding Pages From the Current Annual AIEE Transactions Volume

Statistical Methods Applied to Insulator Development and Manufacture

J. J. TAYLOR
ASSOCIATE AIEE

Synopsis: The science of statistics is highly developed and applicable to a variety of engineering subjects. There remains the task of getting statistical methods out of books and into general use. Towards this end examples are given of problems that relate to high-voltage-insulator manufacture and for which statistical treatment seems desirable.

IN THE MANUFACTURE of high-voltage insulators almost every incentive for statistical analysis is present. The product is produced in large quantity and to meet an exacting service. Product characteristics are determined by tests carried to destruction. Manufacturing control is difficult, because insulators are made in part from natural clays and are processed through incomplete chemical reactions. In service the finished product is exposed to all the vagaries of weather in almost every climate, and is expected to endure for a length of time far beyond a reasonable proof-test interval.

It is apparent, therefore, that the element of chance will be present at every stage of product use and manufacture. To deal effectively with the electrical and mechanical problems of the insulator industry the usual engineering conceptions shown in Figures 1A and 1B should be supplemented by an equivalent group of statistical tools and diagrams.

Statistical Distributions

Many product characteristics that are measured, studied, and specified show a relation between observed values and the frequency with which they occur. If observations are grouped as shown in Figure 2, the distribution is called "normal" and is defined by the value of the arithmetic mean, $\overline{X}$, and by the standard deviation. The standard deviation, σ, is a measure of variability and is the root-mean-square of all the deviations of the individual values from the mean value. It is analogous therefore to the rms value of alternating current.

To carry this analogy further, a sine wave is more often approximated than realized and so also is a "normal" curve. Where the departure is slight, an assumption of normality may introduce no intolerable error and permits the use of many published tables. Nonnormality in a product characteristic does not prove any deficiency in either the product or its control. In some instances, however, direct physical reasons for nonnormal distributions exist and can be ascertained.

In 1934 an excellent statistical manual published by American Society for Testing Materials[1] served as an incentive for the statistical study of a variety of suspension insulators. Frequency diagrams of ultimate-strength tests were plotted from data accumulated in the years preceding. Of the insulator types studied, two were found with distribution curves that proved later to be of particular interest. They are illustrated in Figure 3. A normal curve superimposed on distribution *A* indicates considerable flatness distortion or kurtosis. Distribution *B* is decidedly skewed, with less variability than would be expected in the upper range of observed values.

In both cases a direct physical cause was found for these abnormalities. The flatness of curve *A* resulted from a change in the firing cycle of the porcelain component. Figure 4[7] shows, in an exaggerated manner, how a shift in the average value of test groups produces a flat-top shape when all observations are plotted together.

The abnormality of Figure 3B was caused by a peculiarity of design and requires for its explanation a brief review of insulator construction. A suspension insulator has internal and external metal parts attached with Portland cement to a ceramic shape. In tests of ultimate strength the fracture occurs usually through the ceramic component—a not surprising result since ceramic materials fail at a unit deformation of about 0.1 per cent. The comparable figure for malleable cast iron is 200 times as great. In a design in which the metal sections and the cement matrix areas are liberal, the cement and metal respond under load in accordance with their respective elastic moduli. As a general rule, the elastic modulus of structural materials is exceptionally reliable and consistent; the ultimate strength is variable. One would expect to find, therefore, that the variability of an insulator assembly is determined largely by the variability in ultimate strength of its ceramic component, rather than by erratic performance of the metal parts or cement matrix. The skewness of Figure 3B was attributable to a cap section so thin that its yield point was reached in the upper range of observations. This yield prevented normal support of the ceramic section and development of the potential strength of the strongest ceramic components.

Similar nonnormal distributions to which a physical cause can be assigned are found in high-voltage measurements. Two of these are illustrated in Figure 5. In the parallel arrangement of insulator and sphere gap, a marked lack of symmetry is noticeable in the distribution of

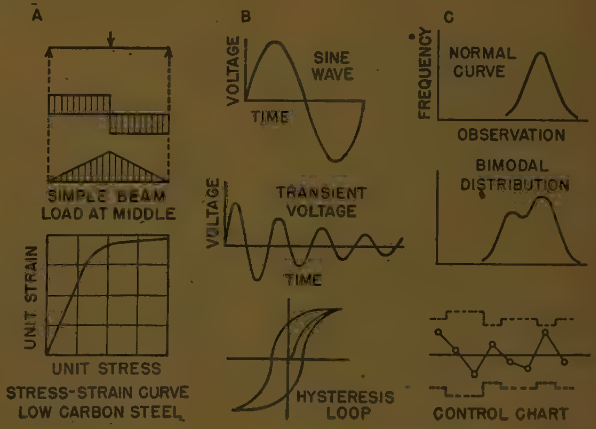


Figure 1. Diagrams of mechanical, electrical, and statistical performance

A. Mechanical
B. Electrical
C. Statistical

Paper 45-89, recommended by the AIEE subcommittee on applications of statistical methods of the Standards committee for publication in AIEE TRANSACTIONS. Manuscript submitted November 20, 1944; made available for printing March 8, 1945.

J. J. TAYLOR is chief engineer, insulator division, Ohio Brass Company, Barberton, Ohio.

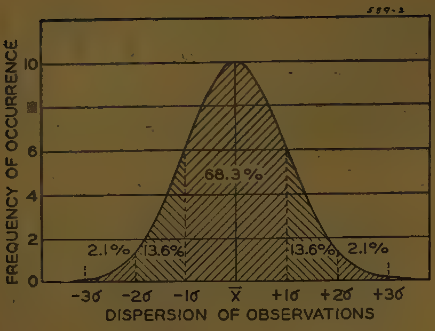


Figure 2. Significant areas and limits of a normal distribution

measurements. The consistent flashover value of the sphere gap cuts off the maximum end of the insulator-flashover distribution pattern.

The opposite type of skewness was observed in tests of imperfect spheres. A sharp projection on the grounded sphere introduced a discontinuity in the otherwise uniform electrostatic field. Flashover could occur from sphere to sphere or sphere to point. In the distribution shown conditions were such that the majority of the flashovers were between sphere and point.

Control of Product Quality

Since a great variety of insulator designs start from a common source of raw materials and go through essentially the same manufacturing process, a measure of quality control can be assured by observations on a few important types. The distribution for one such key insulator is shown in Figure 6. This type of insulator is manufactured in large quantity and, fortunately, has a frequency distribution approximately normal. The design was first produced in 1933. Control limits were not set up until 1937 and were then readily established by a study of accumulated test data.

Daily tests are made on units selected at random from current production and on factory rejects with imperfections that do not influence the characteristic being observed. As shown in Figure 7, the control limits are adjusted for variations in sample size. Sample averages falling beyond the control limits indicate an assignable cause of variation and are a signal for technical inquiry and corrective action.

Table I. Endurance Test of Suspension Insulators

Tested at 80 Per Cent of Average Ultimate Strength

Specimen Number	Placed on Test	Date of Failure	Elapsed Time
1	11-17-38	11-17-38	10 hours
2	11-17-38	3-12-39	115 days
3	11-17-38	1-14-39	58 days
4	11-17-38	11-22-38	5.1 days
5	11-17-38	8-23-39	279 days
6	11-17-38	11-18-38	1.1 days
7	11-17-38	12-8-38	21 days
8	11-17-38	2-13-39	88 days
9	11-17-38	12-3-38	16 days
10	11-17-38	11-17-38	7 hours
11	11-17-38	11-17-38	0.5 hours
12	11-17-38	3-17-42	1,216 days
13	11-17-38	11-21-38	4.2 days
14	11-17-38	11-25-38	8 days
15	11-17-38	11-28-38	11 days

Quality Control in Relation to Routine Tests

Ceramic parts may be inspected visually, sometimes with the aid of penetrating fluids, or aurally by "ringing-out" methods. Attention has been given also to X-ray examination and to nondestructive tests employing supersonic response. Many tests, however, are potentially destructive and would produce failure if increased sufficiently in severity. Dielectric soundness is checked by high frequency or 60-cycle flashover on ceramic parts and assembled units. For certain specialized types of insulators, impulse tests are routine procedure. Routine mechanical tests of assemblies are made at a load level about half the rated ultimate strength and are followed by a dielectric test.

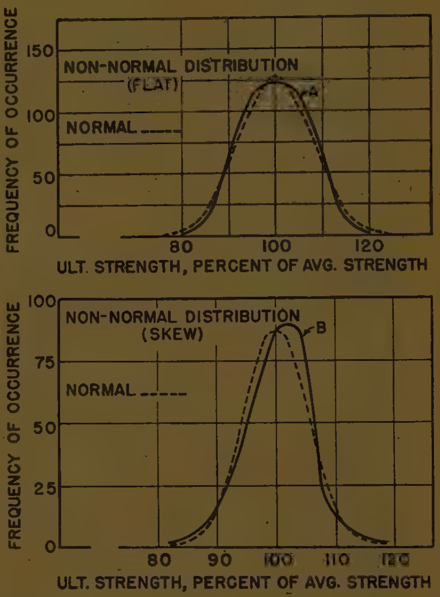


Figure 3. Nonnormal distributions of suspension-insulator strength

As routine tests for both dielectric and mechanical strength are potentially destructive, some compromise is necessary between tests too mild to eliminate defectives and those severe enough to damage an appreciable portion of specimens near the minimum end of the distribution pattern.

Mechanical tests at half the rated strength are not intended to cut off the minimum end of the distribution curve. They are used to give assurance against defectives with potential characteristics far below the minimum level of expectancy.

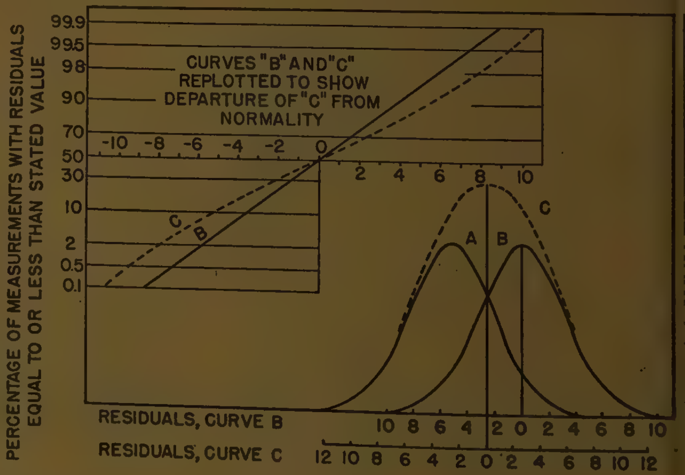
It cannot safely be assumed that high levels of routine test guarantee quality. Such tests may be particularly undesirable on a product manufactured with good factory control but with no great margin in average strength. In Figure 8 two distribution curves are shown that have the same probability of failure at or below the specified minimum level. If both groups of product are proof-tested to eliminate units lying below the minimum line and if the test is harmful to units only slightly stronger than the proof load, then it is likely that such testing will be detrimental to design (*a*) and beneficial to design (*b*). In the first case it damages more units than it eliminates and in the other case eliminates more units than it damages.

It is probable that no amount of proof testing, however drastic, can select from carelessly manufactured material a finished batch as dependable as one made in the first instance with good technical control and given routine tests at reasonable levels. This view implies, in specifications, emphasis on consistency of tested characteristics; a carefully drawn sampling plan; a cumulative record of test history so that past experience can be of real value in interpreting the results of samples under immediate review.

It should be noted also that some product characteristics cannot be given routine tests that will check them. Line and station insulators are designed and design-

Figure 4. The summation of two normal curves with different $\bar{X}$ values is nonnormal

Inset shows curves *B* and *C* with abscissa values plotted against a probability scale of ordinates. A normal curve so plotted gives a straight line



tested to withstand electrical surges of great severity, but so far it has not been practical to surge-test at these extreme levels in routine fashion. Even if it were possible, it might be proved an undesirable practice because of the damaging nature of the tests. In such instances reliance must be placed on methods of quality control which give high-probability assurance that the required characteristic is present.

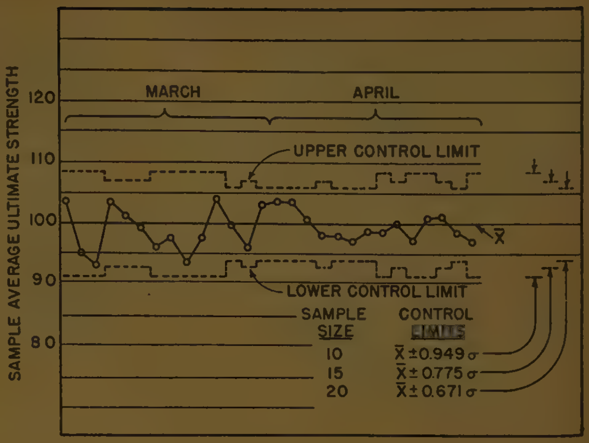


Figure 7. Suspension-insulator control chart

Control is exercised by limiting fluctuation of sample averages with respect to assumed grand average X̄

Estimating and Predicting From Statistical Data

Specifications frequently require certain minimum or maximum characteristics in the product they cover. The best that a manufacturer can do, by conservative rating of his product and by technical control of process, is reduce the probability of a "defective" to some tolerable level. To choose a small sample from a large batch and in that small sample to focus attention on a single value, to the exclusion of other equally significant values and to the exclusion also of the supplier's over-all performance, is certainly uneconomical and may also be unfair. It is quite possible for a chance defective to appear in a product of good quality; and it is more than possible that poor quality will not be detected by looking for a single defective in a small sample. As product characteristics are by nature statistical, assurance of quality in purchasing standards logically might be obtained by stating limiting criteria for the frequency distribution of the quality characteristic in question. Consideration certainly should be given to the average, which is readily established from a small number of samples; to the shift in observed average from batch to batch; and, if additional assurance is needed and when the accumulation of data is adequate, to some measure of the dispersion of values about the average.

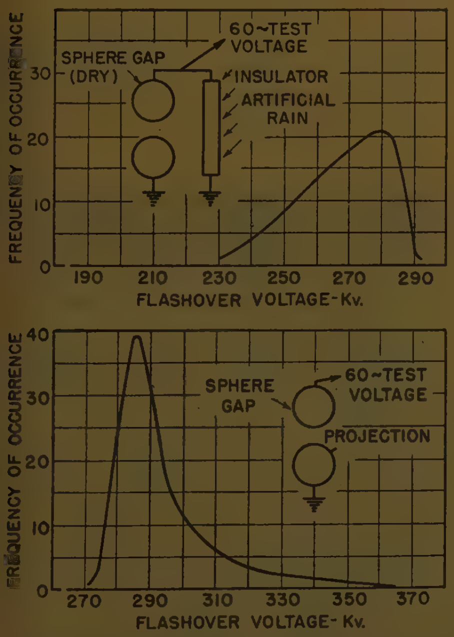


Figure 5. Assymetrical 60-cycle flashover distributions having an assignable cause

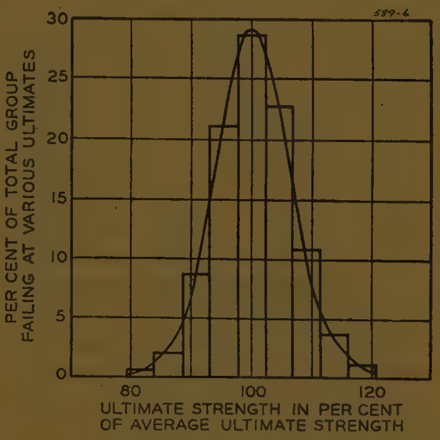


Figure 6. Nine-cell distribution of ultimate strength for a suspension insulator

Although statistical methods cannot be expected to indicate such probabilities as one unit in a thousand being below minimum, they are nevertheless the most effective means for estimating limits beyond which not more than four or five units out of a thousand are likely to lie. Their use is indicated particularly in work on new designs for which test information is meager. Illustrative of the general problem of sampling are trials made from a batch of 1,000 specimens having a normal type of distribution and with the specimens divided into nine distribution "cells." The characteristics of the parent distribution were:

Average—19,500 pounds
Maximum—23,100 pounds
Minimum—15,900 pounds
Standard deviation—1,245 pounds

As this was an artificially prepared batch, the actual minimum was known to be 15,900 pounds; yet samples of nine specimens each showed the appearance of a minimum unit in only ten per cent of the trials. Even with samples of 50 specimens each, an error of at least 1,000 pounds in observed minimum versus batch minimum occurred in 60 per cent of the trials. In 20 per cent of the trials the discrepancy between observed and batch minimum was more than 2,000 pounds. When the samples reported were analyzed statistically and the expected minimum calculated rather than observed, the error, even on the most misleading sample, was only 600 pounds. The most misleading sample was selected from 40 trials in each of which the sample size was 50 specimens.

Correlation of Product Characteristics

In some instances the determination of a product characteristic is inordinately time-consuming or expensive. If such a characteristic can be related intimately with one for which data are already available (or can be readily obtained) three advantages ensue:

1. The desired characteristic may be estimated.

2. Meager data about the desired characteristic may be expanded and confirmed.

3. Both characteristics may become more intelligible.

Since the characteristics sought are statistical distributions, correlation hardly can be expected between individual specimens. At least in tests that are of the destructive type the correlation must be between frequency-distribution diagrams or between terms defining these diagrams. An example of this approach is

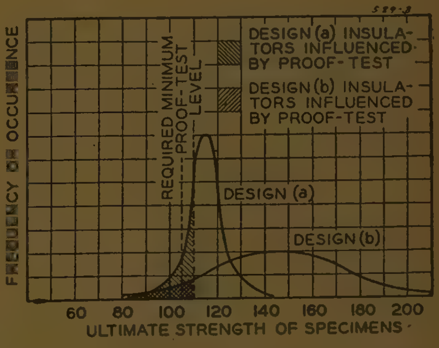


Figure 8. Proof testing in relation to strength distribution

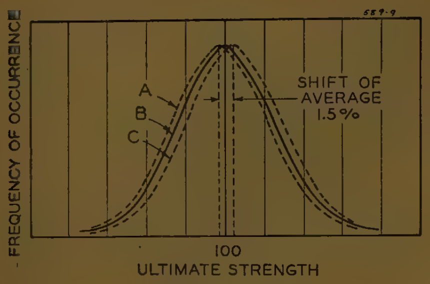


Figure 9. Change in observed average strength caused by different rates of load application

Increase in load per minute:
A—15 per cent of rated strength (minimum)
B—22½ per cent of rated strength
C—30 per cent of rated strength (maximum)

the relation between the ultimate strength of suspension insulators and their performance when endurance-tested, for extremely long intervals of time, by loads of varying severity.

In endurance or time–load testing, insulators are subjected to axial tension for a stated interval of time. In many specifications this is a "withstanding" test in which the failure of any unit is cause for rejection of the lot. To the design engineer, therefore, endurance tests carried to failure are a necessary part of his information about the product. A typical group of endurance-test results is shown in Table I.

It is obvious that interpretive effort has to be made if significant information is to be obtained from these test values. A clue is offered by the shift in strength observed when insulator groups are tension-tested at different rates of load application.

When insulators are tested for ultimate strength, the increase in load per minute may be from 15 to 30 per cent of the rated ultimate strength of the specimen.[2] At these limiting rates the distributions shown in Figure 9 were found. The shift in average strength was about 1.5 per cent.

There is an element of endurance testing in any ultimate-strength test, since the specimen responds to a combination of time and load. Some indication of this is given by Figure 9. The influence of time can be emphasized by rates of load increase that are exceedingly slow.

If the ultimate strength of a particular specimen is designated as 100 per cent, some contribution to the final destruction of the specimen is made by the load levels immediately preceding—99, 98, 97 per cent, and so on. For electrical porcelain loading history up to 85 or 90 per cent of the ultimate appears to make a negligible contribution to failure. This rapid decrease in sensitivity to load implies some type of exponential function. Moreover, since damage by the 99-per-cent, 98-per-cent, and preceding load levels has been assumed, it can be argued that, if it were possible to apply a 100-per-cent load instantaneously yet without shock, failure would not take place immediately.

By a series of tests[3] run at slow rates of load increase it was found that the interval to failure following application of a "100-per-cent" load was about equal to the time spent in going, at a uniform rate of load increase, from the 98-per-cent to the 100-per-cent level. Let us suppose, for example, that an insulator is tested at a 4,000 pounds per minute rate and that failure occurs at 20,000 pounds. The load change from the 98-per-cent to the 100-per-cent value is 400 pounds, and this change is effected in 0.1 minute. The expectation is, then, that, had a load of 20,000 pounds been applied instantaneously and without shock to this particular specimen, failure would have taken place in 0.1 minute.

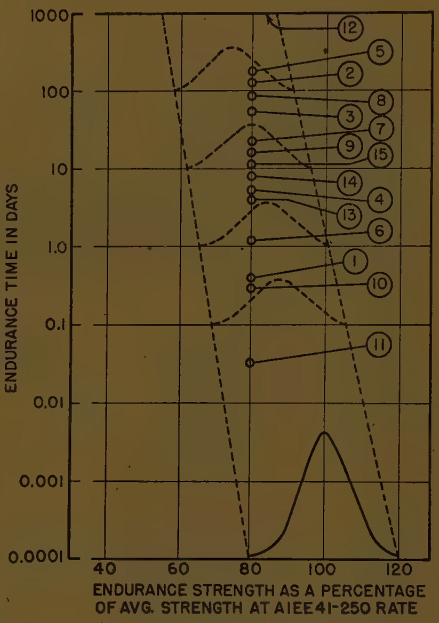


Figure 10. Values from Table I plotted on an endurance-test chart for suspension insulators

In Figure 9, a 100-per-cent change in the rate of load application produced a shift in the distribution pattern of only 1.5 per cent of the observed average ultimate strength. By changing the rates of load increase per minute according to some suitable exponential regression such as 1,000, 100, 10, 1, 0.1, and by testing at each chosen rate a liberal sample of specimens from the same batch of product, a series of distributions is obtained with a three-dimensional relation: observed strength, frequency of occurrence, and loading rate or equivalent endurance time. This expands Figure 9 into the developments shown in Figure 10. On Figure 10 the tabular values for 1938–42 appear rational and significant; they are part of a very much-extended frequency distribution, the plane of which is parallel to the time axis. The model illustrated in Figure 11 shows what might be expected of a complete batch of product if it were subjected to a fixed load and this load maintained without change until every unit had failed.

In concluding this effort at correlation it is interesting to compare original and final interpretations of the same set of data. Following a perfunctory study of the original endurance-test values, reasonable conclusions might be:

1. That the product was extremely erratic.

2. That with better endurance-test strength more consistent intervals to failure would be obtained.

After an analysis of Figures 9, 10, and 11, a very different set of conclusions is reached:

1. The endurance-test observations are consistent with the distribution pattern for ultimate tensile strength.

2. A product with perfect endurance-test resistance, subjected to a continuing load of a size between the limits of its quick-strength pattern would have

(*a*). Immediate failures of minimum strength specimens.

(*b*). Indefinite endurance of maximum-strength specimens.

3. Time–load sensitive materials such as lead would show consistent intervals to failure because of increased angularity be-

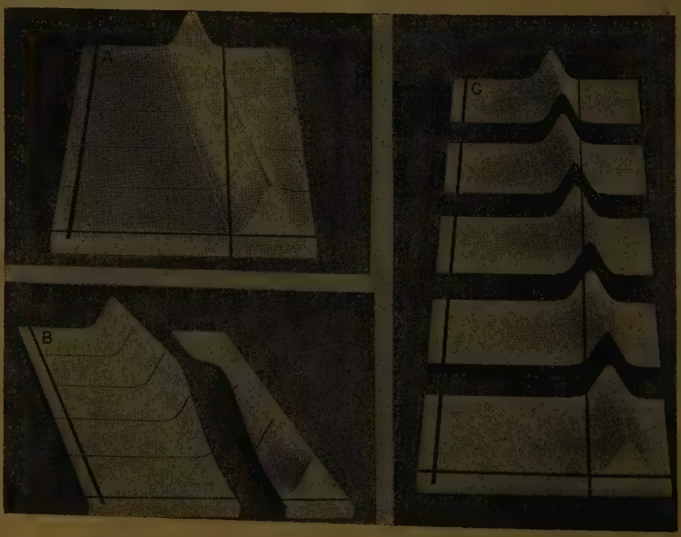


Figure 11. Model of Figure 10

Showing:
A—The three-dimensional nature of endurance strength
B—The elongated distribution parallel to the time axis associated with a specific value of test load
C—The influence of width of the distribution pattern in determining endurance time

tween the central plane of the time–frequency–strength solid and the time axis.

4. Statistical methods aid in building a performance structure from which modest extrapolations can be made.

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Winding-Fault Detection and Location by Surge-Comparison Testing

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Synopsis: Surge-comparison methods of testing insulation were investigated with the objective of locating as well as detecting faults. Experience of other investigators was confirmed and limitations of methods and equipment studied. Supplementary methods and equipment were devised to extend the field of usefulness of this basic type of tester, especially to the testing of very large machines with multiple-parallel windings. Apparatus studied included a-c motors from fractional-horsepower single-phase to very large three-phase types; a large variety of d-c armatures; and numerous armature, stator, and magnet coils. This surge-comparison tester has proved to be suitable for use by electrical manufacturers and by service and repair shops, as it can be used for testing most common types of electric windings.

SURGE-COMPARISON testing of electric windings is a new and very useful tool for detecting and locating insulation faults. Foust and Rohats[1] are to be commended for their great contribution to the art. The surge-comparison principle in combination with the cathode-ray oscilloscope opens the way to numerous possibilities for improving insulation testing equipment and technique. The application of a known and controlled voltage surge with visual comparison of the wave shape makes it possible to study insulation faults as well as to detect them.

The work reported by Foust and Rohats[1] was confined to testing of three-phase windings. The authors have extended the field of usefulness of the surge-comparison tester, applying it in its original form and with important modifications to numerous other types of apparatus. Other apparatus studied included d-c armatures, fractional-horsepower single-phase stators, and large three-phase stators. Means for fault location were developed to supplement the original scheme which included only fault detection. Voltage distribution was studied on several types of apparatus.

Description of Test Equipment

The surge-comparison tester is an electronic device designed for two functions:

1. To apply a voltage stress on turn-to-turn, phase-to-phase, and winding-to-ground insulation in coils and wound apparatus.

2. To detect grounded or short-circuited turns in the windings under test.

The turn and phase insulation is stressed by the application of a repetitive surge voltage to the winding terminals. The surge is produced by a charged capacitor, and a high rate of voltage rise is obtained. Because of this high rate of rise, a relatively high test voltage can be applied to most types of windings and coils. The detection of faults is accomplished by means of a cathode-ray oscilloscope on which is impressed the surge voltage across each part or half of the winding alternately. Two standing waves thus appear simultaneously on the oscillograph screen (screen persistence retains the traces between alternate cycles). A divergence or difference in these two waves reveals a dissymmetry in the winding which generally indicates the presence of a fault. The alternate appearance of the two waves is brought about by a synchronous reversing switch which interchanges the surge and ground connections repeatedly. The two detection voltages also may be obtained from exploring coils or other devices close to the winding under test. The applied surge voltage may be varied from zero to maximum as readily as the voltage of the ordinary 60-cycle test box. No other circuit constants can be conveniently changed.

Test Methods

The surge tester used by previous investigators was designed primarily for testing small three-phase induction motors. Three output terminals were provided for connection to the three terminals of the test motor. The surge and the oscilloscope could be connected to any combination of the three motor terminals by means of a manual transfer switch built into the tester as shown in the diagram of Figure 1A. Nearly all small three-phase motors can be tested for weak insulation and faults in the manner illustrated by this diagram.

Individual coils, single-phase motors, or other pieces having no inherent three-phase symmetry may be tested in pairs as shown in Figure 1B. Here one coil or winding of a type is used as a standard for comparison testing of other similar pieces. This method was applied to all coils and miscellaneous windings tested. A comparison method also was found useful in d-c armature testing. The surge was applied to the commutator at points one brush span apart with a third contact midway between to form the mid-point for the two-winding comparison scheme. Figure 1C illustrates this simple adaptation.

Variations from the connection arrangements of Figure 1A and 1B were made for certain tests and will be described in a discussion of the results. The surge test voltage used was generally made equal to the peak of the specified rms high-frequency test[2] voltage.

Cores or frames of all apparatus tested were always grounded during test for several reasons: to increase the operator's safety, to keep the voltage from conductors to iron at a known value, and to eliminate the effects of minor variations in capacitive couplings. The surge-tester frame also was always connected to a ground.

Results of Tests

The application of the surge tester and the results obtained will be described briefly for each type of apparatus tested.

Armature and Stator Coils

Coils of various sizes from three-turn coils 16 inches long to ten-turn coils 36 inches long were tested as in Figure 1B. Faulty insulation and short-circuited turns were readily detectable in most coils. Examples are shown by the delineation, Figure 2A, of the oscilloscope-

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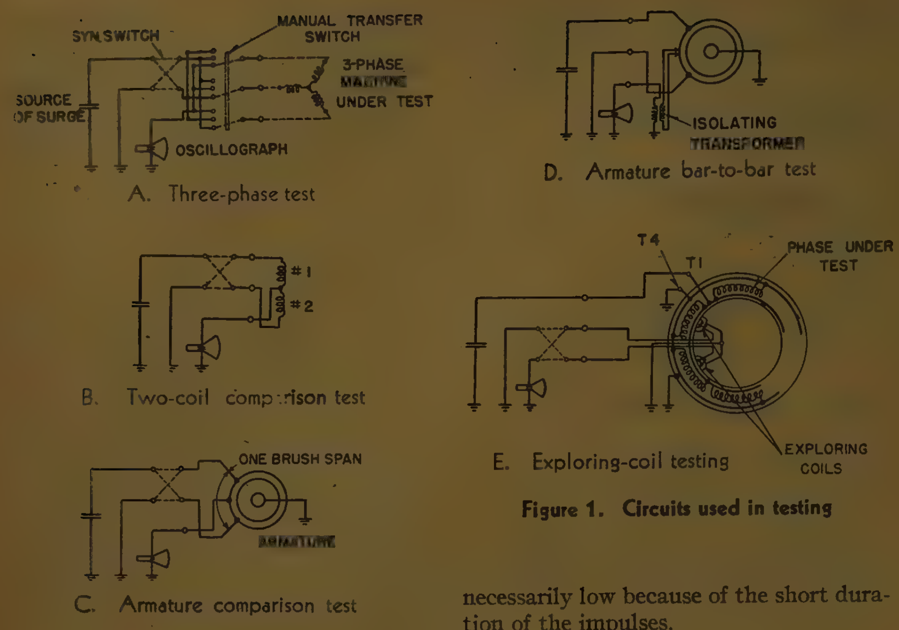

A. Three-phase test

B. Two-coil comparison test

C. Armature comparison test

D. Armature bar-to-bar test

E. Exploring-coil testing

Figure 1. Circuits used in testing

voltage traces of two ten-turn synchronous-motor coils. Corona or arcing present in some coils could be detected readily and the voltage at which it started determined. Burning or heating of faults in coils usually was not great enough to facilitate exact location of the faults. This was due to the fact that the average power output of the surge generator is necessarily low because of the short duration of the impulses.

Field Coils

Many types of field coils were tested using the connection of Figure 1B. Excellent detection of one short-circuited turn was obtained with coils having either air or iron cores. Slightly better results were obtained with the types having the iron pole in place, but as shown by Figure 2B, detection of a short-circuited turn in a 477-turn air-core coil was very satisfactory.

Armature

Numerous types of railway and d-c generator armatures were tested. Dead grounds and short circuits were detected very satisfactorily using the comparison connection, Figure 1C. A typical example is Figure 3A which shows clearly the visual evidence of a dead short-circuited coil midway between the surge and oscillograph contacts on the commutator of Figure 1C. Armatures lying on the floor were tested by this method by use of a portable contact arm. Insulation could be tested completely by moving the contacts once around the commutator. Movement through one brush span served to detect faults as a preliminary test.

The comparison method was found inadequate for locating a short circuit or ground in cross-connected or wave-wound armatures. To overcome this difficulty, the bar-to-bar voltage-measuring circuit of Figure 1D was devised from equipment used with the high-frequency test.[2] With this arrangement, dead short-circuited or grounded coils were located exactly and quickly in any of the armature types tested. Arcing or intermittent short circuits were detected easily by either method, but too few examples of this type of fault were found to permit development of a satisfactory means of locating them.

Single-Phase Motors

The testing of fractional-horsepower single-phase motors was accomplished by

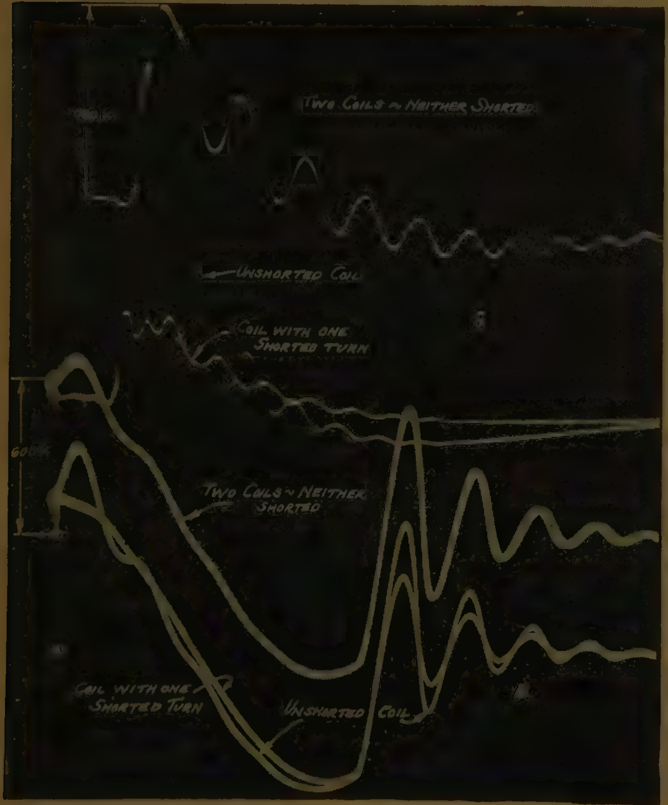

Figure 2. Short-circuited turn detection in stator and field coils

A. Ten-turn stator coils
B. 477-turn field coils. (No iron present)

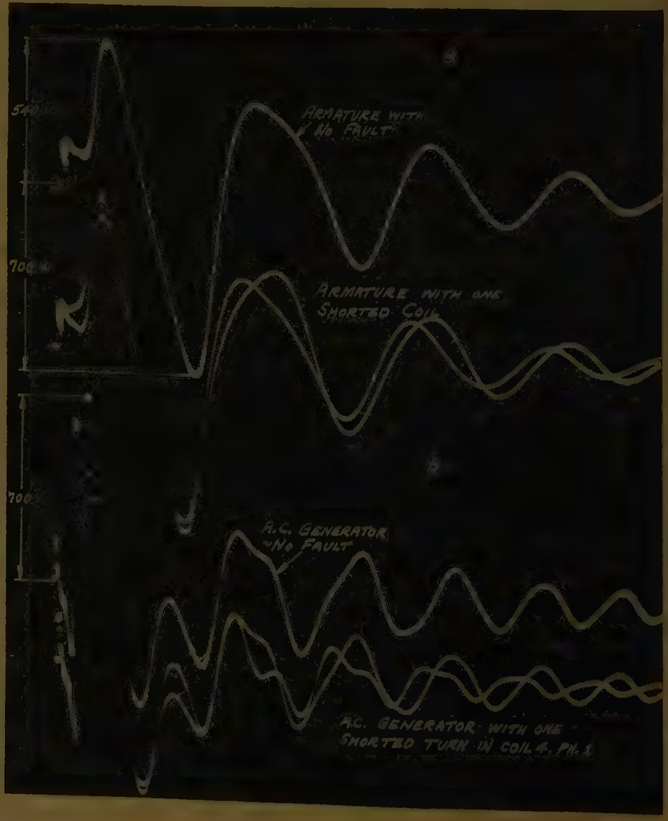

Figure 3. Short-circuited turn detection in a large d-c armature and in an a-c generator

A. Large d-c motor armature
B. Large a-c-generator

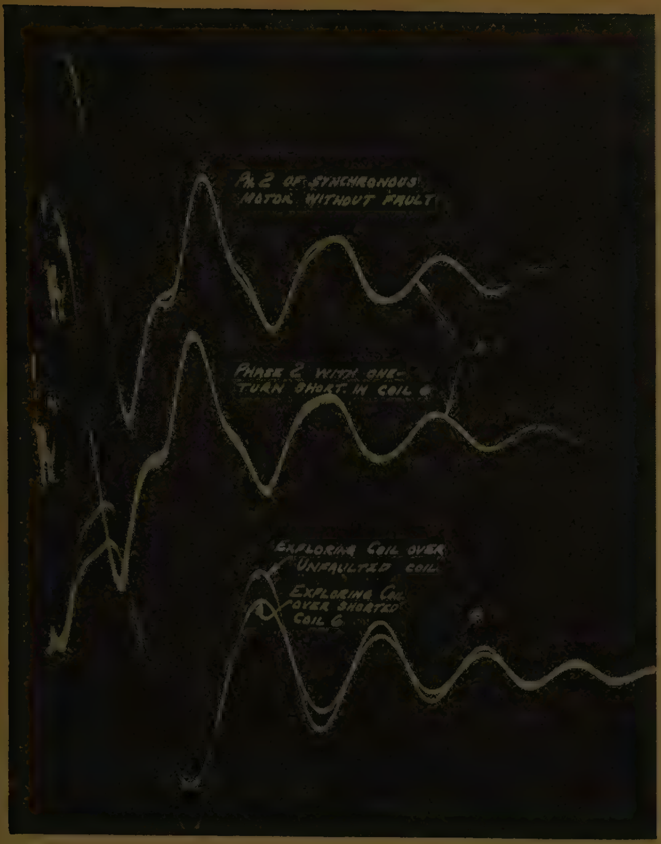

Figure 4. Tests on large synchronous motors

Comparison of detection of a single-turn short circuit in a winding having 12 parallel conductors per phase when tested by the two methods: simple phase comparison and exploring-coil comparison

A. Three-phase comparison test

B. Single-phase test using exploring coils

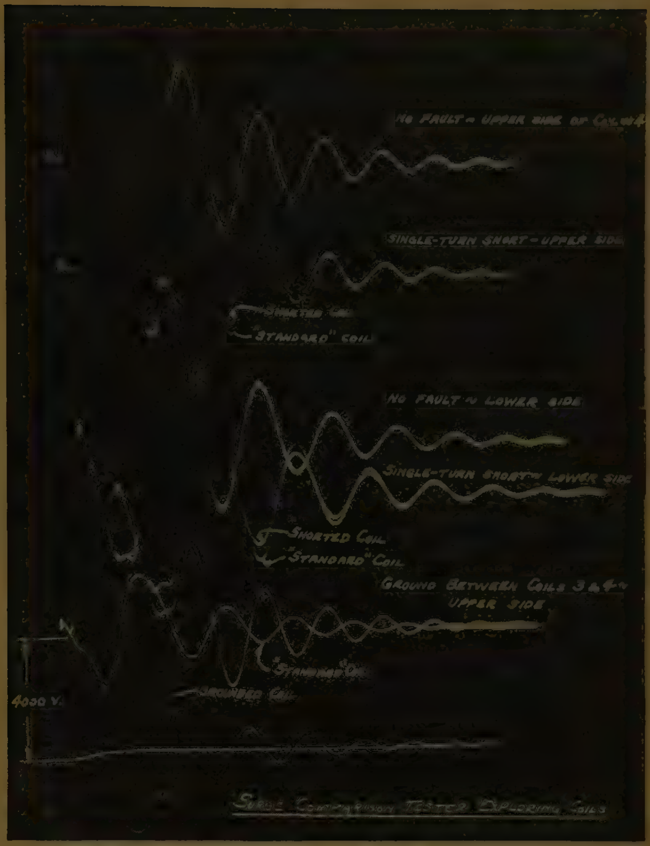

Figure 5. Exploring-coil voltage traces during test on large synchronous motor

One exploring coil over first the upper and then the lower side of a faulted coil in a winding with 12 parallel conductors

comparison either of two similar motors or of the two halves of a single winding. Both of these variations in the comparison technique yielded satisfactory results in the detection of short-circuited or grounded coils. One short-circuited turn in as many as 1,000 total turns was easily noticed.

THREE-PHASE MACHINES

Three-phase motors and generators of various sizes were tested with a surge-comparison tester. The simple phase-to-phase testing method, Figure 1A, permitted unmistakable detection of short-circuited or grounded coils in all windings of relatively few parallel circuits. Figure 3B illustrates the detection of a single-turn short-circuit in a large three-phase winding of 21 turns per phase. Detection of one-turn short circuits also was satisfactory in smaller motors having as many as 600 series turns per phase.

In windings with many parallel circuits, detection as well as location of a short-circuited turn or coil was found impossible with the simple conventional circuit of Figure 1A. This condition is illustrated by Figure 4A, which gives the result of a test on a very large synchronous motor which had 12 parallel circuits per phase, each consisting of eight ten-turn coils. Short-circuiting out one turn in phase *T*1-*T*4 of this winding made a nearly imperceptible difference in the voltage traces which actually made the winding look better than it did with no short circuit. Furthermore, inherent winding dissymmetries in these large motors caused oscillograph trace differences which added to the uncertainty. A close inspection of Figure 4A reveals the fact that the trace differences present in the unfaulted winding are greater than those caused by a short-circuited turn.

Because of these difficulties, the problem of detecting and locating faults in the windings of large three-phase machines with this tester was attacked first by using the simple comparison scheme of Figure 1B to compare the separate parallel circuits before connection to the paralleling rings. This method generally was satisfactory and made detection of a coil having a single short-circuited turn easily possible. Testing of the winding after completion also was desired, however, and the circuit and auxiliary equipment of Figure 1E ultimately were devised. With this method a unidirectional repeating surge voltage is applied to a terminal of one phase of the winding to stress the turn insulation and break down any weak places; the other terminal is grounded. While one phase at a time is thus placed under test, two identical exploring coils are moved by hand around one revolution in the bore of the machine. These coils are attached to the ends of an adjustable arm and so spaced as to be located over two winding coils that lie in identical electrical positions but in adjacent parallel circuits of the phase being tested. Since the two coils in the winding must react alike to the surge voltage if no fault is present, their emitted flux induces identical voltages in the two exploring coils. The induced voltages are compared by means of the oscillograph and synchronous reversing switch in a manner similar to that of the comparison connection of Figure 1B. A fault in either winding coil will cause a reduction in its flux, and hence a difference will appear in the oscillograph traces. By virtue of the complete revolution of the exploring coils, every coil in the phase will be compared with two other coils. The effect of a short-circuited turn in a winding coil under one of the exploring coils is illustrated by Figure 4B. The exploring coils used were wound on C-shaped iron cores. In Figure 5, a complete set of traces is shown for the exploring coils located over upper and lower sides of the faulted coil both with and without the fault of one short-circuited turn. The surge applied to the winding is included at the bottom of the page. This trace shows the extremely high rate of

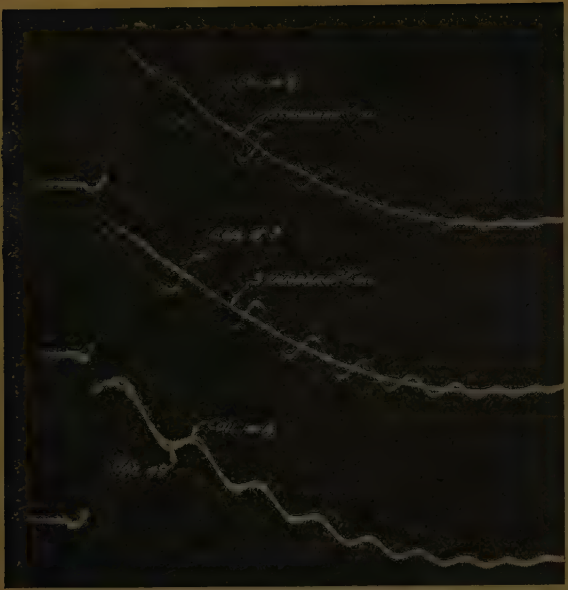

Figure 6. Oscillograms showing the possible effect of variations in varnish treatment on surge voltages

Railway-motor field coils. Coils 8 and 9 were varnish-impregnated; the standard coil was left untreated

voltage rise across the winding (0.3 microsecond to crest).

Ground faults in any device could be detected unmistakably with the original tester circuit and equipment, but the location of grounds in large motors and generators required the use of the exploring coils and the new circuit in the manner just described. With the exploring coils, the location of a ground in any group of series coils could be narrowed down quickly to two adjacent coils. This location of the approximate position of the ground was based upon the fact that coils from the surge application terminal to the ground will induce voltages above normal, whereas those from the grounded coil to the grounded terminal will induce voltages far below normal. The exact coil then was found by opening the winding between the two coils in question and reapplying the surge and exploring coils.

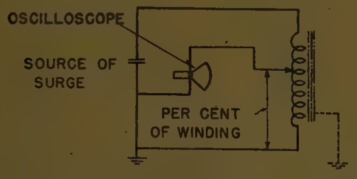

Figure 7. Method of measuring surge peaks

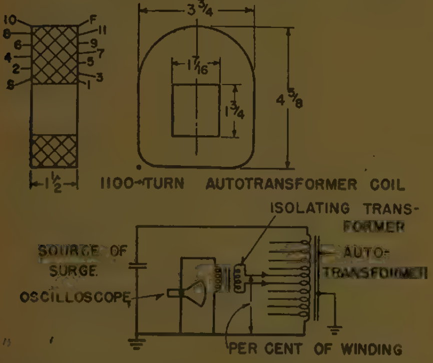

Figure 8. Circuit for determining volts per turn

The connection opened always was one of the two which later would necessarily be broken to remove the grounded coil.

Magnet Coils

Tests were made on air-core magnet coils using the two-coil comparison connection, Figure 1B. One short-circuited turn in as many as 4,000 was readily detected. The maximum number of turns in which one short-circuited turn could be detected was not determined exactly, as it depends somewhat on the size and shape of the coil, but the limit appeared to be about 10,000 turns. Above 10,000 turns only a short circuit of several turns or more could be detected satisfactorily. The application of an appreciable turn-to-turn voltage stress was possible only in coils which had relatively few turns because of the practical limit to the available test voltage. Another and possibly more important obstacle to high turn-to-turn voltages in magnet coils is the fact that most coils are not provided with terminals and leads capable of withstanding the high total voltage which would be required.

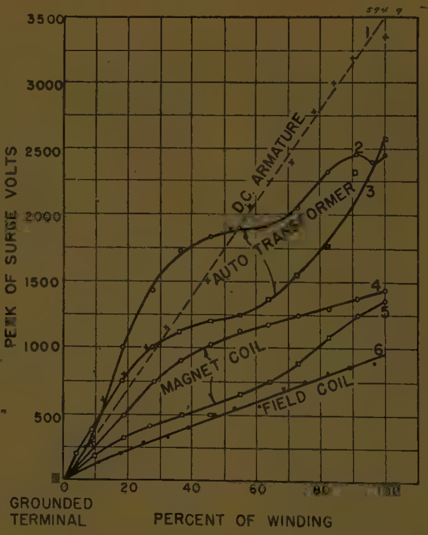

Figure 9. Surge-voltage distribution in four types of coils

Curve 1. Large d-c armature

Curve 2. 1,100-turn autotransformer—surge to inner lead

Curve 3. 1,100-turn autotransformer—surge to outer lead

Curve 4. 35,000-turn magnet coil—surge to inner lead

Curve 5. 35,000-turn magnet coil—surge to outer lead

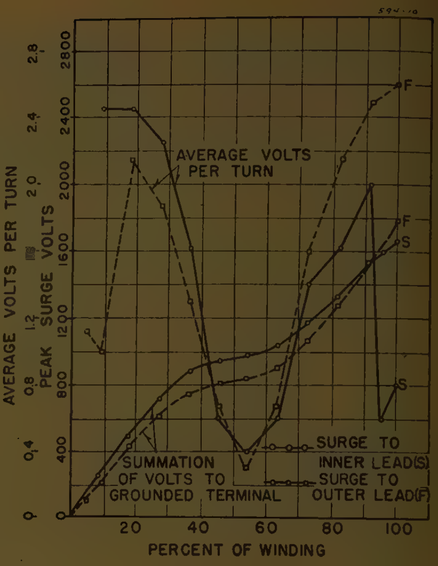

Figure 10. Average volts per turn in an autotransformer calculated from measurements obtained as in Figure 8

General Observations

Winding dissymmetries may cause differences in the voltage traces of two supposedly identical coils or windings even though no faults exist. This was observed often during tests on the apparatus described in this paper. Examples are the traces of Figures 2B and 4A. Many windings and coils seemingly have inherent dissymmetries caused by differences in their varnish treatment, amount of fill, moisture present in the winding, or unknown causes. An extreme example of this is illustrated by tests on three main field coils for a railway motor, Figure 6. Coils 8 and 9 both were impregnated in black solventless varnish and dipped in Thermoset varnish. The only difference in these two coils was in the resin treatment on the glass turn insulation. The standard coil was untreated and had mica-tape turn insulation. It may be seen that the voltage traces of the two treated coils have a superimposed oscillation caused by the presence of the resins (which increase the distributed capacity). The two treated coils, furthermore, are not identical owing to a difference either in the two resins or in the degrees of penetration into the coils. These effects of winding dissymmetries sometimes have caused uncertainty as to whether a fault was or was not present in the device under test. Nearly all examples of this uncertainty, however, were readily eliminated in a practical manner by placing the minimum possible fault, usually one short-circuited turn, in the winding and observing the effect. One short-circuited turn usually produced the greater and more unmistakable effect in most types of armature, field, and magnet coils, and in small machines with few parallel circuits. The inherent dissymmetries of large multiparallel windings, as previously discussed, rendered detection of short-circuited turns by the simple comparison scheme impossible.

Distribution of voltage magnitude across the winding turns of several representative pieces of apparatus was determined. This was first done by using the oscillograph in the tester simply to measure voltage from the grounded terminal of a winding to taps through the winding. Peak voltages only were recorded, and, as these occurred at nearly the same time after application of the surge, time differences were neglected. The method is illustrated by Figure 7. Data obtained in this manner yielded the curves of Figure 9. Additional measurements using a revised circuit, Figure 8, then were made on the autotransformer of curves 2 and 3 of Figure 9. The actual voltages between taps of the transformer were measured and the average volts per turn calculated. These data are plotted in Figure 10. Since this shell-type transformer had been chosen for its extremely nonuniformly distributed capacitance and inductance, its surge voltage distribution curves reveal approximately the worst condition that is likely to be present in any type of winding during this surge test. This uneven distribution, of course, could be improved by minor test-circuit revisions to reduce the steepness of the surge-voltage wave front.[5] The wave applied to this transformer had a time to crest of about 0.2 microsecond.

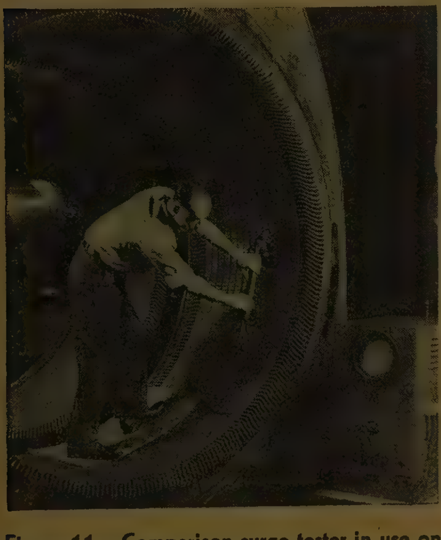

Figure 11. Comparison surge tester in use on large stator winding with many parallel circuits

The surge-comparison tester thus has been tried on numerous samples of all common types of coils and windings. In general, it was found to be a satisfactory test for many insulation-testing applications. The use of the cathode-ray oscilloscope for detection of faults permits accurate measurement of voltage when desired, gives the operator a view of the manner in which the winding is being tested, and, furthermore, reveals any arcing, corona, or other unusual phenomena occurring in the winding. The oscilloscope, however, does not provide a definite quantitative distinction between faulted or unfaulted windings. The interpretation of the oscilloscope evidence, however, could become easy and positive with experience on any particular application.

The problem of fault detection and location has been largely restricted in this paper to faults involving short-circuited turns or coils. Ground-fault detection was found easy and distinct in all windings so that no refined technique was necessary. In fact, any ground fault found in a winding during test had to be cleared up before other faults could be detected as its effect was to upset completely the symmetry of the winding. The location of grounds was attempted only on armatures and large three-phase machines, both of which have been discussed.

No circuit constants of this type of tester need be varied by the operator except the test voltage, which is set by turning a tap switch or a calibrated dial. The wave shape is fixed by the tester circuit components in conjunction with those of the winding on test. The connections for testing are simple and nearly the same for most types of apparatus.

Conclusions

The surge-comparison insulation tester is a highly versatile tool which, with further development, could be applied to many insulation testing problems. It possesses the following desirable or advantageous features:

1. The test voltage may be readily preset to any desired value which will not vary during the testing except with the line-voltage changes. The test voltage may not always be the same for different sizes of windings, however, because of the voltage drop in the protective resistor which becomes appreciable if the device under test has a relatively low surge impedance.

2. Control of the machine is simple and easy. Only the voltage requires setting by the operator, in addition to the usual control knobs for the oscilloscope.

3. Setting up for test usually is reduced to mere connection of three leads. No auxiliary equipment is needed for most testing, and testing instructions are simplified.

4. The surge tester is flexible; large generators or small magnet coils may be tested with no change in circuit constants or connections, except for the testing of windings with many parallel circuits.

5. The cathode-ray oscilloscope gives the operator a constant view of his test voltage and its effect on the winding. Arcing and corona can be detected along with steady-state faults. The oscilloscope also may be used to measure voltage to ground at any point.

6. The voltage stress applied to ground insulation is kept to a limited and known value, because cores and frames can be grounded. Personal safety also is increased by having all parts of a machine except conductors at ground potential.

7. The surge voltage applied has a wave front at least as steep as the wave fronts of most switching and lightning surges which are the sources of dangerous overpotentials in service.

8. The tester can be built as a relatively compact and lightweight unit which can be moved about easily for testing in numerous locations.

9. D-c armatures may be given an insulation and fault-detection test while lying on the floor or in winding frames and need not be placed in a special testing lathe as is now necessary with some conventional methods.

10. Photographic records may be easily made of any test to show unusual results obtained or for future comparison with other tests.

11. Grounded coils in large a-c machines can be located without the puncturing of insulation with probes or the unnecessary breaking of connections.

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Precision Testing of Electric Tachometers

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Synopsis: The magnetic-drag electric tachometer which was developed especially for the measurement of aircraft-engine speed is inherently a more precise measuring device than those instruments which normally have been used as reference standards of speed measurement. This paper describes an electric-tachometer testing system which meets the requirements of high accuracy and is also suitable for large-quantity manufacture of tachometers. The system provides facilities for testing at 22 scale points in a range of speeds from 500 to 4,500 indicated rpm without referring to a standard instrument. The power to drive the tachometers being tested is supplied from tachometer generators which are driven by a d-c motor controlled by a precision tuning fork. The method of operation and the control system are explained.

AMONG the outstanding aircraft instruments developed to meet the urgent needs of the armed forces is the magnetic-drag electric tachometer.[1,2] This tachometer consists of an indicator, Figure 1A, and a three-phase synchronous generator, Figure 2. The indicator is mounted on the instrument panel, and the generator is coupled directly to the aircraft engine. The generator voltage, of frequency proportional to engine speed, operates a synchronous motor in the indicator. The disassembly view in Figure 1B shows the magnet assembly which is rotated by the motor. The magnet assembly is arranged so that an aluminum-alloy disk fastened to a shaft and pointer assembly and supported in independent bearings is displaced by electromagnetic torque against a spring-restoring torque. A pointer deflection thus is produced which is proportional to the speed of the rotating magnets or to the frequency of the generator output, and therefore to engine speed. As the indicator is a frequency-sensitive device, variations in magnitude of the generator output voltage do not influence the accuracy.

The inherent accuracy of the tachometer indicator and high-quantity production required that special test equipment for checking calibration be developed. Early methods of testing tachometer indicators were accomplished by manual control of the speed of a d-c motor driving the transmitting element of the tachometer being tested and by comparison with a reference standard. For each calibration point to be tested, the correct motor speed was adjusted manually according to the indication of the reference standard, and then the indication of the instrument being tested was observed. When a large number of scale points were to be checked, this procedure became lengthy and susceptible to the hazards of serious personal errors. Even more important, however, was the fact that the available standard tachometers, even those of the liquid type, were accurate to only ±0.2 per cent of full-scale calibration. These were not satisfactory for testing the aircraft tachometer, because the required accuracy of calibration as shown in Figure 1C is of the same order of magnitude.

When performed by the early methods of point-for-point comparison with a reference standard, testing is both slow and inaccurate. The well-known synchronizing-fork type of equipment, though accurate to ±0.1 per cent, is not adaptable to high-production testing. A tuning fork in this equipment provides means for manually synchronizing the speed of a d-c machine to which is coupled a tachometer generator through a step-ratio gear train providing a large number of test points.

The development of high-precision standards of frequency measurement[3,7] and automatic speed control has made the means available for vast improvement over old methods of tachometer testing. Described in this paper is a tachometer testing system which meets the requirements set by high-production rates and high accuracy.

Test Requirements

All of the testing operations require that voltage of a correct frequency corresponding to the desired scale point be applied to the indicator. These operations include: adjustment of bearings and adjustment of the full-scale point by reducing the magnetic strength of the rotating drag magnets. The entire calibration through 22 scale values then is checked both for increasing and decreasing frequency as well as for various angular positions of the instrument.

The most desirable method of performing the testing operations is to have available for immediate selection a series of voltage sources of the correct fre-

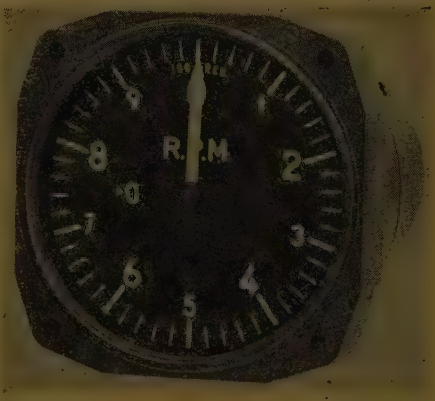

Figure 1A. The magnetic-drag electric tachometer indicator

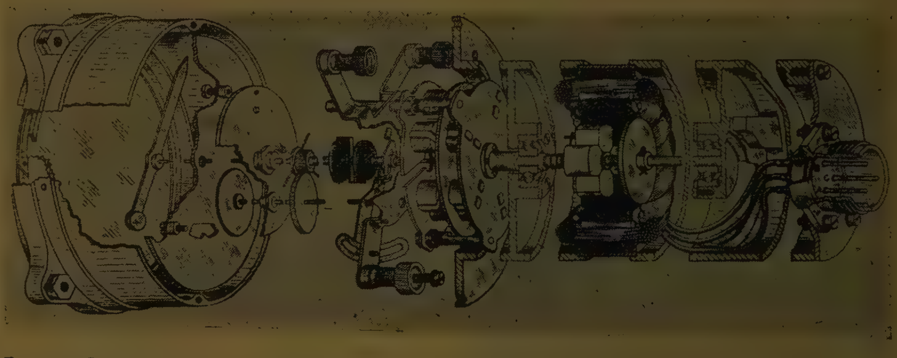

Figure 1B (above). Disassembly view of tachometer indicator

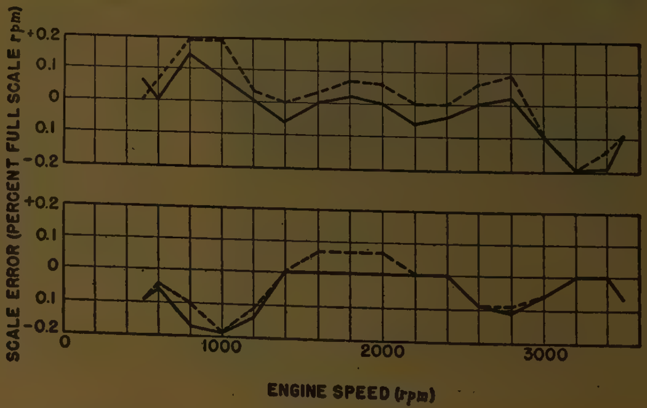

Figure 1C (right). Typical calibration curves

—— Engine speed increasing
– – – Engine speed decreasing

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Figure 2. Tachometer generator which furnishes three-phase power to the indicator

quencies corresponding to the desired scale points. The indicator then can be correctly energized and the adjustments made without reference to a standard. The success of this method requires that each frequency corresponding to a scale point on the indicator be extremely accurate. An allowable error one-tenth that required of the tachometer-indicator calibration would be acceptable, although a higher accuracy, of course, is desirable.

Techniques of frequency measurement using piezoelectric crystals and precision tuning forks maintained at constant temperature have been developed to a high degree. It is now possible to measure frequency over a wide range to a precision of only one part in 10,000,000. By incorporating automatic control with the synchronizing-fork method of tachometer testing and utilizing these new techniques of frequency measurement, it has been possible to obtain testing equipment having an over-all precision of better than ±0.01 per cent.

Test Equipment

The line diagram, Figure 3A, shows the arrangement of the components in this equipment. The principal components are:

1. Test stand.
2. Gearbox and tachometer generators.
3. D-c motor.
4. Pilot generator.
5. Motor field control.
6. 60-cycle frequency standard.
7. Oscilloscope monitor.
8. 60-cycle laboratory standard of frequency using a piezoelectric crystal.

The gearbox, Figure 3B, drives a group of tachometer generators which supply voltage for testing at 22 different frequencies corresponding to the scale points on the tachometer indicator. Gearbox, d-c motor, and pilot generator all are connected to a common drive shaft through a two-to-one, one-half-to-one, one-to-one multiplier designated in the diagram as a gear shift. This unit extends the usefulness of the equipment to more than one instrument scale range. Speed of the drive shaft is maintained constant by control of motor field current by the field-control unit as determined by the deviation in frequency between the pilot-generator voltage and the reference signal from the tuning-fork standard. The pilot generator is a four-pole 60-cycle alternator, and the tuning-fork standard provides the 60-cycle reference signal. Continuous indication of the speed of the main drive shaft is obtained by comparing the frequency of the pilot-generator voltage with a voltage from the laboratory standard of frequency on a cathode-ray oscilloscope.

Terminating in a convenient test stand, Figure 4, are the voltage circuits from the group of tachometer generators. Through a series of interlocked push buttons, the tachometer being tested may be connected to the desired generator and thereby become energized at the precise frequency corresponding to the particular scale point. Reference to a standard tachometer is unnecessary as voltages for the scale range from 500 to 4,500 indicated rpm in the 22 specified steps are selected. The testing operation thus is so materially simplified that it can be accomplished in a few minutes by a relatively unskilled operator. When performed by a skilled operator, according to the early methods of testing, the operation required from 20 minutes to one-half hour. Moreover, the test stand is equipped with a knock-down coil necessary for the adjustment of the full-scale point. This coil is conveniently arranged so that the magnetic strength of the indicator drag magnets may readily be reduced while the instrument is mounted in the test stand.

The equipment was designed to provide for four of the test stands shown in Figure 4. Additional facilities were added by the use of three-phase power amplifiers.

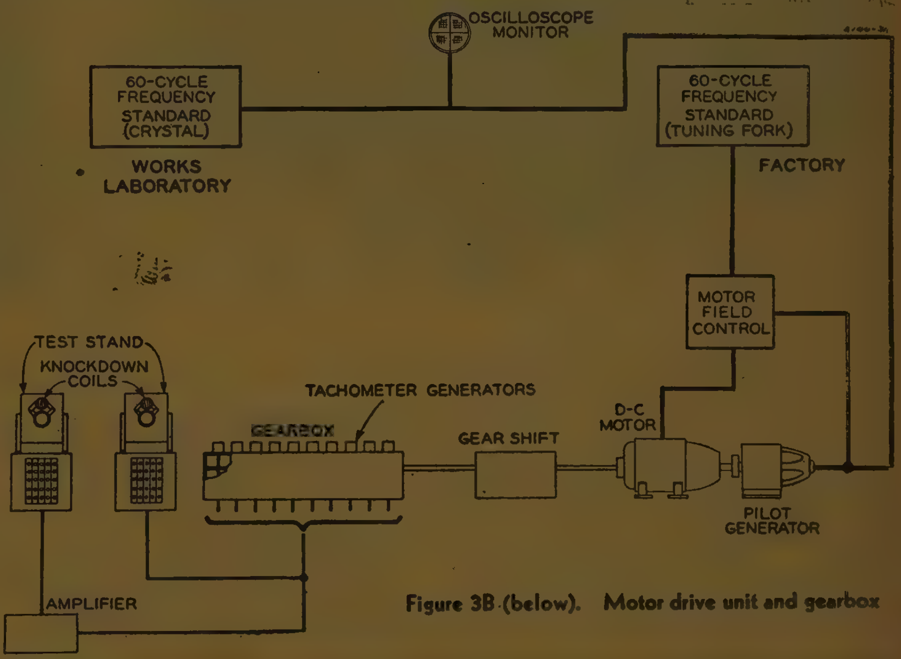
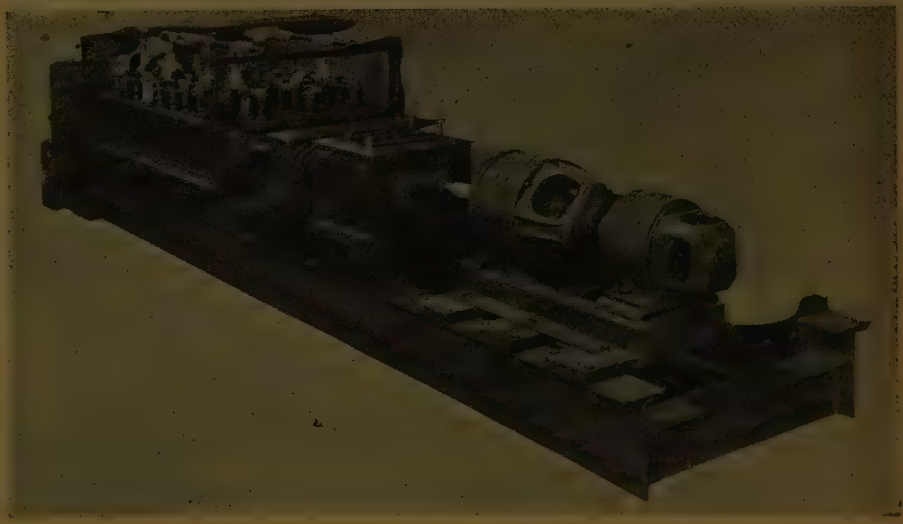


Figure 3A. Line diagram of complete tachometer testing system

Figure 3B (below). Motor drive unit and gearbox



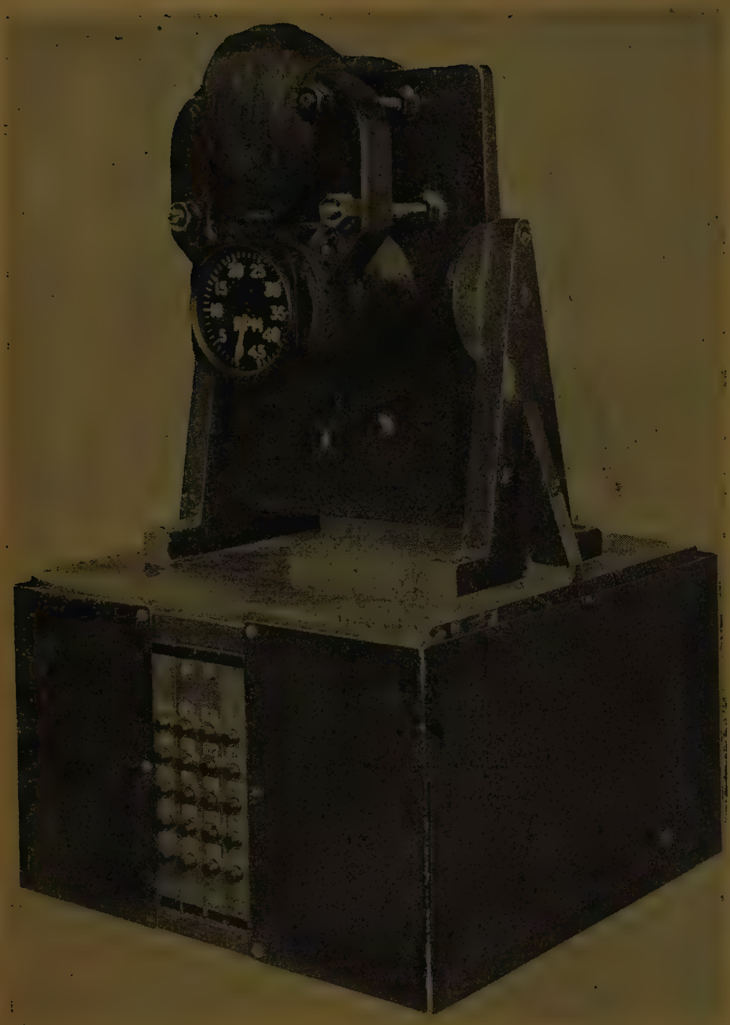


Figure 4. Test stand for calibrating tachometer indicators

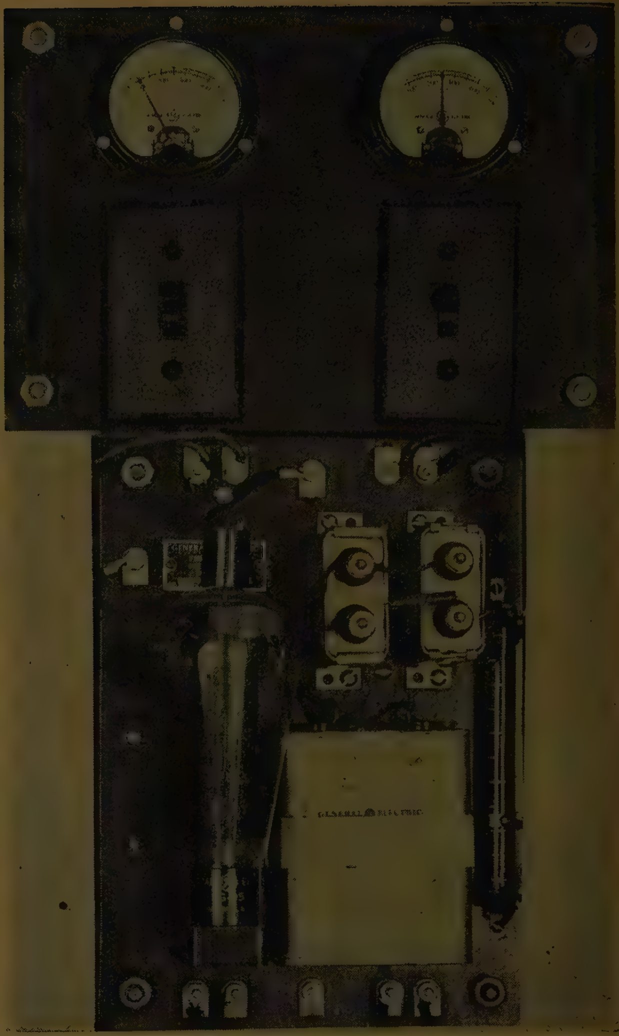


Figure 5. Field-current control panel

Through selector switches the amplifier input is connected to the desired generator, and the test stand operates identically with those stands which are connected directly to the tachometer generators. As shown in Figure 5, the system provides testing facility for a large number of testing stands to be operated simultaneously. Since the amplifiers draw no additional load from the generators, the testing facilities can be extended to meet any desired requirement of increased production. Smaller generators, such as magnetized gears or tone wheels, also may be used to furnish the range of frequencies desired if their output is supplied only to the amplifiers.

A modified testing system using small generators and one amplifier with the associated frequency standard and controlled-speed motor might be applicable in field testing of tachometer indicators. An even simpler arrangement for field testing might be developed by the use of an oscilloscope and frequency standard but with manual control of the d-c motor. The use of the tuning-fork frequency standard thus can be utilized to provide the advantage of high precision in field testing as well as in high production.

Motor Speed Control

Speed control of the d-c motor is obtained by means of a thyratron field control shown in Figure 5 and a precision tuning fork in Figure 6. This system of motor speed control was developed originally for turbine-governor application[6] where large power variations exist. As a precise tachometer testing system, it is subject to the same variables as well as variations in line voltage. But the magnitude of the variations is small and controllable.

The control circuit in Figure 7A shows the motor field windings connected in two arms of a bridge of which the other two arms are equal fixed resistors. Main field current I_f, is supplied to the field windings from the d-c line through two terminals of the bridge circuit, while the control current, I_c, is applied through the thyratron tube to the other two bridge terminals. The current, I_c, flows in a direction to oppose I_f; hence, the net current to the generator field is equal to I_f minus the average value of I_c. Any tendency for the motor to change speed is opposed by a restoring torque produced by a change in I_c, and consequently in the net field current.

The magnitude of I_c is controlled by the phase relation between the voltage E_g applied to the thyratron control grid and the plate voltage E. The voltage

Figure 6. Precision tuning-fork frequency standard

E is obtained through transformer T_1 from the single-phase pilot generator which is connected directly to the d-c motor and gearbox drive shaft. The frequency of its voltage is proportional to the shaft speed. The voltage, E_g, is the vector sum of a standard voltage, E_f, obtained from a tuning-fork standard oscillator shown in Figure 6 and a voltage E_s from the pilot generator. Through the phasing network in Figure 7A, the voltage E_s from T_2 is brought in phase quadrature with the plate voltage E.

Plate- and grid-circuit relations are best described by referring to Figure 7B. As both E and E_s are supplied from the pilot generator, small changes in motor speed will not alter their phase relation. However, small changes in motor speed will be followed by a phase displacement between E and the tuning-fork voltage E_f. Thus, as the motor speed tends to increase, the resultant grid voltage becomes displaced in phase in the direction of the upper arrow in Figure 7B. A shift in phase in this direction causes a decrease in the average value of I_c. As I_c flows in a direction to oppose the main field current, I_f, a net increase in motor field strength results, and the tendency for the motor speed to increase is opposed. The opposite action takes place when the motor speed tends to decrease. The displacement of the grid voltage E_g is now indicated by the direction of the lower arrow in Figure 7B. The resulting increase in the average value of I_c causes the motor field strength to decrease, thus opposing the tendency of the motor speed to decrease.

Accuracy of Calibrating System

The factors which affect the performance of the speed control are:

1. Tuning-fork error.
2. Temperature variations.
3. Load variations.
4. Line-voltage variations.

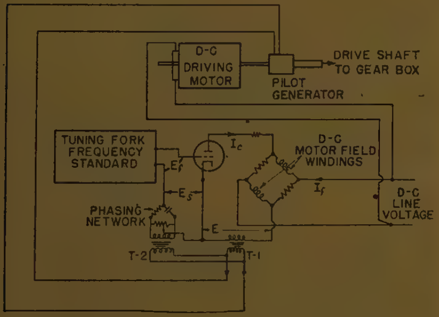


Figure 7A. Field-control circuit

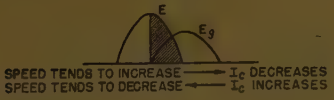

E
E_g
SPEED TENDS TO INCREASE ⟶ I_c DECREASES
SPEED TENDS TO DECREASE ⟵ I_c INCREASES

Figure 7B. Plate- and grid-voltage phase relation

The precision of the system is established principally by the magnitude of the frequency deviation of the tuning-fork oscillator.[8] The tuning fork in this unit is maintained at constant temperature, and its maximum frequency deviation does not exceed ±0.003 per cent.

Temperature, load, and line-voltage variations which operate to exceed the control range of the current I_c, Figure 7A, will cause the system to drop out of synchronism. Figure 8A shows the range in control current for variations in line voltage while the load is constant. Figure 8B shows the control current range when the line voltage is held constant and the load varied. Only temperature variations caused by self-heating of the equipment cause the control range of I_c to be exceeded, and for this condition a compensating adjustment of I_f is made after stable thermal conditions have been established. Variations in load and line voltage which do not operate to exceed the control range of I_c contribute to the over-all error of the system by introducing hunting. This is kept to a negligible value, largely because the line voltage is obtained from an independent supply having good regulation; and the intermittent loading of the tachometer generators as tachometer indicators are alternately connected and disconnected is small.

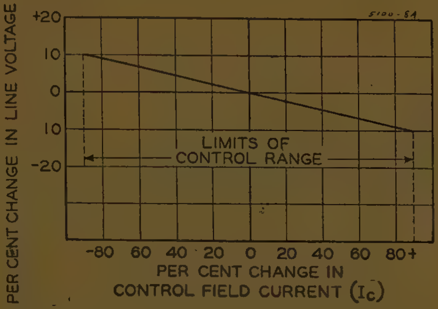


Figure 8A. Control-current range in terms of line-voltage variations for constant load

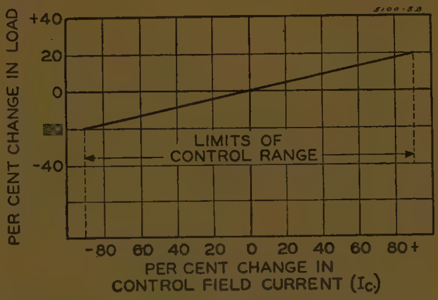


Figure 8B. Control-current range in terms of load variations for constant voltage

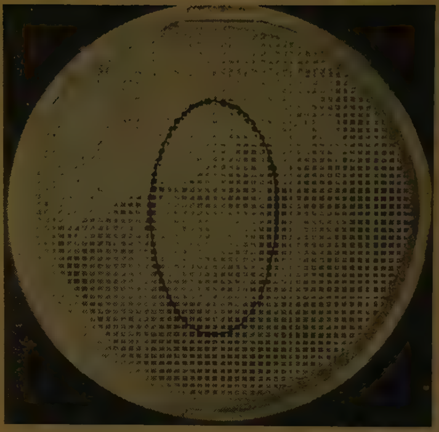


Figure 9. Lissajous figure on oscilloscope screen

A detailed study of the conditions influencing hunting is beyond the scope of this paper. It is evident, however, that the time constant of the control system is dependent almost entirely upon the inductance in the d-c motor field. Consequently, it is necessary that load and line-voltage variations be small. The curves in Figure 8A and 8B indicate acceptable limits of variations of these factors without causing appreciable hunting. By observing the precautions of small load changes and steady d-c line

Figure 10. Group of tachometer-indicator test stands installed in factory

voltage the over-all accuracy of the system is within ±0.01 per cent.

Continuous Reference to Primary Standard

Continuous check of the precision of the base speed and therefore to any one of the test points is obtained by reference to a standard signal from a piezoelectric crystal oscillator. A 60-kilocycle crystal and associated multivibrator stages are standardized daily with standard time signals from the Bureau of Standards radio station WWV. The precision of the 60-cycle output of the oscillator is better than ten parts in 1,000,000. This signal is wired to the factory testing system and connected to the vertical plates of a cathode-ray oscilloscope located prominently in the testing area. Connected to the horizontal plates of the same oscilloscope is the voltage output of the pilot generator in Figure 3A. Figure 9 shows the Lissajous figure formed by the two signals applied to the oscilloscope. The rate of displacement of an element of this figure is a measure of the deviation in frequency of the two applied signals. Displacement of one degree per second of a point on the image formed on the oscilloscope screen corresponds to the over-all error of 0.01 per cent and can be detected readily.

Conclusion

The convenience of operation and precision of the tachometer calibrating system as a factory testing device is apparent. Skill in accomplishing the tachometer calibrating operation is reduced to a minimum and an operator can be trained in a matter of hours. The testing time over early test methods has been reduced from one-half hour to a few minutes per instrument, thus making the large production requirements possible with a minimum of increase in personnel.

The use of such equipment because of its complexity and size is necessarily limited to use in high-production testing. Obviously, such a system is not adequate for field testing of electric tachometers. However, modifications in the applications of the oscilloscope, d-c motor with manual speed control, and the standard-frequency oscillator, can be adapted for small testing requirements. Such modification would retain the advantages of precision over the conventional types of standards for the testing of tachometer indicators.

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A 400-Cycle Aircraft Motor

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Synopsis: The ever-present need for lighter-weight and more compact aircraft electric motors is evident as any reduction in weight or space provides increased fuel, bomb, or pay-load capacity. Reliable 400-cycle motors more compact and approximately 40-per-cent lighter than existing 24-volt d-c motors have been developed recently. Present trends indicate that these motors will be used extensively aboard the airplane of tomorrow. This paper describes some of the 400-cycle motors that have been developed and discusses their requirements with the view of disseminating information so that others may better understand the motors and the performance to be expected from them.

IN THE PAST FEW YEARS considerable thought has been given to proposed methods for reducing the weight of aircraft electric equipment below that possible from existing 24-volt electric systems. The value of high frequency as one avenue through which this could be accomplished was demonstrated by the 400-cycle electric system aboard the *B*-19 airplane built some time ago. This system functioned satisfactorily according to expectations, but the lack of a suitable constant-speed drive for the alternators prevented its adoption for general use, particularly on military airplanes, at that time. Subsequent months witnessed the inception of ideas for driving the alternators at nearly constant speed from the airplane engines. Practicability of these ideas and the need for electric power in excess of that possible from existing systems brought about the selection of a 208/120-volt 400-cycle three-phase four-wire electric system for several military airplanes under consideration.

Heretofore knowledge of the electric motors developed for use on these and other airplanes has been confined, generally, to the few intimately associated with their design. Much of the accumulated information pertaining to their design and application may be useful, particularly to aircraft electrical engineers or others who will apply the motors. Therefore, this paper was written, primarily from a design point of view, for the purpose of disseminating some of this information so that others may better understand the motors and the performance to be expected from them. It contains the following material:

1. A summary of the motor requirements.
2. The principles of operation compared briefly with those of 60-cycle commercial motors.
3. A comparison of characteristics with those of existing d-c aircraft motors.
4. The design factors involved.
5. A discussion of the motor characteristics related to loads encountered aboard aircraft.
6. Illustrations and characteristic curves representative of a line of motors that has been developed.
7. Conclusions.

Requirements

The more important motor requirements mutually agreed upon by the United States Army, Navy, Air Force Command, interested motor manufacturers, and representatives of the aircraft industry are summarized as follows:

1. Dependable operation from −70 to +60 degrees centigrade according to National Aircraft Standards Committee charts at all altitudes to 40,000 feet and in all humidities existing within this altitude range.
2. Reliable operation without overheating on frequencies ranging from 380 to 420 cycles and voltages from 190 to 210 volts when running and from 180 to 210 volts when starting.

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3. Ability to withstand shock and vibration encountered aboard airplanes.

4. High efficiency and power factor to permit minimum weight of associated electric equipment.

5. Small size and light weight consistent with other requirements.

6. A life of 500 hours without undue servicing on the specified duty cycle, preferably more.

7. Simplified construction arranged for easy servicing.

8. Intermittent-rated motors must operate on a duty cycle of one minute on and 20 minutes off. Their locked-rotor torque must be 300 per cent of the rated torque on normal voltage and frequency and at 25 degrees centigrade ambient temperature. The slip at rated torque must not exceed 15 per cent of the synchronous speed.

9. Continuous-rated motors must have 200-per-cent breakdown torque and 160-per-cent locked-rotor torque, both in terms of rated torque on normal voltage and frequency and at 25 degrees centigrade ambient temperature.

Comparison With Other Motors

These motors, most of which are three phase four wire, possess principles of operation similar to those applicable to 60-cycle motors of the same type. Generally the same fundamental laws and formulas applicable to 60-cycle alternating motors are usable although instances occur where it is necessary to modify or extend associated empirical data to embrace new conditions. They differ from the 60-cycle motors chiefly as follows:

1. Higher operating speed.

2. Smaller size and less weight for corresponding horsepower ratings.

3. Better material utilization—electrically, mechanically and thermally—to obtain lighter weight without sacrificing reliability.

4. Ability to operate over increased temperature, humidity, and altitude ranges.

Their characteristics compared with those of existing 24-volt d-c aircraft motors, segregated according to merit, are listed in the following paragraphs.

DESIRABLE CHARACTERISTICS

1. Lighter weight and smaller size for same output rating.

2. Higher efficiency, generally.

3. Simplicity of construction.

4. Absence of commutators and brushes which are difficult to maintain, particularly at high altitude, and which may permit erratic motor operation because of their susceptibility to external vibration.

5. Higher operating voltage and lower currents which permit a reduction in the weight of wiring and generating equipment.

6. Inherently longer life with less servicing.

7. Less speed variation over the normal load range, including that caused by normal voltage and frequency variations. This does not apply if the speed of the direct motor is controlled by a speed regulator.

8. Freedom from explosion-producing properties such as sparking at brushes.

9. Reduced voltage regeneration and hence less danger of damage to motor and system if direction of rotation is instantly reversed.

10. Inherent regenerative braking for overhauling loads.

11. Generally, a lower ratio of starting to running currents, a condition which makes it possible to simplify the design of associated equipment.

12. Easier to apply thermal overload protection.

13. Reduced rotor inertia—hence less braking required to stop motors in the same length of time.

14. Reduced danger of damage or faulty operation from corrosion of vital parts.

15. Moderate ratio of locked-rotor to running torque. D-c motors have locked-rotor torques up to about 1,000 per cent of their rated load torques. Continuous-rated a-c motors according to specifications have 160-per-cent locked-rotor torques. Intermittent-rated 400-cycle motors according to specifications have 300-per-cent locked-rotor torques.

16. Less filtering required to eliminate radio interference.

UNDESIRABLE CHARACTERISTICS

1. Locked-rotor and breakdown torque variations, caused by both voltage and frequency changes rather than voltage changes alone.

2. Lack of speed control.

Many engineers would list the moderate ratio of locked-rotor to running torque as an undesirable characteristic, as any increase in this torque is added assurance that the motors will start their loads. However, it must be remembered that generally increased locked-rotor torque is brought about by increased current which adds weight to the motor and associated equipment. Furthermore, there is never a need for additional assurance if the motor is correctly applied on the basis of starting as well as rated load conditions. It is better to use a larger motor to provide additional starting torque should a specific application require it rather than add a weight penalty to the entire system by building additional torque in all motors.

Basic Motor Constructions Involved

Open 400-cycle motors are most satisfactory for the majority of applications. Normally, they do not possess explosion-producing characteristics and can be ventilated with less complication and without excessive weight increases. Gasoline-pump motors will require an explosion-resisting construction. Some of these will operate submerged with their windings, bearings, and other vital parts directly exposed to the gasoline. A few totally enclosed intermittent motors may be required for engine nacelle or other locations saturated with oils or dirt.

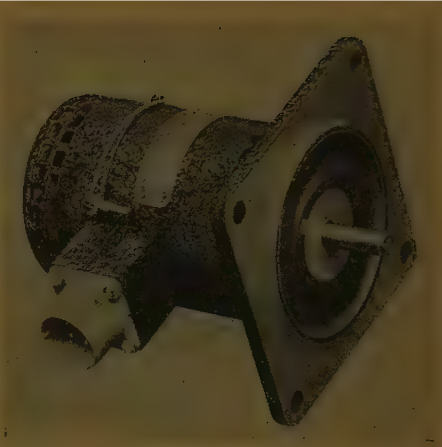

Figure 1. Aircraft motor; three-quarter horsepower, 10,800 rpm, 400 cycle, three phase

Synchronous speeds of 24,000, 12,000, 8,000, 6,000, 4,800, and 4,000 rpm are possible from 2-, 4-, 6-, 8-, 10-, and 12-pole windings, respectively. The four-pole motor will be used almost exclusively. Generally a lighter-weight power unit can be obtained by gearing down to the desired speed rather than increasing the number of poles. Nevertheless six- and eight-pole motors will have a limited use for driving loads such as centrifugal fans and gasoline pumps because of the noise and lubrication problems associated with gears. Lighter-weight motors could be obtained by using two-pole windings, but the lack of suitable bearings capable of withstanding speed approaching 24,000 rpm without freezing at low temperatures prevents using them at this time.

Design Considerations

In designing 400-cycle aircraft motors it is necessary to correlate functions of five different kinds of engineering: namely, electrical, mechanical, thermal, air-flow and lubrication. Individually, these functions are complicated by the high-speed operation, better material utilization, and varying conditions in which the motors must operate. Correlation is particularly difficult, as the interrelation is such that any one function may affect any or all of the others. Usually the electrical studies are given first consideration, followed by fitting the other engineering factors around the basic electrical design. This is desirable, not because of a position of greater importance, but rather because the other functions to a large extent are determined by the electric output and losses.

ELECTRICAL DESIGN

Usually, the electrical design can be determined best by calculating performance characteristics using several different rotor diameters, tooth widths, stacking lengths, and so on, for each stator-punching diameter contemplated. By so doing the less important factors such as different amounts of primary copper, reduced iron thickness, modified ratios of tooth to core section, which affect the characteristics appreciably, are given proper consideration. Also, this procedure permits making allowances for thermal-transfer surfaces and air-flow paths, both of which reduce the electric output. These are involved calculations beyond the scope of this paper. However, it is possible to predict with reasonable simplicity approximate characteristics to be expected by considering what happens when a 60-cycle motor is operated on 400 cycles. The flux should be held at the design value, and if so maintained the applied voltage must be increased in the ratio of the frequency change. This is demonstrated by the commonly used flux formula:

$$\phi=\frac{E\times10^8}{2.22\times f\times W_k\times C_1}$$

where

ϕ = flux in lines per square inch
E = applied voltage per phase
f = applied frequency
W_k = the winding constant
C_1 = primary conductors per phase

Raising the frequency causes the motor reactances to increase in proportion to the frequency change but does not affect the resistance values. Iron loss increases by an amount dependent upon the magnetic material used and the density at which it operates. Available data indicate that the increase is 12.5 times for high-grade silicon iron at a density of 95,000 lines per square inch. Windage losses vary as the cube of the speed. So-called stray losses resulting from tooth pulsations and similar losses vary in different designs but usually are about five per cent of the calculated electric output. Friction losses are small and can be neglected without appreciably affecting the over-all results.

Table I, column 1, contains design data and calculated characteristics for a typical 200-volt 60-cycle three-phase motor. Column 2 contains similar data for 400-cycle operation of the same motor. Column 3 contains similar 400-cycle data for the same motor with increased secondary resistance to obtain 160-per-cent starting torque. For the considerations tabulated in columns 1 and 2 rated load is arbitrarily placed at 50 per cent of the breakdown torque. For column 3 rated load is established such that the slip at this point does not exceed ten per cent of the synchronous speed and such that the breakdown and starting torques are at least 200 and 160 per cent of the full-load torque, respectively. The slip would exceed ten per cent if rated load were placed at 50 per cent of the breakdown torque.

From these data it is noted that rated output increases 9.3-fold when the 60-cycle motor without modification is operated on 400 cycles. The locked-rotor torque decreases to 23.5 per cent of the full-load torque which is far too low for most applications.

Table I. Design Data and Calculated Characteristics for Typical 200-Volt Three-Phase Motors

	Column		
	1	2	3
Design Data			
Applied volts per phase	115.5	770	770
Applied frequency, cycles	60	400	400
Primary resistance per phase, ohms	1.3	1.3	1.3
Secondary resistance per phase, ohms	2.2	2.2	13.95
Primary leakage reactance per phase, ohms	2.7	18	18
Secondary leakage reactance per phase, ohms	2.7	18	18
Magnetizing leakage reactance per phase, ohms	90	600	600
Iron loss, three-phase, watts	30	375	375
Windage loss, three-phase, at 1,800 rpm, watts	5.0	5.0	5.0
Stray-loss factor	0.95	0.95	0.95
Calculated Characteristics			
Full-load output, watts	1,125	10,430	8,200
Full-load speed, rpm	1,645	11,880	10,850
Full-load slip, per cent of synchronous speed	8.6	1.0	10
Full-load efficiency, per cent	81.0	79.5	73.8
Full-load power factor, per cent	90		89.5
Full-load torque, ounce-feet	77	99	85.0
Full-load loss, watts	265	2,720	2,900
Full-load primary I^2R loss, watts	79	156	105
Full-load secondary I^2R loss, watts	97	119	780
Full-load iron loss, watts	30	375	375
Full-load windage loss, watts	4	1,450	1,110
Full-load stray loss, watts	57	620	410
Breakdown torque, per cent of full load	200	200	231
Locked-rotor torque, per cent of full load	161	23.5	161
Locked-rotor input, watts	3,495	5,210	17,185
Locked-rotor primary I^2R loss, watts	1,335	1,805	1,510
Locked-rotor secondary I^2R loss, watts	2,130	3,040	15,330
Locked-rotor iron loss, watts	30	375	375

The rated output increases 7.3 times when the secondary resistance and rated loads are adjusted simultaneously to obtain 160-per-cent starting torque without exceeding ten-per-cent slip at rated load. Standstill and rated load losses increase 5- and 11-fold, respectively. Rotor losses at standstill increase 7.2 times which means that the heating rate of the rotor also increases. Hence the ability of the rotor to withstand locked-rotor conditions is materially reduced, particularly if the resistance increase is obtained by changing the amount of secondary material but not the material itself.

The power required to establish air motion through rotation of the rotor and fans normally constitutes the windage losses. Considering that the rotor portion is usually small, it appears that sufficient cooling exists on 400-cycle operation to permit continuous-rated-load operation. Whether or not this is true depends upon the arrangement of thermal-transfer surfaces and the flow of a cooling medium across these surfaces.

THERMAL AND AIR-FLOW DESIGN

The designs of thermal-transfer surfaces and air-flow paths are coincidental. Two basic arrangements are possible, one for double-end ventilation, the other for through ventilation. Most intermittent-rated motors and continuous-rated motors having large diameters and thin magnetic structures can be cooled successfully by double-end ventilation. When used, cool air brought in through suitable openings is directed such that it impinges on the ends of the windings and magnetic structures at high velocity. Heat generated internally must transfer to the exposed surfaces by conduction for dissipation. Usually these exposed surfaces are not adequate to dissipate the heat of continuous-rated motors having stacking lengths which permit minimum compactness and weight. In these, adequate ventilation can be obtained by conducting air through special openings in the magnetic structure. Best results are obtained by placing these openings, which should be designed for maximum air velocity and heat-transfer surface, where most of the heat is generated. Excellent air paths are obtained by pressing the stator windings back into the slots. This permits air flow directly over the stator and rotor windings as well as sections of the stator and rotor teeth. All of these have good transfer surfaces and are parts in which much of the heat is generated. Also, holes punched in the stator and rotor core sections serve as good air-flow paths.

MECHANICAL CONSTRUCTION

Several equally satisfactory but distinctly different mechanical constructions are possible. Usually that used is deter-

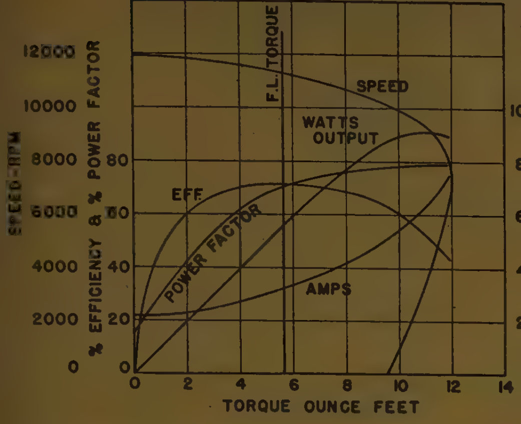

Figure 2. Characteristic curves for three-quarter-horsepower 10,800-rpm 400-cycle three-phase aircraft motor

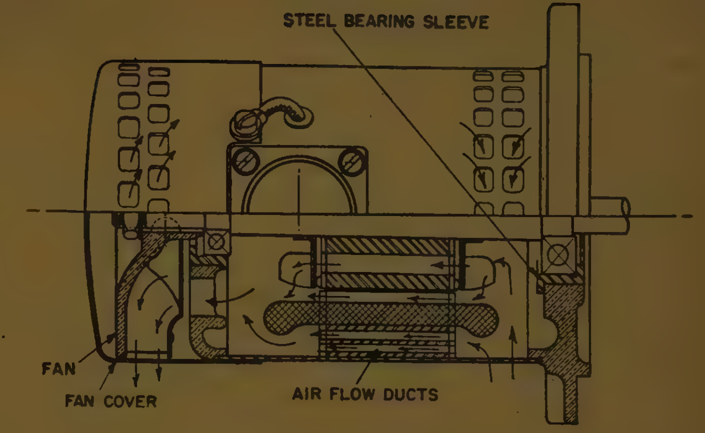

Figure 3. Section view of 400-cycle aircraft motor

mined by the basic requirements and other fundamentally extraneous considerations such as available manufacturing facilities, shop practices, and individual preferences. Such other considerations add to or detract from a design only to the extent of their influence on the making of individual parts or the assembly of these parts into the completed unit. Whether or not a design is satisfactory is determined wholly by weight considerations and ability to withstand burdens encountered over the range of operating conditions throughout its expected life, without destroying other essential characteristics. The most severe burdens are imposed by airplane vibrations. According to specifications vibrations up to 55 cycles per second with an excursion (double-amplitude) of $^{1}/_{16}$ inch may be expected.

The motor shells, end flanges, bearings, shafts, and a few other miscellaneous parts constitute the vital parts entering into the basic mechanical construction of most 400-cycle aircraft motors. Grease seals or other special features are required only for specific applications.

Comparable lightweight shells may be obtained either from low-specific-gravity materials such as aluminum or magnesium or from stronger high-specific-gravity materials such as steel but with the sections reduced. Steel shells with reduced sections permit a better over-all design, particularly in conventional constructions where accurate dimensions are necessary to maintain the radial location of stators slipped or pressed into the shells. These have the same coefficient of thermal expansion as that of the magnetic iron, and therefore the same accurate dimensions exist over the entire temperature range. Aluminum or magnesium shells have coefficients of thermal expansion greatly different from that of magnetic iron. Because of this, satisfactory dimensions at normal temperatures may not remain satisfactory and may cause failure through excessive stresses in the shells and magnetic structures when extreme temperatures below normal are approached.

Owing to limitations in manufacturing processes, the end flanges usually are made of low-specific-gravity materials. Internal rabbets should be used to eliminate failure from excessive stresses at low temperatures when the shells are made of steel.

At present, only ball bearings are satisfactory for the high-speed operation, but sleeve bearings with special lubrication may be used in the future. Bearings of the minimum size necessary for radial and thrust loads and the specified life should be used to prevent excessive torque for overcoming friction at low temperatures.

High-grade steel shafts with hardened extensions are required, particularly where pinions or splines are an integral part of the shaft extensions.

Lubrication

Suitable lubrication capable of withstanding high temperatures without freezing at low temperatures cannot be obtained. Those available lubricants capable of withstanding high temperatures freeze at about −30 degrees centigrade. Those satisfactory for low temperatures fail in a relatively few hours if operation is sustained at about 60 degrees centigrade or above. Lubricants do not remain frozen long after operation is started, and therefore they are objectionable only because of the added torque required to break the bearings loose. Generally, it is better to use the high-temperature lubricants and provide the additional torque required for starting.

Motor Characteristics

Specifications for the motor characteristics were established by considering many loads encountered aboard aircraft, together with anticipated variations caused by voltage and frequency fluctuations and changes in operating conditions. Both intermittent and continuous loads were scrutinized.

Intermittent motors are required principally to operate a variety of mechanisms such as wing flaps, cowl and oil cooler flaps, bomb-bay and turret doors, automatic temperature controls, and armament. However, some are required for driving hydraulic pumps, air compressors, or other miscellaneous applications. Fundamentally these are torque applications in which the power requirements vary almost continuously during the operating period.

Therefore the motors will be given a torque rating approximately equal to but not more than one third of the normal locked-rotor torque. In most of these applications peak torque requirements are considerably less than 200 per cent of the root-mean-square torque. The locked-rotor torques may decrease to about 200 per cent of their normal values under the most severe operating conditions which are brought about by a combination of minimum voltage, maximum frequency, and minimum temperature. Hence, 300-per-cent starting torque as established for the specifications is adequate for most applications. Usually there will be a surplus of torque as the factors entering into the most severe operating conditions do not occur simultaneously.

Continuous-rated motors are needed principally for driving fans, fuel pumps, hydraulic pumps, and air compressors, either directly or through gear reductions. Motorization of fans and fuel pumps of the centrifugal type, which are the devices "most used" usually, entails selecting ratings corresponding to the peak load requirements. Locked-rotor requirements are essentially those of the motor itself in the infrequent applications where gear reductions are used. Motorization of the other devices entails selecting ratings up to about ten times the normal or root-mean-square load if sufficient capacity for peak loads or starting is provided. The occasional or even frequent necessity of selecting ratings in excess of the normal load does not mean that the specifications should require greater breakdown and locked-rotor torques. Motor weights for these applications are the same, regardless of whether or not the ratings correspond to the normal or root-mean-square loads, providing the ratio of breakdown to locked-rotor torque is correct. However, standardizing on increased values for these torques would raise the motor weights as well as that of associated equipment for applications not requiring a reserve for peak loads or starting. The ratio of breakdown to locked-rotor torque, although not ideal for all applications, is nearly correct for the majority of them.

Illustrations and Characteristic Curves

Figure 1 shows a three-quarter-horsepower continuous 400-cycle 200-volt three-phase motor designed to operate in accordance with the requirements previously outlined. The weight of this motor is 3.4 pounds. It is about 40 per cent lighter than a d-c motor having the same horsepower and speed rating. Other 400-cycle and 24-volt d-c motors with corresponding ratings have about the same ratio of weight.

Figure 2 shows characteristic curves plotted from test data taken on the three-quarter-horsepower motor at normal voltage, frequency, altitude, and ambient temperature. Speeds, efficiencies, power factors, amperes, and watts output which are plotted against torques are included.

Figure 3 illustrates schematically the construction used for continuous-rated motors with outputs from one eighth to 25 horsepower. That used for intermittent-rated motors is similar except the ventilation ducts, overhung fans, and fan covers are omitted.

Major design features used in these motors are summarized as follows:

1. Magnetic structures fabricated from high-grade silicon steel with special anneal to permit high flux, good permeability, and low iron loss.

2. Insulation. Class-*B* material for insulating slots and for between-phase protection. Generally the stator windings are of heavy Formex-glass-covered wire to provide greater protection against heat shock. Stator core complete with windings impregnated with special moisture-resisting varnish.

3. Squirrel-cage-type rotors with brazed copper or brass windings. Protected with anticorrosion treatment.

4. Through ventilation in continuous-rated motors for minimum composition and weight. Double end-turn ventilation in intermittent-rated motors.

5. Shells made by expanding thin steel tubing to obtain light weight, rigid construction, and a temperature coefficient the same as that of the stator.

6. Magnesium or aluminum end shields for light weight.

7. Heat-treated alloy steel shafts for strength and dependability. Hardened shaft extensions with or without pinions or splines.

8. Double-shield-type permanently lubricated ball bearings housed in a steel insert in the end-shield castings to assure long life.

9. Protective finishes. Magnesium or aluminum castings treated against corrosion and primed with zinc chromate. Steel parts other than stator and rotor punchings cadmium- or zinc-plated. External parts except mounting surface have black wrinkle finish.

Conclusions

Present trends indicate an increasing demand for 400-cycle electric systems for both military and commercial airplanes. These trends are justified in part by the inherently light-weight rugged and dependable constructions possible in 400-cycle motors.

The information contained in the paper is that which seems most important. Many additional data have not been mentioned; however, it is hoped that the disclosures herein will assist in creating a better understanding of the motors and the performance to be expected from them.

Analysis and Design of a D-C Selsyn System

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Synopsis: At the present time there are several remote-indicating systems in common usage in industry. These types include the conventional a-c Selsyn, a-c Magnesyn system, and the d-c Selsyn. The d-c Selsyn embodies two features which present a distinct advantage in aircraft systems.

1. The system does not require an a-c source of power.

2. The system requires only three wires between transmitter and receiver, whereas all other types of systems require at least five wires leading from the transmitter to the receiver.

To offset somewhat these advantages, all d-c Selsyn systems now in operation have a very slight natural error which, although not important in some applications, becomes very undesirable in applications where a high degree of accuracy is required. In this paper the author presents a rigorous analysis of a conventional d-c Selsyn remote-indicating system. The first part of the analysis shows that a natural error exists in the system, which has a 60-degree cycle and which reaches a maximum value of 1.1 degrees. The error is zero at each 30-degree point and reaches a maximum almost midway between the 30-degree points. The natural error of the Selsyn system arises from the fact that the transmitter resistance is a linear function of the transmitter angle θ. This natural error may be overcome by designing a transmitter resistance in a nonlinear manner. The equation is developed for the necessary variation of the transmitter resistance with the transmitter angle. Furthermore, the shape of the toroid on which this resistor should be wound to give this desired variation, is derived therefrom. The paper further presents optimum values of resistances for both transmitter and receiver, in order that the greatest amount of torque may be transmitted to the receiver for a given amount of power.

Determination of Natural Error in a Conventional D-C Selsyn System

THE INVESTIGATION of the d-c Selsyn embodies two separate questions.

1. If no frictional error is assumed, what are the magnitudes and characteristics of natural errors which may exist in the instrument, and how may they be corrected?

2. For a given amount of power consumption how should the instrument be constructed to give best follower properties from transmitter to receiver; that is, for a given amount of friction, how can the frictional error be reduced to a minimum?

In analyzing the first question, we consider the conventional d-c Selsyn shown in Figure 1. The receiver coils *AB*, *BC*, and *CA* are wound with their magnetic axes at 120 degrees to each other and connected in a delta connection. Points *A*, *B*, and *C* are connected to points *A'*, *B'*, *C'*, respectively, located at 120-degree intervals on the transmitter winding. The battery contacts *D'* and *E'* are on a diameter and may be moved at will.

Ideally, the direction of the resultant field in the receiver should move in exact coincidence with the angle θ in the transmitter. There is no a priori reason for supposing that this situation will exist. Actually, as we shall see, if the resistances in the transmitter are linear with the angle θ, the ideal situation does not exist. The amount of natural error introduced is determined in the following analysis:

We obtain first the current distribution in coils *AB*, *BC*, and *CA* when the contacts *D'* and *E'* are rotated θ degrees from *A'* as shown. $0 \leq \theta \leq 60$ degrees. The transmitter is assumed to have a re-

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sistance of R_1 ohms per degree, or 120 R_1 ohms per leg. The receiver is assumed to have a resistance of 120 R_2 ohms per leg. Each section of resistance in the circuit is numbered as shown in Figure 1, and the equivalent circuit is shown in Figure 2, with corresponding numbers and the value of each resistance shown. The points C, A, and B are marked, since we are interested in the voltages between these points.

To simplify the analysis we replace the delta sections 3-4-7 and 1-2-6 by their equivalent wye sections.

The transformation relationships which replace a delta system R_1-R_2-R_3 by a wye system r_1-r_2-r_3 are given by

$$\left.\begin{aligned} r_1 &= \frac{R_2R_3}{R_1+R_2+R_3} \\ r_3 &= \frac{R_1R_3}{R_1+R_2+R_3} \\ r_2 &= \frac{R_1R_2}{R_1+R_2+R_3} \end{aligned}\right\} \qquad (1)$$

The foregoing relationships leave the current and voltages at the three terminals unchanged.

Figure 3 shows the equivalent circuit after the transformation is made and the resistors 5 and 8 are combined. Values are shown. The resistance of the two parallel branches in Figure 3 are equal; hence half the total current flows in each branch. The system is further reduced in Figure 4, and we obtain for the total current

$$2i = \frac{60E(R_1+R_2)}{5{,}400R_1R_2+R_1^2(-\theta^2+60\theta+1{,}800)} \qquad (2)$$

The current in each branch is then

$$i = \frac{30E(R_1+R_2)}{5{,}400R_1R_2+R_1^2(-\theta^2+60\theta+1{,}800)} \qquad (3)$$

Referring to Figure 3, we have the following voltages:

$$E_{CA} = i\left(\frac{120R_1R_2}{R_1+R_2}\right) = \frac{3{,}600Er}{(-\theta^2+60\theta+1{,}800)+5{,}400r} \qquad (4)$$

$$E_{AB} = -i\left(\frac{R_1R_2(120-\theta)}{R_1+R_2} - \frac{\theta R_1R_2}{R_1+R_2}\right) = \frac{-30Er(120-2\theta)}{(-\theta^2+60\theta+1{,}800)+5{,}400r} \qquad (5)$$

$$E_{BC} = -i\left(\frac{R_1R_2(60+\theta)}{R_1+R_2} - \frac{R_1R_2(60-\theta)}{R_1+R_2}\right) = -\frac{60Er\theta}{(-\theta^2+60\theta+1{,}800)+5{,}400r} \qquad (6)$$

where $r = R_2/R_1$. The currents in the coils are then, from Figure 1,

$$I_{CA} = -\frac{30E}{R_1}\left[\frac{1}{(-\theta^2+60\theta+1{,}800)+5{,}400r}\right] = K\phi_{CA} \qquad (7)$$

$$I_{AB} = -\frac{E}{2R_1}\left[\frac{60-\theta}{(-\theta^2+60\theta+1{,}800)+5{,}400r}\right] = K\phi_{AB} \qquad (8)$$

$$I_{BC} = -\frac{E}{2R_1}\left[\frac{\theta}{(-\theta^2+60\theta+1{,}800)+5{,}400r}\right] = K\phi_{BC} \qquad (9)$$

For a fixed number of turns, the magnetic fields due to each of the coils are proportional to the currents. Figure 5 shows the relative orientation of these fields when the current in each branch is positive. We denote the direction of the resultant field, ϕ, with the x axis by the angle θ'. When $\theta=0$ we see from equations 7, 8, and 9 that ϕ_{CA} and ϕ_{AB} are equal in magnitude, and $\phi_{BC}=0$. ϕ_{CA} is positive, and ϕ_{AB} is negative. Hence when $\theta=0$, $\theta'=0$.

To obtain the value of θ' for any value of θ, we denote the x and y components of the resultant flux by ϕ_x and ϕ_y. From Figure 5 we see that

$$\phi_x = \frac{\sqrt{3}}{2}(\phi_{CA}-\phi_{AB}) \qquad (10)$$

$$\phi_y = -\phi_{BC}+{}^1/_2(\phi_{CA}+\phi_{AB}) \qquad (11)$$

and

$$\tan\theta' = \frac{\phi_y}{\phi_x} \qquad (12)$$

Using equations 7, 8, and 9 and noting that the proportionality constant between flux and current is the same in every case, in conjunction with equations 10, 11, and 12, we obtain

$$\tan\theta' = \sqrt{3}\frac{\theta}{120-\theta} \qquad (13$$

It is important to note that equation 13, which shows how the receiver angle θ' varies with the transmitter angle θ, is independent of the values of the resistances used in the Selsyn. Hence changing the values of the resistances can in no way compensate for natural error in the instrument.

Figure 6 shows a plot of θ' versus θ for 60 degrees. Conditions repeat themselves every 60 degrees, of course, since at 60-degree intervals one battery contact is in coincidence with a coil terminal. We see from the figure that the receiver gives an exact indication of the transmitter every 30 degrees. On one side of this point the receiver lags and on the other side it leads the transmitter.

If we denote by β the difference of the receiver angle and the transmitter angle, we have

$$\beta = \theta-\theta' = \theta-\tan^{-1}\left(\frac{\theta\sqrt{3}}{120-\theta}\right) \qquad (14)$$

It readily may be shown that this error angle is symmetrical in magnitude about the point θ = 30 degrees. Figure 7 shows a plot of β versus θ for $0 \leqq \theta \leqq 60$ degrees.

By differentiating equation 14 with respect to θ and setting the result equal

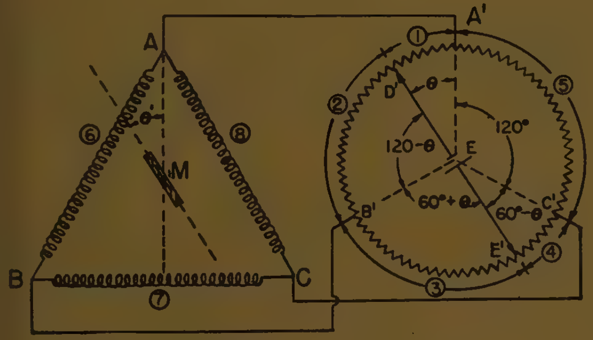

Figure 1 (left). D-c Selsyn transmitter and receiver

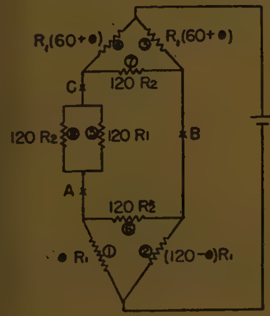

Figure 2 (lower-left). Electric circuit of Figure 1

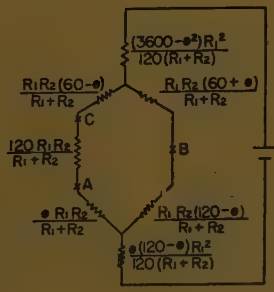

Figure 3 (center). Reduction of circuit in Figure 2

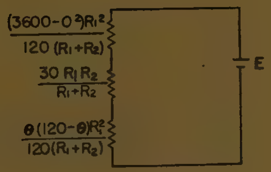

Figure 4 (below). Further reduction of circuit in Figure 2

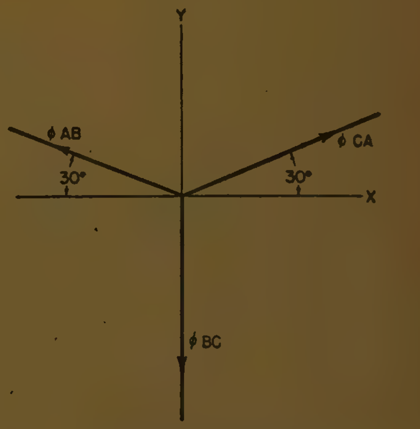

Figure 5. Space relationships of fluxes produced by receiver coils

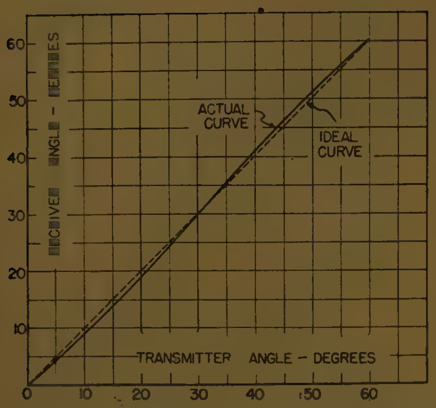

Figure 6. Variation of receiver angle with transmitter angle

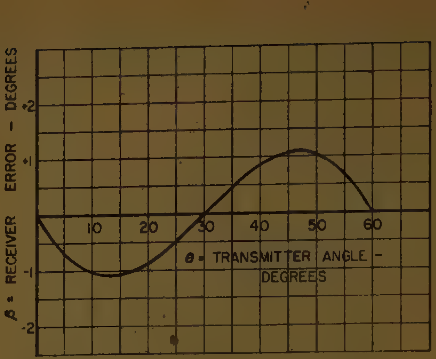

Figure 7. Natural error in Selsyn receiver

$$\beta=\tan^{-1}\left(\frac{\sqrt{3}\,\theta}{120-\theta}\right)-\theta$$

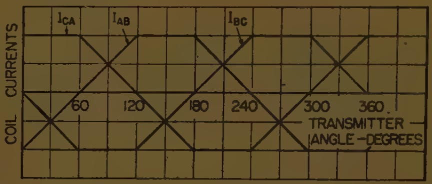

Figure 8. Current distribution in receiver coils, $r=R_2/R_1$

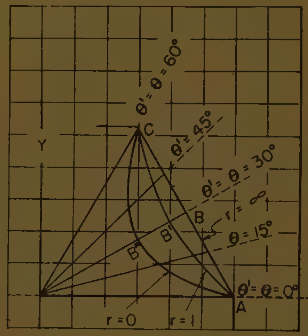

Figure 9. Loci of ends of flux vector for various resistance ratios

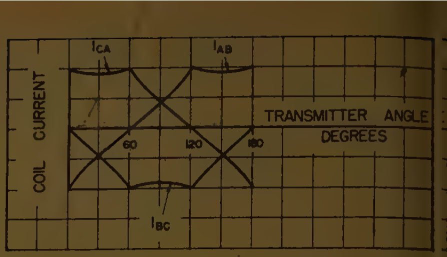

Figure 10. Current distribution in receiver, coils for $R_2/R_1=1$

to zero, we obtain the value of θ for which β is a maximum.* This gives values of θ = 13 degrees 18 minutes and 46 degrees 42 minutes. The corresponding maximum values of β are 1 degree 6 minutes and −1 degree 6 minutes, respectively.

It is also of interest from a design standpoint to investigate the variation of the magnitude of the resultant field as θ is varied, since the friction error will depend on this magnitude. To do this we investigate first the case for $r\to\infty$, that is when the receiver resistance is many times larger than the transmitter resistance. Then $(-\theta^2+60\theta+1{,}800)$ is negligible compared to $5{,}400r$ in equations 7, 8, and 9. These equations become

$$I_{CA}=\frac{E}{180R_2} \tag{7′}$$

$$I_{AB}=\frac{-E(60-\theta)}{10{,}800R_2} \tag{8′}$$

$$I_{BC}=\frac{-E\theta}{10{,}800R_2} \tag{9′}$$

Figure 8 shows the current distribution in the coils for θ from 0 to 360 degrees. Equations 7, 8, and 9 are valid only from 0 to 60 degrees, but the rest of the cycle can readily be seen to be as shown. Substituting the foregoing values in equations 10 and 11, we obtain

$$\phi_x=K\left(2-\frac{\theta}{60}\right) \tag{10′}$$

$$\phi_y=K\sqrt{3}\,\frac{\theta}{60} \tag{11′}$$

where K = constant.

These are the parametric equations of one side of a regular hexagon centered at the origin and having a vertex on the x axis. The plot is shown in Figure 9. The line ABC is the locus of the ends of the flux vector as θ varies from 0 to 60 degrees.

To obtain the locus of the end of the flux vector for any value of r, we simply note that all the currents given by equations 7′, 8′, and 9′ will be equal to the currents given by equations 7, 8, and 9 when the former are multiplied by a factor proportional to $1/(-\theta^2+60\theta+1{,}800+5{,}400r)$. Hence, the vectors ending on the line ABC need only be multiplied by the afore-mentioned factor to give the locus for any value of r. It is to be noted that the angle the flux vector makes with the x axis is θ', not θ, and hence equations 10′ and 11′ multiplied by the foregoing factor should be used to determine the locus. The lines $AB'C$ and $AB''C$ show the locus for $r=1$ and $r=0$, respectively. For comparison purposes, the maximum values in each case have been made to coincide. The actual value of maximum flux depends on R_1 and R_2, but this is discussed in the second part of the report. Figure 10 shows the variation of current in each coil for $r=1$.

It may be seen from Figure 9 that the flux reaches a minimum every 60-degree interval of transmitter rotation. In the analysis which follows, design considerations will be based on making this minimum value of flux as large as possible for given power. As we shall see, this is not the same as making the maximum flux as large as possible.

Optimum Transmitter and Receiver Resistances

For a given amount of power, and a given amount of space in which to wind the receiver coil, it is possible to determine optimum resistances for both transmitter and receiver and hence the wire size to be used in the receiver. It is also possible to determine the best size of receiver coil, consistent with space limitations.

Let us assume that the coil is to have the dimensions shown in Figure 11.

l = length of one turn
a = cross-section area of coil
N = number turns on one coil
A = cross-section area of wire

We then have

$$120R_2=\text{total resistance of coil}=\frac{\rho l N}{A} \tag{15}$$

$$NA=\alpha a$$

where α is a spacing factor. (16)

$$\alpha=0.91$$

We wish to make the resultant flux a maximum when $\theta=30$ degrees, which is the weak flux point. This amounts to making NI in the coil CA a maximum for $\theta=30$ degrees. Using equation 7, we have

$$NI=\frac{30E}{R_1}\,\frac{N}{2{,}700+5{,}400r}$$
$$=\frac{E}{90}\,\frac{N}{R_1+2R_2} \tag{17}$$

Using equations 15 and 16, we eliminate N and obtain

$$NI=K\frac{R_2^{1/2}}{R_1+2R_2} \tag{18}$$

where

$$K=\frac{E}{45}\sqrt{\frac{30\alpha a}{\rho l}}$$

It is evident from equation 18 that K is made large by increasing a, decreasing ρ, or decreasing l for a given supply

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* See Appendix I.

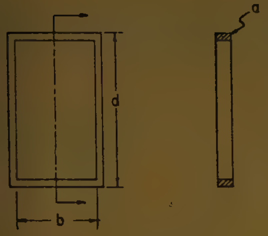

Figure 11. Coil dimensions

l = mean length
One turn
= 2(d + b)

voltage. It is not advisable to decrease l, since the enclosed area of the coil is proportional to torque for given NI. However, for best results, a should be made as large as possible and ρ as low as possible, that is, low-resistivity material should be used.

The maximum power consumed in the circuit is given by

$$P=\frac{E^2}{R_{\min}}$$

where $R_{\min}$ is the least value of resistance between battery terminals. From Figure 4 we see that

$$R_{\min}=\frac{30R_1}{R_1+R_2}(R_1+3R_2)$$

and since

$$P=\frac{E^2}{30}\frac{R_1+R_2}{R_1(R_1+3R_2)} \tag{19}$$

from which

$$R_2=\frac{R_1\left(1-\frac{30P}{E^2}R_1\right)}{\left(\frac{90P}{E^2}R_1-1\right)} \tag{20}$$

Substituting value of R_2 from 20 in equation 18, we obtain

$$NI=K\frac{(-3K'^2R_1^2+4K'R_1-1)^{1/2}}{R_1^{1/2}(K'R_1+1)};\quad K'=\frac{30P}{E^2} \tag{21}$$

By differentiating expression 21 with respect to R_1 and equating result to zero, we obtain, for maximum NI*

$$R_1=\frac{1+2\sqrt{2}}{7K'}=0.547\frac{E^2}{30P} \tag{22}$$

$$120R_1=\text{resistance per leg}=2.19\frac{E^2}{P}$$

and from equation 20

$$R_2=R_1\frac{(3-\sqrt{2})}{(3\sqrt{2}-2)}\doteq\frac{1}{\sqrt{2}}R_1 \tag{23}$$

With the optimum values determined from equations 22 and 23, the size wire for the receiver may be calculated from equations 15 and 16.

Small variations of the receiver resistance from the optimum value will not affect the performance appreciably, although power consumption, of course, will change. This may be seen by writing equation 18 in the form:

$$NI=\frac{K}{R_1^{1/2}}\frac{r^{1/2}}{(1+2r)} \tag{18'}$$

Equation 18 is plotted in Figure 12 for both R_1 constant and R_2 constant. It may be seen that the variation of NI with R_2 is small, whereas the variation with R_1 is greater, in the region of $r=\frac{1}{\sqrt{2}}$.

Design equations based on making the flux a maximum in the strongest point, that is, when $\theta=0$, lead to somewhat different equations. Analysis similar to the foregoing yields the following results:

$$120R=\text{resistance per leg}=2.30\frac{E^2}{P} \tag{22'}$$

$$R_2=\frac{R_1}{\sqrt{3}} \tag{23'}$$

For best performance, equations 22 and 23, not 22′ and 23′, should be used.

Correction of Natural Error in Selsyn System

In the first part of this report it was found that the error is zero in the receiver when the transmitter angle θ (see Figure 1) is 0, 30, 60 degrees, and so forth. Since one of the brushes is in contact with a terminal every 60 degrees, it is obvious that any nonlinear resistance must be symmetrical on either side of every 0, 60, 120 degrees, and so forth. Furthermore, since the natural error for a linear resistance is symmetrical and of opposite sign on either side of every 30-degree point, we may assume that the nonlinear resistance should be symmetrical on either side of 0, 30, 60 degrees, and so on.

The foregoing considerations reduce the problem to one of finding the deviation from linearity for the region from 0 to 30 degrees, making the deviation from 30 to 60 degrees symmetrical about the 30-degree point and of opposite sign, then repeating the cycle every 60 degrees.

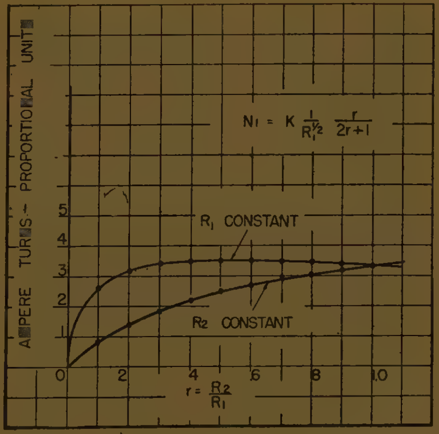

Figure 12. Variation of ampere turns with R_1 and R_2 constant power

Referring to Figure 1, we let the resistance of each transmitter leg be $120R_1$, and the resistance of each receiver leg $120R_2$, as before. For $\theta\leqq 30$ degrees, let the resistance of that portion of the winding marked ① be equal to αR_1. The problem now resolves itself into one of finding α as a function of θ such that the error shall be zero for all values of θ from zero to 30 degrees.

Remembering the symmetry which we have noted, the values of the resistances ① to ⑤ of the transmitter then must be

① $=\alpha R_1$
② $=(120-\alpha)R_1$
③ $=(60+\alpha)R_1$
④ $=(60-\alpha)R_1$
⑤ $=120R_1$

Referring to Figure 2, we see that the afore-mentioned resistances are exactly the same as those shown in Figure 2, with α substituted for θ. In our present situation, then, it is obvious that the receiver angle θ', will be given by equation 13, with α substituted for θ, or

$$\tan\theta'=\frac{\sqrt{3}\alpha}{120-\alpha} \tag{24}$$

In order to have zero error, $\theta=\theta'$, and $\tan\theta=\sqrt{3}\alpha/(120-\alpha)$.

Solving for α, we obtain

$$\alpha=\frac{120\tan\theta}{\sqrt{3}+\tan\theta} \tag{25}$$

This gives the necessary variation of α with θ for zero error and hence the variation of resistance with θ, since the resistance of segment ① is αR_1.

Figure 13 shows the variation of α with θ, along with the linear variation, so that the deviation may be noted.

It is to be especially noted that the variation of α with θ for zero error is independent of the receiver resistance and hence will work for all receivers or paralleled receivers.

To obtain the shape of the developed cylinder which will give a resistance in accordance with equation 25, we refer to Figure 14. $OABC$ represents 30 degrees

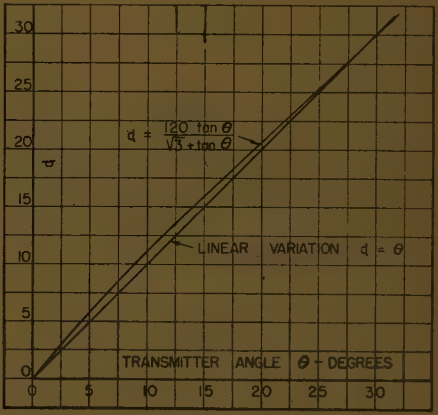

Figure 13. Variation of modification factor with transmitter angle θ

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* See Appendix II.

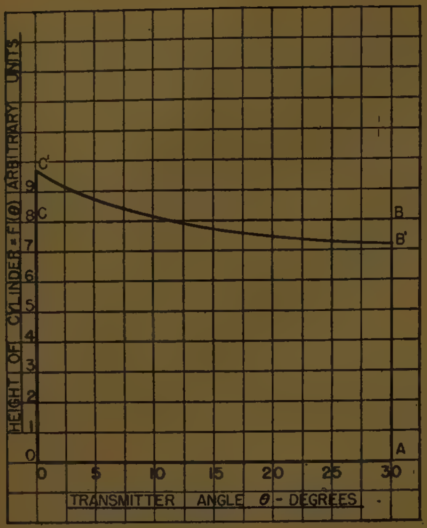

Figure 14. $F(\theta)$ for 0 degrees $\leq\theta\leq$ 30 degrees

of the developed cylinder for a linear resistor. $OAB'C'$ represents the shape of 30 degrees of the developed cylinder for the nonlinear resistor.

Let us denote the equation of $C'B'$ as $F(\theta)$. For a particular value of θ, the ratio of the nonlinear resistance to the linear resistance will be equal to the ratio of the area beneath the curve $C'B'$ to the area beneath the curve CB. This ratio is obtained from the equations of the two curves in Figure 13. Hence we have

$$\frac{\int_0^\theta F(\theta)d\theta}{L\theta}=\frac{120\tan\theta}{\sqrt{3}+\tan\theta}\cdot\frac{\pi}{180\,\theta}$$
$$=\frac{2\pi}{3\theta}\,\frac{\tan\theta}{\sqrt{3}+\tan\theta}$$

where θ is expressed in radians, and $L=OC$ in Figure 15.

If we solve for the numerator in the lefthand side,

$$\int_0^\theta F(\theta)d\theta=\frac{2\pi L}{3}\,\frac{\tan\theta}{\sqrt{3}+\tan\theta} \tag{26}$$

If we differentiate both sides with respect to θ,

$$F(\theta)=\frac{2\pi L}{3}\,\frac{d}{d\theta}\left(\frac{\tan\theta}{\sqrt{3}+\tan\theta}\right)$$
$$=\frac{2\pi L}{3}\,\frac{1}{(\sqrt{3}\cos\theta+\sin\theta)^2} \tag{27}$$

Figure 15 shows $F(\theta)$ plotted against θ from 0 to 30 degrees. $OAB'C'$ is thus the developed shape of 30 degrees of the toroidal resistor. The developed shape for the whole resistor is shown in Figure 15.

Conclusions

In a d-c Selsyn a natural-error cycle exists which has a 60-degree period. The error is positive for 30 degrees and negative for 30 degrees, reaching a maximum of 1 degree 6 minutes at 13 degrees 18 minutes and 46 degrees 42 minutes on the transmitter.

Design equations to give maximum flux in receiver for a given voltage and power are given by

$$\text{Transmitter resistance}=2.19\frac{E^2}{P}\text{ ohms per leg}=R_T \tag{22}$$

$$\text{Receiver resistance}=\frac{1}{\sqrt{2}}R_T\text{ ohms per leg} \tag{23}$$

Furthermore, the cross-sectional area of the receiver coil should be as large as is practical, and the resistivity of the receiver coil as low as possible.

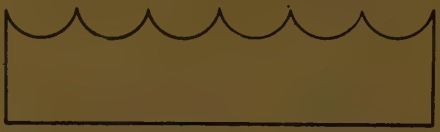

Figure 15. Developed shape of cylinder on which transmitter-resistor may be wound in order to produce zero error

The material and design dimensions of the transmitter coil are immaterial insofar as flux properties are concerned.

The natural error of a d-c Selsyn system may be eliminated completely by winding the transmitter resistance in a nonlinear manner.

The necessary variation of resistance with transmitter angle to give zero error is independent of receiver resistance. This correction for natural error may be attained for all possible loads on the transmitter.

One method of obtaining the desired resistor is to wind it on a cylinder whose developed shape is shown in Figure 16. This is by no means the only way of accomplishing the desired result. Any method which will make the resistance vary in accordance with equation 25 is satisfactory from an operational viewpoint. From a manufacturing viewpoint there may be more desirable methods than that of modifying the shape of the toroid on which the resistor is wound.

Appendix I. Maximum Natural Error in Selsyn

If we use equation 14, and express angles in radians, so that we may differentiate

$$\beta=\frac{\theta}{57.3}-\tan^{-1}\frac{\sqrt{3}\theta}{120-\theta}$$

$$\frac{d\beta}{d\theta}=\frac{1}{57.3}-\frac{1}{1+\left(\frac{\sqrt{3}\theta}{120-\theta}\right)^2}\times\frac{(120-\theta)\sqrt{3}+\sqrt{3}\theta}{(120-\theta)^2}$$

$$=\frac{1}{57.3}-\frac{120\sqrt{3}}{(120^2-240\theta+4\theta^2)}=0$$

for maximum β

If we solve,

$$\left(\frac{\theta}{60}\right)^2+\left(\frac{\theta}{60}\right)+1=\frac{57.3\sqrt{3}}{120}=0.828$$

$$\left(\frac{\theta}{60}\right)^2+\left(\frac{\theta}{60}\right)+0.172=0;$$

$$\frac{\theta}{60}=\frac{1\pm\sqrt{1-0.69}}{2}=0.778\text{ or }0.222$$

$\theta=13.3°, 46.7°$

Appendix II. Conditions for Maximum NI in Receiver

If we use equation 21,

$$NI=K\frac{\Delta}{R_1^{1/2}(K'R_1+1)}$$

where

$$\Delta=[-3K'^2R_1^2+4K'R_1-1]^{1/2}$$

$$\frac{d(NI)}{dR_1}=0$$
$$=K\left[\frac{R_1^{1/2}(K'R_1+1)\cdot{}^1/_2\Delta^{-1}\times(-6K'^2R_1+4K')-\Delta\{R_1^{1/2}K'+R_1^{-1/2}(K'R_1+1)\}}{R_1(K'R_1+1)^2}\right]$$

Setting numerator equal to zero, we have

$$R_1(K'R_1+1)(-6K'^2R_1+4K')=(-3K'^2R_1^2+4K'R_1-1)(R_1K'+K'R_1+1)$$

If we multiply and separate powers of R_1,

$$-7K'^2R_1^2+2K'R_1+1=0$$

$$R_1=\frac{1}{K'}\,\frac{-2\pm\sqrt{4+28}}{14}=\frac{1}{K'}\,\frac{1\pm\sqrt{8}}{7}$$

If we discard negative values of R_1, as meaningless.

$$R_1=\frac{1+2\sqrt{2}}{7K'}=\frac{0.547}{K'}$$

Dynamic Braking With Traction Motors

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ynopsis: Retardation with traction mo-ors, either independently or as an adjunct o mechanical braking, has been utilized in arious forms on practically all types of lectric vehicles from battery trucks to main-line locomotives. In some cases, it is used only to maintain uniform speeds on ong grades and in others to make regular ervice stops. Although the general prin-iples are nearly as old as the electric-trac-ion industry itself, in recent years numerous efinements have greatly extended the use-ulness of dynamic braking by affording improved performance, additional safety, and implification of mechanical-braking sys-ems. This paper discusses the various means by which favorable results have been accomplished, with a minimum amount of additional electric equipment.

The Origins of Dynamic Braking

FROM THE STANDPOINT of basic terminology, "dynamic braking" could cover any and all forms of retardation created by an electric motor acting as a generator. However, in traction parlance, the term is recognized as referring to a system of dissipating the generated energy in resistors carried on the vehicle, as distinguished from "regenerative braking" where the generated energy is returned to the power system, necessarily at line voltage.

In the early days when streetcar speeds and weights outgrew the hand brake, air brakes had not been developed for small equipments, and the need for some form of power-operated braking apparatus led to the application of several kinds of "magnetic" and "dynamic" brakes. The traction motors were used to generate on a closed resistor circuit, the dynamic current producing retardation in the motors themselves and also energizing magnetic track shoes and applying wheel brakes, both at the same time. For standing on grades, the magnetic shoes were energized by line current, becoming a direct "electric" brake.

It was known that a series motor, rotated backwards and short-circuited on itself, would generate current and "buck," as a result of building up circulating current from the residual field magnetism retained in the iron after operating with power. By reversing the field connections with reference to the armature, the motor would "buck" without changing the direction of rotation. Two motors permanently connected in parallel provide a loop circuit which permits both to build up as generators. This action is experienced when one is attempting to tow a four-motor car backwards without first opening the paralleling circuit between motors.

Conventional four-motor equipments can be stopped in emergency, without power, simply by reversing the motors. The result is a cumulative generating effect which builds up suddenly, after some delay, the current and voltage rising to excessive peaks. Before the car comes to a stop, frequently with a series of jerks, one motor usually will slip and start running backwards, being driven as a motor by the other machine, which continues to generate at a diminishing rate until the car slows down to a creeping speed.

In all of the early forms the traction motors were reversed in changing from power to brake operation and were operated all in parallel on a single load circuit consisting of brake magnet coils and variable resistors. A stabilizing connection was made, paralleling the motor fields to equalize the load on all armatures.

Later, additional stability was secured by cross-connecting pairs of armatures and fields, in effect producing separate excitation and better inherent equalizing of the load.

Typical forms of magnet-operated wheel and track brake shoes are illustrated in Figure 1. It is interesting to note that many of the operating advantages claimed for these equipments are as fully attractive today as they were when advocated 40 years ago. These may be summarized as follows:

1. Regulated braking independent of line power.
2. Safety in operation, with an additional powerful braking system.
3. Control of speed down long grades.
4. Use of braking energy for car heating.
5. Braking rates in excess of adhesion limits.
6. Prevention of skidding with locked wheels.

These pioneer systems were very effective and were used extensively abroad but fell into disuse in this country with the development of the simple "straight-air" brake equipment. Some of the limiting factors were the abuse of the electric apparatus, both motors and control, and the excessive weight and mechanical maintenance of magnetic-shoe devices. As a result, there was little advance in the art of "dynamic" braking over a period of several years.

Design and Performance of Motors Used as Series Generators

One of the major factors leading to the revival and extensive use of dynamic braking in recent years has been the improvement in commutating characteristics and overload capacities in the modern lines of traction motors. Normal proportioning of the operating characteristics found in passenger vehicles involve a balancing speed of at least twice the speed at which the motors have full voltage applied at full load. To produce a full-load braking rate in the balancing-speed zone, it is obvious that the generated voltage may be double the rated line voltage. Motors capable of satisfactory commutation under these conditions are in common use. To provide for braking at occasional excessive speeds, it is sometimes desirable to initiate the braking action with the fields shunted so as to limit the voltage to a safe maximum, accepting a reduced braking rate in the high-speed zone.

Inherent stability by inductance and also generous commutating and insulation factors are essential design characteristics.

In the case of four-motor equipments, the motors frequently are wound for half voltage and connected permanently two in series. In the limited space available, it is practicable to build low-voltage commutators and brush rigging with an increased margin for overvoltage operation, thus extending the speed range for full-load dynamic-brake operation.

Motor applications for dynamic braking must take into account grade conditions amounting to 20 pounds of retarding force per ton of total train weight for each one per cent of downgrade, in addition to the stored kinetic energy requiring approximately 100 pounds per ton of retarding force for each unit of miles per hour per second braking rate desired. The actual braking effort required of the motor is reduced by the friction and windage drag of the vehicle. Obviously, the total must be kept within the recognized adhesion limits of the motorized axles and the rated capacity of the motors.

The motor rating must be selected with sufficient margin to perform the combined duty of acceleration and braking. In the case of self-ventilated motors, the extra heating is minimized by the fact that it takes place mostly at high speeds where the ventilating system is most effective.

Where two motor circuits are available for interconnection, they will pick up generating voltage very quickly and give a prompt brake response over a wide range of speed. In the case of single-motor equipments, the response is not so prompt, and field flashing is desirable.

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The amount of exciting current normally is not more than ten per cent of full load, and it can be removed as soon as braking is initiated, so that the added power requirement is negligible.

The typical behavior of a motor in building up the braking load is illustrated by the oscillograph record shown in Figure 2.

Dynamic-Brake-Control Characteristics

Another factor in the rapid extension of the use of dynamic braking has been the development of simplified control systems for initiating and regulating the braking load.

By adopting a permanent connection for each pair of motors whereby in both power and brake the relations of armature and field are interchanged with reference to the line, the series and parallel connections for power and the parallel connection with cross-connected fields for braking can be obtained with a minimum of switching equipment. Furthermore, the transfer from power to brake can be made without operating the reversing devices, and practically all of the switching and resistor equipment required for acceleration also can be used in brake regulation. Figure 3 illustrates in simplified form the relative arrangement of motor and resistor circuits.

In addition to the accelerating resistor steps in each motor circuit which are kept balanced in value at all times, the brake circuit is completed through a common loop which carries the combined current from both motors and includes resistor steps of a value which essentially doubles the total effective resistance in each motor circuit. This total amount is determined by the excess voltage to be absorbed when applying the brake at maximum speed.

In order to secure prompt build-up of the brake without separate excitation, it is customary to close the brake loop circuit through a high resistance each time power is shut off. This recaptures the residual volts before the flux has collapsed and immediately circulates a braking current of low value, not enough to cause any appreciable retarding effect, but sufficient to keep the motors in a responsive state. Brake application then is uniformly prompt, depending on the rate at which resistance is reduced in the loop circuit. This is known as the "spotting" system.

The basic cross-connection of motors, whereby each becomes separately excited, not only produces inherent stability and load equalization, but also avoids skidding, because any tendency on the part of one motor to lower its speed immediately reduces the field of the other motor. This in turn reduces the voltage being applied to the motor which has attempted to slow down by creeping. There is no sudden or complete loss of retardation.

The regulation of braking rates may be controlled either manually or automatically. Locomotives with varying train weights and grade conditions normally would be manually operated and the brake used principally for holding a controlled speed on long grades. Where grade or slowdown brake requirements exceed the locomotive adhesion limits, dynamic braking must be supplemented by the train brakes, and some provision is desirable to prevent applying the air brakes on the locomotive as long as the dynamic brake is operating.

Passenger vehicles in frequent-stop service require uniform braking, sometimes at relatively high rates, all of which is best accomplished with automatic current regulation. The operator's attention then is concentrated on the selection of the desired braking rate, no particular skill being required to produce a smooth stop.

The minimum speed to which full-load braking can be maintained is determined by the internal resistance of the motor added to the resistance of the cables and switching equipment. For the average urban transit vehicle, this speed is two to three miles per hour, below which the brake fades out and disappears.

Selection of the total number of braking notches is governed by the same general factors as the selection of acceleration notches. Passenger comfort, slope of the motor saturation curve, maximum speed of application, bunching of slack in trains, adhesion limits, and minimum fade-out speed all require consideration. A study of Figure 4 shows that at high speeds the braking curves are very steep and can be spaced rather widely apart, whereas at low speeds they are relatively flat and require close spacing to hold the current increments within uniform acceptable limits. Obviously, the higher the braking rate, the greater the number of notches required.

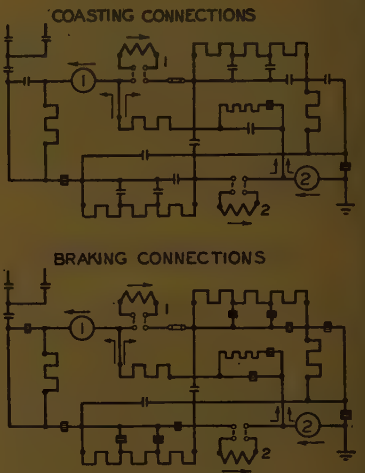

Figure 3. Standard motor circuit connections for power and brake operation, with pre-excitation by "spotting" in the coasting position

Many applications are based on an average braking load somewhat less than the full accelerating load and require the same number of notches for each, even though the speed range in braking may be twice the range over which the accelerating equipment is in operation. The apparatus required to control the braking, in excess of that used in acceleration, thus is held to a minimum.

Each application requires a careful study of the switching system and resistor-step values, if they are to be used to the best advantage in both power and brake operation.

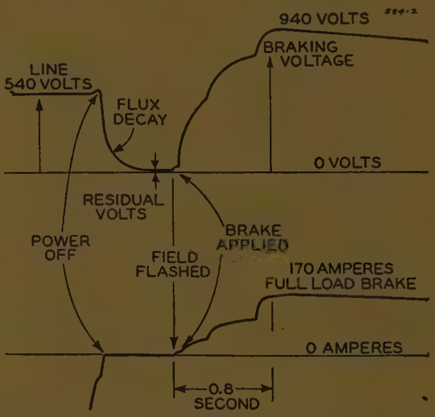

Figure 2. Motor voltage and current record of the transfer from power to brake operation

Figure 1. The Newell streetcar brake of the year 1900, comprising combined track and wheel-brake shoes, both energized with current generated by the traction motors

Advantages and Economies in Operation

In making the fundamental decision as to whether or not dynamic braking should be included in any given application, the following benefits are to be considered:

1. Safety in operation, as provided by an effective brake system, in addition to the regular mechanical brakes.
2. The use of accelerating and braking resistor losses for heating the vehicle.
3. Simplification of the mechanical brake equipment.
4. A marked reduction in wear on brake shoes and wheel rims or brake drums, with less frequent adjustment and replacement.
5. Minimized skidding and side slipping under adverse rail or street conditions.
6. Better feeling of security on the part of operators.
7. Automatic regulation and graduation of the brake in making service stops.

Typical Applications and Results

The PCC Car

No dynamic-brake system has shown such phenomenal benefits as the equipment which is standard on all PCC streetcars, of which there are nearly 3,000 in operation and on order. Without such a system, the cushioned-wheel type of truck could not have been used because of the excessive heating and the massive brake rigging which would have been required to produce the high deceleration rates with mechanical brakes alone. Being independent of line power, this equipment is extremely reliable and makes possible the high schedule speeds for which the vehicle is designed.

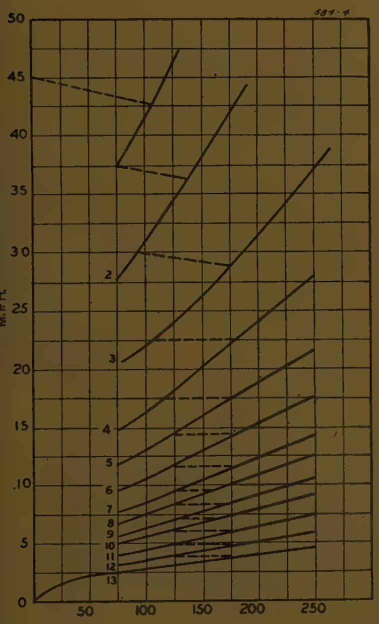

Figure 4. Typical dynamic-brake notching curves for service stops from maximum speed to three miles per hour

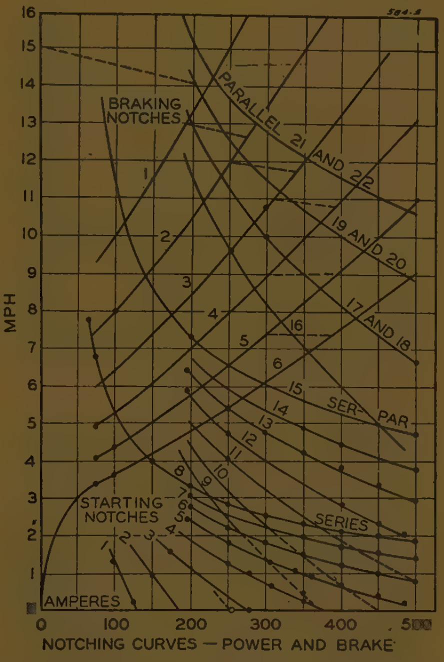

Figure 5. Complete locomotive starting and braking notching curves, with several brake-holding positions

The converted energy of retardation, introduced into the ventilating system in the form of heat, provides adequate car heating without the use of supplemental heaters, in all except the most severe winter conditions.

A simple form of air brake and a magnet track brake supplement the dynamic brake. The latter normally is required only in emergency, and the light duty on the air-brake shoes has extended the life of brake shoes and the adjustment period to at least 15 times that for older types of streetcars.

The multinotch control developed for these cars produces exceedingly smooth brake performance, and deceleration rates as high as four miles per hour per second can be obtained with comfort. The "spotting" system of recapturing residual voltage for quick brake response is used to position the control in proportion to speed during coasting, so that the correct resistance value always is available for immediate full-brake application. The regulation is automatic with a wide selection of rates.

Trolley Coaches

With the prevailing tendencies toward high schedule speed, the necessary braking rates long since have exceeded the limitations of brake-drum and wheel-rim temperatures. Of the more than 3,500 trolley coaches in the United States, nearly half have included some form of dynamic or combined dynamic and regenerative braking. The results in operation compare favorably with those obtained on PCC cars, with an important increase in dependability and safety on slippery pavements. There has been a reduction in brake-block and brake-drum maintenance to about one fourth of that which existed when all braking is done mechanically.

Haulage Locomotives

Braking conditions on material-handling systems, particularly in underground mining, are more severe than in any other branch of the traction industry. Grade and track conditions are extremely variable, and in many cases the cars have no power-brake equipment, all braking duty being concentrated on the locomotive itself. The advantages of a dynamic brake, supplementing the mechanical wheel-brake system, in safety and dependability have been demonstrated repeatedly. In open-pit mining, long grades frequently are encountered, and dynamic braking can be proportioned to absorb at least 50 per cent of the total braking effort required, with a corresponding increase in brake-shoe and wheel life on cars and locomotives.

Another notable application is the use of a dynamic holding brake on Diesel-electric main-line locomotives in high-speed mountain grade service.

Figure 5 shows the combined acceleration and braking performance curves for a haulage locomotive with several manually selected brake-holding notches.

Rapid Transit

In this field dynamic braking has not yet found any general application. However, it is reasonable to anticipate that future extensions will follow the trend toward faster schedules and higher braking rates. The problems associated with brake-shoe dust, wheel wear, and skid flats will become increasingly critical, and dynamic braking will have definite advantages in this respect. Furthermore, uniform braking on all wheels in the train will be assured.

Summary

The many proved benefits to be realized in the use of dynamic-braking systems bear a most favorable proportion to the nominal amount of additional apparatus involved. They are applicable to practically all branches of the land-transportation industry.

Control of Electrical-Insulating Varnishes

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Synopsis: Varnish treatment of electrical equipment is an important step in its manufacture. Varnish coatings protect apparatus from moisture and dirt and maintain the high electrical-insulating level required for insulation. The quality of the varnish film deposited depends upon the condition of the varnish in the processing tanks. Thickness of deposited varnish films varies with viscosity and body. Heavy films interfere with armature-coil sizes and make problems of heat transfer in field coils and magnet coils more difficult. Thin films give inadequate protection from moisture and dirt. Conducting contaminating materials are introduced into the tanks from rehabilitated apparatus and manufacturing atmosphere. These indicate that close control of tank varnish is necessary for consistently high quality of apparatus treated. Methods of control are discussed which include both laboratory tests of the varnish and physical tests which can be concluded at the tank. A system of daily inspection of tanks for proper quality and daily adjustments before use is described. Dielectric properties of the varnish film and the liquid varnish and periodic chemical analysis of samples from each tank reveal the condition of the varnish. A system of portable cold pressure filtering for removing much of the contamination present is described. It should be recognized by the electrical industry that insulating varnishes are complex chemical mixtures, and care should be maintained in their use. It is important that they be under the control and supervision of persons with the proper training and experience. Constant quality control is necessary for satisfactory results.

THE PROTECTIVE CAPACITY of varnish on electric equipment depends upon the quality of the varnish in which the apparatus is treated. Proper maintenance of varnish requires consideration of many factors. Modern synthetic varnishes are a complex combination of resins and solvents which must be properly controlled. The quality of the varnish film depends on many factors including the viscosity–body relationship, the degree of polymerization of the resins, chemical composition of the resin, and the purity of the film. There is a tendency for insulating varnish to become contaminated with various foreign materials in the course of its use. Contaminants seriously impair the insulation value of the varnish and lower the reliability of the electrical equipment treated.

Certain difficulties in manufacture may be overcome by proper control. For example, certain coils treated in varnish of excessive viscosity may result in oversize conditions due to deposition of heavy varnish films. Improper varnish treatments may cause unduly high temperature rises. Troublesome effects of poor varnish treatment may become apparent during factory tests or when the machine is in service.

Chemical Changes in Varnish

Body, viscosity, and specific gravity of varnishes are interdependent. Body is the per cent nonvolatile material (resin, drying oil, plasticizer) in the varnish. Deposition of the nonvolatile material from the solvent forms the varnish film.

Viscosity is a measure of the resistance of a fluid to flow. Viscosity is dependent upon the body of varnish and also upon the degree of polymerization—that is, the average chain length of varnish resin molecules. Polymerization in turn is a factor in the aging of varnish resins. Specific gravity is a measure of the amount of solid material in a given solvent. The amounts of solvent and solids, and the condition of resin may be interpreted by means of these tests.

For example, an increase in viscosity and specific gravity from a given standard usually means a loss in solvent. Adjustments of solvents are easily made. However, a continued increase in viscosity with the percentage solids and specific gravity remaining the same denotes a chemical change in the varnish. This indicates a gradual polymerization or aging of the varnish resin. In practice, the tank varnish becomes thicker and has to be cut back with solvents repeatedly to insure smooth coating. The percentage solids soon becomes low as a result of addition of solvents, and a thin varnish film with poor physical properties is deposited, which does not impart adequate insulation protection.

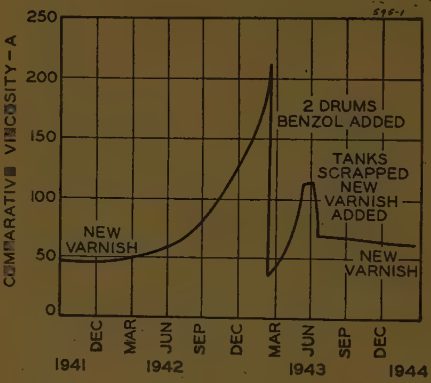

Figure 1. Comparative viscosity of synthetic varnish in toluol; viscosity versus time

Figure 1 gives comparative viscosity of a tank of synthetic varnish over a period of 29 months. Comparative viscosity is a method of comparing the viscosity of a varnish at a particular body to a viscosity at a reasonable standard reference body for the same varnish. Gradual rises in the curve with time show an increase in the degree of polymerization or aging of the resin. Eventually, the physical properties of the film deposited become so poor that the varnish must be scrapped. If one refers to the curve, 50 is the index for varnish in good condition. After nine months there is a steady rise in comparative viscosity to 130 (14 months after start). At this time the physical properties are definitely below standard.

If viscosity and specific gravity of tank varnish are allowed to increase owing to solvent losses without adjusting to a given standard, a thicker varnish coat is deposited. Thick coatings on armature coils result in difficulty in winding in slots because of their being oversize. Too heavy coats deposited on field coils and magnet coils decrease the rate of heat transfer and result in higher temperature rises. Table I gives the thermal conductivity of a magnet coil in degrees rise per watt after successive coats of varnish.

In general, film thickness is dependent upon viscosity, whether viscosity is adjusted by solvent or by heating the varnish. Figure 2 gives varnish build-up versus viscosity of a synthetic varnish (in toluol solvent) deposited on copper, paper, and fiber glass. Note that on glass a rise in viscosity from 35 seconds to 100 seconds deposits a film 30 per cent thicker.

It is apparent that a proper body–viscosity relationship must be maintained in the tanks to insure efficient treating results.

Contamination

In the usual electrical-manufacturing plant the varnish tank is open during use and is located in the center of the manufacturing activities, and the apparatus or coils to be coated are themselves not perfectly clean. Under such conditions a certain amount of contamination is unavoidable.

Previous work by the authors[1] indicates there are three mechanisms of contamination of the varnish. The contaminant may be washed from the piece while it is immersed in the varnish tank. This means the suspension of particles of a

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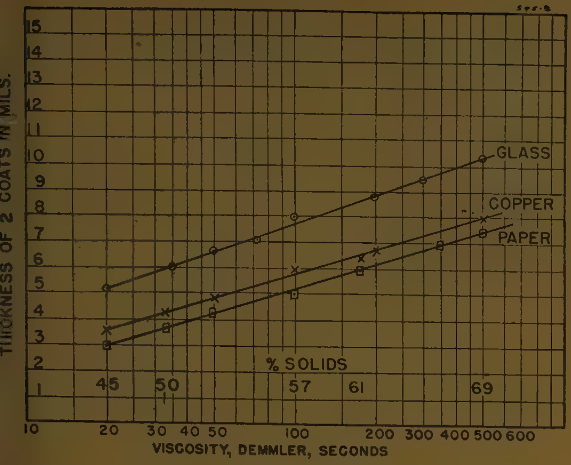
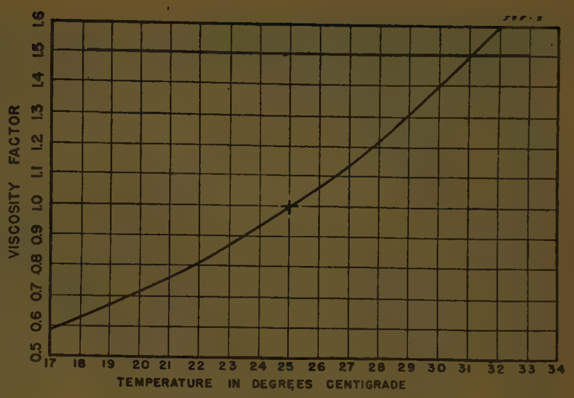


Figure 2 (left). Synthetic alkyd varnish, toluol thinner, build-up of film versus viscosity

All samples drained at 28 degrees centigrade

Figure 3 (right). Correction of viscosity to a base temperature of 25 degrees centigrade

Viscosity at temperature X viscosity factor = viscosity at 25 C



particularly troublesome nature in the varnish. For example, when rehabilitated armatures are being processed, some of the dust and dirt accumulated in field industrial use is washed into the varnish tanks on immersion of the apparatus.

A second means of contamination is through the air. Carbonaceous dust metal chips of small diameter, small droplets of paint from nearby spray booths, and other foreign material may enter the varnish when the tank lid is open. Each varnish-treating room is equipped with overhead cranes and lifts which are additional possible sources of contamination. Also material foreign to the varnish may be added by mistake. The addition of a varnish whose resin is only slightly soluble in the varnish and solvent in the tank is an example. Here the foreign resin would be precipitated out, possibly as troublesome colloidal particles.

Contaminants generally may be classified as conducting and nonconducting. Carbonaceous dust, lint, metal particles, and electrolytes are offenders as conducting contaminants. These often are introduced by apparatus being processed, but can be present due to the manufacturing atmosphere. Nonconducting particles are represented by wood chips and precipitated or polymerized resins, pieces of cotton, and asbestos and mica dust. This type of material comes from pieces of insulation washed off the coils and wound apparatus on immersion in the varnish bath.

Contamination may be classified into three types by use of wet and dry dielectric tests and heat-endurance tests Conducting particles lower wet and dry dielectric strength and generally impair the heat endurance of the varnish film. A second type is identified by a high dry dielectric strength, a low wet dielectric strength, and a low heat endurance. Contamination producing varnish gels or precipitated resins might not lower the dielectric strength but would lower the heat endurance or life of the varnish film.

Disruption of the varnish film by particles of foreign matter seriously impairs the properties of the insulation. On electrical equipment there is no need to mention the effect of conducting or ionizing particles on the dielectric breakdown value, particularly when they become imbedded in the insulation. In some cases, although the particles are not conducting, they cut into the insulation during processing, lowering the breakdown level of the material. In addition, the initial mechanical strength and life or endurance of the varnish film are reduced when foreign materials contaminate the varnish.

Thus, it is important to keep varnish for treating electrical equipment as clean from foreign material as possible. As a base or goal, the electrical properties and mechanical properties should be kept in the same order of magnitude as those of the new varnish.

Varnish Tests

To control the varnish adequate tests must be worked out which reflect the condition of the tank. Some of these are of a type which permit testing at the tank itself, whereas others require sampling of the varnish and testing in a chemical laboratory.

Those test methods which are practical to apply at the tank must be simple and require only a short time to perform. Viscosity and specific-gravity measurements fit easily into a daily inspection routine. Viscosity measurements may be made by one of several methods. The simplest of these involves the use of a cup with a calibrated orifice in the bottom which allows varnish to flow in or out at a definite rate. The time a definite volume of varnish fills or empties from this cup may be taken as a measurement of viscosity. Another method is to fill a convenient-length transparent tube with varnish. A small bubble is left at the top. When turned end for end the bubble rises slowly. The time required, by the bubble to travel a certain distance is a measure of viscosity. However, a number of tubes with liquids of known viscosity may be used. The rate of rise of the bubble in the tube of tank varnish will be approximately the same as for one of the bubbles in a tank containing a liquid of known viscosity. The viscosity of the tank varnish is taken as the same as the liquid whose bubble rises at the same rate.

Viscosities with the cup or tube methods may be reported directly in seconds and will be comparable to other determinations made with the same apparatus. This is the most convenient arrangement.

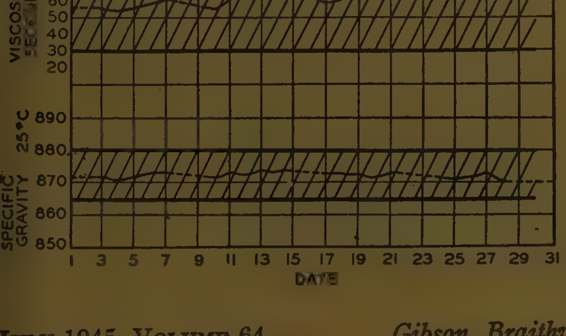


Figure 4. Daily varnish checks in tank of oleo resinous varnish, April 1 through April 30, 1944

Shaded area shows limit specified

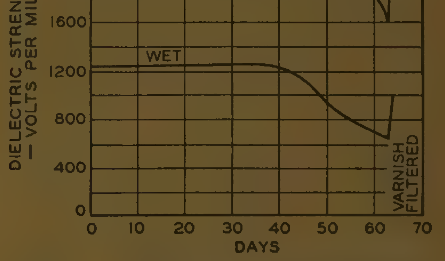


Figure 5. Wet and dry dielectric strength of heat-reactive varnish

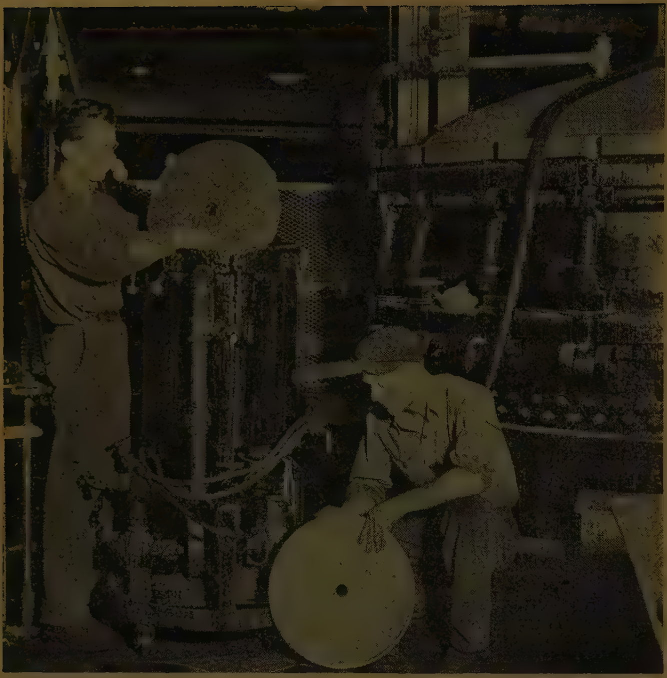


Figure 6. View of the portable pressure filter showing the perforated stainless-steel filter plates

However, conversion to classical viscosity units may be made easily.

Specific gravity is taken easily with a hydrometer. Precautions should be taken to prevent bubbles in the varnish sample from contributing to erroneous results. Care should be exercised that the spindle be read at the surface of the liquid, not at the "high-tide" mark of the varnish.

Both specific gravity and viscosity are dependent upon temperature. Corrections to a standard base temperature are simple and should be made in all cases to provide a common basis of discussion. A value of 25 degrees centigrade is recommended as the standard base (refer to Figure 3 for a typical correction curve). The combination of viscosity and specific gravity, however, give many useful indications. Although samples for body may be taken at frequent intervals, this test requires an oven and despite its simplicity must be classified as a laboratory test.

Samples for laboratory tests should be taken at convenient and regular intervals. A number of tests have proved significant. Body of course is determined easily by evaporating the solvent and weighing the remaining resin. Body determination should be made more frequently than other laboratory tests since the information obtained is useful for correlation with the daily viscosity–specific-gravity determinations.

Table I. Thermal Conductivity of Magnet Coils

Tests Made on Same Coil, With No Outer Coat of Varnish, and With One, Two, and Three Varnish Coats

Each Test Is an Average of Three Temperature Runs

Varnish Treatment	Rise, Degrees Centigrade	Watts Input	Degrees Rise Per Watt
No outer coat	77.5	29.2	2.65
One dip coat	82.5	29.2	2.83
Two dip coats	85.5	29.3	2.92
Three dip coats	86.0	28.4	3.02

Other properties of varnish are not evaluated so easily. Wet and dry dielectric strengths of varnish films on copper panels, hardness of varnish cakes, heat endurance, oil resistance, and drying time all are important.[2] Many of these tests are affected by dust-laden atmospheres, by humidity conditions, and by temperature. They should be performed in a dust-free and moisture- and temperature-controlled room.

Dielectric strengths and heat endurance of varnish films have become important measures of the degree of varnish contamination. Heat endurance is determined by depositing varnish films 0.002 inch thick on copper strips 0.005 inch thick.[2] These are heated continuously at 150 degrees centigrade for accelerated tests, as suggested by Frost and Moses.[3] The time to produce failure of the film on bending about a one-eighth-inch mandrel is reported as the heat endurance of "flexing life." Dielectric tests consist of measuring the dielectric breakdown strength of a 0.002-mil-thick varnish film on each side of a 0.005-inch-thick copper sheet as received dry and after immersion in water for 24 hours.

Hardness measurements are made on varnish cakes at least one eighth inch thick with a Shore durometer *A*.[5] The measurements are taken after three hours bake at 135 degrees centigrade and again after six hours at 135 degrees centigrade. A further durometer measurement after 48 hours in oil at 110 degrees centigrade allows evaluation of the oil resistance of the varnish. These measurements are dependent upon temperature, and tests should be in a controlled atmosphere if possible. Drying time determinations should be made as per American Society for Testing Materials specifications (*D*115–39*T*).[2] A one-mil varnish film is deposited on a copper plate and baked at 110 degrees for baking varnishes. At intervals a strip of kraft paper is pressed upon the surface. The drying time is taken as that time when the paper does not stick to the panel.

From these data a representative picture of the condition of the tank varnish can be drawn. Contamination and chemical changes can be noted quickly and steps taken to compensate for these changes where necessary. Table II consists of a typical report on laboratory varnish tests.

Inspection System

An inspection system is essential to insure proper quality control of tank varnish. Inspectors should be assigned the responsibility of checking and adjusting varnish tanks and inspecting new varnish to be added to tanks. Each active tank should be checked for viscosity and specific gravity daily. It is not enough merely to inspect; tanks must be adjusted daily within definite limits by addition of solvent or new varnish. If tanks are not within the proper limits of viscosity and specific gravity, they should not be used until proper adjustments are made.

Limits of viscosity and specific gravity are set for each type of treatment and each type of varnish in the tank. Daily examinations should be made of the varnish for dirt, varnish skins, or gels. All varnish should come within specifications in all characteristics before addition to tanks. A single drum of overpolymerized varnish can initiate rapid polymerization throughout a large tank which may lead to scrapping. Daily records or curves may be kept of each tank with additions of solvent and new varnish indicated. At the end of each month a report on the condition of the varnish tanks should be made to interested parties. By the use of curves viscosity and specific-gravity trends can be followed and the necessity for adjustments readily seen.

Figure 4 is a record of the inspector's

Table II. Varnish Tests

Sample 58

Body	53.56 per cent
Specific gravity	0.985
Viscosity (Demmler 1) (25 C)	132 seconds
Drying time (ASTM at 110 C)	2 hours
Hardness varnish cake at 25 C (durometer *A*):	
3 hours bake at 135 C	2
6 hours bake at 135 C	24
48 hours in oil at 110 C	29
Flexing life (150 C)	60 hours
Wet dielectric	900 volts per mil
Dry dielectric	1,800 volts per mil

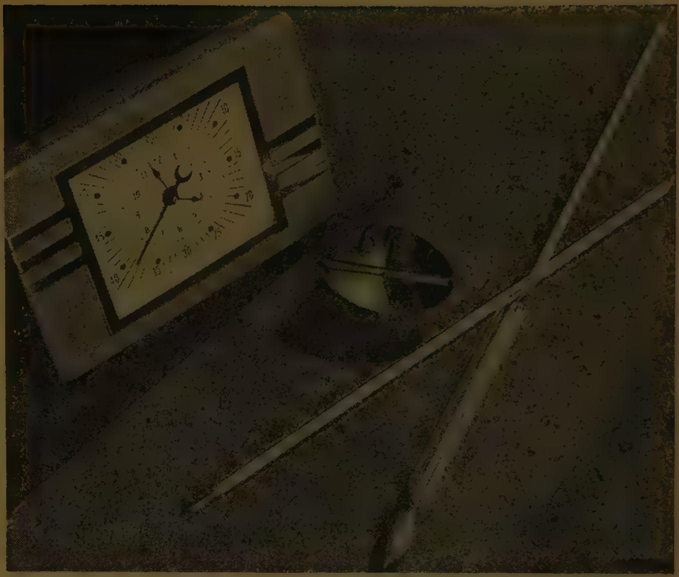
monthly report on specific gravity and viscosity of a tank of synthetic varnish. Note limits of viscosity and specific gravity for that particular tank.

In addition periodic chemical laboratory examinations should be made of every varnish tank to follow changes in electrical and mechanical properties. All of this is insurance against improper treatment of equipment. The benefits are apparent in the improved appearance of coils and wound apparatus and improved performance. Actual economies are hard to estimate.

Varnish and solvent added must be carefully mixed into the tank. Improperly mixed solvent will rise to the top in a separate phase, whereas the more viscous and dense varnish will tend to form a layer at the bottom of the tank. Either phenomenon results in stratified layers each with its own viscosity and body. Such a tank will not deposit consistently good protective films. This condition may be remedied by simple but thorough mixing of each drum into the tank as it is added.

Figure 8. Varnish inspector's test material

These include: stop watch, viscosity measurement cup, thermometer, hydrometer



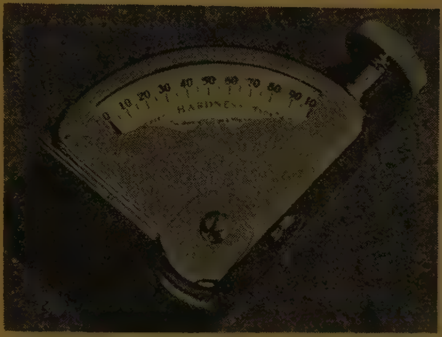


Figure 9. A Shore durometer for testing hardness of the varnish

Filtration

Careful control of the varnish requires an adequate filtration method.[1,6] The materials used in varnish manufacture are critical, and maintenance of the varnish in usable condition is essential. The particle sizes of contaminants range from large pieces of wood to colloidal carbon. Pieces of metal as small as 0.003 inch in diameter have been found. A study of the available methods suggested that the most efficient method of reclamation of the varnish and conservation of critical materials is by cold pressure filtration.

A portable-type pressure filter, consisting of a number of plates arranged vertically and enclosed in a shell, has been found to be satisfactory. Pressures of 15 to 28 pounds per square inch are used, with rates of flow varying from one to five gallons per minute depending upon the pressure, amount of cake, and viscosity of filtrate.

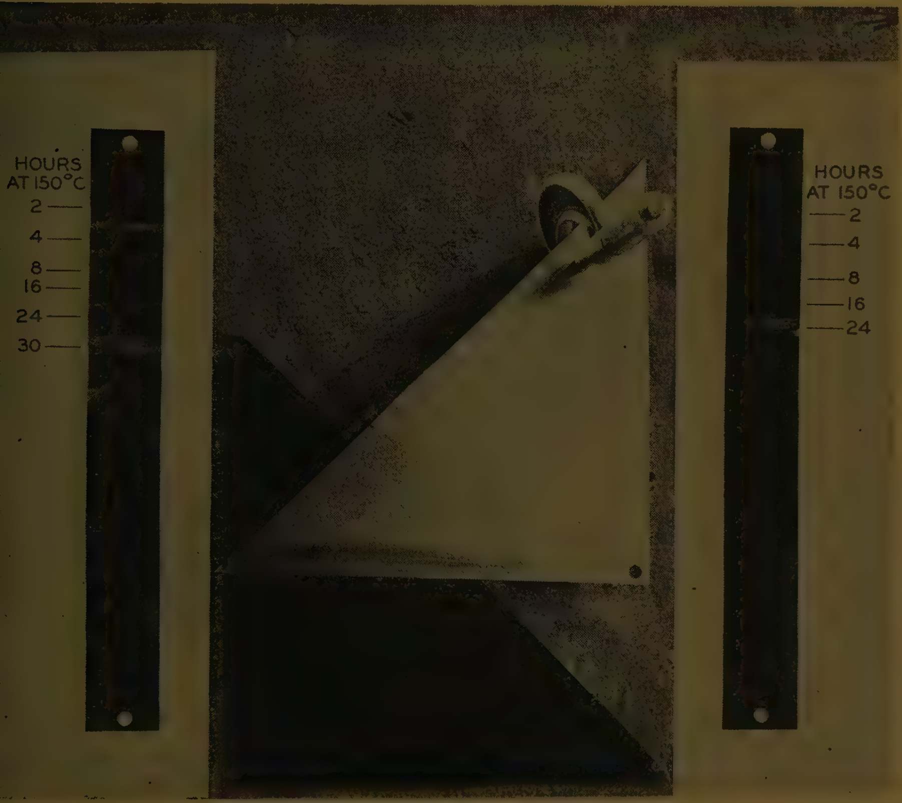
Filtration is carried out using a diatomaceous earth as a filter aid. This is particularly necessary as some of the contaminating material filtered out is a gel and quickly fills up the pores of the filter medium. The addition of filter aid allows a porous cake to build up which is not so easily clogged as the filter medium alone. In order to avoid addition of the filter aid to the large varnish tanks small portable tanks may be set up in which the diatomaceous earth may be mixed continuously with the incoming varnish stream. Varnish is pumped from here through the filters and into a clean tank in the system.

The efficiency of the filtering treatment is judged by a series of chemical and physical tests on samples taken before and after filtration. Percentage of body, viscosity, and specific gravity show whether or not there is a loss of solvent or varnish resin during the filtering operation. The results of the heat-endurance tests and dielectric tests before and after the operation show whether or not harmful foreign materials were present and have been removed.

Improvements in dielectric properties produced by filtering the varnish are

igure 7(below). Dielectric and heat-endurance panel

he dielectric panel has been slit to facilitate easuring thickness of the varnish film. Cracks in varnish film on heat-endurance strips indicate failure of the varnish



remarkable. A typical increase in dry dielectric strength in the order of 30 per cent and in the wet dielectric strength in the order of 70 per cent was found (see Figure 5). Heat endurance and the mechanical properties of the varnish improved from 10 to 20 per cent upon filtering. In one case a badly contaminated sample was filtered with a resulting improvement in heat endurance of over 100 per cent. Plastic asphalt-type varnish improved in this characteristic over 40 per cent after filtering. An increase in heat endurance indicates that particles of colloidal matter which damage the varnish film have been removed. It is apparent that the type of contaminant depends upon the material treated in the tank. The length of time between filtering operations depends upon the use or activity of the tank.

Filtration in itself would be of no avail if the filtered varnish is pumped back to become recontaminated by dirty tanks. Tanks should be cleaned periodically; frequent filtrations provide an opportune time for cleaning.

Conclusions

Thus a system of varnish control which includes testing and inspection of varnish and methods of maintenance is essential in sustaining the quality of insulating varnishes. Such a system will reduce the number of possible sources of electrical failures in shop testing and in the field. In addition it will eliminate certain manufacturing problems. The economies resulting from longer use of the varnish are difficult to estimate. This control and filtration system is a step forward in the conservation of critical varnish materials. It should be recognized by the electrical industry that insulating varnishes are complex chemical mixtures, and care should be exercised in their use. Persons of the proper training and experience are required for their supervision. Constant quality control is necessary for satisfactory use of insulating varnishes.

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Statistical Tools for Controlling Quality

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THE MANUFACTURER is always faced with the problem of getting the maximum output from his available equipment, material, and man power. This means operating with a minimum amount of scrap production or producing a minimum amount of products which fall outside of specification limits and which will require reworking. But since tools of production are subject to wear, and since materials vary in quality or characteristics, the manufacturer must have a means of predicting the probable quality of products he is likely to get sometime in the future.

Having satisfied himself that his quality is satisfactory now, he wants to be able to control or maintain that quality indefinitely, or as long as that particular product is made. This prediction must be clear and definite enough to serve as the basis for action to remove assignable causes which if not removed will result in the production of parts outside of specification limits.

Furthermore, in modern mass production where several tools perform the same operation, as with multicavity molds and gang cutters, the manufacturer wishes to know if there is any significant difference in the quality of products coming from different cavities in the same mold or from different cutters in the same setup involving a number of cutters on the same machine.

The Frequency Distribution

In a manufacturing process, it is desirable to know, as soon as possible after production is started, what the quality of the product is likely to be. The "frequency distribution" is a simple way of obtaining this information. By measuring the quality characteristic in question on a number of pieces produced at the start of a run, a simple computation can be made to show the range within which measurements are most likely to fall for the subsequent production. Comparison of these limits with the tolerance limits shows whether the process should be allowed to continue or if a change in the setup is required. The data obtained from the screw-machine part, shown in Figure 1, illustrate the frequency distribution.

A sample of 50 pieces was taken and each piece was measured for over-all length, which was required to be 4.450 inches ± 0.010 inch. The measurements obtained are shown in Table I.

The data in Table I do not give much information in their present form. However, the data can be reorganized as shown in Table II so as to become very enlightening. A survey of the data shows that exactly 16 pieces were 4.450 inches long, eight pieces were 4.451 inches long, nine pieces were 4.449 inches long, and so on.

Such a table is called a *frequency distribution*. It indicates the number, or frequency of occurrence, of measurements in each size. In practice, the pieces may be tallied as they are measured, as is shown in Figure 2. This gives visual meaning to the frequency distribution. It will be noted that the distribution is bell-shaped and symmetrical about the center, high in the center, and falls off sharply at both ends. The envelope curve of the tallied marks is known as the *frequency curve*. For best-quality production the frequency curve must lie totally within the specification limits.

Figure 3 shows the frequency curve, for this part, in relation to the specification limits. It is evident, in this case, that the process is capable of producing good pieces and will continue to do so provided nothing happens to change the shape or location of the frequency distribution. A periodic check of five pieces per hour on this item, suffices to show whether the frequency distribution remains unchanged. If the distribution does remain unchanged the entire production is considered to be within toler-

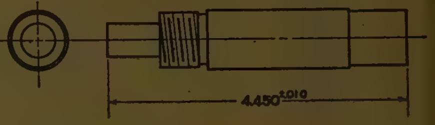

Figure 1. Screw-machine part

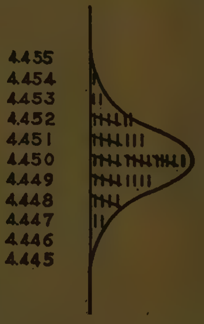

Figure 2. Frequency curve of 50 measurements on screw-machine part shown in Figure 1

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Table I. Fifty Measurements on Screw-Machine Part Shown in Figure 1 (Inches)

1.	4.450	14.	4.451	27.	4.450	40.	4.450
2.	4.449	15.	4.448	28.	4.450	41.	4.449
3.	4.449	16.	4.450	29.	4.448	42.	4.447
4.	4.450	17.	4.451	30.	4.450	43.	4.448
5.	4.452	18.	4.451	31.	4.450	44.	4.449
6.	4.452	19.	4.452	32.	4.450	45.	4.449
7.	4.451	20.	4.454	33.	4.448	46.	4.450
8.	4.452	21.	4.453	34.	4.449	47.	4.451
9.	4.451	22.	4.450	35	4.450	48.	4.450
10.	4.451	23.	4.450	36.	4.450	49.	4.448
11.	4.452	24	4.453	37.	4.452	50.	4.449
12.	4.452	25	4.449	38.	4.450		
13.	4.451	26.	4.447	39.	4.449		

Table II. Frequency Distribution of 50 Measurements on Screw-Machine Part Shown in Figure 1

Length of Piece (Inches)	No. of Pieces
4.455	0
4.454	1
4.453	2
4.452	7
4.451	8
4.450	16
4.449	9
4.448	5
4.447	2
4.446	0
4.445	0

ance limits, and final inspection may be dispensed with. The central location of the frequency distribution with respect to the specification limits and its narrowness compared to the tolerance range guarantee that the product is of high quality.

The shape and position of a frequency curve with respect to the specification limits are indicative of assignable causes which must be corrected if products being produced are to be kept within specification limits. Two types of curves most frequently encountered are shown in Figure 4A and Figure 4B.

Figure 4A is an example of a process for which the band spread, or the variation from part to part, is narrow enough to produce a good product, but the machine is not properly adjusted. In other words, the frequency distribution is not centralized with respect to tolerance limits. As a matter of fact, this distribution was obtained, at an earlier stage of the process when the machine was being set up to produce the part, the distribution of which is shown in Figure 3.

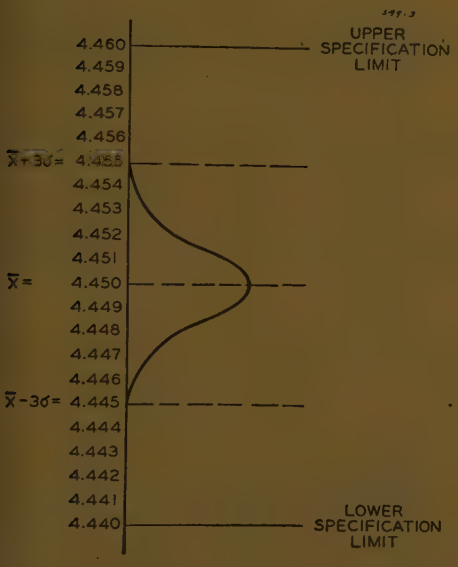
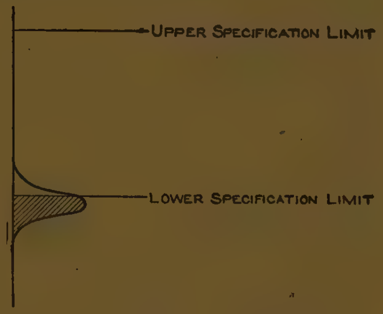
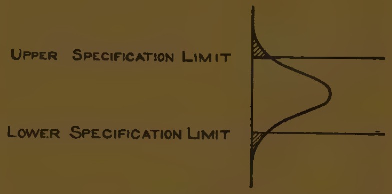


Figure 3. Relative position of frequency curve of the process relative to the specification limits shows good statistical control



A. Process not centralized



B. Band spread too wide

Figure 4. Typical frequency curves showing assignable causes

Figure 4B illustrates a frequency distribution which has a wider band spread than the tolerance range. Such a situation may be easily remedied. A few assignable causes indicated by a wide band spread are worn bearings in the machine, flimsy and unstable fixtures, and excessive variation in hardness of stock.

It should be apparent from the foregoing discussion that a frequency distribution has two characteristics which are of practical importance, the "central tendency" and the "amount of band spread" or degree of dispersion. These characteristics may be represented numerically by the mean, $\bar{X}$, and standard deviation, σ. The standard deviation is merely the root-mean-square value of the deviations. Another characteristic of a frequency distribution is its skewness, or amount of deviation from symmetry, but we are not particularly interested in this at present. A normal process will have a distribution which is nearly symmetrical. Statistical tests exist for symmetry, but one is often able to judge by sight whether or not a given frequency distribution is sufficiently symmetrical for purposes of controlling quality.

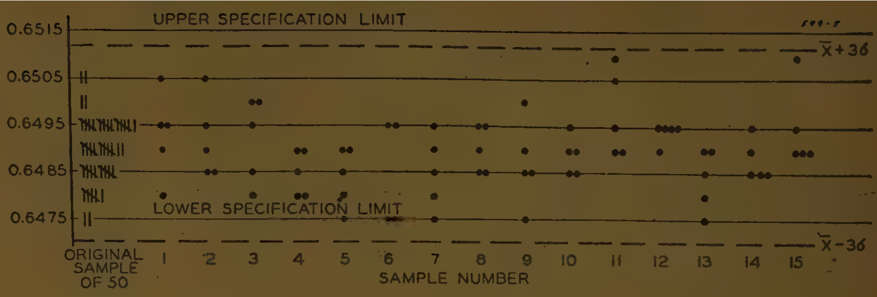


Figure 5. Typical control chart for samples of five pieces, showing individual observations

The Control Chart

After control of quality has been achieved by means of the frequency distribution, and production is under way, it is necessary to maintain this control in order to avoid the production of defective parts. The control chart is the tool used for maintaining this control; it is a take-off from the frequency distribution. It is necessary to calculate the mean, $\bar{X}$, and standard deviation, σ, from the frequency distribution in order to establish limits for the control chart. In many cases a rough approximation of the control limits may be obtained from an inspection of the frequency distribution.

After it has been established that $\bar{X}$ and σ are satisfactory for the process, and a control chart has been constructed, a periodic check of, say, five pieces is sufficient to detect tendencies toward loss of control. The time interval between samples may be an hour or a day, or a sample might represent a batch of material. The choice of samples is governed by such factors as tool wear, skill of operator, closeness of specification limits, and rate of production. Practically all measurements made on samples should fall within a distance of 3σ from $\bar{X}$, or within the "control limits." If any measurements fall more than 3σ from $\bar{X}$, this is a strong indication that a change has occurred in the process at those points which has changed the frequency distribution.

As an illustration, consider the data of Figure 5, which were obtained from a dimension on the breech block of an antiaircraft gun. The dimension is 0.6495 inch ± 0.002 inch. As shown in the figure, a frequency distribution of 50 measurements was obtained and $\bar{X}$ and σ computed. The results of this computation are $\bar{X}=0.6491$ inch and $\sigma=0.0007$

Table III. Factors Used in Computing Control-Chart Limits

Number of Observations in Sample	Factor by Which Average Range $\bar{R}$ Is Multiplied to Obtain 3σ	Factor by Which Average Range $\bar{R}$ Is Multiplied to Obtain $3\sigma/\sqrt{\eta}$ for Limits on Control Chart for Average	Factors by Which Average Range $\bar{R}$ Is Multiplied to Obtain Limits on Control Chart for Range: Upper Limit	Lower Limit	Factor by Which Average Standard Deviation $\bar{\sigma}$ Is Multiplied to Obtain $3\sigma/\sqrt{\eta}$ for Limits on Control Chart for Average	Factors by Which σ Is Multiplied to Obtain Limits for σ Chart: Upper	Lower
3	1.77	1.02	2.57	0	2.39	2.69	0
4	1.46	0.73	2.28	0	1.88	2.33	0
5	1.29	0.58	2.11	0	1.60	2.13	0
6	1.18	0.48	2.00	0	1.41	2.00	0
7	1.11	0.42	1.92	0.08	1.28	1.90	0.10
8	1.06	0.37	1.86	0.14	1.18	1.83	0.17
9	1.01	0.34	1.82	0.18	1.09	1.77	0.23
10	0.97	0.31	1.78	0.22	1.03	1.72	0.27

Table IV. Turbine-Blade-Edge Thickness Above Nominal (Inch)

	Cutter 1	Cutter 2	Cutter 3	Cutter 4
Cam 1	0.004	0.004	0.009	0.016
Cam 2	0.008	0.010	0.015	0.022
Cam 3	0.007	0.008	0.010	0.015
Cam 4	0.009	0.009	0.012	0.018
Cam 5	0.009	0.011	0.022	0.027
Cam 6	0.004	0.007	0.013	0.017

inch. Thus $\bar{X}-3\sigma=0.6470$ inch, and $\bar{X}+3\sigma=0.6512$ inch. Dotted lines are drawn at $\bar{X}-3\sigma$ and $\bar{X}+3\sigma$. These are called "control-chart limits." In this example 15 samples each containing five measurements were then obtained—a sample a day for 15 consecutive days. As is apparent from the chart, all 75 measurements fall within the control-chart limits. The process accordingly is considered to be "in control."

In this particular example, the control-chart limit falls below the lower specification limit by 0.0005 inch. Hence, although the process is consistent—therefore, in control—it is not quite centralized and has a slightly too wide band spread. Accordingly, the process will produce some defective pieces, and adjustments should be made to assure that all pieces are within specification limits. This emphasizes the fact that there are three statistical requisites of a process—control, centralization, and small band spread.

It should be noted that every measurement of the five pieces in each sample actually is plotted on the chart. This is in contrast to the standard control chart on which only averages are plotted rather than individual measurements.

The control chart for individual measurements has the advantage over the control chart for average and range that control-chart limits may be considered to be the limits within which the process actually is producing.

In control charts for averages and ranges, on the other hand, the control-chart limits do not have so much meaning to the man in the shop as they have in the control charts for individual measurements. Indeed, they are often misunderstood as being the actual limits within which the process is producing. For this reason we suggest that at least until a program of statistical control is well established in the shop control charts for individual observations be used.

In a control chart for averages, charts are run simultaneously for "average" and for "range" or "standard deviation." "Range" is simpler to calculate than "standard deviation" and will be used here. In Figure 6 we have such a control chart for the data of Figure 5. The average was computed for each of the 15 groups of five measurements, and the results are plotted on the "average" chart. The ranges are plotted on the "range" chart.

Table V. Showing Calculation of Between-Column Variance and the Ratio of Between-Column Variance to Within-Column Varianc (Inch)

	Cutter 1	Cutter 2	Cutter 3	Cutter
Cam 1	0.004	0.004	0.009	0.016
Cam 2	0.008	0.010	0.015	0.022
Cam 3	0.007	0.008	0.010	0.015
Cam 4	0.009	0.009	0.012	0.018
Cam 5	0.009	0.011	0.022	0.027
Cam 6	0.004	0.007	0.013	0.017
Totals	0.041	0.049	0.081	0.115
$\bar{X}_{cu}$	0.0068	0.0082	0.0135	0.0192

$$\bar{\bar{X}}_{cu}=\frac{\sum_{1}^{4}\bar{X}_{cu}}{4}\doteq 0.0119$$

The between-column variance is

$$\frac{6\Sigma(\bar{X}_{cu}-\bar{\bar{X}}_{cu})^2}{(4-1)}=0.0001911 \text{ inch}$$

The within-column variance is 0.00001350 inch.

$$\frac{\text{Between-column variance}}{\text{Within-column variance}}=\frac{0.0001911}{0.0000135}=14.1$$

Obviously, averages of five measurements obtained from a process will not deviate from each other by as much as will the individual measurements themselves. The control-chart limits for averages accordingly must be drawn much closer together than for individual measurements. For averages of five, the factor of shrinkage is $\sqrt{5}$. The control-chart limits for averages are therefore $\bar{X}-(3\sigma/\sqrt{5})$ and $\bar{X}+(3\sigma/\sqrt{5})$. If the values $\bar{X}=0.6489$ inch and $3\sigma=$ 0.0023 inch are substituted, the control-chart limits become 0.6479 inch and 0.6499 inch. Factors for computing these limits and limits for the range chart are given in Table III.

It should be stated that the control chart frequently is used in such a way as actually to tell the source of trouble as well as the existence of trouble. The reason for this is that most machine-shop operations are essentially so simple that the likely causes of lack of control are

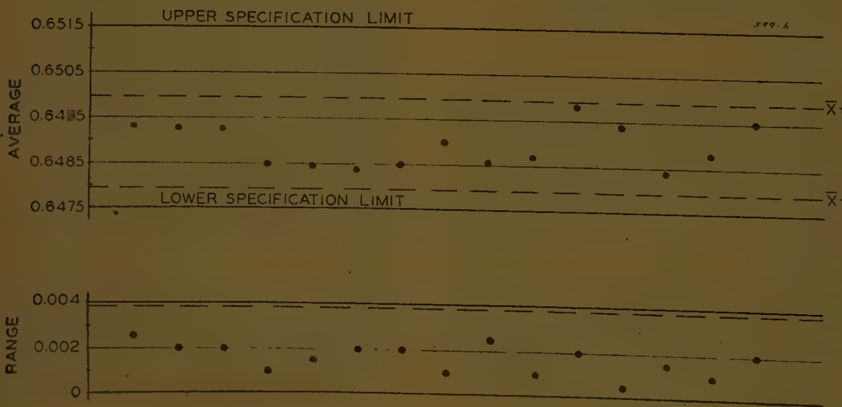


Figure 6. Control chart for "average" and "range"

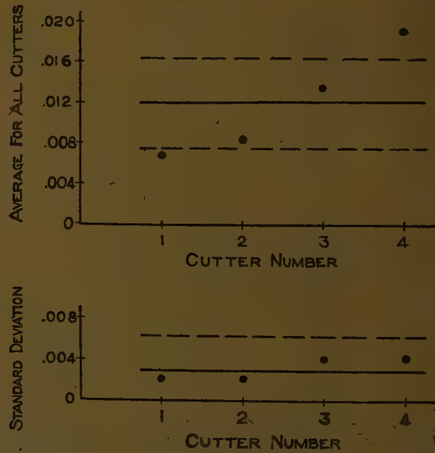


Figure 7. Control chart for between-cutter variance

few in number and are known beforehand, and it is easy to discern which will manifest itself as a lack of control of ranges and which as a lack of control of averages.

For example, we mention briefly a grinding operation on bars of rectangular stock which were running off as much as 0.004 inch. This was considerably excessive. The operation was performed by placing about 50 bars on a magnetic chuck and grinding with a circular motion. Two most likely causes of variation were dirt on the chuck and a variation in the distance to which the grinder was run down each time. A control chart for average and range was kept on the operation by taking groups of five bars at random from the 50 ground at each grinding. The first cause of variation obviously would show "range" out of control on the chart, and the second cause would show "average" out of control. The control chart showed both average and range out of control. By proper cleaning of the chuck, range was brought into control. Average was brought within control when steps were taken to assure that the grinder ran down to the same position each time. With a control chart on the operation to show that the right practice is being continued, the dimension is now held to 0.001 inch.

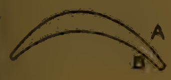


Figure 8. Sectional view of turbine blade

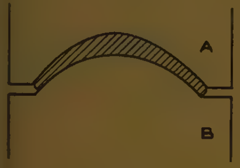


Figure 9. Showing method of gauging edge thickness of turbine blade

The Analysis of Variance

The analysis of variance is another statistical tool which may be used to uncover the existence of assignable causes of variation in a product. The type of shop problem to which this method may be applied is one where several tools or machines perform the same operation. For example, in a multicavity molding operation it may be that, due to differences between cavities, there is a variation in the product significantly greater than that which would exist if all the cavities were alike. It is true that tools which are supposedly alike do not always perform similarly.

Table VI. Data Collected After Regrinding Cutters

Compare With Table IV

Cutter 1 (Inch)	Cutter 2 (Inch)	Cutter 3 (Inch)	Cutter 4 (Inch)
0.002	0.003	0.002	0.003
0.003	0.010	0.004	0.004
0.008	0.006	0.002	0.004
0.003	0.006	0.003	0.003
0.008	0.010	0.010	0.018
0.003	0.007	0.001	0.006

One method of analyzing such a problem is by making a control chart, as shown in Figure 7. The standard deviation is computed for each tool, or cavity, and for the whole process. The control chart shows the average for the process and for each tool. Another statistical method which lends itself to this type of problem is the analysis of variance. We shall now illustrate this method, giving the calculation in detail.

An example of this type of problem is a drum-mill operation on turbine blades. The drum mill consists of a circular drum with six rows of pockets equally spaced around the circumference. Each row contains four pockets, making a total of 24 pockets, each holding a blade; there are six cams, one for each row, and four cutters, each of which cuts six blades, one from each row, as the drum rotates. Variations in the product can be caused by differences between cutters, between cams, or between the pockets themselves.

Figure 8 shows a cross-sectional view of the blade. The drum-mill operation considered consists of forming the surface *A*. This is called the "convex" of the blade. The surface *B*, called the "concave" of the blade, is formed on a previous operation.

Figure 9 shows the method used to measure the form of the blade. A feeler gauge inserted between the sections *A* and *B* of the gauge showed how much stock had to be removed by grinding and polishing. Statistical evidence that this could be controlled showed that the grinding operation was unnecessary. As a result of action taken on the basis of statistical data on this operation, this grinding operation was ultimately eliminated.

The data shown in Table IV were obtained for a single revolution of the drum. To learn whether a significant difference exists between cutters, the data are arranged as shown in Table V. The variance between columns and the variance within columns are then computed.

To Obtain Variance Between Columns

1. Obtain the average of each column. In this example the averages are $\bar{X}_1=0.0068$, $\bar{X}_2=0.0082$, $\bar{X}_3=0.0135$, and $\bar{X}_4=0.0192$, respectively, for the four cutters.

2. Obtain the grand average $\bar{\bar{X}}$ of all the measurements. In this example, $\bar{\bar{X}}=0.0119$.

3. Subtract the respective column averages obtained in item 1 from the grand average in item 2. In this example,

$$\bar{\bar{X}}-\bar{X}_1=0.0119-0.0068=0.0051$$
$$\bar{\bar{X}}-\bar{X}_2=0.0119-0.0082=0.0037$$
$$\bar{\bar{X}}-\bar{X}_3=0.0119-0.0135=0.0016$$
$$\bar{\bar{X}}-\bar{X}_4=0.0119-0.0192=0.0073$$

4. Square the results of item 3. In this example,

$$(\bar{\bar{X}}-\bar{X}\ )^2=(0.0051)^2=0.00002601$$
$$(\bar{\bar{X}}-\bar{X}_2)^2=(0.0037)^2=0.00001369$$
$$(\bar{\bar{X}}-\bar{X}_3)^2=(0.0016)^2=0.00000256$$
$$(\bar{\bar{X}}-\bar{X}_4)^2=(0.0073)^2=0.00005329$$
$$0.00009555$$

5. Add the results of item 4. In this example, the total is 0.00009555.

6. Multiply the result of item 5 by the number of measurements in each column. In this example, there are six measurements in each column.
Therefore, we have $6\times0.00009555=0.00057330$.

7. Divide the result of item 6 by the total number of columns minus 1 (the degrees of freedom $=4-1=3$). In this example, there are four columns. Hence, we divide the result of item 6 by 3 to obtain $0.00057330\div3=0.00019110$. This last figure is the between columns variance.

To Obtain Variance Within Columns

1. Subtract each measurement from the average of the column in which it appears. In this example, the results are:

$0.0068-0.004=0.0028$
$0.0068-0.008=0.0012$
$0.0068-0.007=0.0002$
$0.0068-0.009=0.0022$
$0.0068-0.009=0.0022$
$0.0068-0.004=0.0028$

$0.0082-0.004=0.0042$
$0.0082-0.010=0.0018$
$0.0082-0.008=0.0002$
$0.0082-0.009=0.0008$
$0.0082-0.011=0.0028$
$0.0082-0.007=0.0012$

$0.0135-0.009=0.0045$
$0.0135-0.015=0.0015$
$0.0135-0.010=0.0035$
$0.0135-0.012=0.0015$
$0.0135-0.022=0.0085$
$0.0135-0.013=0.0005$

$0.0192-0.016=0.0032$
$0.0192-0.022=0.0028$
$0.0192-0.015=0.0042$
$0.0192-0.018=0.0012$
$0.0192-0.027=0.0078$
$0.0192-0.017=0.0022$

2. Square the results in item 1. In this example, the results are:

$$(0.0028)^2=0.00000785$$
$$(0.0012)^2=0.00000144$$
$$(0.0002)^2=0.00000004$$
$$(0.0022)^2=0.00000484$$
$$(0.0022)^2=0.00000484$$
$$(0.0028)^2=0.00000784$$
$$0.00002685$$

$(0.0042)^2=0.00001764$
$(0.0018)^2=0.00000324$
$(0.0002)^2=0.00000004$
$(0.0008)^2=0.00000064$
$(0.0028)^2=0.00000784$
$(0.0012)^2=0.00000144$
0.00003084

$(0.0045)^2=0.00002025$
$(0.0015)^2=0.00000225$
$(0.0035)^2=0.00001225$
$(0.0015)^2=0.00000225$
$(0.0085)^2=0.00007225$
$(0.0005)^2=0.00000025$
0.00010950

$(0.0032)^2=0.00001024$
$(0.0028)^2=0.00000784$
$(0.0042)^2=0.00001764$
$(0.0012)^2=0.00000144$
$(0.0078)^2=0.00006084$
$(0.0022)^2=0.00000484$
0.00010284

3. Adding the column totals in item 2, we get a grand total of 0.00027003.

4. Divide the result of item 3 by the total number of measurements minus the total number of columns. In this example, there are 24 measurements and four columns. Hence, we divide 0.00027003 by 20. The answer, 0.00001350, is the within-column variance.

USE OF FIGURE 10 TO DETERMINE WHETHER DIFFERENCE IS SIGNIFICANT

1. Divide the between-column variance by the within-column variance. In this example, the result is 0.00019110÷0.00001350 =14.16.

2. The between-column degrees of freedom is the number of columns minus one. The within-column degrees of freedom is the total number of measurements minus the number of columns. In this example, the between-column degrees of freedom is 3 and the within-column degrees of freedom is 20.

3. Using the between-column degrees of freedom as ordinate and the within-column degrees of freedom as abscissa, locate the curve on Figure 10, passing through their intersection. In this case the point of intersection is almost at 5, and we shall call it 5.

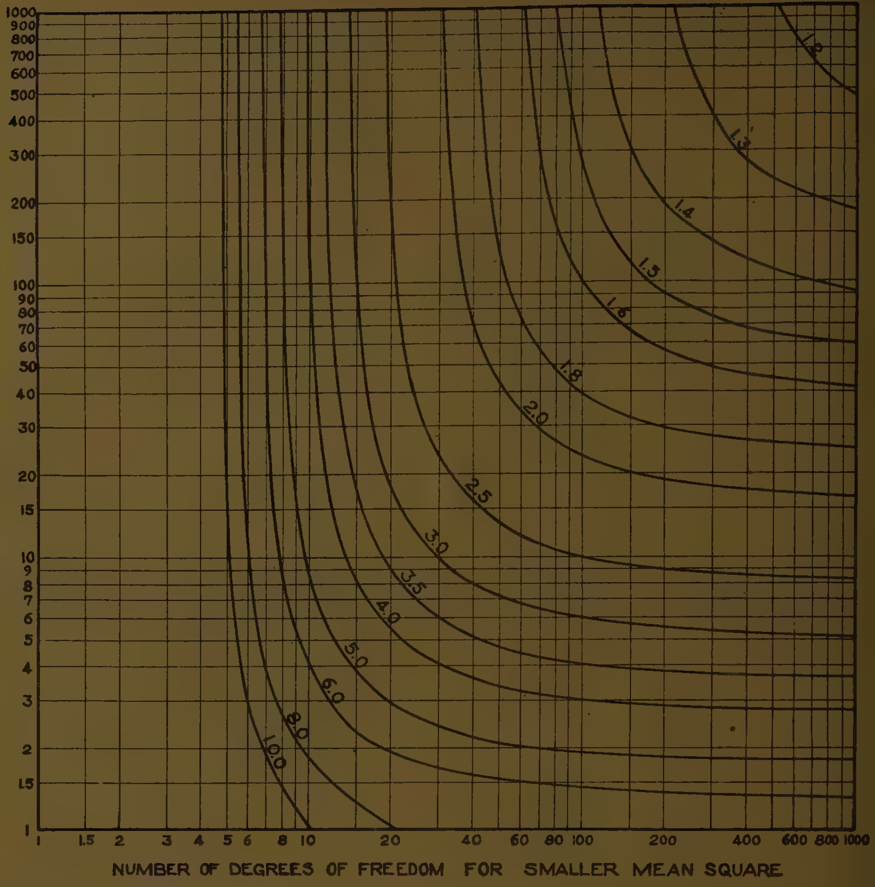


Figure 10. Analysis of variance chart—one-per-cent probability levels

4. If the answer obtained in item 1 exceeds the value on this curve, then a significant difference exists. In this example, the cutters produce significantly different parts, since 14.16 is greater than 5 (14.16>5).

Hence it is necessary to regrind the cutters before running the operation. The cutters were reground and the data on Table VII were obtained. For these data, the between-column variance is 0.0000144 and the within-column variance is 0.0000129. Reference to Figure 10 shows that significant difference between cutters is unlikely. The blades obtained from these cutters were sufficiently uniform to eliminate the grinding operation.

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Basic Concepts of Moving-Magnet-Instrument Rotors

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DURING RECENT YEARS the progress made in the development of permanent-magnet materials has brought about a considerable increase in the use of permanent-magnet rotors in d-c instruments. One such application is in the d-c Selsyn position indicator,[1] another in a newly developed electric instrument,[2] and a third in a new ratio-type instrument.[3] The space limitations in these designs predicated small actuating elements which were obtained by using permanent-magnet moving systems.

In view of these developments and the increasing number of permanent-magnet materials now available, it is appropriate to analyze the basic concepts on which the design of permanent-magnet rotors should be based. Although it is not possible to cover all rotor shapes, the elliptic spheroid, which can be shown to have uniform internal magnetization, lends itself to complete analytical treatment. A general analysis of this shape made in terms of its major and minor axes can be used as a good approximation for many rotor shapes and gives a clear physical insight into the behavior of permanent-magnet rotors.

The demagnetization portion of the hysteresis loops for several permanent-magnet materials are shown in Figure 1A, with their more important properties listed in Table I. These data are based on tests made on ring samples or the equivalent and without the demagnetizing effects of air paths in the magnetic circuit. All practical permanent-magnet applications, however, involve such air paths, and the designer is faced with the problem of applying these ring-sample data to designs subject to the demagnetizing effects of air paths.

The objects of this paper are:

1. To provide the instrument designer with a method for determining from ring-sample data and appropriate demagnetizing factors the intrinsic induction and the magnetic moment of a given rotor design.

2. To show how the value of the minimum field strength necessary to magnetize the rotor fully follows logically from the analysis.

3. To present a graphical method for determining the effect of an externally impressed field on a magnetized rotor.

4. To outline a scientific approach to the selection of materials for such applications.

Demagnetizing Factor of a Ring and Air Gap

The conventional analysis of permanent magnets usually deals with the case of a simple ring and air gap as shown in Figure 2B. The concept of a demagnetizing factor is not introduced as such, although it actually occurs in the relation which is established between the intrinsic induction and the magnetizing force. It is worthwhile, therefore, to re-examine the conventional analysis to identify this factor which is essential to the analysis of the magnetized rotor of elliptic-spheroid shape.

In the conventional analysis,[5] Ampere's law, $\oint H\cdot dl=0$, is applied to the case of the ring and air gap to obtain the expression:

$$H_m l_m + B_g d = 0 \tag{1}$$

where H_m is the magnetizing force in the material in oersteds, B_g is the flux density of the gap in gausses, l_m is the mean length of the magnetic material in centimeters, and d is the air-gap length in centimeters.

The assumption is then made that the leakage flux is negligible so that the flux density in the material,

$$B_m = B_g \tag{2}$$

Equation 1 then becomes

$$\frac{B_m}{H_m} = -\frac{l_m}{d} \tag{3}$$

But in the electromagnetic system of units, which is used throughout this paper

$$B_m = B_{im} + H_m \tag{4}$$

where B_{im} is the intrinsic induction in the material for a given value of H_m as determined experimentally by a ring-sample test. Thus

$$B_{im} = f(H_m) \tag{5}$$

where $f(H_m)$ is the demagnetizing portion of the major hysteresis loop for the continuous ring of the material as shown in Figure 2A.

Equations 3 and 4 then are combined to obtain

$$B_{im} = -\left(\frac{l_m+d}{d}\right)H_m = -\frac{1}{\left(\dfrac{d}{l_m+d}\right)}H_m \tag{6}$$

As indicated in Figure 2C, the equations 5 and 6 can be solved graphically for the values of B_{im} and H_m needed to provide an approximation of the air-gap flux density available from the permanent-magnet circuit shown in Figure 2B. It should be emphasized that the analysis is based on the assumption that the leakage flux is negligible and that the resulting value of flux density is in error to the extent that this assumption varies from actual conditions.

In the graphical solution of equations 5 and 6, the slope of the line OQ, which represents equation 6, is given by the coefficient of H_m. The value of $\dfrac{d}{l_m+d}$ may be defined as the *demagnetizing factor*, K, for this magnetic circuit as it determines the amount by which the intrinsic induction, B_{im}, is reduced from its residual value, OS, to its value, OR, in the magnetic circuit as a result of the demagnetizing effect of the air gap. An increase in the value of K reduces the value of OR.

Demagnetizing Factor of a Permanent-Magnet Rotor

The concept of the demagnetizing factor which has been identified for the relatively simple analysis of ring and air-gap magnets, greatly facilitates the more complex analysis of permanent-magnet rotors. It permits engineers to translate the results of the analysis of a difficult field problem into terms of magnetic-circuit theory which is more widely understood.

In the general case, a permanent-magnet rotor of any shape will not be uniformly magnetized when it is immersed in a uniform magnetic field. For example, the intrinsic induction in a rectangular bar magnet varies from a maximum at its center to a minimum close to its ends. Thus internally the intrinsic induction must be represented as a space function of some selected co-ordinate system. Under such conditions, the concept of a single demagnetizing factor is not valid. This factor becomes a demagnetizing function, to be specified from point to point within the rotor, and the mathematics required to evaluate it is lengthy, difficult, and beyond the scope of this paper.

If, however, a shape is chosen in which the intrinsic induction is uniform, a straightforward analysis using a single demagnetizing factor is possible. Such a shape is an elliptic spheroid, the surface of which is generated by rotating an ellipse about either its major or minor axis to develop the prolate or oblate forms of it

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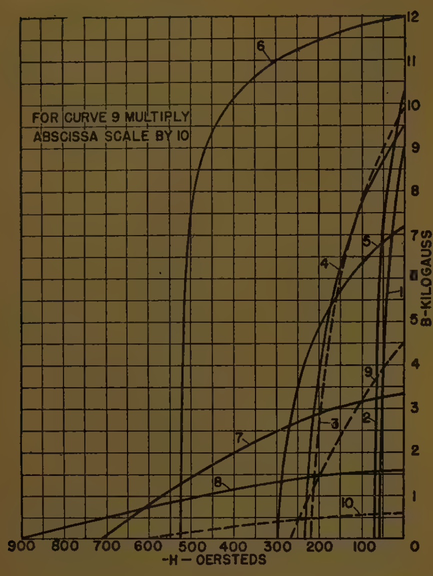
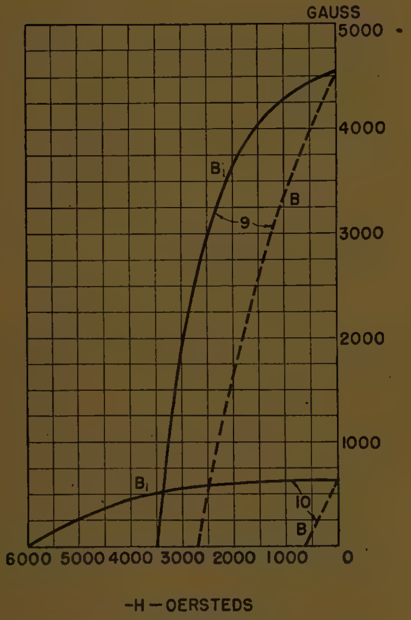


A. Curves for materials numbered 1 through 10 in Table I in terms of flux density, *B*, and demagnetizing force, *H*



B. Curves for cobalt–platinum– and silver–aluminum–manganese alloys in terms of *B* and *H* and also in terms of intrinsic induction, B_i, and demagnetizing force, *H*

Figure 1. Demagnetizing curves for permanent-magnet materials

shown in Figures 3A and 3B, respectively. Uniform intrinsic induction has been proved to occur in magnets of this shape if they are magnetized either along the generating axis or normal to it.[4] This shape has been used successfully as an approximation of many permanent-magnet-rotor designs. In doing so, the designer must distinguish clearly between the generating axis of the elliptic spheroid and the axis about which the rotor turns in the instrument. These axes may either coincide or be normal to one another. In addition, it should be noted that the direction of magnetization is always normal to the axis of rotation in the instrument.

The problem of determining the demagnetizing factor for the elliptic spheroid necessitates solving Laplace's equation $\Delta^2 F=0$ for the magnetomotive force distribution of the rotor and the applied uniform field. In addition, the basic physical relations[5] must hold at the boundary surface. With the magnetomotive-force distribution determined, the flux paths are found and then the demagnetizing factor. Another method of approach[4] recognizes the solution of Laplace's equation required for uniform magnetization by analogy with the gravitational potential theory, and from this solution the demagnetizing factor for the elliptic spheroid is found. In any case the simple use of Ampere's law as outlined for the ring and air gap cannot be used until the flux paths are determined.

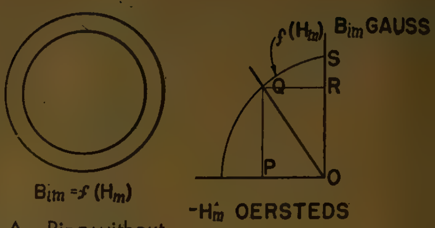
In Appendix I equations are given for the demagnetizing factors for the two forms of the elliptic spheroid, each magnetized either along the generating axis or normal to it. From these equations curves of the demagnetizing factors for values of the ratio of the generating axis to the axis normal to it are shown in Figure 4A. Figure 4B illustrates the elliptic-spheroid approximation of a short cylinder and also of a thin disk. The relationship among the three mutually orthogonal axes, the appropriate axis ratio c/a, and the demagnetizing factor K are indicated in each case.

Determination of the Intrinsic Induction and Magnetizing Force

The demagnetizing factors given by the curves in Figure 4A can be used to determine the intrinsic induction and magnetizing force for elliptic-spheroid rotors of any dimensions. In Figure 3A such a rotor is introduced into a uniform field of magnetizing force H_A. The source of this magnetizing force is assumed sufficiently large so that the presence of the rotor in the field has negligible effect on it. The rotor causes a redistribution of this original field outside the rotor which may be attributed to a distribution of poles on the surface as shown. The pole distribution which is associated with the intrinsic induction exerts a demagnetizing effect on the original field H_A within the rotor, and the final magnetizing force H_x in the material, while still uniform, is reduced to a smaller value than H_A. The reduction in internal magnetizing force is proportional to the intrinsic induction. The constant necessary to determine this proportion is the demagnetizing factor K. Thus

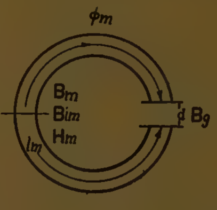
$$H_x=H_A-KB_{ix} \qquad (7)$$

where H_x is the resultant magnetizing force in the material in oersteds, B_{ix} is the intrinsic induction in the direction x in gausses, and K is the demagnetizing factor



A. Ring without air gap for determining fundamental data of Figures 1 and 2

C. Graphical construction for determining operating point *Q* of ring and air-gap magnet



B. Ring and air-gap magnet

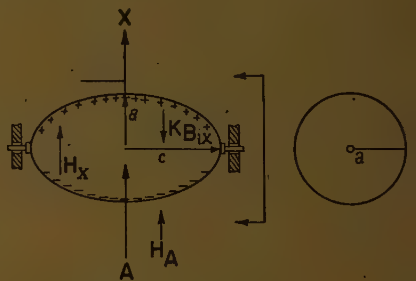
Figure 2. Conventional ring-shaped magnet

which depends on the ratio of the length of axis *c* to that of axis *a*.

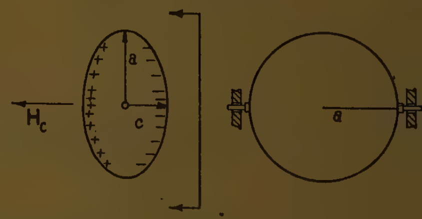
In equation 7 H_A is known, and K can be determined from Figure 4A. Two unknowns B_{ix} and H_x remain to be found. Hysteresis loops for the material based on ring-sample tests represented graphically or in symbols as $B_{ix}=f(H_x)$ furnish the other relationship needed to obtain numerical results. With equation 7 transposed, the problem may be formulated as follows:

$$\left.\begin{aligned}B_{ix}&=(1/K)(H_A-H_x)\\ B_{ix}&=f(H_x)\end{aligned}\right\} \qquad (8)$$

These equations can best be solved graphically to obtain numerical results. In Figure 5 *OM* is laid off equal to the magnetizing force H_A. The line *MP* is of



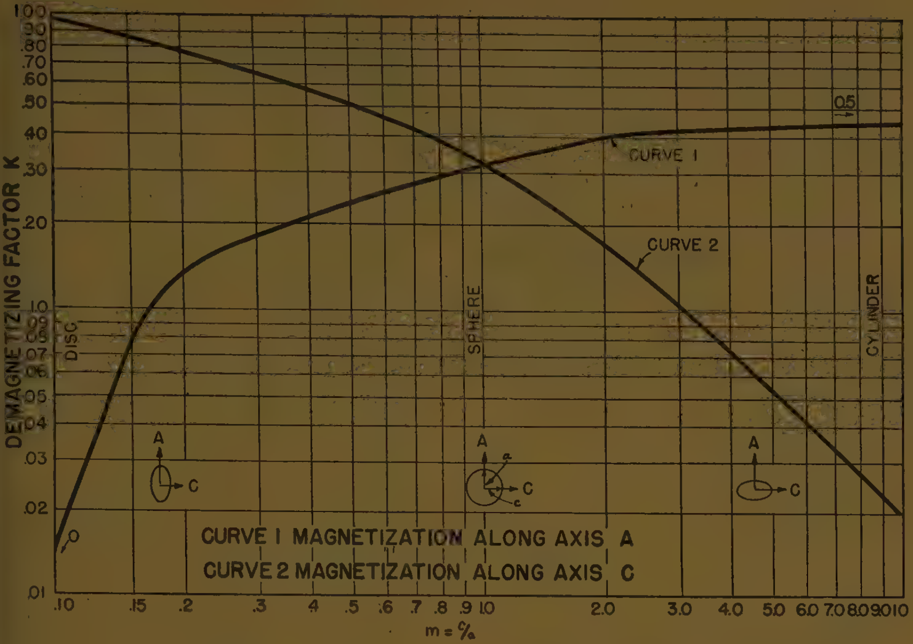
A. Prolate form



B. Oblate form

Figure 3. Elliptic spheroid

The intrinsic induction is always normal to the axis of rotation



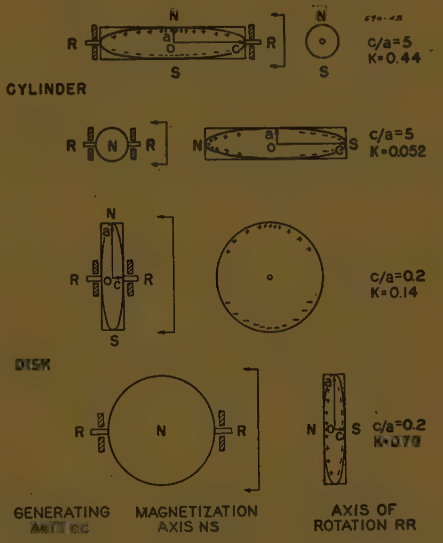


Figure 4

A (above). Curves giving values of the demagnetization for elliptic-spheroid rotors in terms of the axis ratio c/a

B (right). Elliptic-spheroid approximation of a short cylinder and of a thin disk

such slope that PN read on the ordinate scale divided by NM read on the abscissa scale equals $1/K$. As the major hysteresis loop is a multivalued function, there are three solutions P, Q, R, for this particular value of H_A, depending on how the magnetization takes place.

When the rotor is saturated by a large value of H_A which is then reduced to zero, its magnetic characteristics are represented by the point P_1, the ordinate and abscissa of which are the values of the intrinsic induction (B_{ix}) and the demagnetizing force ($-H_x$) for the permanent-magnet rotor. The point P_1 is found by the intersection of a line from the origin having slope $1/K$ and the demagnetizing portion of the hysteresis loop.

The magnetizing force H_A' needed to saturate the rotor can be obtained from the value H_s of this force required to saturate the ring sample. H_A' is determined by the intersection of a line drawn from S having slope $1/K$ and the abscissa. H_A' is larger than H_s because of the self-demagnetizing effect of the rotor shape. Thus the value needed to saturate a thin disk magnetized across its thickness is usually many times that required for the ring sample, and that for a long axially magnetized cylinder is nearly equal to it.

The foregoing analyses have been based on ring-sample hysteresis loops in terms of the intrinsic induction (B_{ix}) and the magnetizing force (H_x) rather than the more usual terms of flux density (B_x) and magnetizing force (H_x). When only the latter are available, B_{ix}–H_x curves are readily obtainable as $B_{ix}=B_x-H_x$. The B_{ix}–H_x relation is preferred for permanent-magnet-rotor analyses, as the intrinsic induction is equal to the magnetic moment per unit volume from which the mechanical torque provided by the rotor for instrument indication can be calculated.

Effect of External Fields on Magnetized Rotors

A properly magnetized rotor of course must be immersed in an external field with which it reacts in order to produce mechanical torque. This external field, however, tends to alter the intrinsic induction of the rotor along minor hysteresis loops. For example, a rotor having an intrinsic induction B_{ix} corresponding to point S_1 on the major hysteresis loop in Figure 6A is immersed in an external field H_u. The relative directions of B_{ix} and H_u may vary from alignment to opposition as indicated by Figures 6B and 6C. In Figure 6B the magnetizing force is increased and the material performance follows the minor loop S_1S_2, returning to S_1 if H_u is reduced to zero. When, as in Figure 6C H_u opposes B_{ix}, the path follows the major loop from S_1 to S_4, but, if H_u is reduced to zero, the path now becomes S_4S_5. Thus the intrinsic induction is permanently reduced to a value corresponding to S_5, and further demagnetization will not occur unless the rotor is exposed to an external field greater than H_u and in opposition to the intrinsic induction. It also has been shown experimentally that, if the rotor has been demagnetized to S_5 as in Figure 6C, no further reduction will occur in the condition shown by Figure 6D. If the minor hysteresis loops which determine points such as S_2 and S_6 are not available from ring-sample tests, they can be closely approximated for high-coercive-force materials by drawing lines parallel to the tangent to the major loop at the point of residual induction (B_r).

Calculation of Mechanical Torque of a Rotor

The intrinsic induction, B_{ix}, determined by the foregoing analyses for an elliptic-spheroid rotor, is by definition the magnetic moment per unit volume, M.[5] The mechanical torque of the rotor can be calculated from this moment, M. In practical designs the value of B_{ix} used for this purpose should be the minimum expected in service as determined by the relation of it to the field H_u, where H_u produces the strongest demagnetization effects as in Figure 6C.

With this value of B_{ix} selected, the magnetic moment in maxwell-centimeters is

$$M=B_{ix}\times(4/3)\pi a^2c \qquad (9)$$

where $(4/3)\pi a^2c$ is the volume of the elliptic spheroid. The mechanical torque for the general case of the elliptic spheroid is therefore

$$T=\frac{M}{4\pi}\times H_u \qquad (10)$$

where T is the torque in dyne-centimeters, M is the magnetic moment in maxwell-centimeters, and H_u is the operating-field intensity in oersteds. In most practical applications motion is so restricted that only coplanar components of vectors M and H_u can produce motion. A discus-

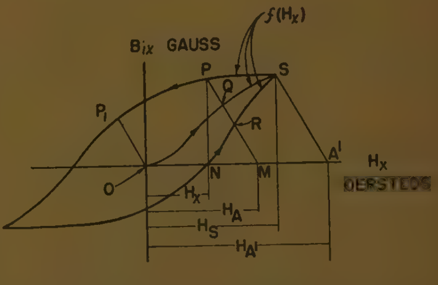


Figure 5. Graphical determination of the magnetizing force in an elliptic-spheroid rotor from the major hysteresis loop

Table I. Permanent-Magnet Materials

Number	Alloy Designation	Coercive Force, Oersteds	Residual Induction, Gausses	Density, Grams Per Cubic Centimeter	Mechanical Properties	Method of Fabrication
1	Chrome steel	63	9,000	8.20	Hard, strong	Hot forge, cast, machine
2	Tungsten steel	70	10,300	8.18	Hard, strong	Hot forge, cast, machine
3	Comol	220	10,000	8.15	Hard, strong	Hot forge, machine, cast
4	Cobalt steel	240	9,500	8.16	Hard, strong	Hot forge, machine, cast
5	Copper–nickel–iron	300	7,200	8.62	Ductile	Cold roll, machine punch
6	Alnico V	525	12,500	7.30	Hard, brittle	Cast grind
7	Copper–nickel–cobalt	710	3,400	8.30	Ductile	Cast, cold roll, machine
8	Sintered oxide	900	1,800	2.76	Hard, brittle	Mold, sinter
9	Cobalt platinum	2,700	4,500	18.0		
10	Silver–aluminum–manganese	6,000*	550	9.00	Ductile	Cold roll, machine punch

* Demagnetizing force required to reduce the intrinsic induction to zero, usually denoted by jH_c

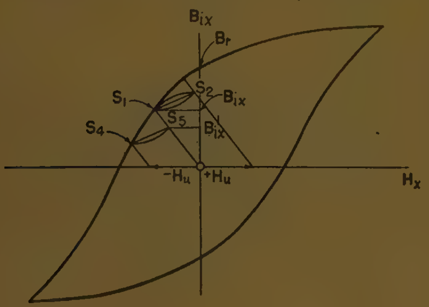
sion of this departure from the electromagnetic system of units is found in Appendix II. Thus equation 10 can be simplified to the following scalar form:

$$T=\frac{M}{4\pi}H_u \sin\theta \qquad (11)$$

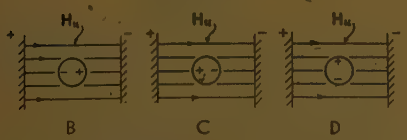
where M is the magnitude of the magnetic moment determined from equation 9 and is directed along the magnetization axis, H_u is the magnitude of the operating-field component normal to the axis of rotation, T is the torque in dyne-centimeters about this axis, θ is the angle between the directions of $\boldsymbol{M}$ and $\boldsymbol{H_u}$.

Practical Rotor Designs

The application of the analysis of the elliptic-spheroid rotor to practical instrument-rotor designs serves to explain the use of this method of approach in selecting the proper permanent-magnet material.



A. Graphical construction for determining the effect of the operating field on a magnetizing rotor



B, C, D. Possible relative directions of the intrinsic induction of the rotor and the operating field

Figure 6

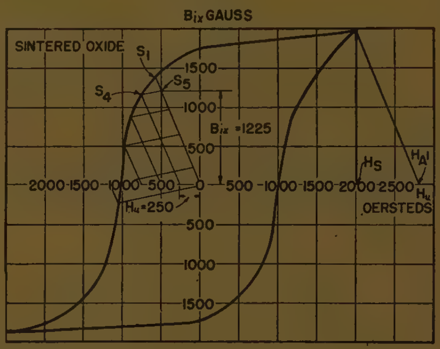


Figure 7. Graphical construction for determining the intrinsic induction of a one-half-by-one-centimeter cylindrical sintered-oxide rotor

In a specific design, the size of the instrument and therefore the permanent-magnet rotor usually is determined by the space limitations of the instrument application. Cylindrical rotors one centimeter long and one-half centimeter in diameter, magnetized diametrically and pivoted on the cylinder axis, may therefore be used in considering designs made of two materials, sintered oxide and silver–aluminum–manganese, which are well adapted to instrument rotors.

Figure 8. Graphical construction for determining the intrinsic induction of a one-half-by-one-centimeter silver–aluminum–manganese rotor

The elliptic spheroid which approximates this shape is the prolate form in which the direction of magnetization is normal to the generating axis and the ratio of major to minor axis, $c/a=2$. From Figure 4 the demagnetizing factor $K=0.41$. This factor then is applied to the hysteresis-loop data obtained from ring-sample tests for sintered oxide in Figure 7 and for the silver alloy in Figure 8. It will be noted that these data are in terms of the intrinsic induction, B_i, and the magnetizing force, H, rather than the more usual B–H relationships. The relative positions of the B–H and the B_i–H curves are shown in Figure 1B for the silver alloy.

In Figures 7 and 8 graphical constructions following the procedure previously explained for the general case of a rotor immersed in an opposed external field (Figure 6C) are used to determine the values of the intrinsic induction B_{ix} corresponding to various values of the external field H_u. In Figure 7, S_1, S_4, S_5 correspond to the same points in the general analysis of Figure 6A. The condition of this field in opposition to the intrinsic induction of the rotor is used, since it is assumed that the rotor is allowed to turn to the position at which this condition can occur.

From the values of B_{ix} thus obtained for sintered oxide and the silver alloy, the corresponding values of magnetic moment M then are calculated from

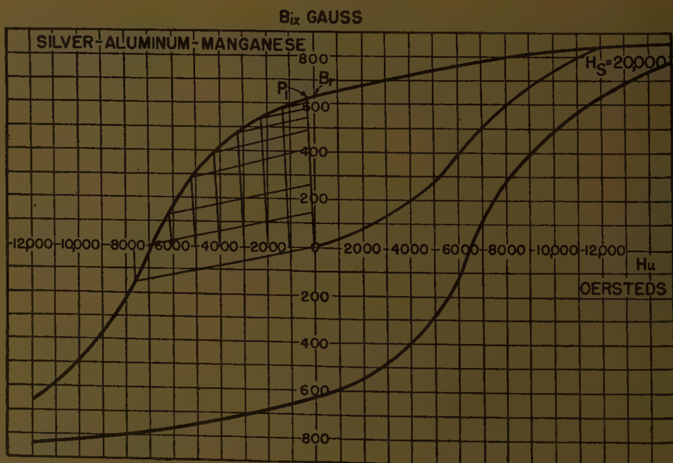
$$M=B_{ix}\times(2\pi a^2c) \qquad (12)$$

where $2\pi a^2c$ is the volume of the cylindrical rotor or in terms of the dimensions chosen,

$$M\text{(maxwell-centimeters)}=\frac{\pi}{16}B_{ix} \qquad (13)$$

These values of M for sintered oxide and the silver alloy are plotted in Figures 9 and 10 for values of the external field H_u. From equations 11 and 13, the mechanical torque for the rotor position at which $\theta=90$ degrees is

$$T_{\theta=90°}\text{ (dyne-centimeters)}=\frac{B_{ix}}{64}H_u \qquad (14)$$



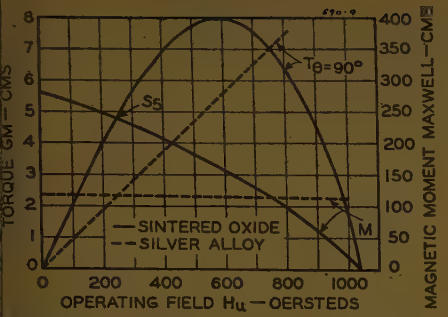


Figure 9. Magnetic moment and 90-degree torque available from a sintered-oxide rotor, and a silver–aluminum–manganese rotor of the same shape operated in field strengths to 1,050 oersteds

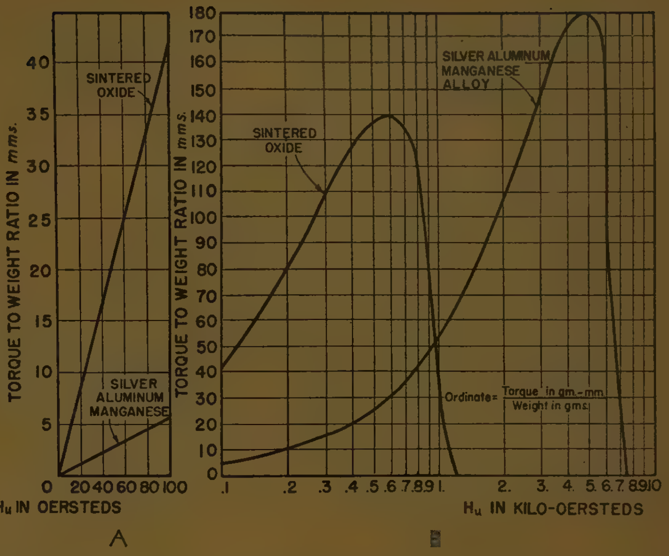


Figure 11. The performance of one-half-by-one-centimeter cylindrical rotors of sintered-oxide and silver – aluminum–manganese materials compared on the basis of torque-to-weight ratio for operating fields of 0 to 100 oersteds in A and 100 to 7,500 oersteds in B

$$T_{\theta=90°}\ \text{(gram-centimeters)} = \frac{B_{ix}H_u}{64\times 980} \qquad (15)$$

The torque values calculated from equation 15 also are plotted in Figures 9 and 10.

The comparison of the torque values for the two materials in Figure 9 shows that higher torques are available from sintered oxide for external fields below 770 oersteds, and for stronger fields the silver alloy is superior. As instrument designers are more often concerned with the torque per unit weight of the rotor rather than its absolute value, Figure 11 is given to compare the materials on this basis and further emphasizes the advantages of sintered oxide for field intensities below about 800 oersteds.

It is also interesting to note that, for field strengths between 3,000 and 6,000 oersteds, the silver-alloy rotor develops even higher torque per unit weight than the sintered oxide used with its most favorable field of 600 oersteds.

The instrument designer also will note that the values of torque to weight are considerably higher than those obtained in moving-coil designs. This is no doubt one of the chief reasons for the increasing popularity of moving-magnet instruments for applications in which sufficient operating field strength can be provided by stationary coils or other means.

In the preceding comparison of sintered-oxide and silver-alloy rotors, the operating field in which the rotor is immersed to produce torque has been assumed to be equal to the maximum field to which the rotor will ever be exposed. If, however, this operating field is exceeded because of overload currents in the coils producing it or other temporary conditions, the rotor will be further demagnetized. To take an extreme case, assume that the silver-alloy rotor while operating normally in a 500-oersted field may be subject to a momentary field of 5,000 oersteds in a direction to oppose the intrinsic induction of the rotor. Once the rotor has been demagnetized by this 5,000-oersted field, subsequent applications of it will cause no further demagnetization. The rotor magnetization therefore can be made permanent for all fields up to this value by subjecting it to an initial equivalent demagnetization. This, however, is accompanied by a reduction in the magnetic moment and torque from the values corresponding to the 500-oersted field. From Figure 10 the magnetic moment would be 77 maxwell-centimeters instead of 119, but a rotor having this reduced moment and operating normally in a 500-oersted field still has a 90-degree torque of 3.1 gram-centimeters. As this corresponds to a torque-to-weight ratio of about 17 millimeters, the rotor demagnetized to stand the momentary 5,000-oersted field should be satisfactory for most applications. This result exemplifies the value of the silver alloy in applications where a permanent magnet must withstand excessive demagnetizing fields.

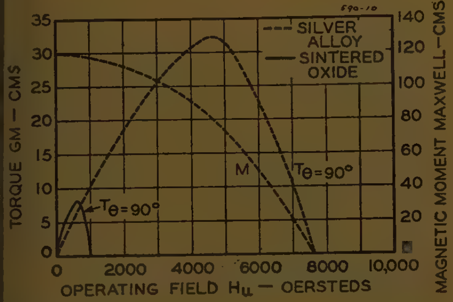


Figure 10. Magnetic moment and 90-degree torque available from the silver-alloy rotor operated in field strengths to 7,500 oersteds

Conclusions

The analyses presented are of value in determining the performance of permanent-magnet rotors for instruments. The assumptions made to simplify the treatment and particularly the use of the elliptic spheroid to approximate cylinders or other actual rotor shapes should be understood clearly by the designer applying the analyses to specific designs. He must continue to exercise sound judgment, especially in the selection of the elliptic spheroid which provides the closest approach to the magnetic conditions existing in the actual rotor under consideration. The writer has used the analyses presented for numerous instrument-rotor designs. The results obtained have agreed with experimental tests of samples of these designs within the accuracy limits of the tests and of the ring-sample data on which the calculations were based.

Materials other than the sintered oxide and the silver alloy already considered can be used for rotors and are preferable under certain conditions. Although economic as well as technical considerations should determine the final choice, the foregoing analyses can be of great assistance in making the selection, as the performance of a given rotor shape made of several different materials can be determined rapidly without recourse to building and testing samples of each. They may also serve as a basis for comparing moving-coil- and moving-magnet-instrument designs and for evaluating various rotor shapes. As the methods of calculation developed are based on well-established electromagnetic theory, it should be feasible to provide similar procedures for other permanent-magnet applications which will continue to increase both within and outside of the field of instruments and measurements.

List of Symbols

B_g—Air-gap flux density in gausses.

B_m—Flux density in the magnetic material in gausses.

H_m—Magnetizing force in the magnetic material in oersteds.

B_{im}—Intrinsic induction in the magnetic material in gausses.

l_m—Length along an assumed flux line in centimeters.

d—Air-gap length in centimeters.

H_x—Magnetizing force acting in the uniformly magnetized material in the direction x.

H_A—The intensity in oersteds of the uniform magnetizing field, before an unmagnetized rotor is immersed in it.

K—The demagnetizing factor (dimensionless).

B_{ix}—Uniform intrinsic induction in gausses in the direction x.

M—Magnetic moment in maxwell-centimeters.

T—Torque in dyne-centimeters.

H_u—Operating-field intensity in oersteds into which a saturated permanent-magnet rotor is introduced to produce torque.

c, a—Semiaxes of an elliptic spheroid in centimeters.

m—Ratio c/a (dimensionless).

Appendix I

For a prolate elliptic spheroid magnetized along the c and a axes, respectively, in Figure 3A, the demagnetization factors are:[4]

$$K_c=\frac{1}{(m^2-1)}\times\left[\left\{\frac{m}{\sqrt{m^2-1}}\log_\epsilon\left(m+\sqrt{m^2-1}\right)\right\}-1\right]$$

$$K_a=\frac{m^2}{2(m^2-1)}\left[1-\frac{1}{m\sqrt{m^2-1}}\times\log_\epsilon\left(m+\sqrt{m^2-1}\right)\right]$$

$$m=c/a$$

For an oblate elliptic spheroid magnetized along the C and A axes, respectively, in Figure 3B, the demagnetization factors are:[4]

$$K_c=\frac{1}{(1-m^2)}\left[1-\frac{m}{\sqrt{1-m^2}}\sin^{-1}\sqrt{1-m^2}\right]$$

$$K_a=\frac{m}{2(1-m^2)}\left[\left(\frac{1}{\sqrt{1-m^2}}\sin^{-1}\sqrt{1-m^2}\right)-m\right]$$

$$m=c/a$$

Appendix II

If M' is the magnetic moment in electromagnetic units (poles times centimeters), and H is the field intensity in oersteds, $T=M'\times H$ gives torque in dyne-centimeters. Now M' has the dimensions unit poles times centimeters, and, since 4π maxwells of flux emanate from a unit pole, the following equality may be written:

$$M' \text{ unit poles}\times\text{centimeters}$$
$$=M\text{ maxwells}\times\text{centimeters}\times\frac{1\text{ unit pole}}{4\pi\text{ maxwells}}$$

whence

$$M'=\frac{M}{4\pi}$$

Thus $T=\dfrac{M}{4\pi}\times H$ will give the torque in dyne-centimeters if M is in maxwell-centimeters, and H is in oersteds. Permeameter values of B_i are measured in gausses or maxwells per square centimeter. But this B_i value is identical to maxwell times centimeters per cubic centimeter, or the magnetic moment in maxwell-centimeters per unit volume. Thus for uniformly magnetized rotors it is convenient to modify the electromagnetic units as given in equations 10 and 11.

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Governor Requirements for Aircraft Alternator Drives

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RECENTLY much engineering thought has been directed toward the use of a-c power systems in very large aircraft.[1,2] To permit operation at substantially constant frequency, when generators are driven by accessory drive shafts of main aircraft engines, variable-ratio transmissions have been designed. The ratio of such a transmission may be varied by means of a governor which maintains generator speed within a narrow range while the aircraft engine speed varies over a wide range. The engine speed range in typical cases may be three to one or greater. The frequency of a typical a-c system as now designed for large military aircraft must be held within about five per cent of normal, except during unusual transient conditions, when somewhat greater variations are permissible. When alternators are to be operated in parallel, differences in engine speeds should not be permitted to cause loss of synchronism of the generators.

The requirements for governing the variable-ratio transmissions differ in several ways from governor requirements for ordinary prime movers for industrial and central-station generators. In the aircraft systems considered, the prime mover is the main airplane engine which drives the propeller. This may be 100 times as powerful as the generator and will vary in speed over a wide range independently of the generator's requirements. Rapid changes in engine speed may be encountered. These conditions indicated that analysis of governor requirements for aircraft alternator drives would involve special study.

As a part of the development program for certain military aircraft, a study was undertaken to establish performance specifications for governors for variable-ratio transmissions. Some of the results of this study are presented in this paper.

The action of the governors should be sufficiently rapid so that the transmission ratios will follow changes in engine speeds closely enough to prevent loss of synchronism or undue variations of frequency. The rates of acceleration or deceleration of the engines determine the response which a governor must have to operate successfully. The effect of sudden acceleration of one engine was investigated for a variety of operating conditions. Accelerations are considered to affect performance more adversely than decelerations, because under transient conditions acceleration of a generator tends to increase its electrical load, whereas deceleration tends to reduce this load. Also, when one generator of a group of paralleled generators is decelerated, its effect on frequency and synchronism is minimized by use of a free-wheeling device in its drive. That is, when an engine is decelerated, decelerating torques are not imposed on the generator by its transmission. As a result of these considerations the study reported in this paper is devoted to the effects of acceleration rather than decelerations of main aircraft engines. The major part of the paper concerns parallel operation, but certain information concerning nonparallel operation is also given.

The results and discussion included may help to give a clear conception of system performance during periods of engine speed changes.

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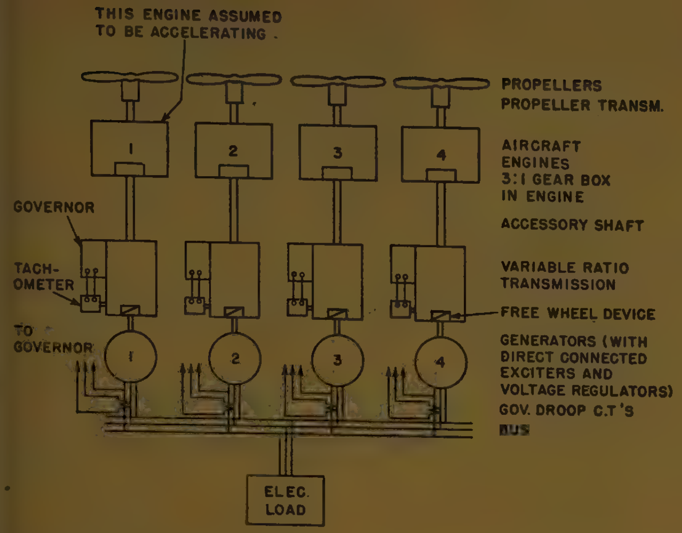


Figure 1. Schematic diagram of a-c power-supply system for airplane

System Arrangement

Figure 1 is a schematic diagram illustrating the arrangement proposed for operating four 40-kva 30-kw 6,000-rpm a-c generators in parallel for supplying 208-wye/120-volt three-phase 400-cycle power for use on an airplane. Each alternator is driven by a variable-ratio transmission, which in turn is driven from the main airplane engine.

Such a transmission may take any of a number of forms. One form consists of an oil pump with continuously variable stroke and a hydraulic motor driven by the variable oil flow. The stroke of the pump is varied by a servomechanism controlled by the governor. Each transmission is driven from an accessory drive shaft of a main aircraft engine. The engine rating is, of course, many times the rating of the alternator and transmission. Each transmission is controlled by its governor which operates to maintain alternator speed in spite of variations in engine speed. A free-wheeling device is included in the output shaft of each transmission. Each governor operates from the speed indication of a tachometer connected to the driving end of the free-wheeling device. An electric power-measuring circuit using a current transformer and a potential connection in each alternator output circuit is connected to the governor in such a way as to cause the generator speed to be reduced somewhat (droop) as the output is increased. Generators are controlled by individual voltage regulators. These regulators are connected to hold constant voltage and to divide reactive kilovolt-amperes properly among generators operating in parallel. Switching equipment in the electric-power circuits is not shown on the diagram.

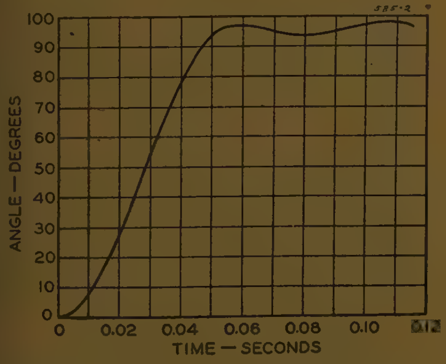


Figure 2. Angle between generators, case 1

Angle shown is the electrical angle between the alternator of the accelerated engine and the other three alternators

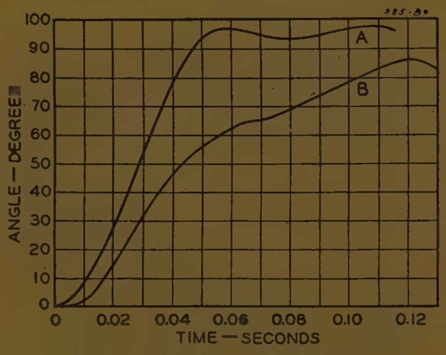


Figure 3. Effect of slip in drive

Angle shown is the electrical angle between the alternator of the accelerated engine and the other three alternators

A—Case 1; no slip in drive

B—Case 2; one-per-cent slip at full load

Engine Acceleration

If the engine speed changes very quickly, the governors cannot be expected to maintain absolutely constant speed on the alternators; instead, the alternator speed will change somewhat until the governors can restore normal frequency. If one engine (say number 1, Figure 1) accelerates while the speed of the other engines remains constant, the accelerated alternator will tend to pull away from the other alternators and may pull out of step. The governor should act sufficiently quickly to prevent the accelerated alternator from pulling out of step.

In all cases studied, one engine was accelerated at a rate of 2,150 rpm per second which was assumed to be the maximum possible. This value was determined by applying rated engine torque to the propeller inertia of a typical engine-propeller combination for a large airplane. The corresponding accessory-drive-shaft acceleration is 6,450 rpm per second. This value is probably much higher than any acceleration which will be encountered in normal operation. The engine acceleration was assumed to appear instantly. In practice the maximum acceleration probably will not appear instantly; the assumption made tends to make the results conservative.

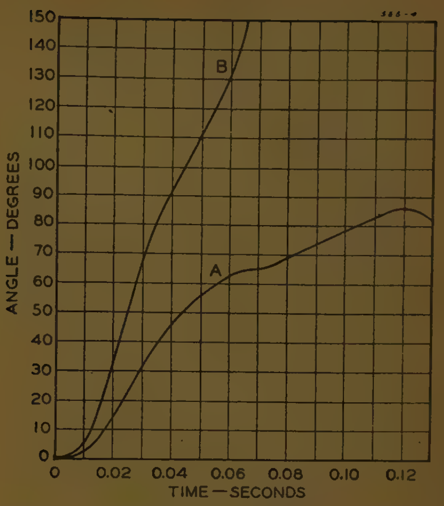


Figure 4. Effect of increasing applied acceleration

Angle shown is the electrical angle between the alternator of the accelerated engine and the other three alternators

A—Case 2; 6,450 rpm per second applied to alternator

B—Case 3; 14,300 rpm per second applied to alternator

The acceleration is considered to be sustained throughout the short time intervals investigated. It was necessary to study only the first few tenths of a second after the sudden appearance of acceleration, since synchronism was either lost or definitely established during this time.

In practice, the accelerations will not last indefinitely, and so the calculations are conservative in this regard.

Operating Conditions

A variety of initial operating conditions was investigated as outlined in the list of cases, Table I. Thus the effect of various factors, such as engine input speed, electric load, parallel versus nonparallel operation, governor action, drive slip, was investigated.

Each drive is considered to be controlled by an individual governor with an individual droop circuit for division of load. The load on paralleled generators was assumed to be balanced prior to the disturbances studied.

The voltage was considered to be controlled by individual generator voltage regulators.

Electric-System Characteristics

The time required for the voltage regulators to operate was considered to be 0.02 second, which is conservatively long. The exciter response was determined from calculated exciter characteristics, and the alternator electrical characteristics were determined from calculated machine constants. Subsequent tests on exciter and alternator have indicated that the calculated characteristics are reasonably correct.

The load on the bus was assumed to be at 0.75 power factor and to be a simple impedance load. The actual airplane electric load will consist of a mixture of motors and static loads. However, the results obtained are considered to give a reasonably accurate idea of the effect of electric load.

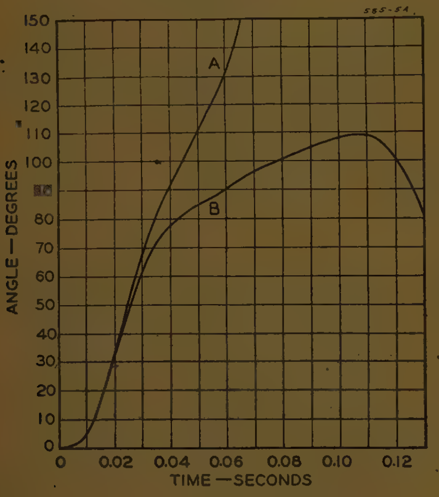


Figure 5A. Effect of governor action

Angle shown is the electrical angle between the alternator of the accelerated engine and the other three alternators

A—Case 3; no governor

B—Case 4; governor operating

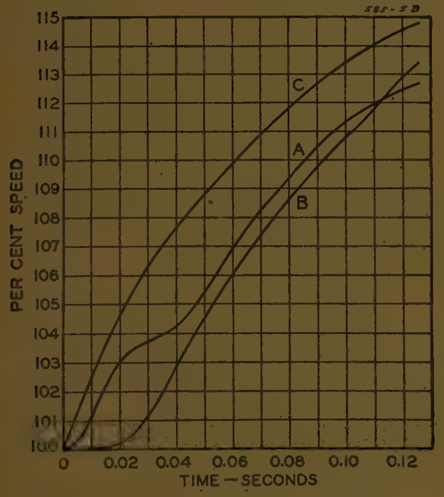


Figure 5B. Speed changes, case 4

A—Speed of alternator of accelerated engine

B—Speed of other three alternators

C—Speed *A* + drive slip = input speed × no-load ratio

Transmission Characteristics

In most of the cases studied, the variable-ratio transmission was considered to have a slip proportional to load such that full-load output causes a slip of one per cent. This value was obtained from information supplied by designers of a typical transmission. The drive ratio is varied by moving a lever. The no-load ratio of the drive is determined by the position of this lever. Depending upon the position of this lever with respect to neutral, the drive will operate either in underdrive (output speed less than input speed) or in overdrive.

Governor Characteristics

The rate at which the governor changes the drive ratio is assumed to be proportional to the alternator speed erro

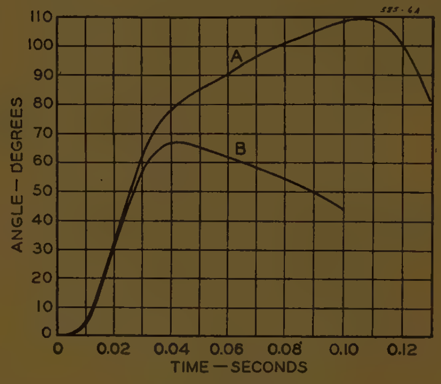


Figure 6A. Effect of increasing governor response

Angle shown is the electrical angle between the alternator of the accelerated engine and the other three alternators

A—Case 4; governor response = 0.0025 (ratio change per second) per rpm error

B—Case 5; governor response = 0.005 (ratio change per second) per rpm error

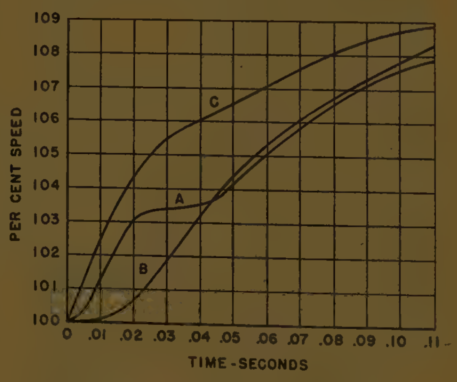


Figure 6B. Speed changes, case 5

A—Speed of alternator of accelerated engine

B—Speed of other three alternators

C—Speed *A* + drive slip = input speed × no-load ratio

This error is the difference between th alternator speed and the reference speec as determined by the governor setting droop circuit adjustment, and the ir stantaneous value of electric load on th governor.

The indicated error is the sum of th

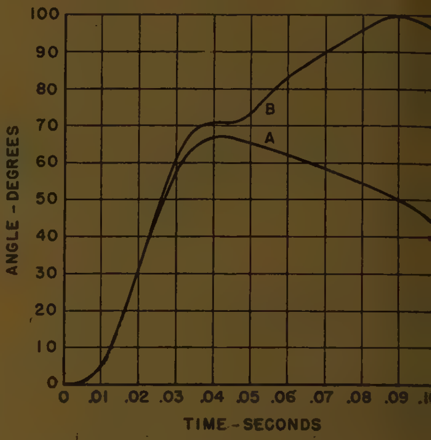


Figure 7A. Effect of electric load

Angle shown is the electrical angle betwee the alternator of the accelerated engine an the other three alternators

A—Case 5; no electric load

B—Case 6; electric load = 67 per cent pe generator

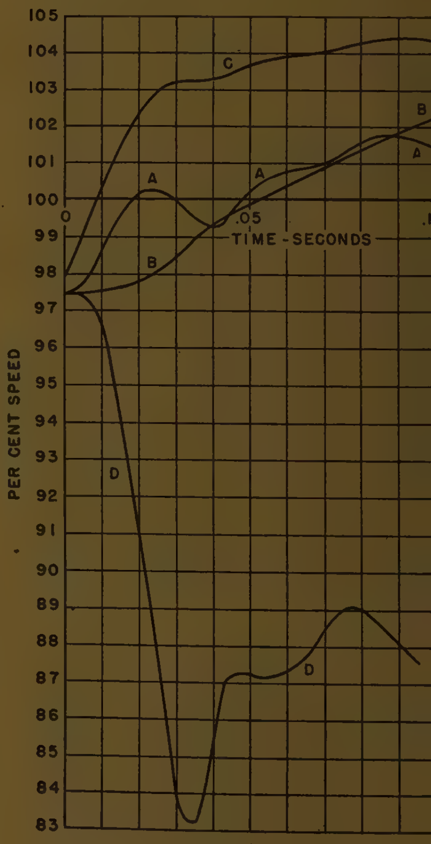


Figure 7B. Speed changes, case 6

A—Speed of alternator of accelerated engin

B—Speed of other three alternators

C—Speed *A* + drive slip = input speed

no-load ratio

D—Speed called for by governor droop circu

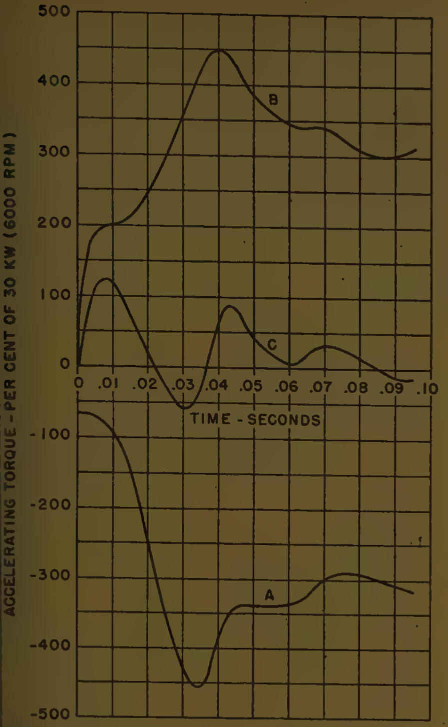


Figure 7C. Torques on generator, case 6

A—Electrical torque on generator of accelerated engine

B—Mechanical torque on shaft of same generator

C—Net torque on same generator

actual error indicated by the tachometer circuit and the apparent error indicated by the droop circuit. Such an apparent error occurs whenever the alternator power output increases (or decreases) as a result of transient conditions. Thus, an overload due to acceleration of an alternator causes the droop circuit to call for a lower speed and thus increases the signal to the governor in the direction of restoring correct speed. The correct speed in [illegible] case is considered to be the proper [illegible]ed for the particular electric load under balanced steady conditions, as determined by the governor setting and droop-circuit adjustment.

In practice, there probably will be a maximum rate at which the governor can change the drive ratio, but it is assumed that the governor is so designed that this is reached only for speed errors outside the range encountered in the cases studied.

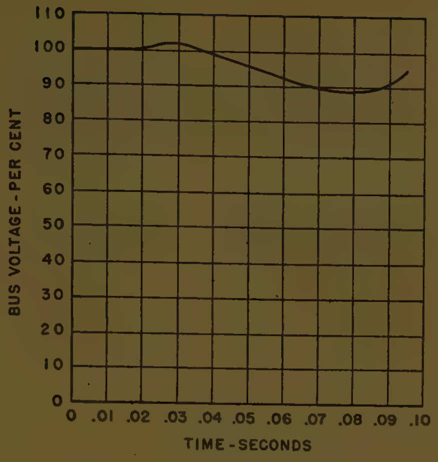


Figure 7D. Bus voltage, case 6

In each case, the governor is assumed to have a definite constant value of response in (drive-ratio change per second) per rpm of indicated error. The value selected is not the same in every case, and so the effect of changes in this value can be ascertained.

The drive ratio is the output speed divided by the input speed.

For the cases studied, there is a three-to-one gear ratio between engine shaft and accessory drive shaft. At an engine rpm of 900 the input speed is 2,700 rpm, so that for an output speed of 6,000 the ratio is 2.22. For this condition a rate of ratio change of 0.005 per second corresponds to a rate of alternator speed change of 13.5 rpm per second.

The governor is assumed to have no time lags in attaining the assumed response values. Actually there will be some time delay between the appearance of a speed error and the start of movement of the transmission ratio-control lever by the governor. Some idea of the effect of delays can be obtained by comparison of cases involving no delay and those including no governor action. These indicate that some delay is permissible and indicate approximately its allowable magnitude.

Cases Studied

The cases studied are outlined in Table I. A review of the results of the calculations for these cases form the basis of the conclusions given in this report.

The constants selected for the calculations are typical of a system for a large airplane using a-c power distribution. These constants are listed in Appendix III. Table II illustrates the manner in which the conclusions were obtained by comparing various results listed in Table I.

Sufficient cases have not been studied to be sure that the most severe conditions have been investigated; however, number 6 is considered to be fairly representative of a very severe case. This is the case of accelerating one engine at the maximum rate when operating at low speed, connected to a loaded alternator which is in parallel with three other loaded alternators.

Analysis of Results

The results of the cases studied have been plotted on Figures 2 to 8. These results may be analyzed by comparing various cases as outlined in Table II.

On several of the figures (Figure 2 for example) the electrical angle between generators is plotted. When the machines are operating at equal loads and excitations, as would be the case for normal operation, this angle is zero. That is, the rotors of the machines rotate at exactly the same speed and pass by a given point in each generator stator at exactly the same instant. If one generator is accelerated, its poles will pass any given point in the stator ahead of the corresponding poles in the other generators, so that the angle between the accelerated machine and the other machines is said to be increased from zero to a positive value. As this angle increases, the accelerated alternator delivers more load and may even furnish power to the other generators. The power transferred between generators tends to slow down the alternator of the accelerated engine and to speed up the other alternators, and thus resists the increase in angle.

Since the power transferred is only a very small fraction of the rating of the airplane engines, it will have practically no effect on the speed of the engines. However, it will cause changes in the slip in the transmissions, and it will cause the governor droop circuits to function in a

Table I. List of Cases

	Figure Numbers			Intial Conditions				Governor Characteristics			Results	
Case	Showing Angle	Showing Speed	Showing Other Quantities	No. of Generators in Parallel	Load Per Generator (Per Cent)	Engine Rpm	Slip in Drive at Full Load (Per Cent)	Full-Load Droop (Per Cent)	Response* (Ratio Per Sec) Per Rpm	Engine Acceleration Rpm Per Sec	Max Angle Between Generators (Degrees)	Max Speed Change (Per Cent)
1	2, 3			4	0	2,000	0		0	2,150	98	
2	3, 4			4	0	2,000	1		0	2,150	86	
3	4, 5A			4	0	900	1		0	2,150	Pulls out	
4	5A, 6A	5B		4	0	900	1	3.75	0.0025	2,150	109	16
5	6A, 7A	6B		4	0	900	1	3.75	0.005	2,150	68	9
6	7A	7B	7C, 7D	4	67	900	1	3.75	0.005	2,150	100	4.2
7		8		Nonparallel	0	900	1		0.005	2,150		12.2

* Governor response is measured in (drive-ratio change per second) per rpm of indicated alternator-speed error.

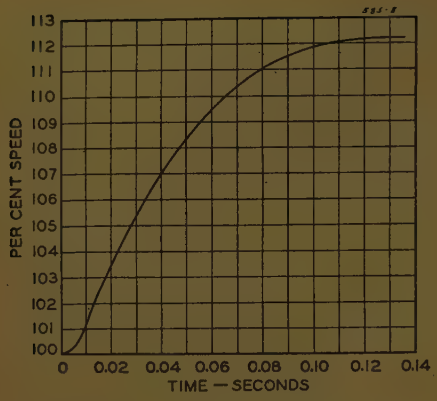


Figure 8. Generator speed, case 7

Speed returns to normal even if engine acceleration is sustained

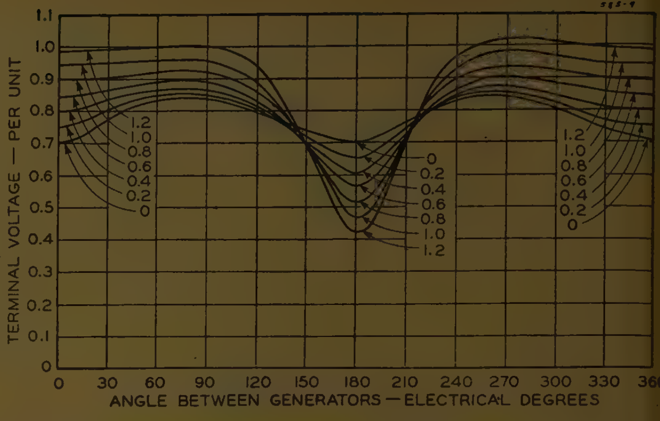
direction to help maintain synchronism. The synchronizing torques also may cause the slower alternators to free-wheel at a speed above their drive output speed, thus tending to minimize alternator speed difference and keep the angle between generators within safe limits. Hence, the synchronizing torques between generators are very helpful in preventing loss of synchronism.

The electrical characteristics of the generators under transient conditions are such that the synchronizing torque increases with angle until an angle of about 130 degrees is reached. At this angle, the torque is several times normal. Beyond this angle the torque falls off rapidly, so that the machine is very likely to pull out of step if the angle exceeds 130 electrical degrees.

The angle for maximum transient torque is greater than 90 degrees because

Table II. Schedule of Comparative Results

For Effect of	Compare	With
Slip in drive	Case 1 (no slip)	Case 2 (1% slip)
Applied acceleration	Case 2 (6,450 rpm per second acceleration applied to alternator)	Case 3 (14,300 rpm per second acceleration applied to alternator)
Governor action	Case 3 (no governor)	Case 4 (with governor)
Governor response	Case 4 (governor response —0.0025 per second per rpm error)	Case 5 (governor response —0.005 per second per rpm error)
Bus load	Case 5 (no load)	Case 6 (67% load per generator)
Nonparallel operation	Case 5 (parallel operation)	Case 7 (nonparallel)



One generator pulling ahead of three others. Impedance load on system, 0.75 power factor, 2.5 times rating of one generator

Figures on curves are per-unit values of armature flux linkages behind direct-axis transient reactance of one generator, value for other generators being unity

Figure 9. Relations between generator terminal voltage, flux linkages, and angles

of the effect of flux trapped by the field winding. Typical transient torque–angle characteristics are given by Figure 10.

In Figure 2 (case 1) the angle between machines does not exceed 100 degrees and returns toward zero after about 0.10 second. Hence synchronism is maintained. Since there is no governor in this case, the frequency, of course, continues to increase as long as the engine accelerates. Free wheeling permits the other three generators to be accelerated by electric power from the alternator of the accelerated engine.

Figure 3 shows the effect of slip in the drive. On curve *B* (case 2) the angle does not increase so rapidly nor reach so high a value as for curve *A* (case 1), so that there is less tendency to pull out and more time for governor action in this case than in the previous case. This shows that slip in the drive is helpful. See Table II.

Figure 4 shows the effect of increasing the acceleration applied to the alternator. On curve *B* (case 3) the angle increases rapidly beyond 130 degrees so that the generator can be seen to be pulling out of step as a result of the acceleration of the engine. It may be noted (Table I) that the engine acceleration is the same as for the previous case (curve *A*), but the drive ratio is higher, resulting in a higher acceleration of the alternator. In case 3 (curve *B*), the free wheeling and drive slip are unable to prevent pull-out.

In Figure 5A (case 4) the angle does not exceed 110 degrees and returns toward zero, so that the action of the governor, which was not present in the previous case, may be seen to be responsible for maintaining synchronism. (Table I shows that the initial conditions for this and the previous case are the same.)

In Figure 5B is shown the speed changes for case 4. The speed of all machines is increased, the other three machines being accelerated (free wheeling) by the electrical synchronizing torque. The effect of the slip in the drive may be observed by comparing curves *A* and *C* of this figure. The frequency may be seen to be approaching a maximum of about 116 per cent at about 0.2 second, after which i will return toward normal. The frequency will return toward normal even i the acceleration is maintained.

Figure 6A shows the effect of increasin governor response. It will be observe that the increased governor response c curve *B* (case 5) reduces the maximu angle reached.

It is evident that, when acceleration are less than the maximum possible, ther will be more time for the governor to mak corrections, so that a governor may hav appreciable time lag and less than th maximum required response, if the ac celeration does not attain its maximu value immediately. It is expected that i practice accelerations will not reach thei maximum values instantly, so that th calculations may be conservative eve though an actual governor will not attai its maximum response instantly. Mor complete data must be obtained befor the net effect of delays in attaining max mum acceleration response and maximu governor response can be determined.

Figure 6B shows the speed changes fo case 5. By comparison of this figure wi Figure 5B (case 4) it will be seen tha greater governor response of case 5 duces the change in frequency resultin from the engine acceleration. The fr quency will return to normal sooner tha in case 4.

Figure 7A shows the effect of electr load. This figure shows that the max mum angle is increased by the addition electric load to the system. On curve (case 6) the alternator does not pull o of step. However, the maximum ang reached in this case is quite large, (10 degrees), so that the generator probabl would pull out if the governor respons were appreciably less. Hence the r sponse for this case (0.005 per second p rpm error) is just about sufficient to mai tain synchronism in this severe case.

Figure 7B shows the speed changes f this case. The effect of the govern droop circuit is clearly shown by curve which shows the speed called for by th circuit as a result of the electric overloa on the generator during the disturbanc

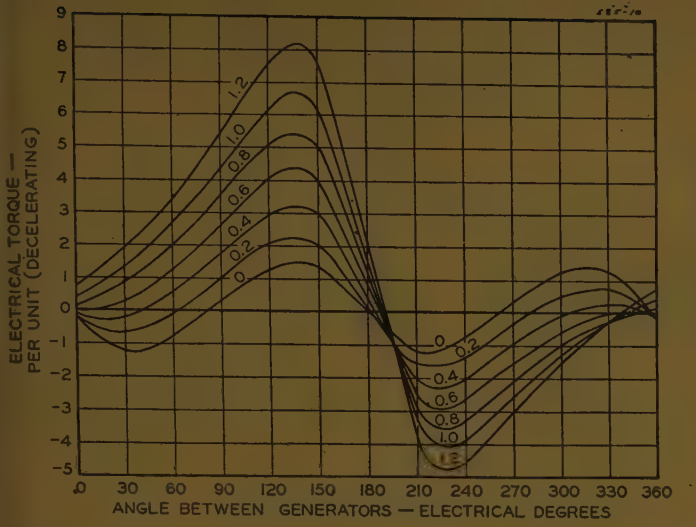


Figure 10. Relations between generator angles and flux linkages and the electrical torque on one generator

One generator pulling ahead of three others Impedance load on system, 0.75 power factor, 2.5 times rating of one generator

Torque values are for generator which is being pulled away from others

Figures on curves are per-unit values of armature flux linkages behind direct-axis transient reactance of one generator, value for the other generators being unity

Torque values given on curves do not include torques due to amortisseur windings

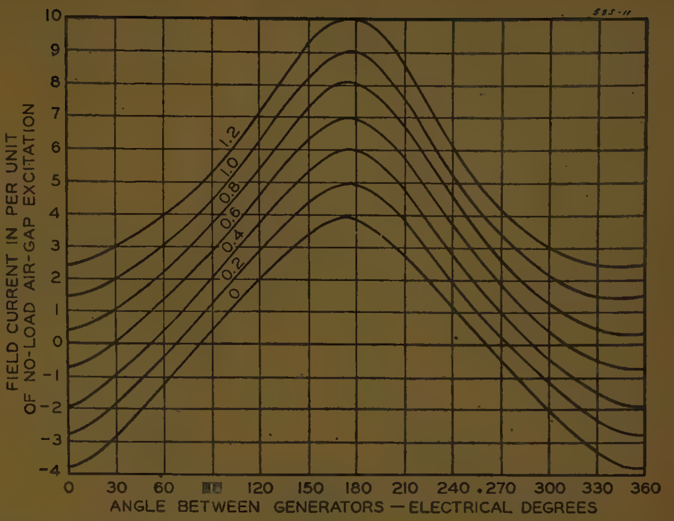


Figure 11. Relations between generator angles and flux linkages and the field current of one generator

One generator pulling ahead of three others Impedance, load on system, 0.75 power factor, 2.5 times rating of one generator. Field-current values are for generator which is being pulled away from others

Figures on curves are per-unit values of armature flux linkages behind direct-axis transient reactance of one generator, value for the other generators being unity

The signal to the governor introduced by this circuit is seen to be greater than the actual speed change of the generator. The selected per-cent droop (3.75 per cent) is less than the value (five per cent) specified by the Army, so that droop circuit actually used may be even more effective if time delays do not hamper its action seriously. The initial speed is 97.5 per cent, because the initial load on the generators causes the droop circuit to establish a speed lower than the no-load speed, which was taken as 100 per cent.

The maximum speed change is less in case 6 (Figure 7B) than in the previous case, because the electric load tends to prevent overspeeding.

Figure 7C shows the torques acting on the alternator of the accelerated engine. The electrical torque (curve *A*) is seen to reach a value of over four times normal due to the angular displacement. This opposes the mechanical torque (curve *B*) exerted by the drive, so that the net accelerating torque (curve *C*) is relatively small and results in only a relatively small frequency error.

Figure 7D shows the bus voltage for this same case (case 6). It will be noted that, when four generators are operating in parallel, one generator may undergo a severe disturbance without causing more than about a ten-per-cent dip in the system voltage.

Figure 8 illustrates the performance during nonparallel operation, when the engine acceleration and governor characteristics are the same as for the preceding two cases. It will be noted that the maximum speed error is greater with nonparallel operation, because there is no synchronizing torque from other generators to minimize overspeeding. The frequency increases to about 112 per cent, after which it returns toward normal.

Conclusions

1. A free-wheeling device in the output shaft of each variable-ratio transmission is very helpful for maintaining parallel operation during periods of engine acceleration. Under favorable conditions such devices permit synchronism to be maintained without any governor action whatever.

As most accelerations are expected to be moderate and of short duration, they normally will produce only momentary unbalances in load division among generators, which will be corrected by governor action and by the cessation of the accelerations.

When free wheeling is employed, the generators usually will resynchronize themselves following a disturbance which causes pullout to occur.

2. Even a small amount of slip (say one per cent at full load) in the transmission is helpful for maintaining sychronism, during transients due to engine acceleration.

3. If the acceleration which appears at the alternator is increased, it becomes more difficult to maintain synchronism. For a given amount of engine acceleration, the alternator acceleration is greatest when the transmission is at maximum overdrive ratio.

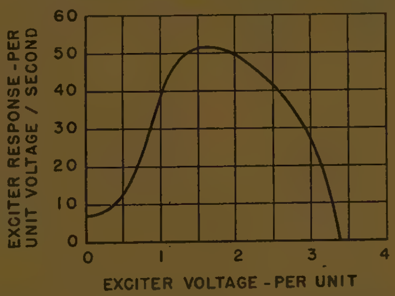


Figure 12. Exciter response—per-unit voltage per second

Exciter voltage is expressed in per unit of the value required for no-load air-gap rated-voltage excitation of the alternator

4. For the maximum accelerations considered, the effects of free wheeling and drive slip are not sufficient to maintain synchronism, so that reliance on governor action is necessary.

5. The effect of electric load on the bus is to increase the required governor response to maintain synchronism during periods of engine acceleration. This is true, because the load reduces the transient electrical synchronizing torque between generators.

6. Parallel operation tends to cause the frequency to be maintained within closer limits when one engine is accelerated, than would be the case for the same acceleration and governor response with nonparallel operation.

7. The rate of increase of acceleration is important in establishing governor requirements, as well as the maximum value of acceleration.

8. Time lag in attaining governor response should be kept to a minimum.

9. The governor droop circuit (which causes the alternator speed to droop with increased load) has a very beneficial effect in maintaining synchronism.

10. For the most severe case studied (case 6) the governor should attain its maximum required rate of ratio change in about 0.03 second to maintain synchronism. This maximum rate of ratio change is 5.3 per second for the assumed maximum engine acceleration of 2,150 rpm per second. The calculations indicate that to attain this rate

of ratio change in 0.03 second would require a governor response of about 0.005 expressed in (drive-ratio change per second) per rpm of indicated alternator-speed error. The maximum indicated alternator-speed error includes about three per cent (180 rpm) actual error, and 14 per cent apparent error, due to the governor droop circuit.

11. The method of calculation used is adaptable to studying a wide variety of cases.

When test data on system performance are available, the method may be used to assist in analyzing test results.

Appendix I. Method of Calculation

The calculations of electrical angle and other quantities versus time were made by a step-by-step process. At each value of time the rates of change of angles, speeds, generator flux linkages, transmission ratios, and exciter voltages were determined. These rates of change were assumed to remain constant for a short interval of time and the increments in these various quantities thus determined. By adding these increments to the original values, the angles, speeds, generator flux linkages, transmission ratios, and exciter voltages after the time interval were calculated. By making use of these new values and the constants of the system, rates of change for the next interval of time were determined. The process was repeated continuously, beginning with the time of first appearance of engine acceleration and continuing until the behavior of the system was ascertained.

In determining the rates of change of the quantities mentioned, certain other quantities, such as terminal voltage and torques, were calculated and plotted. The rates of change were determined using the constants given by Appendix III. The calculations for these rates were made on the basis of the assumptions listed in Appendix II.

The time intervals employed in making calculations in all cases were only 0.005 second or less. The interval used was determined by repeating calculations for one case with successively smaller intervals until further reduction in the interval produced substantially no difference in the result obtained.

It was found convenient to plot the calculated relations between terminal voltage, electrical angle, electrical torques (except amortisseur torque), generator flux linkages, and generator field currents, prior to making the step-by-step calculations, as these relations were independent of time. The calculations for these relations were aided by use of the a-c network analyzer. Typical results are given by Figures 9, 10, and 11.

Appendix II. Assumptions

1. The effect of magnetic saturation is neglected. This is considered justifiable, because the flux-linkage changes involved are relatively small. Methods for including this effect are available for cases requiring them.

2. The effect of amortisseur windings on generator armature currents is neglected. Preliminary calculations showed this effect to be small.

3. Torque due to amortisseur windings is assumed to vary directly with the rate of change of electrical angle with respect to terminal voltage, to be unaffected by rotor angular position, and to vary directly as the square of terminal voltage. Preliminary calculations indicated that this assumption is reasonably correct for the range of speed differences involved.

4. Voltages due to rates of change of direct- and quadrature-axis armature flux linkages are neglected. This assumption is justifiable, because these voltages are small compared to those resulting from rotation.

5. Generator friction, windage, core loss, and exciter torque are neglected.

6. Deviations in speed up to about ten per cent are assumed to have no effect on machine reactances or the ratio between power and torque. The effect of speed variations on terminal voltage is considered, however.

7. Alternators, voltage regulators, and governors are assumed to be identical so that several machines operating in parallel may be lumped into one, provided their engine speeds do not differ.

8. The rate at which the governor changes the drive ratio is assumed to be proportional to the alternator-speed error, within the range of speed changes involved.

9. Voltage-regulator action for small voltage errors (up to three per cent) is neglected. For greater voltage disturbances, the regulators are assumed to act, but to delay exciter response for 0.02 second. The effect of reactive cross-current compensation is neglected. This effect will be small if the regulators and their compensating circuits are adjusted properly.

10. Exciter response is assumed to be in accordance with Figure 12, which was obtained from typical exciter design data.

11. Exciter voltage changes caused by variations in generator speed are neglected.

12. Impedance between generators is assumed to be negligibly small.

13. The speed of the driving engine is assumed to be unaffected by the alternator.

Appendix III. Values of System Constants

The selected values of system constants used for the examples studied were as follows:

Engine Characteristics

Speed range	900–2,700 rpm
Accessory-shaft gear ratio	3:1
Accessory-shaft speed range	2,700–8,100 rpm

Alternator Constants

Rating: 40 kva, 30 kw, 6,000 rpm, 208 wye/120 volts, three phase, 400 cycles	
Open-circuit field time constant	0.10 second
Amortisseur damping coefficient	7.0 (per-unit torque) ÷ (per-unit slip)
Inertia constant, H	0.21 second
Transient reactance, x_d'	0.16 per unit
Synchronous reactance, x_d	1.04 per unit
Quadrature reactance, x_q	0.58 per unit
Armature resistance	0.03 per unit

Exciter Characteristics

Response	Figure 12

Transmission Constants

Slip (when present)	0.010 per unit at 45-horsepower mechanical torque (6,000 rpm)
Input speed range	2,700–8,100 rpm
Output speed rating	6,000 rpm

Governor Characteristics

Speed droop (when present)	0.0375 per unit at 30 kw electrical torque (6,000 rpm)
Response	Up to 0.005 drive-ratio change per second per rpm error

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2. Why Tomorrow's Planes Will Have A-C Electrical Systems, T. B. Holliday. *Electrical Manufacturing*, July 1943.

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A Current-Limiting Fuse for Aircraft Applications

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Synopsis: Although many fuses are available for aircraft applications, it has become increasingly evident that the trend to large aircraft electric systems necessitates a review of existing fuse devices, particularly with regard to their interrupting ability. Such a survey indicates many applications for an improved interrupting device. A new current-limiting fuse has been designed to meet the requirements. This paper describes the device and presents test data to indicate its suitability.

MUCH has been said about electric systems in aircraft including the history af their growth—voltage, available current, frequency, design, protection—as well as predictions of systems to come.[1,2] Articles, discussions, and lectures have indicated a trend that will require improved protective devices, particularly fuses. The fuse is a very widely used overcurrent protective device in aircraft electric systems today, because it provides maximum benefits for a minimum weight, space, and cost. Fuses which have proved satisfactory for present-day aircraft, however, should not be considered suitable for larger aircraft of the future. The definite trend to larger aircraft has resulted in three fundamental changes in the electric system:

1. There is an increasing number of operations being performed electrically, requiring larger amounts of available generated power.
2. Short circuits caused by battle damage in military airplanes or normal failures of connected equipment must be removed rapidly and satisfactorily to prevent system disturbances.
3. The circuits have become more complicated, requiring accurate, consistent, and correct fuse or protective-device operation.

The following general specifications for the required fuse and support are therefore proposed:

1. Adequate interrupting capacity.
2. Accurate and consistent time–current characteristics.
3. One-half cycle or less of arcing time.
4. Uniform contact resistance.
5. Vibration and shock resistance.
6. The foregoing characteristics to be affected only slightly by altitude up to 40,000 feet or ambient temperatures of −70 to +60 degrees centigrade.
7. Small size, compactness, light weight, and easy replacement.

Admittedly, a lot is required of a relatively simple device. Nevertheless, to meet these specifications, a new current-limiting fuse has been designed particularly for aircraft use. This paper discusses this fuse and presents information regarding applications.

Construction and Operation

For several years there has been available for central-station and industrial users a current-limiting power fuse.[3] This fuse, consisting essentially of a pure silver-wire element in an interrupting medium in the form of granular quartz, is unique in the way it operates on short circuits. The fuse limits the magnitude of the short-circuit current, under most conditions, to a value below that which would be produced in a direct short circuit. This action is summarized by the curves in Figure 2.

It was decided that this current-limiting action would be extremely desirable in aircraft electric systems, and therefore a fuse was designed, using this basic principle, which would meet the other specifications, particularly as to size, operating conditions, and vibration. The results are shown in Figure 3 and Figure 4.

The device is built at present in two frame sizes, one for a fuse of 30-ampere maximum rating and the other for a fuse of 60-ampere maximum rating. This division of ratings was arrived at after

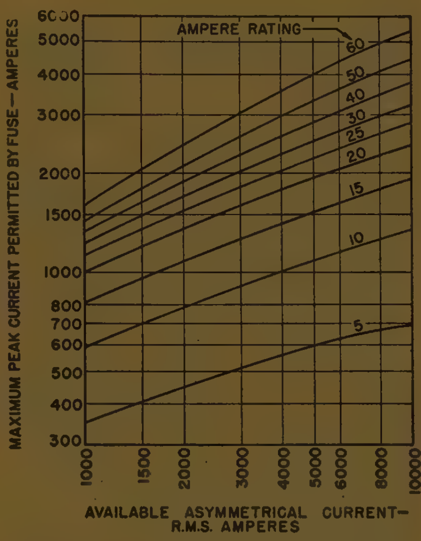


Figure 1. Current-limiting characteristics

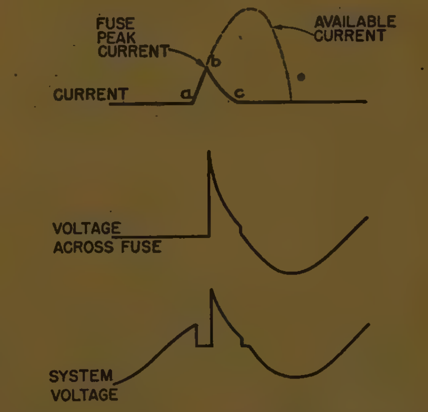


Figure 2. Wave forms of typical oscillogram of current-limiting fuse operation on a 60-cycle circuit

Short circuit starts at *a*, fuse melts at *b*, and arcing and interrupting complete at c

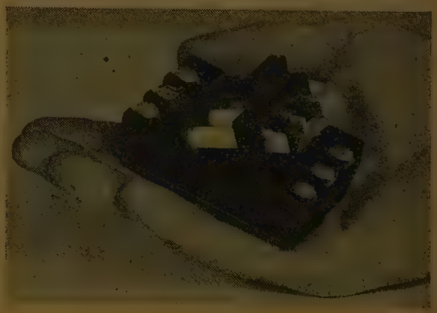
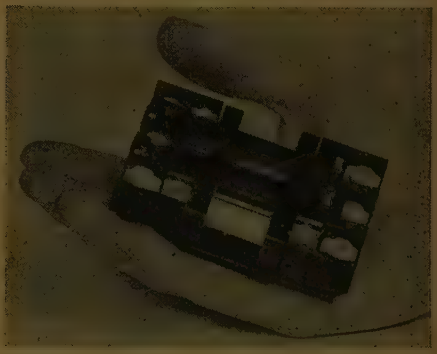
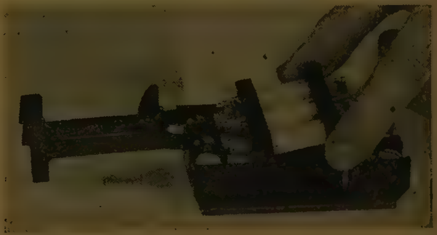


Figure 3. The new current-limiting fuse for aircraft applications

Thirty amperes, 208 wye/120 volts, 400 cycles. Fuse unit partially removed from holder



A. Fuse unit and latching bar are in place



B. Latch bar open and fuse being removed

Figure 4. The new current-limiting fuse for aircraft applications

Sixty amperes, 208 wye/120 volts, 400 cycles

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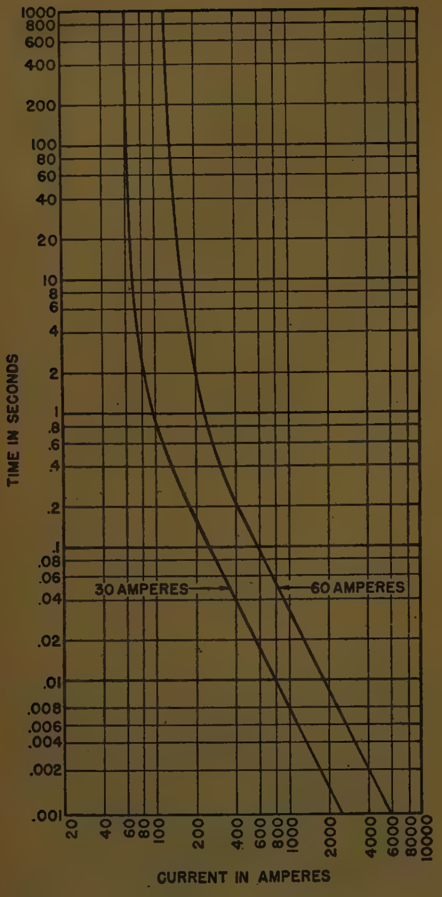


Figure 5. Melting-time–current curves for new current-limiting aircraft fuse

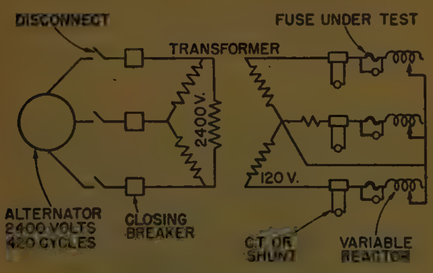


Figure 6. Test-circuit diagram

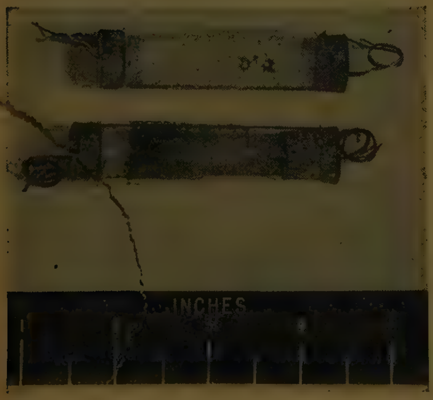
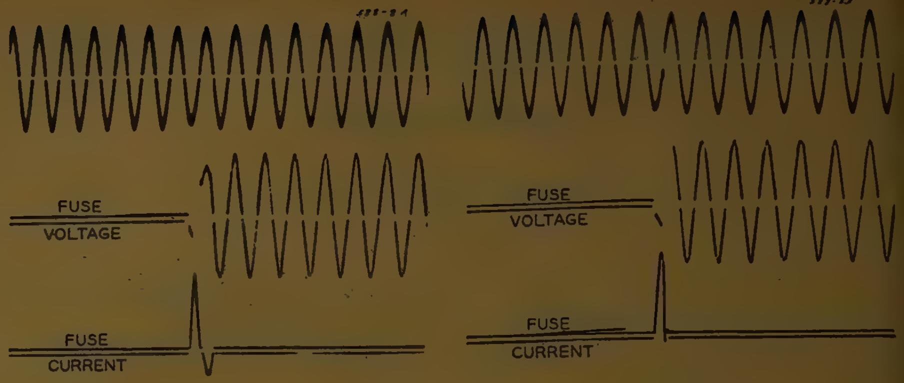


Figure 7. First sample fuses for preliminary tests, following interruption

Upper unit, silver wire, quartz filler. Lower unit, copper wire, air filler



A. Copper wire, air filler

Available current, 3,000 amperes; melting time, 0.00119 second; arcing time, 0.00059 second (Test 2, Table II)

B. Silver wire, quartz filler

Available current, 4,500 amperes; melting time, 0.00059 second; arcing time, 0.00030 second (Test 6, Table II)

Figure 8. Oscillograms for test samples shown in Figure 7

Tests made at 120 volts, 420 cycles

careful consideration of probable load requirements and design limitations of the maximum continuous current which could be carried safely by the terminals, contacts, and the fuse unit itself.

Fuse Units

The two fuse units are similar in construction, each consisting of two molded end pieces with three blade inserts for the contacts, three Pyrex glass tubes, silver-wire elements, and the special granular-quartz filler. The molded parts are made of a compound that will withstand vibration, shock, and the expected extremes of temperature. The blades which mate with contacts in the supports are silver-plated. The Pyrex tubes fit into counterbores in the molded pieces and are cemented in place during assembly. The fusible elements pass through small holes in the blades and are soldered to them with a lead–silver alloy which will not be affected by the operating temperature of the fuse. A brightly colored paint is applied to the inside of the glass tubes on the side towards the top of the fuse. The paint will be changed in color by the heat from the melting of the fuse element to indicate the blown condition of the fuse. The filler material is added through small holes in the fuse ends, after the tubes and elements are in place, and the holes then are sealed with cement. The design described thus provides a three-pole fuse assembly which has several desirable features:

1. When this fuse is used with proposed three-phase electric systems, the fuse units in all phases are replaced simultaneously, assuring that a fuse which might not have melted but was damaged is replaced.

2. There is no chance of one phase being accidently left open.

3. The phases are in separate isolated compartments, entirely preventing phase-to-phase short circuits within the device during the interruption.

The fuse element is made of pure silver wire with a reduced center section to limit the length which melts during interruption. There are, of course, many materials which could be used for the element, but silver was selected as the most suitable after careful investigation. Silver has a low specific resistance; thus a minimum cross section of wire will carry a given current, and there is a minimum amount of material to be dissipated during interruption. Furthermore, its melting and vaporizing action has proved more satisfactory in fuse operation than that of copper, which was also considered. Resistance to oxidation and corrosion are other characteristics in which silver is superior.

In the fuses for aircraft applications which are being considered in this paper, it is essential that ambient temperature have very little effect on the fuse time–current characteristics, for fuses which must co-ordinate may be in locations where ambient temperatures differ by 100 degrees centigrade. Thus, this is another reason why a high-melting-point element is superior to a low-temperature material such as zinc.

Many factors in the application of a

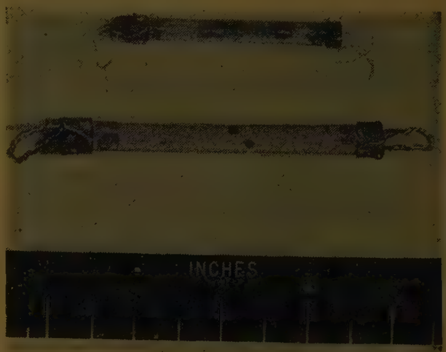
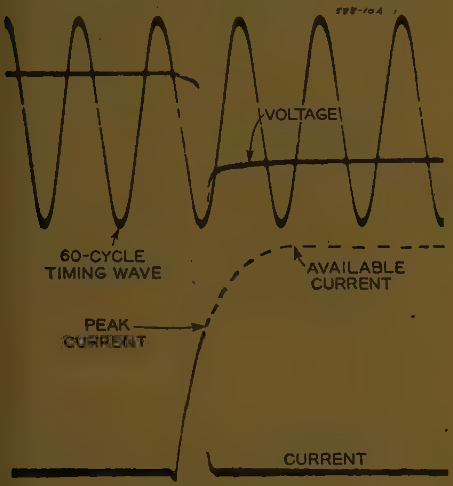


Figure 9. Sample fuses for second set of preliminary tests, after interruption

Upper unit rated 30 amperes; lower unit rated 60 amperes fuse dictate that it normally should be employed only as a short circuit rather than an overload protective device. This is discussed more fully later in the paper, but to meet this condition a fuse with "fast" characteristics is desirable, and this condition is obtained with the current-limiting fuse using a silver-wire element. Figure 5 shows typical time–current curves for 30- and 60-ampere aircraft fuse units.

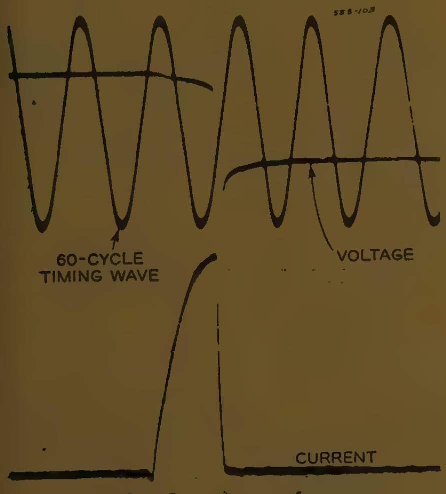
FUSE SUPPORTS

The fuse supports shown in Figures 3 and 4 are similar in that two molded end pieces are fastened to an insulated base to form the support for the terminals and contacts Beryllium–copper spring contacts, silver-plated, are used in the 30-ampere holder in order to provide uniform contact resistance. The fuse unit is latched into the fuse support, to prevent loosening from vibration, by having the blades on one end of the fuse unit assembled under a pin at one end of the support, while a spring backed catch at the opposite end of the fuse support locks over a projection on the other end of the fuse unit. Thus the fuse unit operates similar to the blade of a disconnecting switch and may be readily removed from the support when in the "open" position. The catch must be released before the fuse unit can be moved to the "open" position.



A. Thirty-ampere fuse

Available current, 3,149 amperes; peak current, 1,640 amperes; melting time, 0.0067 second; arcing time, 0.0015 second (Test 5, Table III)



B. Sixty-ampere fuse

Available current, 3,149 amperes; peak current, 2,640 amperes; melting time, 0.013 second; arcing time, 0.0023 second (Test 6, Table III)

Figure 10. Oscillograms of d-c short-circuit tests made at 135 volts

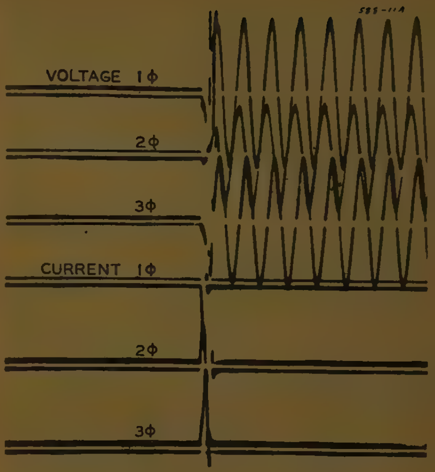
The 60-ampere support differs from the 30-ampere design, in that silver-plated copper contacts, backed by steel compression springs, are used. Furthermore, a hinged latching bar is arranged to swing into closed position after the fuse is inserted. The closing of the latching bar puts added pressure on the contacts, thus assuring uniform contact resistance, and holds the fuse in place against vibration and acceleration. The latching bar is held in place by a catch which is operated in the same manner as the catch described for the 30-ampere holder.

The over-all dimensions and weights of the fuse units and holders, given in Table I, indicate that the device is small and lightweight.

Tests

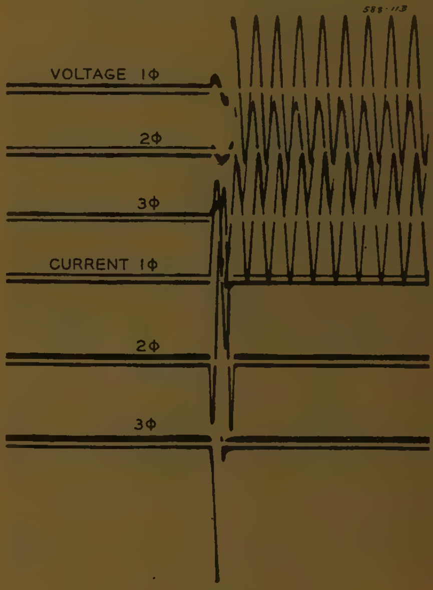
The current-limiting aircraft fuse was developed primarily for use on relatively high-voltage a-c electric systems. Specifically, the anticipated application was on a 208-wye/120-volt three-phase 400-cycle system. Therefore, the majority of the tests were made using this type of circuit. To check for operation of the fuses under other conditions, short-circuit tests also were made on 135-volt d-c circuits, and 120-volt 60-cycle a-c circuits.

Approximately 350 short-circuit tests were made during the development of the new fuse. Except for final check tests with a three-phase circuit, all tests were made with a single-phase 120-volt circuit, arranged as indicated in Figure 6, with a frequency of 420 cycles, which was the nearest available to 400 cycles. The initial series of tests was made using crude sample fuses to determine length of element, filler material, and other design limitations. Two of these samples after interruption are shown in Figure 7, and oscillograms for the two tests, Figure 8, indicate clearly the short arcing time obtained by using a suitable filler material. Table IIA lists the results of some of the preliminary tests made for the purpose of comparing the interrupting characteristics of silver and copper element in air and in quartz filler. Table IIB lists results of tests which demonstrate the current-limiting action of the fuses. Note that tests were made at currents up to 5,600 amperes and 40,000 feet altitude using a suitable pressure chamber. The arcing time was generally long and erratic with the air filler, but consistently short with the quartz filler. Thus the current-limiting fuse will meet the specifications for adequate interrupting capacity and short arcing time.



A. Twenty-ampere fuse

Available current, 3,100 amperes; melting time, 0.00071 second; arcing time, 0.0003 second (Test 2, Table IV)



B. Fifty-ampere fuse

Available current, 3,100 amperes; melting time, 0.0019 second; arcing time, 0.0003 second (Test 4, Table IV)

Figure 11. Oscillograms of final three-phase interrupting tests at 208 wye/120 volts, 420 cycles

Following the preliminary tests, a second series was made using samples with smaller-diameter tubes and elements designed in accordance with the results of the first series of tests. Typical samples for these tests are shown in Figure 9. The results were similar to the first series of tests and confirmed the suitability of the design. Tests also were made using the second set of samples, Figure 9, on a 135-volt d-c circuit. The results were very satisfactory. Figure 10 shows typical oscillograms of the tests, and again the very short duration of arcing should be

Table I. Dimensions and Weights of Fuse Units and Holders

Device	Inches			Weight (Ounces)
	Length	Width	Height	
30-ampere support	$2\frac{3}{4}$	$1\frac{11}{16}$	$1\frac{1}{4}$	1.9
5–30-ampere fuse unit	For 30-ampere support			1.1
60-ampere support	$3\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{9}{16}$	3.9
40–60-ampere fuse unit	For 60-ampere support			1.6

noted. Table III lists some of the d-c test results.

Before final short-circuit tests were made, many long-time melting tests were required in order to determine the exact size of fusible-element wire to obtain the required continuous-current ratings. Two factors govern this determination:

1. The temperature rise of the fuse while carrying the full current.

2. The melting current at some relatively long time, five minutes for example.

It was found that satisfactory results were obtained when the five-minute melting current was about twice the normal full load-current rating of the fuse. A series of tests was then made to determine the five-minute melting current of a range of wire sizes (both one and two wires per fuse tube) and the final wire sizes were determined and checked.

The lower ends (short-time) of the time–current curves were determined from the final short-circuit tests. The various points were combined to form the completed curves which, as shown in Figure 5, indicate a fuse with "fast" characteristics (a slight change in current above the minimum melting current will produce very rapid melting). The lower end of the curve is a straight line with a two-to-one slope.[4] Results of some of the final tests are listed in Table IV, and typical oscillograms are shown in Figure 11.

Table II. Preliminary Interruption Tests at 120 Volts, 420 Cycles, Single-Phase

A. To Compare Effect of Filler Materials

Test Number	Fuse Rating	Element Material	Filler	*Available Current	Altitude (Feet)	Melting Time (Second)	Arcing Time (Second)
1	20	Copper	Air	3,400	1,000	0.00089	0.00040
2	20	Copper	Air	3,000	40,000	0.00119	0.00059
3	22	Silver	Quartz	4,500	1,000	0.00059	0.00030
4	20	Copper	Quartz	3,400	40,000	0.00062	0.00030
5	20	Silver	Quartz	172	40,000	0.128	0.00035

B. To Indicate Current-Limiting Action. Silver Wire, Quartz Filler

Test Number	Fuse Rating	**Available Current	#Peak Current	Closing Angle (Degrees)	Altitude (Feet)	Melting Time (Second)	Arcing Time (Second)
6	22	6,380	4,000	30	1,000	0.00059	0.00030
7	20	5,940	4,100	70	40,000	0.00050	0.00030
8	22	4,250	2,780	75	1,000	0.00094	0.00035
9	25	8,180	4,800	30	40,000	0.00055	0.00030

* Available current is maximum rms current which the circuit can produce in the absence of the fuse, calculated from the rms symmetrical current available and corrected for the actual closing angle.

** Available current is maximum peak current which the circuit can produce in the absence of the fuse, calculated from the rms symmetrical current available and corrected for the actual closing angle.

# Peak current is the maximum instantaneous current permitted by the fuse.

Vibration tests show conclusively that the particular filler material used has no detrimental effects on the elements. After 50 hours of vibration with a frequency of 10 to 55 cycles per second, no reduction in cross section of the element was observed. This may well be due to two factors in the design:

1. The special size and shape of the grains of the filler material.

2. The fact that the filler material is tightly packed in the fuse tubes.

Application

The subject of application of fuses in aircraft electric systems is too broad to be covered in this paper. However, a few suggestions as to the fundamental points are in order.

Good fusing practice dictates that a fuse should be used for short-circuit protection, not overload protection. There are other devices, such as automatic motor protectors[5] and temperature relays, which provide satisfactory overload protection. However, in aircraft systems it is essential that the circuits remain operable at all times, except when they are damaged, in which case the fuse must operate to disconnect that part of the system. The current-limiting fuse which has been described is ideally suited for circuit protection, for its current-carrying ability and time–current characteristics are similar to those of the conductor and are not appreciably affected by differences in ambient temperatures.

Table III. Short-Circuit Tests at 135 Volts Direct Current

# Test Number	Fuse Rating (Amperes)	Available Current (Amperes)	* Peak Current (Amperes)	Melting Time (Seconds)	Arcing Time (Seconds)
1	10	480	296	0.003	0.0007
2	40	480	480	0.042	0.0022
3	20	1,675	870	0.0055	0.0009
4	50	1,675	1,580	0.022	0.0022
5	30	3,149	1,640	0.0067	0.0015
6	60	3,149	2,640	0.013	0.0023

# All tests made at 500 feet altitude.

* Actual maximum peak current permitted by fuse.

Thermal damage to a fuse element also must be considered when one is applying a fuse. Sufficient tests have been made to indicate that a change takes place in any element material when it is heated nearly to its melting temperature and then cooled. To make sure that a fuse is not damaged, it is a safe practice to select a fuse for a given circuit so that the fuse will not have to pass more than 75 per cent of the melting current shown on the time–current curve for the length of time involved. For example, in reference to the characteristics of the 30-ampere fuse given in Figure 5, if the circuit carries a temporary overload of 70 amperes, such a current should not pass through the fuse for more than 0.9 second.

Summary

The short-circuit capacity of aircraft electric systems is increasing constantly so that many available fuses are not satisfactory for the duty imposed on them. Furthermore, higher voltages are being used in order to transmit more power with smaller conductors. A fuse is an excellent short-circuit protecting device, but it must be properly built and correctly applied.

To meet the present and anticipated future circuit growth, a new current-limiting fuse has been developed. This fuse, based on the well-known and time-tested design of high-voltage power fuses, meets the requirements of adequate interrupting capacity, extremely short arcing time, vibration and shock re-

Table IV. Final Interruption Tests at 208 Wye/120 Volts, 420 Cycles

Test Number	Fuse Rating (Amperes)	* 3-Phase Available Current	Altitude (Feet)	# Average Melting Time (Second)	Average Arcing Time (Second)
1	10	3,100	1,000	0.0003	0.0003
2	20	3,100	1,000	0.00071	0.0003
3	40	3,100	1,000	0.0012	0.0003
4	50	3,100	1,000	0.0019	0.0003
5	60	4,580	1,000	0.0018	0.0003
6	60	2,990	1,000	0.0038	0.0004

* Available current is the rms symmetrical current (average three phases) which the circuit can produce in the absence of the fuse.

Melting time is average of three phases. The variation in time was caused by the closing angle of the three phases.

sistances, high altitude and wide-range temperature operation, and small size and weight.

Tests demonstrated that the new fuse will interrupt dependably more than 4,000 amperes at 120 volts, 400 cycles, single-phase alternating current or at 120 volts direct current, with an arcing time of less than one-quarter cycle on the a-c circuit.

To obtain satisfactory operation of aircraft fuses, they should be used for short-circuit protection, not overload protection, and selected so that they will not be damaged by overloads.

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Measurement of Railway-Motor Temperatures in Service

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THE EVER-INCREASING application of Diesel-electric locomotives to the many diversified traffic requirements of the railway systems over the nation is an acknowledgment of the versatility and effectiveness of this type of motive power. It brings to the steam road the high availability of the Diesel engine, the practical adaptation of which to railway-traction requirements is made possible through the electric transmission. The electric generator coupled to the Diesel engine enables the essentially constant torque and speed output of the prime mover to be converted to the widely variable range of torque and speed required for traction by means of electric motors geared to the locomotive drivers. This same transmission is used again when electric braking is applied to reduce or hold train speed.

The application of the Diesel-electric locomotive is then actually based on the speed-tractive-effort or braking characteristics of the transmission, the size of which for a given duty is dictated by economic considerations and the greater over-all advantages of utilizing to full extent an optimum standardized equipment of given continuous rating. However, tractive effort far in excess of continuous rating can be exerted by the smoothly applied motor torque to drivers with all the locomotive weight available for adhesion, and this capacity for overload is limited only by the heating of the windings.

Temperature of windings therefore is a prime consideration in the application of Diesel motive power wherever loads exceeding the continuous rating are encountered to see that the maximum permissible limits are not exceeded or maintained over a period of time to shorten unduly the insulation life. To this end, the silent movement of an ammeter needle over its scale is, psychologically to the man trained in steam operation, a poor exchange for the attention-compelling blast of a laboring steam-locomotive exhaust as an indication of exerted effort. There is an unquestioned finality when a steam locomotive stalls with its load that finds no adequate equivalent in the electric drive. The steady unemphasized beat of the Diesel exhaust continues, irrespective of the overload coupled to the locomotive which it moves with an apparent effortless ease with no sensible external impression of the winding's rising temperature or its proximity to the limit, until smoke and smell of burning insulation announce the damage to equipment.

The Diesel-electric locomotive does not know when to quit; nevertheless its ability to handle overloads is dependent upon the thermal capacity of the electric apparatus so that the duration, for which a given overload may be maintained with safety, is determined by the temperature of the windings preceding its application. Where ruling grades or speed of operation limit train weights, the load is generally below the continuous rating of the equipment, so that low winding temperatures provide a useful margin of overload capacity. This is an advantageous attribute of the electric drive in railway operation when it is correctly utilized.

It is essential to know motor temperatures just before a given overload to be able to determine the permissible duration for such duty. Temperature measurements taken in the intended service will be a check on the assumptions made during application study. It is often difficult to make proper assumptions of variables when making such studies for a difficult service. When steam helpers are required to move heavy trains over long grades, it is necessary at times to reduce Diesel-engine speed and power input to the motors in order to accommodate differences between the speed–tractive-effort characteristics of the steam and Diesel locomotives. If traction-motor blowers are belt-driven, the volume of ventilating air then is decreased, so that even operation at the continuous rating may be questioned. Temperature measurements are useful to obtain data and checks on such operation.

The use of cut-and-try overload application methods in service on narrow margins without knowledge of apparatus temperature is an uncertain undertaking the inadequacy of which may not be immediately evident, but will show up in time by increased maintenance cost and frequent motor or generator failures. Severe overheating in a single instance may be sufficient to shrink wedges and relax bands, without causing immediate failure, but subsequent movement of the winding under influence of vibration will cause the insulation to wear through and fail by short circuit or ground even before complete deterioration of insulation is achieved.

Various means have been developed in the past to give a continual indication of motor temperature, but none have been favored by general use. Until such time as temperature-indicating instruments are developed for railway use reliable enough to warrant general application, the measure of motor temperatures by the resistance method is the most reliable and accurate means available today. It is a recognized and familiar means of temperature determination by electric-apparatus manufacturers and is common on their test floors where the personnel is trained and skilled in its use.

This same method has been developed for the railway field by a series of evolutionary experiences until the technique and required skill have been reduced to a point where measurements can be made under these adverse conditions by an operator and the help of men unskilled in laboratory or test-floor methods. The results equal and sometimes surpass in speed, accuracy, and uniformity those obtained under more favorable conditions on the test floor, and it becomes a practical tool for general field use where questions pertaining to motor temperatures have a bearing on railway operation.

Method

The method selected for determination of average motor winding temperature is dependent upon the known change in electrical resistance of a copper conductor with temperature and is expressed by the

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relation $T=(R/r)(234.5+t)-234.5$. The initial or cold resistance, r, of each component winding in question is measured after the motors have stood without power long enough to allow all motor parts to cool to a uniform ambient temperature, t, which is measured directly with mercury thermometers placed upon the motor parts and checked with each other and surrounding ambient temperature. The hot winding resistance, R, is measured at prearranged stops on the main line with the train and at the point where the temperature of the motors in the actual service is questioned. The temperature, T, is then readily evaluated from the corresponding cold resistance, r, at temperature, t.

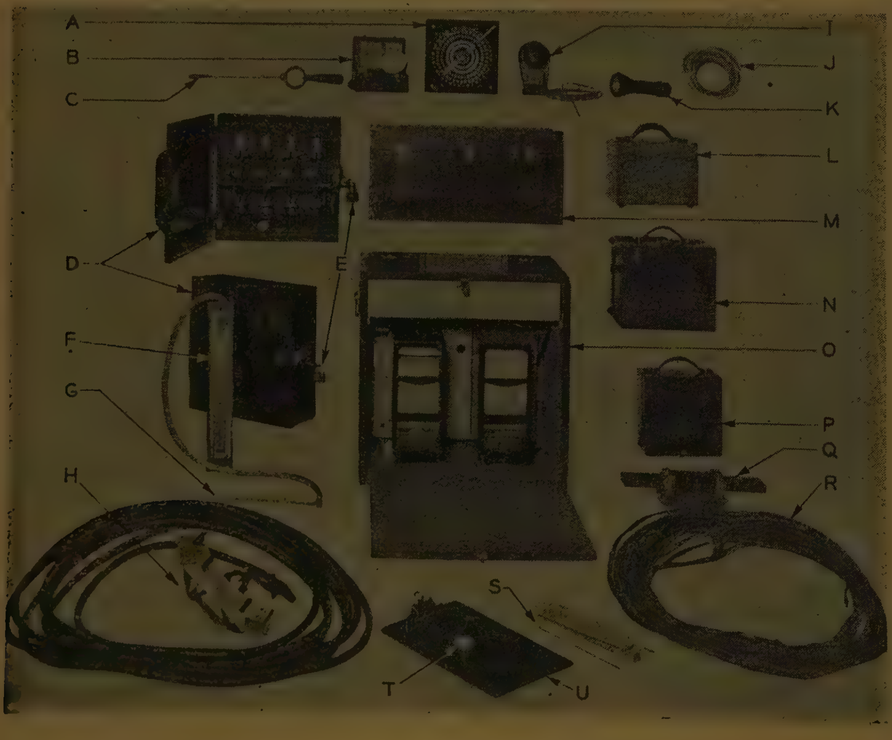
It is necessary to remove power from the motors and bring the train to a stop before these resistances can be measured. During this interval the motors are cooling, and the temperature of the winding as determined is for the instant of time when the resistance is obtained. This temperature is lower than the actual temperature of the winding at the instant when power was removed by the amount of cooling taking place during this elapsed time. However, by continuing to measure the resistance of the winding, as it cools with respect to a time started at the instant of power removal, it is possible to plot these values and draw a cooling curve through the related points. By extending this curve back to zero time, the most probable actual temperature of each winding is determined at the instant of power off and is well within the limits of accuracy for all practical purposes.[1]

Although temperature is the primary consideration determining motor life, it is expedient to plot the data in terms of temperature rise so that performance can be compared between successive tests under different conditions. Measurements, therefore, must be made of the ambient temperature during the test of the traction-motor ventilating air, as well as of the air surrounding the locomotive. The volume of ventilating air delivered to each motor being tested also must be determined, as well as the power input to the motors, so that rms current values can be calculated for the train tonnages hauled.

The success of the method outlined is dependent upon practical means for making the necessary measurements and upon a test crew organized and equipped to make these measurements without causing serious loss of time to the scheduled run of the train.

Figure 1. Instruments and special equipment required for measurement of railway-motor temperatures in service

Used to obtain operating data
A—Thermocouple selector switch
B—Hand tachometer
C—Hand pyrometer
D—Motor-field-resistance selector switches opened and closed
E—Padlocks
F—U-tube manometer
G—Static pressure tube
H—Armature-resistance contact yoke with flexible cable
I—Signal bell
J—Thermocouples
K—Flashlight
L—Thermocouple potentiometer
M—Special vibration-proof self-balancing double bridge
N—One-thousand-volt voltmeter
O—Special vibration-proof recording-chart ammeter and voltmeter
P—Millivoltmeter
Q—One-thousand-ampere shunt
R—Millivolt leads calibrated with shunt and meter
S—Centigrade mercury thermometer
T—Stop watch
U—Clip board



Instruments

The instruments and their connections are shown in Figures 1 and 2.

The Resistance Bridge

The traction motors in common use with Diesel-electric road locomotives are of the series type having armature, commutating, and series exciting fields of low resistance. The Kelvin double-resistance bridge circuit is ideal for the measurement of these low-resistance values. A vibration-proof galvanometer or its equivalent is required to obtain a balance on the bridge. This element of vibration requires careful attention in the selection and preparation of the instrument, as it is advantageous to keep the locomotive Diesel power plant running at idling speed during the stop for resistance measurement so that a quick train start can be made at the conclusion of measurements, and to keep the air compressors in operation to recharge the air-brake system after bringing the train to a stop. Even though the one power plant close to the instrument is shut down, enough vibration will come through from the remaining engines, through the couplers between units, to cause erratic behavior of the standard galvanometer when one is attempting to obtain a balance of the bridge.

A vibration-proof self-balancing portable instrument was constructed for this special purpose by the Erie works laboratory of the General Electric Company for the engineering department to be used in checking railway-motor application and has demonstrated ability in actual service to obtain data with speed, uniformity, and accuracy surpassing that obtained by previously used methods.

Connections to Motor Fields

The motor field windings are connected to the instrument with temporary wiring and suitable switches, so that resistance measurements can be made quickly. The instrument station should be located in the cab as close to the motors as possible so as to hold the interconnecting leads to a minimum. A pair of leads, a current and a voltage lead, is run to each terminal point of the winding and should be securely connected or soldered. The series field winding and one end of the commutating winding can be connected to the end of the motor cable in the junction box where they are joined with the locomotive power cable. They must connect to the motor end of the terminal to avoid the terminal contact resistance. The connection of the other end of the commutating winding can be made in-

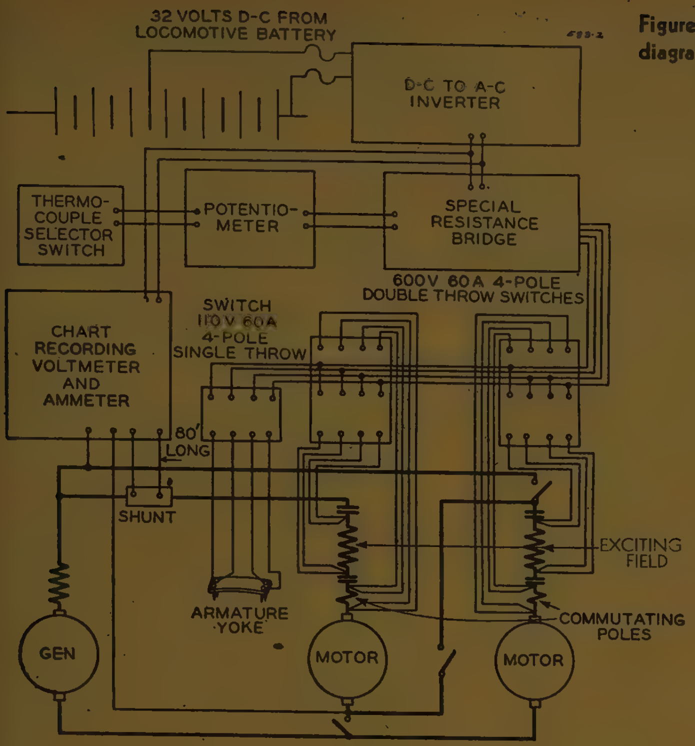

32 VOLTS D-C FROM LOCOMOTIVE BATTERY
D-C TO A-C INVERTER
THERMO-COUPLE SELECTOR SWITCH
POTENTIO-METER
SPECIAL RESISTANCE BRIDGE
600V 60A 4-POLE DOUBLE THROW SWITCHES
SWITCH 110V 60A 4-POLE SINGLE THROW
CHART RECORDING VOLTMETER AND AMMETER
80' LONG
SHUNT
EXCITING FIELD
ARMATURE YOKE
COMMUTATING POLES
GEN
MOTOR
MOTOR

Figure 2. Connection diagram for measurement equipment

side the motor where it joins the brush holder. These two instrument leads should be brought out of the frame without interfering with the motor inspection cover as it will have to be removed for armature-resistance readings and then quickly replaced. A hole may be drilled through a noncritical section of the frame to let these leads through, to be closed at the conclusion of the test. These leads should be run and secured alongside one of the main motor cable, as far as the junction box under the locomotive floor, where it can join the bundle of leads coming off the terminals and going to the instrument station. This bundle of instrument leads should have additional protection where it passes through and bends over structural members to avoid damage to the insulation.

Connections to Motor Armature

The armature resistance is obtained by selecting a span of the armature winding to which connection is made by contacts spaced apart by the desired number of commutator segments and held in fixed relationship by a special yoke applied by hand pressure to the commutator through the motor inspection opening.

This yoke has a locating lip to engage the undercut mica space between two adjacent commutator segments to align the contact points correctly on the segment at each end of the selected span of bars and prevent their coming in between two segments to cause erratic measurements. The number of segments included in the span should be as large as the space between brush holders permits without cramping the yoke, so as to obtain a maximum value of resistance for measurement. The contacts are individually spring-loaded and slide in parallel ways to accommodate slight inequalities in the hand pressure applied to the yoke.

Connection between contacts of the yoke and the instrument station is made through a multiple-wire flexible cable long enough to reach through a cab window down to the motors in the locomotive truck. This equipment is coiled and carried in the cab between stopping points for measurements.

Resistance Selector Switches

The switches are 600-volt four-pole double throw, with flexible jumpers connected around the hinge point of the blade. They should be completely enclosed with provision for padlocking the switch in the open-circuit position. This is a safety precaution, as the instrument leads are connected with the high-voltage power circuit when the locomotive is in operation, and will serve to isolate the instruments and personnel from harm. It also permits wiring to be installed during layovers between runs, thereby keeping the locomotive available for service while preparation for the test is being made. It also permits removal of instruments after a test run until results are checked. The switches should be located within reach of the instrument table so that the bridge can be switched from one connection to another in prearranged sequence.

Communication System

A signal system is necessary between the man at the instrument station and the men in position at the motors when measurements are in progress because of their remote location and the magnitude of noise level with Diesel engines idling. A push button fixed at the instrument station and a simple electric bell properly protected against injury during handling are used for this purpose, interconnected with flexible leads long enough so that they can be lowered from the cab to a man on the road bed. The signal code used is as follows:

One bell—make contact with yoke on armature.

Two bells—hold contact; measurement is in progress.

Three bells—remove contact.

An ammeter placed in the battery circuit of the bridge indicates the instant a circuit is established through the armature and informs the resistance bridge operator to proceed with measurement, while the man making the contact is informed to hold it by signaling two bells. A failure to establish contact with all four points on the commutator simultaneously is evidenced by failure of the ammeter to indicate if the current circuit is open or by an off-scale swing of the bridge if the voltage or drop circuit is open. In this case, the operator continues to call for contact by repeating the one-bell signal.

Arrangement for Power-Input Measurement

The power input to the motors is measured with a voltmeter and ammeter with respect to time, the interval of which is dependent upon the profile of the railroad and the operation being studied. This is a laborious and tedious procedure which may be eliminated by installing chart-recording instruments to keep a graphic record of the test. Recording instruments should be equipped with special marker pens, energized periodically by means of a Telechron-driven contact-making switch to put a reference mark on each graph. A push button also should be provided to enable operating the marker pens at will for special purposes, such as the passage of significant locations. In addition, the chart must be clearly marked with notes to aid analysis after these tests are completed.

The recording instruments must be shockproof-mounted to obtain a smooth graphic record. The indicating voltmeter and ammeter should be retained for a periodic calibration check of the recording instrument. All instruments must be secured to the cab structure to prevent displacement when curves at high speed are being negotiated.

The ammeter shunt must be connected to the motor power circuit on the low-voltage or grounded side of the circuit to minimize hazard to personnel. The leads between the indicating ammeter and shunt must be calibrated together with the shunt and recording instrument before being installed in the locomotive. One side of the voltmeter always will be on the high side of the power circuit and must be safeguarded.

1. With fuses small enough to prevent fire in case of accidental short circuit.

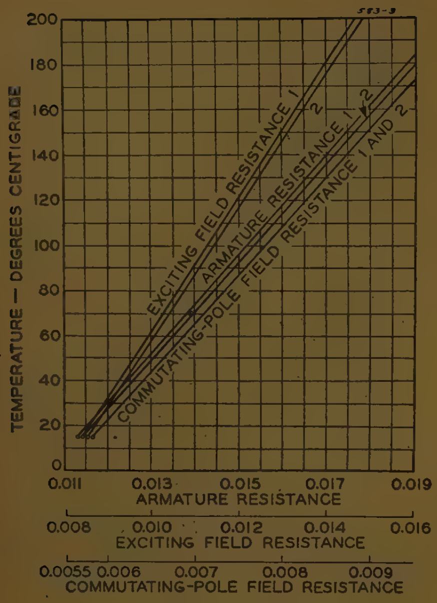


Figure 3. Typical temperature–resistance curves for motors 1 and 2

Resistance at 25 Degrees Centigrade

	Number 1	Number 2
Armature	0.01175	0.01190
Exciting field	0.00875	0.00890
Commutating pole	0.00605	0.00605

2. With a cutout switch in the event it is necessary to remove or make adjustments to the voltmeters.

A small rotary inverter operating on 32 volts direct current provides alternating current at 110 volts, 60 cycles, from 16 cells of the locomotive storage battery to operate the self-balancing bridge, the recording instrument, and the Telechron timer for the marker pens. The circuit between the storage battery and the inverter should be protected with fuses.

INSTALLATION OF THERMOCOUPLES

Thermocouples made up from copper and copnic wire serve to measure the temperatures at remote locations inaccessible while under way, such as the roof and road-bed ambient temperature, traction-motor-armature and axle-bearing, frame, and air-out temperatures. Thermocouples are connected to a selector switch which becomes the cold junction and should be equipped with a thermometer so that the measuring potentiometer connected with it can be corrected for variations in temperature of the selector switch which may fluctuate with cab temperature.

The potentiometer may be set up in shockproof mountings so that the galvanometer can be balanced, or it may be connected with the balancing element of the special resistance bridge which is unaffected by vibration. This is feasible as thermocouple readings usually are taken while under way in between test stops.

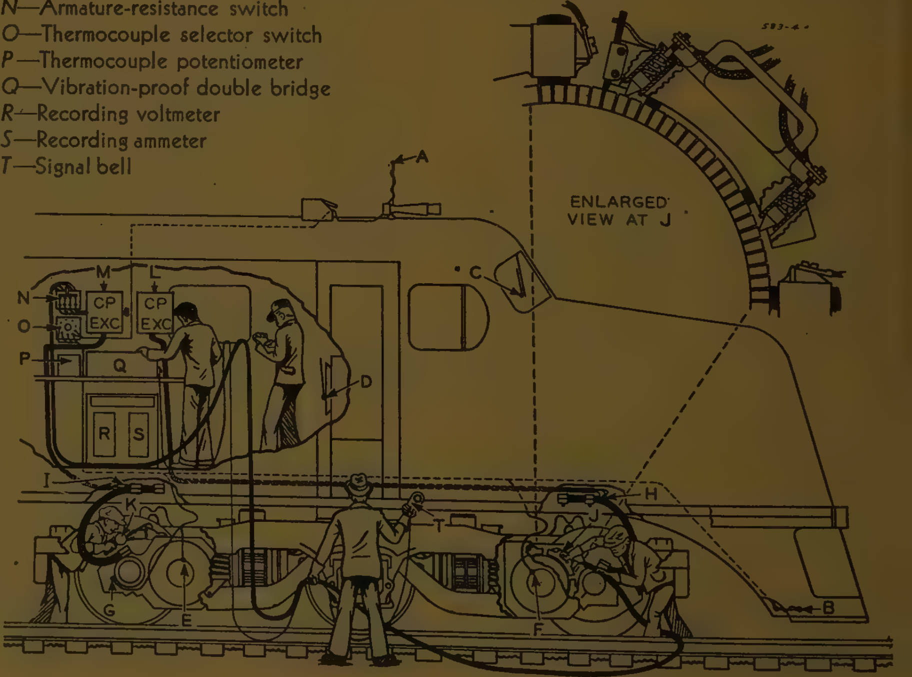
The roof-air-temperature thermocouple fastened to a length of spring wire should project above the boundary layer, but care must be taken that it stay within the clearance diagram when operating in an electrified zone to avoid contact with the high-voltage overhead system. The road-bed-temperature thermocouple is secured to a step bracket on the pilot or other structure entering air near the track level. Thermocouples reading frame or bearing temperatures must be kept out of the wind stream by protecting the junction end from its influence. They must be secured in tight contact with the member to be measured. Thermocouples for roller-bearing temperatures should be clamped between the outer race of bearing cap if opportunity permits; otherwise, a spring wire with the thermocouple junction soldered to its end may be inserted through the grease pipe to make contact with the bearing or the bearing cap metal in contact with the bearing.

MEASUREMENT OF COMMUTATOR TEMPERATURES

Commutator temperatures are measured with a contact-type hand pyrometer. This is an open-junction thermocouple instrument with inherent ambient compensated millivoltmeter reading directly in degrees centigrade.

Figure 4. Position of personnel when taking temperature measurements

A—Roof-air-temperature thermocouple
B—Road-bed-air-temperature thermocouple
C—Windshield air thermometer
D—Traction-motor blower air-in thermometer
E—Pinion-end armature-bearing-temperature thermocouple
F—Commutator-end armature-bearing-temperature thermocouple
G—Axle-bearing-temperature thermocouple
H—Number-1-motor field-resistance leads
I—Number-2-motor field-resistance leads
J—Armature-resistance yoke
K—Commutator-temperature pyrometer
L—Number-1-motor field-resistance selector switch
M—Number-2-motor field-resistance selector switch
N—Armature-resistance switch
O—Thermocouple selector switch
P—Thermocouple potentiometer
Q—Vibration-proof double bridge
R—Recording voltmeter
S—Recording ammeter
T—Signal bell



Measurement Procedure

MEASURING COLD RESISTANCE

The cold-resistance value of the motor windings is a first consideration after instruments and wiring are installed as it is basis for subsequent temperature determination. To permit all windings to cool off to a uniform temperature and to equalize with that of the entire motor may require 12 to 24 hours and will present a real problem unless the layover between runs is of sufficient duration for this purpose. Arrangement should be made to cut out the motors selected for measurement and to keep the blowers running before the train arrives at the terminal, when they are no longer required. This may gain two or three hours of cooling time. If electrically driven blowers are used, they should be kept running with the motors or power plant cut out.

As soon as the locomotive is spotted in a final position for servicing, mercury thermometers can be applied to the frames, field coils, pole tips, armature core, and commutator risers. It is well to caution against the application of putty directly on the brush surface of the commutator, as it may present some difficulty in cleaning without at the same time destroying the commutating film, thereby altering conditions for subsequent tests. Comparative ambient temperature may be obtained by placing a thermometer on the track rail head with putty over the bulb, or by placing a thermometer in a small can filled with oil so as to stabilize fluctuation in air temperature when large doors are opened in the locomotive service shop or roundhouse, or when the wind is variable when the locomotive is standing in a railroad yard. Cold-resistance measurements can be started as soon as comparative observations of thermometers show reasonable agreement. Record the date, time, temperatures, with resistance values, and check at intervals.

When armature cold resistance is being measured, the armature yoke should be applied on several alternative segment positions available, and, after the armature thermometers are removed, the locomotive may be moved sufficiently to present different sections of the commutator, the measurements on which should check reasonably with one another. If a wide variation is detected, the reason for it should be ascertained and corrected. The section showing disagreement may be explored by making bar-to-bar measurements to see if a high-resistance fault exists in the armature windings or its connection to the commutator risers. Motors will operate without any visible sign of distress on the commutator and yet may have appreciable variation in bar-to-bar resistance; although this is permissible for operation, it would not be acceptable for a temperature test, as the data obtained may be erratic if the train happens to stop with this fault in the accessible section of the commutator for the temperature measurement. It may prove a wise precaution to bar-to-bar-test the armatures of the motors before selecting them for temperature tests, as a visual inspection will only detect faults which have progressed to an advanced stage.

Cold-resistance readings again should be measured at the conclusion of all testing and checked with the initial readings to ascertain that no changes had occurred during the testing period. For this purpose, resistance values should be evaluated at 25 degrees centigrade as a common basic temperature for comparison.

TEMPERATURE–RESISTANCE CURVES

The work of reading temperatures will be facilitated greatly by drawing temperature–resistance curves for each winding in each motor being tested (Figure 3). Since this curve is a straight line within the range of temperatures under consideration, it is only required to plot the cold-resistance values at their measured temperature and calculate the value of resistance for 200 degrees centigrade as being the extreme value when hot. A straight line is drawn through these two points, and any intermediate value can be read directly from the curve.

DETERMINATION OF COOLING-AIR VOLUME

The air delivered to the motors by the blower is best measured by drilling the commutator inspection cover to admit a static pressure tube into the commutator chamber, and then reading the air pressure in inches of water on a manometer. The readings should be taken at idling, one-half and full speed of the blower, and at the same time the speed of the engine should be measured. When tests are concluded, engine and blower speed again should be measured to see that conditions remained the same during the testing period. Record date, time, speed, temperature of air at blower intake, and barometric pressure, if the data are to be used for comparative reference.

A similar procedure for electrically driven blowers should be followed, except that speed would be dependent upon blower voltage instead of engine speed as with the belt-driven blower.

The volume of air to the motors is determined from a pressure–volume curve usually available for the type of motor being tested. If one is not available, it can be prepared by blowing a similar motor through a calibrated pipe to determine the volume delivered for corresponding pressures in the commutator chamber as outlined by blower manufacturer's testing code.

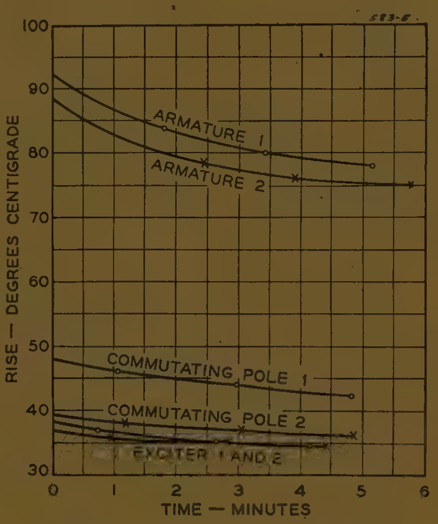


Figure 5. Typical cooling curves

Total train weight 1,224 tons (13 cars + locomotive) rms current 449 amperes for 67 minutes or total distance of 81 miles

O=motor 1 X=motor 2

Rises Based on 42 Degrees Centigrade

	Number 1		Number 2	
	Temperature	Rise	Temperature	Rise
Armature	135	93	131	89
Commutating field	89.5	47.5	81.5	39.5
Exciting field	80.0	38.0	79.0	37.0
Commutator	110	68	105	63

MAKING THE SERVICE TEMPERATURE MEASUREMENTS

Before the test run is made the locomotive should be moved from over the service pit onto a section of track similar to the main-line road bed. Assign the test crew to their stations, and outline the testing procedure and the duties for each station. The entire sequence of the test should next be rehearsed, including the making of measurements, and the entire operation timed from start to finish. This practice will prove invaluable to the men by relieving tension and instilling confidence and teamwork in preparation for the actual measurements on the main line.

The following program of events is performed in sequence (Figure 4):

1. Just before the arrival at the designated stop for heating measurements, a prearranged signal from the head end is given when power is no longer required to make the stopping point. Padlocks are removed from the switches, and all equipment is made ready.

2. As soon as power is shut off at the head end and the engines come to idling speed, a stop watch is started at the instrument station, and the electric-driven blower or the Diesel engine belted to the blower, as the case may be, is shut down completely.

3. As soon as the train comes to a stop, it is made safe (removal of reverser handle or other means to lock power off) by a man at the head end who then leaves the cab and takes a position on the road bed at the truck while two other men, one for each motor, climb under the locomotive and remove the motor inspection covers.

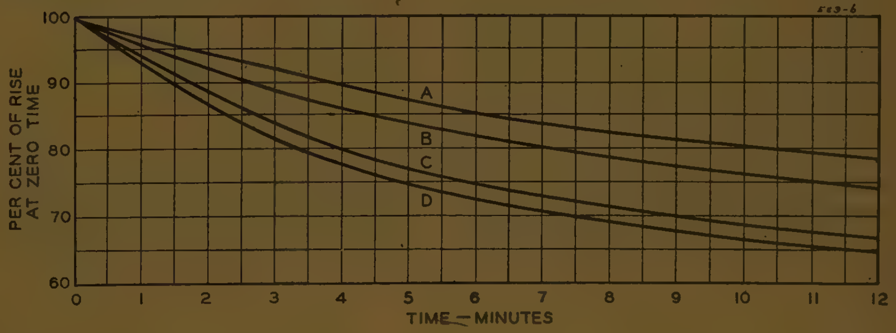
4. While step 3 is taking place, the two men at the instrument station unwind cables and pass the armature yoke, signal bell, and pyrometer down to the man on the road bed adjacent to the truck, who retains the bell and gives the resistance yoke to one man at the motors and the pyrometer to the other.

5. Resistance measurements are made in the following order by two men at the instrument station:

(*a*). Motor 1 commutating field, then exciting field, followed by the field measurements on motor 2.

Figure 6 (below). Average armature-cooling curves obtained from heat-run data

	Amperes	Rpm	
A	650	1,300	Continuous
B	675	778	Continuous
C	1,000	450	30 minutes
D	1,130	374	13 minutes



(*b*). Signal for armature contact to be made and held on armature 1 while commutator 2 temperature is measured. Signal as soon as resistance is obtained, and the man on the road bed will pass the yoke from motor 1 to motor 2 and the pyrometer from motor 2 to motor 1 and make a record of commutator temperature. As soon as contact is established on armature 2, the instrument station will acknowledge by signal to hold contact while resistance is measured, after which the concluding signal is given. Meanwhile record is made of commutator 1 temperature.

(*c*). The fields of motors 1 and 2 again are measured while the yoke and pyrometer are exchanged by the man on the track in readiness for repeating armature and commutator temperatures. At least three readings on fields and armatures of each motor should be obtained in six minutes time from the instant the power is off, which may be broken down into an average of two minutes to make the stop and get the first armature contact and four minutes for all subsequent measurements. As each resistance value is called off by the instrument operator, the other man notes the time and enters the value on a prepared data sheet.

6. As soon as measurements are concluded, the cables are pulled into the cab, motor covers are replaced, and both men climb from under the locomotive; when all is clear the shutdown engine or blower is started and the run is resumed.

7. The commutator temperatures which were recorded by the men on the track are transferred to the data sheet at the time values corresponding with the opposite motor's armature-resistance values.

8. During the run, and especially for a period of time preceding a stop for winding-temperature measurements, readings are taken and recorded in the log of air temperature on the roof and road bed, and at traction-motor blower intake. Readings also are made periodically of motor-frame, air-out, and bearing temperatures. The chart-recording instruments are inspected to see if they are inking, and calibration is checked with readings on the indicating instruments. At the head end of the locomotive, windshield temperature, speed, location on the railroad, and controller notch position are recorded with respect to time so that they may be related to the readings in the test log.

For the purpose of obtaining these readings, the test crew is reassigned to secondary duties, so that there is a division of labor, but co-ordination of all data at one point is insured. The make-up of the train in tons and number of cars for each test run should be recorded with the train identification and direction of travel.

Determination of Operating Temperature

THE COOLING CURVE

The resistance value can be read off directly as temperature from the resistance-temperature curve and by subtracting the motor-blower air-in temperature the rise of the winding is obtained, corresponding with the time of measurement. These values of rise are plotted with respect to time, and a smooth exponential curve is drawn through related points to zero time, and the intercept with the temperature axis is the most probable temperature rise of the corresponding winding at the instant of power removal (Figure 5). The importance of getting the first readings as soon as possible after power off becomes evident for it minimizes the extent of extrapolation of the curve.

The cooling-curve technique also is applied to the hand-pyrometer commutator-temperature readings to arrive at the probable rise just before power off.

THE USE OF AVERAGE COOLING CURVES

Occasionally, circumstances will prevent immediate measurement of armature resistance and a prolonged cooling period will result before the first reading is obtained. The extrapolation of these delayed data to zero time is aided by the use of average armature-cooling curves (Figure 6). These curves are prepared from factory test data on the same type of motor for several different conditions of power input and are expressed in terms of per-cent temperature rise with respect to time. Generally, heavy loads of short duration have steeper cooling-curve shapes than continuous runs. This is caused by the steep temperature gradient between the copper and the iron core which lags behind during the short heating-up period, whereas during a continuous run their temperature becomes more nearly equalized.

The proper cooling shape is selected by evaluating the power input for the period preceding the shutdown in service and identifying it with a corresponding curve shape, or by interpolating between the two curves coming closest. From this curve is read, for the elapsed cooling time at which the first service reading was taken, the percentage value of the actual measured temperature rise, from which is prorated the 100 per cent value at zero time.

DETERMINATION OF RMS CURRENT

The rms current value is obtained from the chart made by the recording ammeter by dividing the graph into intervals of time, and the average current value for each interval is squared and multiplied by that time interval. The sum of these products is divided by the total time, to get the average current squared for the run. The square root of this value is the rms current value for the test.

ADAPTATION OF METHOD TO VARYING CONDITIONS

It is not always practical to obtain the full desired train tonnage without delaying the test, or during the test run it may be desired to check performance with one power plant hypothetically broken down. These situations can be approximated by purposely reducing the locomotive power by multiples of the number of motors until the desired prorated train tons per motor is obtained. The temperature measurements then are made under this artificial condition. For example, if two out of eight motors are cut out of a locomotive in an 18-car test train having a combined weight of 1,680 tons, the load per motor will be increased from 210 to 280 tons. The heating of the motors then would be equivalent to that of a 25-car train of 2,240 tons total weight.

The method, procedure, and equipment also are applicable to measurements of generator temperature which can be made in a multiple-unit locomotive by shutting down the one power plant for a few minutes to obtain measurements without the necessity of stopping the train.

The recording-chart instrument with calibrated instruments affords data on engine horse-power output by reference to a generator characteristic for the generator efficiency values over the operating range from which the horsepower input to the generator can be calculated.

Conclusion

Temperature of locomotive electric apparatus is more than a question for academic consideration; it is an active economic factor, never at rest and continually exerting its influence with time on the life of the equipment. The penalties of rash application or abusive operation are inescapable.

The effect of elevated temperatures on organic insulation materials follows the approximate rule of half the life for each 8–12-degree-centigrade increase in temperature.[2] The indisputable and direct approach to correct utilization or application of motive power is with a full knowledge of electric-apparatus temperature. The testing facilities and method outlined provide these necessary data for railroad operating and motive-power departments, as well as engineering information of application and design value. It is a means whereby a railroad can take the measure of its tough spots and with which the engineer can correct his shots.

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2. REPORT ON GENERAL PRINCIPLES FOR RATING OF ELECTRICAL APPARATUS FOR SHORT-TIME, INTERMITTENT, OR VARYING DUTY, AIEE Standard 1*A*, September 1941.

TRANSACTIONS SECTION

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Synopsis: With the selection of the 208Y/120-volt three-phase 400-cycle a-c system by the Army Air Forces for large aircraft, there has arisen the need for appropriate switchgear. In order to be applicable to the electric systems of such planes, switchgear must be able to perform its normal and emergency functions under the encountered conditions of temperature, pressure, humidity, vibration, and acceleration. This paper describes a 120-ampere remotely controlled electrically operated air circuit breaker designed for use as a bus sectionalizing, ring tie, or generator breaker on the 400-cycle grounded-neutral system. The interrupting rating of this breaker is 3,800 amperes at all pressures from sea level to 50,000 feet altitude. The closing solenoid and shunt trip coils have been designed to operate over a wide range of ambient temperature and control voltage. Weight and size, being primary considerations in aircraft equipment, have been held to a minimum.

TO MEET the demands of increased electric loads in aircraft, the Army Air Forces has selected the 208Y/120-volt three-phase 400-cycle grounded-neutral a-c system as applicable to large aircraft.[1] Supplementing this system will be found a 24-volt d-c source for control power and some small loads. There are many reasons why an a-c system of this voltage and frequency has been selected for large aircraft. However, it is beyond the scope of this paper to treat them. A complete study of the factors entering into the selection of electric systems for aircraft has been presented in another paper.[2]

Heretofore, only small amounts of 400-cycle power, by comparison, have been used on aircraft. Equipment for the generation, distribution, and utilization of large amounts of 400-cycle power has been recently developed or may be still undergoing development. The weight and size of this equipment, it is specified, must be held to a minimum. Circuit breakers must be able to perform their normal and emergency functions under these conditions:

1. Altitude—from sea level to 50,000 feet, or a pressure range of from 14.7 to 1.68 pounds per square inch.
2. Temperature — ambient - temperature range from 60 to −70 degrees centigrade at sea level and from −40 to −65 degrees centigrade at 50,000 feet altitude.
3. Humidity—a relative humidity of from 0.5 to 100 per cent.
4. Vibration and acceleration—an amplitude of 0.030 inch at all frequencies from 10 to 50 cycles per second.

An air circuit breaker, custom built for aircraft to operate under these conditions, has been developed for the 400-cycle system. Figure 1 shows several views of this breaker designed for use as a bus sectionalizing, ring tie, or generator breaker. The weight of this breaker is five pounds and the over-all size is $7^1/_4$ inches high, $4^1/_8$ inches wide, and $4^{17}/_{32}$ inches deep. All parts are enclosed within a dust-resisting molded phenolic case on the back of which are the line, load, auxiliary-switch, and control-circuit terminals. The breaker may be mounted with these terminals either toward or away from the panel. In either position, the terminals are covered with insulating barriers.

Throughout the breaker, parts have been plated in accordance with accepted aircraft standards to prevent corrosion. Precautions have been taken to prevent moisture absorption and fungus growth by applying protective finishes to nonmetallic parts.

In the development of a circuit breaker, it is the requirements of the interrupter in terms of pressure, wipe, separation, and acceleration of the contact tips which determine the design of the operating mechanism and in turn the design of any closing and tripping devices. The interrupter, therefore, will be described first.

The Interrupter

It is required that the interrupter and contacts of this breaker be able to do the following:

1. Withstand vibration and acceleration as specified without physical damage or excessive heating.
2. Close and interrupt maximum available short-circuit current in the 208Y/120-volt three-phase grounded-neutral 400-cycle system at any altitude from sea level to 50,000 feet.
3. Carry 120 amperes continuously, 165 amperes for five minutes, and 245 amperes for five seconds without excessive temperature rise.

Before the interrupter and contact structure for a circuit breaker can be designed there are two dependent variables to be evaluated, namely, the type of contact material and the arc-quencher construction to be used.

Table I. Data for Representative Three-Phase Interrupting Tests Made at 230Y/133 Volts, 420 Cycles

Breaker Current, Rms Amperes	Maximum Arc Duration in Milliseconds at Pressures Equivalent to the Indicated Altitudes		
	1,000 Ft	20,000 Ft	50,000 Ft
80	3.7	2.5	1.2
120	2.5	1.2	1.2
250	3.7	2.5	1.2
310	5.0	2.5	3.7
750	2.5	2.5	2.5
1,700	2.5	2.5	2.5
2,800	3.7	3.7	3.7
3,800	6.2	5.0	5.0

CONTACT MATERIAL

The contact material has a direct bearing on arc duration and the operating temperature, welding current, and deterioration of the contacts. The influence of the contact material on arc duration is determined by:

1. Melting temperature of contact material.
2. Ionization potentials of the metal vapors derived from contact-material decomposition.

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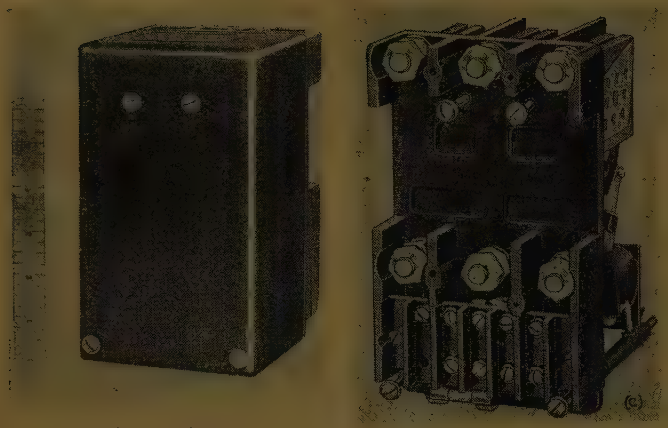
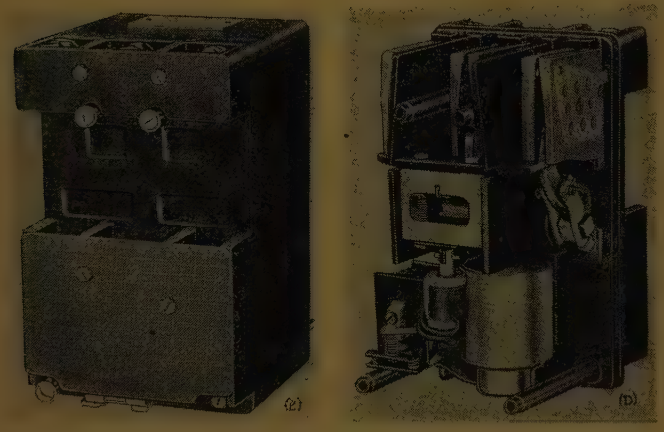


A—Front right view

C—Rear left view without covers showing load, line, and control-circuit terminals



B—Rear left view

D—Front right view without covers

Figure 1. A 208Y/120-volt 120-ampere three-pole 400-cycle electrically operated aircraft circuit breaker

3. Instantaneous temperature of contacts at time of parting.

The following factors determine the contact operating temperature:

1. Operating temperature of the member to which the contact itself is attached.
2. Resistivity of the contact material.

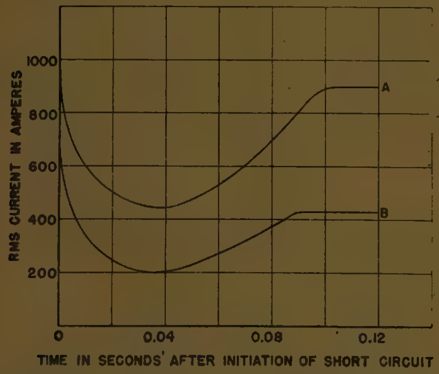


Figure 2. Short-circuit characteristics of 40-kva 208Y/120-volt three-phase 400-cycle aircraft alternator

Curves represent envelope of 400-cycle symmetrical current. First peaks may be increased because of offset. Voltage regulator operating in exciter field

A—Line-to-ground short circuit
B—Three-phase short circuit

3. Resistance of the point of contact of the movable and stationary members. This is a function of the contact pressure and the formation of chemical compounds on the surface of the contacts.
4. Composition and mass of material surrounding the contact structure. It is this material which determines the amount of heat energy removable from the contacts at any given operating temperature.

The instantaneous value of current at which a given set of contacts will weld is determined by:

1. Physical properties of contact material, namely, composition, melting temperature, thermal conductivity, and hardness.
2. Contact temperature, that is the transient temperature attained for any given value of current.
3. The instantaneous pressure present for any given operating current.

The following determine the deterioration of the contacts when interrupting both normal and short-circuit currents:

1. Magnitudes of the arc current.
2. Arc duration.
3. Melting temperature of contact material.
4. Hardness of the contact material.
5. Presence of gas-producing substances included in contact composition.

In order to determine the type of contact material which best meets the requirements of low operating temperature, high welding current, short arc duration, and minimum contact deterioration, approximately 1,100 tests were made using 14 types of contact material. These tests were made at ten per cent above the rated voltage and at pressures equivalent to altitudes ranging from 1,000 to 50,000 feet using 16 different types of arc quenchers. The type of contact material selected as best fulfilling these requirements is a sintered mixture of silver and tungsten.

ARC-QUENCHER DESIGN

The design of the arc quencher has a direct bearing upon the magnitude of voltage and current which can be successfully interrupted, the arc duration for any combination of voltage and current, and the atmospheric conditions of humidity, temperature, and pressure under which the interrupter will function satisfactorily. The type of mechanism needed to operate the contacts and the size and weight of the arc quencher are dependent upon its design.

Since the neutral of the 208Y/120-volt three-phase 400-cycle system is solidly grounded, it is unlikely that a sustained voltage of more than ten per cent in excess of 120 volts will appear across a given breaker pole. Consequently, 132 volts will be considered the rms operating voltage of each arc quencher.

The system is supplied by four 40-kva 0.75-power-factor 400-cycle alternators. The short-circuit characteristics of the alternator, exciter, and voltage regulator combined are shown in Figure 2. Inspection of Figures 2 and 3 shows that the maximum fault current results from a single-phase line-to-ground fault. From Figure 3, it is evident that with any possible location of such a fault, the maximum short-circuit current which a single pole of any breaker may be called upon to interrupt is that contributed by three generators and the connected motors. Studies of the proposed system,

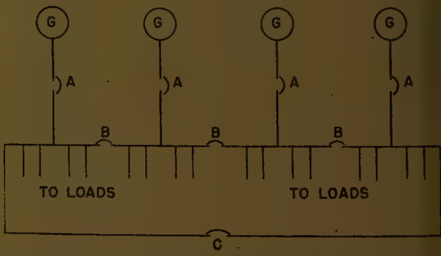


Figure 3. Proposed power circuit for 400-cycle aircraft system, using remotely controlled electrically operated circuit breakers

G—208Y/120-volt 40-kva 0.75-power-factor 400-cycle alternator
A—Generator breaker
B—Bus-sectionalizing breaker
C—Ring-tie breaker

which take into account circuit impedances, indicate that the interrupting rating of the breaker, 3,800 amperes, will not be exceeded.

A test circuit was set up as shown in Figure 4 for determining, at first, the best type of arc quencher, and later, the over-all performance of the complete breaker under single-phase and three-phase fault conditions. Test currents ranging from 3 amperes to 4,000 amperes for both single- and three-phase operation were obtainable on a 230Y/133-volt 420-cycle system with a system power factor of approximately 15 per cent. All interrupting tests were made in a chamber capable of being evacuated to an equivalent pressure less than that encountered at 50,000 feet altitude.

Because the arcing time of most conventional 120-volt air circuit breakers increases materially at the decreased atmospheric pressures encountered at high altitude, it seemed necessary at first to use a compression-type interrupter. An interrupter of this type is capable of generating its own internal pressure. However, interrupting tests indicated that over the operating range of currents and pressures of this breaker, a converging-slot-type arc quencher performs as well as the compression-type interrupter. The advantages of the lightweight, mechanically simple, converging-slot-type arc quencher over the heavier, more complex, compression-type interrupter made it ideally suited to this type of aircraft application.

Two views of this converging-slot-type arc quencher with the lower portion of one side removed are shown in Figure 5. A front right view of the breaker is shown in Figure 6 with the cover, one phase barrier, and half of the right-pole arc quencher removed. The movable and stationary contact members together with the magnet frame are so arranged that the arc stream and its terminals are driven toward the top of the breaker as it is shown in Figure 6. The contour of the inside of the arc quencher has been made such that unnecessary extension of the arc is avoided to obtain a maximum amount of cooling in a minimum amount of time. The deterioration of the arc quencher and the contacts is thereby minimized during interruption.

Table I is a tabulation of data of representative three-phase interrupting tests made at 230Y/133 volts, 420 cycles, and pressures equivalent to altitudes from 1,000 to 50,000 feet. The maximum arc duration in milliseconds is shown for various currents and altitudes. Oscillograms showing typical breaker operation when interrupting three-phase 420-cycle short-circuit currents of approximately 2,900 amperes and 3,600 amperes at corresponding pressures equivalent to 50,000 feet and 20,000 feet altitude are shown in Figures 7 and 8. Continuous-current-carrying tests made at 120 amperes and short-time overload tests made at 165 amperes and 245 amperes indicate that the thermal capacity of the breaker is adequate for use on this system.

Operating Mechanism

A latching-type solenoid-operated toggle mechanism is used to close the hinged contacts against a heavy opening spring. The contacts are opened by energizing the shunt trip coil, the plunger of which releases the latch and permits the opening spring to pull the contacts rapidly to the open position. It is necessary that the contacts be parted as quickly as possible to minimize the over-all interrupting time.

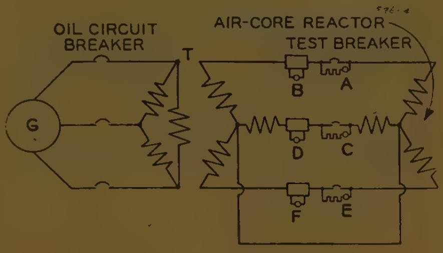


Figure 4. Test circuit used for interrupting tests

Letters *A–F* indicate magnetic oscillograph vibrators:

A—Voltage across pole 1
B—Current, pole 1
C—Voltage across pole 2
D—Current, pole 2
E—Voltage across pole 3
F—Current, pole 3
G—Alternator, 3,000 kva, 2,400 volts, three phase, 420 cycles
T—Transformer, 600 kva, 2,400 delta–208Y volts (60-cycle rating)

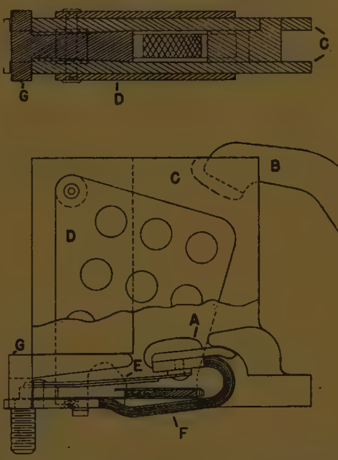


Figure 5. Top and side views of converging-slot-type arc quencher for 208Y/120-volt 120-ampere three-pole 400-cycle aircraft circuit breaker

Lower portion of one side removed

A—Stationary contact
B—Movable contact
C—Arc chute
D—Magnet frame
E—Leaf spring
F—Flexible braid
G—Shield

No means have been provided for closing or opening the breaker manually, nor have any overload tripping devices been incorporated in the breaker.

In Figure 9 is shown the relative size of the subassembly of the shunt trip device and the closing solenoid. Because of the design of these two devices and the fact that the coils have only a short-time rating, the work output per unit of weight is high for both of them. The ratio of the

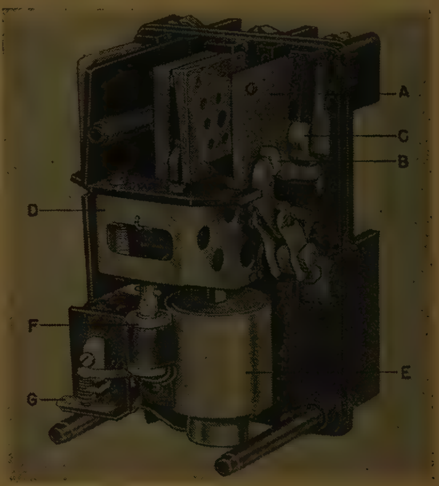


Figure 6. Front right view of breaker

One phase barrier and half of right-pole arc quencher removed showing the contact and arc-quencher construction

A—Arc quencher
B—Movable contact
C—Stationary contact
D—Mechanism
E—Closing solenoid
F—Shunt trip
G—Cutoff switch for closing solenoid

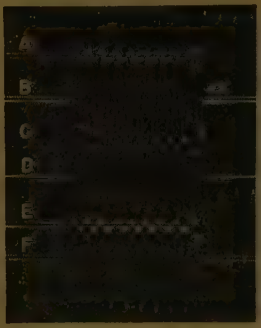


Figure 7. Magnetic oscillogram of a three-phase interrupting test at 420 cycles and a pressure equivalent to 50,000 feet altitude

A—Voltage across pole 1, 133 volts
B—Current, pole 1, 2,860 rms amperes
C—Voltage across pole 2, 133 volts
D—Current, pole 2, 3,230 rms amperes
E—Voltage across pole 3, 133 volts
F—Current, pole 3, 2,750 rms amperes

product of the force at the beginning of the stroke and the length of the stroke to the weight is 3.1 inch-pounds per pound for the closing solenoid and 3.3 inch-pounds per pound for the shunt trip device. This same ratio for a continuously rated commercial solenoid is 0.34 inch-pounds per pound. Weighing only 19 ounces, the closing solenoid develops more than 45 pounds at the point near the end of its stroke where the overtravel of the plunger begins. The shunt trip device weighs four ounces.

The pull curve of the closing solenoid has been so shaped that even at as low a voltage as 18 volts, measured at the terminals of the battery, with 0.25 ohm in series with the coil to represent line resistance, the breaker closing time is 0.027 second. Higher closing speed could be obtained at this low closing voltage only at the expense of increased weight of the closing solenoid. The minimum tripping voltage is 12 volts measured at the terminals of the battery with 0.25 ohm resistance in series with the shunt trip coil.

Control Circuit

A cutoff switch connected in series with the coil of the closing solenoid prevents it from overheating after the closing impulse has been delivered to the mechanism. This switch is actuated by the solenoid plunger in the overtravel portion of its stroke after the mechanism has been fully closed. As long as the closing circuit is energized after the breaker has been closed, the cutoff switch, shown in Figure 10, is maintained in the open position by a holding coil. As this holding coil is continuously rated, it is possible to use either a momentary- or a maintaining-contact switch to control the closing of the breaker. When using the latter type of switch, the closing circuit must be de-energized after tripping and before reclosing the breaker. With this control scheme, it is impossible for the breaker to pump should it be closed against a fault with the closing circuit energized.

Included as an integral part of the breaker is a four-stage auxiliary switch. Each stage or set of contacts of this switch is a small snap-action vibration-proof contact mechanism. One stage is used as the auxiliary contact in series with the shunt trip to open its coil circuit the instant the breaker primary contacts open. The other three sets of contacts may be arranged individually to be either normally open or normally closed and used with external relays for breaker control. The terminals of these contacts are brought out to the back of the breaker with the close, trip, and ground terminals as shown in Figure 1C.

The three control-circuit coils have a tropical treatment. Polyvinyl-chloride-insulated wire is used for all internal control-circuit connections to guard against corrosion.

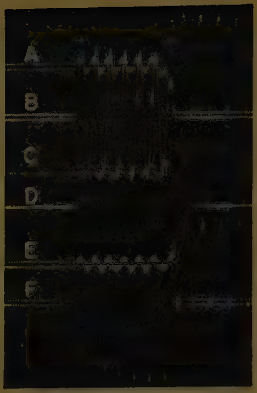


Figure 8. Magnetic oscillogram of a three-phase interrupting test at 420 cycles and a pressure equivalent to 20,000 feet altitude

A—Voltage across pole 1, 133 volts
B—Current, pole 1, 3,510 rms amperes
C—Voltage across pole 2, 133 volts
D—Current, pole 2, 3,940 rms amperes
E—Voltage across pole 3, 133 volts
F—Current, pole 3, 3,420 rms amperes

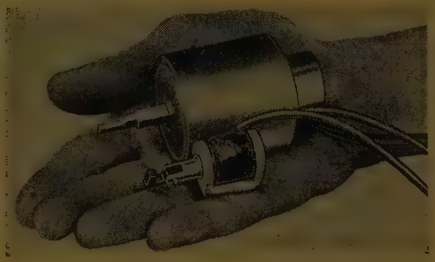


Figure 9. Shunt-trip and closing-solenoid subassembly

Operating Characteristics

High tripping speed of the breaker is very important in that it helps materially to reduce the over-all interrupting time. High closing speed is desirable particularly when automatic means of synchronizing are used on the system.

Oscillographic timing tests have been made in a 25-degree-centigrade ambient temperature to determine the speed of closing over a range of control voltage of from 18 to 28 volts and the speed of tripping over a range of from 12 to 28 volts. The operation of the breaker has been found to be satisfactory over these same voltage ranges while being subjected to vibration with an amplitude of 0.030 inch at all frequencies from 10 to 50 cycles per second. For all operation tests, both closing and tripping, a 0.25-ohm resistor was connected in series with the ground terminal of the breaker control circuit to simulate line resistance in the plane.

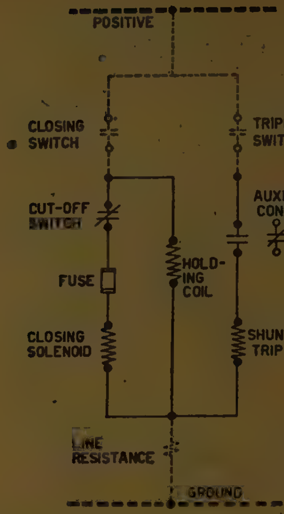


Figure 10. Control-circuit diagram, 208Y/120-volt 120-ampere three-pole 400-cycle electrically operated aircraft circuit breaker

Nominal d-c control voltage 24 volts

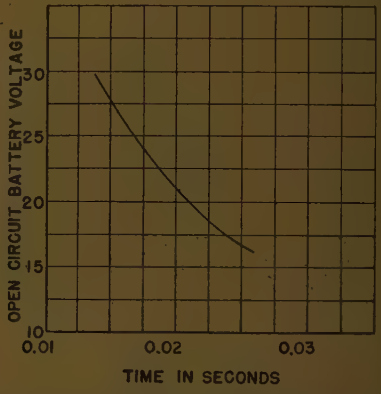


Figure 11. Closing characteristic, 208Y/120-volt 120-ampere three-pole 400-cycle electrically operated aircraft circuit breaker

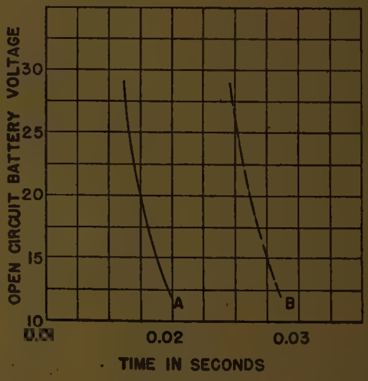


Figure 12. Opening characteristics, 208Y/ 120-volt 120-ampere three-pole 400-cycle electrically operated aircraft circuit breaker

A—Breaker primary contacts part
B—Breaker primary contacts fully open

Although the resistance of the coils increases somewhat when the breaker is placed in an ambient temperature of 60 degrees centigrade, the breaker closes satisfactorily at 18 volts and trips at 12 volts, both voltages being the open-circuit control voltage.

The closing characteristics, time from the instant the closing circuit is energized until the breaker primary contacts meet versus the open-circuit control voltage,

Oil Stresses in Impregnated-Paper Insulation

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RECENT EXPERIMENT on fully impregnated cable-type paper insulation, has shown that under certain conditions use of a paper of low density results in a higher dielectric strength than does the use of a paper of higher density.[1] It has also been shown that failure of such insulation usually begins in the oil channels between successive turns of paper tape.

The former result is in apparent conflict with general experience that the denser a given substance or medium, the higher the dielectric strength. But the conflict is apparent only, for impregnated paper is not a homogeneous substance, and failure begins first in its weaker constituent, namely, the oil. High-density paper with its higher dielectric constant throws a higher stress into the oil channels and the critical or breakdown stress is reached at a lower applied voltage than when low-density paper of lower dielectric constant is used.

The following approximate analysis indicates that under certain conditions in a single-conductor cable of circular cross section, the use of high-density paper next the conductor and low-density paper outside should result in a higher over-all dielectric strength than that of the same cable insulated throughout with either low- or high-density paper. The higher specific inductive capacity of the high-density paper shifts a greater proportion of the over-all voltage to the outer less dense paper thereby reducing the stress at the conductor surface. This reduction under certain conditions may cause the critical oil-channel stress to fall below that for the same cable insulated throughout with low-density or high-density paper.

Over-All Average Stress at Conductor Surface

Let a, b, and c be the radii of the conductor and of the outer surfaces of the denser and less dense papers respectively; K_{ab} and K_{bc} the measured dielectric constants of the two impregnated papers when assembled separately as cable insulation, and V_{ab} and V_{bc} the division of the total voltage, V_o, between them. Then we have:

$$V_{ab}+V_{bc}=V_o \qquad (1)$$

$$\frac{V_{ab}}{V_{bc}}=\frac{K_{bc}}{K_{ab}}\cdot\frac{\log(b/a)}{\log(c/b)} \qquad (2)$$

$$V_{ab}=\frac{K_{bc}}{K_{ab}}\cdot\frac{\log(b/a)}{\log(c/b)}(V_o-V_{ab}) \qquad (3)$$

$$=\frac{A}{1+A}\cdot V_o;\quad A=\frac{K_{bc}\log(b/a)}{K_{ab}\log(c/b)} \qquad (4)$$

Also, let E_a and E_b be the mean values of the radial stresses at the surface of the conductor, and at the outer surface of the inner denser paper respectively; then we have

$$\left.\begin{aligned}E_a&=\frac{V_{ab}}{a\log(b/a)}\\ E_b&=\frac{V_{bc}}{b\log(c/b)}\end{aligned}\right\} \qquad (5)$$

and from equation 2

$$\frac{E_a}{E_b}=\frac{V_{ab}}{V_{bc}}\cdot\frac{b}{a}\cdot\frac{\log(c/b)}{\log(b/a)} \qquad (6)$$

$$=\frac{K_{bc}}{K_{ab}}\cdot\frac{b}{a} \qquad (7)$$

Also

$$E_a=\frac{V_o\cdot A}{(1+A)a\log(b/a)}$$

$$E_a=\frac{V_o}{a\left[\frac{K_{ab}}{K_{bc}}\cdot\log(c/b)+\log(b/a)\right]} \qquad (8)$$

In equation 8 E_a is a mean or average value of the electric stress at the conductor surface. Its value is readily determined from measured values of V_o and of the dimensions a, b, and c, and of the specific inductive capacity of each of the papers when assembled separately as cable-type insulation. For an insight as to how the value of E_a varies as the proportion of dense to lighter papers varies, we may consider two papers each five mils thick; paper A (specific gravity 0.74; $K = 3.78$) and paper D (specific gravity 1.18; $K = 4.91$). Also in order to limit the number of variables, let us consider the particular case in which the total thickness of insulation is equal to the diameter $2a$ of the central conductor: thus $c = 3a$; in Figure 1 the curve E_a shows how E_a varies for all values of b. We note that for $b = a$ (all A paper) and for $b = 3a$ (all D paper), the values of E_a are the same and at their greatest. Introduction of any small amount of D paper inside of

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are shown graphically in Figure 11. At the nominal control voltage, 24 volts, the closing time is 0.02 second.

The opening characteristics, time from the instant the shunt trip is energized until the breaker primary contacts part and until they are fully open versus the open-circuit control voltage, are shown in Figure 12. The time from the parting of the contacts until they reach the fully open position, 8.5 milliseconds, is independent of the control voltage. The arcing time at pressures equivalent to altitudes up to 50,000 feet and currents up to 3,800 amperes rms, from Table I, never exceeds 6.2 milliseconds. The right-hand curve of Figure 12, contacts fully open, shows the interrupting time of the breaker versus control voltage at 50,000 feet altitude and rated interrupting current.

Conclusions

Designed primarily for the 208Y/120-volt 400-cycle a-c system selected by the Army Air Forces for large aircraft, this breaker in its application is the counterpart of industrial air circuit breakers.

The outstanding features of this breaker are:

1. Satisfactory and reliable operation under extreme conditions of temperature, pressure, humidity, vibration, and acceleration normal to aircraft.
2. Adequate interrupting capacity for the proposed 400-cycle aircraft system.
3. Short closing and tripping times over a wide range of control voltage.
4. Light weight and small space requirements.
5. Protection of parts against fungus growth and corrosion.

Future increases in aircraft electric loads will undoubtedly call for more extensive systems using breakers of higher continuous and short-circuit ratings. The experience gained in the design of this breaker indicates that these demands can be met without sacrificing any of the present features.

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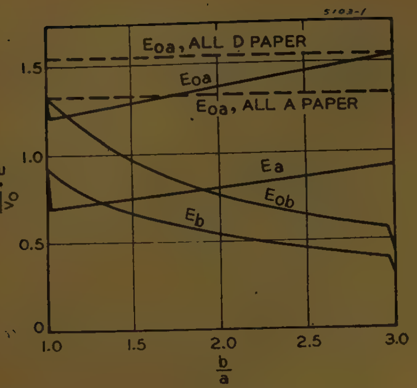

Figure 1. Stresses in a graded cable, denser paper inside, range of b/a from 1 to 3

Paper *D*, inside, specific gravity 1.18
Paper *A*, outside, specific gravity 0.74
25 per cent overlay, $n=4$
E_a—Average stress at conductor surface
E_{oa}—Stress in oil channel of paper *D* at conductor surface
E_{ob}—Stress in oil channel of inside layer of paper *A*
E_b—Average stress at surface of separation of the two papers

A paper, that is, for values of b slightly above a, E_a is sharply reduced and then increases approximately linearly to its maximum at $b=c=3a$.

Similarly, the average stress at the surface of separation of the two papers (equation 7) is plotted as curve E_b. Its maximum value occurs at $b=a$ (all *A* paper) and is the same as the maximum value of E_a.

Figure 2 shows similar variations of the several stresses for values of b/a of from 1 to 2, that is for insulation wall thickness equal to the radius of the conductor.

Stress in Oil Channels

So far E_a and E_b are independent of the thickness and overlay of the paper tapes, except as these affect the measured values of K_{ab} and K_{bc}.

But we have taken the stress in an oil channel at the conductor as a measure of dielectric strength, and so it remains to determine the value of this stress, E_{oa}, in terms of some measurable quantity, for example, E_a. Efforts to derive the completely accurate value for E_{oa} lead to difficult and cumbersome expressions. We can proceed however, much more easily and with perhaps equal accuracy if we make two assumptions:

(*a*). That the pattern of overlay of tapes repeats itself regularly through the insulation wall.

(*b*). That the electric stress in the central region of the oil channel and immediately overlaying paper tapes is radial and uniform.

Under the first assumption the insulation wall may be considered as made up of a number of coaxial cylindrical capacitors, each having the same radial thickness, the same number of paper layers, the same overlay of tapes, and the same width of oil channel; also, for any one type of paper the dielectric of each elementary capacitor will have the same value of specific inductive capacity; or in other words K_{ab} and K_{bc} in the respective papers would have the same value if measured over any one or more of the characteristic elementary capacitors.

Assumption *b* is warranted by the fact that the oil channel is usually from 10 to 20 times as wide as it is thick (0.005 inch). We fix our attention on the central part of the channel where the stress is very nearly uniform and at its maximum value.

With all dimensions of the single-conductor cable and the applied voltage given, these assumptions enable us to compute, to a close approximation the value of the voltage, V, across the innermost characteristic capacitor, and further to derive the value of the stress in the innermost critical oil channel in terms of the average surface stress E_a of equation 8, as follows:

The characteristic capacitor will have a number of layers, n, depending on the overlay, and any oil channel adjacent to the conductor will be in series radially with $n-1$ paper tapes. Let us consider only that elementary portion of the innermost capacitor for which an oil channel is next to the surface of the conductor. Then, if we make the not too violent assumption that the stress in the central region of the channel is radial, we may compute the stress in the oil channel from the relations in a cylindrical capacitor with an inside layer of oil, and above it $n-1$ layers of impregnated paper.

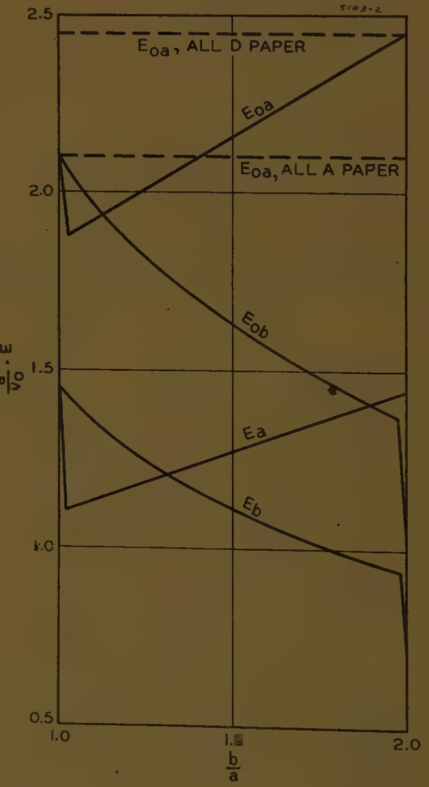

Figure 2. Stresses in a graded cable, denser paper inside, range of b/a from 1 to 2

See subcaption of Figure 1

If a is the radius of the conductor, and $a_1, a_2 \ldots a_n$ the radii of the successive paper layers, and V is the voltage over the innermost capacitor, and K_o and K_p the dielectric constants of oil and impregnated papers respectively, then remembering the assumption *b* and formula 4, we find V_{oa}, the voltage over the oil channel,

$$V_{oa}=\frac{BV}{B+1}$$

where

$$B=\frac{K_p \log (a_1/a)}{K_o \log (a_n/a_1)} \tag{9}$$

and the stress E_{oa}, in the oil channel and at the surface of the conductor,

$$E_{oa}=\frac{V_{oa}}{a \log (a_1/a)}$$

$$=\frac{V}{a\left[\log (a_1/a)+\frac{K_o}{K_p}\log (a_n/a_1)\right]} \tag{10}$$

Also since the innermost capacitor in each paper is typical of all:

$$E_a=\frac{V}{a \log (a_n/a)}=\frac{V}{a \log\left(\frac{a+nt}{a}\right)} \tag{11}$$

where t is the thickness of the paper tape.

Whence

$$E_{oa}=\frac{E_a \log\left(\frac{a+nt}{a}\right)}{\log (a_1/a)+\frac{K_o}{K_p}\log (a_n/a_1)}=GE_a \tag{12}$$

$$G=\frac{\log\left(\frac{a+nt}{a}\right)}{\log (a_1/a)+\frac{K_o}{K_p}\log (a_n/a_1)} \tag{13}$$

the stress in an oil channel at the surface of the conductor.

For a given conductor, paper, and overlay in assembly G is a constant; when two papers are used, G has two values, one for each paper as determined by the two values of K_p, the specific inductive capacity of the impregnated paper. The values of G will always be greater than the average stress E_a at the conductor surface, if as is commonly the case K_p is greater than K_o.

In Table I values of G are given for several values of a and n, within the range of cable practice, and for each of the papers *A* and *D*. As will be seen, the values of G are practically independent of the conductor radius within the range 0.125 inch to 0.500 inch but increase with increase in n, the numbers of layers of paper in the "characteristic capacitor", or pattern of assembly. The greatest

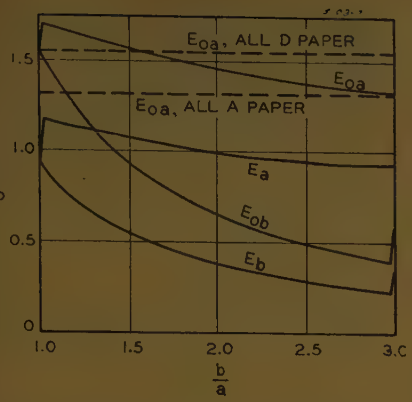

Figure 3. Stresses in a graded cable, denser paper outside, range of b/a from 1 to 3

Paper *A* inside, paper *D* outside; see subcaption of Figure 1

value of G, within the range examined, is 1.70, D paper with $n = 4$ (25 per cent overlay), and the smallest 1.26, A paper with $n = 2$ (50 per cent overlay). These then are the factors by which the values of E_a and E_b plotted in Figure 1 are to be multiplied to give the stresses in the oil channels of the innermost layers of the inside denser paper and the outside less dense paper respectively. The resulting curves E_{oa} and E_{ob} have been plotted in Figure 1; also, in horizontal dotted lines, for the purposes of comparison, the values of the stresses in the oil channels at the conductor surface when the insulation is entirely of D paper and of A paper respectively.

We note from Figure 1 that in the special case there considered ($n = 4$, $b/a = 1$ to 3), for $b/a = 1$ (all A paper) and for $b/a = 3$ (all D paper) the average stress E_a has the same value as is well understood. However, in both cases the maximum oil stress is higher than E_a—the average maximum stress, 1.7 times for the denser paper D and 1.46 times for the less dense paper A (see Table I); the resulting difference is indicated by the horizontal dotted lines at the top of the figure.

This confirms qualitatively, and in terms of simple physical principles, the experimental result found by the author that in cable-type fully impregnated paper insulation a denser paper has a lower dielectric strength than a less dense paper.

When both types of paper are used in the same insulation wall, and, as we have assumed, the denser paper is on the inside, we see at once that for values of b/a between 1 and 1.75 the maximum oil stress E_{oa} in the denser paper is lower than the maximum oil stress in the case when the entire wall consists of the less dense paper. Thus properly proportioned graded insulation may have a lower maximum oil stress and a higher dielectric strength than either of its constituents when used singly for the same wall thickness of insulation.

Table I. Values of G for papers of two values of density (*D* and *A*)

Radius of Conductor, a, Inch	n=2		n=3		n=4	
	D	A	D	A	D	A
0.126	1.37	1.26	1.65	1.37	1.68	1.45
0.25	1.37	1.26	1.66	1.37	1.70	1.46
0.375	1.40	1.29	1.62	1.41	1.67	1.43
0.500	1.38	1.28	1.61	1.37	1.70	1.45

The stress E_{ob} in the inside oil channels of the outer less dense paper is always less than that in the inside channels of either paper used alone. However, in the graded case we note from Figure 1 that for values of b/a between 1 and 1.12, E_{ob} rises above E_{oa}, that is to say, breakdown is more likely to occur in the inside layers of the outer less dense paper.

Figure 2 shows that similar relations with only small variations in values occur when the total insulation wall thickness is one-half the diameter of the conductor, that is, the range of b/a is from 1 to 2.

It is of interest to note the relations which arise when in the graded cable the positions of the two grades of paper are reversed, that is, when the low-density paper is placed on the inside, adjacent to the conductor. These relations are shown in Figure 3 for a variation of b/a from 1 to 3, and so may be compared with those of Figure 1. As seen, for all values of b/a, that is, for any radial depth of the less dense paper, the stress in the oil channels next to the conductor is higher than if the low-density paper were used alone; up to $b/a = 1.55$ the stress is higher than if the high-density paper were used alone. The higher dielectric constant of the outer high-density paper shifts such a large fraction of the total voltage to the low-density paper that the lower oil–paper stress ratio of the latter is surpassed over the entire range.

The methods used in the foregoing analysis are approximate at several points, and several conditions obtaining in cable insulation as manufactured have not been introduced, as for example the stranding of the central conductor, the spiral course of the oil channels, and space-charge action in the oil channels. It is believed, however, that the bearing of these elements will be much the same relatively in each of the several cases compared, and that their influence would appear only in a possible attempt at a quantitative derivation of stress values.

The experiments referred to in the opening paragraph were conducted at 60 cycles, on thoroughly impregnated insulation in which there was at no time any evidence of internal gas pockets. Discussion has suggested, therefore, that the clear evidence of a higher dielectric strength of a lower density over a higher density paper should be restricted to thoroughly impregnated insulation of the oil-filled or other similar type, and not be extended to so called solid-type insulation, in which it is generally admitted that following load cycles, or due to other cause, gas pockets or layers are present, and that the range of breakdown stress values is substantially lower than those encountered in the author's tests. It is also suggested that a different result might be found even in insulation of the oil-filled type in breakdown tests of impulse type.

Some comment on this type of discussion will be found in the paper by Whitehead and Kopper.[1] It is not, however, within the purpose of this paper to continue that discussion, but rather to show in terms of simple physical principles and of the experiments referred to, that the use of high-density paper in cable insulation should result in a higher dielectric strength if used in combination with lower density paper in suitably proportioned graded insulation, than would result in the use of either grade of paper alone.

It is also fair to emphasize that the principles utilized herein, depending as they do only on variations of dielectric constant, must apply to all types of cable insulation, oil-filled, solid, gas-filled, or other, insofar as the values and distribution of dielectric stress are concerned. Whether or not complete breakdown is a function of maximum dielectric stress only, is another question, discussion of which also falls outside the scope of this paper.

The results of this study are offered as a possible explanation of the recent conflict of opinion as to the influence of high-density paper in cable insulation.

Conclusions

1. In cable-type impregnated-paper insulation the use of all high-density paper results in a higher maximum stress in the oil channels than does a low-density paper.

2. The use of both high- and low-density paper, with the former inside, as in a graded cable, may result in a lower maximum oil stress, and so a higher dielectric strength, than if all low-density paper or all high-density paper is used.

Reference

1. THE DIELECTRIC STRENGTH AND LIFE OF IMPREGNATED PAPER—IV, J. B. Whitehead, J. M. Kopper. AIEE TRANSACTIONS, volume 64, 1945, April section, pages 171–7.

The Role of Frequency in Industrial Dielectric Heating

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MEMBER AIEE

A TENDENCY has developed to look upon frequency as a variable in dielectric heating which may be increased at will in order to increase the rate of heating quite independently of other factors. No doubt this tendency has been fostered by the acceptance without knowledge of its derivation of the equation for rate of dissipation of high-frequency energy in a dielectric, wherein the frequency term appears to the first power and in the numerator. The purpose of this paper is to analyze more carefully the role of frequency in industrial dielectric heating. The expression for the rate of heating will be critically examined, particularly with reference to the direct influence of frequency and its indirect influence through certain of the other variables of the heating equation that are not independent of frequency. In the light of this and other known effects of frequency, an attempt will be made to determine the practical upper limit of frequency for large-scale industrial dielectric-heating installations. It is hoped that, as a result of this discussion, there will result in the future a more precise use of the term frequency as related to dielectric heating. It should be pointed out that, in order to express the effect of frequency quantitatively, it has been necessary either to assume the ideal condition of pure L and C or to assume reasonable values of capacitance to ground and of distributed L and C. For this reason it has been possible in certain instances only to establish limits.

It is apparent from the analysis to follow that:

1. None of the variables in the heating equation is completely independent of frequency or its effects.

2. As frequency is increased, load capacitance must be decreased to allow tuning and to avoid short-circuiting the generator.

3. At higher frequencies load length must be kept at a minimum to eliminate the possibility of standing waves.

4. Increased frequency accentuates the load-shunting effect of stray capacitance and seriously reduces the over-all efficiency of the dielectric heating.

Therefore, it appears likely that the frequency band from 10 to 30 megacycles will prove best adapted for large-scale industrial dielectric heating, that is for load capacitances up to 500 micromicrofarads and for output power as high as 100 kw.

Equations for Development of Dielectric Heat

An expression for the heat developed in a dielectric by the application of an alternating voltage may be derived readily from the theory of the imperfect capacitor, represented schematically in Figure 1. Whenever there is an imperfectly insulating material to be heated between the plates of a capacitor, the latter may be considered as a capacitance with either a resistance R_s in series or a resistance R_p in parallel. These resistances are defined as

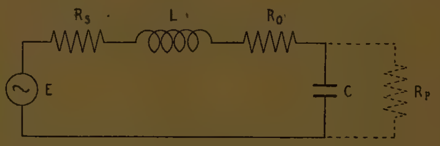

Figure 1. Equivalent circuit of dielectric heating load

E—Oscillator
C—Capacitance
R_s—Effective series resistance
R_p—Effective parallel resistance
L—Residual inductance
R_o—Residual resistance

$$R_s=\frac{\cos\theta}{\omega C} \qquad (1)$$

$$R_p=\frac{1}{\omega C\cos\theta} \qquad (2)$$

where

$\cos\theta$ = power factor of material heated and $\cos\theta \leqq 0.1 = \sin\delta = \tan\delta$
C = capacitance of load
$\omega = 2\pi$ (frequency)
E = alternating voltage

In addition there is associated with the capacitor a certain residual inductance L and resistance R_o. As, for simplicity in this analysis, the effect of leads will be ignored, the L term may be neglected and R_o reduces to the effective resistance of the capacitor plates.

If the concept of effective series resistance is employed, the power dissipated in the capacitor may be determined from an examination of the phase diagram of Figure 2 where

I = current flowing
E = voltage impressed
$R = R_s + R_o$, or total resistance

From Ohm's law it may be stated:

$$\text{Power}=\frac{E^2}{R}=\text{heat per unit time}=H$$

Therefore, the heat developed in the resistance R may be expressed:

$$H_R=\frac{{E_R}^2}{R}=\frac{E^2\cos^2\theta}{\dfrac{1}{\omega C\tan\theta}}=\omega CE^2\cos\theta\sin\theta \qquad (3)$$

where

$$\theta=\tan^{-1}\left(\frac{1}{\omega CR}\right)$$

$\cos\theta$ = power factor of capacitor

Equation 3 is the general equation governing the development of heat in an imperfect capacitor. If desired, $\sin\theta$ may be replaced by $\sqrt{1-\cos^2\theta}$.

If the effective resistance of the capacitor plates may be neglected, an expression for the purely dielectric heat developed within the plates may be derived as follows:

$$H=\frac{{E_R}^2}{R_s}=\frac{E^2\cos^2\theta}{\dfrac{\cos\theta}{\omega C}}=\omega CE^2\cos\theta \qquad (4)$$

where

$\cos\theta$ = power factor of material heated ($\leqq 0.1$)

Equation 4 may be verified by considering the effective parallel resistance R_p as the dissipative element.

For plane parallel electrodes and negligible effective resistance equation 3 takes the form:

$$W=\frac{5}{9}k\cos\theta f\left(\frac{E}{d}\right)^2\times 10^{-12} \qquad (5)$$

where

W = heat in watts per cubic centimeter
k = dielectric constant of material heated
$\cos\theta$ = power factor of material heated ($\leqq 0.1$)

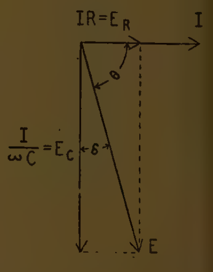

Figure 2. Phase diagram for dielectric-heating load

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Credit is due to I. J. Barsey for assistance in certain of the calculations and to G. E. Gard and K. D. Lawson for advice on the practical aspects of frequency in dielectric heating.

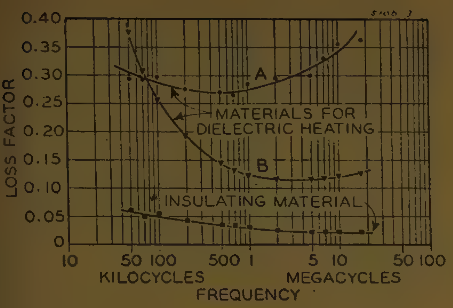

Figure 3. Variation of loss factor with frequency for typical materials to be heated (A and B), and a typical insulator

f = frequency in cycles per second

$\frac{E}{d}$ = voltage gradient in volts per centimeter

Equation 3 may be evaluated for any electrode configuration whose capacitance may be expressed although the heating will be uniform only in those instances where the electric field is uniform.

Examination of equations 4 and 5 shows that the magnitude of the heat generated is determined collectively by

(*a*). The geometry of the load.

(*b*). Its electrical properties.

(*c*). The characteristics of the source of high-frequency power.

The appearance of C in equation 4 clearly establishes the dependency of the heating upon load geometry. Equation 5 for plane parallel electrodes, perhaps the most frequent situation in industrial dielectric heating, includes dielectric constant and power factor which are, of course, inherent electrical properties of the material being heated. Furthermore, both equations contain frequency and voltage terms, which are determined not altogether independently by the oscillator and the coupling to the load circuit.

From a cursory analysis of equation 5, it is reasoned that increased frequency will allow somewhat decreased voltage to accomplish equivalent heating or linearly decreased time of heating if the voltage is maintained constant. As a result considerable emphasis has been placed on higher frequency for industrial dielectric heating to gain these advantages. This emphasis in many instances, has been misplaced since

1. The acquirement of greater skill has allowed high-frequency voltages up to 25 kv to be insulated.
2. The remaining terms in the heat equation are not independent of frequency.
3. Increased frequency causes serious limitations in itself.

Variables in Heat Equation not Independent of Frequency

In equations 3, 4, and 5 no terms are independent of frequency except the numerical constants, although certain quantities are more directly a function of frequency than others. Both dielectric constant, k, and power factor, $\cos\theta$, are direct functions of frequency and incidentally of temperature as well. The product of these two quantities is known as the loss factor of the material in question, and it has been plotted versus frequency in Figure 3 for two materials suitable for industrial dielectric heating, A and B, and for one material suitable as a high-frequency insulator. Although attention has been called by some to frequencies at which maxima and minima occur in the loss-factor curve, this fact is not of great value in shortening heating cycles because, even in approaching such points, a given percentage change in frequency normally results in a much smaller percentage change in loss factor. In general, it may be said that relatively good high-frequency insulators exhibit a loss-factor characteristic that slowly diminishes as frequency increases, whereas materials suitable for high-frequency heating may either increase, decrease, or go through maxima and minima as frequency increases. Although variations of loss factor in a given material seldom warrant choice of a particular frequency to increase the heating rate, large variations between different materials may require higher frequencies to heat the better insulators.

When a parallel resonant circuit containing an imperfect capacitor, as is the case in a dielectric-heating load circuit, is tuned to resonance, it presents to the oscillator an approximate resistive impedance:

$$Z_R=10^6/(2\pi fC\cos\theta)\text{ ohms} \qquad (6)$$

where

f = frequency in megacycles
C = capacitance in micromicrofarads between electrodes
$\cos\theta$ = power factor of dielectric

This is true because at resonance the parallel circuit offers infinite impedance, but shunted with the effective parallel resistance of the dielectric, R_p. Consequently, the impedance, Z_R, in equation 6 is merely R_p, since the latter is much less than infinity for conditions that allow the development of appreciable dielectric heat.

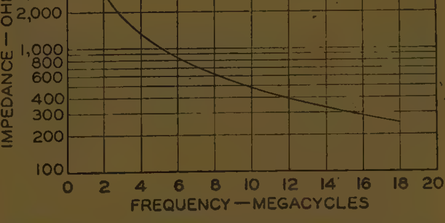

Figure 4. Variation with frequency of impedance for a dielectric-heating load (material A, Figure 3) tuned to parallel resonance

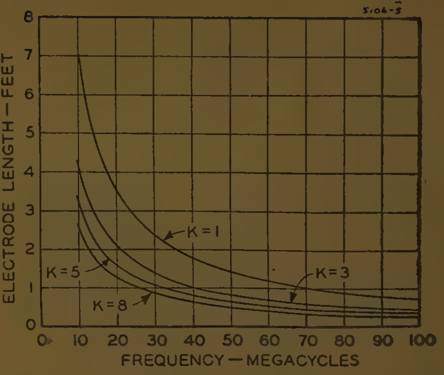

Figure 5. Maximum tolerable electrode length allowing ten-per-cent voltage variation as a function of frequency

K = dielectric constant

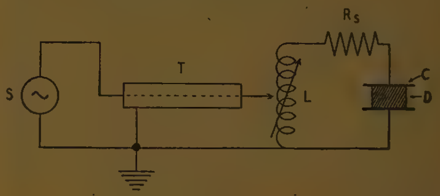

Figure 6. Tuned load circuit for dielectric heating

S—Oscillator
T—Coaxial transmission line
L—Tuning inductance
R_s—Effective series resistance
C—Electrodes
D—Material to be heated

In Figure 4 Z_R has been plotted as a function of frequency for a 36-by-36-by-3-inch (100-pound) load of material, A, presented in Figure 3. For this load the capacitance varies from 388 to 330 micromicrofarads over the frequency range considered. The impedance across the load is seen to diminish markedly with frequency.

The power transfer to and voltage drop across a load necessary for heating depend upon the impedance match between the output of the oscillator and the load circuit. As an increase in frequency will reduce load impedance, a reduced voltage will exist across the load, thereby minimizing power transfer, other factors remaining the same and the load capacitance having been tuned to the new frequency. Consequently, voltage may not be maintained constant across a load to shorten a heating cycle as frequency is increased unless adjustments are made to maintain the impedance match. If the oscillator is assumed to have an output impedance of 300 ohms and the load to be tuned with a high Q parallel resonant circuit, Figure 4 shows that the impedances would be matched for the load in question at 15.5 megacycles if the terminals of the oscillator were connected directly across the load. At much higher frequencies some sort of impedance-matching network would be required. There are, of course, practical limits beyond which impedance matching may be of no avail in overcoming the load short-circuiting effect of increased frequency.

Furthermore, the geometrical factors included in the capacitance term of equations 3 and 4, particularly length of electrodes, may not be chosen independently of frequency. If the electrode length approaches one-quarter the wave length of the high-frequency voltage employed, it will act as an open-ended transmission line upon which a quarter-wave-length standing wave is maintained. In the case of longer electrodes, half- or full-wave-length standing waves may exist or any multiples thereof. This, of course, will result in nonuniform generation of dielectric heat.

In the case of such a standing wave the voltage varies sinusoidally along the electrode, that is:

$$V=\sin \omega t \qquad (8)$$

where

$\omega=2\pi f$
$t=$time

For an electrode the voltage at any point may be expressed:

$$V=V_{max}\cos 2\pi ft$$

where

$$t=\frac{x\sqrt{k}}{c}$$

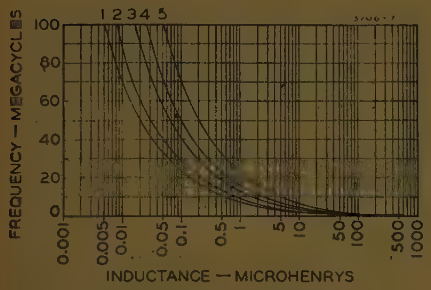

Figure 7. Ideal tuning inductances required as a function of frequency for loads of specified capacitance

1—C=500 micromicrofarads
2—C=300 micromicrofarads
3—C=150 micromicrofarads
4—C=100 micromicrofarads
5—C= 50 micromicrofarads

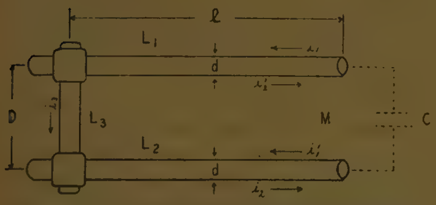

Figure 8. Schematic drawing of tuning stub

L_1, L_2—Parallel conductors
L_3—Movable short-circuiting bar
i_1, i_2, i_3—Primary currents
i_1', i_2'—Induced currents
C—Distributed capacitance

so that

$$V=V_{max}\cos 2\pi f\left(\frac{x\sqrt{k}}{c}\right) \qquad (9)$$

in which

$V_{max}=$maximum voltage
$x=$distance from point of maximum voltage
$k=$dielectric constant of medium between electrodes
$c=$velocity of light
$f=$frequency

If it is assumed that a ten-per-cent voltage variation may be tolerated along the greater dimension of a rectangular electrode, it is possible to calculate from equation 9 the maximum permissible electrode length as a function of frequency. These lengths are plotted in Figure 5 and are seen to be considerably reduced as frequency is increased, particularly for media with high dielectric constant. For the data of Figure 5 the generator is assumed to be connected to the mid-point of the shorter edge of the rectangular electrode.

Limitations of Increased Frequency

As the frequency employed is increased, the maximum capacitance, or load, that may be tuned is decreased, because practical tuning inductances may not be devised below a certain minimum inductance value. In order to absorb energy from the oscillator in the arrangement shown schematically in Figure 6, the load must be tuned to approximately resonance at the frequency of the source in accordance with the basic condition for resonance:

$$f=\frac{1}{2\pi}\frac{1}{\sqrt{LC}} \qquad (10)$$

where

$f=$resonant frequency
$C=$capacitance of load
$L=$inductance of tuning element

Speaking practically, the load is best tuned just above resonance on the capacitative side as operation at this point is more stable and efficient than either at resonance or on the inductive side of resonance. As shown in Figure 6 for parallel resonance, probably the most common load-circuit connection in industrial dielectric heating, this means the load presents nearly maximum impedance to the source, a condition favoring the high voltage across the load electrodes necessary to produce inphase heating current. In a series-resonant arrangement tuned near resonance, the load presents nearly minimum impedance giving rise to high transmission-line currents and corresponding losses. This undesirable feature, together with the fact that in a series arrangement the voltage across the capacitor is greater than the impressed voltage by a factor, Q, which frequently causes voltage breakdown, has led to the more widespread use of the parallel connection. However, equation 10 holds for the series case; and, if the voltage across the electrodes is identical, the facts of this analysis apply to either parallel or series connection unless otherwise stated.

Equation 10 defines the resonant frequency strictly for the case of zero resistance in the load circuit. Although the residual resistance of such a parallel circuit reasonably may be neglected, the effective series resistance, R_s, of the dielectric should be considered, leading to a shifted resonant frequency defined:

$$f_r=\frac{1}{2\pi}\frac{1}{\sqrt{(L-CR_s^2)C}} \qquad (11)$$

However, for the materials whose dielectric properties are shown in Figure 3 (that is, $\cos\theta\leq 0.1$) this correction is negligible. If C is assumed constant the change in frequency may be defined as

$$\frac{f_r}{f}=\sqrt{1+\cos^2\theta} \qquad (12)$$

where

$f_r=$resonant frequency with resistance
$f=$resonant frequency with zero resistance
$\cos\theta=$power factor of material

More important is the fact that the term CR_s^2 in equation 11 diminishes as the temperature of the dielectric is increased owing presumably to changes in k and $\cos\theta$ with temperature. Because of incomplete information regarding the effect of temperature on these dielectric properties, it can only be said that, as the temperature of the load is increased, less tuning inductance is required.

It is evident, from equation 10 that, as the load capacitance is fixed, at least at the beginning of the heating cycle, it is necessary to reduce the inductance of the tuning element in order to resonate the fixed load at higher frequencies. In Figure 7 are plotted, from equation 10

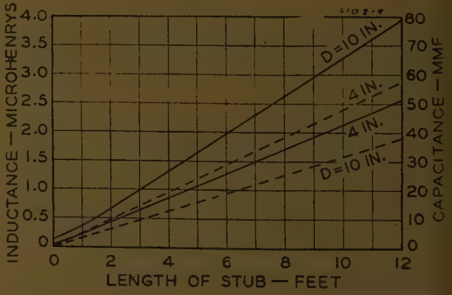

Figure 9. Calculated L and C characteristics of tuning stub of Figure 8 as a function of conductor length and separation

D=conductor separation
Conductor diameter—1³/₈ inches
Solid line—Inductance
Dashed line—Capacitance

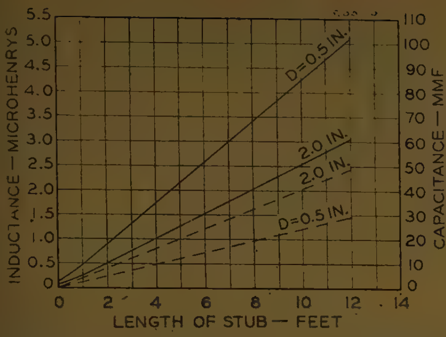

Figure 10. Calculated *L* and *C* characteristic of tuning stub of Figure 8 as a function of conductor length and diameter

D=conductor diameter
Conductor separation—8 inches
Solid line—Inductance
Dashed line—Capacitance

for pure *L* and *C*, the inductances required to tune loads of 50 to 500 micromicrofarads over a frequency range from 10 to 100 megacycles. Although it is recognized that these frequencies are shifted by the effective series resistance of the dielectric and the latter's rise in temperature during the heating cycle, they are sufficiently accurate for illustrative purposes. So far as temperature rise of dielectric is concerned, the inductances presented are maximum required values for a particular load geometry. The 500-micromicrofarad capacitance corresponds approximately to a 2.25-cubic foot 108-pound load of material *A* in Figure 3 at room temperature. Although ideally inductances of the order of magnitude of those indicated would be possible, there exists, however, a practical lower limit for an inductance suitable to tune a load dissipating a large amount of high-frequency power.

Such loads have been successfully resonated by means of a so-called tuning stub, that is, two parallel copper pipes connected with a sliding cross piece or short-circuiting bar, as is illustrated in the schematic drawing of Figure 8. In Figure 9 are plotted the *L* and *C* characteristics for tuning stubs from 0 to 12 feet in length, separated four and ten inches, respectively, and made from $1^3/_8$-inch-outside-diameter copper pipe. Variation of *L* and *C* as a function of conductor diameter is shown in Figure 10. The calculations, for which the details are presented in the appendix are ideal to the extent that zero capacitance to ground has been assumed for the stub. It is apparent, from an examination of Figures 9 and 10, that 0.1 to 0.15 microhenry is the minimum calculated inductance that may be expected from such an arrangement. Furthermore, the tuning stub does not act as an inductance under all circumstances, an additional limitation on the maximum capacitance that may be tuned at a specified frequency. As there is always capacitance associated with the inductance of a stub, the latter has a natural or resonant frequency. When the frequency of the voltage applied to the stub is in excess of the natural frequency, the stub presents a capacitative impedance and is no longer useful as a load-tuning element. At intermediate frequencies the effective inductance of the stub may be computed from the approximate formula:

$$L_{eff}=\frac{L}{1-\gamma^2} \tag{13}$$

where

L=calculated inductance
γ=ratio of actual to resonant frequency (accurate for $\gamma<0.8$)

In Figure 11 has been plotted the resonant frequency of the stub in question with a conductor spacing of ten inches. The frequencies corresponding to each length are the maximum frequency at which the stub presents an inductive impedance. Of course, if some capacitance to ground had been assumed, these maximum frequencies would be reduced.

The data of Figures 9 and 11 and experience have led to the conclusion that a straight piece of tubing one foot long may be taken to represent the practical minimum inductance that may be employed as a load-tuning element. The calculated inductance of such a length of tubing $1^3/_8$ inches outside diameter is 0.149 microhenry. In practice there arises the necessity of shielding such an element to avoid interfering radiation. A cylindrical grounded shield of eight-inch radius introduces a capacitance to ground of 6.91 micromicrofarads, from which information the effective inductance may be computed as a function of frequency. From equation 10 the maximum effective capacitance that can be tuned can be calculated. If an inherent inductance of 0.00006 microhenry per micromicrofarad is assumed in the load of Figure 1, its geometrical capacitance may be computed from the relation:

$$C=\frac{C_{eff}}{1+4\pi^2f^2C_{eff}} \tag{14}$$

Based on these assumptions, the maximum geometric capacitance that may be tuned by conventional methods has been plotted as a function of frequency in the curve labeled "practical" in Figure 12. These maxima are to be compared with those of the curve designated "ideal" in Figure 12 computed for the case of pure *L* and *C*. It is apparent that, if the 500-micromicrofarad 2.25-cubic foot 108-pound load already referred to is to be tuned by ordinary means, the frequency should not exceed approximately 20 megacycles.

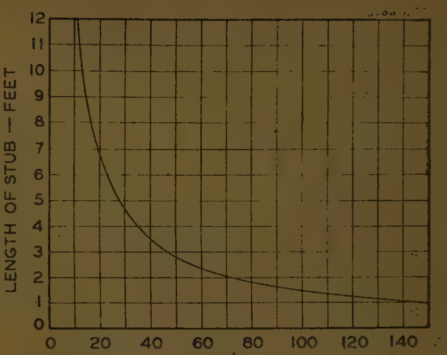

Figure 11. Resonant frequency of tuning stub of Figure 8 as a function of conductor length

Conductor separation—10 inches
Conductor diameter—$1^3/_8$ inches

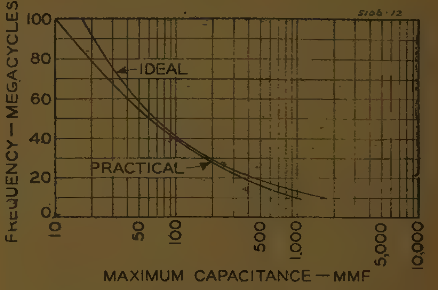

Figure 12. Maximum tunable load capacitance as a function of frequency, if 0.149-microhenry minimum practical tuning inductance is assumed

Ideal—Pure *L* and *C*
Practical—Residual parameters

There are means by which the maximum tunable capacitance may be increased, but such means are not feasible in all cases. The capacitance of a load may be halved by connecting a similar capacitance in series; but if the additional capacitance is air-insulated it robs part of the available voltage from the load, and if it is a duplicate load the power requirements are doubled, which may prove uneconomical. Bierwirth and Hoyler[1,2] have ingeniously tuned a 700-micromicrofarad load at 45 megacycles by means of multiple-tuning stubs each of minimum inductance, which incidentally tune out the standing waves that would otherwise appear on the load electrodes. However, such a scheme is of limited application.

It should be emphasized that the impedance across a capacitative load is diminished as the frequency is increased. Quantitatively, this impedance is expressed for the general case:

$$Z_C=\frac{1}{2\pi fC}\sqrt{\cos^2\theta+1} \tag{15}$$

where

f=the frequency employed
C=capacitance
$\cos\theta$=power factor of medium

Relatively small stray capacitances to ground frequently exist in a practical dielectric-heating load. As the medium is usually air ($\cos\theta=0$), for low frequency

Z_C is large and may be ignored. However, as f is increased Z_C decreases, which effectively adds to the capacitative load to be tuned. Therefore, not only is there a maximum C that may be tuned but also a minimum capacitative load that practically may be presented in an effort to compensate for increased frequency.

Finally the over-all efficiency of an arrangement of circuits as shown in Figure 6 for dielectric heating is seriously reduced as the frequency is increased; and this is an important consideration in so expensive a mode of heating. The efficiency should be kept between 50 and 60 per cent for economic operation. Not only are there resistive and radiation losses throughout the circuits, but an inherent limitation is the efficiency of the oscillator power tubes. The plate efficiency of available power tubes is reduced as the frequency of the input is increased largely on account of

1. Dielectric losses in glass walls and elsewhere.

2. Increased grid dissipation because of higher circulating currents due to lower grid impedance and because of long electron transit times.

3. Increased plate dissipation because of a phase shift between grid voltage and plate voltage due to long electron transit time.

Although plate efficiency data are not available, rated reduction in power input for several oscillator tubes that may be used for industrial dielectric heating with output power up to 100 kw has been plotted versus frequency in Figure 13. A conventional triode will function with 60-70-per-cent plate efficiency up to 20 megacycles. Above this frequency losses increase until at approximately 100 megacycles losses equal input power and the tube ceases to oscillate. It is apparent that, if higher frequency is required for shorter heating cycles, limited power output must be anticipated but not necessarily with reduced input power.

Choice of Frequency

In conclusion it must be emphasized that in reality the choice of frequency for a dielectric-heating application is a compromise among

1. The geometry of the load.

2. The electrical characteristics of the material constituting the load.

3. The oscillator tubes available.

Frequency must be selected with these factors clearly in mind, as well as the desire for minimum duration of heating cycle.

Broadly speaking, as high a frequency as is practical, available, and economical should be used. In particular, highest frequency is suited to very small loads, thin sections not capable of insulating high voltages, and materials with low loss factor. There is a limit to the extent that frequency may be increased to accommodate thin sections because, as thickness diminishes, load capacitance increases, presenting a serious tuning problem. Emphasis on high frequency to obtain short heating cycles may frequently be misplaced because

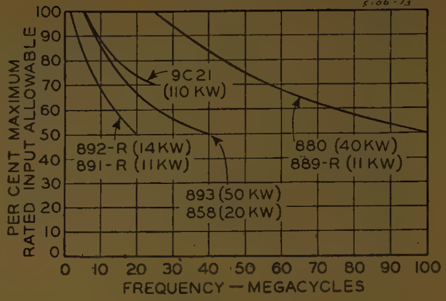

Figure 13. Allowable percentage of maximum rated input as a function of frequency for certain power tubes suited to dielectric heating

1. Extraneous circumstances often set a practical lower time limit to acceptable heating cycles.

2. Too rapid heating may have a deleterious effect on the material.

3. Increased production rate may not be economical unless accompanied by increased volume of production.

This analysis of the role of frequency in industrial dielectric heating indicates that at least for several years to come the range of frequency from 10 to 30 megacycles only will be utilized for large-scale dielectric heating, that is, for powers up to 100 kw. Changes in dielectric properties with elevated temperature will make close frequency control difficult unless means of automatic tuning are devised. Therefore, users and suppliers should coordinate their research and development activities with these facts in mind.

Appendix. Calculation of L and C for Tuning Stub

The total inductance L_T of the tuning stub shown schematically in Figure 8 may be calculated from the relation:

$$L_T = L_1 + L_2 + L_3 - 2M \qquad (16)$$

where

L_1 = self-inductance of first conductor
L_2 = self-inductance of second conductor
L_3 = self-inductance of short-circuiting bar
M = mutual inductance between first and second conductors

It is apparent that the relatively small mutual inductance between the short-circuiting bar and the other conductors has been neglected. The mutual-inductance term has been subtracted since, as may be seen from the currents indicated in Figure 8, the induced currents i_2', i_1' oppose the existing ones i_1, i_2.

The self-inductance in microhenrys of a tubular conductor of length l centimeters and diameter d centimeters may be expressed by the equation:[3]

$$L = 0.002l\left(2.303 \log_{10}\frac{4l}{d} + \frac{d}{2l} - 1 + \mu\delta\right) \qquad (17)$$

where $\delta \approx 0$ for copper tubing employed at frequencies above one megacycle. The mutual inductance in microhenrys of two parallel conductors of length l centimeters separated by a distance D centimeters from center to center as in Figure 8 may be computed from the relation:[3]

$$M = 0.002\left(2.303l \log_{10}\frac{l+\sqrt{l^2+D^2}}{D^2} - \sqrt{l^2+D^2} + D\right) \qquad (18)$$

When equations 17 and 18 are substituted in equation 16, the total inductance L_T of the tuning stub may be simplified to the form:

$$L_T = 0.004l\left(2.303 \log_{10}\frac{4lD}{d(l+\sqrt{l^2+D^2})} + \frac{d+2\sqrt{l^2+D^2}-2D}{2l} - 1\right) + L_3 \qquad (19)$$

Where L_3 is the self-inductance of the short-circuiting bar, which may be computed from the relation:

$$L_3 = 0.002D\left(2.303 \log_{10}\frac{4D}{d} + \frac{d}{2D} - 1\right) \qquad (20)$$

The values of tuning-stub inductance plotted in the curves of Figures 9 and 10 were computed from equations 19 and 20.

The tuning stub in question is, of course, not pure inductance but has a certain distributed capacitance between the parallel conductors that has been represented in the schematic drawing of Figure 8. If the effect of the short-circuiting bar is neglected and the conductors are assumed to be infinitely removed from ground, this capacitance in micromicrofarads may be computed from the relation:

$$C = \frac{0.1208l}{\log_{10}\frac{2D}{d}} \qquad (21)$$

where l is length of conductors in centimeters and D is their separation in centimeters, center to center. The values of capacitance for the tuning stub plotted in Figures 9 and 10 were computed from equation 21.

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Rewinding Commutator-Type Traction Armatures

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THE AMERICAN RAILROADS achieved more than twice as many passenger-miles and hauled twice as many ton-miles of freight in 1943 as in the corresponding period of 1918. Incomplete reports indicate increases for 1944. This enormous job is being done with smoothness and dependability and with far less congestion than was possible with the equipment and personnel of World War I.

Locomotives with electric transmissions, which include straight electric, Diesel-electric, and gasoline motorcars, and their attendant electric equipment, have played an important part in this accomplishment. With the increased number of electric locomotives in use, the longer hours of operation, and the heavier loads, old maintenance problems have been accentuated; new ones have arisen. Down the long list of these problems is one entitled "Armature Rewinding."

The object of this paper is to review for those experienced in the art of armature rewinding and to cite for the newcomer some of the important and frequently overlooked factors which must be considered to restore a damaged armature to its original condition and to modernize and improve, consistent with current practices. The factors will be discussed under the following headings:

1. Analysis of failure.
2. Cleaning.
3. Magnetic inspection.
4. Commutator.
5. Rewinding materials.
6. Armature windings.
7. Banding.
8. Impregnating.
9. Balancing.

Railway armatures present an interesting though difficult problem as they must withstand the severe strains imposed upon them by varying loads at varying speeds,[1] plus the several combinations of vibration and strains resulting from train motion and roadbed shock, and often with rapidly changing ambient temperatures.

Analysis of Failures

Traction motors are of necessity rugged in design because of inherently severe duty requirements. The kind of service is such that road failures must be kept at a minimum, not only because of the direct cost but because of the consequent expense incurred when traffic movement is obstructed. In most industrial plants a replacement motor or part can be installed in a comparatively short time. With motive power, however, the point of operation is not fixed and failures may occur many miles from the nearest repair or exchange facility.

In spite of equipment design and operating precautions taken to prevent failures, they will happen. To minimize these failures, an analysis of the cause of each must be made and steps taken to lessen the possibility of a recurrence.

Figure 1. View of vapor cleaning tank

Powerful cleaning action is effected by hot vapors condensing on the object to be cleaned by virtue of:

1. Strong solvency of cleaning fluid
2. Distilled vapor is full strength, undiluted by dissolved grease
3. 188 degrees Fahrenheit temperature helps to clean and to cook off paints and varnishes

Causes of armature failure can be divided into two classes:

1. Those which can be attributed to faulty operating conditions.
2. Those originating within the motor due to mechanical or electrical defects.

Causes of armature failures may be numerous and varied; however, some of the major and frequently occurring troubles are discussed in the following paragraphs. The armature winding may be completely roasted because of a mechanical fault causing excessive overload; it may be roasted because of heat resulting from friction generated when worn-out bearings or journals allow the armature to rub the pole pieces. The armature winding or commutator may be grounded, short-circuited, or open-circuited; bands may fail owing to overspeeding, but in all cases the cause can be assigned to either the category of faulty operation or to that of a defect in the motor proper.

- Most failures traceable to a motor defect originate first as a mechanical failure. Improperly seasoned armature insulation caused by incorrect baking and hot banding procedure may permit relative movement of wires or coils which will cause actual mechanical destruction of the insulation and result in an electrical failure. Faulty commutator seasoning on high-speed traction motors, weak brush-holder spring tension, and short or binding brushes are only a few additional things to consider when analyzing armature failures. The advantage of a thorough investigation to determine the cause of each failure cannot be stressed too strongly.

Cleaning and Preparation of Core

The proper cleaning of cores to be rewound goes beyond removing the old coils and scraping out the slots. Grease must be removed, old paint stripped, vent ducts cleared, and the outer surfaces of laminations cleaned. One of the most effective methods of accomplishing these aims is by the use of hot vapor cleaning—a process termed by the sheet-metal industry "degreasing." The equipment consists of a heated tank with a few inches depth on the bottom of the liquid cleaning solution (normally trichlorethylene) and a water-cooled condensing coil around the tank about halfway up the side wall.[2]

In operation, an armature to be cleaned, with its old winding intact, is lowered into the gaseous medium existing between the boiling pan and the condensing coils. As the vaporized cleaning solution is kept at a temperature of 188 degrees Fahrenheit, much higher than the object being cleaned, the vapor condenses upon the object, attacking grease, dirt, and varnish. Condensation is so rapid that the fluid flows from the work in a steady stream, carrying with it dissolved materials. Upon first seeing this action, one would think that liquid was being fed onto the object from an invisible pipe line.

When the entire armature is placed in the tank, not only is the grease and dirt removed but the insulating and bonding varnishes are softened, facilitating the removal of the old winding. This procedure minimizes the hazard of lamination damage when extremely tight coils must be stripped from the slots.

After the temperature of the work has reached that of the vapor, condensation ceases and the cleaning process stops. The

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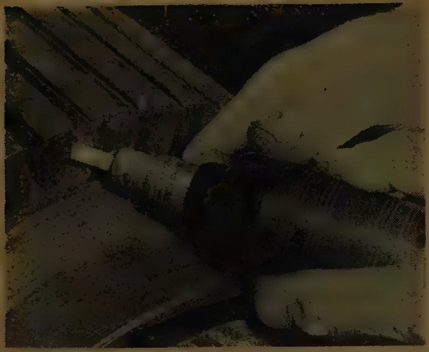

Figure 2. Clearing surface-fused laminations

armature may be withdrawn from the cleaning tank, and a few strokes with a wire brush removes loose residue and flakes of varnish. If the slots have been dirty, it may be necessary, after removing the coils, to return the stripped core to the cleaning tank for another treatment, but a second cleaning is only occasionally required.

The solvents normally used in this type of apparatus have little or no dissolving effect upon some types of coating materials such as shellac and certain synthetic enamels. The vapors have no effect on water glass (silicate of soda) or other standard lamination-insulating varnishes. Tests reveal that, after these lamination-insulating materials have been suspended in the degreasing vapors for ten minutes, neither their surface smoothness nor adherence to the metal has been affected. A mica commutator segment and cone ring given the same treatment were found to have a very small increase in weight, which was caused by the shellac softening under heat and allowing some vapor to collect. This increase in weight was lost after cooling took place, with no damage to the mica splittings or the shellac bond.

Over a period, repeated dips and bakes build multiple layers of baking varnish on the laminations, in the radial vents, and in the longitudinal core ducts. This insulating coating retards heat transfer and restricts air flow. The removal of this varnish in the "degreaser" will restore original ventilating efficiency to the armature. The elimination of grease from commutator, mica, and cone rings provides a clean surface to which insulating and protective coatings may be applied.

Core laminations of armatures which have failed frequently are found to have been damaged by arcing in slots between conductors and core. Also, the outer surface of core may have been defaced by pieces of conductors or bands being caught between armature and pole faces. Where any of the damage is severe, new core stacking may be necessary. However, after most armature failures, core stacking will not be required.

A very effective method of removing the fused surfaces in the core slots is to grind them out with a high-speed portable tool as shown in Figure 2. Sufficient grinding should be done to clean up all short circuits between laminations. Otherwise, hot spots may be expected to occur in operation, their severity depending principally upon the amount of fusion, the relative location, the magnitude of the armature current, and the frequency.

On older-type armatures having core bands it is often desirable to true up the banding grooves before rewinding. They are often burred over by the tangled mass of banding wire and copper that sometimes gets between the armature surface and the pole piece. An often used practice for accomplishing this is to drive a hard maple block into each slot, filling to the bottom of the band groove, and then, with the armature turning in a lathe, true up burred-over laminations at edges of band groove, taking light cuts with a high-speed tool-post grinder.

Magnetic Inspection

Magnetic inspection is the term applied to the process of examining ferrous metal pieces for flaws by magnetizing the piece and dusting fine iron particles or flowing over the piece a solution with iron particles in suspension. Since different magnetic poles are formed at the cracks, a concentration of the magnetic particles will collect so that the defects can be found by visual inspection.[3]

Magnetic inspection, although not new, has come into common use only since the great forward developments of aircraft, which must, of necessity, use materials efficiently to achieve lightness and safety. Railway equipment, although not limited to the same extent as that of aircraft, is subjected to repeated stresses that may cause costly failures. Armature shafts continuously are subjected to twisting torques while rotating. Improved alloy steels are used for these shafts; however, a metal that is fatigue-proof is not available. Shafts, before complete failure, will develop fatigue cracks. The application of magnetic inspection to traction armature shafts is comparatively recent, and only a limited amount of data are available. The available data indicate that many factors must be evaluated, such as shaft design, the load or torque on the shaft, and frequency of application of loads. The users of traction-motor equipment must compile data on the fatigue of shafts in order that shafts which have reasonable possibility of fatigue cracks may be tested by magnetic inspection.

The general practice in the magnetic inspection of traction armature shafts is to apply the test to exposed surfaces without pressing the shaft from the armature core. One class-1 western railroad has found that most fatigue cracks are detected on the tapered pinion fit adjacent to the bearing fit. This finding has resulted in the following handling procedure for all armatures that are removed from the motor frame for any reason:

1. Remove the pinion and antifriction bearings.
2. Clean all exposed shaft surfaces, removing all grease and dirt.
3. Magnetic-test and inspect for fatigue cracks, fractures, and defects.
4. Check for shaft trueness and alignment.
5. Renew defective shafts.

Fatigue cracks in armature shafts are not limited to the zone between the bearing and pinion fit. If a complete check is wanted, the shaft must be removed and tested over its full length.

Various types of equipment are available for performing magnetic inspection. Some use alternating current, some direct current from storage batteries, and others rectified alternating current for excitation of the magnetic structure.[4] Any type excitation is generally suitable for inspecting traction-motor armature shafts for fatigue cracks. The equipment for checking consists essentially of a magnetic yoke assembly with adjustable pole pieces for contacting the ends of the shaft and an excitation coil with control for allowing current to flow in the coil for comparatively short intervals (0.00001 to 3 seconds), similar to that shown in Figure 3. Magnetic particles in liquid suspension are easier to use, as the solution may be pumped or flowed over the shaft.

Magnetic testing of the wheels, axles, and locomotive crank pins of rolling stock has been used by many railroads for a

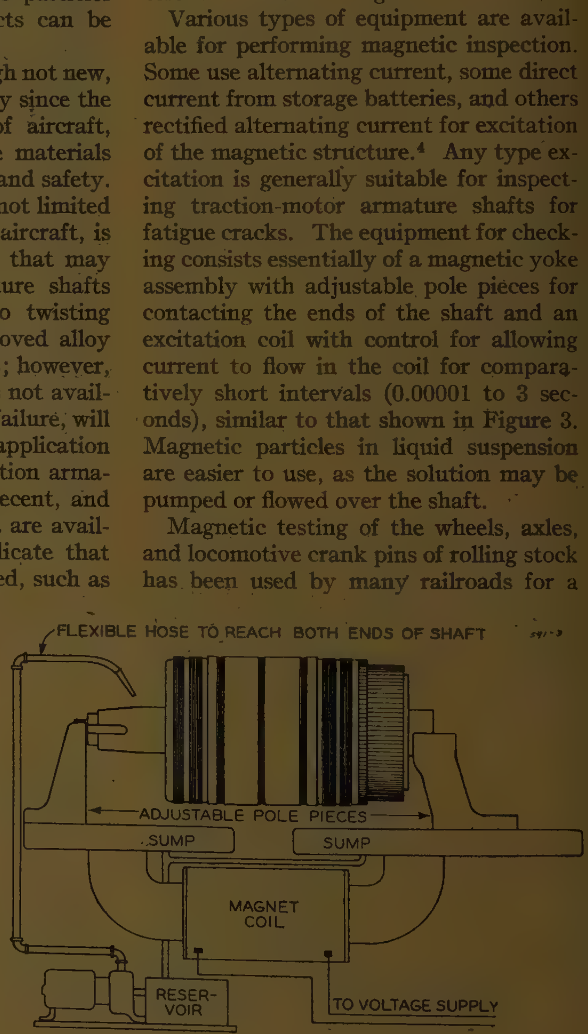

Figure 3. Traction armature in position for magnetic inspection of shaft

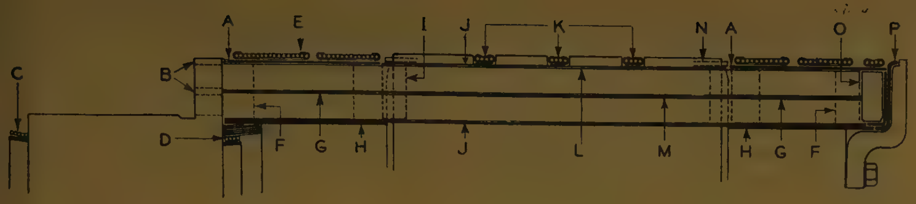

Figure 4. Guide for armature-rewinding materials

See Table III

A—Insulation under bands
B—Commutator-neck fillers
C—Commutator V ring
D—Mica seal
E—Band wire and clips
F—Insulation between coil-end turns
G—Insulation between layers
H—Coil-rest insulation
I—Formed U slot-end insulation
J—Slot insulation
K—Core bands and insulation
L—Strip filler
M—Slot-insulation strip separator
N—Slot stick
O—Coil-connection insulation
P—End-shield insulation

number of years. Some railroads are effectively using the same equipment for testing armature shafts.

Commutators

The condition of a commutator may indicate a real story to the experienced operator. A close examination is, therefore, of prime consideration so that corrective measures can be taken if necessary.

National Electrical Manufacturers Association Standard *RP*4-20 specifies that the Brinell hardness shall be 75 for bars of 0.125 inch and less at the thick edge, 70 for thickness of 0.125 inch to 0.250 inch, and 65 for bars over 0.250 inch. When a commutator shows evidence of having been very hot, it is good practice to test the hardness of the bars and to replace those which are too soft. It is also important on commutator of high peripheral speed that the amount of bar stock depthwise be checked so that there will be no danger of the bar bowing outward because of centrifugal forces. When there is doubt as to the depth of the bar remaining, a check with the commutator manufacturer is warranted. If the depth is less than a safe value, the commutator must be renewed.

After the winding is removed from the armature, the commutator slots are cleaned for the new coil leads, all burrs and fins are removed, and the commutator is given a ground test with the proper test voltage as specified by AIEE Standards. The authors have been unable to find a published Standard for a bar-to-bar test but have used very successfully a value of 220 volts alternating current, 60 cycles.

If the *V* rings are loose or have been disturbed, the commutator should be seasoned before the armature again is placed in service. Many different practices are followed in the seasoning of commutators, and most of them have good points. The object of seasoning is to obtain a tight commutator that will not develop high and low bars after being placed in service. To accomplish this objective we must apply the same elements that produce distortion in service, namely, heat and centrifugal force. The commutator assembly may be rotated in a heating rig and the bolts uniformly tightened at intervals. For the required uniformity in tightening the clamping bolts, it is recommended that a torque wrench be used.

As little copper as possible is removed from the commutator in turning and truing. The point of the tool should be rounded so that a smooth surface results. Final surfacing can be done with a stationary stone mounted in the tool post with the armature running at a high speed. A final polish may be given with a hand stone or very fine sandpaper. The commutator should be true within 0.001 inch for any high-speed motor. Where greater precision is wanted, the commutator may be ground with a carborundum wheel while the armature is rotated at full speed in its own bearings in a special grinding rig.[5]

Undercutting of commutator mica segments first came into use over 35 years ago, and, though at first it was not accepted, there is today little occasion to doubt its merits. However, expert care must be exercised in the operation, or the merits of undercutting are lost. It is very important that all mica fins be removed, as they are frequently the source of unsatisfactory operation. After undercutting, the rough corners of the copper bars must be removed.

After the commutator has been finished, it should be cleaned thoroughly and the armature blown out to remove all the copper chips and dust. It should be inspected carefully and then given a ground test according to AIEE Standards.

Table I. Banding Tension

Brown and Sharpe Gauge	Diameter (Inches)	Grade-C Steel and 18-8 Stainless Steel *Tension (Pounds)	Phosphor Bronze *Tension (Pounds)
8	0.128	850	600
10	0.102	600	400
12	0.081	400	250
14	0.064	250	160
16	0.051	160	100

* When double-deck bands are used, reduce the tension values by ten per cent for the top band.

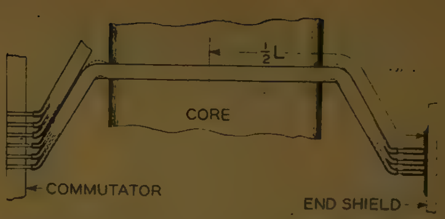

Figure 5. Increase in length of armature-coil bars from copper expansion

One-half length at 20 degrees centigrade = 17.000 inches

One-half length at 125 degrees centigrade = 17.036 inches

One-half length at 200 degrees centigrade = 17.058 inches

The protection of the outside surface of the *V*-ring mica that projects beyond the commutator can be accomplished readily by applying a coat of shellac, running over this a single layer of Fiberglas braid, and coating repeatedly with red enamel, sandpapering between coats. This will result in a very smooth surface which will minimize the collection of dust and oil.

Rewinding Materials

An often overlooked important element of armature rewinding is engineering planning of the rewinding materials. Rewinding materials include all insulation, solder, band wire, and like material, exclusive of the coils. Figure 4 gives the location of various items of rewinding material, and the table therewith will serve as a general guide to good practice in the use of material with AIEE class-*A* and class-*B* windings. Precision armature rewinding necessitates complete drawings of the armature assembly with detailed instructions on the method of application of each item of rewinding material. Information gathered from the analysis of failures is a part of the engineering data required for preparation of these rewinding specifications. Each item of rewinding material must be engineered for its job.

Too often certain items of insulation are applied to an armature, because it has been customary to use insulation in that particular spot on similar equipment. Frequently overlooked in applying rewinding insulation are:

1. Formation of pockets for trapping carbon dust and other foreign material.
2. Obstruction of ventilation.
3. Formation of dead air pockets.
4. Use of materials that are heat insulators.

These factors weigh heavily in the selection of proper rewinding materials. As an example, the material under the bands is preferably of a cushioning nature such as

Figure 6. Sectional view of band at anchor clip

varnished asbestos cloth or paper, in order that the material will not chafe and disintegrate with winding movements resulting from expansion, contraction, acceleration, and deceleration.

Economical rewinding requires that many items of the insulation be precut and fabricated. Volume production permits the use of tools and dies for shaping the rewinding material to greater accuracy than is possible by hand cutting and shaping. Better-quality workmanship may be expected by the use of packaged precut rewinding material, along with complete instructions for its use.

Armature Windings

The most important factors to be considered in the design, fabrication, and installation of armature and equalizer coils are copper fatigue and insulation deterioration. The number of turns, span, connections, copper size, and arrangement, and minimum dielectric requirement of windings are factors fixed for a given set of electrical characteristics. Changed operating requirements or need for modernization may necessitate the altering of one or more of these factors. When such is the case, it is an electrical-design problem, and complete design analysis should be made.

Copper fatigue and the resultant wire breakage in armature windings can be attributed to many causes, and there are equally as many devices for countering the supposed causes. There are, however, some fundamentals that must be followed to minimize copper failures.

The basic copper must be of the highest quality and the most fatigue-resistant available. Several premium coppers having superior properties are obtainable under various trade names. Soft annealed copper is generally accepted as desirable; any severe strains set up in the copper grain structure by edge bending, swaging, and similar cold working should be relieved by annealing. Surface scratching and nicking the copper must be avoided, as well as excessive pounding while forming the coils.

Table II. Acceptable Limits of Unbalance

Maximum Speed (Rpm)	Amplitude Unbalance at Bearings (Inch)		
	Fair	Good	Very Smooth
600	0.004	0.0025	0.0013
900	0.003	0.0016	0.001
1,200	0.002	0.0015	0.0007
1,500	0.0016	0.001	0.0006
1,800	0.0013	0.0008	0.0005
2,400	0.0009	0.00051	0.0003
3,000	0.0006	0.0004	0.0002
3,600	0.0005	0.0003	0.0002

The foregoing is particularly true at positions in armature windings where vibration and bending forces are concentrated, such as at the leads back of the commutator, at the coil corners near the end of the core, and at the clips or knuckles. Prominent among the several causes of bending forces is the linear expansion and contraction of the copper wire.

Figure 5 shows the amount of increase in length of an armature-coil bar from expansion caused by increased temperature. The dotted loop at the end of the slot portion illustrates coil distortion at a position where a large portion of this increased metal length is relatively free to be relieved. After the armature body has absorbed some heat, the increased copper length relative to the core is then a function of the difference in coefficients of expansion of the copper and the steel of the armature. It is not so much the difference between the coefficients of expansion of the two metals that causes the trouble but rather the wide difference in their temperature. The increase in copper length from expansion relative to the core is greatest when heavy starting currents flow through a cold armature; however, there is a continual variation in coil length during operation, caused by the variable-type load on traction motors.

This changing of coil length and resultant bowing of the coil at the end of the slot section not only contributes to copper fatigue but also disturbs the coil insulation; and the same heat that causes the expansion and contraction accelerates thermal aging of the insulation. In time, flexible coil insulations will be destroyed by heat and movement.

All modern traction armature windings are fabricated with class-*B* insulation, having an AIEE (Standard 11) temperature rating of continuous operation at 120 degrees centigrade above ambient. Often, higher temperatures are reached in operation, and the best of insulating materials are strained to their limit.

Class-*B* insulation covers a wide variety of combinations of inorganic substances in conjunction with organic bonds and backing materials. Some maintain their flexibility after being subjected to prolonged high temperatures, whereas others become brittle and lose their ability to withstand movement. The backing materials normally used are organic papers and fabrics; however, in some instances inorganic paper and cloth are used. There are applications where each of the different backings may have advantages. In recent years Fiberglas cloths, some as thin as 0.00175 inch and 0.002 inch, have proved highly successful for class-*B* backing materials. Along with the step-up in inorganic content of class-*B* material, we now have a new heat-stable organosilicon-oxide polymer in the form of varnishes and binders.[6] Although too early for exhaustive field tests, the performance to date on traction armature coils is most favorable, and an entirely new line of insulating materials has resulted. The temperature limits of these are in excess of present AIEE class-*B* ratings.

Equal in importance to the copper and insulation consideration is the proper installation and physical fit of the armature windings. The size of the slot portions must be held to a very close tolerance and must be correct for a tight drive fit without the necessity of adding filler materials. However, rather than have coils loose in the slot, it is preferable to use even organic fillers, though mica is the normally accepted material. A tight-fitting coil may be driven into the slots with less hazard of damage by painting with an insulating varnish just prior to installation. The wet varnish serves as a lubricant and, after it sets, provides a bond between the coils and the core.

The coils must be shaped so that when they are installed the preformed copper will lie in its normal position without setting up strains and binding points.

The point at which the coil slot section leaves the core lamination is hazardous in any winding and is even more so on railway-type armatures. The distortion from expansion shown in Figure 5 and the forces caused by vibration and by inertia of the end windings concentrate combined motion and pressure between the coil and the ends of the core lamination. The slots of many armatures are recessed at their ends to accommodate a U-shaped piece of heavy paper or fabric, thus affording a cushion and method of distributing the strain. Where recesses in the core may not have been provided, another effective method of minimizing strain concentration is to taper the core slot from $^1/_{32}$ to zero inches for about one-half inch back into the core, rounding off the outer edges.

The placement of the various insulations should be executed in accordance with carefully prepared drawings especially for the armature under consideration and made along the lines as shown in Figure 4.

Banding

A properly cleaned and insulated armature core and the correct use of insulating materials and coils must be complemented by carefully considered banding procedure. The quality of the finished armature may be lowered, unless the same careful planning is given to the banding method, as has been followed in the other steps of armature rewinding.

To calculate the load which the bands must carry, the usual procedure is to consider as separate problems the stresses in each of three sections of the winding, that is, the pinion end, the core section, and the ommutator end section. The stresses in each of these sections can be calculated by the empirical formula:[7]

$$S=3.25\times\left(\frac{\text{rpm}}{1,000}\right)^2 WD$$

where

S = stress on the bands in pounds
W = weight of coils, including insulation and band wire in the particular section under consideration
D = armature diameter in inches

For core bands use a constant of 2.8 instead of 3.25.)

After the value of S has been determined for each end and for the core section (except when slot wedges are used), a suitable-size wire is selected. The allowable tension per wire for the wire size chosen is given in Table I. Divide this value into the stress to get the number of wires required in each band. On coil ends use double-deck bands, if necessary, to accommodate the required number of wires.

Steel wire having a tensile strength of 240,000 to 270,000 pounds per square inch generally is used for banding armatures. However, to reduce commutation voltage and band losses, a nonmagnetic band wire is desirable on higher-speed traction armatures. Nonmagnetic stainless-steel wire is used in preference to phosphor bronze because of its greater tensile strength, 225,000 to 256,000 pounds per square inch, as compared to 130,000 to 150,000 pounds per square inch for phosphor bronze. The band wire must be of such quality and toughness that it can be bent 180 degrees on itself without breaking.

The anchoring of the ends of the band wire is most important, and it should be such that it will hold even after the solder may have melted and thrown off.[8] A frequently used method satisfying this requirement of fastening the band-wire ends is shown in Figures 6 and 7, which are, respectively, a section and a plan view. This method is applicable principally to bands on the end windings, because core-band grooves normally do not have sufficient space for the double folded-over clip. A good method for applying core bands is, first, to insulate each band groove with a layer of saturated 0.010-inch asbestos paper, over which is placed a strip of 0.015-inch tinned copper. Apply the band on the tinned-copper strip, using standard folded-over band clips, and solder to form a solid mass of band wire, copper strip, and clips.

The intermediate band clips normally are made from tinned copper and are usually spaced five or six inches apart, except at the beginning and ending of the band, at which point the clips are placed closer together. The number of clips must be an odd number that is not a multiple of any whole number divisor of the number of poles in the machine or closer than ten per cent (more or less) of the number of poles.[9]

Temporary "hot banding" is necessary on traction armatures to insure against lack of solidity that may lead to premature failure. Hot banding is accomplished by heating the armature to 250 degrees Fahrenheit before soldering the leads into the commutator and before brazing coil-connection clips where they may be used. A temporary band is run over the hot armature from the commutator riser to the pinion end of the windings. The end windings are protected with a cushioning pressboard strip under the band, and a filler strip of metal, fiber, or hard wood is placed in each slot projecting above the slot.

In the case of a wedged armature, install band clips so that the core portion of the temporary band can be removed for wedging with the end bands in place. After the slot wedges are in and before removing the temporary end bands, solder and braze the windings as required and then dip and bake the armature according to established standards. After the armature has cooled to room temperature, remove the temporary bands and apply the band insulation and permanent bands.

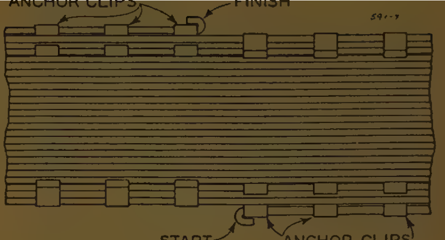

Figure 7. Plan view of beginning and ending of band wire showing method of anchoring

It is very important that the tension be uniformly applied. There are a number of tension devices on the market which control and uniformly apply any prescribed tension, the amount of which should not be a matter of guess. The values in Table I have proved satisfactory.

Table III. Guide for Armature Rewinding Materials

See Figure 4

Type	Position	Use With AIEE Class-A Coils	Use With AIEE Class-B Coils
Core	Coil rests	Pressboard or tapings of varnish cloth and cotton tape	Flexible mica, oiled asbestos paper or 0.030-inch varnished glass cloth, tapings of glass tape, molded mica rings
	Coil shield	Rope paper and varnished cloth	Flexible mica and 0.030-inch varnished glass cloth, molded mica rings
Slot	Troughs or cells	Fish paper or high-density rag paper	Preferably none—use uncut mica if filler is needed
	U end pieces	Rag pressboard	Treated asbestos tape
	Bottom fillers	Pressboard, fish paper, or high-density rag paper	Sheet bakelite, oiled asbestos paper, or molding mica
	Coil separators	Pressboard, fish paper, or high-density rag paper	Sheet bakelite, oiled asbestos paper, or molding mica
End	Between layers	Treated duck or rag pressboard	Oiled asbestos paper
	Between leads and knuckles	Treated duck or high-density rag paper	Flexible mica or 0.030-inch varnished glass cloth or Mica-Glas combination
	Between knuckles or rear coil connections	Treated duck or high-density rag paper	Flexible mica or 0.030-inch varnished glass cloth or Mica-Glas combination
	Between coil end turns	Treated duck or high-density rag paper	Glass, asbestos, or canvas-base bakelite, molding mica
Wedges or sticks	Coil retainer or protective strip	Canvas-base bakelite	Asbestos-base bakelite, glass-base bakelite, or canvas-base bakelite
Band	On core	Fish paper, rag paper, or rag pressboard	Uncut mica or varnished asbestos paper
	On coils	Treated duck, fish paper, or high-density rag paper	Flexible mica and oiled asbestos paper or varnished asbestos cloth
Miscellaneous	Commutator seal	$^1/_{16}$-inch flax cord painted with Red Commutator Enamel	$^1/_{16}$-inch glass braid painted with Red Commutator Enamel
	Band wire	Tinned steel, grade *C*-1, or tinned 18-8 stainless steel	Tinned steel grade *C*-1 or tinned 18-8 stainless steel
	Band clip	Tinned steel or tinned copper	Tinned steel or tinned copper
	Solder	50 per cent tin—50 per cent lead or Alloys upward to 100 per cent tin	100 per cent tin, 95 per cent tin—5 per cent antimony, or high-temperature alloy
	Flux	Nonacid rosin flux	Nonacid rosin flux
	Dipping varnish	Synthetic black baking	Synthetic black baking
	Spray varnish	Black oilproof air-dry	Black oilproof air-dry

Except on phosphor-bronze band wire, a high-temperature soldering alloy or high-tin-content solder should be used when possible. Care must be exercised to prevent annealing of the band wire during the soldering process, and the wire should not be heated to values exceeding those below:

Phosphor bronze—450 degrees Fahrenheit

Plain steel—600 degrees Fahrenheit

Nonmagnetic steel—1,000 degrees Fahrenheit

Solders having a liquefaction point higher than 450 degrees Fahrenheit should not be used on phosphor-bronze band wire.

Impregnation

The proper varnish treatment ranks close to the top of the list of those things that may be done to improve the quality of a winding, and yet it is one of the least expensive to execute. Consequently, it would be imprudent to use anything but the highest-grade insulating varnish and the best method for treating and baking.

Most of the varnish manufacturers have a top-grade varnish; and although there may be many formulas for producing such a material, the generally preferred properties are:

1. Long life at elevated temperatures.
2. Deep-drying ability.
3. Only slight resoftening under heat.
4. Toughness.
5. Resistance to oil, moisture, and chemical attack.
6. Good wet and dry dielectric strength.
7. Satisfactory viscosity for proper draining and build-up.
8. Reasonably safe flash point.

Vacuum-pressure impregnation usually is accepted as the best method of application for the initial treatment followed by one or two dips. The impregnating cycle may require 10 to 12 hours elapsed time, 1 hour preheat, 3 hours vacuum, 3 hours pressure, and 3 hours draining. During this treatment, the innermost parts of the winding become saturated and coated, even though more than half the varnish forced in may drain or "bleed" out during baking. The additional varnish dips fill surface cracks and voids, providing a protective film over the entire winding.

A good impregnation and dip treatment bonds and solidifies the entire winding, resulting in:

1. Increased heat conduction.
2. Sealing out dirt, oil, and other foreign materials.
3. Improved and prolonged life of insulation.
4. Less damage from mechanical shock.
5. Moisture resistance.

The armature should be baked after impregnation and after each dip in a well-ventilated, preferably recirculating-air-type, oven at from 250 to 275 degrees Fahrenheit. The time, of course, depends upon the drying properties of the varnish, characteristics of the oven, and the physical size of the armature. A typical baking cycle for a 400-horsepower high-speed traction armature with 18-inch-diameter by 16-inch-long core, using a high-grade synthetic varnish of the afore-mentioned general properties is as follows:

1. Preheat in oven one hour at 250 degrees Fahrenheit.
2. Allow to cool to 90 degrees Fahrenheit before placing in vacuum tank.
3. Bake 12 hours at 275 degrees Fahrenheit after impregnation.
4. Bake 12 hours at 275 degrees Fahrenheit after first dip.
5. Bake 18 hours at 275 degrees Fahrenheit after second dip.

It is usually desirable to permit the armature to cool to approximately 100 degrees Fahrenheit before each dip. This procedure gives a heavier film build-up and minimizes polymerization of the heat-reactive varnish in tank.

Dynamic Balancing

Traction drives are using higher-speed motors to obtain better efficiency and lighter weight as is evidenced by the increase in peripheral speeds from 8,000 to 12,000 feet per minute during the past 25 years. Motors operating at these speeds must have precision-built dynamically balanced rotating members. An unbalanced armature may destroy antifriction bearings, break armature-coil leads, damage gearing, and cause bad commutation, and, in addition may have undesirable physiological effect.

Modern-designed rotating elements usually have provisions for fastening the dynamic-balancing weights, though the methods may vary widely. Some units have grooves on both the pinion and commutator ends into which the counterbalancing weights are electrically welded. Others require the fastening of the weight by bolts secured against loosening by lock plates. Practically all traction-motor armatures are dynamically balanced by their maker after assembly of the spider, laminated core, and commutator, and before the application of the windings. This minimizes the amount of balancing weights required after the armature is wound and finished. Various types and designs of machines are available for dynamic-balancing operations.

Figure 8 illustrates a traction-type armature mounted in a balancing machine for dynamic balancing.

Experience of the operator is a large factor in the quality and cost of the operation, irrespective of the machine. The tolerances to which the armatures are dynamically balanced affect the cost. The various balancing machines do not have indicators measuring in the same units such as ounce-inches or thousandths-of-inch amplitude. Table II gives limits of unbalance that have proved acceptable.[10] The values in this table are in thousandths-of-an-inch amplitude as measured at the motor bearings. (Amplitude is the extent of the vibratory movement measured from one extreme to the oppo-

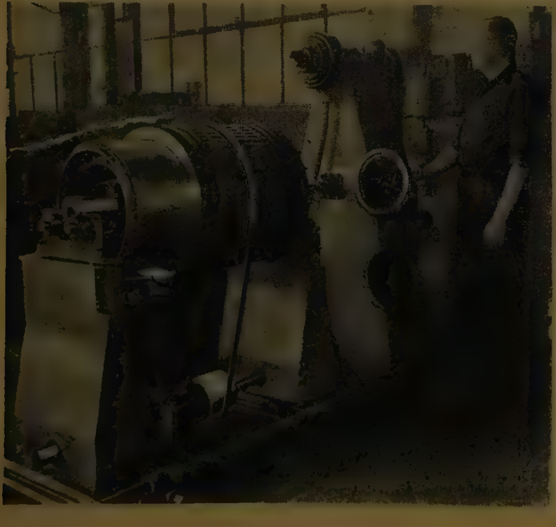

Figure 8 (left). Traction armature in dynamic-balancing machine

Figure 9 (right). Traction motor being checked for unbalance with vibrometer site.) The values indicated are applicable when the armature is revolving at maximum rated speed in its own bearings.

Checks on the amplitude of unbalance may be made with a simple vibrometer of the type shown in Figure 9, which consists of a dial indicator mounted in a heavy base. The heavy base provides high inertia allowing the dial-indicator case to remain substantially stationary in space while the indicator plunger is free to follow the linear motion of the bearing. Other types of simple portable indicators are available for these measurements.

Conclusions

Restoring damaged armatures to their original condition and making improvements where needed involves many factors and combinations of many arts. Some of the considerations are peculiar to the armature repairing trade, requiring the application of engineering techniques of comparable order to those used in the building of new armatures if new-armature efficiency is to be maintained.

By their omission it is not the intention to minimize the importance of other equally essential, though better publicized, factors as: Process and final testing, soldering and brazing, and record keeping.

The methods and desired results discussed are not all new, but it is hoped this paper has correlated some of the older methods and injected some new ones that will result in better armature repairing.

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Per-Unit Impedances of Synchronous Machines

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THE ANALYSIS of synchronous machinery is dependent upon a correct and complete knowledge of the machine impedances. The progress of synchronous-machine analysis has been characterized by the recognition of an increasing number of rotor circuits and by an accompanying need for accurate numerical data on the impedances of these additional rotor circuits. The modern equivalent-circuit method of analysis demands an almost limitless array of impedances involving not only mutual reactances but mutual resistances as well. As the application engineer, confronted by more complex application problems, has created more complete equivalent circuits, he has demanded of the design engineer accurate numerical data on the impedances of additional rotor circuits previously ignored. The design engineer himself has been confronted with more complex design problems which have made him directly cognizant of the need for more complete impedance data.

It is the purpose of this paper to present generalized formulas by means of which the per-unit impedances of the new additional rotor circuits may be calculated directly from the physical impedances (in ohms and henrys), and by means of which these new per-unit impedances can be made consonant with the per-unit impedances of rotor circuits previously analyzed.

The technical literature contains several articles which give design formulas for many machine impedances, but these articles are all characterized by the desire to present the impedance formulas in a per-unit form immediately applicable to routine design calculations. This practice is commendable to some degree, since correct numerical impedances usually are obtainable from these published formulas, but in a broader sense is to be deplored. The published per-unit formulas not only tend to obscure the physical characteristics of the various impedances, but they also imply that all per-unit impedances are unique and single-valued when expressed "in per unit of the machine kilovolt-amperes and voltage." This paper will show, however, that, if those impedances which are measurable from the machine terminals are neglected, all machine impedances are multivalued when expressed only in per unit of the machine kilovolt-amperes and voltage, and all these multiple values may be correct when defined properly in terms of rotor-current bases.

It is general practice at the present time to derive the per-unit impedances of the various circuit elements in an individual and detached sense. For instance, an impedance measurable from the stator is converted to per unit by dividing the physical value by the selected unit value of stator ohms X_{ao} (universally taken as phase voltage divided by phase current); the per-unit impedances of the various elements in the multiple rotor circuits are obtained by transferring the corresponding physical impedances to the stator by some effective stator–rotor turn ratio, and dividing the transferred values by X_{ao}. The various rotor-circuit impedances published by any one investigator usually are consistent among themselves when studied as a detached group, but the per-unit impedances published by different investigators do not always agree when placed in apposition, because the different investigators do not necessarily select the same effective turn ratio.

Such ambiguity could be predicted by noting that the turn ratio of a synchronous machine is an elusive quantity, as there is no unique value which could be called "the" turn ratio. For instance, one turn ratio could be defined as the ratio of the series turns per stator phase to the rotor turns per pole; a second turn ratio could use the effective series turns per stator phase including the K_pK_d factor; still another ratio could use the mathematical turns per pole in the fundamental components of the stator- and rotor-excited flux waves; and there are still other turn ratios, all of which have strong physical characteristics arguing for their use. It is not surprising, therefore, that published formulas for rotor-circuit impedances are not in strict accord

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Table I

e_a, e_b, e_c	in per unit of	e_{ao}
ψ_d, ψ_q	in per unit of	ψ_{ao}
i_d, i_q	in per unit of	i_{ao}
Ψ_{fd}, Ψ_{fq}	in per unit of	Ψ_{fdo}
Ψ_{nd}, Ψ_{nq}	in per unit of	Ψ_{xdo}
I_{fd}, I_{fq}	in per unit of	I_{fdo}
I_{nd}, I_{nq}	in per unit of	I_{xdo}

since these formulas of necessity employ some turn ratio. The question then arises as to which of these ratios is correct, or, if there is no unique correct value, which turn ratio should be used to obtain the per-unit impedances for synchronous-machine analysis; and to what degree are these various impedances affected by the turn-ratio selection.

The importance of the role played by the selected turn ratio in the magnitude of the per-unit impedances is physically self-evident, but the equal importance of the rotor-circuit current selected as unity is not so evident. By means of X_{afd}, however, it can be proved as follows that the selection of the turn ratio is tantamount to the selection of unit rotor-circuit current. X_{afd} is defined as the voltage induced in each stator phase per ampere field current, and X_{afd} is the corresponding per-unit value. As normal stator voltage is universally taken as unity, however, the product $X_{afd}I_{fd}$ must be unity at normal open-circuit air-gap-line stator voltage which shows that the unity field current I_{fdo} is not submissive to free selection after the turn ratio has been selected.

The relation which exists between the turn ratio and the unit field current suggests that these are equivalent and that rather than selecting a turn ratio it should be just as practicable to select a unit field current and have the turn ratio be derived therefrom. This is proved in the analysis of this article, not only for the field-winding circuit but for any of the rotor circuits, and, as unit rotor-circuit currents are visualized more easily than turn ratios, they are employed in the generalized impedance formula which are presented in this article.

The foregoing discussion has not answered the question as to which turn ratio should be used in the per-unit impedance formulas, but the emergence of the rotor-circuit unit currents shows that the per-unit impedances and the unit currents must be obtained simultaneously. The per-unit impedance formulas must be obtained in terms of the ampere-inch impedances and the selected unit currents. To obtain these results it is ineluctable that the synchronous-machine equations be derived first in the physical ampere-inch system and converted as a group into per-unit equations. The per-unit impedance formulas thereby obtained will then show which of the limitless series of unit currents should be used.

The method previously outlined is presented in this paper. The per-unit impedance formulas thereby obtained are given in the following section, and are in terms of the selected unit currents. The flexibility and generality of these expressions are discussed in the subsequent section, "Conclusions."

Results

Formulas for the per-unit impedances of a three-phase synchronous machine are given in the following in terms of the "ampere-inch" impedances and the "base-current ratios." The ampere-inch impedances are defined as the circuit impedances in physical units such as amperes, inches, seconds, as contrasted to the per-unit impedances which are pure numerics. The base-current ratios are defined as the ratio of the rotor-circuit currents selected as bases to $^3/_2$ the peak value of rated stator-phase currents. As previously discussed, the selection of base-current ratios is equivalent to the selection of stator-rotor turn ratios. The degree to which each impedance is dependent upon the base-current ratios is apparent from the following expressions. A consistent per-unit system with reciprocal per-unit mutual impedances will be obtained from these formulas. All terms are defined in the nomenclature.

$$x_d=\frac{L_d}{L_{ao}} \tag{1a}$$

$$x_{ad}=\frac{L_{ad}}{L_{ao}} \tag{1b}$$

$$X_{afd}=X_{fad}=\frac{3}{2}\frac{L_{afd}}{L_{ao}}\left(\frac{I_{fdo}}{\frac{3}{2}i_{ao}}\right) \tag{2a}$$

$$X_{and}=X_{nad}=\frac{3}{2}\frac{L_{and}}{L_{ao}}\left(\frac{I_{xdo}}{\frac{3}{2}i_{ao}}\right) \tag{2b}$$

$$X_{ffd}=\frac{3}{2}\frac{L_{ffd}}{L_{ao}}\left(\frac{I_{fdo}}{\frac{3}{2}i_{ao}}\right)^2 \tag{3a}$$

$$X_{nnd}=\frac{3}{2}\frac{L_{nnd}}{L_{ao}}\left(\frac{I_{xdo}}{\frac{3}{2}i_{ao}}\right)^2 \tag{3b}$$

$$X_{nfd}=X_{fnd}=\frac{3}{2}\frac{L_{nfd}}{L_{ao}}\left(\frac{I_{fdo}I_{xdo}}{\frac{9}{4}i_{ao}^2}\right) \tag{3c}$$

$$X_{nmd}=X_{mnd}=\frac{3}{2}\frac{L_{nmd}}{L_{ao}}\left(\frac{I_{xdo}}{\frac{3}{2}i_{ao}}\right)^2 \tag{3d}$$

$$r=\frac{r}{X_{ao}} \tag{4}$$

$$R_{ffd}=\frac{3}{2}\frac{R_{ffd}}{X_{ao}}\left(\frac{I_{fdo}}{\frac{3}{2}i_{ao}}\right)^2 \tag{5a}$$

$$R_{nnd}=\frac{3}{2}\frac{R_{nnd}}{X_{ao}}\left(\frac{I_{xdo}}{\frac{3}{2}i_{ao}}\right)^2 \tag{5b}$$

$$R_{nfd}=R_{fnd}=\frac{3}{2}\frac{R_{nfd}}{X_{ao}}\left(\frac{I_{fdo}I_{xdo}}{\frac{9}{4}i_{ao}^2}\right) \tag{5c}$$

$$R_{nmd}=R_{mnd}=\frac{3}{2}\frac{R_{nmd}}{X_{ao}}\left(\frac{I_{xdo}}{\frac{3}{2}i_{ao}}\right)^2 \tag{5d}$$

The per-unit values of the rotor-circuit voltages are given by the following formulas:

$$E_{fd}=\frac{E_{fd}}{e_{ao}}\left(\frac{I_{fdo}}{\frac{3}{2}i_{ao}}\right) \tag{6a}$$

$$E_{nd}=\frac{E_{nd}}{e_{ao}}\left(\frac{I_{xdo}}{\frac{3}{2}i_{ao}}\right) \tag{6b}$$

$$p=\frac{1}{\omega}p \tag{6c}$$

The direct-axis field-winding current as obtained in any per-unit system defined by the foregoing formulas will be in per unit of the selected base current I_{fdo}, but the currents of all the additional rotor circuits are expressed in per unit of the base current of the *x*th direct-axis additional rotor circuit, with the *x*th circuit submissive to free selection. When advantages are obtainable by the introduction of individual base currents for any or all of the additional rotor circuits, these new bases may be introduced by the methods originating in this article. It is evident from the foregoing formulas, however, that a change in the base currents will result in a change in the per-unit impedances.

Only the direct-axis impedances are given explicitly in the preceding formulas. The quadrature-axis impedances can be obtained from the direct-axis expressions previously given by merely replacing the *d* subscripts by *q*; this will express all the quadrature-axis currents in terms of the selected quadrature-axis base currents. It is often advantageous, however, to express the quadrature-axis currents in terms of the direct-axis bases since the latter are more easily determined: in this case, the quadrature-axis per-unit impedances are obtained from the direct-axis formulas previously given by first replacing all the *d* subscripts by *q*, and then placing the quadrature-axis current bases equal to the direct-axis bases.

Conclusions

Given in the foregoing are conversion formulas for converting ampere-inch impedances into per-unit impedances. These expressions were obtained by converting a *system* of ampere-inch equations into a *system* of per-unit equations, rather than by deriving the per-unit impedance of each circuit element as a detached independent entity. The only condition imposed during the derivation of the conversion formulas, and it is important to note that in a per-unit system this is an externally imposed condition, was that the per-unit mutual impedances must be reciprocal.

The most important result given by the foregoing formulas is the proof that no unique value of base-current ratio is demanded by the general per-unit system. The base rotor-circuit currents are entirely submissive to free selection, conditioned only by the restriction that when any value of rotor-circuit current has been selected as a base it must be used in the calculation of the machine impedances in the consistent method defined by the foregoing formulas.

The preceding expressions definitely prove, however, that the numerical values of many of the per-unit impedances are dependent upon the value of the rotor-circuit current which has been selected as a base. It is not surprising, therefore, that different numerical values may be published for the same machine impedance, as the base currents are not necessarily equal in the various publications. Before the results of different investigations are compared, the impedances must be reduced to a common current base. This can be done by reducing the impedance expressions to forms analogous to those already given as the base-current ratios will then be identifiable.

Not all the per-unit impedances are dependent upon the base-current ratios as is shown by equations 1a, b, and 4. If the stator base is defined as rated kilovolt-amperes and rated voltage, the impedances 1a, 1b, and 4 are invariant. That these are all measurable from the stator terminals is suggestive that all such impedances measurable from the stator terminals (the definitive impedances) are similarly invariant. This is proved in Appendix I wherein the $x_d(p)$ function[1,2]—the operational machine impedance viewed from the stator—is given in terms of the foregoing conversion formulas. It is there shown that the base-current ratios may be canceled out leaving $x_d(p)$ and $x_q(p)$ as invariant impedances. Hence, all impedances measurable from the stator terminals, such as x_d, x_d', x_2, r, are independent of the values of rotor-circuit currents selected as bases. The impedances which are not invariant are those internal impedances which cannot be measured from the machine terminals.

The invariance of the definitive impedances is of great importance in system studies, and so forth, wherein only the machine terminal phenomena are desired, but, in those more general problems for whose solution the internal organization of the machine is indispensable, all the machine impedances are employed, and the base rotor-circuit currents used in the calculation of the machine impedances must be known, and the corresponding base-current ratios must be employed in the impedance calculations in the consistent method defined by the foregoing formulas.

The second and concluding part of this paper presents a review of the base-current ratios which have been used, generally implicitly, in contemporaneous technical literature and suggests the adoption of a standard or preferred rotor-circuit current as a base value so as to make various published results consonant with each other.

Derivation of Generalized Per-Unit Impedance Formulas

Ampere-Inch Linkage Equations

Using the inductance coefficients defined in the nomenclature, the stator-phase linkages due to the rotor-circuit currents in the direct and quadrature axes can be written directly as in equations 7a, b, respectively.

$$10^{-8}\psi_{a1}=+(L_{afd}I_{fd}+L_{a1d}I_{1d}+L_{a2d}I_{2d}+\ldots)\cos\theta_a \quad (7a)$$

$$10^{-8}\psi_{a2}=-(L_{afq}I_{fq}+L_{a1q}I_{1q}+L_{a2q}I_{2q}+\ldots)\sin\theta_a \quad (7b)$$

The corresponding linkages in the b phase and c phase may be written from equations 7a, b by replacing θ_a by θ_b and θ_c, respectively.

The stator phase linkages due to the stator currents can be written as in equation 7c. This ampere-inch expression can be derived in a manner analogous to the per-unit expression of Park.[3]

$$10^{-8}\psi_{a3}=-\frac{2}{3}\left(\frac{L_d+L_q}{2}\right)\left(i_a-\frac{i_b+i_c}{2}\right)-\frac{1}{3}L_O(i_a+i_b+i_c)-\frac{2}{3}\left(\frac{L_d-L_q}{2}\right)\times(i_a\cos 2\theta_a+i_b\cos 2\theta_c+i_c\cos 2\theta_b) \quad (7c)$$

The corresponding linkages in the b phase and c phase may be obtained from equation 7c by cyclic rotation of the phase currents and phase angles.

The linkages in the direct-axis field winding due to the rotor currents in the direct-axis may be written as in equation 8a by means of the self- and mutual-inductance coefficients defined in the nomenclature.

$$10^{-8}\Psi_{fd1}=L_{ffd}I_{fd}+L_{f1d}I_{1d}+L_{f2d}I_{2d}+\ldots \quad (8a)$$

The corresponding equations for the linkages in the direct-axis additional rotor circuits can be written from equation 8a by analogy. The linkage equations for the quadrature-axis circuits can be written from equation 8a by substituting q for d.

In the physical ampere-inch system, all the mutual-inductance coefficients must be reciprocal. The linkages in the direct-axis and quadrature-axis field windings due to the stator currents thus may be written as in equations 8b, c directly from equations 7a, b, respectively.

$$10^{-8}\Psi_{fd2}=-L_{fad}(i_a\cos\theta_a+i_b\cos\theta_b+i_c\cos\theta_c) \quad (8b)$$

$$10^{-8}\Psi_{fq2}=+L_{faq}(i_a\sin\theta_a+i_b\sin\theta_b+i_c\sin\theta_c) \quad (8c)$$

The linkages in the additional rotor circuits can be written from equations 8b, c by analogy.

The complete stator and rotor linkages will be simplified considerably if the direct-axis and quadrature-axis linkages, defined as in equations 9a, b, are first introduced.

$$\psi_d=+\frac{2}{3}(\psi_a\cos\theta_a+\psi_b\cos\theta_b+\psi_c\cos\theta_c) \quad (9a)$$

$$\psi_q=-\frac{2}{3}(\psi_a\sin\theta_a+\psi_b\sin\theta_b+\psi_c\sin\theta_c) \quad (9b)$$

The i_d and i_q equations can be obtained from equations 9a, b by substituting i for ψ everywhere.

Introducing equations 9a, b into the preceding equations gives the complete direct-axis linkages of equations 10a, b.

$$10^{-8}\psi_d=L_{afd}I_{fd}+L_{a1d}I_{1d}+L_{a2d}I_{2d}+\ldots-L_di_d \quad (10a)$$

$$10^{-8}\Psi_{fd}=L_{ffd}I_{fd}+L_{f1d}I_{1d}+L_{f2d}I_{2d}+\ldots-\frac{3}{2}L_{fad}i_d \quad (10b)$$

The linkage equations of the direct-axis additional rotor circuits can be written from equation 10b by analogy. The complete quadrature-axis linkages can be obtained from the direct-axis linkages by substituting q for d.

Note that the mutual-inductance coefficients between the rotor and stator as obtained in equations 10a, b are no longer reciprocal because of the presence of the factor $^3/_2$. This nonreciprocity has been introduced by the definitions 9a, b, since with these definitions unit stator-phase current will give only two-thirds units of i_d or i_q.

Per-Unit Linkage Equations

A per-unit system consistent in both per-unit impedances and per-unit linkages and currents will be obtained by converting the system of ampere-inch equations represented by equation 10a, b into a corresponding per-unit system. The various quantities will be expressed in terms of the base values given in Table I.

This paper expresses all the rotor quantities in per unit of the base values of the field winding and the xth additional rotor circuit.

Only the first three base quantities in Table I are thus far specified, since these are to be taken as the values corresponding to rated stator kilovolt-amperes and voltage. The base rotor quantities can be specified by introducing the following relations between the various base currents and linkages:

$$k_{1f}\boldsymbol{\psi}_{ao}=10^8\boldsymbol{L}_{afd}\boldsymbol{I}_{fdo} \qquad (11a)$$

$$k_{2f}\boldsymbol{\Psi}_{fdo}=10^8\boldsymbol{L}_{ffd}\boldsymbol{I}_{fdo} \qquad (11b)$$

$$k_{1d}\boldsymbol{\psi}_{ao}=10^8\boldsymbol{L}_{axd}\boldsymbol{I}_{xdo} \qquad (11c)$$

$$k_{2d}\boldsymbol{\Psi}_{xdo}=10^8\boldsymbol{L}_{xxd}\boldsymbol{I}_{xdo} \qquad (11d)$$

Dividing the linkage equations $\boldsymbol{\psi}_d$, $\boldsymbol{\Psi}_{fd}$, $\boldsymbol{\Psi}_{1d}$, $\boldsymbol{\Psi}_{2d}$, and so forth, by equations 11a, b, c, d, respectively, gives the per-unit equations of 12a, b, c, d.

$$\psi_d=k_{1f}I_{fd}+k_{1d}\frac{\boldsymbol{L}_{a1d}}{\boldsymbol{L}_{axd}}I_{1d}+ k_{1d}\frac{\boldsymbol{L}_{a2d}}{\boldsymbol{L}_{axd}}I_{2d}+\ldots-\frac{10^8\boldsymbol{L}_d\boldsymbol{i}_{ao}}{\boldsymbol{\psi}_{ao}}i_d \qquad (12a)$$

$$\Psi_{fd}=k_{2f}I_{fd}+k_{2f}\frac{\boldsymbol{L}_{f1d}\boldsymbol{I}_{xdo}}{\boldsymbol{L}_{ffd}\boldsymbol{I}_{fdo}}I_{1d}+ k_{2f}\frac{\boldsymbol{L}_{f2d}\boldsymbol{I}_{xdo}}{\boldsymbol{L}_{ffd}\boldsymbol{I}_{fdo}}I_{2d}+\ldots-\frac{1.5\times10^8\boldsymbol{L}_{afd}\boldsymbol{i}_{ao}}{\boldsymbol{\Psi}_{fdo}}i_d \qquad (12b)$$

$$\Psi_{1d}=k_{2d}\frac{\boldsymbol{L}_{1fd}\boldsymbol{I}_{fdo}}{\boldsymbol{L}_{xxd}\boldsymbol{I}_{xdo}}I_{fd}+k_{2d}I_{1d}+ k_{2d}\frac{\boldsymbol{L}_{12d}}{\boldsymbol{L}_{xxd}}I_{2d}+\ldots-\frac{1.5\times10^8\boldsymbol{L}_{a1d}\boldsymbol{i}_{ao}}{\boldsymbol{\Psi}_{xdo}}i_d \qquad (12c)$$

$$\Psi_{2d}=k_{2d}\frac{\boldsymbol{L}_{2fd}\boldsymbol{I}_{fdo}}{\boldsymbol{L}_{xxd}\boldsymbol{I}_{xdo}}I_{fd}+k_{2d}\frac{\boldsymbol{L}_{21d}}{\boldsymbol{L}_{xxd}}I_{1d}+ k_{2d}\frac{\boldsymbol{L}_{22d}}{\boldsymbol{L}_{xxd}}I_{2d}+\ldots-\frac{1.5\times10^8\boldsymbol{L}_{a2d}\boldsymbol{i}_{ao}}{\boldsymbol{\Psi}_{xdo}}i_d \qquad (12d)$$

In order that the per-unit equations should retain the physical features of the ampere-inch system and not degenerate into an array of mathematical expressions, the per-unit mutual-inductance coefficients must be made reciprocal. Note that this is an applied condition which removes the nonreciprocity introduced by the definitions 9a, b. The *k* quantities accordingly must satisfy the relations given by equations 13a, b, c.

$$k_{1f}=\frac{1.5\times10^8\boldsymbol{L}_{afd}\boldsymbol{i}_{ao}}{\boldsymbol{\Psi}_{fdo}} \qquad (13a)$$

$$k_{1d}=\frac{1.5\times10^8\boldsymbol{L}_{axd}\boldsymbol{i}_{ao}}{\boldsymbol{\Psi}_{\text{[illegible]}}} \qquad (13b)$$

$$k_{2f}\frac{\boldsymbol{L}_{f1d}\boldsymbol{I}_{xdo}}{\boldsymbol{L}_{ffd}\boldsymbol{I}_{fdo}}=k_{2d}\frac{\boldsymbol{L}_{1fd}\boldsymbol{I}_{fdo}}{\boldsymbol{L}_{xxd}\boldsymbol{I}_{xdo}} \qquad (13c)$$

Eliminating between equations 11a, b, c, d and 13a, b, c, d results in the following explicit expressions for the *k* quantities:

$$k_{1f}=\frac{3}{2}\frac{\boldsymbol{L}_{afd}}{\boldsymbol{L}_{ao}}\left(\frac{\boldsymbol{I}_{fdo}}{\frac{3}{2}\boldsymbol{i}_{ao}}\right) \qquad (14a)$$

$$k_{1d}=\frac{3}{2}\frac{\boldsymbol{L}_{axd}}{\boldsymbol{L}_{ao}}\left(\frac{\boldsymbol{I}_{xdo}}{\frac{3}{2}\boldsymbol{i}_{ao}}\right) \qquad (14b)$$

$$k_{2f}=\frac{3}{2}\frac{\boldsymbol{L}_{ffd}}{\boldsymbol{L}_{ao}}\left(\frac{\boldsymbol{I}_{fdo}}{\frac{3}{2}\boldsymbol{i}_{ao}}\right)^2 \qquad (14c)$$

$$k_{2d}=\frac{3}{2}\frac{\boldsymbol{L}_{xxd}}{\boldsymbol{L}_{ao}}\left(\frac{\boldsymbol{I}_{xdo}}{\frac{3}{2}\boldsymbol{i}_{ao}}\right)^2 \qquad (14d)$$

$$\frac{\boldsymbol{\psi}_{ao}}{\boldsymbol{\Psi}_{fdo}}=\frac{\boldsymbol{I}_{fdo}}{\frac{3}{2}\boldsymbol{i}_{ao}} \qquad (15a)$$

$$\frac{\boldsymbol{\psi}_{ao}}{\boldsymbol{\Psi}_{xdo}}=\frac{\boldsymbol{I}_{xdo}}{\frac{3}{2}\boldsymbol{i}_{ao}} \qquad (15b)$$

Substituting equations 14a, b, c, d into equations 12a, b, c, d results in the system of per-unit equations of 16a, b, c, d with the per-unit impedances given by the generalized formulas of 1a, b, 2a, b, and 3a, b, c, d. The impedances also may be written in terms of base linkages rather than base currents by means of equations 15a, b.

$$\psi_d=X_{afd}I_{fd}+X_{a1d}I_{1d}+X_{a2d}I_{2d}+\ldots-x_di_d \qquad (16a)$$

$$\Psi_{fd}=X_{ffd}I_{fd}+X_{f1d}I_{1d}+X_{f2d}I_{2d}+\ldots-X_{fad}i_d \qquad (16b)$$

$$\Psi_{1d}=X_{1fd}I_{fd}+X_{11d}I_{1d}+X_{12d}I_{2d}+\ldots-X_{1ad}i_d \qquad (16c)$$

$$\Psi_{2d}=X_{2fd}I_{fd}+X_{21d}I_{1d}+X_{22d}I_{2d}+\ldots-X_{2ad}i_d \qquad (16d)$$

The quadrature-axis equations can be obtained from equations 16a, b, c, d by substituting *q* for *d*. The foregoing equations may be reduced to those of Park[1] by the methods of reference 4.

Ampere-Inch Rotor-Circuit Voltage Equations

Conversion formulas for the rotor-circuit resistances and voltages are obtained from the rotor-circuit voltage equations. These are given in the ampere-inch system by equation 17 and are converted to per-unit in the following section.

$$\boldsymbol{E}_{fd}=10^{-8}\boldsymbol{p}\boldsymbol{\Psi}_{fd}+\boldsymbol{R}_{ffd}\boldsymbol{I}_{fd}+\boldsymbol{R}_{f1d}\boldsymbol{I}_{1d}+\boldsymbol{R}_{f2d}\boldsymbol{I}_{2d}+\ldots \qquad (17)$$

The equations for $\boldsymbol{E}_{1d}$, $\boldsymbol{E}_{2d}$, and so forth, can be written from equation 17 by analogy. The quadrature-axis equations can be obtained from equation 17 by substituting *q* for *d*.

Per-Unit Rotor-Circuit Voltage Equations

The rotor-circuit voltage equations are converted to per-unit equations by dividing the equation for $\boldsymbol{E}_{fd}$ by $\boldsymbol{\Psi}_{fdo}$, and by dividing the analogous equations for $\boldsymbol{E}_{1d}$, $\boldsymbol{E}_{2d}$, and so forth, by $\boldsymbol{\Psi}_{xdo}$. The system of per-unit equations thereby obtained is given by equations 18a, b, c.

$$\frac{\boldsymbol{E}_{fd}}{\boldsymbol{\Psi}_{fdo}}=10^{-8}\boldsymbol{p}\Psi_{fd}+\frac{\boldsymbol{R}_{ffd}\boldsymbol{I}_{fdo}}{\boldsymbol{\Psi}_{fdo}}I_{fd}+ \frac{\boldsymbol{R}_{f1d}\boldsymbol{I}_{xdo}}{\boldsymbol{\Psi}_{fdo}}I_{1d}+\frac{\boldsymbol{R}_{f2d}\boldsymbol{I}_{xdo}}{\boldsymbol{\Psi}_{fdo}}I_{2d}+\ldots \qquad (18a)$$

$$\frac{\boldsymbol{E}_{1d}}{\boldsymbol{\Psi}_{xdo}}=10^{-8}\boldsymbol{p}\Psi_{1d}+\frac{\boldsymbol{R}_{1fd}\boldsymbol{I}_{fdo}}{\boldsymbol{\Psi}_{xdo}}I_{fd}+ \frac{\boldsymbol{R}_{11d}\boldsymbol{I}_{xdo}}{\boldsymbol{\Psi}_{xdo}}I_{1d}+\frac{\boldsymbol{R}_{12d}\boldsymbol{I}_{xdo}}{\boldsymbol{\Psi}_{xdo}}I_{2d}+\ldots \qquad (18b)$$

$$\frac{\boldsymbol{E}_{2d}}{\boldsymbol{\Psi}_{xdo}}=10^{-8}\boldsymbol{p}\Psi_{2d}+\frac{\boldsymbol{R}_{2fd}\boldsymbol{I}_{fdo}}{\boldsymbol{\Psi}_{xdo}}I_{fd}+ \frac{\boldsymbol{R}_{21d}\boldsymbol{I}_{xdo}}{\boldsymbol{\Psi}_{xdo}}I_{1d}+\frac{\boldsymbol{R}_{22d}\boldsymbol{I}_{xdo}}{\boldsymbol{\Psi}_{xdo}}I_{2d}+\ldots \qquad (18c)$$

Introducing equations 15a, b into equations 18a, b, c will give the per-unit equations 19a, b, c with the per-unit resistances defined by the generalized formulas of 5a, b, c, d, and with the per-unit rotor-circuit voltages given by the generalized formulas 6a, b. The per-unit differential operator is given by equation 20.

$$E_{fd}=p\Psi_{fd}+R_{ffd}I_{fd}+R_{f1d}I_{1d}+R_{f2d}I_{2d}+\ldots \qquad (19a)$$

$$E_{1d}=p\Psi_{1d}+R_{1fd}I_{fd}+R_{11d}I_{1d}+R_{12d}I_{2d}+\ldots \qquad (19b)$$

$$E_{2d}=p\Psi_{2d}+R_{2fd}I_{fd}+R_{21d}I_{1d}+R_{22d}I_{2d}+\ldots \qquad (19c)$$

$$p=\frac{1}{\omega}\boldsymbol{p}=\frac{1}{\omega}\frac{d}{dt}=\frac{d}{d(\omega t)} \qquad (20)$$

Nomenclature

The quantities given in the following are per-unit values (unless in bold-face type) in a consistent per-unit system with reciprocal mutual impedances. The bold-face characters used throughout the text are defined as given in the following but are in the "ampere-inch" system.

The ampere-inch system is defined as the system in which all quantities are expressed in their physical dimensions, such as amperes, inches, seconds. Impedances such as *R* and *L* must be calculated initially in the ampere-inch system before a per-unit value can be obtained.

The rotor circuits are separated into the main field-winding circuits and the "additional" rotor circuits. The additional rotor circuits comprise all the rotor circuits except the main field winding.

Subscript Notation

The subscript notation is as follows:

a, *b*, *c* = armature phases
f = field-winding circuit
m, *n*, *x* = additional rotor circuits
d, *q*, *0* = direct, quadrature, and "zero" axes
o = base quantities

The quantities *L*, *R*, and *X* use a triple subscript notation in which the first two subscripts refer to the circuits in which these impedances act, and the third subscript locates the (*d* or *q*) axis.

$_a, e_b, e_c$ = stator-phase voltages, peak values
e_{ao} = peak value of rated stator-phase voltage
E_{fd} = field-winding circuit voltage
E_{nd} = voltage in nth additional rotor circuit
$_a, i_b, i_c$ = stator line currents, peak values
$_{ao}$ = peak value of rated stator line current
$_d, i_q, i_O$ = stator current in d, q, and O axes, respectively
I_{fd}, I_{fq} = current in field-winding circuits
I_{fdo} = base value of field current
I_{nd}, I_{nq} = current in nth additional rotor circuits
I_{xdo} = base value of current in additional rotor circuits
L = mutual inductance between the circuits specified by the first two subscripts
L_{ao} = base (stator) inductance =

$$\frac{10^{-8}\psi_{ao}}{i_{ao}}=\frac{X_{ao}}{\omega}$$

L_{ad}, L_{aq} = inductance of armature reaction with the stator magnetomotive force in the direct and quadrature axes, respectively
L_d, L_q, L_O = synchronous inductance; direct, quadrature and zero axes, respectively
p, p = differential operator $\frac{d}{dt}, \frac{d}{d(\omega t)}$ respectively
r = stator resistance per phase
R = mutual resistance between the circuits specified by the first two subscripts
X = mutual reactance between the circuits specified by the first two subscripts
X_{ao} = base reactance = ωL_{ao}
x_{ad}, x_{aq} = reactance of armature reaction
x_d, x_q = synchronous reactance
x_2 = negative-sequence reactance
$x_d(p)$ = operational impedance viewed from stator terminals
$\theta_a, \theta_b, \theta_c$ = angle between center-line of direct-axis (main pole) and center line of stator phase indicated by subscript
ψ_a, ψ_b, ψ_c = stator phase flux linkages, peak values
ψ_{ao} = peak value of rated stator-phase linkages
ψ_d, ψ_q = stator linkages in d and q axes, respectively
Ψ_{fd}, Ψ_{fq} = linkages in field-winding circuits
Ψ_{nd}, Ψ_{nq} = linkages in nth additional rotor circuits
$\omega = 2\pi$ times frequency

Appendix. The $x_d(p)$ and $x_q(p)$ Functions

The $x_d(p)$ function is defined as the operational impedance as viewed from the stator terminals. In a general per-unit system, the $x_d(p)$ of a machine with only one additional rotor circuit in the direct axis is as given by equation 21a.[2]

$$x_d(p)=x_d-\frac{\left[p^2(X_{11d}X_{afd}^2-2X_{f1d}X_{a1d}X_{afd}+X_{ffd}X_{a1d}^2)+p(X_{afd}^2R_{11d}+X_{a1d}^2R_{ffd})\right]}{\left[p^2(X_{11d}X_{ffd}-X_{f1d}^2)+R_{11d}R_{ffd}+p(X_{11d}R_{ffd}+X_{ffd}R_{11d})\right]} \tag{21a}$$

Substituting the per-unit formulas given under "Results" into equation 21a gives $x_d(p)$ in terms of the ampere-inch impedances as in equation 21b.

$$x_d(p)=\frac{L_d}{L_{ao}}-\frac{1.5}{L_{ao}}\times\frac{\left[p^2(L_{11d}L_{afd}^2-2L_{1fd}L_{a1d}L_{afd}+L_{ffd}L_{a1d}^2)+p(L_{afd}^2R_{11d}+L_{a1d}^2R_{ffd})\right]}{\left[p^2(L_{11d}L_{ffd}-L_{1fd}^2)+R_{11d}R_{ffd}+p(L_{11d}R_{ffd}+L_{ffd}R_{11d})\right]} \tag{21b}$$

Since equation 21b is independent of any base-current ratios $x_d(p)$, is invariant when expressed in per unit of the machine kilovolt-amperes and voltage, and is independent of the base rotor-circuit currents; $x_q(p)$ is similarly invariant. The foregoing proof can be extended to include any number of additional rotor circuits.

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Application of Quality Control to Resistance Welding

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Synopsis: This paper describes a system of quality control of resistance welding that has been used successfully for the past few years in manufacturing switchgear cubicles and equipment. Standard samples of material identical with production parts are periodically inserted in the welding machine and welded without disturbing the settings and then tested to destruction in a torsion device. The diameter, torque, and angle of twist at failure all are measured and combined into a single figure indicative of weld quality. This number is plotted on control charts, and corrective action is taken whenever the control limits are exceeded. By setting these limits well within allowable values the quality of production parts is assured.

QUALITY CONTROL of resistance welding, in shops handling a large variety of work, has been simply and effectively accomplished. The methods of testing and control give assurance of uniform weld quality on a process where visual inspection leaves considerable to be desired, making possible the use of resistance welding on assemblies where high quality must be maintained at all times.

Standardization of the welding procedure is essential to the statistical control of the process. Suitable tables, giving the proper settings of the welding-machine controls, must be available to operators so that adjustments can be made to secure the best possible results for the conditions of each job. These settings are too important to be determined by any approximate methods. It is, likewise, essential that the equipment itself and its controls be adequately maintained to insure that, at a given setting, welds of uniform quality are produced. An adequate supply of electric power is necessary to insure consistent performance.

The quality-control procedure described in this article was developed over a considerable period of time. This report will give a brief picture of the method and its application. The mathematical details involved in the development of the quality index and statistical-control limits are not intended to be within the scope of this paper.

The structural stability of assemblies which are made with the use of resistance welding depends chiefly upon:

1. The quality of the parts used.
2. The quality of the welded section.

The quality of the parts used is controlled easily by usual methods of inspection and quality control. Attention, therefore, is directed entirely to the quality of the welded joints. Primarily, we are interested in the strength and ductility of the welds—the two things which are most difficult to determine without destruction of the fabricated parts. We also are interested in the appearance of the weld, the proper dimensions of the welded assemblies, and the change in dimensions caused

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by the welding. These can be determined easily by visual inspection and measurement.

In determining the best method for testing the strength and ductility of resistance welds where it is not practical to test to destruction, we have chosen to make test specimens of material identical in quality and thickness to the regular production parts but of a suitable size for testing. These samples are made with the machine settings identical to those under which the production work is being done. These samples are made at frequent intervals to insure that the quality of the welds made by the machine is maintained at the required quality level. From the results of the tests, a quality-control index is determined, and the average value is charted to give a continuous record of performance.

On highly repetitive jobs it is only necessary to establish the proper welding procedure once and then continue on that basis for the duration of the job. However, when the welding conditions are variable, as in the case of fabrication of switchgear equipment, it is necessary to have a specific procedure for each combination of materials. Figure 1 is an example of a cubicle frame showing the many combinations of materials that may be encountered in this type of structure.

In the absence of basic fundamental data for such conditions, it was necessary to undertake an extensive program to determine the effects of the several variables, such as: material type and thickness; welding current, pressure, and time; proximity of adjacent welds; and overlap.

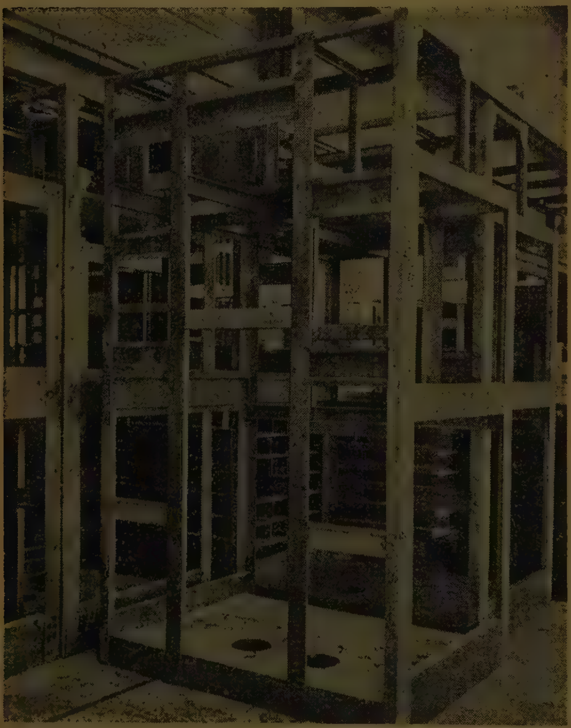

Figure 1 (left). Switchgear cubicle fabricated by spot welding—rear view

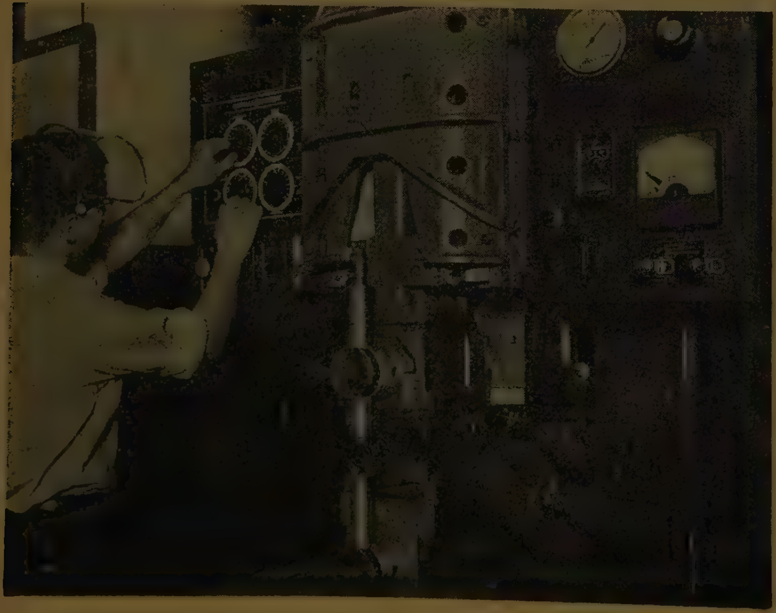

Figure 2 (below left). Control apparatus for resistance-welding machine

Figure 3 (below right). Resistance-spot-welded test specimen being placed in testing machine

Many thousands of tests were made on weld specimens in which these factors were independently varied. From this study it was possible to establish the proper welding procedures and the physical properties of the welds. This information was given to both the engineering and the manufacturing sections and is prominently posted on each welding machine.

With a standardized welding procedure, it is obviously necessary to have properly calibrated reliable welding machines. Our machines are equipped with thyratron synchronous controls to assure that, once the machine has been set for a definite procedure, all of the welds made will be consistent. Figure 2 shows a typical machine. Note that the adjusters for the thyratron control panel, the current regulator, the sequence panel, and the air circuit breaker, as well as the pointer stop ammeter and pressure gauge, are conveniently located on the front of the machine, readily accessible to the operator.

By means of plug-in jacks, the calibrations of the machines are checked at regular intervals; thus, it is necessary only for the operator to set the controls, using the applicable welding procedure.

Once the machine has been adjusted, good consistent welds should be obtained. However, it is necessary to check the results periodically to guard against carelessness on the part of the operator, improper functioning of the welding machine, or inconsistencies in the parts.

The quality of a resistance weld is dependent largely upon three factors: appearance, strength, and ductility; and it is these factors which the quality-control system must control. The necessity for maintaining the appearance of the welds is obvious. This factor is controlled by measuring the tip indentation on spot welds and by comparing with visual standards. The strength of the weld is essential as the design of the structure is based on the established minimum weld strength. The ductility of a weld is essential as it is an indication of the resistance to shock and fatigue strength.

At the present time no method has been found to determine the ultimate strength and ductility of a weld except by a destructive test.

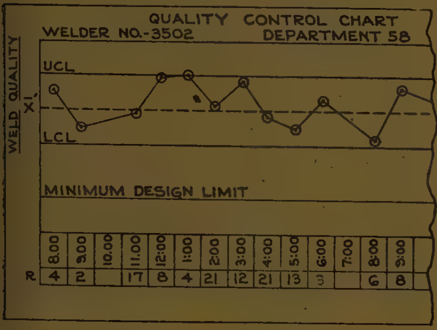

Figure 4. Control chart

In the investigation undertaken to determine the optimum welding procedures, it was found that the most satisfactory method of test for spot welds on heavy steels (one-eighth inch thick and over) was torsional testing. A special machine for this purpose was designed and built in the Philadelphia works laboratory of the General Electric Company. This machine subjects the weld to torsional shear about the center of the weld, and the ultimate torque and angle of twist at the ultimate torque are measured. Figure 3 shows one of the torsional testing machines now being used by the inspection and quality-control division. This torsional testing gives a simple and accurate means of determining the weld strength and ductility, since the strength is proportional to the ultimate torque and the weld diameter (which is easily measured). The ductility is proportional to the angle of twist.

These factors are combined to give a quality-control number which is an index of the quality of the weld. By the use of this comparative index in statistical control, it is possible to show the changes in process performance.

Because of limitations in the skill and ability of the control operators available for this work and the necessity of limiting the time required for testing, a simple, quick, and effective control method is essential to the success of this program.

The control-chart method of controlling quality as covered by the American Standards Association publications Z1.1, Z1.2, and Z1.3 was adopted for this work using a chart for averages ($\bar{X}$) with a modified range (R) chart. After a few weeks of trial testing, during which the process was checked for control by a study of the frequency-distribution charts on data collected, it was found that a sample lot of three would be adequate for control of this process. Control limits were established for the grades of steels in common use. Control-chart scales were developed for each grade of steel so that one set of control limits would be common for all combinations of materials. This simplified the control chart and made possible a form of report which is a composite picture of the quality level being maintained in an operating division on all spot welding, regardless of how diversified the work.

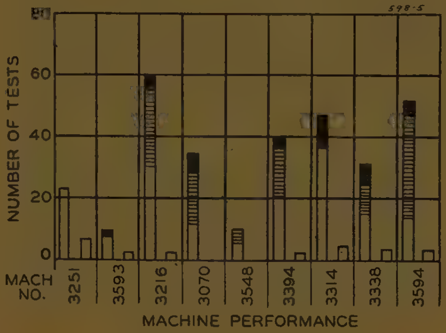

Figure 5. Weekly machine performance

The control-chart scales are so proportioned that all limit lines coincide with common lines on the chart, with the distance from the central line to the upper and lower limits divided into a suitable number of equal divisions. The scale distance from the lower control line to the minimum design-limit line and the distance from the upper control line to the upper limit of the chart are similarly divided. Averages below ten are plotted as zero and averages above 60 are plotted as 60. Distribution diagrams for the various grades of steels thus will be shown in proper relationship to the limit lines on the chart, and the composite diagram will be an indication of the over-all control of the process.

The weld test specimens are made from drop-off pieces of materials currently being used in the shop. A supply of these samples is kept in suitable containers in the welding areas. A test is taken on each welder in operation once each hour. Three samples of the same combination of materials and thicknesses are welded under the same conditions of machine control setting being used on the production work at the time of test. No adjustment of the machine is permitted at the time of test, as control is based upon machine performance under production conditions.

Control charts shown in Figure 4 are placed on each machine, and the results of tests are posted after each test. A record sheet is filled in by the control operator and becomes a permanent record of performance in the welding sections. The machine chart is forwarded to the foreman's office the following morning, and the record sheet is forwarded to the central control office for analysis.

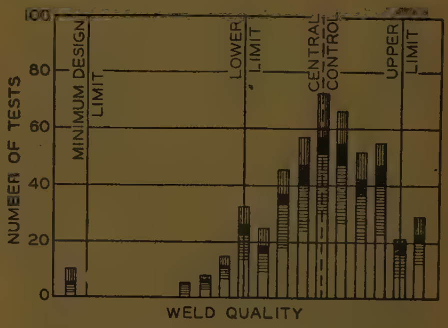

Figure 6. Weekly weld-quality distribution

The daily records are posted on a weekly record sheet in convenient form to show the frequency-distribution totals for all welding combinations based on machine and process performance. When completed, this record sheet contains complete information for the weekly report which is distributed to the department heads, manufacturing superintendents, welding engineers, quality-control specialists, and inspection supervisors.

The weekly report gives a complete analysis of weld quality control by showing the relative performance for each machine in the section, Figure 5, and frequency-distribution diagrams, Figure 6, showing the relative effect of materials and welding methods on the over-all level of weld quality.

This method of controlling weld quality has given excellent results. Proof tests on welded structures made under controlled conditions have proved the effectiveness of the control. Special test structures have been made and subjected to severe shock conditions without any weld failures. Resistance welding, properly controlled, becomes a safe dependable fabrication process. Quality control of this process can insure strong dependable welds of uniformly high quality.

Treatment of High-Altitude Brushes by Application of Metallic Halides

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Synopsis: It has been found that the rapid wear of brushes under high-altitude conditions can be prevented by the treatment of the brushes with a suitable metallic halide. Carbon, electrographitic, and metal-graphite brush grades all respond to such treatments. A great many nondeliquescent metallic iodides, bromides, chlorides, and fluorides have been tested and found to prevent brush dusting on the heavy-duty electric units of aircraft. These treatments also have been applied with considerable success to earth-surface machines operating under severe commutating conditions. A mechanism by which this class of chemical compound may react to maintain a satisfactory low-friction film on a commutator is proposed.

IT IS BELIEVED that the manufacturers and operators of commutating electric machines and the producers of brushes for them are in complete agreement on only two points:

1. They agree that a film on the commutator is essential for long brush life.
2. The film on a commutator should be as uniform as possible on the entire commutator surface if optimum commutation is desired.

The precise chemical nature of this essential film is still a subject of discussion. The mechanism of its formation is also not certain. The film is extremely thin, and its weight per unit area is so small that an accurate chemical analysis is difficult. Recently Van Brunt and Savage[1] reported a typical film to consist largely of cuprous oxide and carbon (with a number of minor ingredients totaling only 12 per cent). They calculated the film thickness as 5.4×10^{-6} centimeters of which 2.1×10^{-6} centimeters was copper oxide. These workers and many others stress the complex composition of commutator films. It is the thesis of the present paper that an *ideal* commutator film is composed substantially of cuprous oxide (the oxide of monovalent copper) and that all other substances which are present in *actual* films are either nonessential or are definitely harmful.

The color of the track produced on a copper commutator by a given brush grade is a measure of the thickness of the commutator film. This thin protective nonmetallic layer holds the brushes from direct contact with the metal and thus prevents rapid wear. This film should develop to a uniform thickness under all brushes if they are all alike and if the operating conditions are the same for all brushes. Experience has shown that a high uniformity in the color of all of the brush tracks on a commutator goes with an excellent machine performance. Uniformly colored brush paths indicate uniform current division between brushes. Streaking and threading within a single brush track (which again reveal differences in film thickness) may be diagnosed as caused by nonuniform current distribution within the brushes themselves. On an *ideal* commutator there will be no bar-edge burning; streaking and threading will be absent; and the color of all commutator bars will be exactly the same.

An *actual* commutator never meets these desired standards of uniformity. The brush tracks are never exactly alike in color, and a critical eye can always detect some evidence of streaking, no matter how slight. Though an electrographitic brush is expected to develop a rich satiny chocolate-brown polish on the copper, there is wide disagreement as to the depth or shade of color which is most desirable.

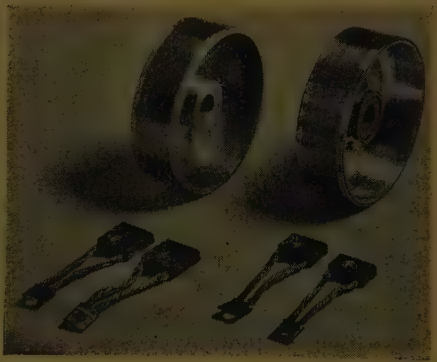

Figure 1. Copper slip rings and brushes after test in high-altitude test chamber

Early in World War II it was reported that brushes wore rapidly at times on certain airplane equipment when they were operated at altitudes above 20,000 feet. Apparently the commutator film, normally maintained at low altitudes, might disappear quite suddenly at high altitudes, and rapid brush wear, or brush dusting, then would follow through abrasion.

A discussion of this failure of commutator films under the abnormal atmospheric conditions prevailing at high altitudes is best prefaced by a consideration of the mechanism by which these films may normally be formed and maintained.

Clean metallic copper cannot be exposed to air without acquiring a tarnish film. At room temperature this film gradually increases in thickness, although the rate of growth is so slow that the film remains transparent and practically colorless for a long time. (Vernon[2] has found that the film on copper heated in air for one hour at only 130 degrees Fahrenheit has a thickness of 1.05×10^{-7} centimeters.)

The commutator is "turned to size" as one of the final steps in the manufacture of an armature. During this turning operation the copper is heated by the work done upon it. As copper is oxidized more rapidly at higher temperatures, each spot of the metal, as it emerges from under the turning tool, acquires a thicker film than it would get if it had been exposed at room temperature. In addition this initial film is further thickened during the seating of the brushes. The coefficient of friction between unseated brushes and a copper commutator is normally high. As the brushes are worn to a seat under light electric loads, their faces become polished. At the same time the film upon the commutator increases in thickness as its surface is burnished by the mechanically loaded brushes under which it moves. If the brushes have been properly chosen, if the machine is well designed, and if the running-in conditions are correct, the film will gradually increase in color, thickness, and polish, and the friction between brushes and commutator will gradually decrease.

However, the formation of a commutator film by electrically loaded brushes is a relatively complex dynamic process and is by no means so simple as the description in the preceding paragraph might indicate. The heat appearing at the brush-commutator interface is generated not only by friction but also by the electric energy dissipated at this interface. (The coefficient of friction between a high-grade well-seated electrographitic brush and the burnished copper-oxide commutator film is often two tenths or higher. The double contact drop for this same type of brush will range from $1^1/_2$ to 3 volts. The actual value for each of these properties will vary with the electric load, the brush grade, and the condition of the commutator.) At any given instant the electron transfer between the commutator and a brush takes place only at a few highly localized areas on the brush face. When the electric load is

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added to the already present mechanical load these spots become very hot. These overheated active areas of the brush being in close contact with the commutator will react chemically with the copper-oxide film to form metallic copper and carbon monoxide. Thus the electron transfer areas on a brush face become chemically reducing agents and defilming rather than filming agents.

When one of the minute areas of freshly reduced metallic copper emerges out from under the brush and contacts the air, it immediately is reoxidized. But as this same area soon passes under another brush, it may perhaps be reduced again before the oxide film has been re-established to normal thickness. However, under normal operating conditions at any given instant the active electron transfer area is small; thence a freshly oxidized spot may pass under several brushes without again being reduced, thus allowing the film to be built up to equilibrium thickness during the repeated periods of air exposure. Nevertheless, with continued operation, an area on a commutator is frequently reduced and again reoxidized. Therefore the oxide film should be regarded as dynamic rather than static.

The character of the cuprous-oxide film on a commutator will depend on many variables: the properties of the brushes, the shape of their faces, the angle at which they contact the commutator, the spring loading placed upon them, the area of the commutator covered by them, their operating temperature, the temperature of the commutator, and the purity of the ventilating air all play essential roles in controlling the thickness and quality of the film which is developed. The film, as previously explained, is a dynamic, alterable structure. It is possible that a slight change in operating conditions will alter it drastically overnight. A seemingly insignificant change in the surroundings may affect it; for example, a shift in the wind direction may bring a trace of hydrogen sulphide into the room. A leaking gas tunnel has also been responsible for the malperformance of a group of machines in a steel mill. The edges of the commutator in such contaminated atmospheres became covered with a continuous gray film of copper sulphide, and the brush tracks were blotched with irregular patches of this compound scattered in between the normal cuprous-oxide areas. Copper sulphide may completely replace cuprous oxide as a commutator film, if the hydrogen sulphide in the air reaches the concentrations sometimes found in a kraft paper mill. Brush wear is always excessive under such conditions. Generally it can be said that anything will impair the performance of a machine and affect brush wear which alters the ability of the air to oxidize copper and thus maintain a continuous well-polished cuprous-oxide film on its commutator.

It is well known that the surface of a piece of copper, which initially had a high polish, will be roughened by exposing it to repeated oxidation and reduction. However, continued operation of a commutator under ideal operating conditions increases rather than diminishes the finish of the surface. It is believed that a properly chosen brush polishes the oxide film as it is formed and prevents the roughening which otherwise would follow as a result of the repeated oxidation and reduction. Thus the *ideal* brush is an excellent burnishing tool: it "flows" rather than abrades the commutator film. *Actual* brushes, however, to a greater or lesser degree, fail to maintain a burnished continuous layer of cuprous oxide which is substantially free of other materials. Frequently the film becomes very rough in service and so nonuniform in its composition and properties that cleaner brushes are added to remove this undesirable film. Or, as a last resort, the commutator may be "re-turned" and the development of a new film attempted, probably with the aid of another brush grade.

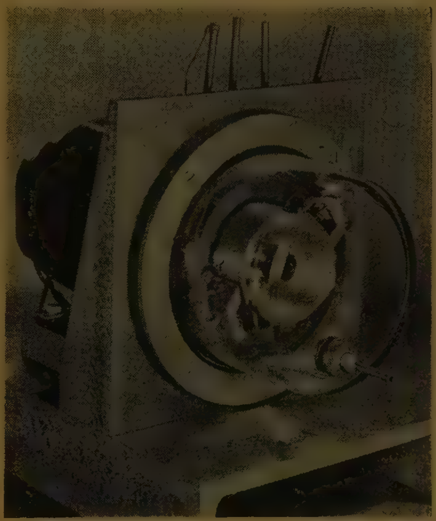
Figure 2. A simple high-altitude test chamber for testing slip rings

In view of the foregoing discussion it can be easily understood why the heavy-duty airplane generators present such a difficult problem to the designer. As these generators must be as light as possible, their commutators and brushes have to be loaded to the limit. Heavily loaded commutators retain an oxide film only with difficulty even in ground service or in low-altitude flying. If the bombers are flown to an altitude of 33,000 feet, where the oxygen content of the air is approximately one fourth of that on the earth's surface, the commutator film naturally will not be maintained.

Lack of the active oxidizing agent, however, is not the only cause for the loss of the essential cuprous-oxide film in high-altitude flying. The water content of the rarefied air, is only one-thousandth that normally fed to electric machines. The extreme dryness of the ventilating air is a major cause of the disappearance of the film. The beneficial effects of water vapor usually are ascribed to lubrication of the brush-commutator interface by water vapor adsorbed in the film or brush. Possibly the true role of the water is that not of a lubricant but of a catalytic agent promoting the oxidation of copper by the oxygen of the air. As soon as the film on the commutator disappears, the brushes and copper grind one another away in a very short time.

Figure 1 shows two copper slip rings and brushes tested in our laboratory high-altitude tester. Both sets of brushes were seated on the rings while these were running in normal laboratory air. At the end of the seating period, the air was changed to simulate the pressure and dryness of the atmosphere at an altitude of 40,000 feet. After 15 minutes of operation, under electric load, one pair of brushes wore out. The faces of the brushes and their path on the copper ring were badly roughened. The second pair of brushes, tested under the same conditions for a testing time of seven hours, showed no measurable wear. The polish of the brush faces and of the brush track was unchanged. The first pair of brushes were of high-quality electrographitic material which has given excellent performance on electric machines under normal service. The second pair of brushes were from the same stock but had been treated for high-altitude operating conditions.

Epidemics of rapid brush wear occur frequently on ground-operated machines also. Such epidemics often, if not always, are associated with an unusually low water concentration in the air. The trouble often is avoided by feeding steam into the air of a generator room during extremely cold weather. However, if high-speed exciter brushes suddenly disappear in a central station, although repercussions may be heard in the front office, a standby unit usually can be thrown on the line. Or, if the brushes chatter and disintegrate on the motors of a 90-mile-an-hour locomotive during subzeor weather, while the passengers may miss engagements, they still stay alive. But if the brushes on the fully loaded generators of a fighting bomber wear out, and the turrets cannot revolve and a dozen other units fail to function, the crew on that bomber may not survive.

When rapid brush wear was shown to be a real hazard to high-altitude flying, means to combat it were sought in many places. To study brushes under simulated high-altitude conditions it was necessary to design suitable testing equipment. A simple slip-ring tester is pictured in Figure 2 and diagrammed in Figure 3. Several of these "testing chambers" have been in constant use for several years in testing hundreds of

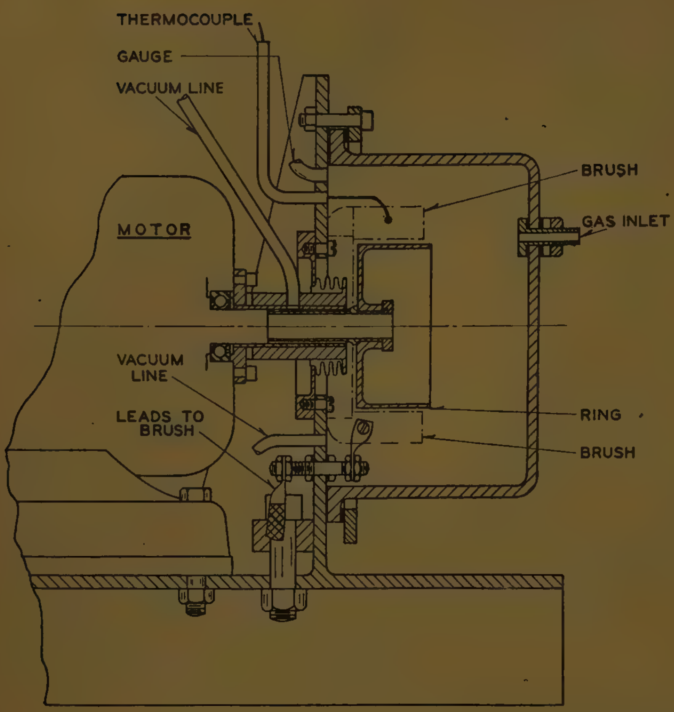

Figure 3. Cross section of test chamber shown in Figure 2

brushes. The tester consists of a copper slip ring $3\frac{1}{4}$ inches in diameter which is mounted on the shaft extension of a 3,500-rpm motor. This shaft enters the glass chamber through a close-fitting carbon tube. This tube is separately evacuated to a pressure low enough to prevent room air entering the chamber along the shaft. The pressure in the testing chamber is maintained at the desired "test altitude" while dry air is passed through to sweep out the impurities. With this tester the brushes can be operated under any mechanical or electric load throughout a wide range: the angle of the brushes can be changed, and the positive and negative brushes can run on the same or separate paths. The temperatures of the brushes and the double contact drop are recorded continuously. This tester has proved extremely useful and very reliable: it has the added advantage over most laboratory dusting testers that it gives results on the pessimistic rather than on the optimistic side.

This tester aided greatly in the search for a brush grade which would maintain a film on the commutator under all service conditions. It soon was found that for each brush grade there was a correlation between the "test altitude," the electric load, and the dusting tendency. A certain brush grade was giving long life in actual flight test. It was possible to predict on the basis of the evidence in Figure 4 that a moderate increase in the electric load in flight would cause this brush to fail. This prediction soon was verified when further flights proved that a load of 50 amperes or more would wear out in three hours the same brushes which promised a life of several hundred hours when the load was limited to 35 amperes or less.

Tests of a large number of untreated brushes failed to find any existing grade which would not dust in the laboratory tester. Therefore as the next step attention was turned to brushes impregnated with various organic compounds as lubricants. The use of paraffin and other waxes in brushes to lower their friction against copper is well known. Unfortunately, all of the organic treatments, which seem to behave as lubricants, were ineffective for this purpose as they are too volatile to stay in the brushes for more than a few hours when subjected to heavy loading.

After organic lubricants had failed to solve this problem, it was suggested by L. W. Chubb that an inorganic lubricant, molybdenum sulphide, be tried. This compound is stable, has a low vapor pressure at moderately high temperatures, and has been applied successfully as a solid lubricant in a number of specialized fields. It resembles graphite and certain other solids in having crystals of layer lattice structure.

Brushes treated with molybdenum sulphide failed to develop a characteristic dark cuprous-oxide film but transferred a thin film of a sulphide itself to the commutator. This acted as a protective lubricating film which was held tenaciously by the copper, and generators equipped with molybdenum-sulphide-treated brushes have shown low brush wear on high-altitude flights. However, sooner or later the sparking which is always present under the face of a brush, even when commutation is apparently black, must convert some of the molybdenum sulphide to molybdenum carbide. The latter is very hard and causes severe grooving of the commutator.

We arrived at the conclusion that a solid film having lubricating properties would be helpful. In accordance with this theory much thought was given to providing these solid lubricating films on commutators. It occurred to us to try lead iodide. This experiment yielded wholly unexpected and gratifying results. From previous experience it was expected that this soft yellow solid would behave like molybdenum sulphide by coating the copper with an adhering lubricating film. However, much to our surprise this did not happen. Instead the lead-iodide-treated brushes developed a characteristic dark satiny film that seemed identical with the normal cuprous-oxide film. This film then became burnished as it passed successively under the brushes and stayed on the surface even in air of low oxygen and water-vapor content. It was found that lead-iodide-treated brushes are able to maintain a perfect film even in the most adverse conditions. This we presume is made possible by the formation of an intermediate compound, cuprous iodide, which being extremely unstable is readily oxidized to cuprous oxide even in an atmosphere where the normal oxidation of copper would not take place.

If this explanation is correct, the rate at which a copper commutator can be oxidized has been increased greatly, even in air of low oxygen content and with almost no water-vapor content. After the favorable results obtained with lead iodide, it seemed reasonable to suppose that other iodides which do not have layer-lattice structure and which are not solid lubricants also should aid in the maintenance of an oxide film and thus prevent brush dusting. Actually many iodides have been tested in the laboratory and have been found to be effective.

An even more convincing evidence for the effectiveness of iodine compounds is the prevention of dusting by only a trace of iodine vapor. Under simulated-high-altitude conditions, untreated brushes, which dusted in the absence of iodine, showed low wear and developed an oxide film on a copper ring when iodine vapor was introduced.

As bromine vapor also prevents brush dusting, studies were extended to cover all metallic halides as a class. Tests were made of typical iodides, bromides, chlorides, and fluorides, and all were found effective. It is believed that each of them thermally decomposes at the brush-commutator interface to form the corresponding metal and free halogen. The latter may recombine, either with its original metallic partner, or with the carbon of the brush, or with a bare copper spot on the commutator.

However, the copper halides differ greatly in their thermal stability, the iodide being the least stable and the fluoride the most stable compound. The comparatively high stability of copper fluoride should slow down the rate at which it is oxidized. Under operating

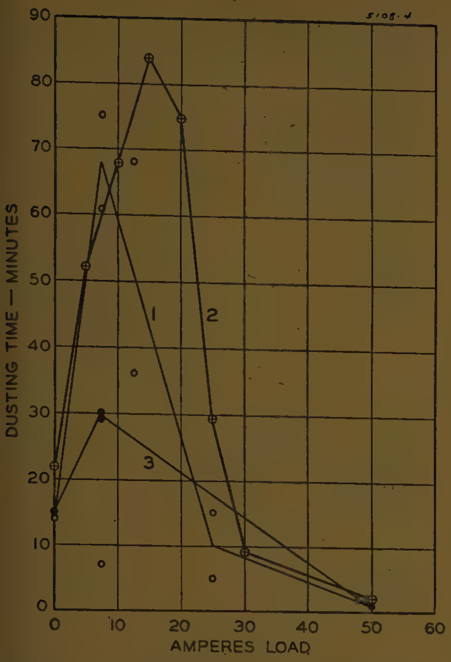

Figure 4. Amperes of load versus dusting time for an untreated hard carbon brush

Curve 1 at pressure equivalent to 30,000 feet; curve 2 at 35,000 feet; curve 3 at 40,000 feet. This brush does not dust at 25,000 feet

conditions this might lead to the existence of a film that contains substantial amounts of both the oxide and the fluoride of copper. It is therefore preferable to choose a treatment that, under the given operating conditions, develops a film substantially free of any compound other than cuprous oxide.

All of the metallic halides which have been tested as brush treatments are capable of limiting brush dusting under simulated-high-altitude conditions. Those selected for actual flight test have effectively limited brush wear. However, there are many practical difficulties in the use of many of these compounds. For example, certain halides, which are water soluble, but not deliquescent, and which contain no water of crystallization, can be put in the brushes from a water solution. However, these water-soluble halides accidentally could be leeched out of the brushes when the ventilating air duct connected to the generator happens to direct water onto the commutator and brushes. That is the reason why the easily soluble halides are less desirable as treating agents.

Difficulties also arise in the case of halides which are thermally unstable to such a degree that dissociation of the compound takes place, not only at the brush-commutator interface, but throughout the brush itself. The ideal halide should be thermally stable to temperatures up to those attained in the face of the brush. Another major objection to a great many halides is the fact that they adsorb water and dissolve in it. Or, one or more of the thermal-decomposition products may deliquesce in air or react so as to leave an undesirable residue in the brush or on the commutator.

The lead formed by the thermal decomposition of lead iodide may remain in the brush either as lead or as lead oxide. Some of it may be mechanically transferred and enter as an impurity into the copper-oxide film on the commutator. In any case, the physical and chemical properties of the solid residues are such that they are not detrimental to commutation. A number of the other metallic halides that were tested gave nonuniform films and high contact drops even though they did prevent actual brush dusting.

The war needs necessitated the early adoption of a treated brush. Lead iodide was chosen in preference to others as the most promising halide treatment for the first large-scale application. Most of the *P*-1 generators now in service have been equipped with Stackpole brushes containing this treatment. Nevertheless, tests with other halides have continued both here and elsewhere, and it is believed that at the present time all heavy-duty generators placed on high-altitude bombers are fitted with brushes containing one of the metal halides as an antidusting agent.

A number of practical problems had to be solved in the initial application of the lead-iodide treatment. The temperatures at which electrographitic brushes are fired are so high that the iodide must be placed in the plate after manufacture. This may be done either by immersing the brushes in the fused salt or by carrying out metathetic reactions in the plate by successive immersion in the proper chemical solutions. The latter operation must be done with great care, as trace amounts of excess reagents or products of reaction often affect brush performance disastrously, and the leeching out of small concentrations of such compounds is very laborious.

Many of the halides can be incorporated into metal-graphite brush grades at the time of molding. Some halides are so unstable that they react with the copper in the brushes at all practical molding temperatures. While lead iodide can be molded successfully into metal-graphite grades, care must be taken in the curing of the brushes; otherwise undesirable reactions take place between the copper and the iodide.

Brush treatments which are capable of maintaining an oxide film on a heavily loaded commutator operating under the adverse conditions encountered in the upper air certainly warrant a trial in difficult brush applications at more normal levels. Man power and material limitations have severely handicapped the execution of such tests. However, in such severe service as is imposed on the brushes of high-speed electric locomotives halide treatments definitely have improved commutator conditions and increased brush life. Recently a group of test brushes was removed from a locomotive after 85,000 miles of high-speed service. The brushes are in excellent condition, and the wear has averaged only ten mils per thousand miles of service.

Exploratory applications have been made in other fields where commutating conditions are unusually difficult. Each application presents a new problem. In all cases, the brush grade and the type and the amount of treatment must be properly matched to the machine and to the operating conditions. Though progress necessarily has been slow, experience has now been accumulated to prove that this new purely chemical approach to the old problem of forming and maintaining a satisfactory copper-oxide film on commutators and slip rings will prove as effective in many ground applications as it was in its initial one, namely, the solution of the high-altitude brush problem.

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Chemical Contamination as Causes of Electrical-Insulation Failure

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Synopsis: Electrical-insulation failures were investigated and found to be due to chemical contamination with ionizable materials. Examples of such contamination are certain types of hand creams, soldering fluxes, and germicides in lubricating oils. Contamination mechanisms often involve operations which are otherwise insignificant in manufacturing processes. The effect of these materials on the electrical characteristics of the insulation is reported. The need for close correlation of industrial-hygiene manufacturing processes, insulation design, and quality control is emphasized.

ELECTRICAL-INSULATION troubles sometimes occur which baffle the observers. Frequently the failure is attributed to some entirely innocent cause. Several such failures when carefully studied were found to be caused by the presence of ionizable substances which when dry were not harmful but when exposed to humidity were a constant hazard in the presence of a voltage differential. The insidious behavior of such materials in insulation may allow apparatus to pass a high-potential test and subsequently cause failure during a less severe test or in service. Furthermore, the presence of ionizable materials near a damaged spot in insulation increases the hazard manyfold. As insulation ages and the protective varnish coating deteriorates, these contaminants become a more active hazard to insulation. Harmless-appearing substances such as hand creams and soldering fluxes may be sources of chemical contaminants which cause inestimable damage.

As a consequence, investigations of such failures must be thorough and include each component material down to and including the glass, mica, cotton, and other material which make up the insulation. Careful attention should be paid to recent changes in methods or technique and even to safety and hygiene campaigns. The examples which follow illustrate only a few mechanisms of chemical contaminations.

Soldering Fluxes

Several years ago trouble was experienced with a number of cases of short-circuited commutators as a result of unauthorized use of zinc-chloride soldering flux. One commutator in particular for an experimental motor was an outstanding example. After several short circuits occurred, the bar-to-bar insulation resistance was checked. Seventy bars out of 300 had far below normal insulation resistance, which were about 0.1 megohm instead of above ten megohms. Upon application of a 60-cycle dielectric test it was found that bars with low insulation resistance stood a 200-volt test but generally broke down at 400 volts. Bars with normal insulation resistance withstood a 600-volt test and failed at not less than 800 volts.

Mica segments were removed from the spots having low insulation resistance for chemical tests in comparison with uncontaminated mica segments. A 15-gram sample of each mica segment was boiled with 15 milliliter of distilled water and the conductivity of the solution determined as shown in Table I.

This test clearly indicated the presence of ionizable contaminants in the low-resistance mica segments which increased the conductivity of the extracted solution by a factor of more than ten to one as compared to a similar extract of uncontaminated insulation. Chemical analysis of the solutions clearly indicated the presence of zinc and chlorine in the low-resistance solutions, whereas the control specimens had less than one part per million of either.

Erikson[1] points out the effects of a combination of soldering flux and moisture on fine-wire magnet coils. Failures were due to corrosion of copper by fluxes which caused open circuits. Failures to ground and short circuits due to electrolytes from the corroded copper and from the soldering fluxes themselves forming conducting paths are also possible in this type of application.

Fluxes are available in many forms. The acid type, zinc and ammonium chloride, are available in water solution or in the form of a paste using a petrolatum or other grease vehicle. Milder fluxes include rosin and alcohol solutions and rosin–petrolatum pastes. An example of a zinc-chloride water solution on mica V rings has been discussed previously. Zinc chloride, the electrolyte on these V rings, was deposited from water solution throughout the insulation. Under humid conditions these electrolytes formed a conducting path which leads to the burning of the mica as seen in Figure 1. Zinc- and ammonium-chloride fluxes in water contribute to corrosion of copper.

The effect of zinc chloride on insulation under moist conditions is summarized in Table II. These results were determined on sample coils similar to those shown in Figure 2 and on mica by a jig similar to that shown in Figure 3. Sample coils had been immersed in zinc-chloride solution and others in distilled water for two days and then allowed to stand in air for

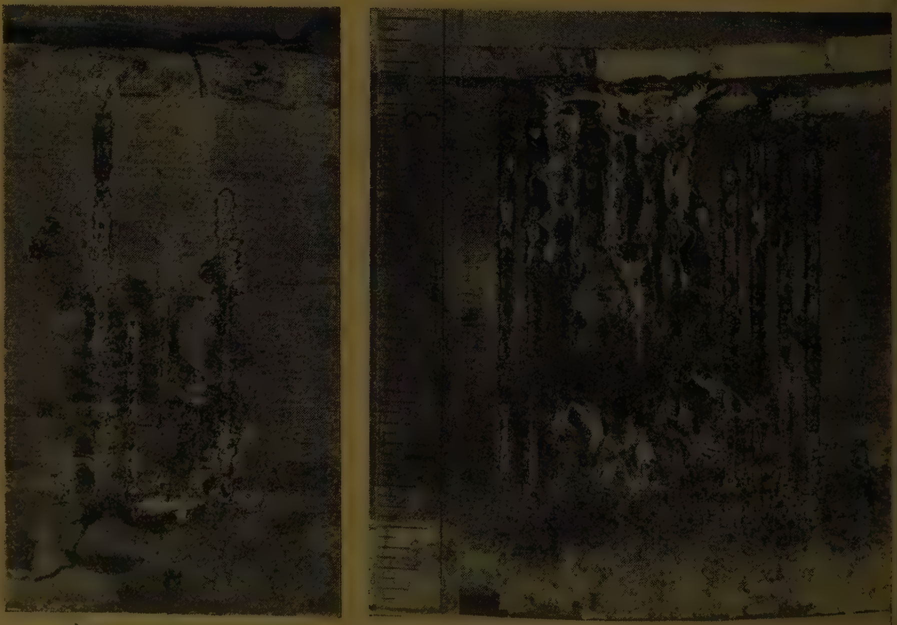

Figure 1. Typical failures of commutator mica V rings due to acid flux on segments

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The authors are grateful for the contributions and criticisms of D. L. Gibson, R. A. Lubker, R. L. Ferguson, and P. W. Hernick, all of Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

Table I

Segment Identification	Resistance (Ohms)
2	600
13	1,000
20	1,200
21	950
23	950
24	1,210
35	590
Uncontaminated control segment	13,600
Uncontaminated control segment	16,000
Fresh distilled water	35,000

Table II. Insulation Resistance in Megohms

Wire Insulation Used	Soaked in Flux for 48 Hours, Stood in Air 7 Days	Soaked in Distilled Water for 48 Hours, Stood in Air 7 Days
Asbestos	0.15–43	4,000
Glass	15 –29	1,800
Glass (sliver)	3.8 –18	3,000
Double cotton	16 –39	72
Mica*	0.25– 0.30	30–75

Fluxed samples were soaked in the following solution for 48 hours:

Zinc chloride	1 pound
Ammonium chloride	2 ounces
Water	3.3 gallons

These together with the distilled-water-soaked samples were allowed to stand one week in the air before testing.

* Aged one week at 100-per-cent relative humidity.

seven days. Insulation resistances of the coils then were taken and are reported in Table II.

Insulation resistance of cotton-covered wire was poor after this treatment in distilled water but was even lower after the acid flux treatment. The asbestos-insulated wire after treatment in flux had an insulation resistance of 0.5 per cent of that treated in distilled water. Mica behaved in a manner similar to the wire insulation.

Zinc chloride or ammonium chloride may be mixed into a petrolatum vehicle as a paste. In this form corrosion and electrical failures due to use of acid fluxes may be apparently reduced. The dangerous electrolytes still are introduced into the insulation; their effects are merely postponed. Under humid conditions, particularly after the petrolatum vehicle has disappeared, the zinc and ammonium chlorides become conductors. They lower insulation resistance and decrease breakdown strengths of insulation. This leads to failures in service rather than failures in tests during manufacturing the latter which might occur if water solutions of these fluxes were used.

Rosin and alcohol, another type of soldering flux, are relatively safe for use near insulation. No electrolytes are deposited, and the corrosion problem is greatly reduced. This flux has been in use many years without causing failure epidemics.

Each worker should be informed thoroughly as to the proper flux for his application. Acid-type fluxes may be used on parts which are not insulated if the flux deposits are washed carefully from the metal after completion of the operations. No form of soldering flux which contains zinc chloride, ammonium chloride, or other electrolytes should be tolerated for use near insulation. Rosin in alcohol solutions, or other organic nonelectrolytic types of flux, should be used in all doubtful cases. The security in using an electrolyte in grease mixture as a flux is false. It should be remembered that these electrolytes will ionize and become conducting under humid conditions, and, in addition, many of these substances are hygroscopic and thus will become conducting in only moderate humidity conditions.

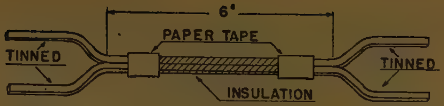


Figure 2. Coil for testing soldering fluxes on insulation

Hand Creams

The use of hand creams in shop practice was suspected as a cause of insulation failures. Protective-hand-cream use was being encouraged in varnish-treating rooms, in coil-insulating, and wire-insulating departments as insurance against dermatitis. Such creams are effective in prevention of irritation due to varnish solvents and adhesives. However, investigation proved many creams contained conducting materials which are corrosive to copper or harmful to insulation.

Schwartz[2,3] points out that ointments fall into a number of classes:

1. Simple vanishing-cream types which fill the pores with soap and facilitate removal of soil when washing after work.
2. A type which leaves a thin film of resin or wax on the skin and thus prevents the irritant from direct contact with the skin.
3. A third type which fills pores and covers the skin with a harmless fat that repels water-soluble irritants and prevents entrance of petroleum oils, greases, and coal-tar derivatives.
4. A fourth type which contains nonirritant chemicals intended to detoxify the irritants.
5. Another form which causes inert powders to adhere to the skin, forming a physical barrier against skin irritants.

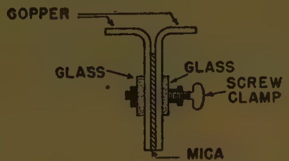


Figure 3. Device for testing soldering fluxes on mica

A series of determinations was made to discover the conducting properties of the various hand creams in use in the shops and the effect of ointments and humidity on ointment-contaminated insulation. Six-mil filter paper was wrapped around five-eighths-inch-diameter glass tubes. Various creams were rubbed into the paper, and the insulation resistance was determined between two electrodes $1^1/_2$ inches apart. These tests were conducted at 20-, 50-, and 97-per-cent relative humidity. Test bars also were prepared of asbestos-insulated wire onto which various creams had been applied. These wires were tested for insulation resistance between strands and for dielectric breakdown.

It was found that those creams which had soap bases and were therefore alkaline in reaction became conducting at higher humidities and lowered insulation resistances. This is reflected by the lower insulation resistances of certain hand creams upon the porous paper tubes (Table III). Hand creams lowered the insulation resistance and dielectric breakdowns when test bars of asbestos-insulated wire were subjected to hand creams and humidity (Table IV). These humidity conditions approximate summer weather conditions. The hand creams which proved most troublesome

Table III. Effect of Hand Creams on Insulation Resistance in a Humid Atmosphere

	At 20–30-Per-Cent Relative Humidity (Megohms)	At 50-Per-Cent Relative Humidity (Megohms)	At 97-Per-Cent Relative Humidity (Megohms)
A	950	17	0.095
B	Over 1,000,000	22,000	1.3
C	200,000	200	0.06
D	Above 200,000	8,000	2.5
E	Above 200,000	6,500	4.6
F	Above 200,000	20,000	7.7
G	Very high	150,000	12

Conductance in water solution was measured in a cell with platinum electrodes by an a-c conductivity bridge at 20 degrees centigrade after suspending the creams in 20 parts of water

	Resistance in Ohms
Distilled water	156,000
A	975
B	2,600
C	750
D	4,370
E	19,300
F	4,500

A = alkaline soap base
B = neutral grease
C = alkaline soap base
D = neutral grease
E = lanolin and olive oil
F = alkaline soap base
G = blank paper

A contains Bentonite clay.

C is a clear solution in water when hot, setting to a gel on cooling, like soap. Both these contain water and are mildly alkaline.

C has a tendency to turn copper green.

B contains grease and no water, with a mineral filler. It is neutral in reaction.

D and *E* are neutral greases.

F is alkaline in reaction. These do not contain water.

Table IV. Insulation Resistance and Dielectric Breakdown on Asbestos Wire Test Bars

Cream Used	(Megohms) Insulation Resistance		Dielectric Breakdown In Volts (Failed at)	
	At Room Condition	After Conditioning at 70-Per-Cent Relative Humidity	At Room Condition	After Conditioning at 70-Per-Cent Relative Humidity
A	500– 1,400	8 and 14	1,500	2,000
B	11,000–13,000	120 and 130	2,000	2,500
C	1,800– 2,000	0.08 and 0.08	2,000	800–800
D	13,000–13,000	5 and 4.6	2,000	2,500
No cream	1,400– 2,300	10 and 8.5	2,500–2,000	2,000–2,500

A = alkaline soap base *B* = neutral grease *C* = alkaline soap base *D* = neutral grease

Test conditions:

Strands rubbed with protective hand creams, formed into straight-side test bars, and hot pressed.

All tests made between two conductors at room conditions (20–30-per-cent relative humidity), and after conditioning 48 hours at 70-per-cent relative humidity, and 25 degrees centigrade.

Test bars were allowed to stand one week before testing.

were of the alkaline soap-base vanishing-cream type.

One of these soap-base hand creams was applied to copper panels which were dipped into varnish. The cream tended to dissolve in the varnish, thus contaminating it, and to melt and run during the baking operation. As a consequence a very poor film resulted which suffered a 75-per-cent decrease in dry dielectric strength over a control panel prepared clean and free of hand cream.

On the basis of these data, the hand creams used in manufacturing were limited to the types which leave a thin film of resin or wax on the hands but which are neutral in reaction. The use of these hand creams is not detrimental to insulation if moderation is practiced. However, each hand cream used should be investigated thoroughly by the engineers and industrial-hygiene department before its use is authorized.

Germicide Additions to Lubrication Oils

An excellent example of the type of failures produced by contamination of insulation with ionizing materials is shown in Figure 4. This was found to be due to the addition of a very small percentage (0.4 per cent) of germicide to the compound used for lubricating the blocks for pressing armature coils to size. Tests showed that the addition of one per cent of the lubricating compound to the germicide raised the conductivity of distilled water by a factor of ten, and the addition of one per cent of the germicide raised the conductivity of distilled water by a factor of 200. Chemical analysis of the coil-insulation and winding material adjacent to this failure disclosed no inorganic salts (sodium zinc, chloride, or sulphates). The germicide used was of the phenolic type. A phenol coefficient of at least 18 (Federal Druggists Association method) was specified. This is not, however, a measure of the compound's conducting properties. However, phenolic-type compounds are acidic and hence conductors; organic germicides of this type are chemical relatives of carbolic acid. Insulating materials pressed in molds lubricated with the compound having germicide were found to have high conductivity when humidified. The evidence appeared to be conclusive that this was the cause of the creepage failure shown in Figure 2.

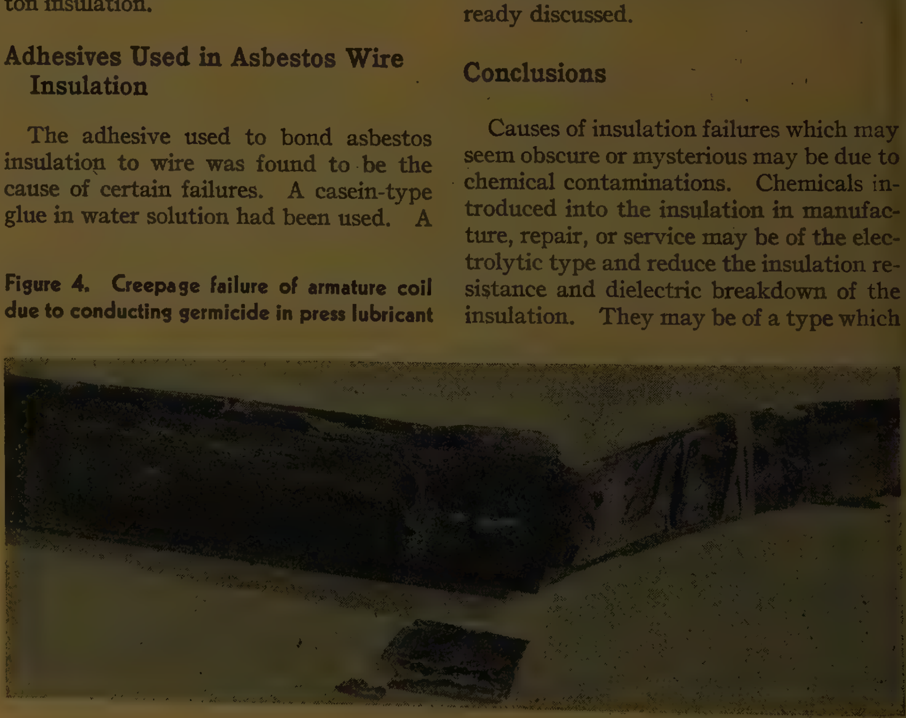
Motors Operating in Chemical Plants

A more obvious type of failure due to chemical contamination occurred in service. A large motor with class-*A* insulation was installed in a magnesium plant. After eight months service this motor was found to have three grounded field coils. Scrapings from the windings were analyzed by microchemical methods. The material was found to be chiefly magnesium chloride and other hygroscopic salts. This contaminant did not deteriorate the varnish film but formed a conducting path through cracks in the film and around the leads into the coil. Magnesium chloride also will decompose cotton insulation.

Adhesives Used in Asbestos Wire Insulation

The adhesive used to bond asbestos insulation to wire was found to be the cause of certain failures. A casein-type glue in water solution had been used. A change in formulation by the manufacturer had introduced inorganic salts. These, sodium carbonate and sodium phosphate, are electrolytes and in humid conditions corroded the copper. The insulation turned green, and failures resulted during standard proof tests. Inadequate drying of the glue contributed to the corrosion and allowed ionization to take place which led to early test failure of many of these coils. This situation was corrected by use of an organic-type synthetic adhesive which contained no inorganic salts.

Figure 4. Creepage failure of armature coil due to conducting germicide in press lubricant



Carbon-Dioxide-Type Fire Extinguishers

Proper construction of varnish tank systems should include some fire-control device. Frequently fire extinguishers are so arranged that they will release automatically in case of fire. In some cases, the extinguishers accidentally will set off and spray the surface of the varnish with the extinguisher's contents. Foamite-type extinguishers often are used which consist of a small container of an inorganic acid and a larger one of sodium carbonate. When mixed, carbon-dioxide gas will be formed which propels the mixture from the extinguisher and sprays the varnish with a mixture of electrolytes: unreacted acid, unreacted sodium carbonate, carbon dioxide, water, and the sodium salt of the inorganic acid. This contaminates the varnish with electrolytes and water, thus lowering its insulation qualities. Filtration methods[4] will not remove this type of contaminant. The use of this contaminated varnish would introduce electrolytes into the insulation which would prove as detrimental to the machine as the other types of ionizable chemical contaminants already discussed.

Conclusions

Causes of insulation failures which may seem obscure or mysterious may be due to chemical contaminations. Chemicals introduced into the insulation in manufacture, repair, or service may be of the electrolytic type and reduce the insulation resistance and dielectric breakdown of the insulation. They may be of a type which

A New Outdoor Air Switch and the Principles Involved in Its Design

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THE OUTDOOR air switch is a universally used device which through the years has evolved into a few well-known types. Increasing preference has been shown for the rotating-insulator vertical-break switch, which has become almost a standard for outdoor service. A new switch of this type is described, and a discussion of the problems associated with its design is presented.

It may seem, at first glance, that the design of air switches should have been reduced to a few simple calculations and a few hours' work on the drawing board. As a matter of fact, their correct design requires a long period of development, including many calculations and laboratory tests. It is a comparatively simple task to design a switch that fulfills the essential requirements when new, but it is much more difficult to design an economically practical switch that will retain its characteristics after years of use outdoors where sun, wind, ice, and dust prevail and where corrosion is always at work. In the course of development of a new outdoor switch, a thorough study was made of the basic problems involved without regard to existing opinions on the subject. A complete review and analysis of past and present-day switch designs followed to determine the relative merits and defects of each. This study led to new approaches to some of the problems including the use of recently developed materials.

Design Problems

Reduced to its essentials a switch consists of two terminals, a blade hinged on one terminal to open and close the circuit between the terminals, a suitable base for mounting, and insulation between the terminals and ground. Past experience with air switches indicates that the contacts between the blade and the terminals are likely to be the chief source of trouble. For this reason, the subject of contacts must be given first consideration in the design of any switch. That there is a wide divergence of opinion on the subject is shown by the fact that almost every known type of contact can be found on present-day switches.

The blade mechanism—that is, the means for converting rotation of the insulator into the desired blade motion—is the second consideration. The type of blade mechanism selected is the determining factor in the ability of the switch to operate with ease under both normal and adverse conditions. The choice of a mechanism for performing this operation is influenced by the type of contacts adopted, since certain types of contacts require a more elaborate mechanism than others, if the switch is to operate easily and reliably.

Fortunately, the insulation no longer presents a problem to the outdoor-switch designer. As a result of excellent work of standardization committees, electrical and mechanical properties of outdoor air switch and bus insulators, as well as minimum spacings for switch insulators, all have been standardized. The resulting advantages to user and manufacturer are obvious.

Contacts

There are only two major requirements of air-switch contacts:

1. They must carry their rated current without reaching injurious temperatures, not only when new, but also after many years of service.

2. They must carry without injury the maximum short-circuit current of the system to which they are applied, until this current is interrupted by a circuit breaker. This ability also must be maintained for the life of the switch.

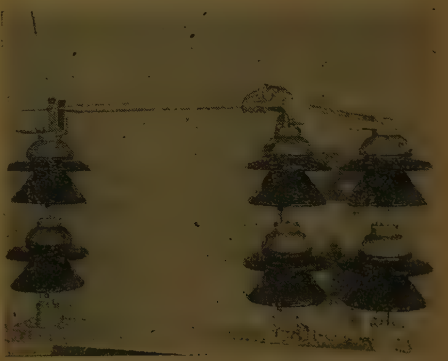

Figure 1. The new disconnecting switch, 69-kv 600-ampere rating

Low contact resistance is necessary to meet both of these requirements. As early as 1892[1] the importance of the total pressure in regard to its effect on contact resistance was realized. In 1913 F. W. Harris[2] showed that the contact resistance varies inversely with the total pressure and is substantially independent of the

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is harmless when dry and at room temperatures, but which will become conducting under high humidity conditions, particularly when hot. The contaminant may cause metallic corrosion which produces electrolytes, and this tends to accelerate failure. Failures due to contaminations may occur in epidemics in factory testing or may not occur until the machine is in field service.

Close co-operation between the manufacturing, hygiene, and engineering departments is necessary to control this problem. Each worker should be educated in the importance of his operation on the final product and the ever-present danger that he may introduce dangerous chemicals into insulation from apparently harmless sources. All precautions should be taken, particularly in the use of new materials, to see that toxicity and irritation are eliminated, but not at the expense of insulation quality. Hygiene campaigns should be co-ordinated with insulation engineering to protect both the workers and the apparatus quality.

Both materials in everyday use and those which are new material should be investigated thoroughly as possible sources of contaminations. Use of chemicals even for insignificant-appearing jobs should be reviewed. Review of soldering fluxes, for instance, is essential.

Changes in manufacturing technique should be considered carefully. A small change may cause more damage than more obvious changes. Possible sources of chemical introductions into insulation should be investigated thoroughly, and the harmful effects of this type of contamination should be kept ever in mind.

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area. This was confirmed some years later by A. R. Enger,[3] who found experimentally that the temperature rise of switch contacts was not increased appreciably by slight misalignment of the blade in the switch jaw, since this decreased the contact area but increased the unit pressure. In 1928 a mathematical relation between the contact resistance and the total pressure was developed by W. Schaelchlin.[4] He showed that

$$R = cP^{-k}$$

where R is the contact resistance, c a constant depending on the material and shape of the contact, P the total pressure, and k a constant which may have any value from 0.5 (if the number of contact points remains constant as the pressure increases) to 1.0 (if the number of contact points increases linearly with the pressure).

The shape of contacts is important, inasmuch as it affects unit pressure and the amount of material immediately adjacent to the contact. There are three general shapes of contacts: plane or surface contact, as between two planes; point contact, as between a sphere and a plane; and line contact, as between a cylinder and a plane.

The surface contact when new and clean is entirely adequate for both rated and fault current. However, because of low unit pressure it is extremely susceptible to corrosion. While formerly widely used on outdoor air switches, the surface contact now is used chiefly for low-current indoor knife switches.

The point contact, used to some extent in air switches,[5] meets the requirements of high unit pressure and low contact resistance and is completely adequate for carrying rated current. However, under fault conditions the small amount of metal adjacent to the contact limits the rate at which heat is conducted away and thereby limits the short-circuit current that can be carried without damage.

The line contact has found increasing use in recent years and has proved very satisfactory. B. W. Jones[6] determined experimentally that, with copper contacts and pressures between 10 and 500 pounds, one inch of line contact has only one half to one third the resistance of a surface contact one inch square. This results in excellent characteristics under normal load currents. The amount of material adjacent to a line contact available for heat conduction is inherently greater than that adjacent to a point contact and is, therefore, able to carry the heat away from the contact more rapidly with the result that much higher fault currents can be carried without damage. This principle is used widely in the high-pressure single-line switch contact and the modern medium-pressure, multiple-line contact.[7]

The material for contacts must be selected on the basis of its resistivity, chemical activity, hardness, and annealing temperature. The initial contact resistance varies directly with the resistivity of the contact material.[2] However, this resistance tends to increase with age, due to the formation of surface films. The base metals, such as copper and its alloys, form a high-resistance oxide film when exposed to air, the rate of oxidation being comparatively low at room temperature, but becoming more rapid at elevated temperatures. The noble metals do not oxidize in air, but silver, which is a widely used contact material, does react with sulphur in the atmosphere to form silver sulphide. However, this film has comparatively low electric resistance, is thin, and is easily rubbed off. The high electric and thermal conductivity of silver make it an ideal material for indoor or enclosed locations where a medium-pressure contact is satisfactory.

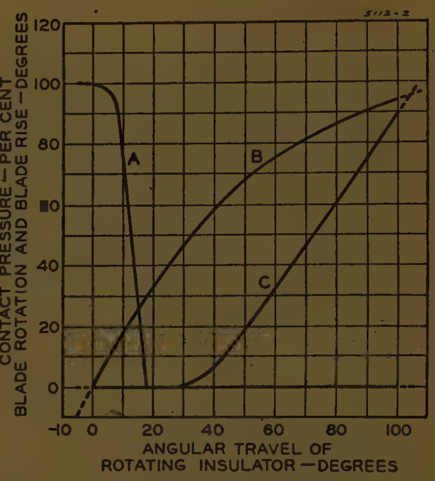

Figure 2. Curve showing relative motion of moving parts

A—Contact pressure in per cent
B—Blade rotation in degrees
C—Blade rise in degrees

Contacts exposed to the weather must meet additional requirements, since dirt, dust, and corrosion accumulate more rapidly and since there is likelihood of ice formation. Here the contact must have sufficient pressure to break through coatings of ice, dirt, and corrosive films. Even in enclosed locations, rubbing action, desirable on silver, is essential on copper and its alloys in order to remove corrosion from the surface. In high-pressure rubbing contacts soft metals, such as silver and copper, are subject to excessive wear and galling, and harder alloys must be used. These alloys must not anneal when heated to temperatures of several hundred degrees centigrade, which may be reached under short-circuit conditions.

Both flexible shunts and contacts are used on the hinge end of outdoor switches at the present time. Shunts are subject to oxidation and corrosion, particularly in sulphurous industrial atmospheres, and the combination of corrosion and flexing may break the fine wires until eventually the shunt breaks completely. The same conditions that cause rapid deterioration of shunts also cause rapid deterioration of exposed contacts. At the break jaw contacts necessarily must be left exposed. At the hinge connection, however, it is not necessary to use either a shunt or an exposed contact, since the contacts can be completely enclosed, protecting them from the elements and, to a great extent from corrosive atmospheres.

Blade Mechanisms

There are two chief requirements of blade-operating mechanisms:

1. They must permit easy operation of the switch, even after a number of years in service with infrequent operation.

2. They must permit operation of the switch when it is covered with ice.

The effort necessary to operate switches is due entirely to friction, except in the opening operation on small switches where the weight of the blade is not counterbalanced. The only essential friction is that produced by the rubbing motion in contacting the switch. This motion should be separated from the blade-elevating motion and accomplished with maximum mechanical advantage. All other friction is incidental to switch operation and should be reduced to a minimum. For this reason, the number of pins and bearing points should be as small as possible. In normal service wear on a disconnecting switch can be neglected and, without maintenance, its life depends on freedom from corrosion of bearings and contact surfaces. Even though bearings must be made of materials that are corrosion resistant, they still should be designed with ample clearances so that corrosion will not cause seizing.

The earliest rotating-insulator vertical-break switches were merely hook-stick-operated switches with the hook-stick eye omitted and with a third rotating insulator added. A crank at the top of the rotating insulator raised and lowered the blade. In 1924 Koppitz conceived the earliest of the modern dual-motion switches. These switches utilize a second motion, usually a continuation of the first (blade-closing) motion to apply contact pressure during or after entry of the blade into the break jaw. This contacting motion makes use of a much greater mechanical advantage than is available for the blade-closing motion. The dual-motion switch made possible the high-pressure switch contact as known today.

Contact pressure can be applied in a number of ways. In some switches the blade is pushed forward into the break contact. In others contact tips are expanded by two blade members to engage the break jaw. Still others use a flat blade tip which is rotated in a resilient break jaw to establish contact pressure. Though each of these has its advantages, it must be kept in mind that each additional bearing is one more point to corrode and stick; each additional link or lever is one more part which may break or become frozen in ice. The mechanism should, therefore, be as simple and contain just as few moving parts as possible.

Description of Switch

The switches designed as a result of these studies are unique in their simplicity with only three moving parts and one exposed contact. The 69-kv 600-ampere pole unit shown in Figure 1 is typical of the line of switches, which are of the rotating-blade type. The rotating-blade motion was chosen because it permits application and release of contact pressure at the break jaw with a single blade member and with maximum mechanical advantage. It obviates the necessity for a number of parallel links at the hinge end, and it eliminates the possibility of cantilever stress on the jaw insulator due to switch operation. It further permits large deflections of the break-jaw insulator stack due to stresses from connecting lines or busses without affecting operation of the switch.

As the switch is opened, the blade is rotated until the contact pressure is fully released, then with the contact completely free the blade is elevated to the open position. In closing the reverse sequence is followed. The contacting motion is accomplished with high mechanical advantage to overcome the high frictional forces. After contact pressure is released, these forces are no longer present, and the blade is raised easily. This complete release of contact pressure before vertical movement of the blade is essential for easy operation. The relations between insulator rotation, blade rotation, and blade rise are shown in Figure 2. This ideal sequence of blade motions has been accomplished by the use of independent means for performing the two functions.

The mechanism for obtaining this sequence of motions is shown in Figure 3. An operating arm, bolted to the top of the rotating insulator stack and pivoted in the hinge support, engages two separate parts to give the independent motions. The blade-elevating motion is obtained through the action of the roller at the end of the operating arm in a slot on the inside surface of the mechanism housing. This slot is designed so that the portion corresponding to the first 27.5 degrees of insulator rotation lies on the circle traversed by the roller in revolving about the center line of the insulator stack. The housing and axis of the blade therefore remain stationary, while the roller rolls in this portion of the slot, rotating the blade only on its own axis. The next portion of the slot is inclined so that the blade is accelerated uniformly to its final velocity, which is maintained to the full open position by uniform rotation of the insulator.

The contacting motion is accomplished through a forked crank rigidly fastened to the hinge end of the tubular blade, engaging the operating arm in such a way that rotation of the arm results in rotation of the blade. The physical relation of the arm and the fork is such that one degree of movement of the rotating insulator twists the blade approximately two degrees when the blade is within the jaws. The rapid initial twisting of the blade permits complete disengagement of the contacts before the elevating movement of the blade begins. Figure 4 clearly shows the sequence of these motions.

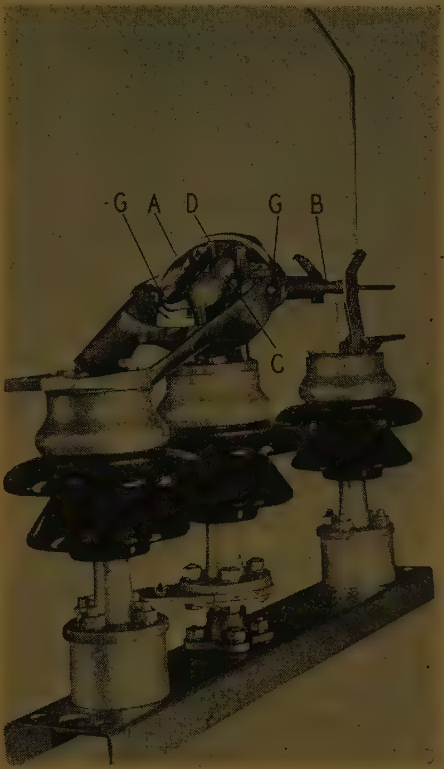

Figure 3. Early model of the new switch with special cutaway housing, which shows the mechanism for operating the blade (hinge contact not shown)

The tubular blade *B* is mounted in the mechanism housing *A*, free to rotate on its own axis. The fork *D* is part of the blade and rotates with it. The housing *A* rotates about the horizontal axis formed by the hinge bearings *GG*. The operating arm *C* extends through the fork *D*, and a roller at its outer end rolls in a groove in the inner surface of the housing *A*. To operate the switch the center insulator is rotated by turning the lever near the base. This rotates the operating arm *C*, which is bolted to the top of the rotating insulator. During the first 27.5 degrees of rotation of the operating arm, the arm engages the fork and rotates the blade to release the contact pressure on the jaw at the far end of the switch. Then the roller strikes the inclined portion of the groove, and, since the roller is constrained to revolve in a horizontal plane, the housing is rotated until the blade reaches a vertical position

Figure 5 is a top view of the center portion of the switch indicating the current path and showing the general arrangement of the hinge contacts. One end of the internal contact member bears against the heavy hinge stud to form a self-aligning annular-line contact. The other end bears against a machined surface on the fork to form a straight-line contact, which is also self-aligning. Contact pressure is supplied by a compression spring with adequate deflection so that the loss of contact pressure would be negligible even under the most severe conditions of contact wear. The hinge stud is threaded in the tie casting and locked against turning by a special locking washer and lock nut, which maintain intimate high-pressure contact at this point.

The mechanism housing completely encloses the hinge contacts and protects the moving parts from ice formation, so that the switch can be operated readily under heavy ice and sleet conditions. Furthermore, because the parts are protected from sun, rain, dust, and harmful atmospheric gasses, corrosion is very greatly reduced inside the enclosure, and lubrication is unnecessary.

Troubles due to annealing of work-hardened nonferrous materials, such as hard copper and phosphor bronze, have been eliminated by the use of precipitation-hardened (heat-treatable) alloys at the break contact. The jaw is beryllium cobalt copper, a comparatively new but already well-known and proved material. Under short-circuit conditions beryllium cobalt copper will not be affected at temperatures which would completely anneal work-hardened copper or phosphor bronze. This guarantees that the hardness of 90 Rockwell *B* and tensile strength of 90,000 pounds per square inch will be maintained for the life of the switch. As the physical characteristics of the material will be unchanged throughout the life of the switch, the jaw itself is used as the spring member as well as the current-carrying part.

The formed contact over the flattened end of the blade is chromium copper. This alloy, developed within the past decade, is similar to beryllium cobalt copper in that it is also precipitation-hardened. Though its strength and hardness are not so great, it has an electric conductivity 80 per cent that of copper, as compared with 45 per cent for beryllium cobalt copper. Hence, it is an ideal material for contacts in which extremes of hardness and deflection are not required.

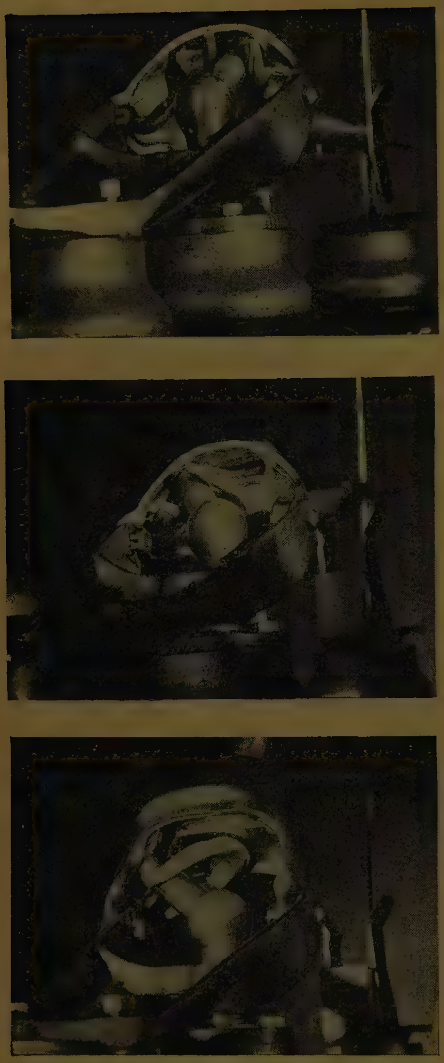

Figure 4. Three views of the switch of Figure 3 showing the blade in various positions

The beryllium–cobalt–copper jaw and chromium–copper contact tip permit high-pressure rubbing line contact capable of breaking up and wiping away coatings of dirt or oxide. The break jaw has adequate deflection to shatter ice formations. The dissimilarity in the contact materials assures a minimum of wear and eliminates the possibility of galling. These factors, together with the nonannealing characteristics of the alloys, assure that the break contact characteristics will remain unchanged after years of severe service even under extreme climatic or atmospheric conditions.

Ease of operation has been considered essential in the design of these switches. The enclosed blade mechanism just described provides high mechanical advantage during the contacting operation where friction is greatest. Friction has been reduced elsewhere by the use of a minimum number of moving parts and bearings. The rotating insulator columns of switches above 15 kv are equipped with either dry-type ball bearings or with roller bearings having pressure fittings for easy lubrication. In the higher-voltage and higher-current ratings, the switches are provided with springs to counterbalance the weight of the blades. The springs work on an arm attached to the under side of the mechanism housing and are mounted within the hinge support for complete protection from the weather.

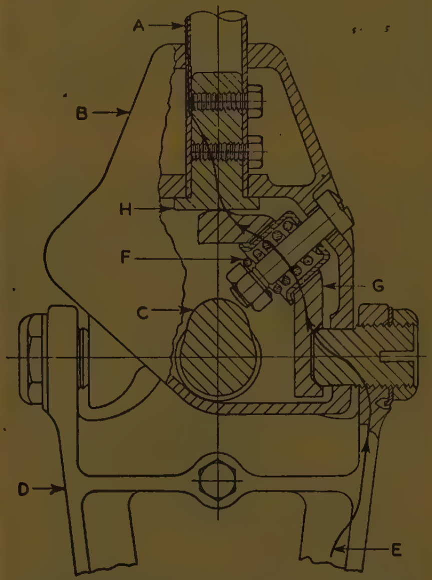

Figure 5. Partial top view of the switch showing construction of hinge contacts

A—Blade
B—Mechanism housing
C—Operating arm
D—Hinge support
E—Current path
F—Spring
G—Internal contact member
H—Fork

Tests

In the development of this switch numerous models were built and subjected to tests to check their performance under normal and extreme operating conditions. Temperature-rise tests were made on all ratings to insure that the switches would carry their rated current with a temperature rise of less than 30 degrees centigrade. Pole units were operated as many as 10,000 times to check wear on contacts, bearings, and operating mechanism parts. Short-circuit tests were made on all ratings to insure that the switch would carry its rated short-circuit current without mechanical injury and without damage to the contacts. To check their ability to operate properly under ice and sleet conditions, various ratings of switches were operated when covered with over a half inch of ice. Tests have been made to verify the effectiveness of the smooth contours in reducing corona and radio interference. Though flashover and withstand tests are considered insulator characteristics, 60-cycle wet and dry tests and impulse tests have been made to check over-all switch characteristics.

Summary

The new outdoor air switch strikes a new note in simplicity, as it has only three moving parts. All parts are simple and rugged with stresses kept to safe values for the materials employed. Hinge contacts and live parts with relative motion are protected from rain, dust, corrosion, and ice by a housing having smooth contours to minimize corona losses and radio interference. Critical adjustments and alignments have been eliminated. The sole exposed contact, at the break end, utilizes new materials which can withstand severe overcurrents and years of weathering without damage. Because of these features the new switch offers improved ease of operation and maximum reliability under all conditions.

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Design of Induction-Heating Coils for Cylindrical Nonmagnetic Loads

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EQUATIONS are presented for the calculation of the required number of coil turns to match a given power source for solenoidal coils with coaxial cylindrical loads of nonmagnetic materials in electromagnetic induction circuits. The equations apply to both solid and hollow loads. A correction factor is established for coil shortness providing the coil length is equal to or greater than the coil diameter. Also given are equations for power distribution and impedance values. The accuracy of the calculations is substantiated by experimental results.

At the present time induction heating of nonmagnetic materials is used in annealing, forging, brazing, melting, heating for shrink-fitting, and other purposes.

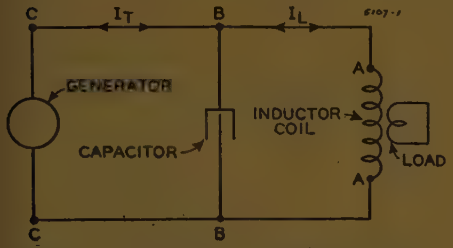

Figure 1. Conventional induction heating circuit using motor generator set

In any application a proper match must be obtained between the heating circuit and power source. In the case of multiturn coils connected directly to the output of a constant-frequency generating device, if the coil has too few turns, the output voltage is low at the rated output current of the generator. Consequently rated power output from the generator cannot be realized without employing some other matching device (transformer or series reactor). If the coil has too many turns, at rated generator output voltage, the current is low. To obtain the desired match to a given power source with a direct connected inductor, a specific number of coil turns must be used for any given physical dimensions. In the past determining the proper number of turns has been a cut-and-try process with many users of induction-heating equipment.

Baker[1] has presented equations for calculating the electrical characteristics of solenoidal coils with coaxial cylindrical loads of nonmagnetic conductors. The equations apply with accuracy to solid loads where the axial length of the coil is large relative to its inside diameter.

He also gives an equation for the power input to hollow cylinders where "the depth of current penetration" is several times the wall thickness. The purpose of the present paper is as follows:

1. To extend the range of equations applying to solid cylinders, so that accurate results may be obtained when the coil has an inside diameter of the same order of magnitude as its axial length.

2. To extend the range of equations applying to hollow cylinders so that accurate results may be obtained regardless of the cylinder wall thickness.

3. To present the equations in a form such that the required number of coil turns may be found to match a given power source.

4. To present tabulated values of experimental and calculated results over the proposed application range of these equations.

Definition of Symbols

Symbols used in appendixes are defined therein.

A = coil resistance divided by N^2, from equation 6
a_o = outside diameter of load, inches
a_m = average diameter of load, inches, = $a_o - t_2$
B = load resistance divided by N^2 from equations 9, 11, and 13
b_1 = inside diameter of inductor coil, inches
C = internal reactance of coil divided by N^2, from equation 7
D = internal reactance of load divided by N^2, from equations 10, 12, and 14
d_1 = reference depth for inductor, measure of current penetration, inches, from equation 15
d_2 = reference depth for load, measure of current penetration, inches, from equation 16
E = air-gap reactance divided by N^2, from equation 8
E_c = voltage across capacitor terminals BB, volts, from equation 23
E_L = voltage across inductor coil terminals AA, volts
E_T = voltage across generator terminals CC, volts
E_R = voltage rating of capacitor, volts
f = frequency cycles per second
h = radial thickness of inductor coil, inches
I_L = current through inductor coil, amperes, from equation 22
I_T = generator output current, amperes
K_1 = coil shortness correction factor, from equation 17
K_2 = resistance coefficient for thick wall and solid rods from Figure 5
K_3 = resistance coefficient for thin-wall rods from Table I
K_4 = reactance coefficient for thin-wall rods from Table II
K_5 = Nagaoka's constant[2] from Figure 4
L = axial length of inductor coil, inches
N = total turns in inductor
n = efficiency of power conversion through inductor to load, from equation 20
P_A = power dissipated in transmission line resistance R_A in watts, from equation 24
P_L = power input to inductor coil and load in watts
P_s = power dissipated in load in watts, from equation 19
P_T = total power input at capacitor terminals in watts, from equation 25
ρ_1 = resistivity of conductor material in inductor, ohms inches
ρ_2 = resistivity of load material, ohms, inches
s = space factor of inductor coil
t_1 = wall thickness of conductor in coil in inches
t_2 = wall thickness of load in inches
R_A = resistance of transmission line from capacitor terminals BB to coil terminals AA in ohms
R_p = resistance of inductor coil itself in ohms, from equation 1
R_s = resistance of load as reflected in inductor coil in ohms, from equation 2
W = axial width of conductor in inductor coil in inches
X_A = reactance of transmission line from capacitor terminals BB to coil terminals AA in ohms
X_o = reactance due to flux in air gap between coil and load in ohms, from equation 5
X_p = reactance due to flux between inside and outside radius of inductor coil in ohms, from equation 3
X_s = reactance due to flux within the load as reflected in inductor coil in ohms, from equation 4

Circuit Description

The conventional induction-heating circuit used with rotating generators is shown in Figure 1. The purpose of the capacitor is to adjust the power factor of the generator load as viewed from terminals CC to approximately unity. The power factor as viewed from terminals AA may be considerably in the lag. Adjusting the capacitor to a value such that load power factor as viewed from the generator terminals is unity, results in a higher current I_L flowing through the coil circuit than the generator

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output current I_T, allowing a given load power dissipation at minimum generator output current for a given generator terminal voltage.

The circuit from capacitor terminals through load is resolved as shown in Figure 2 when the intercircuit capacitance is negligible, as it generally is for induction-heating circuits of 15,000 cycles or below. The power accomplishing the load heating is dissipated in R_s. The power dissipated in R_A, the transmission-line resistance, and R_p, the inductor-coil resistance, should be kept as low as possible for most efficient heating. The power dissipated in the capacitor is generally negligible.

The physical detail of an induction-heating circuit consisting of a solenoidal coil and coaxial cylindrical load is shown in Figure 3. The inductor is shown as a single-layer solenoid wound with a tubular conductor of rectangular shape. A multilayer coil could be used but would result in less efficient power transfer to the load. The conductor is made tubular to permit the flow of a coolant to carry away the heat dissipated in the coil. A round conductor could be used but generally results in a somewhat less efficient heating circuit.

In the inductor-coil circuit, the values R_p, X_p, X_o, X_s, and R_s determine the power dissipated for a given value of E_L. For given coil and load dimensions these values may be adjusted to obtain the desired power by changing the number of turns in the inductor.

Circuit Equations and Their Limitations

Equations

For derivations of the equations which follow see references 1 or 5 for the impedance values of solid cylinders, Appendix I for the impedance of hollow loads, and Appendix II for coil shortness correction factor.

Impedance values in coil circuit in ohms are

$$R_p = N^2A \qquad (1)$$

$$R_s = N^2B \qquad (2)$$

$$X_p = N^2C \qquad (3)$$

$$X_s = N^2D \qquad (4)$$

$$X_o = N^2E \qquad (5)$$

where

$$A = \frac{3.14\rho_1 b_1}{sL}\left[\frac{1}{d_1}+\frac{1}{b_1}\right] \qquad (6)$$

$$C = \frac{3.14\rho_1 b_1}{sLd_1} \qquad (7)$$

$$E = \frac{15.7f(b_1^2-a_o^2)K_1 10^{-8}}{L} \qquad (8)$$

Equations 9 and 10 are used to determine B and D for solid rods. For hollow rods the equations used to determine B and D

Table I. Resistance Coefficient, K_3

For Hollow Rods Where Ratio of Wall Thickness to Reference Dimension $\frac{t_2}{d_2} < 2.0$ and $\geqq 0.5$

t_2/a_o \ t_2/d_2	0.500	0.522	0.545	0.569	0.595	0.621	0.648	0.677	0.707	0.74	0.77	0.805	0.88	1.00	1.19	1.41	2.00
0	2	1.93	1.86	1.78	1.70	1.64	1.58	1.51	1.45	1.39	1.34	1.29	1.20	1.08	0.97	0.92	0.94
0.025	1.96	1.89	1.83	1.76	1.70	1.64	1.58	1.51	1.45	1.39	1.34	1.29	1.21	1.10	0.99	0.93	0.95
0.0500	1.77	1.75	1.72	1.68	1.64	1.58	1.54	1.49	1.44	1.39	1.34	1.30	1.23	1.11	1.04	0.95	0.96
0.0648	1.60	1.60	1.59	1.58	1.56	1.53	1.50	1.46	1.42	1.37	1.33	1.29	1.23	1.12	1.04	0.95	0.96
0.077	1.45	1.47	1.48	1.48	1.48	1.47	1.45	1.43	1.39	1.36	1.32	1.29	1.23	1.13	1.04	0.96	0.96
0.084	1.35	1.38	1.40	1.42	1.44	1.43	1.42	1.40	1.37	1.34	1.32	1.28	1.22	1.13	1.04	0.97	0.96
0.092	1.24	1.29	1.32	1.35	1.37	1.38	1.38	1.36	1.35	1.33	1.31	1.27	1.21	1.13	1.04	0.97	0.96
0.100	1.14	1.19	1.23	1.27	1.30	1.32	1.34	1.34	1.34	1.32	1.30	1.27	1.21	1.13	1.04	0.98	0.97
0.109	1.03	1.08	1.13	1.17	1.21	1.24	1.26	1.27	1.28	1.27	1.27	1.25	1.20	1.13	1.05	0.98	0.97
0.119	0.91	0.97	1.03	1.08	1.14	1.17	1.20	1.22	1.24	1.24	1.24	1.23	1.19	1.13	1.05	0.99	0.97
0.130	0.80	0.86	0.92	0.98	1.03	1.08	1.12	1.16	1.18	1.19	1.20	1.20	1.18	1.13	1.06	0.99	0.97
0.141	0.68	0.75	0.82	0.88	0.94	0.99	1.04	1.08	1.12	1.14	1.16	1.17	1.17	1.13	1.06	1.00	0.98
0.154	0.57	0.62	0.68	0.75	0.83	0.89	0.94	1.00	1.04	1.08	1.11	1.13	1.16	1.13	1.07	1.01	0.98
0.168	0.46	0.53	0.58	0.63	0.72	0.79	0.86	0.92	0.97	1.02	1.06	1.09	1.14	1.13	1.08	1.01	0.99
0.183	0.38	0.43	0.48	0.54	0.60	0.66	0.73	0.81	0.88	0.94	1.00	1.04	1.09	1.13	1.09	1.02	0.99
0.200	0.28	0.33	0.38	0.43	0.49	0.56	0.62	0.70	0.77	0.83	0.91	0.97	1.06	1.12	1.10	1.03	1.00

Note: Values below heavy lines not checked experimentally.

Table II. Reactance Coefficient, K_4

For Hollow Rods Where Ratio of Wall Thickness to Reference Dimension $\frac{t_2}{d_2} < 2.0$ and $\geqq 0.5$

t_2/a_o \ t_2/d_2	0.500	0.545	0.595	0.648	0.707	0.77	0.84	0.92	1.00	1.09	1.19	1.30	1.41	1.54	1.68	2.00
0	0.33	0.36	0.39	0.43	0.47	0.51	0.55	0.60	0.65	0.70	0.75	0.80	0.86	0.91	0.95	0.99
0.0063	0.43	0.44	0.45	0.48	0.51	0.54	0.57	0.61	0.66	0.71	0.75	0.80	0.86	0.91	0.95	0.99
0.0088	0.47	0.46	0.47	0.49	0.52	0.55	0.58	0.62	0.67	0.71	0.76	0.81	0.86	0.91	0.95	0.99
0.0125	0.53	0.51	0.51	0.52	0.54	0.56	0.59	0.63	0.67	0.72	0.77	0.82	0.86	0.91	0.95	0.99
0.0177	0.61	0.57	0.56	0.56	0.57	0.59	0.61	0.64	0.69	0.73	0.77	0.82	0.87	0.91	0.95	1.00
0.0210	0.67	0.62	0.59	0.59	0.59	0.60	0.62	0.65	0.69	0.74	0.78	0.83	0.87	0.91	0.95	1.00
0.0250	0.73	0.68	0.63	0.62	0.61	0.62	0.63	0.66	0.70	0.74	0.78	0.83	0.87	0.91	0.95	1.00
0.0297	0.80	0.72	0.68	0.66	0.64	0.64	0.65	0.69	0.71	0.75	0.78	0.84	0.87	0.91	0.95	1.00
0.0354	0.89	0.79	0.74	0.70	0.67	0.67	0.67	0.70	0.72	0.76	0.79	0.84	0.88	0.92	0.95	1.00
0.0420	0.99	0.89	0.80	0.75	0.72	0.70	0.70	0.72	0.74	0.77	0.80	0.84	0.88	0.92	0.95	1.00
0.0500	1.09	0.98	0.88	0.82	0.76	0.74	0.73	0.74	0.75	0.78	0.81	0.85	0.89	0.92	0.95	1.00
0.0595	1.20	1.07	0.97	0.88	0.82	0.78	0.76	0.76	0.78	0.80	0.83	0.86	0.89	0.93	0.96	1.00
0.0707	1.31	1.18	1.07	0.96	0.89	0.84	0.81	0.80	0.80	0.82	0.84	0.88	0.90	0.93	0.96	1.00
0.084	1.40	1.30	1.19	1.07	0.97	0.90	0.86	0.84	0.84	0.84	0.86	0.89	0.91	0.93	0.96	1.00
0.100	1.46	1.38	1.28	1.18	1.07	0.99	0.93	0.89	0.87	0.88	0.88	0.90	0.92	0.94	0.96	0.99
0.119	1.49	1.46	1.37	1.27	1.17	1.08	1.01	0.96	0.92	0.91	0.90	0.92	0.93	0.95	0.97	0.99
0.141	1.44	1.46	1.43	1.36	1.28	1.20	1.11	1.04	0.99	0.96	0.94	0.94	0.95	0.96	0.97	0.99
0.168	1.36	1.42	1.44	1.42	1.38	1.30	1.22	1.14	1.07	1.02	0.98	0.97	0.97	0.97	0.98	0.99
0.200	1.23	1.34	1.40	1.44	1.44	1.41	1.34	1.26	1.17	1.10	1.04	1.01	0.99	0.99	0.98	0.99

Note: Values below heavy lines not checked experimentally.

depend on the relation t_2/d_2 and are as follows:

For hollow rods where $t_2/d_2 \geqq 2.0$ and for solid rods

$$B=\frac{3.14\rho_2 a_o K_2 K_1^2}{Ld_2} \tag{9}$$

$$D=\frac{3.14\rho_2 a_o K_1^2}{Ld_2} \tag{10}$$

For hollow rods where $0.5 \leqq t_2/d_2 < 2.0$

$$B=\frac{3.14\rho_2 a_m K_1^2 K_3}{Ld_2} \tag{11}$$

$$D=\frac{3.14\rho_2 a_o K_1^2 K_4}{Ld_2} \tag{12}$$

For hollow rods where $t_2/d_2 < 0.5$

$$B=\frac{15.7fa_m^2K_1^2 10^{-8}}{L}\times\frac{\dfrac{2d_2^2}{a_m t_2}}{1+\dfrac{4d_2^4}{a_m^2 t_2^2}} \tag{13}$$

$$D=\frac{15.7fa_m^2K_1^2 10^{-8}}{L}\left[\frac{a_o^2}{a_m^2}-\frac{1}{1+\dfrac{4d_2^4}{a_m^2t_2^2}}\right] \tag{14}$$

The following equations are for determining values for use with the proceeding equations:

$$d_1=3{,}160\sqrt{\frac{\rho_1}{f}} \tag{15}$$

$$d_2=3{,}160\sqrt{\frac{\rho_2}{f}} \tag{16}$$

$$K_1=K_5\left(1-\frac{a_o^2}{b_1^2}\right)+\frac{a_o^2}{b_1^2} \tag{17}$$

The required number of coil turns to obtain given power P_L at given voltage E_L are

$$N=E_L\sqrt{\frac{A+B}{P_L[(A+B)^2+(C+D+E)^2]}} \tag{18}$$

Power dissipated in load, P_s in watts, is

$$P_s=P_L\left[\frac{B}{A+B}\right] \tag{19}$$

Efficiency of power conversion through inductor to load is

$$n=\frac{B}{A+B} \tag{20}$$

Capacitor rating for unity power factor at capacitor-terminal input is

$$\text{kva}_{\text{rated}}=\frac{(P_L+P_A)E_R^2}{1{,}000E_c^2}\times\frac{(C+D+E)N^2+X_A}{(A+B)N^2+R_A} \tag{21}$$

Current through inductor coil, I_L, amperes is

$$I_L=\frac{E_L}{N^2\sqrt{(A+B)^2+(C+D+E)^2}} \tag{22}$$

Capacitor voltage, E_c, volts is

$$E_c=I_L\sqrt{[N^2(A+B)+R_A]^2+[N^2(C+D+E)+X_A]^2} \tag{23}$$

Power dissipated in transmission line, P_A in watts is

$$P_A=I_L^2R_A \tag{24}$$

Total power input at capacitor terminals (neglecting power loss in capacitor), P_T in watts, is

$$P_T=P_L+P_A \tag{25}$$

LIMITATIONS

For an accuracy of ± ten per cent the foregoing equations will apply in the range of the following limitations:

1. Single-layer solenoidal coil wound with rectangular conductor.
2. Cylindrical load telescoped in coil, coaxial with coil.
3. Length of load equal to or greater than length of coil with ends of load flush with ends of coil or extending out of ends of coil.
4. Ratio of inside diameter of coil to length $b_1/L \leqq 1.0$.
5. Ratio of coil thickness to length $h/L \leqq 0.07$.
6. Ratio of conductor wall thickness to inductor reference dimension $t_1/d_1 \geqq 1.35$.
7. Inductor-coil space factor $\geqq 0.70$.
8. Permeability of coil and load material relative to a vacuum $=1.0$.
9. Ratio of inside diameter of coil to reference dimension of coil $b_1/d_1 \geqq 3.5$.
10. Ratio of outside diameter of load to reference dimension of load $a_o/d_2 \geqq 2.0$.
11. Intercircuit capacitance negligible.

Coil Design and Use of Equations

PHYSICAL DIMENSIONS

The length and diameter of the inductor coil are determined by the dimensions of the piece to be heated, the heat pattern desired, and the allowable clearance between inside of coil and outside of load. For a heating operation in which the piece or pieces of metal to be heated are fed continuously through the inductor, the length of the coil is determined by whether surface heating or through heating is desired and the amount of power utilized. For surface heating the coil is made short to realize high power densities, whereas for through heating a relatively long coil gives more satisfactory results. To minimize coil I^2R loss and required kilovolt-amperes of condenser, the inside diameter of the coil should be kept as small as possible. However, in the case of multiturn coils with a substantial potential difference between one end of the coil and the other, sufficient space must be allowed for an insulation coating between conductor and load piece, in addition to the necessary mechanical clearance. Also allowance must be made for expansion of the load when heated. The diameter of the load used in calculation should correspond to that at the particular load-material temperature used for load-resistivity determination.

The conductor wall thickness t_1 should be equal to or greater than 1.35 d_1 (d_1 is measure of current penetration) for effi-

Table III. Calculated Impedance Values Versus Experimental Results for Solenoidal Coils With Coaxial Cylindrical Loads of Nonmagnetic Materials at 10,000 Cycles Per Second*

			Case 1	2	3	4	5	6	7	8	9	10
Inductor, copper	Resistivity, ρ_1, microhm, inches		0.73	0.73	0.71	0.73	0.73	0.76	0.76	0.76	0.76	0.76
	Total turns, N		10	10	10	10	10	20	20	20	20	20
	Inside diameter, b_1, inches		3.010	3.010	3.010	3.010	3.010	1.127	1.127	1.127	1.127	1.127
	Length over turns, L, inches		3.0	3.0	3.0	3.0	3.0	5.95	5.95	5.95	5.95	5.95
	Conductor	Width, W, inches	0.252	0.252	0.252	0.252	0.252	0.248	0.248	0.248	0.248	0.248
		Height, h, inches	0.252	0.252	0.252	0.252	0.252	0.248	0.248	0.248	0.248	0.248
		Wall, t_1, inches	0.042	0.042	0.042	0.042	0.042	0.044	0.044	0.044	0.044	0.044
Load	Material		Copper	Brass	18–8	18–8	18–8	18–8	18–8	18–8	18–8	18–8
	Resistivity, ρ_2, microhm, inches		0.75	2.68	31.2	29.9	28.7	28.9	28.0	29.3	28.5	30.2
	Outside diameter, a_o, inches		2.500	2.500	2.500	1.005	1.001	1.005	1.001	1.000	0.625	0.375
	Wall, t_2, inches		0.131	0.034	0.133	0.135	0.061	0.135	0.061	Solid	Solid	Solid
Impedance values, ohms	Coil resistance, R_p	Calculated	0.0094	0.0094	0.0093	0.0094	0.0094	0.0074	0.0074	0.0074	0.0074	0.0074
		Measured	0.0091	0.0090	0.0090	0.0095	0.0093	0.0073	0.0073	0.0073	0.0073	0.0073
	Load resistance, R_s	Calculated	0.0057	0.0169	0.0485	0.0099	0.0122	0.0372	0.045	0.0290	0.0137	0.0032
		Measured	0.0056	0.0162	0.0465	0.0102	0.0121	0.0360	0.045	0.0280	0.0132	0.0029
	Total reactance, $X_p+X_s+X_o$	Calculated	0.148	0.148	0.170	0.328	0.333	0.071	0.090	0.0702	0.115	0.130
		Measured	0.144	0.143	0.165	0.316	0.326	0.0756	0.095	0.0755	0.119	0.133
Load reference dimension, d_2			0.0274	0.0518	0.177	0.173	0.170	0.170	0.167	0.171	0.169	0.174
Ratio, t_2/d_2			4.78	0.657	0.75	0.78	0.359	0.795	0.365			

* For further details see text.

cient heating. Generally, if the wall thickness is made less than 1.35 d_1, the coil losses will be increased.

LOAD RESISTIVITY ρ_2

The resistivity of load material may vary considerably during the heat cycle. If the heat operation is of the continuous type, the required turns may be based on average values of load resistivity and outside diameter. If the load remains fixed in the coil during heating, separate calculations should be made for maximum and minimum temperature conditions. When the generator is to be loaded to its rated power output, then the greatest number of turns obtained should be used. This allows rated generator output at one end of the heat cycle without exceeding the generator rating at the other end of the heat cycle.

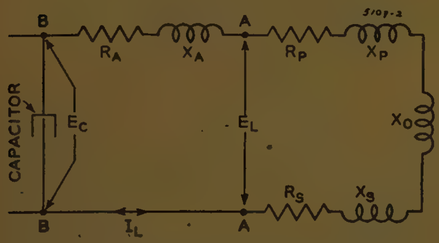

Figure 2. Equivalent electric circuit from capacitor through load

CONDUCTOR RESISTIVITY ρ_1 AND COIL SPACE FACTOR

The coil resistivity used in calculation should correspond to that at the actual operating temperature of the coil conductor. In determining the required turns it is necessary to assume some value of space factor. After the turns are calculated, a check should be made to find if the assumed space factor can be realized with available tubing. If not, a new calculation should be made on the basis of the obtainable space factor. Over the range of the prescribed limitations, greatest efficiency is obtained by making the space factor as close to unity as possible. In the case of multiturn coils with sufficient insulation for terminal voltages of 800 volts or less at 15,000 cycles or below, it is generally possible to realize a space factor of 0.80.

SELECTION OF FREQUENCY AND POWER REQUIREMENT

When the required power input to the load piece (based on past experience or a complete thermal calculation) is known, the necessary power input to the coil to produce this load power is found by dividing load power by efficiency of power conversion. This efficiency is given as follows when the appropriate expressions for A and B are substituted in equation 20:

$$n=\frac{1}{1+\frac{1}{sK_1^2}\frac{\rho_1}{\rho_2}\frac{b_1}{a_o}\frac{d_2}{d_1'}\frac{1}{K_2}}\quad\left(\text{solid loads or }\frac{t_2}{d_2}\geqq 2.0\right)\quad(26)$$

$$n=\frac{1}{1+\frac{1}{sK_1^2}\frac{\rho_1}{\rho_2}\frac{b_1}{a_m}\frac{d_2}{d_1'}\frac{1}{K_3}}\quad\left(\text{hollow loads }0.5\leqq\frac{t_2}{d_2}<2.0\right)\quad(27)$$

$$n=\frac{1}{1+\frac{1}{sK_1^2}\frac{\rho_1}{\rho_2}\frac{b_1}{a_m}\frac{d_2}{d_1'}\left[\frac{t_2}{d_2}+\frac{4d_2^3}{a_m^2t_2}\right]}\quad\left(\text{hollow loads }\frac{t_2}{d_2}<0.5\right)\quad(28)$$

where

$$d_1'=\frac{d_1}{1+d_1/b_1}$$

For most applications of induction heating the reference depth of the coil d_1 is small compared to the coil diameter b_1. Therefore d_1', is approximately equal to d_1, and for this condition the ratio of d_2/d_1' is nearly constant as frequency is varied. The only quantities which change appreciably with frequency in equations 26, 27, and 28, are K_2, K_3, and the expression in brackets respectively. Inspection and comparison of these equations will show:

(*a*). Where equation 26 applies power conversion efficiency increases with K_2 and therefore frequency (see Figure 5). In an analysis of efficiency and kilovolt-ampere requirements Stansel has advocated the use of frequencies at least sufficient to make[3]

$$\frac{a_o}{d_2}=3.54\quad(29)$$

He points out, however, that there is no objection to higher frequencies, if economic factors warrant their use.

(*b*). Where equation 27 applies, power-conversion efficiency increases with K_3.

(*c*). Where equation 28 applies, power-conversion efficiency increases as the bracket expression decreases.

(*d*). Comparison of equations 26 and 27 and reference to Table I will show that for hollow loads with wall thicknesses less than 0.13 of the outside diameter greatest electric efficiency is realized by selecting a frequency to make K_3 maximum or to make the bracket expression of equation 28 minimum. This frequency is given approximately by

$$f=\frac{34.6\times10^6\rho_2}{a_mt_2}\quad(30)$$

This is the frequency for minimum value of the bracket expression in equation 28 and corresponds very closely to the frequency for maximum K_3.

COIL VOLTAGE E_L

The transmission-line impedance between capacitors and coil terminals should be kept relatively small for efficient power transmission. The effect of this impedance is to reduce the voltage E_L across coil terminals with respect to capacitor voltage E_c. In the solution of the foregoing equation for coil turns, E_L may be assumed to be equal to 0.9 E_c; then it usually will be economically feasible to design a transmission line to meet or better this condition.

Calculated Impedance Values Versus Experimental Results

In Table III a portion of the experimental results obtained is compared to calculated values in order to substantiate the use of equations 1 through 25 over the proposed range of limitations. All data given in Table III were obtained at 10,000 cycles per second (±0.5 per cent) with sinusoidal voltages and currents. All loads used in obtaining the data in this table were 18 inches long, located coaxial with the inductor, and extended through the inductor, so that ends of load were either flush with ends of coil or extending out of ends of coil. With the load extending out of both ends of the coil and then moved axially through the coil no change in impedance values was obtained up to the point where one end of the load was flush with the end of the coil. However, with further axial movement, such that the load did not extend fully through the coil, the impedance values changed. This condition of the load not extending fully through the coil represents a separate problem and is not considered in this paper.

Experimental impedance values were arrived at from voltage and current measurements made with thermocouple-type instruments. For this purpose the primary of the current transformer was

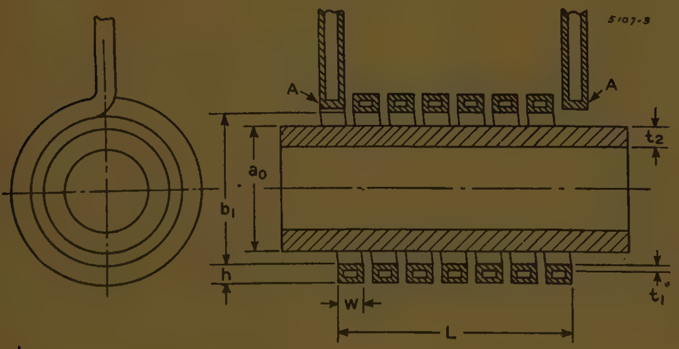

Figure 3. Inductor coil and load

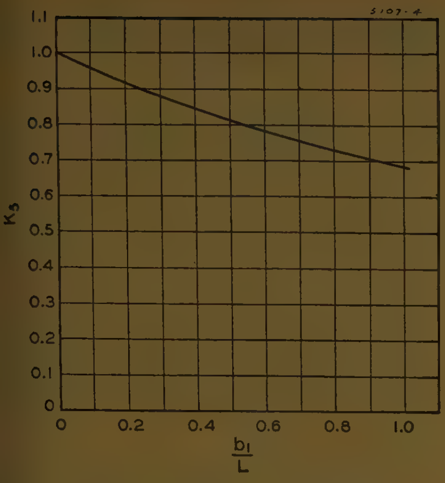

Figure 4. Nagaoka's constant $K5$

connected in series with the inductor coil (Figure 1), and the voltmeter was connected across AA. The total generator output was measured by a wattmeter connected at CC. This wattmeter was calibrated against thermocouple instruments for unity power-factor loads. The capacitor BB was adjusted so that the reactive volt-amperes measured at CC were negligible. The input to the coil was obtained by subtracting power losses in condenser and connectors from the generator output. These losses never exceeded 10 per cent of the output of the generator.

Where hollow loads were used, primary and secondary resistances were measured calorimetrically. Cooling water was circulated through the load and coil. Power dissipation was obtained by flow temperature-rise methods. Measured generator output less capacitor and connector losses was found to agree within ±3 per cent with measured power to the cooling water. In making resistance measurements where solid loads were used it was observed that:

1. Such loads could not be held at a constant low temperature, unless a surrounding water jacket were provided. This would make it impossible to separate coil and load losses where the air gap was small.

2. Direct calorimetric methods were therefore impracticable. However, the accuracy of the electrically determined input to the coil had been substantiated by experiments with hollow loads.

3. For the particular coil used in cases 6 to 10 (Table III) experiments with hollow loads had demonstrated that the nature of the load had little effect upon the resistance of the coil. Therefore, the total resistance of the coil with load inserted, R_p+R_s, was obtained from the electrically measured watts to the coil. R_s then was found by subtracting the value of R_p measured for hollow loads.

The operating temperature of the load was determined by averaging thermocouple measurements over the conducting volume. The resistivity at this temperature was measured with a Kelvin bridge. The test sample was an open ring cut from a cylindrical section of the load. The ring was heated to the desired temperature in an oven.

Conclusions

1. The proposed equations apply over the given range of limitations with sufficient accuracy for calculating inductor coils for induction heating of nonmagnetic cylinders.

2. For hollow cylindrical loads with wall thicknesses less than 0.13 of outside diameter a definite frequency exists at which maximum power-conversion efficiency is obtained (see equation 30).

3. It is important to remember that the voltage across the coil is not necessarily equal to the generator output voltage due to the effect of transmission lines.

Appendix I. Expressions for Impedance—Hollow Loads

Units

Identical definitions apply to quantities previously defined, except that units in Appendix I are in the absolute or centimeter-gram-second electromagnetic system.

Consider a long solenoidal coil AA (Figure 1) wound closely about its load. The flux density on the outside of the load or in the air gap is

$$H=4\pi\frac{N}{L}I_L \tag{31}$$

Load With Wall Thickness Small Relative to Reference Depth

At any given time current density in the load may be assumed constant. Therefore, concentrated circuit theory applies.

Let

R_2 = resistance of load

$=\frac{\pi a_m}{Lt_2}\rho_2$ abohms

X_2 = reactance of load

$=2\pi f\left(\frac{4\pi}{L}\right)\left(\frac{\pi}{4}a_m^2\right)$ abohms

X_M = mutual reactance between load and coil

$=NX_2$ abohms

d_2 = reference depth

$=\frac{1}{2\pi}\sqrt{\frac{\rho_2}{f}}$ centimeters

The primary impedance associated with secondary current flux is:

$$\frac{X_M^2}{R_2+jX_2}=\frac{2\pi^3a_m^2N^2f}{L}\times\frac{1}{\frac{R_2}{X^2}+j}$$

The primary impedance associated with primary current flux inside the load is:

$$j\,\frac{2\pi^3a_o^2N^2f}{L}$$

Adding, the total primary impedance due to flux inside the load gives:

$$R_s+jX_s=\frac{2\pi^3a_m^2N^2f}{L}\left[\frac{\frac{R_2}{X_2}-j}{\frac{R_2^2}{X_2^2}+1}+j\,\frac{a_o^2}{a_m^2}\right] \tag{32}$$

where

$$\frac{R_2}{X_2}=\frac{\rho_2}{2\pi^2a_mft_2}=\frac{2d_2^2}{a_mt_2} \tag{33}$$

When one converts this to the ohm-inch system and separates into real and imaginary parts, equations 2 and 4 are obtained, B and D being given by equations 13 and 14. (Under the assumptions of Appendix I, K_1 is unity.)

Loads of Intermediate Wall Thickness

The current density in the load is neither constant at any given time nor the same as for a solid load, at the same depth beneath the surface. Let

i = rms current density in abamperes per cm^2

Φ = rms flux in maxwells

H = flux density in lines per cm^2

r = distance from axis of load in centimeters

$k=2\pi\sqrt{2f/\rho_2}$

$x=\sqrt{jkr}$

o and I refer to outside and inside of load (except where subscript 0 denotes a Bessel function of zero order)

$r_m={}^1/_2(r_1+r_o)$ centimeters

Consider an element of inside radius r, outside radius $r+dr$, and unit length, and apply Ohm's law. Then

$$2\pi\rho_2ri+j2\pi f\Phi=0 \tag{35}$$

The rate at which flux density changes with radius is

$$\frac{dH}{dr}=-4\pi i \tag{36}$$

By definition

$$\Phi=2\pi\int_0^r rHdr \tag{37}$$

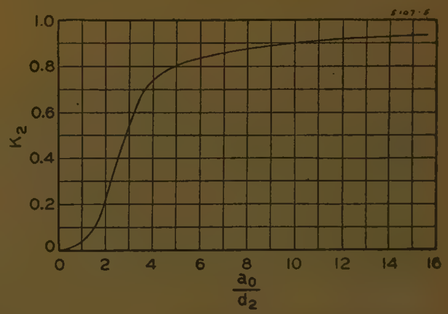

Figure 5. Load-resistance coefficient $K2$

Values of K_2 for $\frac{a_0}{d_2}>15$

$\frac{a_0}{d_2}$	K_2
30	0.97
100	0.99
α	1.0

Differentiating equation 35 with respect to r and making use of equations 36 and 37 gives

$$\frac{d^2H}{dr^2}+\frac{1}{r}\frac{dH}{dr}-jk^2H=0 \tag{38}$$

Equation 38 is a standard form of Bessel's equation of zero order.[4] The general solution is

$$H=C_1I_{0(\chi)}+C_2K_{0(\chi)} \tag{39}$$

Where $I_{0(\chi)}$ and $K_{0(\chi)}$ are modified Bessel functions of zero order, of the first and second kinds. Substituting the boundary conditions for the inside radius into equation 35 and making use of equation 36

$$\psi(\chi_I)C_1+\phi(\chi_I)C_2=0 \tag{40}$$

where

$$\left.\begin{aligned}\psi(\chi)&=-\frac{\rho_2\chi}{2}I_0'(\chi)+j2\pi^2fr^2I_{0(\chi)}\\ \phi(\chi)&=-\frac{\rho_2\chi}{2}K_0'(\chi)+j2\pi^2fr^2K_{0(\chi)}\end{aligned}\right\} \tag{41}$$

Inserting the boundary conditions for the outside radius into equation 39 gives

$$H_o=I_{0(\chi_0)}C_1+K_{0(\chi_0)}C_2 \tag{42}$$

Solving equations 40 and 42 for C_1 and C_2 and substituting into equation 39 gives

$$H=\frac{1}{1-m}\left[\frac{I_{0(\chi)}}{I_{0(\chi_0)}}-m\frac{K_{0(\chi)}}{K_{0(\chi_0)}}\right]H_o \tag{43}$$

where

$$m=\frac{\psi(\chi_I)K_{0(\chi_0)}}{\phi(\chi_I)I_{0(\chi_0)}} \tag{44}$$

The total flux inside the rod is

$$\Phi_o=\pi r_I^2H_I+2\pi\int_I^{r_o}rHdr \tag{45}$$

Integrating $I_{0(\chi)}$ and $K_{0(\chi)}$

$$\int rI_{0(\chi)}dr=\frac{r^2}{\chi}I_0'(\chi)\quad \int rK_{0(\chi)}dr=\frac{r^2}{\chi}K_0'(\chi)$$

Therefore, from equations 43 and 45

$$\Phi_o=\frac{1}{1-m}\pi r_I^2H_o\left[\frac{I_{0(\chi_I)}}{I_{0(\chi_0)}}\left(1-\frac{2}{\chi_I}\cdot\frac{I_0'(\chi_I)}{I_{0(\chi_I)}}\right)-m\frac{K_{0(\chi_I)}}{K_{0(\chi_0)}}\left(1-\frac{2}{\chi_I}\cdot\frac{K_0'(\chi_I)}{K_{0(\chi_0)}}\right)\right]+\frac{1}{1-m}\cdot\frac{2\pi r_0^2}{\chi_0}H_o\left[\frac{I_0'(\chi_0)}{I_{0(\chi_0)}}-m\frac{K_0'(\chi_0)}{K_{0(\chi_0)}}\right] \tag{46}$$

From equations 41 and 44

$$m=\frac{K_{0(\chi_0)}}{I_{0(\chi_0)}}\cdot\frac{I_{0(\chi_I)}-\dfrac{2}{\chi_I}I_0'(\chi_I)}{K_{0(\chi_I)}-\dfrac{2}{\chi_I}K_0'(\chi_I)} \tag{47}$$

Therefore the first expression in brackets in equation 46 vanishes, leaving

$$\Phi_o=\frac{2\pi r_o^2H_o}{\chi_0}\cdot\frac{1}{1-m}\left[\frac{I_0'(\chi_0)}{I_{0(\chi_0)}}-m\frac{K_0'(\chi_0)}{K_{0(\chi_0)}}\right] \tag{48}$$

The flux density at the outside of the load with unit current in the coil is $4\pi N/L$. The impedance of the coil associated with flux inside the load is $j2\pi fN$ times the corresponding Φ_o, or

$$R_s+jX_s=j\frac{16\pi^3fN^2r_o^2}{L\chi_0}\cdot\frac{1}{1-m}\times\left[\frac{I_0'(\chi_0)}{I_{0(\chi_0)}}-m\frac{K_0'(\chi_0)}{K_{0(\chi_0)}}\right] \tag{49}$$

For sufficiently high values of χ the following approximations may be made:

$$I_{0(\chi)}=\frac{1}{\sqrt{2\pi}}\cdot\frac{e^{\chi}}{\sqrt{\chi}}\qquad K_{0(\chi)}=\sqrt{\frac{\pi}{2}}\cdot\frac{e^{-\chi}}{\sqrt{\chi}} \tag{50}$$

Therefore:

$$\left.\begin{aligned}\frac{I_0'(\chi)}{I_{0(\chi)}}&=1-\frac{1}{2\chi}\approx 1\\ \frac{K_0'(\chi)}{K_{0(\chi)}}&=-1-\frac{1}{2\chi}\approx -1\end{aligned}\right\} \tag{51}$$

Also, from equation 47

$$m\approx e^{-2(\chi_0-\chi_I)}\left(1-\frac{4}{\chi_I}\right) \tag{52}$$

Substituting equations 51 and 52 into equation 49 and dropping terms in $1/\chi_I$ in the numerator gives

$$R_s+jX_s=j\frac{16\pi^3fN^2r_o^2}{L\chi_0}\times\frac{1+e^{-2(\chi_0-\chi_I)}}{1-e^{-2(\chi_0-\chi_I)}\left(1-\dfrac{4}{\chi_I}\right)} \tag{53}$$

For loads with thin walls, equation 53 should agree with equation 32. Due to the approximations introduced by equations 51 and 52, this condition is more nearly satisfied if $4/\chi_I$ in the denominator is replaced by $4/\chi_0+4t/r_0$. An improvement in accuracy also results from the use of a correction factor r_m/r_0 for R_s. After these changes are made, equation 53 may be expressed

$$R_s=\frac{\pi a_m\rho_2N^2}{Ld_2}K_3\qquad X_s=\frac{\pi a_o\rho_2N^2}{Ld_2}K_4 \tag{54}$$

where K_3 and K_4 are, respectively, the real and imaginary parts of

$$(1+j)\frac{1+e^{(1+j)\frac{2t_2}{d_2}}}{e^{(1+j)\frac{2t_2}{d_2}}-1+\dfrac{8t_2}{a_o}\left(\dfrac{1-j}{2}\cdot\dfrac{d_2}{t_2}+1\right)} \tag{55}$$

Appendix II. Correction for Coil Shortness

1. The theoretical relations derived in Appendix I assume an inductor coil and load of dimensions such that the reluctance of the return magnetic path outside the coil is negligible in relation to that of the magnetic path between the coil and load. Under these conditions it can be shown easily by Ampere's integral law that the magnetic intensity, H, within the air gap is the same as would exist in an infinitely long coil with the same number of ampere turns per unit length. Therefore, no coil-shortness correction is required if the air-gap area is sufficiently small.

2. If the air-gap area amounts to practically the entire area inside the coil (that is, if the space taken up by the load is negligible) the air-gap reactance is:

$$X_o=K_5X_o'$$

where X_o' is the air-gap reactance which would be calculated by neglecting the reduction in flux density due to coil shortness, and K_5 is Nagaoka's constant.[2]

3. The correction introduced in case 2 involves a reduction of $(1-K_5)$ of X_o'. The correction required in case 1 is zero. In intermediate cases it appears reasonable to subtract (air-gap area÷total area of coil) $(1-K_5)$ of X_o'. This gives

$$\begin{aligned}X_o&=X_o'\left[1-\left(1-\frac{a_o^2}{b_1^2}\right)(1-K_5)\right]\\ &=K_1X_o'\end{aligned}$$

where K_1 is defined by equation 17.

While X_o is directly proportional to the air-gap flux density, H_o, the I^2R loss and time rate of change of magnetic energy in the load are proportional to H_o^2. Therefore, a coil-shortness correction of K_1^2 is required for R_s and X_s.

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A Tachometer–Accelerometer–Vibrometer for Instrument-Gyroscope Testing

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Synopsis: This paper gives an account of the development, operating principles, and use of a possibly unique instrument that has been and is of considerable value on instrument-gyroscope manufacturing and repair production lines. It is used to indicate gyroscope-rotor speed when the rotor is brought up to 1,000 rpm for bearing adjustment. Then, by the turn of a switch, the instrument indicates the rate of deceleration as the bearing is adjusted, thus permitting accurate adjustment of bearing pressures. A third position of the same selector switch gives tachometer operation up to 15,000 rpm for vibration tests. Excessive vibration is indicated visibly and audibly through another amplifier, independently of the selector-switch position.

IN the manufacture of gyroscopic instruments for aircraft it has in the past been the practice to adjust the contact pressure of the ball bearings on which the rotor turns to a satisfactory working value by a coast test. At the point in assembly procedure at which this adjustment is to be made, the rotor is enclosed in a housing (in the case of the horizon gyroscope) as shown in Figure 1. The coast test was made by running the gyroscope up to substantially normal revolutions per minute as indicated by applied air pressure (occasionally checked by Strobotac), turning the air off, and measuring the coast time. This coast time was required to be between 8.5 and 13.5 minutes. This seems like a rather wide tolerance and in fact it was, but in spite of this it sometimes required several trials to get a proper adjustment.

A further objection to setting the bearings by clocking the coast times was the fact that rejections from the operation frequently ran as high as 50 per cent. As there were a number of other operations in the manufacture of gyroscopes that resulted in 50 per cent rejections this one did not seem outstandingly bad. Since some of the rejections were definitely caused by the burning or battering of the bearing surfaces by running at full speed with the rotor too tight or too loose, it seemed possible to improve the process by making the coast test from a lower speed.

At lower speeds (as determined by operating air pressure) coast time was not a reliable measure of bearing pressure, evidently because the starting speed was not determined accurately. It was proposed that equipment be developed to measure the speed of the coasting rotor by determining the frequency of the tone generated by the buckets after the air supply had been shut off; two such measurements a minute apart would give an accurate measure of bearing pressure and save a great deal of time in making adjustments. This development resulted in a workable device, but it was subject to the disadvantages of a low sound level at low speeds and standing waves at high speeds.

A workable photoelectric system was also produced, using a photoelectric tube, amplifier, synchronous motor, and tachometer; a stroboscopic pattern on the rotor was visible through holes in the end of the casing (see Figure 1). There were objections to any method which required that the rotor of the horizon gyroscope be exposed. In the completed instrument the holes in the rotor housing were covered by a cap secured to the housing by four screws. Some manufacturers made bearing adjustments with a ring assembled to the housing in an attempt to reproduce any distortion of the case that would be caused by the final assembly with the cap, and thus eliminate the effect of this distortion on bearing adjustment.

Another possible solution to the problem was suggested by a hole in the rotor housing, intended eventually for the caging screw (filled during the coast test with a plug) through which a probe could readily be brought up close to the rotor buckets. By energizing this probe with several hundred volts of direct current through about two megohms resistance, the variation in electrostatic capacitance between the probe and the buckets as the rotor turned would cause a variation in the potential of the probe that could be impressed on the grid of an amplifier tube. This voltage variation was amplified, divided down in frequency[1] to the range of the synchronous motor and amplified further to run the motor; a tachometer[2] driven by this motor was, in effect, geared to the gyroscope rotor.

First Acceleration-Measuring Model

Because it was the rate of change of rotor speed that was to be measured, and as there are a number of electric devices which are by their very nature such that they give a time differential, it seemed logical to measure this differential directly instead of measuring the speed at the beginning and end of a time interval.

It seemed desirable to start the coast at about 1,000 rpm, which would correspond to 60 cycles or 1,800 rpm for the synchronous motor to be used, and would allow over one minute of adjusting time before the speed dropped 50 per cent. The operation of the synchronous motor with the previously developed amplifier circuit was not reliable below 30 cycles. With a voltage change of two volts per minute from a standard tachometer magneto and with appropriate meters (which were not available), a meter–capacitor circuit would have given adequate results but there would have been something of a problem of adequately protecting the meter from severe overcurrents due to rapid speed changes.

Some 0–500 microammeters were available so an amplifier circuit was designed around these meters, and arranged to give a more expanded coast-time scale than would have been available with the direct-measurement system.

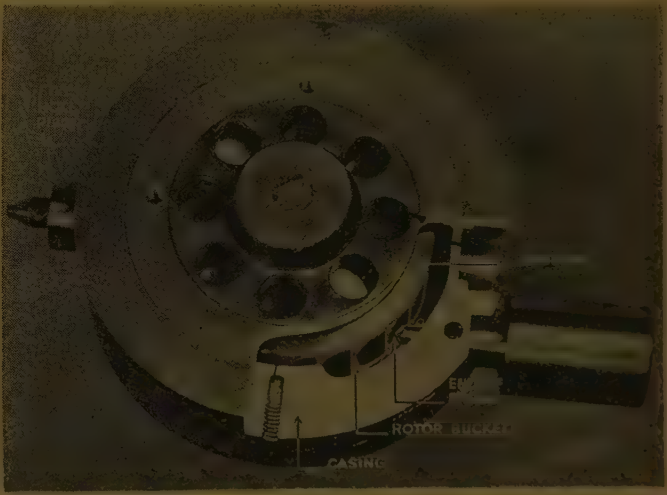

Figure 1. Bottom view of gyroscope-rotor assembly cut away to show rotor buckets and electrostatic probe

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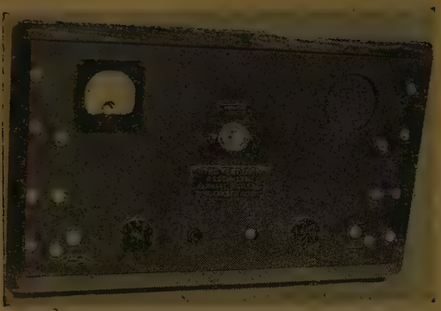

Figure 2. Front view of gyroscope test unit

This instrument is a combination tachometer, accelerometer, and vibrometer

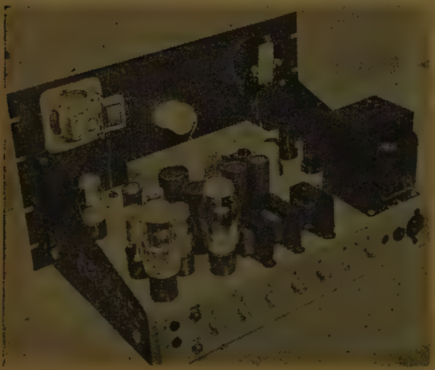

Figure 3. Chassis of gyroscope test unit showing calibration adjustments and test jacks

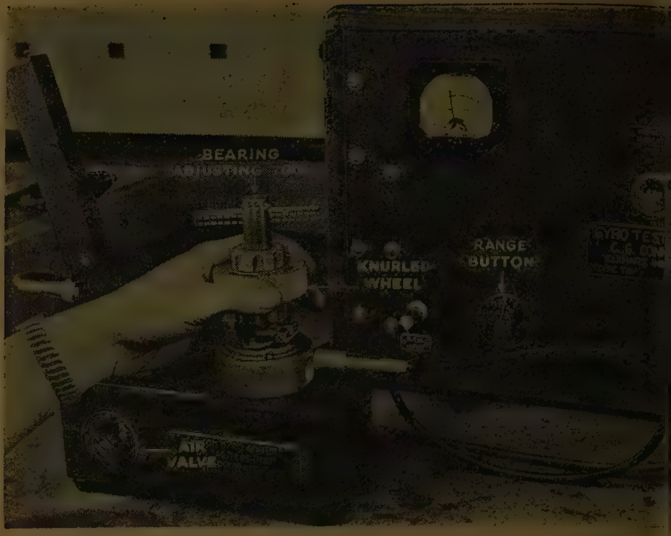

Figure 5. View of completion of bearing adjustment by coast test

This apparatus was developed and tested quite thoroughly. It was found practicable to set the coast time to within 0.5 minute of any desired value, and in the case of the horizon gyroscope this could be done in an over-all time of less than two minutes per instrument. Since the gyroscope was not run above 1,000 rpm until the bearings were properly adjusted it seemed reasonable to believe that the only rejections from such an operation would be those due to defective bearings. As nearly as it could be determined, the maximum error in deceleration measurements with this equipment was about 1.5 per cent. There was no error between the gyroscope and the synchronous motor, and the principle subsequent error was due to the voltage change on the regulated side of a constant-voltage transformer. See appendix for details of test method.

These results were so encouraging that it seemed certainly worthwhile to make some additional units, but it was found that certain items required a year for delivery. It was, therefore, decided to continue the development in order to eliminate the scarce items and reduce the size of the apparatus.

Final Model, Tachometer, Accelerometer, Vibrometer

It was found possible to obtain a pulsating unidirectional voltage having an average value that was linearly related to the speed of gyro rotor from an appropriate vacuum-tube circuit. Most of the pulsation was smoothed out in a resistance–capacitance filter, and the differentiating accomplished in the same way as when the direct current was supplied from a magneto. It was necessary to use substantially more circuit damping than in the earlier equipment, and in consequence the readings lagged behind the adjustment by an appreciable time: roughly two seconds. Other differences worth noticing were that the equipment was noiseless after the moving machinery had been eliminated, was less than half as large since it was also possible to eliminate several frequency-divider circuits and the error in measuring decelerations was approximately twice as great as in the earlier model, but still of no consequence (at least in this application). Figure 2 shows a front view of this instrument, Figure 3 a view of the chassis.

CIRCUIT DETAILS

As there are some elements of novelty in the circuit used, it will be described in some detail. The tachometer–accelerometer part of the circuit is shown in Figure 4, in which S_1, S_2, S_3, and S_4 constitute a four-pole three-position range switch, shown in the 1,000-rpm tachometer position. The central position is for coast test and the third position provides a 15,000-rpm tachometer.

There is a transient of perhaps 15 seconds duration associated with any abrupt change in the grid voltage of V6. Since V6 is concerned only with the coast test and the coast test is always preceded by use as a 1,000-rpm tachometer it is possible to eliminate this abrupt change, and with it most of the starting transient, by tying together the first and second positions of S_2 as shown.

Tubes V1 and V2 constitute a voltage amplifier with a gain of about 10,000. This permits satisfactory operation with the probe as much as 0.015 inch from the rotor. Tube V3 is a limiter and tube V4 is a clipper whose plate voltage is a square wave of bucket frequency, say 400 cycles per second at 1,000 rpm if the rotor has 24 buckets. The principal use of V3 is to keep the meter quiet when there is no input on the rotation channel. The capacitor from plate to grid of V3 (shown dotted) furthered this purpose. This

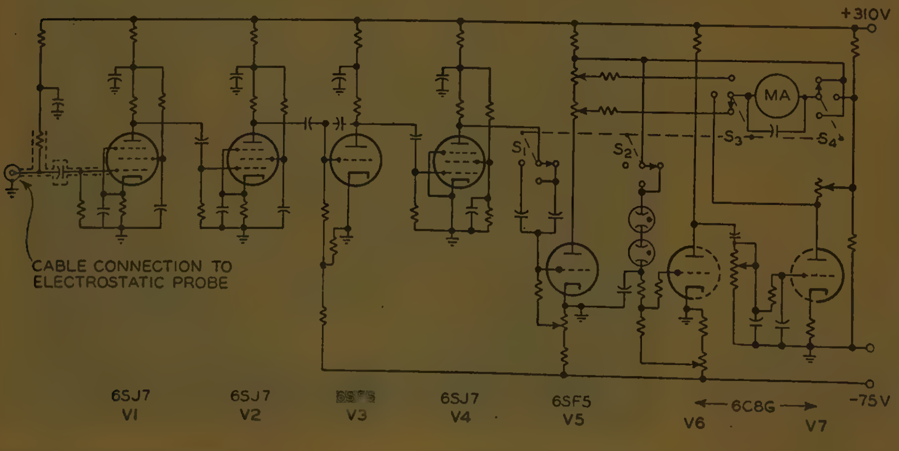

Figure 4. Schematic circuit diagram of tachometer–accelerometer circuits

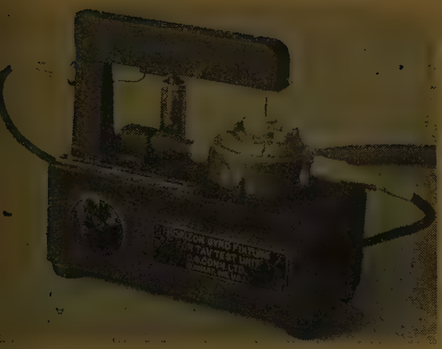

Figure 6. Horizon-gyroscope test fixture with vibration pickup in operating position

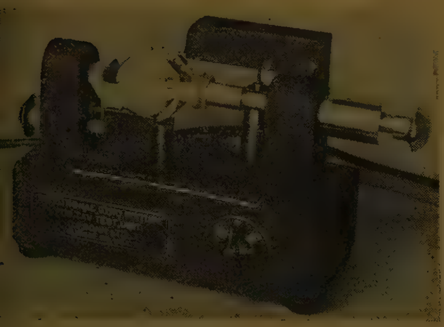

Figure 7. Test fixture for directional gyroscope

capacitor was very small and it has been found expedient to eliminate it by rearranging some of the wires.

Tube *V*5 is biased to cut off and coupled to *V*4 through a capacitance–resistance arrangement so that each stoppage of plate current of *V*4 causes a pulse of plate current in *V*5. This pulse of plate current in *V*5 is of such short duration that the average plate current of *V*5 is directly proportional to the speed of the gyroscope (over a wide range) and is read on a meter calibrated directly in revolutions per minute. Because the average plate current of *V*5 is proportional to the gyro rotor speed, the average voltage at the *V*5 plate will increase as the speed decreases, and speed versus average voltage will be linear. This average is an assumed average over a few cycles.

It is desirable at this point to be able to couple directly to the grid of the next stage and at the same time to reduce the magnitude of the voltage pulsation involved without losing any of the change n the average voltage. A glow-discharge device operating at low currents is essentially a constant-potential-drop device and furnishes a satisfactory coupling.[2] A capacitor in the grid circuit of *V*6 is a satisfactory first-averaging device.

Tube *V*6 is used in measuring deceleration as a voltage amplifier to magnify he change in average voltage and incidentally to reverse the direction of the change. A differentiating circuit is connected to the plate of *V*6 and the output of this differentiating circuit goes through a resistance–capacitance filter to the grid of tube *V*7. The plate current of tube *V*7 is then an inverse measure of the deceleration, so that longer coast times are indicated by higher deflections of the meter.

Advantage is taken of the curvature of the grid-voltage–plate-current characteristic of *V*6 to obtain a constant coast-time reading from the normally slightly decreasing deceleration. Since the plate voltage of *V*5 increases with decreasing rotor speed, the grid of *V*6 will accordingly become more positive. As the grid of *V*6 becomes more positive (within the normal operating region) the amplification factor increases. Therefore the normal gradual decrease in *de/dt* at the plate of *V*5 is considerably compensated for by a gradual increase in the amplification applied to the small changes in average voltage by *V*6. This results in *de/dt* at the plate of *V*6 being substantially constant during the useful part of the coast period (1,000 to 500 rpm).

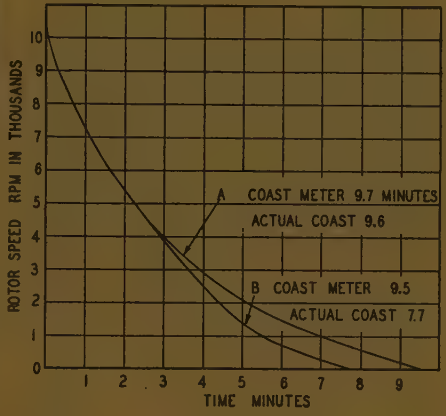

Figure 8. Coast curves of two directional-gyroscope rotors

Curve *A* is typical, and curve *B* exceptional on account of the shape at the lower end

The vibration circuit has not been shown as it consists only of a conventional audio amplifier and loudspeaker, with a *G*10 neon lamp that can be set to glow at any predetermined vibration level. The power supply has not been described because it is also entirely conventional.

Method of Use

In the actual operation of the equipment, for instance with a horizon gyroscope, the rotor assembly is placed in position so that air can be supplied to the inlet provided, and the cord is attached to the electrostatic probe that is already in place. The air valve is turned on with the range switch of the tachometer-accelerometer-vibrometer unit in the low position, and the speed is brought up to 1,000 rpm or slightly higher. Air is cut off and the range switch is turned to the coast position. The meter will swing off scale to the right and then come back to the appropriate coast reading in about five seconds. This reading will usually be too high or too low and it is necessary to loosen the bearing locknut, adjust the bearing, and reset the locknut before reading again. The meter will settle down on a new reading within a couple seconds after the locknut is reset. Figure 5 shows a picture taken after satisfactory adjustment. An experienced operator will usually have the proper bearing adjustment made by the time the rotor has slowed to 800 rpm and will then observe the constancy of the coast time reading for another 20 or 30 seconds. A slow random variation over a range of one minute or less is to be expected. A downward drift of as much as one minute is a definite indication of a defective bearing as will be explained later.

After the bearing tension is properly adjusted, the gyroscope may be safely operated at normal speed for vibration and bearing-noise test. For these tests a crystal pickup with a hardwood needle is dropped into position and lowered by a dash pot (Figure 6). Any vibrations are amplified by a conventional audio amplifier and reproduced by a loudspeaker. A neon lamp in the audio circuit can be set to glow when the vibration level is too high, thus making the test objective. The lamp adjustment is independent of the audio gain for the loudspeaker; the latter is controlled by a volume control on the front of the instrument.

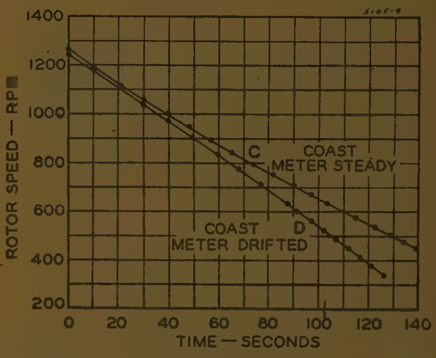

Figure 9. Low-speed coast curves for two gyroscopes

Curve *C* is normal, and curve *D* is abnormal, being convex upward. This is indicated on test by a pronounced downward drift in the coast reading

The use of the equipment in adjusting directional gyroscopes is to a considerable extent the same as that described above for horizon gyroscopes, but the holding fixture is different, as shown in Figure 7. In this case, both rotor bearings are adjustable so the rotor is held horizontally so that both are accessible.

The bearing tension is adjusted as before, and then the gyroscope is checked for possible misalignment of rotor bearings by running the speed up to 2,000 or 3,000 rpm with the pickup in place. Misalignment to the extent usually encountered gives a recognizable rumble or beat in the loudspeaker and can nearly always be corrected by simultaneously adjusting both bearings to the right or left. After this correction it is necessary to readjust the bearing tension to the proper coast-time reading. After misalignment has been corrected, the vibration check at normal speed is made.

PERFORMANCE IRREGULARITIES

In Figure 8 are shown coast curves of two directional gyroscope rotors. Curve *A* is typical of nearly all rotors, and shows the agreement between coast-meter reading and coast time that is usual. Curve *B* shows the performance of a (probably) defective assembly. The coast-meter readings were nearly the same and properly so since they were taken below 1,000 rpm where the curves are nearly parallel. It seems obvious that in such a case, the reading of the coast meter is a better measure of bearing pressure than the actual coast time.

Curve *C* of Figure 9 is in effect an enlargement of the low speed end of curve *A*, and shows normal performance in that the curve is convex downward. This corresponds to a slowly decreasing rate of deceleration. The circuit of the coast meter is so arranged as to compensate for this curvature and give a constant coast-time reading. Curve *D* is abnormal in being convex upward; this causes the meter indicated coast time to decrease as the rotor slows down, the change amounting to one or two minutes or more. Investigations of rotors that showed this characteristic have invariably shown defective bearings.

Conclusion

The practicability of an electronic instrument for measuring accelerations as low as 0.7 radian per second per second with instrument errors limited to a few per cent has been convincingly demonstrated by numerous installations, both in factory production lines and in field repair depots. There is no apparent reason why even smaller accelerations cannot be measured with greater accuracy if the need should arise, and by the present method. The instrument described has been entirely adequate for the present job.

Appendix. Deceleration Measurements by Coast Curves

As a check on apparent irregularities in coast-meter readings it was necessary to provide an independent method of measuring deceleration. This was accomplished by measuring time with a synchronous clock having a one revolution-per-second sweep, and measuring rotor bucket frequency with the Conn chromatic stroboscope.[3,4] The chromatic stroboscope is an instrument for measuring frequencies from 32 to 4,067 cycles per second by a stroboscopic method with a precision of ±0.05 per cent. This frequency range is in effect distributed logarithmically over a scale about 336 inches long. The error in reading a slowly varying frequency in the range of interest here is around 0.1 per cent since a stroboscopic pattern must be held stationary by slowly adjusting a variable-frequency tuning fork. The combined error in time and frequency measurements over a ten-second period is around two per cent but over a period of 100 seconds should be below 0.25 per cent.

Calibration

The factory calibration procedure is quite straight forward. The grid bias of *V5* is set at an appropriate value (about −8 volts) after a five-minute warm-up. Then the lower *V5* plate potentiometer is set so that on the 1,000-rpm range the meter reads 1,000 rpm with 400 cycles per second (five millivolts or more) input. The upper *V5* plate potentiometer is then set so that on the 15,000-rpm range the meter reads 10,000 rpm at 4,000-cycles per second (50 millivolts or more) input. This completes the speed calibration.

The coast calibration is made by adjusting the *V7* plate rheostat until the meter reads at a reference line (about 90 per cent of full scale) with no signal input. Then the *V6* grid bias is set to give −0.5 volt on the grid at 200 cycles per second input and checked for appropriate change at 400 cycles per second. Then the grid potentiometer of *V7* is set so that coast-time readings correspond to actual coast times for a number of gyroscopes.

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Selenium Power Rectifiers for A-C Aircraft Systems

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Synopsis: Increasing electric-power requirements for large aircraft have brought about the introduction of a-c power systems. Two major systems have been developed. In the first power is generated at variable frequency and converted to direct current; in the second power is generated at substantially constant frequency, and both a-c and d-c power are used. In both systems selenium rectifiers can be used as converters. Rectifier units developed to supply 28.5-volt regulated d-c power, which include step-down transformers and voltage-regulating equipment, are described in this paper. Ratings vary from 100 to 800 amperes.

AS THE SIZE and complexity of modern aircraft have increased, the demand for electric power to operate accessories has grown tremendously. The larger size also means that the distance between engine-driven generators and their load has increased. When the 28.5-volt d-c power system is used, large heavy conductors are necessary to keep voltage drops within operating limits. To reduce conductor weight to a practical value and provide a more flexible power system, many of the still larger aircraft of the future will use power generated at higher voltage as alternating current.[1] The Army Air Forces have standardized on a three-phase 208-volt 400-cycle system, where constant frequency is used. Variable-frequency systems also are being considered for some applications.

Generation of power at 208 volts reduces the weight of conductors to a fraction of that required for 28.5-volt power. Alternators are lighter and simpler than d-c generators of the same kilowatt output. With a-c generators brush problems are reduced, and, where simple induction motors can be used, motor brush troubles are eliminated.

Even with the constant-frequency system, where the fullest economical use can be made of alternating current directly, there is a definite need for appreciable amounts of d-c power. If batteries are used, they must be charged. Many in-

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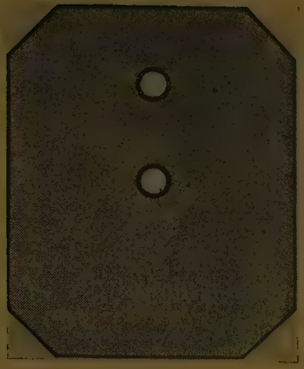


Figure 1. Selenium-rectifier cell for aircraft use

Weight 1.75 ounces. Over-all size: 6 by $7^{5}/_{16}$ inches

struments now are designed to operate on direct current; with 28.5-volt d-c power available present designs of such instruments may be used, saving the time and man power required to develop new types.

If 28.5-volt control power is used, present low-voltage control switches can be used on crowded instrument panels. Some apparatus, although it can be designed to operate on alternating current, may be lighter or have more satisfactory operating characteristics if operated on direct current, for example, contactor coils.[2] In such cases the weight and efficiency of the apparatus required for both a-c and d-c operation must be determined. Then, before it is decided whether to use alternating or direct current, the cruising range of the airplane, the required duty cycle, and the amount of d-c power required for other applications must be taken into account.

To provide the required d-c power from the a-c system the selenium rectifier offers outstanding advantages. It does not have the high voltage drop of an electronic tube nor the moving parts and brushes of a rotating machine. It is simple, reliable, and easy to maintain. For three-phase power input a selenium rectifier can be designed for 85-per-cent efficiency. The weight of a complete unit including transformer and voltage-regulating equipment compares favorably with that of a motor generator set of the same rating.

Selenium rectifiers can be used with transformers to obtain any desired output voltage. Input may be single phase or polyphase. The widest present application is for 28.5-volt d-c units with three-phase input.

Selenium-Rectifier Cells

A selenium-rectifier cell is made by applying a thin layer of selenium to a base plate which is usually iron or aluminum. After heat treatment a low-melting-point alloy known as the "counterelectrode" is sprayed over the selenium surface. The resultant cell will permit current to flow freely from the selenium to the counterelectrode and block almost entirely the flow from counterelectrode to selenium. Characteristics of selenium-rectifier cells have been discussed thoroughly elsewhere.[3] Although special cells have been developed for aircraft use, they are manufactured in the same way and have the same general characteristics as commercial cells now on the market.

The weight of any apparatus for aircraft use is important. Since the heaviest part of the selenium cell is the base plate, this plate should be as light as possible. The size of the cell affects the over-all weight; a large size will reduce the dead weight of inactive areas and the necessary connecting hardware. Large cells also have lower leakage current per unit area. Size is limited by manufacturing facilities and mechanical considerations. Figure 1 shows a cell developed to meet aircraft requirements. This cell has 36 square inches of active rectifying area, the base plate is aluminum alloy 0.020 inch thick, and the finished cell weighs 1.75 ounces.

Current density in a selenium rectifier normally is limited by temperature rise and permissible aging when cooled by natural convection. With forced-air cooling, however, efficiency and voltage drop impose practical limits. As operating life for aircraft applications generally is 1,000 hours or less, aging is relatively unimportant. Tests have shown that to maintain a constant d-c output voltage of 30 volts, it is necessary to raise the a-c input voltage to the rectifier from an initial value of 26.8 volts to 27.5 volts after 1,200 hours of operation, an increase of 2.6 per cent.

Rated current of the cell shown in Figure 1 is 50 amperes d-c output from a three-phase bridge circuit for each cell in parallel per leg. This current density is considerably higher than that used for commercial rectifiers and is based on an adequate flow of cooling air. The voltage rating of a three-phase bridge rectifier is 15 volts per cell in series. Higher-voltage rated cells have been produced but are difficult to manufacture, aging characteristics are less desirable, and ability to withstand overvoltage is not so good.

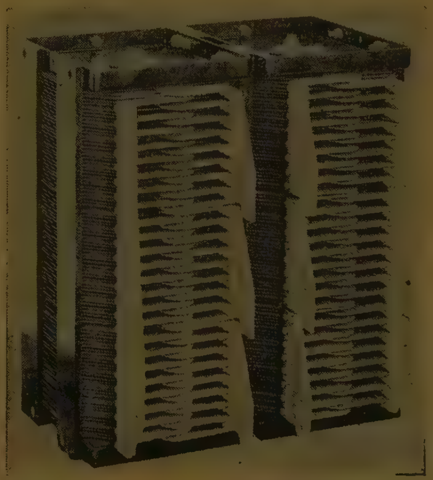


Figure 2. Selenium-rectifier stack assembly, rated 800 amperes 30 volts d-c output. Weight 35 pounds

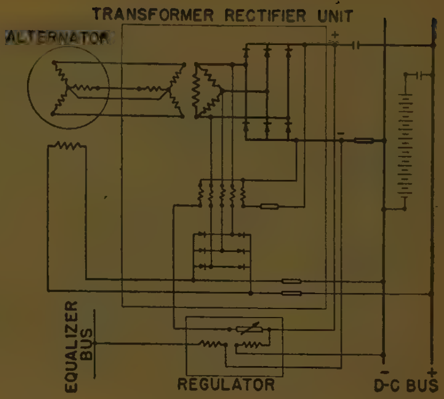


Figure 3. Diagram of variable-frequency power-supply system, showing alternator, transformer, main rectifier, field rectifier, saturable reactor, and voltage regulator

Selenium-Rectifier Stacks

Single rectifier cells act as "check valves" or half-wave rectifiers. For high efficiency a number of cells are connected in one of several possible circuits.[3] The three-phase bridge circuit is used most widely on three-phase power systems. It is simple to build mechanically, and only three connections are made to the a-c supply. If properly designed, a simple arrangement of connecting hardware can be used, and weight can be reduced to a minimum.

Figure 2 shows the three-phase selenium-rectifier stack assembly built for a unit rated 800 amperes 30 volts d-c output. In this assembly 192 cells are used, weighing 21 pounds or 0.87 pound per kilowatt. The complete stack assembly with all connecting and mounting hardware weighs 35 pounds or 1.46 pounds per kilowatt. Cells and connectors are assembled on insulated studs and held under pressure by nuts external to the end brackets. Elements are spaced to obtain maximum cooling effect with minimum air flow.

Cooling

The maximum design temperature of rectifier cells should not exceed 80 degrees centigrade. As Air Corps specifications call for operation in an ambient temperature of 60 degrees centigrade at sea level, sufficient air must be provided, either from the slip stream or a self-contained fan, to limit temperature rise to 20 degrees. Because of this low permissible rise, maximum volume of cooling air is required under sea-level conditions.

As the maximum operating temperatures of reactors, transformers, and fan motors are much higher than that of the

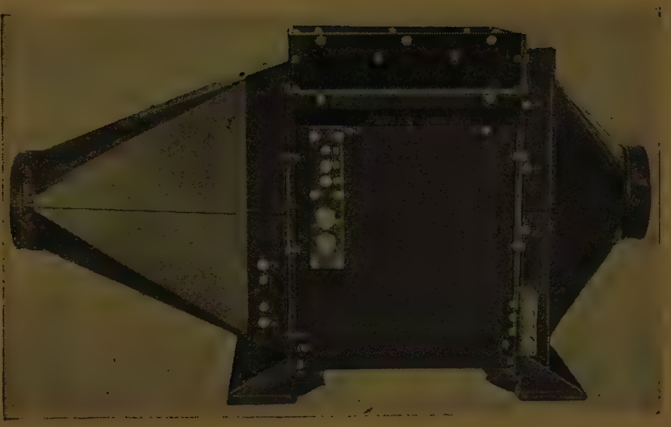
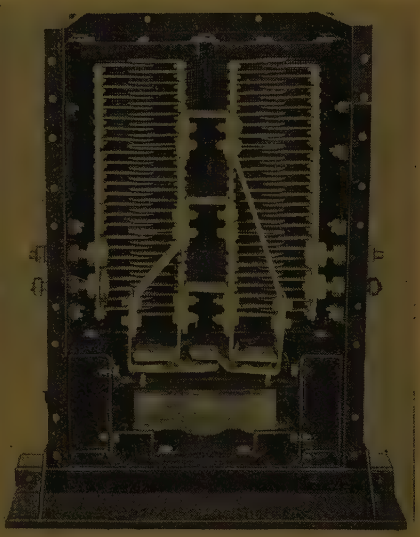


Figure 4. Complete rectifier unit used in variable - frequency system rated 800 amperes 30 volts d-c output

Figure 5 (below). Internal arrangement of rectifier stacks and transformer in rectifier unit of Figure 4



rectifier, the initial permissible rise is greater, and more cooling air generally will be required at high altitudes. Adequate air must be supplied to these auxiliaries to insure safe operation at the highest altitude for which the equipment is intended. With a constant volume of air through a fan-cooled rectifier the load on the fan motor decreases with altitude, so that requirements are less severe than those for a constant-load motor. For a given volume of air forced through the motor, however, a limiting altitude is reached beyond which it will overheat.

Selenium rectifiers have a negative temperature coefficient of resistance, and as the temperature falls the resistance increases.[3] The decrease in resistance of copper wiring and transformer windings partially compensates for this increase.[4] Operating temperature of the rectifier can be raised and resistance lowered under extreme low-temperature conditions by restricting the flow of cooling air. For large rectifiers to be used on high-altitude airplanes sufficient gain in efficiency may be made to justify the inclusion of automatic air-regulating devices.

Units with self-contained cooling fans are more satisfactory for operation within pressurized areas as blast-cooled units would require pressure-tight ducts and casings. There is some danger of damaging blast-cooled units by operating them on the ground with no cooling air available, unless air-flow protection is provided. Fan-cooled units with the fan motor connected directly to the input terminals eliminate this danger, as the motor will be energized at the same time as the rectifier. The major advantages of blast-cooled rectifiers are their lighter weight and the elimination of all moving parts.

Transformers and Reactors

In voltage-regulated power-rectifier equipments for aircraft transformers are used to provide the proper input voltage. Saturable core reactors connected in series with rectifiers and controlled by voltage regulators are used for voltage control. Both transformers and reactors for these applications use special core materials, and high-flux densities are permissible. They may be of the high-temperature type, with operating temperatures of over 200 degrees centigrade.[5] Copper conductors are cadmium-plated to prevent oxidation at high temperatures. Joints are silver-soldered. Glass fiber and glass tapes are used for insulation and impregnated with a special high-temperature resin. Forced-air cooling makes high-current densities possible.

Design of the reactor and transformer must be co-ordinated carefully with that of the rectifier and voltage regulator. Output-voltage variations are caused by changes in load, input voltage, frequency, and temperature. The saturable reactor must have sufficient range in reactance to permit the voltage regulator to hold constant voltage for extreme operating variation of the foregoing factors.

Rectifiers for Variable-Frequency Systems

If alternators are direct-connected to main engines, system frequency varies with engine speed. As the range of speed variation is usually more than two to one, direct application of a-c power is difficult.[6] Parallel operation of alternators is out of the question.

By furnishing a rectifier for each alternator and using direct current, all generating units can be made to feed powe to the same bus. As far as distribution is concerned, this is a d-c system similar to those used at present. However, powe is generated with alternators whose weight is a fraction of that of d-c generators of the same kilowatt output and is transmitted to the rectifiers, which may be located near the load center, at alternator voltage, reducing low-voltage wiring and bus work to a minimum.

Figure 3 is the diagram of a variable-frequency system in which all of the alternator output is rectified. All rectification, transformation, and voltage-regulating equipment except the voltage regulator proper are included in the casing shown in Figure 4. This includes step-down transformer, main power rectifier, saturable reactor for controlling alternator field current, and a field rectifier. Rated d-c output is 800 amperes 30 volts input is three phase, 350 to 1,000 cycles, with maximum alternating voltage 250 volts. Combined efficiency of the transformer and main rectifier is 81 per cent at full load. Weight of the complete rectifier unit is 135 pounds, and the 32-kva alternator developed for this system weighs 50 pounds, a total weight for the system of 185 pounds or 7.7 pounds per kilowatt. This compares with a weight for engine-mounted d-c generators of 5.2 pounds per kilowatt. Obviously the alternator-rectifier system cannot be justified on the basis of weight of equipment alone. However, it should be noted that the alternator can be mounted directly on the engine, whereas a d-c machine of the same rating would be separately mounted, requiring additional weight for support. In addition, the saving in weight of conductors from generator to load center will be appreciable. With further development it is probable that the weight of the rectifier unit can be reduced considerably.

The voltage-regulating system is similar to that of a self-excited d-c machine in that field power is taken from the alternator output. As alternator field current is of the order of 50 amperes, it cannot be handled directly by a standard aircraft-type voltage regulator, and some type o

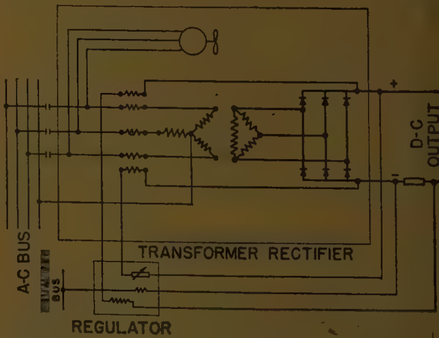


Figure 6. Diagram of rectifier unit used in constant-frequency system, showing saturable reactor, transformer, fan, rectifier, and voltage regulator

d-c current amplifier must be used. In this case the amplifier consists of a three-phase saturable core reactor in series with a three-phase bridge-connected selenium rectifier. Alternating current is supplied to the reactor input from the low-voltage side of the main transformer. The impedance of the saturable reactor varies with the current in its saturating coil which is controlled by a standard-type regulator. The output current of the saturable reactor is rectified and supplied to the alternator field. With this arrangement the voltage regulator controls currents up to ten times the current it can control directly. To make operation more stable a d-c damping winding is connected across the main rectifier output. An equalizing winding is provided on the regulator so that rectifiers may be paralleled, using the same scheme as that used with d-c generators.[7]

Under low-temperature conditions with the engine running at minimum speed, residual voltage may not be sufficient to cause the alternator to build up. This is due largely to the increased rectifier resistance at low temperatures, added to the field resistance and the impedance of the saturable reactor. To provide for positive build-up the field is connected across the main bus through current-limiting resistors. With the bus energized by a battery or other d-c source, sufficient field current will flow to initiate the build-up cycle. When the voltage of the field-supply rectifier reaches normal, current flow through these resistors is reduced to a negligible value.

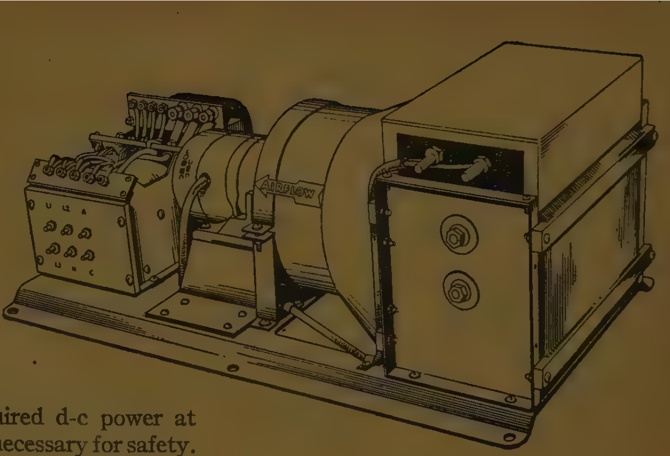
The arrangement of transformers and main rectifier inside the casing is shown in Figure 5. To keep the center of gravity low, the transformer is mounted near the base. Cooling air flows in parallel paths through the transformer and rectifier. Automatic shutters regulate the flow of air through the rectifier stack; they are adjusted to begin to open when the temperature of the exit air from the rectifier reaches 50 degrees centigrade. At low temperatures, the gain in rectifier efficiency is considerable; by raising the operating temperature from 0 to 50 degrees centigrade, the efficiency is increased by six per cent. Air flow through the transformer is unrestricted. Cooling air is supplied from the slip stream and a pressure of six inches of water must be maintained at the air inlet.

Rectifiers for Constant-Frequency Systems

By maintaining a reasonably constant system frequency most of the generated power can be utilized as alternating current. If alternators are driven from the main engines a constant-speed coupling must be provided which will maintain proper alternator speed regardless of engine speed. Alternators may be paralleled.[8]

Figure 7. Aircraft rectifier unit for constant-frequency system, rated 200 amperes 30 volts d-c output

Note open construction; only rectifier and fan blade are enclosed



To furnish the required d-c power at least two rectifiers are necessary for safety. The number and ratings of rectifiers should be chosen so that all vital equipment taking power from the d-c bus can be operated with the largest unit out of service.

Figure 6 is the diagram of a unit designed for operation with the constant-frequency system. A saturable core reactor, transformer, rectifier, voltage regulator, and fan are included. For blast-cooled units the fan is omitted. Output voltage is controlled by a standard aircraft-type voltage regulator and a saturable reactor. In general, the regulating system is the same as that of the 800-ampere unit, but in this case the entire rectifier output is controlled instead of alternator field current. An equalizer winding is provided on the regulator to permit parallel operation of rectifiers. The voltage regulator is separately mounted and can be located at any convenient point.

In Figure 7 is shown a transformer rectifier unit rated 200 amperes 30 volts d-c output designed to take three-phase power from a 208-volt 400-cycle system. Cooling air is supplied by a magnesium-alloy fan mounted on the shaft of a 7,600-rpm induction motor. This unit will operate under all conditions encountered from a 60-degree-centigrade ambient temperature at sea level to an altitude of 40,000 feet. To save weight no enclosing case is used. The fan is located inside the air duct extending from the rectifier stack and discharges air over its driving motor to the transformer and reactor. Weight complete is 47 pounds.

Figure 8. Aircraft rectifier unit for constant-frequency system rated 100 amperes 30 volts d-c output

Cooling air supplied from external source

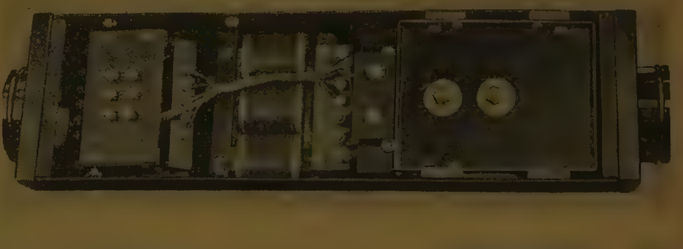
Another unit developed for the constant-frequency system is shown in Figure 8. It is rated 100 amperes 30 volts d-c output, is blast-cooled, and requires an air pressure of three inches of water at the inlet. All parts except the voltage regulator are mounted inside the casing; the compact arrangement of parts is shown in Figure 9. Over-all dimensions are $6\frac{1}{2}$ by $6\frac{5}{8}$ by 26 inches, and weight is 28 pounds.

Operation

Operating characteristics of these saturable-reactor-controlled rectifier units are similar to those of a d-c generator. They have a drooping volt–ampere characteristic due to the regulation inherent in the reactors, transformers, and rectifiers. As load increases, the voltage regulator keeps output voltage constant by increasing saturating current in the reactor. After the saturating current has reached its maximum value, further increase in load will reduce output voltage. This, in turn, reduces saturating current, lowering still further the output voltage. The result is that a critical point is reached beyond which operation becomes unstable and the voltage breaks down. In the case of the variable-frequency system, short-circuit currents are low, as the a-c supply voltage also breaks down. Constant-frequency units, on the other hand, have short-circuit currents of the order of 150-per-cent load current if a-c input voltage is maintained. Current in this latter case is

Figure 9. Aircraft rectifier unit, same as Figure 8

Cover removed, showing compact arrangement of parts



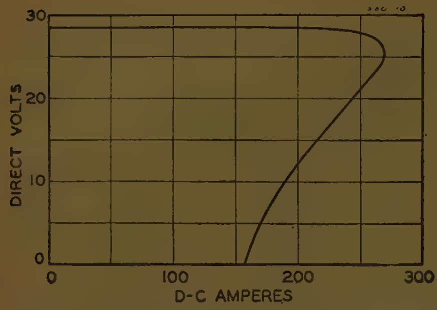


Figure 10. Volt–ampere characteristic curve of aircraft rectifier unit shown in Figure 8

limited primarily by the maximum reactance of the saturable reactor.

Figure 10 shows the volt–ampere characteristic of the 100-ampere unit of Figure 8, with the voltage regulator set to hold 28.5 volts. Input voltage and frequency are held at normal values. Short-circuit current under these conditions is 160 per cent and maximum current 270 per cent of rated value. The effect of a short circuit applied with the same rectifier carrying rated load current and the regulator set to hold 30 volts is shown in the oscillogram of Figure 11. Instantaneous peak current was 411 per cent which decayed to a steady-state value of 165 per cent in $^1/_{60}$ of a second. Instantaneous peaks as high as 600 per cent have been noted under no-load initial conditions. The values of peak and steady-state currents depend on alternating voltage, frequency, load, and regulator setting.

Maximum load and short-circuit currents are increased if a separate source of reactor-saturating current is provided. The volt–ampere characteristic then resembles that of a separately excited d-c generator. For the constant-frequency system this source of current may be a small "exciter" rectifier.

A "compounding" series winding may be added to the saturable reactor through which all or part of the output current flows. By proper design any degree of compounding may be obtained. Compounding reduces the required resistance range of the voltage regulator but adds to the weight of the reactor and complicates the wiring.

Division of load between units, even those of different ratings, is satisfactory. Figure 12 shows the load division between the 100-ampere and 200-ampere units of Figures 7 and 8 under conditions of normal voltage and frequency. The division will vary somewhat, depending on regulator settings and input voltage and frequency.

All of the units described can be overloaded for a short time. With maximum ambient temperature for which the apparatus is designed they are rated at 150-per cent normal current for three minutes. At lower ambient temperatures this overload rating can be increased. All overloads should be applied with caution as the thermal capacity of the rectifier is low. These units never should be operated at higher than rated voltage, or the cells may be damaged permanently.

All data so far obtained on these rectifiers are the results of extensive ground tests, including tests in altitude chambers. No flight-test data are as yet available, but it is expected that thorough flight tests will be made in the near future.

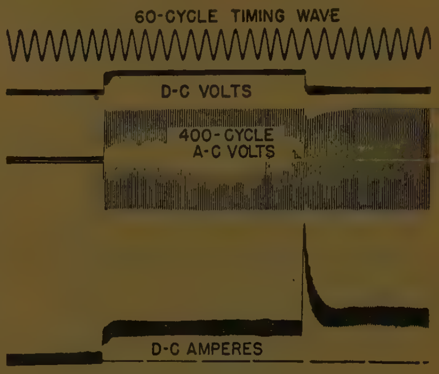


Figure 11. Oscillogram of short circuit applied to rectifier unit shown in Figure 8

Regulator set to hold 30 volts. Rectifier was carrying rated load when short circuit was applied

Applications of Unregulated Rectifiers

For some applications where the d-c load is substantially constant it may be practical to use selenium rectifiers without voltage regulation. Such rectifiers may be designed to operate directly from the a-c line or with suitable transformer to provide any desired direct voltage. Variations in direct voltage will be only slightly greater than those of the system alternating voltage.

In all the regulated units described the transformer weighs more than the rectifier stack, so that the use of rectifiers operated directly from the a-c lines offers attractive weight-saving possibilities. This scheme has one serious drawback; only three direct voltages are available, 220, 170, and 100 volts. These are obtained by the use of a three-phase bridge-connected rectifier, a single-phase bridge connected line to line, and a single-phase bridge connected line to neutral, respectively. Some saving in transformer weight can be made by the use of autotransformers. In either case the d-c circuit is electrically connected to the a-c lines, and neither d-c terminal can be grounded if the a-c system is grounded at any point.

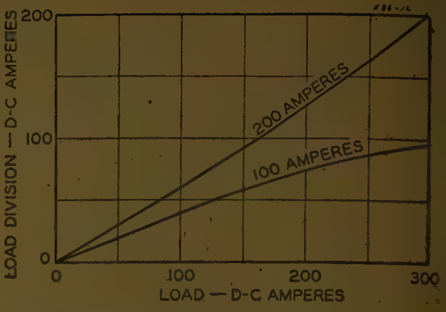


Figure 12. Distribution of load between rectifier units shown in Figures 7 and 8

Rectifiers paralleled through 0.5-volt dropping resistors. Regulators set for 30 volts

In any case where the rectifier supplies a fixed load no d-c switch is necessary. The rectifier may be connected permanently to its load and all switches, contactors, and protective devices located in the a-c line.

Conclusion

It must be kept in mind that the selection of an electric-power system is an individual problem for each airplane. The size of the airplane, its ceiling, its maximum cruising range, and its purpose all must be considered. Power requirements for a cargo airplane, a passenger airplane, and a bomber will differ widely even when all are the same size and general design. Larger airplanes of all classes are being considered for the future, and many of them will have auxiliary power generated as alternating current. In most cases considerable amounts of d-c power still will be required, and in supplying this power the selenium rectifier will find a wide field of application.

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Static Voltage Regulator for Rototrol Exciter

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THE CONCEPT of a generator voltage regulator as an amplifier which produces a large correction in field current for a small departure from normal voltage is quite well known.[1] In the conventional regulator, such as a direct-acting type, this amplification is secured partly in the regulator and partly in the exciter. A rotating amplifier-type regulator provides the amplification almost entirely in the rotating machines. Exciters may be used to obtain two or more stages of amplification, or a single machine having high amplification such as a Rototrol exciter may be used. An example of two-stage excitation is a system with the regulator working in a pilot exciter field.

With high amplification available in the excitation system, voltage regulation can be secured by arranging the control field of the exciter or Rototrol[2] to raise or lower in response to any deviation of generated voltage from the standard of reference.

The particular Rototrol referred to in this paper is a d-c generator operated on the linear part of its saturation characteristic and completely self-excited by shunt and series fields. As such, it will support itself at any voltage from zero up to the point of saturation. It has one or more control fields which run it up or down in voltage as required and keep it operating stably at any desired point.

The present development was undertaken on the basis that a battery could not be used for the standard of reference and that the voltage reference must be inherent in the circuit. Also vacuum tubes could not be used; there must be no moving parts except for the rotating machines, and these should have air gaps approximately as large as in conventional d-c generators.

The choice of voltage reference was narrowed down to "intersecting impedance characteristics." This is the general name given to circuits that detect and respond to voltage level in the following manner. At normal voltage two impedances are adjusted to draw equal or proportional currents; at least one impedance is nonlinear. At higher voltage one impedance draws more current than the other, and the difference current is in a direction indicating voltage above normal. It can be used for control. At lower voltage the same impedance draws less current than the other and the difference thus is reversed, indicating voltage below normal. The currents are rectified before comparison.

Static Voltage-Regulating Circuits

A static type of circuit for the purpose of regulating the voltage of an a-c generator has been used previously,[3] a three-core saturable reactor being employed for amplification as part of the static circuit. Similar circuits using Rototrols for the amplification are illustrated in Figure 1.

The voltage-measuring circuit of Figure 1A consists of a resistor path I_R and a saturating-reactor path I_X, with the current in each path rectified and fed into the control fields of the Rototrol. This illustrates a circuit using a Rototrol pilot exciter, an exciter of conventional design, and an a-c generator. The Rototrol control fields are connected to be equal and opposite magnetically, when the a-c generator voltage is normal at the regulated value. Under this condition the Rototrol output voltage is maintained at the required value by its self-energizing shunt and series fields. If the alternating voltage rises or falls, the control fields adjust the Rototrol as needed to restore normal voltage.

Figure 1B shows the intersecting impedance characteristics, R for the resistor circuit, and X for the reactor circuit. In regulating circuits of this kind means are provided for raising or lowering the point of intersection or "normal" voltage, to allow adjustment of the value which the regulator is set to maintain. Figure 1C shows the same static regulator except used with a Rototrol exciter, thus eliminating the pilot machine. The Rototrol exciter is designed to have relatively high amplification, so that the required output to the field of the a-c generator may be controlled by a small amount of energy in the control field.

Other static circuits are shown in Figure 2. The generator terminal voltage is balanced against a reference voltage, and the differential which appears when the generator voltage deviates from normal is applied to the Rototrol field or fields. For the reference voltage a separate source either direct or alternating current may be used, or, if none is available, a constant potential device may be employed, energized by the voltage of the generator with which the regulator is used. This device is designed to provide essentially constant alternating voltage on its output terminals in spite of relatively large changes in its input voltage.

Figure 2A shows a static regulator controlling a Rototrol exciter which has two control fields. These are connected to have equal and opposite magnetic effect when the a-c generator voltage is at the desired normal value. Should load be added to the generator, thus causing drop in the terminal voltage, the "raise" field predominates and raises the Rototrol armature voltage thus restoring the alternating voltage to normal. The Rototrol control fields again reach a balance.

Figure 2B shows a balanced-voltage circuit arranged to feed a single control field. In this case the control field normally has no current flowing through

Table I. Sample Calculation of E_f/E_R versus E_x/E_R

Refer to Figure 6

These Calculations Are for the Case, $R'=0.2R$ and $R_f=0.3R$

$E=1.00$, $E_{eq}=\dfrac{R_f}{R+R_f}=0.2308$

$R_{eq}=\dfrac{RR_f}{R+R_f}=0.2308R$, $\dfrac{R_{eq}}{R'+R_{eq}}=0.534$

1	2	3	4	5	6	7
θ	$E_x=\sin\theta$	$E_d=\cos\theta$	$E_R= E_d+E_{eq}= E_d+0.2308$	E_x/E_R	$E_f=\dfrac{R_{eq}E_R-E_{eq}}{R'+R_{eq}}= 0.534E_R-E_{eq}$	E_f/E_R
0	0	1.0000	1.2308	0	0.428	0.347
10	0.1736	0.9848	1.2156	0.143	0.419	0.347
20	0.3420	0.9397	1.1705	0.292	0.395	0.337
30	0.5000	0.8660	1.0968	0.456	0.355	0.324
40	0.6428	0.7660	0.9968	0.645	0.302	0.303
50	0.7660	0.6428	0.8736	0.878	0.235	0.269
60	0.8660	0.5000	0.7308	1.186	0.160	0.219
70	0.9397	0.3420	0.5728	1.640	0.0752	0.131
80	0.9848	0.1736	0.4044	2.432	−0.0148	−0.0366
90	1.0000	0	0.2308	4.333	−0.1076	−0.466

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it. By reason of this, it is evident that there are no losses and no heating in this field so long as the a-c generator voltage remains at the normal regulated value. This permits using a minimum of material in the frame of the Rototrol, as winding space and winding losses are less than in the case where continuously energized equal and opposite control fields are employed.

Requirements of New Regulating Circuit

In the development described in this paper, generators having ratings exceeding 1,000 kva at 1,200 rpm were to be controlled, and conventional exciter air gaps were desirable for mechanical reasons. This fixed the control-field energy requirements. Previous designs of static regulators such as described hereinbefore would have been excessive in size and losses, had they been designed to provide adequate control power. It became necessary to make a thorough analysis of the design of such devices to secure optimum energy output for a given voltage departure from normal.

Average three-phase (or positive-sequence) voltage response was required. This further increased the energy requirements, as the positive-sequence network size must be adequate to carry the larger power of the measuring circuit. This problem was solved by deducting negative-sequence voltage from line voltage rather than filtering positive-sequence voltage directly in the conventional manner.

Considerable error in regulated voltage due to frequency was tolerated on earlier static regulators. In the new development it was desired to regulate closely for the same voltage level over a plus- or minus-five-per-cent departure from normal frequency.

To meet these requirements a static regulator of high efficiency was devised. Figure 3 shows a regulator of this type. The following paragraphs describe this regulating network and give its theory and performance.

Circulating-Current-Type Static Network—Theory

One arrangement of the circulating-current type of static-regulating network is shown in Figure 4A. A saturating reactor and resistor are used for the nonlinear and linear branches, respectively. The voltage gradient may be viewed as shown in Figure 5. At normal voltage the voltage drops in the resistor, KR, and reactor, X, are equal and the control-field voltage is zero, as in Figure 5B. As the input voltage is raised or lowered, the saturating-reactor voltage remains nearly constant and the control-field voltage becomes positive or negative, respectively. This is shown in Figures 5A and 5C.

Performance Calculation

Calculations were made to determine the most efficient design of this type of network. First it was necessary to develop a method of calculating the power output from the network as a function of the parameters X, R, R', and R_f of the network and connected control field of Figure 6. Also, a method was developed for the calculation of the direct voltage on the rectifiers to determine whether R' and $(1-k)\,R$ were large enough to keep both rectifier voltages positive over the necessary range of operation (usually for a plus- or minus-ten-per-cent departure from normal line voltage).

Calculations then were made using these methods to determine the relative values of the parameters X, R, R', and R_f which would produce optimum control-field input power as a function of line-voltage departure from normal, with minimum energy consumed in the network itself.

This method of calculation assumes pure sine-wave currents and voltages in the a-c circuits and smooth direct current in the d-c circuits. It is recognized that the actual wave shapes are considerably distorted and that an empirical factor determined from test must be used to allow for this difference. This factor was found to be of the order of 0.5 in field current. The analytical method used with this empirical factor serves as a very satisfactory basis of direct design. The method also affords valuable insight into the operation of the network and a close approach to optimum efficiency which then can be further refined by experimental work.

Method of Calculation

The problem may be stated as follows: *Given* the saturation curve of the reactor X, such as Figure 7, and the values of R, R', and R_f.

Required to determine field voltage, E_f, as a function of the applied line voltage.

The network of Figure 6A contains a-c and d-c circuits and a nonlinear element, and hence, cannot be solved by conventional methods of analysis.

The saturating reactor can best be treated by assuming a value of current, reading the corresponding voltage from the saturation curve, and then calculating the currents and voltages throughout the balance of the network.

As the alternating voltages being dealt with are about to be rectified in single-phase full-wave rectifiers, it is convenient to use their average rather than their rms magnitudes. Then no factor need be applied when passing from alternating to direct current through the rectifier.

Figure 6C shows a vector diagram

Table II. Sample Calculation of Field Power From Static Regulating Network

Refer to Figure 6A

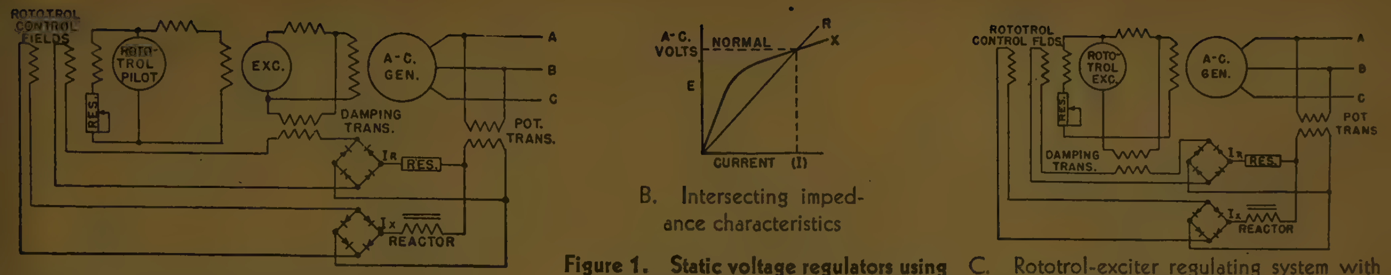
The Subscript *n* Denotes the Value of a Quantity at the Normal Regulated Voltage Condition. Calculation Based on: $R'=0.2R$ and $R_f=0.3R$

Let $E_{xn}=1.000$ $I_{xn}=1.000$ $X_n=1.000$ $E_n=E_{xn}/\cos\sin^{-1}\dfrac{R'}{R}=E_{xn}/0.98=1.02$ $R=E_n/I_{xn}=1.02,\ R'=0.204,\ R_f=0.306$

$$P_n=E_nI_{xn}=\frac{R'+R}{R}=1.02\times1.00\times1.20=1.224 \qquad R_{eq}=\frac{R_f}{1+R_f/R}=\frac{0.306}{1.300}=0.2354 \qquad R'+R_{eq}=0.4394$$

$$E=\frac{R}{R'+R_{eq}}\,E_R-\frac{R}{R_{eq}}\,E_f=2.320E_R-4.33E_f$$

1 E_x	2 I_x (Figure 7)	3 $E_R = I_x(R'+R_{eq}) = 0.4393I_x$	4 E_x/E_R	5 E_f/E_R (Figure 8)	6 $E_f =$ (3)×(5)	7 $E = 2.320E_R - 4.33E_f$	8 E/E_n P.U. Line Voltage	9 $P_f = E_f^2/R_f = E_f^2/0.306$	10 $\frac{100\,P_f}{P_n}$ Per Cent Field Power	11 $\frac{100}{(10)}\left[\frac{1-(8)}{0.1}\right]^2$	12 $\frac{E_f}{E_n}$
0.890	0.467	0.2054	4.33	−0.466	−0.0958	0.890	0.872	0.0299	2.440		0.0937
0.915	0.555	0.2440	3.75	−0.334	−0.0815	0.920	0.902	0.0217	1.770	54.3	0.0800
0.950	0.707	0.3110	3.06	−0.178	−0.0554	0.961	0.942	0.0100	0.820		0.0538
1.000	1.000	0.4394	2.28	0	0	1.020	1.000	0	0		0
1.025	1.189	0.5225	1.960	+0.064	0.0334	1.069	1.048	0.00365	0.298		0.0328
1.050	1.414	0.621	1.690	+0.119	0.0739	1.121	1.100	0.01785	1.458		0.0725
1.075	1.682	0.739	1.455	+0.160	0.1180	1.188	1.163	0.0455	3.72		0.1160



A. Rototrol-pilot-exciter regulating system

B. Intersecting impedance characteristics

C. Rototrol-exciter regulating system with opposed control fields

Figure 1. Static voltage regulators using intersecting impedance characteristics

which is a convenient representation of the complete circuit. To understand it, note first that Figure 6A can be simplified to Figure 6B using Thevenin's theorem.[4] The values of E_{eq} and R_{eq}, in terms of E, R, and R_f are shown by the equations in Figure 6. Note that in the d-c circuit of Figure 6B, the voltage drop across R' and R_{eq} in series is the numerical sum of E_d and E_{eq}. Also note that E_d is obtained from E by subtracting the I_xX drop vectorially from E. The current I_x must be in phase with E_d as the circuit below E_d in Figure 6B appears as pure resistance or in phase. Hence, the drop I_xX must be at right angles to E_d, and this fixes the form of the vector diagram Figure 6C. Note that the relationships in the d-c part of the circuit all are treated as though in phase.

If we start with the current I_x, the voltage E_x is readily determined from the saturation curve, Figure 7. E_R is obtained by multiplying I_x by $(R' + R_{eq})$. E is known and E_{eq} determined from the equation in Figure 6. It remains to find the field voltage, E_f. This can be done most easily by letting θ vary and calculating E_x/E_R and E_f/E_R for various values of θ, and plotting E_f/E_R as a function of E_x/E_R as shown in Figure 8. From these curves the value E_f/E_R and then E_f readily can be determined. A derivation and sample calculation of these curves for $R' = 0.2R$ and $R_f = 0.3R$ is given in Appendix I. Figure 8 gives curves for $R' = 0.2R$ for various values of R_f.

A derivation and sample calculation also is given in Appendix II for field input voltage and power as a function of the departure of line voltage from normal. Figure 9A shows field power and voltage as a function of line-voltage deviation from normal, for the range of operation in which the rectifier voltages are above zero. Figure 9B shows the field power over an extended range of line voltage drop. Another curve, Figure 10, has been drawn for various values of R_f and R', giving watts input to the network per watt input to the field, for a ten-per-cent a-c line voltage drop. This curve is based on several calculations similar to Table II of Appendix II. It indicates that the best performance is obtained with $R' = 0.2R$ and $R_f = 0.3R$. However, the field resistance, R_f, may be varied 40 per cent from this value without reducing the control-field input, at ten-per-cent voltage drop, more than 20 per cent.

A capacitor or a linear inductance may be substituted for the resistor to form the linear branch. The capacitor has the advantage of minimum power loss and correction of power factor, which reduces the required size of potential transformers and compensators.

The foregoing analysis applies equally well whether the impedance kR consists of a resistor, capacitor, or linear reactor having the same ohmic value. With the assumption of smooth direct current, phase relationships are not reflected through the d-c circuit. If the branch, kR, is a capacitor or a reactor, the curves of Figure 10 should be interpreted as giving the summation of volt-amperes (arithmetic sum) in branches, R', $(1-k)R$, and kR, per watt in the control field.

It may be more significant in this case to give the *total* network volt-amperes (arithmetic sum of volt-amperes in each part) per watt in the control field. This can be done by substituting the total volt-amperes for P_n in Table II. The summation of network volt-amperes is given by

$$P_n' = E_n I_{xn}\frac{R'+R}{R}+1$$

For the example worked out in Table II, P_n' would be 2.224 and the network volt-ampere scale would be 2.224/1.224 times the network watt scale of Figure 10.

The summation volt-amperes as used in the foregoing are of significance chiefly in providing a measure of size of the network. The volt-amperes drawn from the source also are important in determining the design of the potential transformer, sequence network, compensator, and so forth, through which this burden is drawn. The input volt-amperes may be obtained by vector addition of the volt-amperes of parts. Use of a capacitor for the linear branch then largely offsets the saturating-

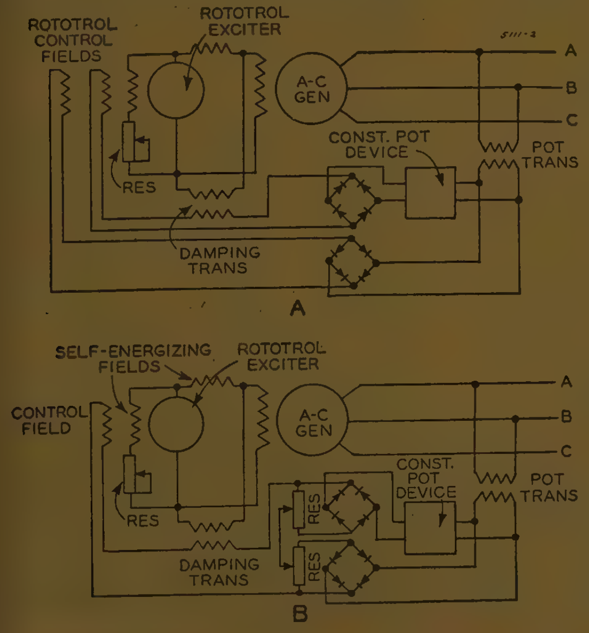


Figure 2 (left). Static voltage regulators using fixed reference potential

A. Opposed control fields

B. Balanced-voltage network and single control field

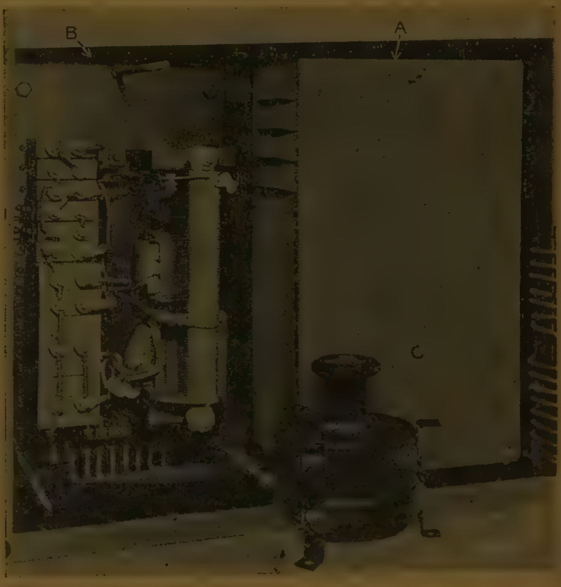


Figure 3 (right). Static-Voltage-Regulator units

A—Potential and compensator unit

B—Voltage-regulator unit

C—Voltage-adjusting rheostat

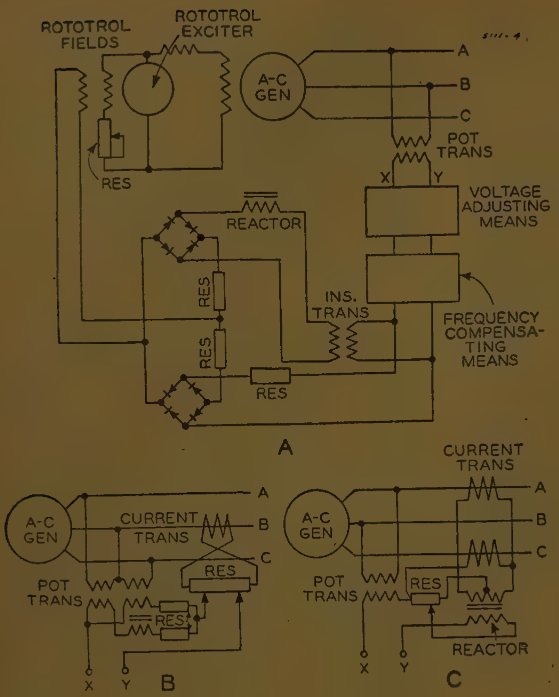


Figure 4 (left)

A. Static voltage-regulating system using circulating-current network

B. Conventional positive-sequence network and reactive-droop compensator

C. Current-energized sequence network including reactive-droop compensation

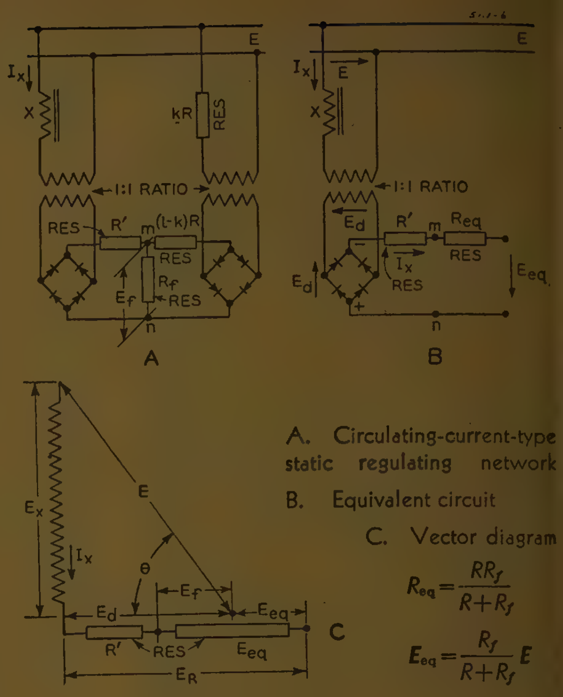


Figure 6 (right)

A. Circulating-current-type static regulating network

B. Equivalent circuit

C. Vector diagram

$$R_{eq}=\frac{RR_f}{R+R_f}$$

$$E_{eq}=\frac{R_f}{R+R_f}E$$

reactor volt-amperes at the normal regulated voltage and minimizes the input volt-amperes to the measuring circuit.

Positive-Sequence Response

The static-regulator volt-ampere parts are determined by the power requirements of the control field as outlined in the foregoing section. In the present development this burden proved to be much larger than the normal burden of a conventional regulator element. In order to provide positive-sequence response the regulator current in the past has been drawn through a positive-sequence network or filter.[5] However, this device increases in size directly with the burden supplied through it; consequently, some other plan for obtaining positive-sequence response was needed.

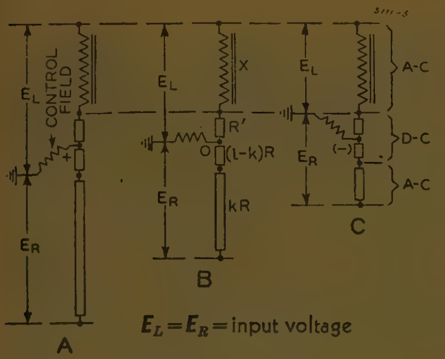


Figure 5. Approximate voltage gradient in circulating-current-type static regulating network

A—Overvoltage—control-field voltage positive

B—Normal voltage—control-field voltage zero

C—Undervoltage—control-field voltage negative

The positive-sequence voltage can be derived by subtracting the negative-sequence voltage from line voltage. This is accomplished by passing the secondary currents of the current transformers through a network proportional to the negative-sequence impedance of the generator. The derived negative-sequence voltage then is deducted from the line voltage. The previously used voltage filter and the new current filter are indicated in Figures 4B and C, respectively. The remainder of the regulating circuit is the same in either case and is as shown in part A. The current-energized sequence filter required about $1/_{60}$ of the volt-amperes of parts as compared with the voltage-type sequence network of Figure 4B. Tests showed it to be accurate and reliable. One potential and two current transformers are used in place of two potential transformers and one current transformer.

Reactive-Droop Compensation

With the single potential transformer and current-energized sequence network shown in Figure 4C, the reactive-droop compensator for parallel operation becomes a resistor in the phase-*C* current and thus can be combined with the similar resistance of the sequence network. Thus, the resistor in Figure 4C serves this double purpose.

Frequency Compensation

As the frequency is varied the static measuring circuit can be kept in balance only by varying the voltage applied thereto in a predetermined fashion. As it is desired to regulate for the same generator voltage with some departure from normal frequency, it is necessary to introduce between the potential transformer and the measuring circuit an impedance which will produce the necessary voltage variation with frequency on the measuring circuit. Network synthesis studies developed the proper combination of inductances and capacitances for this purpose. As the inductance required was a series device, it was obtained simply by proportioning the sequence network to have this value of inductance as viewed from the output terminals.

Conclusions and Test Results

A static regulator was designed and built using the methods outlined in the paper. This regulator, whose circuit is shown in Figures 4A and C was designed for use with a Rototrol exciter and 1,500-kva generator. Tests have demonstrated it to be a highly satisfactory solution to the regulating problem outlined at the beginning of the paper.

The sensitivity was well within the design goal of plus or minus one per cent under steady or slowly changing load conditions. With the generator terminal voltages unbalanced due to the application of a single-phase load, the regulator maintained the average voltage within the sensitivity limits. For variations of frequency within reasonable limits, the regulator showed only small deviation from normal voltage.

Under sudden application of a 50 per-cent rated current load, at low power factor, the voltage dip[6] was only slightly in excess of the initial transient-reactance drop in the generator. The voltage returned to normal in less than one-half second. The action was smooth with very good damping.

Parallel-operation tests at rated power factor showed the automatic wattless load division to be within six per cent of rated generator current up to full rating. This excellent paralleling was obtained with reactive-droop compensation of 5.5 per cent at full-load zero power factor, which is 3.3 per cent at full-load 0.8 power factor.

Appendix I. Curve of E_f/E_R Versus E_x/E_R

With reference to Figure 6C, a curve of E_f/E_R versus E_x/E_R can be calculated, using θ as a parameter. E can be taken as 1, since ratios of voltages are to be determined and the actual values are immaterial. The steps in the computation are as follows:

1. Values of θ are taken from 0 to 90 degrees as the parameter.
2. Then E_x is equal to $E\sin\theta=\sin\theta$, and
3. E_d is equal to $E\cos\theta=\cos\theta$
4. The resistor branch is represented by an equivalent voltage, E_{eq}, and an equivalent internal resistance, R_{eq}, where

$$E_{eq}=\frac{R_f}{R+R_f}\ E=\frac{R_f}{R+R_f}$$

and

$$R_{eq}=\frac{RR_f}{R+R_f}$$

The total voltage across R_{eq} and R' is

$$E_R=E_d+E_{eq}$$

5. The ratio of E_x/E_R is then computed.
6. The field voltage, E_f, is a potentiometer fraction of E_R, minus E_{eq}. That is:

$$E_f=\frac{R_{eq}}{R'+R_{eq}}\ E_R-E_{eq}$$

7. The ratio E_f/E_R is then computed and plotted as a function of E_x/E_R.

Note that $\theta=90$ degrees is the limit of performance, according to this theory, because, as shown in Figure 6C, the rectifier voltage, E_d, becomes zero at $\theta=90$ degrees.

A sample calculation follows; the results are plotted in Figure 8 for the ratio $R_f/R=0.3$.

Appendix II. Circulating-Current-Type Static-Network Field Input Power and Rectifier Voltages

With reference to Figure 6, and saturation curve, Figure 7, the steps in a power computation are as follows:

The field power P_f and voltage E_f are to be computed for a given departure of line voltage from normal, and a given set of proportions R', R_f, and R. This calculation cannot be made directly. It is necessary to start with a given reactor voltage E_x, and calculate the field power and line voltage corresponding. After repeating for several values of E_x, curves of line voltage and field power can be drawn from which the field power can be read for the desired line voltage.

It is assumed that the network is so proportioned that the field voltage and power are zero at normal line voltage.

This requires that at balance:

$R=\sqrt{(R')^2+X^2}$, that is, equal impedances in the two branches up to the point of field connection. Thus, at balance the reactor voltage is slightly less than the line voltage. It is

$$E_{xn}=E_n\cos\sin^{-1}(R'/R)$$

where the subscript, n, denotes the value of a quantity at the normal balanced condition or

$$E_n=E_{xn}/\cos\sin^{-1}(R'/R)$$

In other words, if the reactor voltage is taken as unity at the balance point, the normal line voltage, E_n, is slightly over unity as given by the foregoing equation.

Procedure

The steps of procedure are then as follows:

The basis is stated; that is, R' and R_f in terms of R from which

$$R_{eq}=\frac{RR_f}{R+R_f},\ \text{and}\ (R'+R_{eq})$$

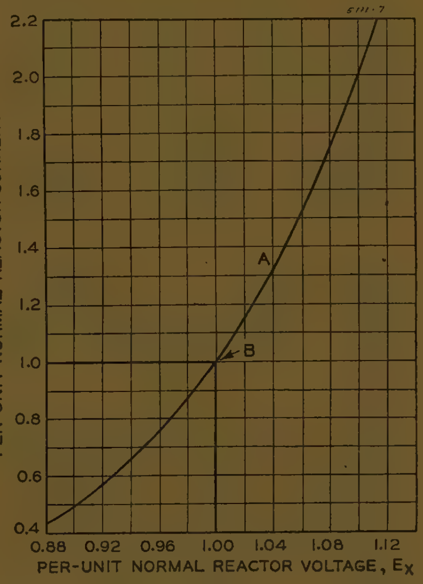


Figure 7. Typical saturation curve of saturating reactor

$$I_x=2^{\left(\frac{E_x-1.0}{0.10}\right)}$$

A—Current doubles for every ten-per-cent increase in voltage

B—Balance point or normal operating point

are determined. The coefficients in the equation for E in terms of E_R and E_f are then determined.

$$E=E_R\frac{R}{(R_{eq}+R')}-E_f\frac{R}{R_{eq}}$$

The normal line voltage is calculated from the normal reactor operating point voltage as follows:

$$E_n=E_{xn}/\cos\sin\,(R^1/R)$$

where $E_{xn}=1.0$.

And finally the normal network power is calculated from

$$P_n=E_nI_{xn}\frac{R'+R}{R},\ \text{where}\ I_{xn}=1.0$$

The steps in the calculations are as follows, item numbers being the same as the column numbers in the example, Table II.

1. A value of E_x is selected.
2. The corresponding current, I_x, is determined from the saturation curve (see Figure 7).
3. This current flows through the resistance $(R'+R_{eq})$ in series and produces the voltage drop E_R, where, $E_R=I_x(R'+R_{eq})$.
4. From this value of E_R, the ratio E_x/E_R is calculated.
5. The ratio E_f/E_R is determined from a curve obtained as described in Appendix I and shown in Figure 8.
6. The field voltage then can be calculated as follows:

$$E_f=E_R(E_f/E_R)=\text{Item }3\times\text{item }5$$

7. E is then calculated as follows: From Figure 6, it can be seen that E_{eq} equals the voltage on R_{eq} minus E_f.

Since

$$E_{eq}=\frac{R_f}{(R+R_f)}E$$

this may be written

$$\frac{R_f}{(R+R_f)}\ E=\frac{R_{eq}}{(R'+R_{eq})}\ E_R-E_f$$

but since

$$\frac{R_f}{R+R_f}=\frac{R_{eq}}{R}$$

$$E=\frac{R}{(R'+R_{eq})}\ E_R-\frac{R}{R_{eq}}\ E_f$$

8. The per-unit line voltage is then calculated, E/E_n.

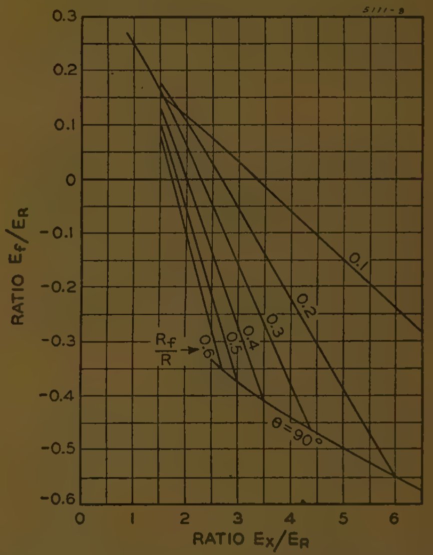
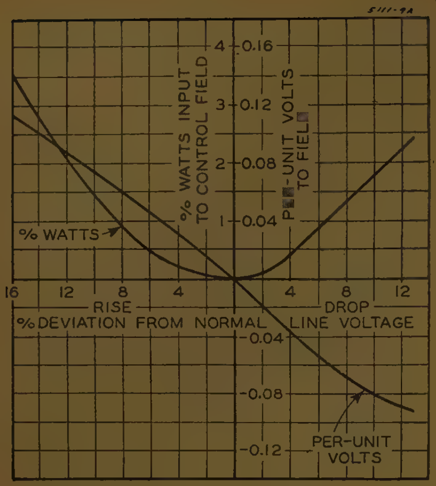


Figure 8. Curves of E_f/E_R versus E_x/E_R for use in computing power output of static regulating network

$R'=0.20R$



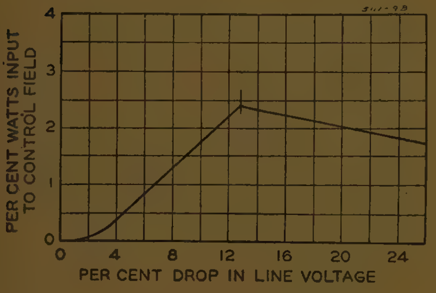
A. Normal range

$R'=0.20R$

$R_f=0.30R$

Per-cent watts are based on network watts at normal voltage

Per-unit field volts are based on average normal line volts



B. Extended range

$R'=0.20R$

$R_f=0.30R$

Per-cent watts are based on network watts at normal line voltage

Figure 9. Calculated static regulating circuit performance—control field

Input power versus line-voltage deviation from normal

9. The power consumed by the field is E_f^2/R_f.

10. The per-cent field power then can be calculated as $100P_f/P_n$. This is also the field watts per 100 watts drawn from the line by the network.

11. Or the network watts per field watt at ten-per-cent line voltage drop may be calculated,

$$\frac{100}{\text{Item }10}\left[\frac{1-(\text{Item }8)}{0.1}\right]^2$$

This calculation should be made for the line voltage (column 8) corresponding most nearly to ten-per-cent voltage drop.

12. The field voltage in per unit of the average value of normal line voltage is given by E_f/E_n.

Table II gives a sample calculation by this method, the results of which are plotted in Figure 9A.

For drops in line voltage beyond the point at which the reactor rectifier goes to zero, the field power and voltage can be calculated setting E_d equal to zero. The field power for this extended range of voltage deviation is plotted in Figure 9B.

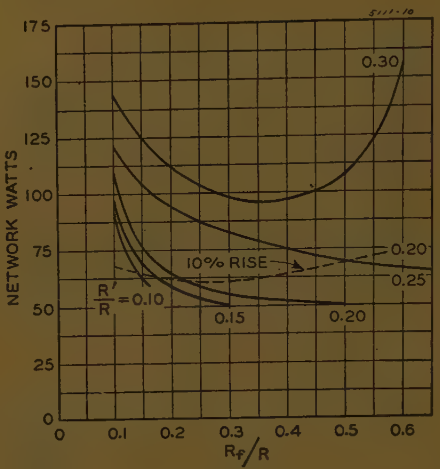


Figure 10. Calculated static-network power requirements

Network watts per watt input to control field at ten-per-cent line voltage drop (except as noted)

Rectifier Voltages

In the calculations for field power and voltage, of which a sample is given in Table II, the reactor current I_x is obtained. This current, rectified, also flows through R'. Thus the equations for rectifier voltages are

Reactor Rectifier—Refer to Figure 6C.

$$E_d=E_f+I_xR'$$

Resistor Rectifier

$$E_d'=-E_f+(I_x-I_f)(1-k)R$$

At line voltage ± ten per cent from normal the rectifier voltages should be checked since the performance as calculated only holds true if the rectifier voltages are positive up to these limits, (or to whatever limits of operation the network is designed).

For example, if we refer to Table II, for $R'=0.2R=0.204$, and $R_f=0.3R=0.306$ at 90-per-cent line voltage, which is the point at which E_d is lowest, the reactor rectifier voltage is

$$E_d=E_f+I_xR'$$
$$=-0.0816+0.555\times0.306$$
$$=-0.0816+0.1698=+0.0882$$

This indicates that R' is sufficiently large and could be reduced somewhat. It must be remembered that harmonics have been neglected and that these calculations therefore must be used with considerable margin.

If it is desired only to ascertain that the reactor rectifier voltage is positive for a particular line voltage drop, no calculations are needed beyond those illustrated in Tables I and II. As long as the E_x/E_R value used in Table II corresponds to θ less than 90 per cent in Table I, the reactor rectifier voltage is positive, as explained previously. Thus it is convenient to stop the curves of Figure 6, prepared from calculations like Table I, at the boundary $\theta=90$ degrees. They cannot then be used beyond the proper operating limit.

The resistor rectifier voltage decreases as the line voltage drop increases to ten per cent. At ten-per-cent high (a point available in Table II) if the resistor rectifier voltage is just zero, then

$$E_d'=-E_f+(I_x-I_f)(1-k)R$$

$$0=-0.0739+\left(1.414-\frac{0.0739}{0.036}\right)(1-k)1.02$$

or

$$(1-k)=\frac{0.0739}{1.173\times1.02}=0.0617$$

that is, $(1-k)$ must be about 6.2 per cent in order that the resistor rectifier voltage remains positive up to ten-per-cent voltage increase.

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TRANSACTIONS SECTION

Preprint of Corresponding Pages From the Current Annual AIEE Transactions Volume

Statistics as an Aid to Engineering Judgment in the Manufacture of Lightning-Arrester Blocks

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IN THIS PAPER we show how modern statistical methods may be used as an aid to engineering judgment in determining the causes of variability in a complex manufacturing process.

Assistant Secretary of War for Air Lovett has said that "a statistician is a man who can lead you directly from a preconceived idea to a foregone conclusion." If properly interpreted, this statement contains an element of truth. But it is not the kind of statement which should be made under oath. For while it is the truth and nothing but the truth, it certainly is not the whole truth. The whole truth might be phrased in similar language as follows: "Among a given class of preconceived ideas, the statistician can tell, with a definite degree of certainty, which subclass constitutes foregone conclusions."

In our particular example on lightning-arrester blocks, the set of preconceived ideas are the variables in the process of manufacturing blocks which were thought to affect the electrical characteristics of blocks. The foregone conclusions are those among these variables which actually do affect significantly the electrical characteristics of the blocks. The term "significantly" is used here in an engineering sense—not a statistical sense.

Production Procedure

A brief description will be given of the production and testing of blocks.

The blocks consist of silicon carbide and a binder. The electrical characteristics of the silicon carbide are the determining factor of fitness for use.

The silicon carbide and binder are mixed in suitable proportions. After proper mixing, the material is screened and stored and is ready for molding into blocks.

The amount of material to be molded per block is weighed, by machine, directly into the mold with the bottom punch in place. The top punch then is inserted and the block pressed. The blocks then are subjected to a 24-hour baking cycle.

The ends of the blocks are sprayed with copper, and the lateral surfaces are coated with an insulating compound.

Figure 1 shows a lightning-arrester block cut through the middle.

Testing Procedure

The blocks are subjected to a life test and a volt-ampere characteristic test. We are here interested in the latter only. The functioning of a lightning-arrester block depends upon the fact that its resistance does not vary linearly. It has high resistance for low currents and low resistance for high currents. Figure 2 shows the volt-ampere characteristics of a lightning-arrester block taken by a cathode-ray oscillograph. The block is subjected to a surge whose crest value is 1,500 amperes. The time of the surge is ten microseconds from zero current to maximum and 20 microseconds to half value. The volt-ampere characteristic is read on a cathode-ray oscillograph. The maximum voltage attained across the block is called the "maximum rise" of the block.

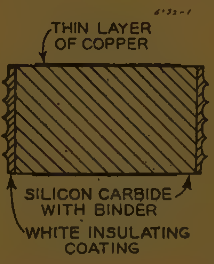

Figure 1. Lightning-arrester block

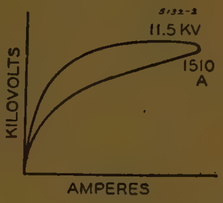

Figure 2. Volt-ampere characteristic curve of lightning-arrester block

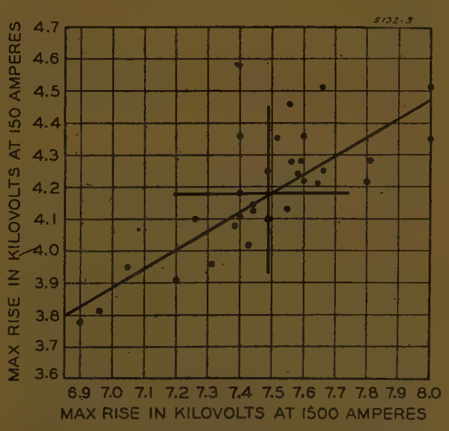

Figure 3. Correlation between maximum rise at 150 amperes and 1,500 amperes

Variables

Among the known factors which could cause electrical variations are such things as weight, length, and density of blocks; variations in the manufacturing techniques, and the character of the raw silicon carbide; and variations in the testing equipment.

Correlation Between Two Tests

As the voltages across the block at both high and low currents are important in the functioning of the blocks, it is of value to know the degree of correlation between these voltages.

Accordingly, for a set of 30 blocks, maximum rise was obtained for a 150-ampere surge and for a 1,500-ampere surge. The data were plotted on a scatter diagram as shown in Figure 3. The regression of Y on X is given by the equation $Y = 0.5X$. The calculation given in Table I shows that the regression is highly significant.

The 0.01 probability level for F with 1 and 28 degrees of freedom is about 8.0. Hence the regression is highly significant.

It may be of interest to note that the dependence of Y on X may be inferred from the fact that only two of the 30 points in Figure 3 fall in the second and fourth quadrants. The chance that this will happen for uncorrelated data is one in 2,000,000. For that reason, plus the fact that the variance from the regression line is small, the test for a 150-ampere surge was eliminated.

Effect of Molding Technique on Electrical Characteristics

To determine the effect of variations in molding technique on electrical characteristics of blocks, a batch of material was mixed and divided into four portions

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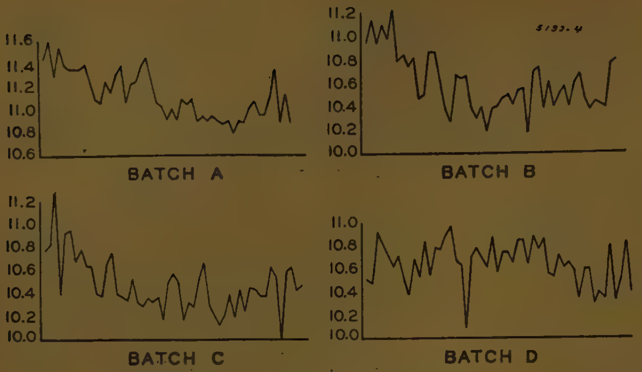
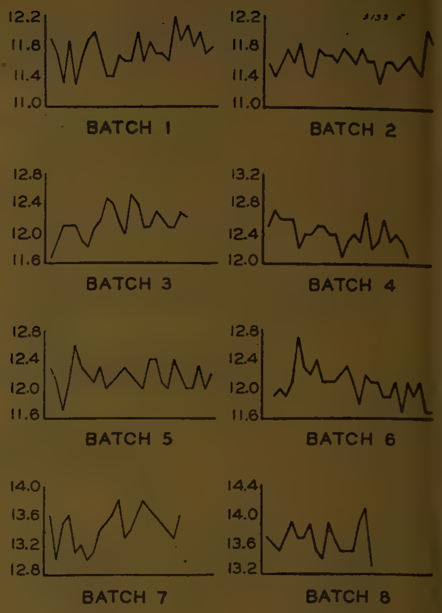


	Batch *A*	Batch *B*	Batch *C*	Batch *D*
$n\sigma^2=$	2.0113	3.0102	3.3774	1.8208
$(n-1)\delta^2=$	1.1691	2.3625	2.3576	3.1626
$\delta^2/\sigma^2=$	0.585	0.787	0.693	1.738

Figure 4 (left). Trends shown by data obtained in experiment on molding technique

Figure 5 (right). Effect of aging on stabilizing electrical characteristics of blocks



	Batch 1	Batch 2	Batch 3	Batch 4	Batch 5	Batch 6	Batch 7	Batch 8
$n\sigma^2=$	1.53	0.75	0.99	0.65	0.88	1.38	1.16	0.77
$(n-1)\delta^2=$	2.59	1.47	1.10	1.08	1.52	1.68	1.56	1.52
$\delta^2/\sigma^2=$	1.69	1.96	1.11	1.67	1.73	1.22	1.34	1.98

from each of which 50 blocks were molded under different conditions. These will be called batches *A*, *B*, *C*, and *D*.

Batch *A* was molded in the laboratory under controlled conditions. Batches *B*, *C*, and *D* were molded in the shop. For *B* the attempt was made to control the variables. For *C* no attempt at control was made, but the operator was under observation. Batch *D* was molded without letting the operator know that it was a special batch.

Incidentally, since batches *A*, *B*, and *C* were molded immediately after mixing, and batch *D* was molded several days later, this experiment shows the effect of aging on maximum rise. Thus a trend definitely exists for batches *A*, *B*, and *C*, although it is not so evident for batch *D*. See Figure 4. The statistic δ^2/σ^2 has values of 0.585, 0.787, 0.693, and 1.738 for batches *A*, *B*, *C*, and *D*, respectively. The 0.01 and 0.05 probability values for this statistic for 50 observations are 1.40 and 1.58, respectively.

The following application of the χ^2 test was made to see whether significant differences exist between the variances of the afore-mentioned four batches.

Table I

	Sum of Squares	Degrees of Freedom
Total	1.0099	29
Regression	0.6881	1
Error	0.3218	28

$$F=0.6881\div\frac{0.3218}{28}=60$$

Table II

Before Aging	After Aging
0.585	1.69
0.787	1.96
0.693	1.11
	1.67
	1.73
	1.22
	1.34
	1.98
$\bar{X}=0.69$	$\bar{Y}=1.59$

$$\chi^2 \cong \frac{2.3026}{C}(n \log S'^2 - \Sigma n_i \log S_i'^2)$$

with $K-1$ degrees of freedom, where $S_i'^2$ is the estimated *i*th variance with *n* degrees of freedom and S'^2 is the estimated total variance with *n* degrees of freedom.

$$C=1+\frac{1}{3(k-1)}\sum\left(\frac{1}{n_i}-\frac{1}{n}\right)$$

In this case,

$$\chi^2 \cong \frac{2.3026}{1.01}(196 \log 0.0462 - 49 \log 0.00000349)$$

$$=2.3\ (-261.73+267.40)$$

$$=13.04$$

But the 0.01 probability level for χ^2 with three degrees of freedom is 11.34.

The conclusion drawn from these data is that the variables of the shop molding practice actually do affect the volt-ampere characteristics of blocks. Further attempts to determine these factors have not been made at this time, primarily because a control chart showed that within-batch variations were small compared to between-batch variations. Maximum effort accordingly was devoted to eliminating causes of between-batch variations. As the effect of variations in molding technique will most likely manifest itself within a batch, a critical study of these variations was left until some future date.

Effect of Aging on Maximum Rise

In the preceding section it was found that material molded without being aged first exhibited trends in maximum rise. The indication was that if material was aged for a suitable length of time no such trends existed. Of course standard practice based on long experience and engineering knowledge was to age the material before using. However, statistical analysis brings a definiteness into the picture which otherwise seems to be lacking and sometimes refutes accepted practice. For this reason, the data shown in Figure 5 were obtained for eight batches aged for what was considered to be a sufficiently long period of time. The data are for every eighth block in order of production. The values of δ^2/σ^2 for the eight batches were respectively 1.69, 1.96, 1.11, 1.67, 1.73, 1.22, 1.34, and 1.98. That trends exist even after aging is evident from the fact that all eight values obtained are less than two, which is the expected value of this statistic if there are no trends. However, the trend obtained after aging is significantly less than that obtained before aging as shown by the *t* test in Table II.

$$t=\frac{\bar{Y}-\bar{X}}{\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\frac{\Sigma(\bar{X}-X)^2+\Sigma(\bar{Y}-Y)^2}{n_1+n_2-2}}}$$

$$=\frac{90}{\sqrt{(1/8+1/8)\frac{(0.02+0.7550)}{9}}}$$

$$=\frac{0.90}{0.198}=4.54\text{, 9 degrees of freedom}$$

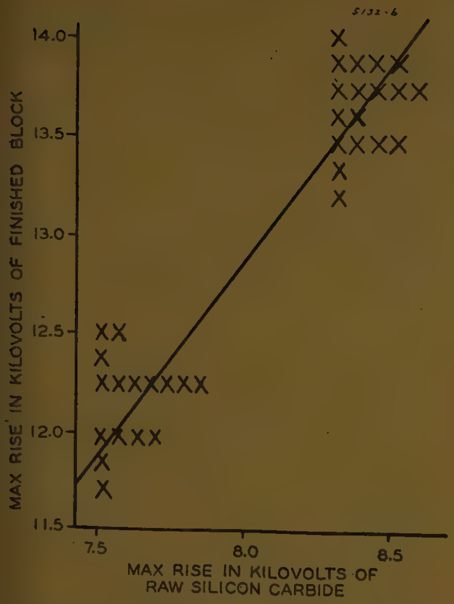


Figure 6. Effect of maximum rise of raw silicon carbide on maximum rise of finished blocks

The 0.01 probability level with nine degrees of freedom is 3.25. Hence the difference is significant.

Effect of Baking Temperature on Blocks

Sixteen blocks were taken from the same batch. Eight of the blocks selected at random were baked in an oven at the standard temperature. The other eight blocks were subjected to a baking cycle whose maximum temperature was 75 degrees centigrade lower. The data are shown in Table III. A value of t = 1.89 was obtained. This falls between the 0.05 and 0.10 probability level of the t distribution. A retest was made with 16 more blocks. A value of t = 3.63 was obtained. This value is higher than the 0.01 probability level, which shows that oven temperature should not be allowed to deviate by as much as 75 degrees centigrade.

To determine whether or not significant differences in electrical characteristics exist between blocks baked in various positions in the oven, 32 blocks were randomized and placed in groups of four in eight positions in the oven. The data are shown in Table IV. An analysis of variance performed with these data yields a value of F = 0.696 with seven degrees of freedom for the between-position variance and 16 degrees of freedom for the error term (within-position variance). This test indicates strongly that significant differences do not exist between positions in oven as far as electrical characteristics are concerned.

Table III

Hot Oven	Cool Oven
First Test	
12.94	13.92
13.17	13.67
13.14	12.64
12.64	12.83
12.81	13.36
13.42	13.57
12.90	13.14
12.67	13.22
$\overline{X}=12.96$	$\overline{Y}=13.29$

$$t=\frac{13.29-12.96}{\sqrt{\frac{1/4(1.6809)}{14}}}=\frac{0.33}{17}=1.94$$

Hot Oven	Cool Oven
Retest	
12.53	12.53
12.37	13.22
12.48	13.01
12.77	12.97
12.52	12.96
12.81	13.03
12.76	12.82
12.52	13.43
$\overline{X}=12.60$	$\overline{Y}=13.00$

$$t=\frac{13.00-12.60}{\sqrt{\frac{1/4(0.6697)}{14}}}=\frac{0.40}{0.11}=3.63$$

Table IV

	Position							
	1	2	3	4	5	6	7	8
	12.3	12.6	12.3	12.3	12.4	12.4	12.3	12.3
	12.1	12.3	12.0	12.2	12.4	12.2	12.0	12.4
	12.2	12.2	12.1	12.1	12.2	12.2	12.2	12.3
	11.7	12.1	12.2	12.2	12.0	12.0	12.3	12.1
Average	12.075	12.3	12.15	12.2	12.25	12.2	12.2	12.275

$\overline{\overline{X}}=12.20$

Total sum of squares = 0.86 — 31 degrees of freedom

Between positions = 4(0.03625) = 0.145 — 7 degrees of freedom

Error = 0.715 — 24 degrees of freedom

$$F=\frac{0.145/7}{0.715/24}=0.696$$

Effect of Clearance Between Mold and Punch on Electrical Characteristics of Blocks

There had been some indication in the shop that blocks molded with large clearance between mold and punch had different electrical characteristics from blocks molded with small clearance. Although there was no apparent reason for this difference, a test was made to determine whether or not such a difference actually exists.

Fourteen blocks taken from the same batch of material were molded as special blocks; every other block was molded with a certain large clearance between molds and punches. The remaining blocks were molded with a certain small clearance between molds and punches. The results are shown in Table V. A value of t = 10.1 was obtained. This shows significant difference. That a significant difference exists also may be seen from the fact that every block molded with large clearance had a higher maximum rise than every block molded with small clearance.

Characteristics of Raw Silicon Carbide

The electrical characteristics of raw silicon carbide are obtained by getting the volt-ampere curve for a sample of the raw material. It would be of value to verify the degree of correlation between the maximum rise of raw material and finished product although theoretically the two should be proportional. However, at the time the test was run only two varieties were available. One had a maximum rise of 7.5 kv, the other 8.2 kv. The data are plotted in Figure 6 and show that the maximum rise of finished blocks is roughly proportional to the maximum rise of raw silicon carbide. The test should be repeated with batches of silicon carbide whose electrical characteristics vary along the scale to determine the regression lines.

Table V

Large Clearance	Small Clearance
12.6	12.1
12.6	11.9
12.8	12.0
13.2	11.9
13.0	11.9
12.6	11.8
13.0	11.9
$\overline{X}=12.825$	$\overline{Y}=11.9375$

$$t=\frac{0.8875}{\sqrt{\frac{0.43(1/4)}{14}}}=10.1$$

Conclusion

As applied to lightning-arrester blocks, statistics helped engineering judgment by verifying that some of the variables which had been thought to affect electrical characteristics actually did so. This led to emphasis of control of these factors at the expense of others which had been thought to affect maximum rise but were found, by statistical analysis, to have a less significant effect. As a consequence of this study a more uniform product is being made, and an appreciable reduction in the scrap loss during manufacturing has resulted.

Statistical methods now are being further applied to determine superior designs. The main tool being used for this purpose is the designed experiment.

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New Transmission-Line Diagrams

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IN THE DESIGN and operation of transmission lines the following quantities are of primary importance: sending-end power, receiving-end power, sending-end voltage, receiving-end voltage, power angle between sending-end and receiving-end voltage. Numerous diagrams have been developed for the solution of transmission-line problems, graphically illustrating relations between some of the afore-mentioned quantities. Among these, the modified Evans and Sels charts, based on the general circuit constants $ABCD$, have found wide acceptance.[1,2,3] In these charts the determination of the power at one end with the power given at the other end requires a transfer of points from one chart to another. It is one of the purposes of this paper to supplement the modified Evans and Sels diagrams in such a manner that this transfer of points can be eliminated. This can be accomplished by relating power at one end to power at the other end directly, without intermediate quantities such as voltage ratio and power angle. The diagrams developed are essentially admittance-circle diagrams, relating admittance at one end to admittance at the other end.

It is a further purpose of this paper to develop corresponding circle diagrams for the impedances, directly relating the impedance at one end to the impedance at the other. These diagrams might be useful in conjunction with impedance-relaying problems where a transfer through voltage ratio and power angle is not essential.

The modified Evans and Sels charts apply to fixed circuit constants. For each new set of values $ABCD$, a new chart must be prepared. Numerous charts have been developed to eliminate this shortcoming, resulting in universal transmission-line charts. These charts permit the determination of the performance of any line with arbitrary constants. In many of these charts it is necessary, however, to execute certain constructions to arrive at solutions. It is a further purpose of this paper to develop a universal transmission-line chart based on general circuit constants from which all the desired values can be read directly.

The following equations and notations will be used:

$$E_S=AE_R+BI_R \qquad (1)$$

$$I_S=CE_R+DI_R \qquad (2)$$

$$AD-BC=1$$

$$\frac{E_R}{E_S}=\left|\frac{E_R}{E_S}\right|\underline{/-\delta}$$

where bars enclosing a quantity indicate magnitude.

δ = angle that E_S leads E_R = power angle

Vector power usually is defined by its components P, Q. In order to simplify the following formulas a single symbol denoting vector power representing a complex quantity is introduced:

$p=P+jQ$ = power vector

$\frac{p_S}{|E_S|^2}=Y_S=G_S+jB_S$ = sending-end (input) admittance

$\frac{p_R}{|E_R|^2}=Y_R=G_R+jB_R$ = receiving-end (load) admittance

$\frac{p_S}{|E_R|^2}=Y_{RS}$; $\frac{p_R}{|E_S|^2}=Y_{SR}$

Circle Diagram for the Calculation of Power at One End From Power at the Other End

The modified Evans and Sels charts are essentially admittance diagrams. For practical purposes, however, they are labeled in terms of power divided by the square of voltages. As the use of admittances produces more compact equations, admittance quantities will be employed freely, the results later being interpreted in power quantities. The four charts are based on the following equations:

$$\frac{p_S}{|E_S|^2}=Y_S=\frac{D}{B}-\frac{1}{B}\left|\frac{E_R}{E_S}\right|\underline{/-\delta} \qquad (3)$$

$$\frac{p_R}{|E_R|^2}=Y_R=-\frac{A}{B}+\frac{1}{B}\left|\frac{E_S}{E_R}\right|\underline{/\delta} \qquad (4)$$

$$\frac{p_S}{|E_R|^2}=Y_{RS}=\frac{D}{B}\left|\frac{E_S}{E_R}\right|^2-\frac{1}{B}\left|\frac{E_S}{E_R}\right|\underline{/-\delta} \qquad (5)$$

$$\frac{p_R}{|E_S|^2}=Y_{SR}=-\frac{A}{B}\left|\frac{E_R}{E_S}\right|^2+\frac{1}{B}\left|\frac{E_R}{E_S}\right|\underline{/\delta} \qquad (6)$$

The four corresponding diagrams are shown in Figures 1A, 1B, 1C, 1D. Diagrams of Figures 1A and 1B are the most widely used because the power circles are concentric. Figures 1C and 1D consist of eccentric circles, which limits their use considerably. Referring to Figures 1A and 1B, these diagrams permit the determination of the power and voltage quantities at one end from the power and voltage quantities at the other end. For instance, if it is desired to determine P_S+jQ_S for a given P_R+jQ_R and a known receiving-end voltage E_R, $P_R/|E_R|^2$ and $Q_R/|E_R|^2$ are computed, locating a point in the receiver-end chart, Figure 1B. This point determines the value of $|E_S/E_R|$ and δ. The corresponding point with identical $|E_S/E_R|$ and δ next is located in the sending-end diagram, Figure 1A whereupon the values $P_S/|E_S|^2$ and $Q_S\div|E_S|^2$ can be read, and P_S, Q_S, and E_S can be computed. The necessity of transfer of points is common to all these modified diagrams since, essentially, they relate an admittance to the voltage ratio and power angle. It can be eliminated only if sending-end admittance and receiving-end admittance are brought in direct relation, as follows:

$$\left.\begin{aligned}Y_S&=\frac{I_S}{E_S}=\frac{CE_R+DI_R}{AE_R+BI_R}=\frac{C+DY_R}{A+BY_R}\\ Y_S&=\frac{D}{B}-\frac{1}{B^2\left(Y_R+\frac{A}{B}\right)}\end{aligned}\right\} \qquad (7)$$

or, in vector-power form:

$$\frac{p_S}{|E_S|^2}=\frac{P_S+jQ_S}{|E_S|^2}=\frac{D}{B}-\frac{1}{B^2\left(\frac{P_R+jQ_R}{|E_R|^2}+\frac{A}{B}\right)} \qquad (7a)$$

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In equation 7, Y_S is represented as an analytic complex function of the independent variable Y_R. Analytic functions have interesting properties which are particularly useful in the analysis of electric-circuit diagrams and which are described in detail in mathematical texts.[4,5,6] Analytic functions map the plane of the independent variable into the plane of the dependent variable in such a manner that corresponding infinitely small configurations are similar. Such mappings are called conformal mappings. Conformal mapping retains the angle between corresponding intersecting lines, in particular, an orthogonal system of lines in one plane transforms into another orthogonal system in the other plane.

Y_S is called a linear fractional transformation of Y_R. Since the mappings produced by this function have been discussed in detail before,[4] it will be sufficient to indicate the steps leading to the final $Y_S=f(Y_R)$ diagram. Analyzing the expression $1/(Y_R+A/B)$, the Y_R plane with origin OY_R and axis G and B is drawn (Figure 2). Addition of A/B produces the Y_R+A/B plane with origin O. In this plane, lines B_1=constant and G_1=constant are drawn. It is well known that the expression $1/(Y_R+A/B)$ can be obtained by a process of complex inversion. Inversion transforms circles into circles. Specifically, the straight lines B_1=constant and G_1=constant are transformed into the circles B_1=constant and G_1=constant respectively, as shown. Additional lines G=constant and B=constant all intersect at O and produce an orthogonal system of circles. Centers of the G=constant circles are located on the real axis, centers of the B=constant circles on the imaginary axis. The diameter of the circles is the reciprocal of the shortest distance of the straight lines from the origin and readily can be computed as shown in Figure 2. The final expression of Y_S requires multiplication of $1/(Y_R+A/B)$ by $-1/B^2$ and addition of D/B, resulting in Figure 3, which lists in detail the steps necessary to arrive at the circles $G=P_R/|E_R|^2$=constant and $B=Q_R\div|E_R|^2$=constant in the $P_S/|E_S|^2+jQ_S\div|E_S|^2$ plane. If numerous circles representing varying $P_R/|E_R|^2$ and $Q_R/|E_R|^2$ values are drawn, $P_S/|E_S|^2+jQ_S/|E_S|^2$ can be determined readily as co-ordinates of the intersection point of circles representing $P_R/|E_R|^2$ and $Q_R/|E_R|^2$ values applying to the particular problem. If Figure 3 is used as a separate chart, the additional use of the modified Evans and Sels chart, Figure 1B, is required to determine the voltage ratio necessary for the calculation of the sending-end power and for the determination of the power angle. It is apparent that the separate use of the chart of Figure 3 has no advantages over previous charts. However, if this chart is superimposed upon the modified Evans and Sels sending-end power chart, Figure 1A, what appears to be a useful new chart is produced.

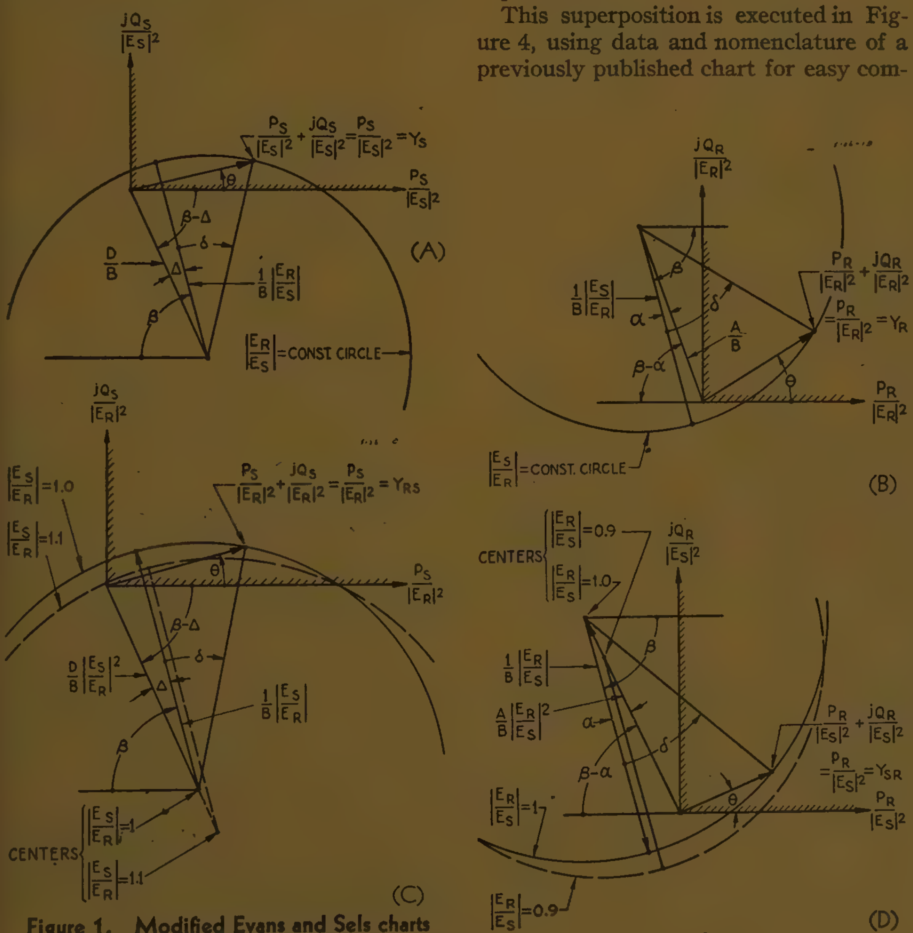

$\frac{jQ_S}{|E_S|^2}$
$\frac{P_S}{|E_S|^2}+\frac{jQ_S}{|E_S|^2}=\frac{P_S}{|E_S|^2}=Y_S$
$\frac{P_S}{|E_S|^2}$
θ
β-Δ
δ
$\frac{D}{B}$
Δ
$\frac{1}{B}\left|\frac{E_R}{E_S}\right|$
β
(A)
$\left|\frac{E_R}{E_S}\right|$=CONST CIRCLE
$\frac{jQ_S}{|E_R|^2}$
$\left|\frac{E_S}{E_R}\right|$=1.0
$\left|\frac{E_S}{E_R}\right|$=1.1
$\frac{P_S}{|E_R|^2}+\frac{jQ_S}{|E_R|^2}=\frac{P_S}{|E_R|^2}=Y_{RS}$
θ
$\frac{P_S}{|E_R|^2}$
β-Δ
δ
$\frac{D}{B}\left|\frac{E_S}{E_R}\right|^2$
Δ
$\frac{1}{B}\left|\frac{E_S}{E_R}\right|$
β
CENTERS
$\left|\frac{E_S}{E_R}\right|$=1
$\left|\frac{E_S}{E_R}\right|$=1.1
(C)
$\frac{jQ_R}{|E_R|^2}$
$\frac{1}{B}\left|\frac{E_S}{E_R}\right|$
β
α
δ
$\frac{A}{B}$
β-α
$\frac{P_R}{|E_R|^2}+\frac{jQ_R}{|E_R|^2}$
$=\frac{P_R}{|E_R|^2}=Y_R$
θ
$\frac{P_R}{|E_R|^2}$
$\left|\frac{E_S}{E_R}\right|$=CONST. CIRCLE
(B)
CENTERS
$\left|\frac{E_R}{E_S}\right|$=0.9
$\left|\frac{E_R}{E_S}\right|$=1.0
$\frac{jQ_R}{|E_S|^2}$
$\frac{1}{B}\left|\frac{E_R}{E_S}\right|$
$\frac{A}{B}\left|\frac{E_R}{E_S}\right|^2$
β
α
δ
β-α
$\frac{P_R}{|E_S|^2}+\frac{jQ_R}{|E_S|^2}$
$=\frac{P_R}{|E_S|^2}=Y_{SR}$
θ
$\frac{P_R}{|E_S|^2}$
$\left|\frac{E_R}{E_S}\right|$=1
$\left|\frac{E_R}{E_S}\right|$=0.9
(D)

Figure 1. Modified Evans and Sels charts

This superposition is executed in Figure 4, using data and nomenclature of a previously published chart for easy comparison.[8] As can be seen, all receiving-end power circles intersect in the center of the $|E_R/E_S|$=constant circles. Since all circles P_R=constant, Q_R=constant intersect at right angles, the intersecting points are determined accurately. The chart is therefore more convenient to use than the synchronous-machine charts which it resembles.

The chart is equally useful in determining receiving-end quantities with sending-end quantities known, the steps being reverse of those indicated in the preceding problem. Essentially, the advantages of the Figure 4 chart are that power and voltage at either end can be determined from power and voltage at the other end by the use of a single chart, eliminating the transfer of points required on previous charts.

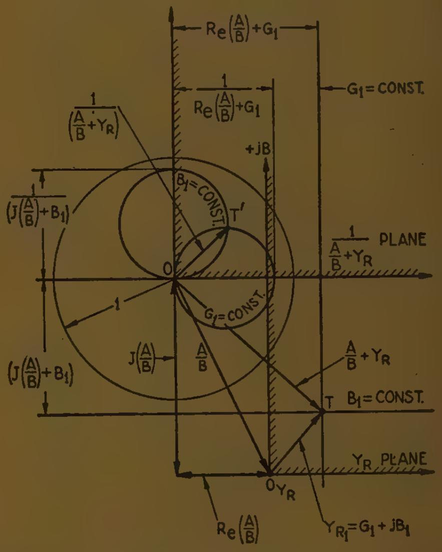

$\mathrm{Re}\left(\frac{A}{B}\right)+G_1$
$\frac{1}{\mathrm{Re}\left(\frac{A}{B}\right)+G_1}$
G_1= CONST.
$\frac{1}{\left(\frac{A}{B}+Y_R\right)}$
+JB
$\frac{1}{\left(J\left(\frac{A}{B}\right)+B_1\right)}$
B_1=CONST.
T′
O
$\frac{1}{\frac{A}{B}+Y_R}$ PLANE
1
G_1=CONST.
$J\left(\frac{A}{B}\right)$
$\frac{A}{B}$
$\frac{A}{B}+Y_R$
$\left(J\left(\frac{A}{B}\right)+B_1\right)$
T B_1=CONST.
Y_R PLANE
O_{Y_R}
$\mathrm{Re}\left(\frac{A}{B}\right)$
$Y_{R_1}=G_1+JB_1$

Figure 2. Complex inversion of Y_R+A/B

In a similar manner Y_R can be expressed as a function of Y_S:

$$Y_R=\frac{p_R}{|E_R|^2}=\frac{P_R+jQ_R}{|E_R|^2}=-\frac{A}{B}-\frac{1}{B^2\left(\frac{p_S}{|E_S|^2}-\frac{D}{B}\right)} \qquad (8)$$

A diagram similar to Figure 4 results, mapping the sending-end plane upon the receiver plane. Corresponding to the Evans and Sels diagrams, Figures 1C and 1D, two additional diagrams exist for the transformation of the $P_S/|E_R|^2=Y_{RS}$ plane into $P_R/|E_S|^2=Y_{SR}$ plane and vice versa. It will be sufficient to list the formulas for one of these transformations,

$$Y_{RS}=f(Y_{SR})$$

$$\frac{P_S+jQ_S}{|E_R|^2}=\frac{D}{B}\left|\frac{E_S}{E_R}\right|^2-\frac{1}{B^2\left(\frac{P_R+jQ_R}{|E_S|^2}+\frac{A}{B}\left|\frac{E_R}{E_S}\right|^2\right)} \qquad (9)$$

The resultant diagrams are still circle diagrams, however, the circles do not intersect in one point. They have similar practical limitations as do the diagrams of Figures 1C and 1D.

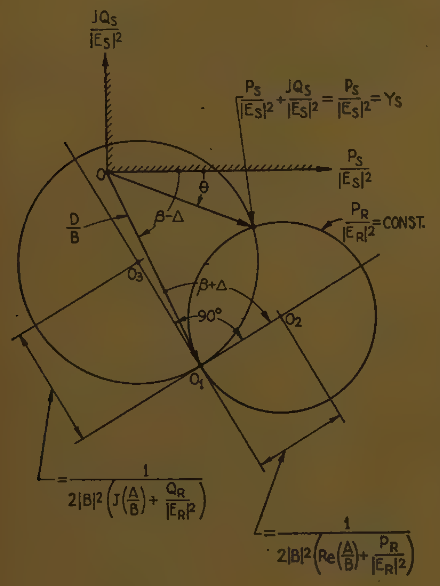

$\frac{jQ_S}{|E_S|^2}$
$\frac{P_S}{|E_S|^2}+\frac{jQ_S}{|E_S|^2}=\frac{P_S}{|E_S|^2}=Y_S$
$\frac{P_S}{|E_S|^2}$
O
θ
$\frac{D}{B}$
β−Δ
$\frac{P_R}{|E_R|^2}$=CONST.
O_3
β+Δ
90°
O_2
O_1
$=\frac{1}{2|B|^2\left(J\left(\frac{A}{B}\right)+\frac{Q_R}{|E_R|^2}\right)}$
$=\frac{1}{2|B|^2\left(Re\left(\frac{A}{B}\right)+\frac{P_R}{|E_R|^2}\right)}$

Figure 3. (above) Construction of circle diagrams relating receiving-end power to sending-end power

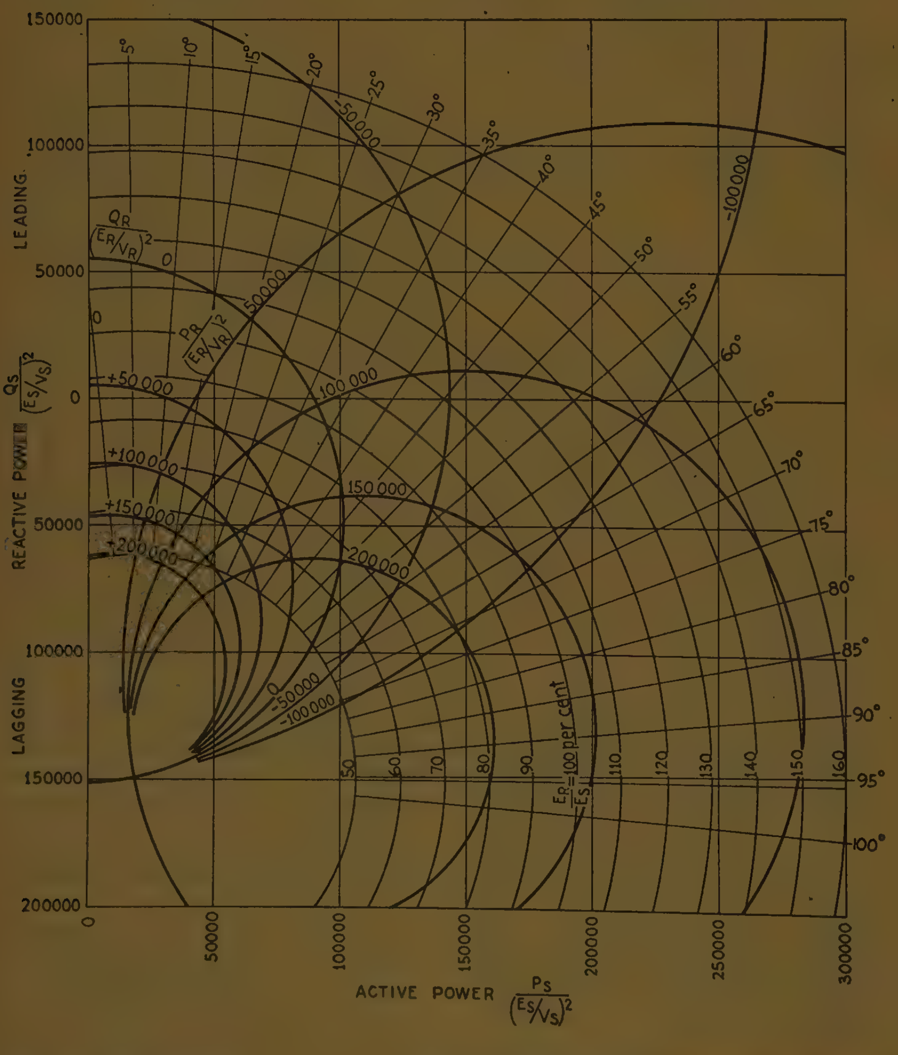
Circle Diagram for the Calculation of the Impedance at One End From the Impedance at the Other End

In conjunction with protection problems involving impedance relays with phase-angle control, circle diagrams relating impedances to $|E_R/E_S|$ and δ have found wide application.[7] These diagrams follow readily from equations 3, 4, 5, and 6. It will be sufficient to analyze but one specific case.

$$Z_S=\frac{1}{Y_S}=-\frac{B}{\left(\left|\frac{E_R}{E_S}\right|\underline{/-\delta}-D\right)} \tag{10}$$

Since Z_S is a linear fractional transformation of the independent complex variable $|E_R/E_S|\underline{/-\delta}$, straight lines and circles in the plane of the independent variable transform into circles in the Z_S plane. In particular, the circles $|E_R/E_S|$ = constant and the lines δ = constant transform into circles. Since the lines $|E_R/E_S|$ = constant and δ = constant are orthogonal, they will produce an orthogonal system of lines in the Z_S plane.[8] A similar diagram can be developed for the impedance Z_R. If it is required to determine the value of Z_S for a given value of Z_R, it is necessary to determine the values of $|E_R/E_S|$ and δ from the Z_R diagram and transfer the point to the Z_S diagram. This procedure is analogous to that required in the Evans and Sels diagrams.

Figure 4. Chart relating receiving-end power to sending-end power, voltage ratio, and power angle



In the following, circle diagrams will be developed for the direct determination of one impedance from the other, without transfer through the intermediate variables $|E_R/E_S|$ and δ. Since these diagrams closely resemble the admittance diagrams developed previously, it will be sufficient to consider one specific case, for instance, the mapping of Z_R into Z_S. Referring to equations 1 and 2, it follows:

$$Z_S-\frac{E_S}{I_S}=\frac{AE_R+BI_R}{CE_R+DI_R}=\frac{AZ_R+B}{CZ_R+D}$$

$$=\frac{A}{C}-\frac{1}{C^2\left(Z_R+\frac{D}{C}\right)}$$

and with $Z_R=R_R+jX_R$:

$$Z_S=\frac{A}{C}-\frac{1}{C^2\left(R_R+jX_R+\frac{D}{C}\right)} \tag{11}$$

As in the admittance diagrams where B = constant and G = constant lines mapped into circles, the foregoing equation transforms lines R_R = constant and X_R = constant into circles which intersect at one point. A particularly simple diagram results when the phase of the circuit constant C equals 90 degrees, as is shown in Figure 5. From such a diagram including numerous R_R = constant and X_R = constant circles the value of Z_S is determined as the intersecting point of appropriate R_R and X_R circles. This type of diagram might find application in conjunction with special impedance-relaying problems where voltage ratio and power angle are of secondary importance.

Universal Transmission-Line Charts

The modified Evans and Sels charts apply to one specific set of $ABCD$ constants. They are simple to construct and furnish information concerning a specific line very quickly. As has been pointed out, the diagrams of Figures 1A and 1B are used most commonly. Figures 1C and 1D diagrams are used rarely as they do not furnish information conveniently, ordinarily requiring construction of particular δ lines. There is, therefore, still a need for charts which furnish sending-end power from receiving-end power and sending-end voltage, and vice versa, in a direct and convenient manner, and without construction.

Furthermore, in planning new transmission lines, the preparation of charts according to Figures 1A and 1B for numerous $ABCD$ values is time consuming. Universal charts, giving data for any line of arbitrary $ABCD$ constants, are preferred. Most of the existing charts, however, again require constructions to arrive at solutions. In the following, four charts similar to the Evans and Sels charts will be developed which will permit the quick calculation of power and voltage quantities for any $ABCD$ constants without construction and with a minimum of computation. These charts also will eliminate the shortcomings inherent to the charts shown in Figures 1C and 1D.

The principal handicap against the modified Evans and Sels charts being suitable for universal charts is the fact that each new value of B and D introduces a new distance and, more particularly, produces a change in angular relations. Different diagrams result even if $|D|$ and $|B|$ do not change, but Δ and β only. This suggests that the method used in Figure 1, representing the angular relations, should be modified, and that more emphasis should be placed on the angular co-ordinates. For instance, it seems indicated to identify sending-end power, not by $P_S=P_S+jQ_S$, but by $|p_S|$ and the power-factor angle θ_S. Furthermore, the power-factor angle θ_S is not as suitable as the angle ϕ_S to which it is related by $\phi_S=\beta-\Delta+\theta_S$. ϕ_S will be given preference in the following. In Figure 6A, the chart Figure 1A is reproduced with emphasis on these angular quantities. p_S is represented by $|p_S|=$ constant circles (only one shown) and $\phi_S=$constant lines (only one shown). As can be seen, each set of values δ and $|E_R/E_S|$ determines a corresponding set of values for $|p_S|$ and ϕ_S.

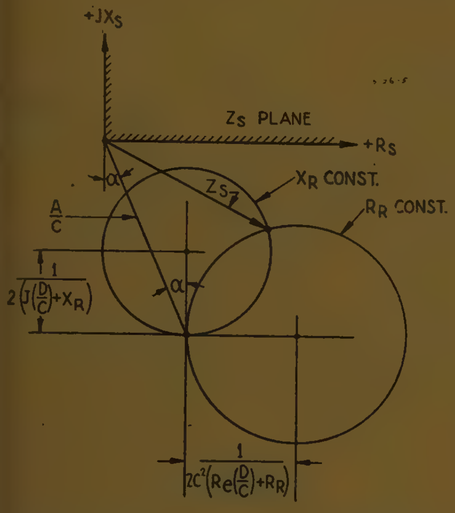

+JX$_S$
Z$_S$ PLANE
+R$_S$
X$_R$ CONST.
R$_R$ CONST.
Z$_S$
α
α
$\frac{A}{C}$
$\frac{1}{2(J(\frac{D}{C})+X_R)}$
$\frac{1}{2C^2(Re(\frac{D}{C})+R_R)}$

Figure 5. Diagram relating receiving-end impedance to sending-end impedance

In the following chart, Figure 6B, a new orthogonal co-ordinate system for δ and $|E_R/E_S|$ is introduced with δ as abscissa and $|E_R/E_S|$ as ordinate. If the values of $|p_S|$ and ϕ_S corresponding to various values of δ and $|E_R/E_S|$ are determined from Figure 6A and entered in Figure 6B, a useful chart results if the $|p_S|=$constant and $\phi_S=$constant lines are indicated. In this manner $|P_S|$ and ϕ_S can be determined for any given values of δ and $|E_R/E_S|$ and conversely. In order to arrive at a universal diagram, equation 3, representing Figure 6A, is divided by $|D/B|$, resulting in

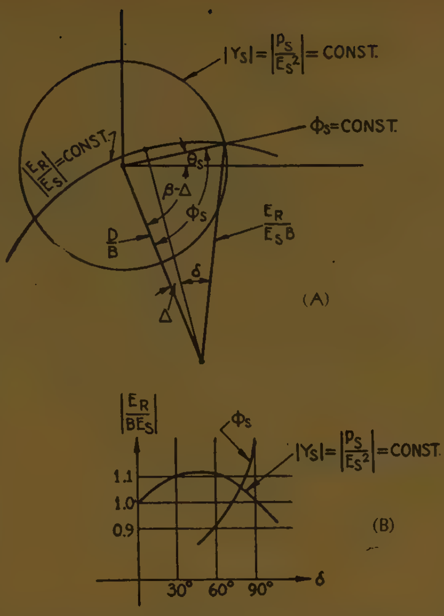

$|Y_S|=\left|\frac{P_S}{E_S^2}\right|=$CONST.
ϕ_S=CONST.
$\left|\frac{E_R}{E_S}\right|$=CONST.
θ_S
β-Δ
ϕ_S
$\frac{D}{B}$
$\frac{E_R}{E_S B}$
δ
Δ
(A)
$\left|\frac{E_R}{BE_S}\right|$
ϕ_S
$|Y_S|=\left|\frac{P_S}{E_S^2}\right|$=CONST.
1.1
1.0
0.9
30°
60°
90°
δ
(B)

Figure 6. Transformation of conventional power diagram into E_R/E_S, δ diagram

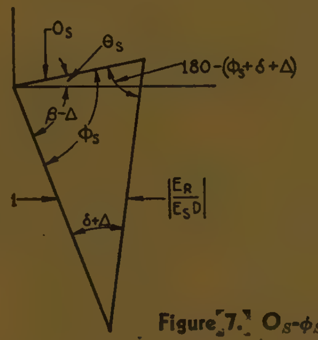


Figure 7. O_S-ϕ_S diagram

$$p_S\left|\frac{B}{E_S^2D}\right|=-\left(\left|\frac{E_R}{E_SD}\right|\underline{/-\delta}-1\underline{/\Delta}\right)\underline{/-\beta}$$

Denoting the real quantity $|p_SB\div E_S^2D|=O_S$, the Figure 7 diagram results, from which the following trigonometric relations can be established for the determination of the lines $O_S=$constant and $\phi_S=$constant in the $|E_SD/E_R|$, δ co-ordinate system:

$$\cos\,(\delta+\Delta)=\frac{1+\left|\frac{E_R}{E_SD}\right|^2-O_S^2}{2\left|\frac{E_R}{E_SD}\right|} \tag{12}$$

$$\left|\frac{E_SD}{E_R}\right|=\frac{\sin\,(\phi_S+\delta+\Delta)}{\sin\,\phi_S} \tag{12a}$$

Figure 8 shows the resulting plot covering a wide range of values. This chart represents conditions at the sending end and corresponds to the modified Evans and Sels chart, Figure 1A. For the receiving-end chart corresponding to Figure 2A the following formulas can be derived when $|p_RB/E_R^2A|=O_R$, $\phi_R=\beta-\alpha+\theta_R$.

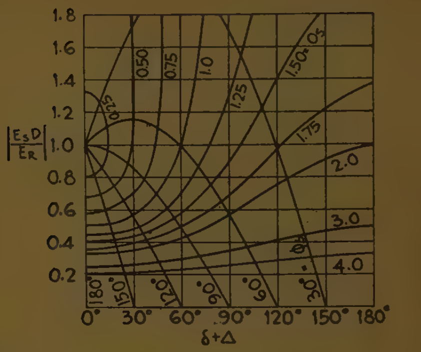


Figure 8. Universal transmission-line chart in E_R/E_S, δ co-ordinates

E_S and E_R in kilovolts

$\delta=$power angle between E_S and E_R

$\phi_S=\beta-\Delta+\theta_S, \theta_S=$power-factor angle

$O_S=(\text{kva})_S\times|B/E_S^2D|$

$$\cos\,(\delta-\alpha)=\frac{1+\left|\frac{E_S}{E_RA}\right|^2-O_R^2}{2\left|\frac{E_S}{E_RA}\right|} \tag{13}$$

$$\left|\frac{E_RA}{E_S}\right|=\frac{\sin\,(\phi_R-\delta-\alpha)}{\sin\,\phi_R} \tag{13a}$$

Comparison of equations 13 and 13a with equations 12 and 12a brings out the fact that identical plots are produced in slightly different co-ordinate systems. Figure 8 therefore could be used readily as receiving-end chart as well, with $|E_RA/E_S|$ as ordinate and $(\delta-\alpha)$ as abscissa. This feature brings out the inherent symmetry in the choice of the co-ordinates. In order to avoid confusion it appears advisable, however, to physically separate the two charts. Figure 9 shows one possible method suitable for practical use. The left-hand side of this chart is marked for receiving, the right-hand side for sending-end quantities. Figure 10 shows a section of chart of Figure 9 which covers the range of most common operating conditions. The use of these charts is illustrated best in conjunction with a practical example.

Given a transmission line with the following constants,

$A=0.917\underline{/1.06^\circ}=D$

$B=153\underline{/78.56^\circ}$

$C=0.00106\underline{/90^\circ}$

Determine sending-end power, power factor, and voltage if 75 megawatts at 99-per-cent power-factor leading is delivered at the receiving end at 154 kv.

Solution:

$(\text{kva})_R\ \cos\ \theta_R=(\text{kva})_R\times0.99=75$ megawatts; $(\text{kva})_R=75.75$ megavolt-amperes, $\theta_R=8.1^\circ$

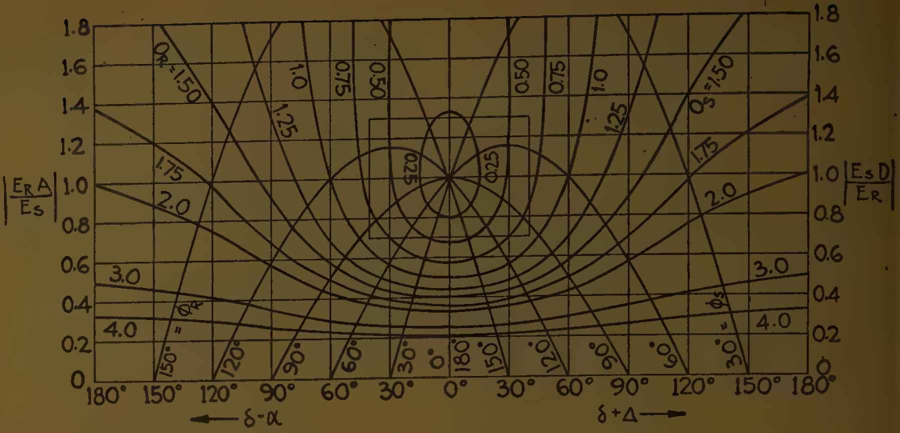


Figure 9. Universal chart for power and voltage given at one end of line

$\phi_R=\beta-\alpha+\theta_R,\ O_R=(\text{kva})_R\left|\frac{B}{E_R^2A}\right|$ $\qquad$ $\phi_S=\beta-\Delta+\theta_S,\ O_S=(\text{kva})_S\left|\frac{B}{E_S^2D}\right|$

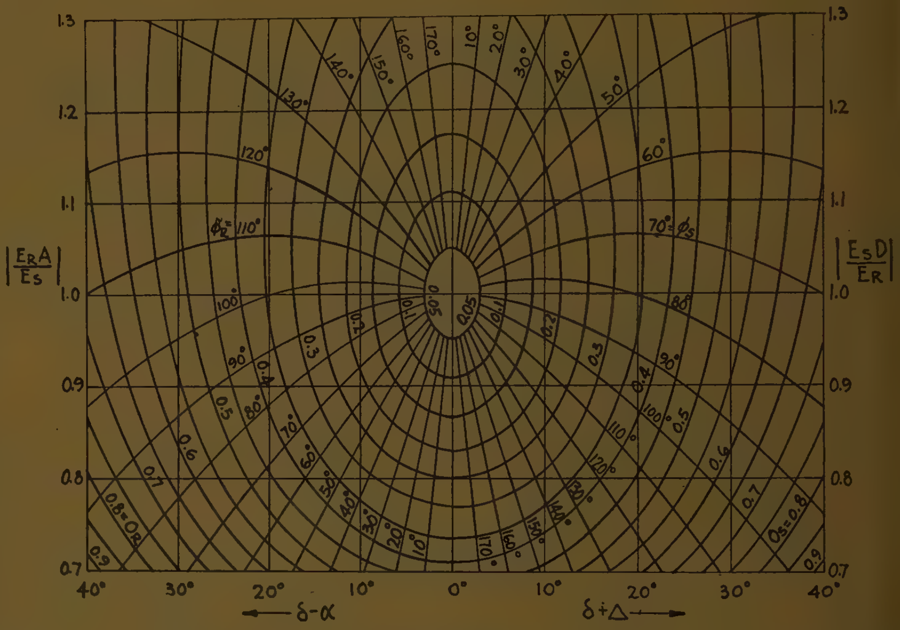


Figure 10. Universal chart for power and voltage given at one end of line, normal operating range

$\phi_R=\beta-\alpha+\theta,\ O_R=(\text{kva})_R\left|\frac{B}{E_R^2A}\right|$ $\qquad$ $\phi_S=\beta-\Delta+\theta,\ O_S=(\text{kva})_S\left|\frac{B}{E_S^2D}\right|$

$$O_R=\frac{(\text{kva})_R|B|}{154^2|A|}=0.533,\quad \phi_R=8.1°+78.56°-1.06°=85.6°$$

With the foregoing values, the operating point is located in the left-hand side of Figure 10, giving

$$|E_RA/E_S|=0.856$$
$$\delta-\alpha=27.2°$$

from which

$$|E_S|=\frac{0.917}{0.856}\times 154=165\text{ kv}$$
$$\delta=27.2°+1.06°=28.26°$$

Next, computation of $|E_SD/E_R|=165\times 0.917/154=0.983$ and $\delta+\Delta=29.32$ furnishes the operating point for the right-hand side of the Figure 10 chart, which corresponds to

$$O_S=0.51=\left|\frac{(\text{kva})_SB}{E_S^2A}\right|$$

giving

$$(\text{kva})_S=83.1\text{ megavolt-amperes}$$
$$\phi_S=77.7=\beta-\Delta+\theta_S$$

giving

$$\theta_S=-0.4°,\ \cos\theta_S\approx 1$$

From the foregoing example it can be seen that this universal chart furnishes the desired information quickly and with a minimum of calculations.

Whereas the charts in the first two sections of this paper were obtained as mappings of analytic complex functions, the universal charts are obtained by a purely geometrical process and do not represent such mappings. The orthogonal system of O_S = constant and ϕ_S = constant lines of Figure 7 does not map into an orthogonal system, as can be seen by inspection of Figure 9. An orthogonal system could be obtained if Figure 7 were mapped according to the function $1_g|E_R/E_SD|\underline{/\delta+\Delta}$; however, this would produce a logarithmic scale for the ordinate which is less practical than the linear scale employed in Figure 9.

Charts of Figures 9 and 10 are applicable for the solution of problems where power and voltage at one end of the transmission line are given. Similar universal charts can be developed with power given at one end and voltage at the other. Since the derivation of the charts is similar to that of the preceding ones, it will be sufficient to list the relations corresponding to equations 12 and 13. The following notations and abbreviations are used in deriving them:

$$\phi_R=\beta-\alpha+\theta_R$$
$$O_{RS}=(\text{kva})_S\left|\frac{DB}{E_R^2}\right|$$
$$O_{SR}=(\text{kva})_R\left|\frac{AB}{E_S^2}\right|$$

resulting in

$$\cos(\delta+\Delta)=\frac{\left|\frac{DE_S}{E_R}\right|^2+\left|\frac{DE_S}{E_R}\right|^4-O_{RS}^2}{2\left|\frac{DE_S}{E_R}\right|^3} \tag{14}$$

$$\left|\frac{E_SD}{E_R}\right|=\frac{\sin(\phi_S+\delta+\Delta)}{\sin\phi_S} \tag{14a}$$

$$\cos(\delta-\alpha)=\frac{\left|\frac{AE_R}{E_S}\right|^2+\left|\frac{AE_R}{E_S}\right|^4-O_{SR}^2}{2\left|\frac{AE_R}{E_S}\right|^3} \tag{15}$$

$$\left|\frac{E_RA}{E_S}\right|=\frac{\sin(\phi_R-\delta+\alpha)}{\sin\phi_R} \tag{15a}$$

The resulting charts are shown in Figure 11, covering a wide range of values, and in Figure 12, applying to the normal operating range. Since appearance and use of the charts is similar to the previous ones, it will be sufficient to compute one example.

Referring to the afore-mentioned transmission line it is required to determine the receiving-end voltage and sending-end power when 50 megavolt-amperes at 96-per-cent power-factor lagging is delivered at the receiving end with the sending-end voltage maintained at 160 kv.

Solution:
As given,

$$\cos\theta_R=0.96$$

therefore

$$\theta_R=-16.3°$$
$$\phi_R=\beta-\alpha+\theta_R=78.56°-1.06°+(-16.3°)=61.2°$$

$$O_{SR}=(\text{kva})_R\left|\frac{BA}{E_S^2}\right|=\frac{51.2\times153.16\times0.917}{160^2}$$
$$=0.286$$

Referring to the left-hand side of chart Figure 12, the intersection of $\phi_R=61.2$ and $O_{SR}=0.286$ furnishes

$$|E_RA/E_S|=0.77$$

from which

$$|E_R|=\frac{0.77\times160}{0.917}=134.2\text{ kv}$$

$$\delta-\alpha=19°$$

from which

$$\delta=19°+1.06°=20.06°$$

To find the sending-end power, compute

$$|E_SD/E_R|=\frac{160\times0.917}{134.2}=1.09$$

$$\delta+\Delta=1.06°+20.06°=21.12°$$

Referring now to the right-hand side of Figure 12

$$\phi_S=66.3° \quad O_{RS}=0.43$$

giving

$$(\text{kva})_S=\frac{0.43\times134.2}{153.16\times0.917}$$
$$=55\text{ megavolt-amperes}$$

$$\theta_S=66.3°-78.56°+1.06°$$
$$=-11.2°,\ \cos\theta=0.981\text{ lagging}$$

Sending-end power $55\times0.981=54$ megawatts.

From the foregoing example it can be seen that the desired information can be obtained quickly. It is particularly interesting to note that in this universal type of diagram none of the difficulties inherent in the modified charts of Figures 1C and 1D are encountered.

Additional data follow readily from these charts. The dotted line in Figure 10 indicates the optimum-power-limit line, corresponding to the envelope of the nonconcentric circles in the conventional diagram. This line corresponds to the line $O=1$ in Figure 10 and therefore can be constructed readily.

The power-limit parabola which is the locus of the maximum power transmitted at regulated potentials in the conventional diagram coincides with the constant-power-angle line where $\delta=\beta$.

Power-angle curves can be developed quickly from these charts.

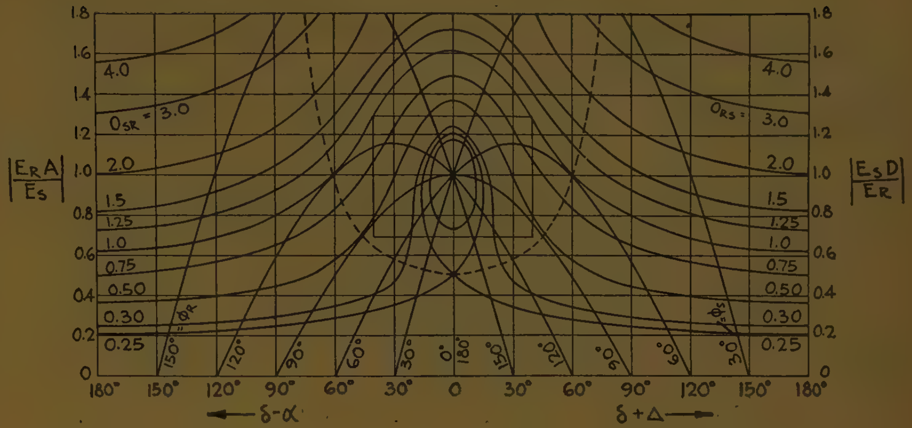


Figure 11. Universal chart for power given at one end and voltage at other end

$$\phi_R=\beta-\alpha+\theta_R,\quad O_{SR}=(\text{kva})_R\left|\frac{BA}{E_S^2}\right| \qquad \phi_S=\beta-\Delta+\theta_S,\quad O_{RS}=(\text{kva})_S\left|\frac{BD}{E_R^2}\right|$$

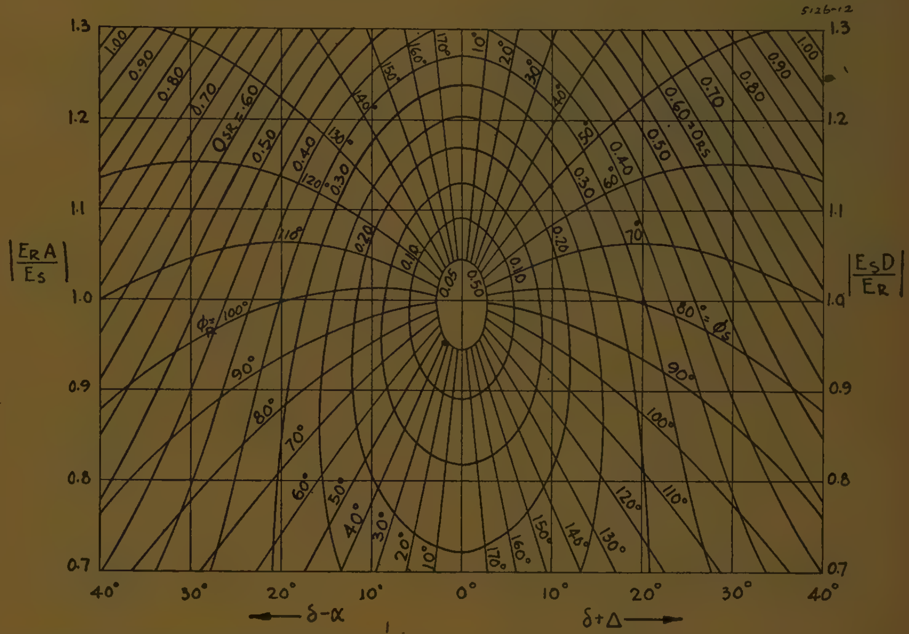


Figure 12. Universal chart for power given at one end and voltage at other end, normal operating range

$$\phi_R=\beta-\alpha+\theta,\quad O_{SR}=(\text{kva})_R\left|\frac{BA}{E_S^2}\right| \qquad \phi_S=\beta-\Delta+\theta,\quad O_{RS}=(\text{kva})_S\left|\frac{BD}{E_R^2}\right|$$

Conclusions

1. A circle diagram is developed, relating power at one end to power at the other end

2. A circle diagram is developed, relating impedance at one end to that at the other.

3. Universal transmission-line charts are developed based on a new co-ordinate system from which the essential data pertaining to any line readily can be computed. The difficulty encountered in the conventional charts with power and voltage given at different ends is eliminated by this type of chart.

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The S-Function Method of Measuring Attenuation of Coaxial Radio-Frequency Cable

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DURING the last few years coaxial radio-frequency cable has been purchased in large quantities by the Army and the Navy for use in aircraft and surface vessels as radio and video transmission lines. Inspection of this cable has been carried out at the factories in accordance with the requirements of the applicable specifications, the latest of which is the Joint Army–Navy Specification *JAN-C-17*. These specifications require frequent measurements, at the factories, of the characteristic impedance and attenuation during production.

Characteristic impedance generally has been determined from results of capacitance and velocity of propagation tests on samples of cable approximately five feet long, by means of a formula similar to equation 32 of this paper. Accuracy of measurements by this method easily can be kept to within one per cent.

Attenuation measurements generally have been made at one or more frequencies such as 100 megacycles or 400 megacycles, by a method involving samples approximately 100 feet long. It has been shown by a series of comparison tests that results of measurements by this method on different equipments and at different laboratories may vary by as much as 20 per cent on one sample. This can be partially attributed to the use of nonstandard equipment and to the highly skilled technique required in making the measurements.

A new method of measuring attenuation, known as the *S*-function method, is here proposed. This method has been found to be simpler, faster, and more convenient than any other in general use. Also, it required samples which are shorter and the lengths of which are less critical than do other methods so far proposed. For example, the best length of radio-frequency cable *RG-8/U* for a 100-megacycle test is 11 inches. Although a knowledge of the characteristic impedance is required, an error in its value generally will not affect adversely the attenuation results, which is in contrast to some methods using resonant lengths of cable, which are subject to errors in direct proportion to the relative error in the supplied value of the characteristic impedance. This method, however, is limited by the design of presently available equipment to frequencies from about 1 to 100 megacycles.

Measurement Procedure

To measure the attenuation of a cable, cut a sample of total length approximately equal to (see Figure 1A)

$$d_0 = \frac{144V}{f_{mc}} \text{ feet} \tag{26}$$

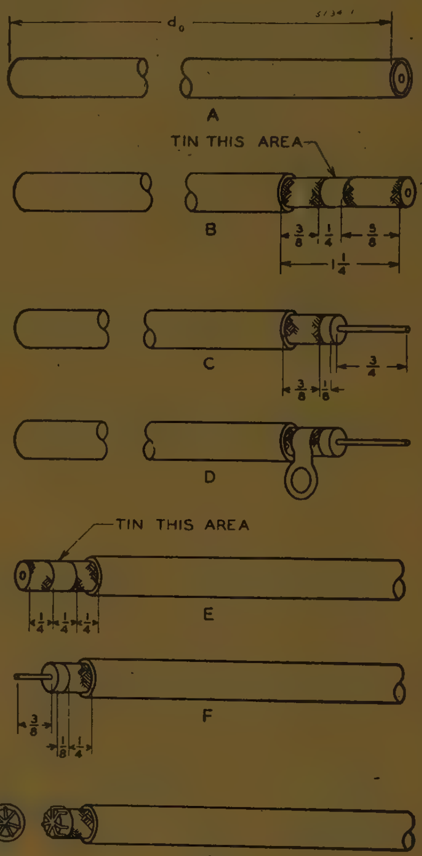

Figure 1. Preparation of the cable sample for measurement

where

V = relative velocity of propagation of cable
f_{mc} = test frequency in megacycles

Strip back outer covering for about 1¼ inches from one end and apply solder to the outside of the braid in a circular band as shown in Figure 1B. Cut through the braid and insulation to the inner conductor at about three-quarters inch from the end (Figure 1C), exercising care not to cut into the inner conductor. Solder a lug to the braid (Figures 1D and 3), making certain that a large part of the lug surface makes contact with the braid and that the entire region around the lug is well soaked in solder. An iron with about 200 watts capacity should be used. Strip back the covering at the other end of the sample for about three-quarters inch (Figure 1E), and soak solder into the braid in a circular band. Cut through the braid and insulation as shown in Figure 1F, and bend back the central strands radially and solder to the braid (Figures 1G and 3). It is important that each strand and this entire end of the braid be well soaked in solder to insure a low-resistance connection. It has been found that making only "spot" contacts has the effect of raising the values obtained for attenuation, *N*, by as much as ten per cent.

The prepared sample is connected to the *Q* meter (Figures 2, 5A, and 5B), which is operated in the usual manner for testing coils, and values of *f*, *Q*, and *C* recorded. These are applied to the solid curve of Figure 6 to obtain the value of *S*, which is inserted in the following formula for attenuation:

$$N = \frac{S}{Qd} \text{ decibels per 100 feet} \tag{20}$$

As a guide in subsequent sample cutting, the dashed line of Figure 6 gives the ratio of preferred sample length (d_0) to the actual sample length (d).

At the lower frequencies it may become necessary to employ auxiliary capacitors. These should be of the silver mica type, and are connected to the "Cond" terminals of the *Q* meter. The amount of external capacitance required is determined easily from the fact that the total capacitance in micromicrofarads required to resonate a cable sample of length d_0 will be

$$C_{\mu\mu f} = \frac{120{,}000}{f_{mc}Z_0} \tag{29}$$

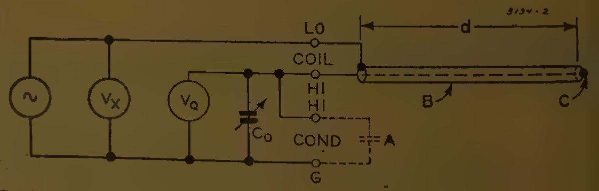

Figure 2. Schematic diagram of the measuring circuit

Standard *Q* meter such as Boonton type 160*A* or 170*A*
A—Auxiliary silver-mica capacitor as required
B—Cable sample under test
C—Inner conductor soldered to outer conductor at end

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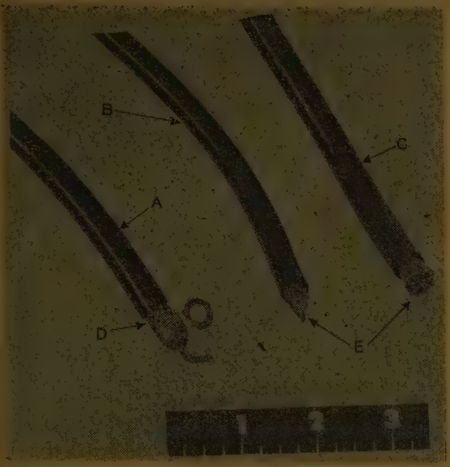

Figure 3. Prepared ends of cable samples for attenuation measurement

A. Radio-frequency cable *RG-8/U*
B. Radio-frequency cable *RG-7/U*
C. Radio-frequency cable *RG-8/U*
D. Connection to *Q* meter
E. Short-circuited ends of cable samples (should be well soldered in this fashion to insure minimum resistance and inductance)

where

f_{mc} = test frequency in megacycles
Z_0 = characteristic impedance of the cable in ohms

From equation 30 it may be noted that the lowest frequency at which 50-ohm cable can be tested on a type-160*A* *Q* meter without external capacitance is six megacycles.

Following is a typical example of data obtained in making an attenuation measurement on a sample of radio-frequency cable *RG-8/U:*

d = 3.0 feet
Z_0 = 52 ohms
f_{mc} = 30 megacycles
Q = 89
$C_{\mu\mu f}$ = 82 micromicrofarads
S = 274
N = 1.03 decibels per 100 feet

Appraisal of the Method

A typical sample of coaxial cable, about 200 feet long, was tested at several government and commercial laboratories by various methods at 100 megacycles. The results are listed in Table I. It will be seen that the result obtained by the *S*-function method agrees closely with the average of the results of the other methods.

In Figure 7 results obtained on a sample of radio-frequency cable *RG-7/U* at frequencies from 1 to 100 megacycles by the *S*-function method have been plotted and compared with nominal values listed in the Joint Army–Navy Specification *JAN-C-17*. The measurement results lie almost in a straight line on "log–log" paper.

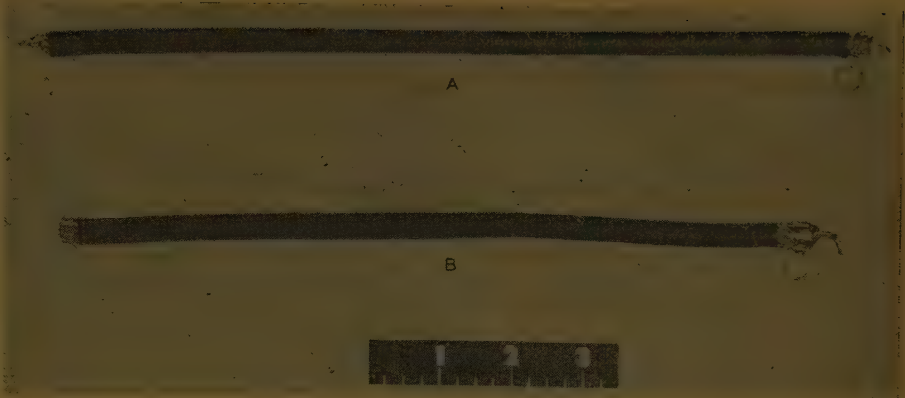

Figure 4. Cable samples prepared for attenuation measurement at 100 megacycles

A. Radio-frequency cable *RG-7/U*   B. Radio-frequency cable *RG-8/U*

By use of equations 18 and 33 it can be shown that if the sample length, d, is held to within ten per cent of the length d_0, prescribed by equation 26, a ten-per-cent error in Z_0 will cause less than a two-per-cent error in attenuation, N, and likewise a 20-per-cent error in Z_0 will cause less than a six-per-cent error in N.

Tests were attempted at 200 megacycles, but in each case the results were from 10 to 15 per cent higher than values obtained from other methods or by extrapolation from lower frequencies. This is probably due to losses in the *Q* meter and its terminals, and possibly to some radiation. At frequencies below one megacycle, for most types of cable now in use, the *Q* values are too low to be read accurately. Consequently, it is recommended that for presently available equipment this method be limited to frequencies from 1 to 100 megacycles.

Definitions of Terms

α = attenuation constant of the cable sample in nepers per foot
β = wave-length constant of cable sample in radians per foot
$\gamma = \alpha + j\beta$
c = velocity of light in vacuum in feet per second $= 983.57 \cdot 10^6$
C = distributed capacitance of cable in farads per foot
C_0 = capacitance required to resonate cable at test frequency in farads
d = length of cable sample in feet
d_0 = preferred length of cable test specimen in feet
ϵ = base of naperian logarithms
f = test frequency in cycles per second
G = distributed conductance of cable in mhos per foot
$j = \sqrt{-1}$
L = distributed inductance of cable in henrys per foot
$M = Z_0/X_{sc}$, introduced for convenience
M_0 = value of M for least error in N caused by an error in Z_0
N = attenuation of cable in decibels per 100 feet
P = power factor of the cable dielectric

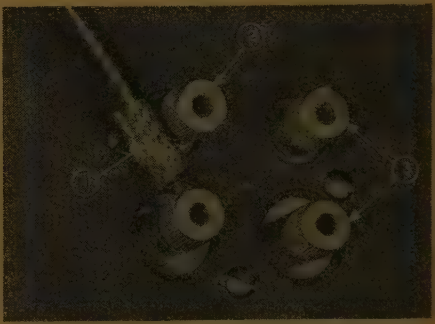

Figure 5A. Connection of cable samples to Boonton type 160*A* *Q* meter for attenuation measurement by the *S*-function method at frequencies from 1 to 35 megacycles

1. Care must be taken that outer braid clears all metal except *LO* terminal by at least one-eighth inch
2. *LO* terminal
3. Connect silver-mica auxiliary capacitor to these terminals if required (usually unnecessary above six megacycles)

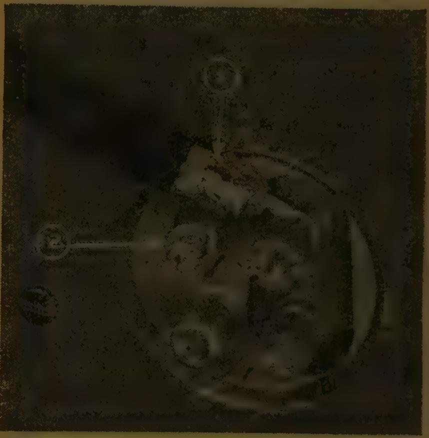

Figure 5B. Connection of cable sample to Boonton type-170*A* *Q* meter for attenuation measurement by the *S*-function method at frequencies from 35 to 100 megacycles

1. Care must be taken that outer braid clears all metal except *LO* terminal by at least one-eighth inch
2. *LO* terminal

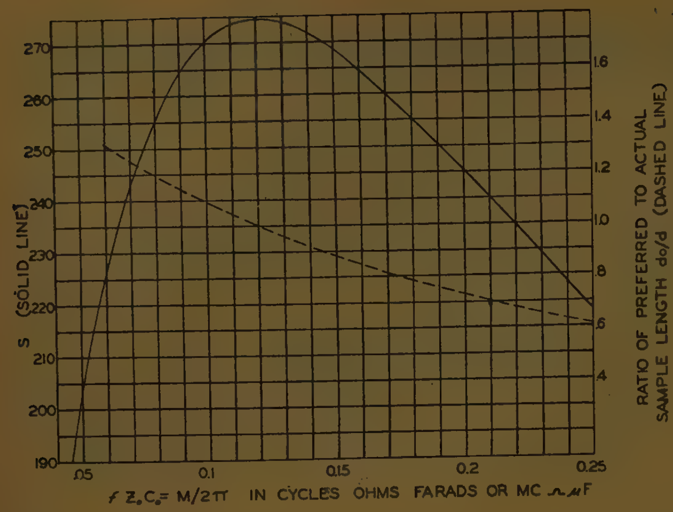

Figure 6. Plot of $S = \frac{2{,}000 \log_{10}\epsilon}{M+1/M+1/\text{arccot } M}$ (solid line), and $d_0/d = 0.923 \div \text{arccot } M$ = ratio of preferred to actual sample length (dashed line)

$Q = X_{sc}/R_{sc}$
R = cable-conductor resistance in ohms per foot
R_{sc} = input resistance of short-circuited cable sample in ohms
S = function of M defined in equation 18 and plotted in Figure 6
V = ratio of velocity of radio-frequency potential in cable to velocity of light
ω = angular velocity of test voltage in radians per second
X_{sc} = input reactance of short-circuited cable sample in ohms
Z_0 = absolute magnitude of cable characteristic impedance
$\underset{\cdot}{Z}_0$ = complex cable characteristic impedance in ohms
Z_{sc} = input impedance of short-circuited cable sample, in ohms $= R_{sc} + jX_{sc}$

Derivation of Formulas

From fundamental transmission-line theory is obtained the following group of formulas[3]:

$$\alpha = {}^1/_2(G\sqrt{L/C} + R\sqrt{C/L}) \tag{1}$$

$$\beta = \omega\sqrt{LC} \tag{2}$$

$$\underset{\cdot}{Z}_0 = \sqrt{L/C}/\underline{{}^1/_2(G/\omega C - R/\omega L)} \tag{3}$$

From basic capacitor theory[4]

$$G = P\omega C \tag{4}$$

$$P = G/\omega C \tag{4a}$$

If we divide equation 1 by equation 2,

$$\alpha/\beta = {}^1/_2(G/\omega C + R/\omega L) \tag{5}$$

Subtracting equation 5 from equation 4a yields

$$P - \alpha/\beta = {}^1/_2(G/\omega C - R/\omega L) \tag{6}$$

Combining equation 3 and equation 6 produces

$$\underset{\cdot}{Z}_0 = \sqrt{L/C}/\underline{P - \alpha/\beta} \tag{7}$$

which, by definition, becomes

$$\underset{\cdot}{Z}_0 = Z_0/\underline{P - \alpha/\beta} \tag{7a}$$

In practice $P \ll 1$ and $\alpha/\beta \ll 1$, so that the following approximation is accurate:

$$\underset{\cdot}{Z}_0 \cong Z_0[1 - j(P - \alpha/\beta)] \tag{7b}$$

From transmission-line theory:[5]

$$Z_{sc} = \underset{\cdot}{Z}_0 \tanh \gamma d \tag{8}$$

Substituting equation 7b in equation 8 yields

$$Z_{sc} = Z_0[1 - j(P - \alpha/\beta)](\sinh 2\alpha d + j \sin 2\beta d)/2(\cosh^2 \alpha d - \sin^2 \beta d) \tag{9}$$

As these tests are made on short lengths of cable such that $\alpha d \ll 1$, the following approximation of equation 9 is justified:

$$Z_{sc} = Z_0[2\alpha d - (P - \alpha/\beta)\sin 2\beta d - j \sin 2\beta d]/2 \cos^2 \beta d \tag{9a}$$

By definition and from equation 9a,

$$Q = X_{sc}/R_{sc} = \sin 2\beta d/[2\alpha d - (P - \alpha/\beta) \sin 2\beta d] \tag{10}$$

$$1/Q = (2\alpha d/\sin 2\beta d) - (P - \alpha/\beta) \tag{10a}$$

$$X_{sc} = Z_0 \sin 2\beta d/2 \cos^2 \beta d = Z_0 \tan \beta d \tag{11}$$

By definition and from equation 11,

$$M = Z_0/X_{sc} = \cot \beta d \tag{12}$$

$$\beta = (\text{arccot})\ M/d \tag{12a}$$

$$1/\sin 2\beta d = {}^1/_2\sqrt{2 + \tan^2 \beta d + \cot^2 \beta d} \tag{13}$$

If we combine equations 10a, 12, and 13,

$$1/Q = \alpha d\sqrt{2 + M^2 + 1/M^2} - (P - \alpha/\beta) \tag{14}$$

From equations 14 and 12a we obtain

$$\alpha = (1/Q + P)/d(M + 1/M + 1/\text{arccot } M) \tag{15}$$

$$N = (2{,}000 \log_{10}\epsilon)\alpha \tag{16}$$

so that

$$N = (1/Q + P)2{,}000 \log_{10}\epsilon/d(M + 1/M + 1/\text{arccot } M) \tag{17}$$

Let

$$S = 2{,}000(\log_{10} \epsilon)/(M + 1/M + 1/\text{arccot } M) \tag{18}$$

This is plotted in Figure 6. Then

$$N = S(1/Q + P)/d \tag{19}$$

American Society for Testing Materials specifications require polyethylene to have a power factor of less than 0.005 at 1 megacycle and at 100 megacycles. Measurements made on various samples of radio-frequency cable *RG-8/U* with *Q* meters showed power factors of the order of 0.0002 at 10 megacyles and of 0.0003 at 100 megacycles. Consequently, for most work involving cables with this insulation, the assumption $P \ll 1/Q$ is justified, in which case equation 19 becomes

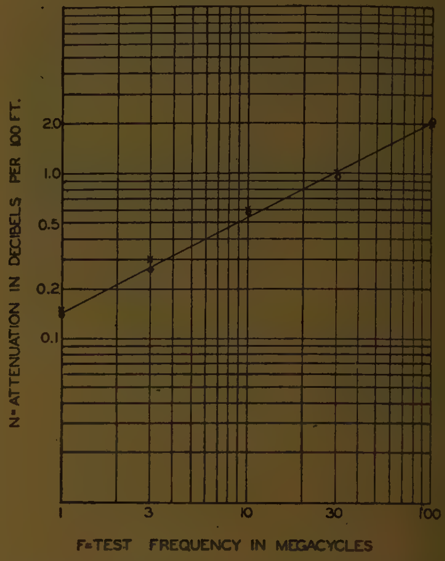

Figure 7. Results of attenuation measurements on radio-frequency cable *RG-7/U* by the *S*-function method (θ) compared with engineering data listed in Joint Army–Navy Specification *JAN-C-17*, paragraph E-3 (X)

⊙ Results of S-function test on typical sample
× Engineering data listed in joint Army–Navy specifications

$$N = S/Qd \tag{20}$$

This assumption introduces an error in opposition to the effect of losses in the Q meter and cable-sample terminals, so that even in cases where P is as great as two or three per cent of $1/Q$, equation 20 in practice may yield results as accurate as does equation 19.

From equation 18 is obtained

$$1/S = (M + 1/M + 1/\text{arccot } M)/2{,}000 \log_{10}\epsilon \tag{21}$$

From inspection of equations 12, 19, and 21, it is seen that the value of d resulting in the least possible error in N introduced by a given error in Z_0 would result from the condition:

$$d(1/s)/dM = 0 \tag{22}$$

Solving equations 21 and 22 for M_0 yields

$$M_0 = \cot(M/\sqrt{1 - M})^4 \cong 0.7567 \tag{23}$$

From equations 23 and 12a, the length of cable which would give least error in N by an error in Z_0 is

$$d_0=(\text{arccot } M_0)/\beta=0.923/\beta \qquad (24)$$

From fundamental theory,[6]

$$\beta=2\pi f/Vc \qquad (25)$$

so that equation 24 becomes

$$d_0=Vc(\text{arccot } M_0)/2\pi f=144.5\cdot 10^6 V/f \qquad (26)$$

In the case of a cable with $V \cong 0.65$, such as those with polyethylene or polyisobutylene dielectrics, this becomes about

$$d_0=94\cdot 10^6/f \qquad (26a)$$

For cables with air dielectrics, $V=1.0$, in which case

$$d_0=144\cdot 10^6/f \qquad (26b)$$

If the measurement of X_{sc} is made on a Q meter,

$$X_{sc}=1/\omega C_0 \qquad (27)$$

from which, with equation 12,

$$M=Z_0\omega C_0=2\pi f Z_0 C_0 \qquad (28)$$

and the setting obtained for the preferred length of cable will be, from equations 23 and 28,

$$C_0=M_0/\omega Z_0 \cong 0.12/fZ_0 \qquad (29)$$

For 50-ohm cables, such as radio-frequency cable RG-$8/U$,

$$C_0=2{,}400\cdot 10^{-6}/f \qquad (29a)$$

Table I

Method	Laboratory	Decibels Per 100 Ft
Diode voltmeters at ends of long terminated sample[1]	E	1.65
	S	2.32
Tuned circuit-coupled-oscillator and diode voltmeter (most commonly employed at radio-frequency cable factories)	E	2.01
	T	2.13
	B	2.0
Short resonant lengths[2] (d=19 in.)	S	2.14
(d=38 in.)	S	2.17
(d=59 in.)	S	2.14
The S-function method (d=11 in.)	S	2.13

It will be found in practice that the lowest frequency at which tests by this method can be made on the type 160A Q meter without auxiliary capacitors is limited by the size of the tuning capacitor, whose highest usable capacitance is approximately 400 micromicrofarads. This frequency is given by equation 28, and for a 50-ohm cable would be

$$f=2400\cdot 10^{-6}/400\cdot 10^{-12}=6\cdot 10^6 \text{ cycles per second}=6 \text{ megacycles per second} \qquad (30)$$

Obviously, the frequency range can be extended downward by connecting low-loss fixed capacitors to the capacitor terminals of the Q meter, so that the total capacitance at the center of the dial range will be approximately that given by equation 29.

From equation 7 and by definition of Z_0,

$$Z_0=\sqrt{L/C} \qquad (31)$$

which, with equations 2 and 25, yields

$$Z_0=1/VcC \qquad (32)$$

From equations 12a and 24,

$$d_0/d=(\text{arccot } M_0)/\text{arccot } M \cong 0.923/\text{arccot } M \qquad (33)$$

This has been plotted as a dashed line in Figure 6.

References

1. Coaxial-Cable-Attenuation Measurements at 300 Megacycles, H. H. Race, C. V. Larrick. *General Electric Review*, volume 44, September 1941, pages 507–10.

2. A Method of Measuring Attenuation of Short Lengths of Coaxial Cable, Chandler Stewart, Jr. *Proceedings* of the Instititute of Radio Engineers, volume 33, January 1945, pages 46–8.

3. Communication Engineering (second edition), W. L. Everitt. 1937. Page 118.

4. The Radio Engineers' Handbook (first edition), F. E. Terman. 1943. Page 110.

5. Reference 3, page 145.

6. Reference 3, page 108, equation 23.

Line Drop Produced by Resistance Welders

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NONMEMBER AIEE

Synopsis: Resistance-welding machines normally form a very reactive pulsating load on a power system which sometimes produces undesirable line regulation. The regulation can be calculated, and, if it is too high, it can be reduced by operation directly on 2,300 volts or by power-factor-correction means. From a line-regulation point of view, the system of 2,300-volt operation is usually the most satisfactory. The effect of electronic controls on line drop is also discussed.

Large currents flowing across the junction between two metals evolve heat because of the contact and material resistance. Both the resistance and current are controlled in a resistance-welding machine so that the heat will be just right to produce a fusion weld. The contact resistance is controlled by proper surface cleaning and control of the contact forces by the welding machine. Current is controlled as to duration and magnitude by timing, heat-control circuits, and welding-transformer taps. The currents involved are very large, being between a few hundred and a few hundred thousand amperes, depending on the welding operation.

It is often necessary for mechanical reasons to have long arms on the welding-transformer secondary. This structure will have an inductance of the order of one microhenry. This inductance, although small, will be the main impedance to the welding current. In order to pass 10,000 amperes through this inductance at 60 cycles the transformer secondary must produce about four volts. The transformer will thus involve 40 kva just for this relatively small welding current.

Welding currents are normally of

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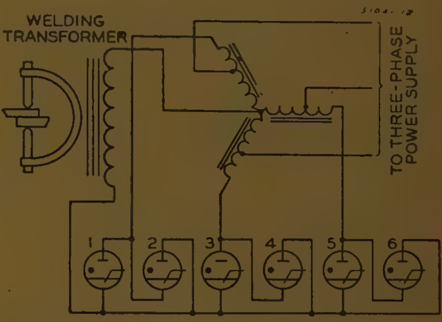

Figure 1a. Power circuit of three-phase to single-phase low-frequency converter

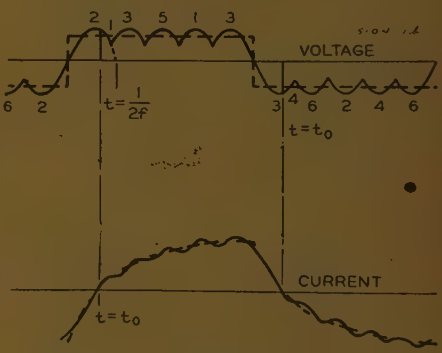

Figure 1b. Voltage and current waves

The solid lines are the actual voltage and current. The dashed lines are the values assumed in the mathematical analysis in the appendix. Numerals indicate which ignitrons are conducting short duration. In production there ordinarily will be several pulses of current per second. This large, pulsating, low-power-factor (usually 0.25 to 0.4) single-phase load frequently requires a power system designed from a regulation point of view entirely without regard for thermal characteristics.

Line-Drop Calculations on Uncorrected Systems

The most common complaint occasioned by line regulation concerns light flicker. The eye is extremely sensitive to flicker if it occurs 2 to 12 times per second, less than one-per-cent voltage fluctuation being objectionable. The voltage drop producing flicker is not the reduction in rms voltage as read on a voltmeter but is the largest average reduction lasting for a time interval of about 0.02 second, a time determined by the physiology of visual response. This time interval varies greatly with different people and even with one person under different conditions. Some welder installations produce enough regulation to upset control circuits. It is desirable to know what the regulation will be for a system before installing a welder.

Regulation can be calculated by the relationship (developed in Appendix I):

$$\text{Per cent line drop} = 100\,\frac{\Delta E_{line}}{E_{line}}$$

$$= \frac{H^2 \cdot P}{100(kva)_{sc}}(\cos\theta_2 + \sin\theta_2 \tan\theta_1) \quad (1)$$

where

E_{line} = line voltage

H = electronic heat control setting = per cent maximum rms current

P = kilowatts dissipated during a weld at 100 per cent heat

$(kva)_{sc}$ = short-circuit kilovolt-amperes of power-supply system

θ_1 = phase angle of total circuit impedance

θ_2 = phase angle of power system impedance

A given welding operation will require a certain value of the kilowatts, P, dissipated during the weld. It is of interest, however, to see how the other parameters enter into the line drop. The power factors or impedance phase angles enter into the line drop critically. The phase angle of the power system, θ_2, is normally between 60 and 85 degrees so that the term $\sin\theta_2 \tan\theta_1$ is of primary importance in this factor. The total impedance of the circuit is largely that of the load so that θ_1 is almost entirely determined by the load power factor. The phase angle θ_1 is usually greater than 65 degrees, thus a slight improvement in power factor may reduce the line drop very greatly as the tangent varies rapidly in this region. Since low-load power factor is largely due to the inductance of the secondary loop, the loop should be designed with as low an inductance as is compatible with other considerations.

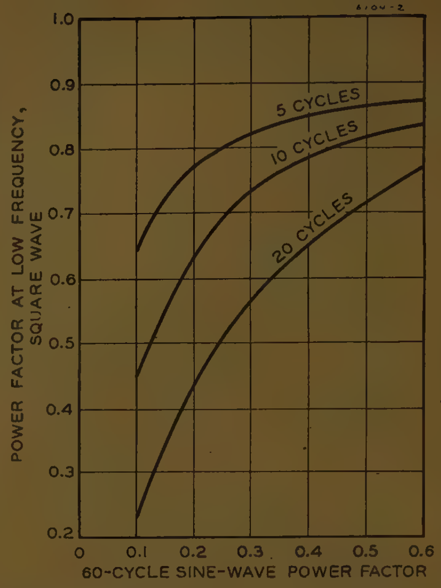

Figure 2. Comparison of the power factor at 60-cycle operation with the power factor of the same machines at low-frequency square-wave operation

It is apparent from the formula that it is desirable to have the short-circuit kilovolt-amperes of the power system, as seen from the welder, as large as possible. On low-voltage power systems the short-circuit characteristics are determined largely by the step-down transformers which limit the short-circuit current greatly. This limitation is avoided and the short-circuit kilovolt-amperes greatly increased by working directly from the high-voltage distribution lines. Welder controls for direct operation on 2,300-volt distribution systems are now available. Operation from these may be direct or may be through a transformer used for the individual welder only in which case the step-down transformer should be considered to be a part of the welding system. Line drop for 2,300-volt operation may be as little as ten per cent of its value for low-voltage operation. Though 2,300-volt operation is usually desirable it is not always possible. Some power-factor-correction means therefore may be required.

Low-Frequency Systems

Certain electronic frequency-changing circuits draw almost balanced three-phase power from the line and have a low-frequency single-phase output. Because power factor is increased when the load is operated at low frequency a double advantage can be had from such an electronic circuit. There are a number of circuits, all fundamentally alike, designed to produce single-phase low-frequency power from a polyphase system. By a system of rectifiers, current is first passed through the load in one direction, then, after one-half cycle of low frequency, the current is passed through the load in the opposite direction.

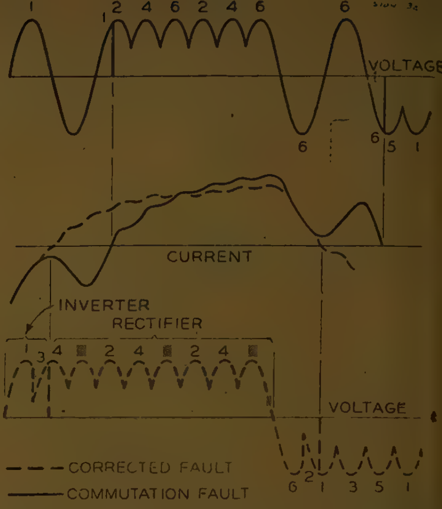

Figure 3a. Current and voltage waves for corrected and uncorrected commutation faults

Numerals indicate which ignitrons of Figure 1a are conducting

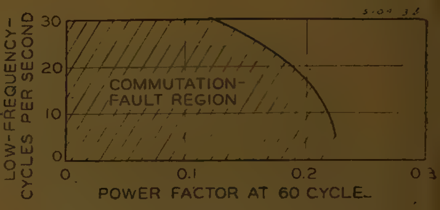

Figure 3b. Region of commutation fault

Figure 1 shows a circuit for three-phase to single-phase conversion with resultant voltage and current waves. The low-frequency voltage wave can be approximated very closely by a square wave plus some high harmonics. The current wave will have these harmonics but only to a very slight degree due to relatively high reactive impedance to these high frequencies. The power factor and other properties of the secondary loop resulting from low-frequency operation can be calculated. (See Appendix II.) The power factor of the entire welder depends on the type of rectifier and the regulation of the transformer involved. Figure 2 shows power-factor curves for a low-frequency welding system having good transformer regulation, being used at full heat. Heat control, which involves delaying the ignition of the rectifier ignitrons, reduces the power factor greatly.

Voltage and Current Reversal

Each ignitron carries current until another ignitron with properly applied voltage takes over this task or until the load current becomes zero. When it is desired to reverse the voltage no subsequent ignitron is fired to which the current can

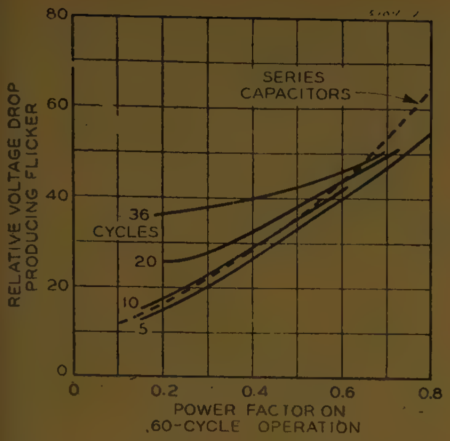

Figure 4. Light flicker for low-frequency and series-capacitor systems compared with 60-cycle uncorrected systems (equal 100 on voltage scale)

The voltage drop producing light flicker depends on the physiology of the eye

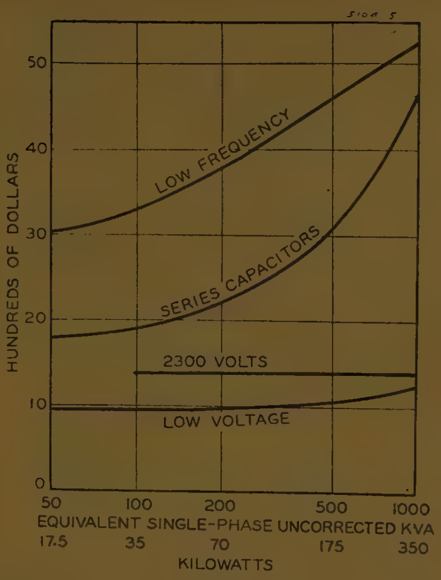

Figure 5. Cost of controls for various systems having a resistance-to-inductance ratio giving 35-per-cent power factor at 60 cycles

commutate so that the last ignitron fired in each voltage pulse carries current until the current ceases.

Before the last ignitron extinguishes, the voltage of the corresponding rectifier-transformer secondary winding reverses. The current is then flowing against the applied voltage and is quickly reduced in magnitude, but continues to flow against the applied voltage because of the reactive nature of the load. When the current finally reaches zero the ignitron extinguishes. The operation can best be seen by referring to Figure 1b. Tube 3 is the last tube to be fired in the upper portion of the voltage wave shown. It continues to carry the current until time t_0, at which instant the current is zero. At that time, or very shortly thereafter, tube 4 is ignited. This tube applies the voltage to the loop in such a way as to reverse the direction of the current. The subsequent firing order is shown in the figure.

If the load is so reactive that $t_0 > {}^1/_{120}$ second, the rectifier transformer voltage again reverses and the current will be built up again instead of being reduced to zero. Therefore the ignitron will continue to conduct. This condition, known as a commutation fault, is shown in Figure 3a. The boundary of the commutation-fault region (calculated in Appendix II) is shown in Figure 3b.

Commutation Fault and Limitations and Correction

Commutation fault introduces drawbacks to the use of low-frequency systems. During one or more cycles (on supply-frequency basis) the entire load is single phase, resulting in a momentarily increased voltage drop of two lines. Though this momentary drop will have a very small value when read on an rms meter it may cause objectionable light flicker. The drop that must be considered is the drop on the loaded phase lines.

It is possible to avoid commutation fault by a different method of operation. Refer again to Figure 3a. The dashed curve shows a system corrected for commutation fault. At the time tube 3 (left of the corrected-voltage-wave, lower part of the figure) is fired, its anode is more positive than that of tube 1 so that the current can commutate to it readily. Very shortly after the commutation is completed, however, the applied anode voltage of tube 3 goes negative, thus the current continued to be opposed. This process is repeated until the current reaches zero. The operation described is that of an inverter. Power from the reactive energy of the electrode loop has thus been fed back into the power-distribution system. After the current reaches zero, the system is again operated as a rectifier. This inverter–rectifier operation eliminates any single-phase operation such as occurs during a commutation fault. The power-factor-correction curves of Figure 2 are based on this inverter–rectifier system.

Series-Capacitor Systems

The power factor of a welder can be corrected by inserting capacitors in series with its primary winding. The capacitance is of such a value that it tunes to resonance, or close to it, with the inductance of the welding transformer. Thus the capacitor and welding transformer in series have no 60-cycle reactive component. The current is limited entirely by the reflected resistance of the welding-transformer secondary winding.

At full heat a series-capacitor-corrected welder operates on unity power factor. When reduction of current by electronic heat control is used, the ignitrons are fired later in the cycle than at 100 per cent heat. This makes the 60-cycle component of current lag behind the line voltage, therefore making the load appear reactive. Because the current is large at 90 per cent heat and is lagging behind the line-voltage the regulation is actually greater than at 100 per cent heat. As the heat is reduced below 90 per cent the regulation improves.

The inductance of the system may be altered by adjusting the secondary loop for different welding operations, or, very slightly, by varying the amount of iron in the secondary loop. Any large inductance changes require readjustment of the capacitor bank if perfect power-factor correction is desired.

Light-Flicker Comparisons

Figure 4 compares the light flicker produced by series-capacitor and low-frequency systems with the flicker resulting from uncorrected-power-factor systems. The curves are based on a power line with 0.3 power factor. Series capacitors correct the power factor completely at 100 per cent heat, but when the ignition is delayed harmonics are introduced which destroy this perfect correction. As a result, maximum line drop occurs at about 90 per cent heat, for which the series-capacitor curve is shown. The drop produced by the low-frequency system was on the basis of full heat. At reduced heat the system utilization is poor and rectifier power factor differs very appreciably from unity so that the greatest drop will be at reduced heat. The drop produced by low-frequency operation will, of course, pulsate at a low frequency to which the eye is very sensitive.

Metallurgical Considerations in Power Factor Correction Problems

Power-factor correction alters one metallurgical factor, the heat generated at the weld being dependent in a different way on work resistance. A slight change in resistance at the weld on uncorrected systems has only a very slight effect on the current as the current is largely inductively limited. In the low-frequency system the current is almost entirely resistance limited. In series-capacitor circuits the welding transformer gets its voltage by virtue of a resonant condition, the primary voltage on the welding transformer being, as a result, approximately inversely proportional to secondary resistance. The secondary current is thus dependent on secondary resistance as it is in low-frequency systems.

The resistance of the work depends on surface conditions, electrode pressure, and materials being welded. The power delivered to the spot, I^2R, will vary with the work resistance depending on how the current varies with resistance. In low-power-factor uncorrected systems, where the weld current is independent of resistance, the power dissipated at the weld is directly proportional to the weld resistance. In corrected systems the heating is inversely proportional to the secondary resistance. The quality of the weld is determined to a large extent by the power dissipation at the spot and therefore on the power factor of the welding machine.

Cost

Both the low-frequency systems and the series-capacitor systems are expensive. The cost depends on many factors, among them: duration of "on" and "off" periods, power factor before correction, type of control, and power required. Figure 5 shows how the cost of the controls varies. The cost is dependent on the power factor before correction and on the power demand. To put the systems on a comparative basis, the power demand is plotted as that for an equivalent uncorrected system.

Conclusions

The line drop of a system can be readily calculated for a given welder. If the line drop turns out to be too great there are several alternatives possible to reduce it, the most satisfactory and least expensive usually being operation on 2,300 volts.

Not only power line but also welding properties must be considered in determining the desirability of using a power-factor-correction system. The dependence of power dissipated at the weld on the power factor is pronounced. No general statement can be made as to the most desirable power factor but it should be pointed out that power-factor correction may be, on some jobs, very undesirable from a welding point of view. Depending on the welding job it might be desirable or undesirable to use power-factor-correction systems.

Series-capacitor systems are less expensive than low-frequency systems for the same improvement in light flicker and kilovolt-ampere demand, though they are still single phase. If the welding loop is to be varied appreciably in any dimension there must also be adjustments of the series capacitors if perfect power-factor correction is to be maintained. Low-frequency systems do not require readjustment with change of the welding loop, but may be subject to commutation fault.

Appendix I. Voltage Drop for Single-Phase Uncorrected-Power-Factor Electronically Controlled Welder

An oscillogram of the voltage and current waves of a heat-controlled wave appear as shown in Figure 6a. The inductive nature of the load causes the current to continue to flow even after the voltage has reversed in direction. It will be noted, on examination of the voltage wave, that it is the sum of the two voltages shown in b and c of Figure 6. Figure 6b is the voltage supplied by the power system and Figure 6c is the voltage generated by the electronic controls. By this division of the electromotive force it is possible to treat the circuit as though it were composed entirely of linear impedance elements. The circuit then has two voltage sources, the power system and the electronic-control system, as shown in Figure 7a.

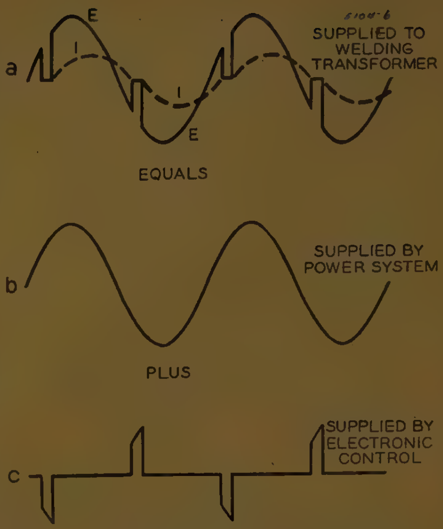
E
I
a
SUPPLIED TO WELDING TRANSFORMER
I
E
EQUALS
b
SUPPLIED BY POWER SYSTEM
PLUS
c
SUPPLIED BY ELECTRONIC CONTROL

Figure 6. Analysis of applied welder voltage

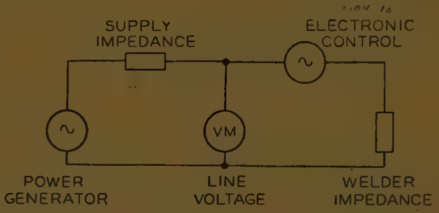

Figure 7a. Schematic circuit with assumption of linear elements

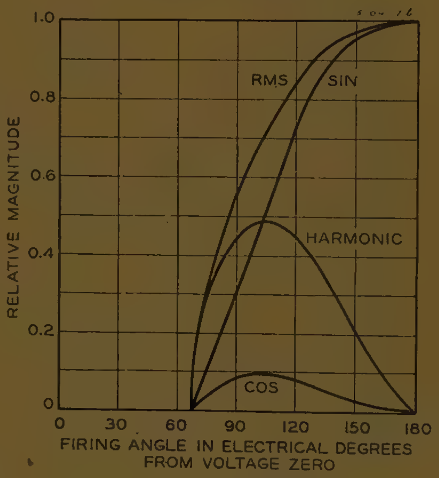

Figure 7b. Typical output of electronic generator

Supply voltage = −1.0

The voltage generated by the electronic-control system, though very rich in harmonics, has a considerable line-frequency component. Figure 7b shows an analysis of the voltage generated by the electronic-control system as a function of the phase of firing. To determine the total applied fundamental-frequency voltage the two voltages must be added vectorially. The harmonic voltages have as their sole source the electronic-control system. They, therefore, tend to increase the rms line voltage. Their contribution to the rms voltage turns out to be negligible, however, as their contribution is less than one tenth of one per cent. In these calculations, therefore, they have been neglected entirely.

Equation 1 was derived on this basis. It is interesting to note that the line drop is the same for input cut down by electronic heat control as for reduction of P by tap adjustment.

Appendix II. Calculation of Three-Phase to Single-Phase Low-Frequency Welder Characteristics

Calculations of power factor on the basis of a perfect square-wave generator. Actual power factors may be only 60 per cent of F_f.

$$F_f = \text{kw}/\text{kva} = 1{,}000P/VI_{rms} \tag{2}$$

where

F_f = power factor at low frequency, f

P = kilowatts

V = square-wave voltage

I_{rms} = low-frequency rms current

$$I_{rms}^2 = \frac{1{,}000P}{R_f} \tag{3}$$

where

R_f = resistive component of low-frequency impedance

$$F_f = \frac{I_{rms}R_f}{V} \tag{4}$$

The current resulting from the square voltage wave will have an exponential form for each half cycle given by

$$i = \frac{V}{R_f}\left[1 - \frac{2\epsilon^{-at}}{1+\epsilon^{-a/2f}}\right] \tag{5}$$

Where f is the square-wave frequency, a is the R/L ratio of the secondary loop and is given by

$$a = 377\left(\frac{1}{F_{60}^2} - 1\right)^{-1/2}$$

where F_{60} is the power factor for the given R/L ratio when operated on a 60-cycle sine-wave voltage. The time t is measured from the instant of voltage reversal.

From the definition of I_{rms}

$$I_{rms}^2 = \frac{2fV^2}{R_f^2}\int_0^{\frac{1}{2f}}\left[1 - \frac{4\epsilon^{-at}}{1+\epsilon^{-a/2f}} + \frac{4\epsilon^{-2at}}{(1+\epsilon^{-a/2f})^2}\right]dt \tag{6}$$

Evaluation of the integrals gives

$$I_{rms}^2 = \frac{2fV^2}{R_f^2}\left[\frac{1}{2f} - \frac{4}{a}\,\frac{1-\epsilon^{-a/2f}}{1+\epsilon^{-a/2f}} + \frac{2}{a}\,\frac{1-\epsilon^{-a/f}}{(1+\epsilon^{-a/2f})^2}\right] \tag{7}$$

Coincidence-Factor Relationships of Electric-Service-Load Characteristics

CONSTANTINE BARY
MEMBER AIEE

Synopsis: This paper contains a demonstration of the existence in practice of relationships between group coincidence factors and number of consumers, and between group coincidence factors and consumers' load factors. Empirical relationships developed from test observations are analyzed from theoretical considerations, formulated in a general manner, and qualified as to the significance of the quantitative values.

THE STUDY of electric-service loads may be divided into two major parts; one dealing with the gradual ebb and flow of loads which are integrated over specified intervals of time measured in terms of minutes, and the second dealing with momentary surges of load which last for short intervals of time measured in terms of seconds. The load characteristics of each of these parts in turn may be subdivided into four distinct components; one taken as at the terminals of the individual appliances and applications, the second one as at the service entrance of individual consumers of electric service, the third one for the diversified conditions of groups of consumers or groups of specified appliances and applications, and the fourth for all classes of consumers or for all classes of appliances and applications, which results in what may be called the "system" conditions.

This paper will be confined to a discussion of the interrelation of some of the factors of the first part of the subject. It will outline two fundamental relationships which have been found in practice to exist between the probable behavior of coincidence factors of individual consumers' loads and

1. A change in the number of consumers.
2. A change in their individual load factors.

These basic relationships were selected for this paper because they have the greatest significance in the engineering and economic features of central-station supply and because comparatively little is known of their probable behavior.

Diversity factor, which is a measure of the degree of diversity present between maximum loads of individual parts of a system, is defined by the AIEE as "the ratio of the sum of the maximum power demands of the subdivision of the system or part of the system to the maximum demand of the whole system or part of the system under consideration measured at the point of supply." For purposes of this discussion, however, it will be easier to visualize and deal with the reciprocal of this relation, that is, with the coincidence in maximum power demands. Its measure is expressed as a "coincidence factor" and will be defined as the ratio of the maximum coincident total power demand of a group of consumers to the sum of the maximum power demands of the individual consumers comprising the group, both taken at the same point of supply and for the same period of time.

While the diversity factor can never be *lower* than unity, the coincidence factor can never be *greater* than unity and, therefore, in dealing with the coincidence factor, we deal in values between zero and unity, while in the case of diversity factor we deal in values between unity and infinity. Since a large part of the facilities required to render electric service depends upon the coincident or diversified maximum demands of groups of consumers, it is evident from the foregoing definition that the higher the coincidence factor, the larger becomes the amount of facilities required per kilowatt of the noncoincident maximum demands of individual consumers, and the lower the coincidence factor, the smaller it becomes.

Major Factors Affecting the Coincidence in the Use of Service

The manner in which electric service is used by individual consumers or groups of consumers, and thus the degree of coincidence in this use, is determined by many factors. Among the more important ones may be mentioned population habits, community and business practices, weather and other climatic conditions, design of utilization equipment, and methods employed in the control of use of the service rendered. The significance of these factors may become clearer from the following discussion.

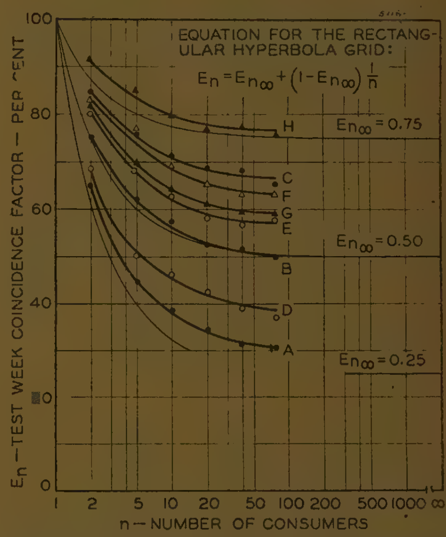

Figure 1. Probable trend in relation between group coincidence factor and number of consumers in group

Based on one-half-hour integrated demands in December reflecting 1938 and 1939 test data, and for a range in average of consumers' monthly load factors, as follows:

Domestic, without electric ranges:
A—12-per-cent load factor
B—23-per-cent load factor
C—31-per-cent load factor

Domestic, with electric ranges:
D—10-per-cent load factor
E—16-per-cent load factor

Retail light and power, industrial:
F—17-per-cent load factor

Retail light and power, commercial:
G—18-per-cent load factor
H—28-per-cent load factor

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but

$$\frac{1-\epsilon^{a-/f}}{(1+\epsilon^{-a/2f})^2} = \frac{1-\epsilon^{-a/2f}}{1+\epsilon^{-a/2f}} = \frac{\epsilon^{a/4f}-\epsilon^{a/4f}}{\epsilon^{a/4f}+\epsilon^{-a/4f}}$$

$$= \tanh\frac{a}{4f}$$

Therefore, by equations 4 and 7

$$F_f^2 = 1 - \frac{4f}{a}\tanh\frac{a}{4f} \qquad (8)$$

Calculation of Commutation Fault

A commutation fault occurs if t_0, the time for the current to reach zero after voltage reversal, is greater than $1/120$ second.

From equation 5

$$i = 0 = \frac{V}{R_f}\left[1 - \frac{2\epsilon^{-at_0}}{1+\epsilon^{-a/2f}}\right] \qquad (9)$$

For $t_0 > 1/120$ second, the condition of commutation fault,

$$\epsilon^{a/120}(1+\epsilon^{-a/2f}) < 2 \qquad (10)$$

or

$a < 84 \qquad F_{60} < 0.22$

A fault occurs if inequality 10 holds.

Population Habits and Community and Business Practices

Population habits and community and business practices exert a powerful influence upon the degree of coincidence in the consumers' use of electric service. The difference in the time when people get up; when they go to bed; how, when, and what they eat; where they live; the holidays they observe; where, when, and at what they work; the amount of leisure they have; and, generally speaking, how they live, work, and play on different days of the week and in different seasons of the year; all of this is reflected in the degree of coincidence of consumers' use of electric service. These conditions are not subject to precise mathematical formulations, and their resultant effect will be different for different communities and different climatic, social, and political conditions. But for any given community, operating under a given set of social and political conditions, the resultant effects of population habits and community and business practices upon the manner of use of electric service usually fall into some general pattern and thus result in some determinable values of the degree of coincidence of individual consumer's use of service. These forces are generally too strong to be subject to control by individuals, although major policies of service-supply may have some minor influences upon them. They are, however, very susceptible to social and political controls, as, for example, by the general behavior of the community during a war as contrasted to its behavior in peacetime; by the artificial changes in the setting of clocks from the normal setting on sun time; by social and political revolutions and by the lack or the presence of regimentation of the people's lives.

Weather and Other Climatic Conditions

Weather and other climatic conditions also exert a powerful influence upon the degree of coincidence in the consumers' use of electric service. For example, the advent of severe darkness because of an oncoming storm during otherwise daylight hours may create a full coincidence of the power and lighting loads, wiping out the normal diversity between their individual maximum demands and thus increasing the coincidence factor. A severe cold spell causing the almost continuous operation of individual heating plants increases the extent of coincidence in the use of electric service in connection with such plants. A severe hot spell, causing an almost continuous operation of individual air-conditioning plants and other forms of cooling and ventilating equipment, increases the extent of coincidence in the use of electric service in connection with these appliances. These forces are beyond the control of individuals. They are not subject to exact mathematical determinations. But they are susceptible, in general, to appraisals from available weather records for average, maximum, and minimum conditions.

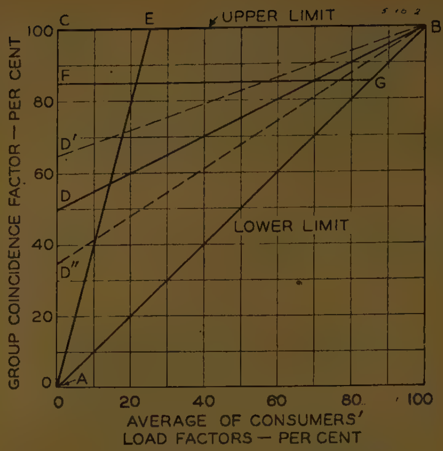

Figure 2. Illustration of the significance of certain lines on chart of coincidence factors versus consumers' load factors

Design of Utilization Equipment

The design of utilization equipment has a considerable influence upon the degree of coincidence of consumers' use of electric service. For example, generally speaking, to perform a given task through expenditure of a given amount of energy, devices which are "overcapacitied" may have lower coincidence factors than devices which are "undercapacitied," although the coincident maximum demand of the former may be higher than the coincident maximum demand of the latter. Devices designed for full automatic operation under intermittent cycles may have lower coincidence factors than those designed to accomplish the same purpose under manual operation. These design features are well within the control of design engineers, although it should be realized that the effects of different designs cannot be ascertained from laboratory tests, but must be obtained from extensive research and tests of sample applications in actual use.

Methods of Control

Methods for artificial control in the use of service also have important influences upon the degree of coincidence of consumers' demands. Generally speaking, a group of identical devices operating without any restrictive control will experience a smaller coincidence in their individual maximum demands than the same group of devices operating under a predetermined control cycle, even though the group of uncontrolled devices may possess features which are undesirable from other considerations affecting the system while the controlled devices may not have such undesirable features.

Notwithstanding the complexity of forces which affect the degree of coincidence in the use of electric service, experience has demonstrated the existence of two general phenomena relative to the behavior of group coincidence factors. One is the relationship between group coincidence factors and the number of consumers in a group, and the other, which only recently has received closer attention, is the probable relationship between group coincidence factors and average load factors of individual consumers of a group. These constitute the two basic coincidence-factor relationships of electric-service-load characteristics and are of fundamental significance in central-station service supply. They find applications in engineering and economic studies dealing with load forecasting for system planning, design of distribution systems, provision of service facilities, development of utilization appliances and applications, determination of cost to serve for widely different characteristics of service requirements, design of rates, and determination of service-sales-promotion policies for best economic results.

Coincidence Factor Versus Number of Consumers

In a general way the coincidence-factor-versus-number-of-consumers relationship may be formulated as follows:

For consumers of equal size, with no artificial restrictions placed in the way of their service use, and with all other things equal, the degree of coincidence of the maximum power requirements of a group of consumers decreases along a path resembling that of a rectangular hyperbola from unity for one consumer in a group to values approaching the asymptote of the rectangular hyperbola for an infinite number of consumers in a group.

Figure 1 depicts a mathematical grid of the simplest equation for a rectangular hyperbola and demonstrates that the results of actual test observations for various types of consumers of the domestic, commercial, and industrial classifications in general follow the mathematical relationships of the assumed empirical equation reasonably well. It also illustrates two very important conditions present in this relationship in practice: first, that the convergence of the relationship toward the asymptote is comparatively rapid for all values of $E_{n\infty}$ indicating that beyond 100 consumers in a group the change in coincidence factors is relatively small, and, secondly, that the convergence to the asymptote is more rapid for the higher values of $E_{n\infty}$ than for the lower ones.

The practical significance of these conditions is found in the fact that statistically stable results, as to diversified-load characteristics, may be expected in the practical range of $E_{n\infty}$ when as few as 100 consumers are involved in any one group, and results of sufficient engineering stability can be obtained even from groups of only 30 to 50 consumers for values of $E_{n\infty}$ of 20 per cent and higher.

Coincidence Factor Versus Load Factor

The subject of the probable behavior of coincidence factors of groups of consumers with a change in their individual load factors only recently received closer attention throughout the electric-utility industry. In the past there existed a feeling among students of this problem that, in a general way, the extent of coincidence in time of the load requirements of a group of consumers of electric service ordinarily increases as the load factors of the individual consumers of a group increase. But, in general, the practical nature of the relationship was not known. It was in 1937 that the author first demonstrated the existence of a probable relationship between coincidence factors and individual load factors of groups of consumers over the complete load-factor range.[1] Subsequently it has been studied, in a thorough manner, by a subcommittee (F. M. Terry, chairman)[2] and H. E. Eisenmenger rationalized and verified the shape of the author's original empirical curve, in a classical manner, from mathematical considerations.[3] The following discussion will elaborate upon some of the more important theoretical aspects of this subject and will expand upon the author's original findings in the light of new and additional information secured since the original demonstration of the relationship.

Theoretical Considerations

It is obvious that a group of consumers, each having a 100-per-cent load factor, has a coincidence factor equal to unity. Furthermore, it may be demonstrated that a group of many consumers, each having a load factor approaching zero, and being unrestricted in the manner of operation throughout the entire period under consideration, will have a coincidence factor which will approach the value of one divided by the number of consumers, and which will approach zero for small demand intervals and for a very large number of consumers in a group. Thus, with the two extreme values known, it may be expected that, for a large number of consumers and under natural conditions of load behavior, there will exist some probable relation which will connect these two points.

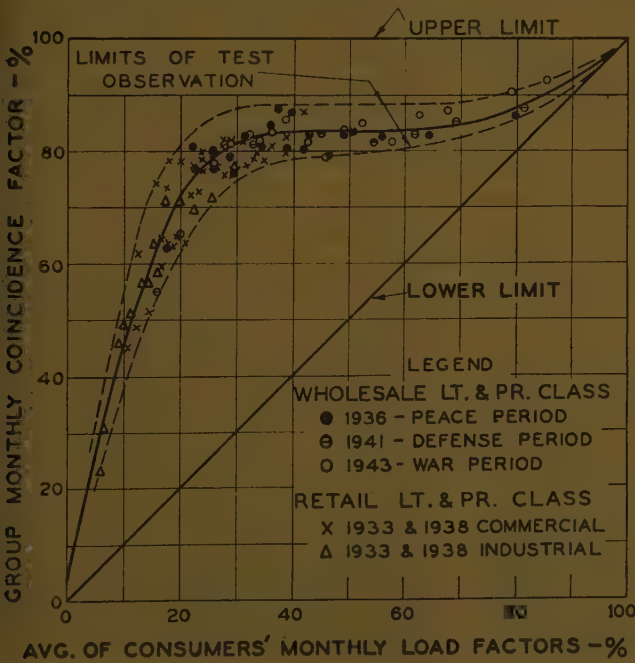

Figure 3. Empirical trend in the relation between coincidence factor and load factor

Based on one-half-hour integrated demands in December for groups of 30 consumers

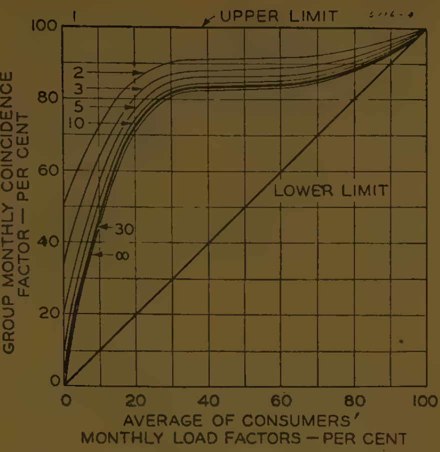

Figure 4. Empirical trend in the relation between coincidence factor and load factor for different numbers of consumers in group

Based on one-half-hour integrated demands in December

Numbers affixed to curves indicate number of consumers in group

It also may be demonstrated that under no condition can the value of the coincidence factor of a group of consumers, each having the same load factor, drop below the load factor, nor can it, by definition, exceed the value of unity. With these limits set, the probable relations should lie between these limits marked *AB* and *CB* in Figure 2. Theoretically, of course, for any given value of consumers' load factors, the group coincidence factor can have any value between these limits.

From theoretical considerations of an equal probability in the influence of the two limits upon the coincidence factor of a group of consumers with a given load factor, one may expect to find the path of the probable relationship to follow the line *BD* whose points for a given load-factor value will be equidistant from the limits of the possible relationships depicted by lines *AB* and *CB*. Straight lines emanating from *B* above or below line *BD*, for example, line *BD′* or *BD″*, produce relationships with unequal weights in the probability of the influence of the upper and lower limits upon a given group coincidence factor, the line above the line *BD* giving greater weight to the influence of the upper limit, and the lines below giving greater weight to the influence of the lower limit.

Actual experience has demonstrated, however, that the most probable relationships between group coincidence factors and individual consumers' load factors do not follow the straight lines depicting certain predetermined and constant relative weighing of the values of the upper and lower limits over the entire range of

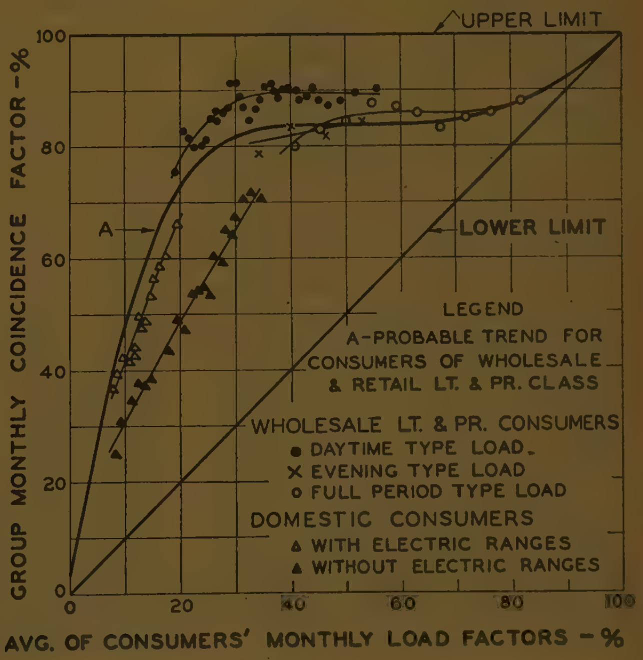

Figure 5. Comparison of empirical trends in the relations between coincidence factors and load factors of different classes of consumers

Based on one-half-hour integrated demands in December for groups of 30 consumers reflecting 1936 test data for the wholesale light and power consumers and 1939 test data for domestic consumers load factors. *The existence of the probable relationships can be obtained only from a large number of test observations of the actual behavior of loads of different classes of consumers.*

At this point it may be well to pause to consider the theoretical significance of certain lines and slopes depicted in Figure 2. It may be demonstrated that any straight line, such as *AE*, from the origin upward and above the lower limit line *AB*, represents the path of the coincidence-factor-versus-load-factor relationship for the theoretical conditions when the predominant and, therefore, the maximum use of service for a large group of consumers is confined to a certain portion of the entire period under consideration, and when all the changes in the consumers' load factors are accomplished through changes in energy use in that period. Hereinafter it will be referred to as the period of high load probability. The shorter this period, the steeper the slope of the straight line, so that, for example, line *AC* represents the shortest period or the full coincidence of maximum demands of all consumers in a group. Line *AB*, however, depicting the lower limit of the relationship or, as it may be called, the "best possible line," represents the longest period of high load probability as it applies to conditions when the loads of all consumers have the same probability of occurring throughout the entire period under consideration. The measure of the length of this period of high load probability in relation to the total period under consideration is obtained by projecting upon the abscissa the point of intersection of such a line as *AE* with the upper limit of the possible relationship indicated by line *CB*.

A horizontal line such as *FG* drawn from the ordinate or from any line such as *AE* toward the "best possible line" will be the path of the coincidence-factor-versus-load-factor relation when the change in the individual consumers' load factors is accomplished outside the original period of high load probability of the group. It should be noted, however, that as this period gets filled up with increased energy use of individual consumers, it in itself causes an extension of the original period of high-load probability until the consumers' load factors reach values at which their group coincidence factor intersects the "best possible limit," line *AB*, at which point the entire period under consideration becomes the period of high load probability.

EMPIRICAL RELATIONSHIP FROM TEST DATA

Figure 3 presents the results of research conducted by the author over a period of many years in establishing the general features of a probable relationship between coincidence factors and load factors of groups of consumers as determined from the service loads of one large electric utility system operating in metropolitan, suburban, and rural territories in the eastern part of the United States.

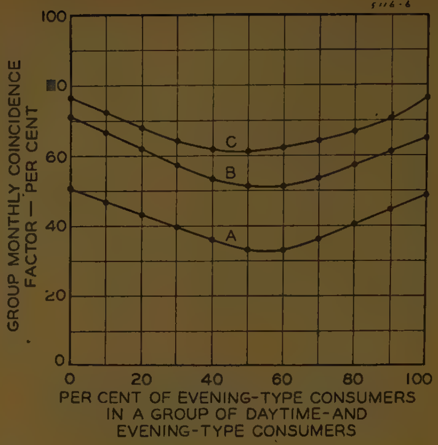

Figure 6. Illustration of effects of different mixtures of load types, for groups of 30 consumers, upon group coincidence factors of the retail light and power class reflecting 1938 test data

A—Consumers with 10-per-cent monthly load factors

B—Consumers with 20-per-cent monthly load factors

C—Consumers with 30-per-cent monthly load factors

The test observations were obtained generally for the month of December over a period of several years from integrated half-hourly load records and from a large number of consumers of several classes of the general service. They include test data for almost a decade before World War II, data during the defense period of this country and, finally, data at the height of this country's war-production activities. The procedure of analysis was uniform throughout. Consumers studied were arranged into groups, each falling into specified rather narrow load-factor limits. Each group consisted of 30 consumers, a sufficient number to produce representative results of the coincidence factors. The maximum load values of all consumers in each group were placed on the same relative-size basis either by grouping consumers with maximum demands of about the same magnitude, or by expressing each half-hour demand of a consumer in per cent of the consumer's maximum half-hour demand. Coincidence factors shown in Figure 3 were obtained from an analysis of individual consumer's daily load curves.

The relationship for the industrial and commercial consumers of the wholesale and retail classes of service, for which observations were available for practically the entire range of monthly load factors, starts out from a value close to zero* at a load factor approaching zero, rapidly increases along almost a straight line to a coincidence-factor value of about 75 per cent at 20-per-cent load factor, thereafter bends sharply to the right and rises slowly to 84 per cent at 40-per-cent load factor and then remains substantially fixed at that level of coincidence factor until reaching a load factor in excess of 70 per cent beyond which it turns up sharply and approaches the "best possible line" in the ultrahigh load-factor range.

Thus, from the foregoing theoretical considerations, the broad generalized behavior of the coincidence-factor-versus-load-factor relationship in practice appears to follow three distinct phases, although, as may be expected, the conditions obtained toward the end of the first phase merge with the conditions obtained at the beginning of the second phase, and the conditions obtained at the end of the second phase merge with the conditions obtained at the beginning of the third phase.

Phase one, obtained in the low-load-factor range, appears to indicate that, for a given classification of consumers, the high probability for the occurrence of the consumer's predominant energy use is confined to a certain well-defined portion of the entire period under consideration, and the predominant changes in the individual-consumer's-load factors are accomplished through changes in the extent of energy use almost entirely in that period.** Thus, in this load-factor region, the coincidence factor rises rapidly and almost in direct proportion to the rise in individual-consumer's-load factors. This phase of coincidence-factor-versus-load-factor behavior approximates that depicted by the theoretical line *AE* in Figure 2. Projecting upwards the portion of the empirical line which approaches a straight line from zero, we find it intersects the upper limit of the possible relationship at a point which corresponds to an individual consumer's monthly load factor of about 22 per cent, indicating that the length of the period of high-load probability is 22 per cent of the total period under consideration. Since the entire period is the month of December, the length of the high load probability thus becomes about 164 hours, or slightly more than four weeks of 40 working hours each.

Phase two, obtained in the medium-load-factor range appears to indicate that beyond individual consumers' monthly load factors of 30 per cent, a further improvement in their load factors is accomplished almost entirely *outside* the period of the group's original high load probability. Thus in this load-factor range the coincidence factor remains substantially fixed, and its behavior with respect to load factor approximates that depicted by the theoretical line *FG* in Figure 2. This phase of the relationship was of utmost significance in appraising the combined load requirements of industrial consumers during this war as a result of substantial changes in the industrial-consumers' load factors through the addition of the second and third shifts and through the extension of the working periods into Saturdays, Sundays, and holidays. Actual experience through this war period, and as indicated by the test points in Figure 3, substantiate the fundamental nature of the empirical relationship developed from actual test observations before the war.

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* For 30 consumers in a group used in this study, the starting point at load factor approaching zero is actually 1 ÷ 30 = 0.0333.

** Actually, minor changes must be occurring outside of this period because the empirical relationship as determined by test observations in the low load-factor range is not strictly a straight line, bu is a curve sloping toward the lower limit of the possible relationship, thus indicating the presence of small horizontal component, in addition to a majo upward one.

Phase three, obtained in the ultrahigh-load-factor range, appears to indicate that beyond an 80-per-cent monthly load factor the entire period under consideration becomes the period of high load probability for the occurrence of the consumers' predominant use of service so that further improvement in consumers' load factors are accomplished at random throughout this entire period. Thus in this ultrahigh-load-factor region the coincidence-factor-versus-load-factor relationship approaches the lower limit of the possible relationship or what may be called the "best possible line."

Present indications point to the fact that this general relationship is basic and fundamental over the entire range of load-factor values, but the numerical values obtained in practice will be different, not only for different classes of consumers, or different appliances and applications, but also for the same types of consumers and appliances in different communities.

The slope of the line in the low-load-factor range depends upon the relationship between the length of period of high load probability of the classification to the total length of the entire period under consideration. That relationship is determined largely by factors mentioned at the beginning of the paper such as population habits, community and business practices, weather and other climatic conditions, and, in addition, also depends upon the relative composition of the types of load represented by a given classification.

Based on the foregoing discussion it is now possible to formulate qualitatively the coincidence-factor-versus-load-factor relationship as follows:

> For a large number of consumers, of equal size, with no artificial restrictions placed in the way of their service use, and with all other things equal, the degree of coincidence of the maximum power requirements of individual consumers in a group changes with the change in their individual load factors. In the low-load-factor range the coincidence factor increases rapidly with the increase in the individual consumers' load factors. Its rate of change is substantially constant and is inversely proportional to the relative length of the high load probability of the class. In the medium range of load factors the magnitude of the group coincidence factor remains substantially fixed, until very high-load-factor values are reached, when the relation between load factor and coincidence factor approaches the lower limit of the possible relationship.

Care should be exercised in the interpretation and the use of this relationship and the following qualifications must be borne in mind:

1. It is an empirical relation and is based on the conditions and experience of one utility system supplying a given community. Other communities with different population habits, different definition as to what constitutes a class of service, different weather and other climatic conditions, may differ in the actual magnitudes of the coincidence factors shown throughout the entire load-factor range.

2. The relation is confined to consumers of substantially the same size, taking the same class of service, operating at the same load factors, and always taken in sufficient numbers for each type, size, and load factor to produce representative results on a coincidence factor of the group. It is obvious that, unless these qualifications are observed fully, different coincidence factors may be obtained.

3. The coincidence factors obtained from Figure 3 are those for individual consumers within a group applied to monthly conditions. There will be additional coincidence factors between different groups of any one class of service, for longer periods than a month, and between different classes of service of a system.

It should be noted also that the foregoing empirical relationship depicted the coincidence factors which, when applied to the consumers' individual maximum monthly demands, would, by definition, produce the coincident maximum demand of the group without indicating its time occurrence. Additional information is required for translating these coincident maximum group demands to load values for different hours of the day or for different days of the month. Such information can be secured only through extensive study of load-curve shapes of individual groups or classes of consumers.

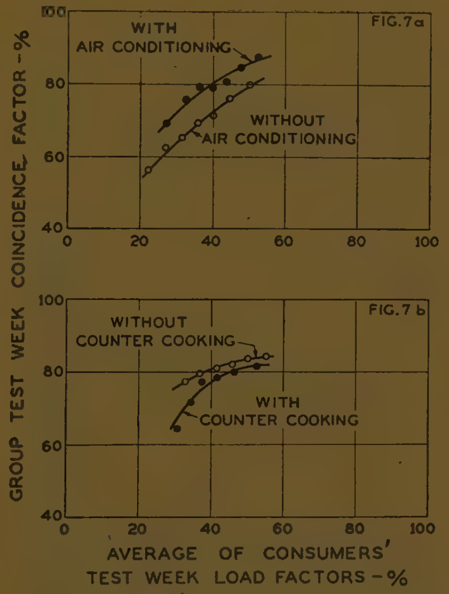

Figure 7. Illustration of the effect of the addition of certain appliances upon the coincidence factors of consumers of the commercial class reflecting 1940–1941 test data

Effect of Number of Consumers on the Coincidence-Factor-Versus-Load-Factor Relationship

The results of practical observations reflected in Figure 3 are based upon 30 consumers in a group. Obviously, for one single consumer in a group, by definition the coincidence factor is unity no matter what the load factors, and for an infinite number of consumers, from Figure 1, the coincidence factor may be expected to be lower than for groups of 30 consumers, throughout the entire load-factor range.

Referring to Figure 1, it is possible to construct a set of curves depicting the coincidence-factor-versus-load-factor relationships for various numbers of consumers in a group ranging from one to infinity. These relationships are shown in Figure 4, employing the equation of Figure 1 and the coincidence-factor-versus-load-factor relationship of the commercial and industrial classification shown in Figure 3. It should be noted that these relationships too, insofar as numerical values are concerned, are subject to the same qualifications as the relationship depicted in Figure 3.

The Influence of Different Classes of Consumers, or of Consumers With Different Types of Loads, Upon the Coincidence-Factor-Versus-Load-Factor Relationship

Figure 5 shows the coincidence-factor-versus-load-factor relationship for domestic consumers without electric ranges, and for domestic consumers with electric ranges, in comparison with the relationship for consumers of the commercial and industrial classification shown in Figure 3.

It will be noted that the relationships for the domestic consumers follow the general trend of the commercial and industrial consumers in the lower-load-factor range, although at a lower level. No test observations were available for these classes of consumers in the medium- and high-load-factor ranges, although it is believed that the extended relationship beyond the available test observations will merge with the horizontal portion of the relationship for the commercial and industrial consumers and thereafter follow it to the lower limiting value. The slope of the relationship for the domestic-service class without ranges is lower than that for the commercial–industrial classification. This would indicate that the period of high load probability for the domestic consumers is relatively longer than for the commercial and industrial ones, since the use period of domestic consumers without electric ranges can extend easily to twice the number of hours per week than that of consumers of the commercial and industrial classes whose individual load factors correspond to those of the domestic consumers, especially since Saturdays, Sundays, and holidays are days of fairly high domestic activity insofar as the use of electric service is concerned. For some of the appliances, the electric refrigerator for example, the entire period under consideration becomes the period of high load probability. However, the difference in the coincidence factors of the domestic and commercial classes of service practically disappears with the introduction of the electric range load in the domestic class because the period of high load probability of the range, with its predominant load requirements, is confined to much shorter periods during the day.

Figure 5 also shows the individual coincidence-factor-versus-load-factor relationship for three major types of consumers in the wholesale light and power classification, in comparison with the general relationship for the commercial and industrial consumers. For purpose of this comparison a classification depending upon the period of maximum average use of energy was employed. The daytime-type consumer was defined as one having maximum average use between 9 a.m. and 4 p.m.; the evening type as having the maximum average use between 4 p.m. and 9 p.m.; and the full-time type as having about an equal use in each of these periods with a resulting high load factor. This classification of load types caused the industrial consumers to fall principally into the daytime and full-time groups, whereas the commercial consumers fell principally into the evening group.

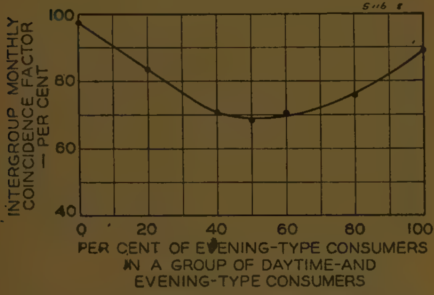

Figure 8. Illustration of the effect of different mixtures of load types upon the intergroup coincidence factor of the retail light and power class reflecting 1938 test data

It will be noted that the coincidence-factor-versus-load-factor relationships for individual types of consumer loads of a given classification follow the general trends of the formulated relationship, but the numerical values of the coincidence factors of the individual types are generally higher than those of the entire class.

Figure 6 illustrates the effect on group coincidence factors of a different mixture of load types for given consumers' load factors. The curves on this chart show the variation in group coincidence factors of a mixed group of thirty consumers with variation in the degree of mixture of daytime and evening load types, averaging 10-per-cent, 20-per-cent and 30-per-cent load factors. They bring out the fact that higher coincidence factors are obtained for groups of consumers homogeneous as to load type than for heterogeneous groups, and the greatest deviation between homogeneous and nonhomogeneous groups is found at the lower load factors.

Figure 7 shows the coincidence-factor-versus-load-factor relationships for two special types of commercial consumers. From Figure 7a it will be observed that the coincidence-factor-versus-load-factor relationship for commercial consumers *with* air conditioning is higher than for the same consumers *without* air conditioning. The difference is substantial in the range of consumers' load factors commonly encountered in practice (between 20 per cent and 40 per cent).

From Figure 7b it will be observed that the coincidence-factor-versus-load-factor relationship for commercial consumers *with* electric counter cooking equipment is lower than for the same consumers *without* the counter cooking equipment, indicating the beneficial effects of this load upon consumers' load characteristics. It should be noted, however, that one of the reasons for this is due to the fact that, for the consumers tested, the relative size of the connected electric counter cooking load did not exceed three times their other connected load.

Intergroup Coincidence Factors

It is apparent that with any grouping of consumers there will exist, in all probability, an additional factor of coincidence between the groups. In general, when consumers are subdivided into small groups, the degree of coincidence in consumers' use of service of individual groups will be greater than when they all are combined into one big group. In addition, when, as a result of the difference in the average load factor of each group, there is a difference in the load-curve shapes of the various groups, an additional coincidence factor between groups becomes a practical certainty.

The intergroup coincidence factor may be defined as the ratio of the actual coincident maximum demand of all consumers of a given class to the sum of the diversified demands obtained by the application of the coincidence-factor-versus-load-factor relationship to the individual consumers' load characteristics.

The intergroup coincidence factors for consumers homogeneous as to load typ may be expected to be higher than fo the entire group of consumers of divers types of load. Figure 8 illustrates th trend in the intergroup coincidence factor for various combinations of daytime and evening-loading-type consumers bringing out the point that higher intergroup coincidence factors will be obtainec for groups with highly homogeneous type loads, than for groups with comparatively high heterogeneous load types.

From the indicated curves of Figures 3 4, and 5, and actual test-load-curve data the intergroup coincidence factors were found to vary from between 93 per cent and 100 per cent for the commercial and industrial classes of consumers, and from 92 per cent to 95 per cent for consumers of the domestic classification with anc without electric ranges.

Conclusion

From the foregoing discussion, it i possible to make the following significan observations:

1. The degree of coincidence in the use o electric service depends upon many factors Among the more important ones are population habits, community and business practices, weather and other climatic conditions design of utilization equipment, and methods used in the control of service.

2. Numerical values of coincidence factors will differ for different load applications, different classes of consumers, and different composition of load types in a given classification. They can be obtained with any degree of certainty only from actual tes observations, and must be checked periodically to detect the effects of changing conditions. In view of the complexity of forces influencing group load characteristics, it i obvious that no single mathematical formula with a given set of constants can be right for all conditions. Mathematical considerations can serve only as a means for rationalizing the results of practical observations.

3. In general, the degree of coincidence in the maximum requirements of electric service of a group of consumers homogeneous as to some controlling characteristic decreases along a path resembling that of a rectangular hyperbola from a value of unity for one consumer in a group to values approaching the horizontal asymptote of the rectangular hyperbola for an infinite number of consumers in a group. The convergence of the relationship toward this asymptote is comparatively rapid for all asymptotic values of coincidence factors, and i more rapid for the higher values than fo the lower ones.

Statistically stable results in coincidence factors may be expected in the practica range of load factors for as few as 100 consumers in a group and results of sufficien engineering stability even can be obtainec from groups of 30 to 50 consumers.

4. In general, the degree of coincidence i the maximum requirements of electri service of groups of consumers changes witl the change in their individual load factors

Synchronous Starting of Generator and Motor

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A SYNCHRONOUS motor, generator, or condenser can be started from rest, by means of shaft torque, induction-motor (amortisseur winding) torque, or synchronizing torque. This paper presents data on the last method of starting. The system studied consists of a prime mover driving a synchronous generator, and a synchronous machine driven electrically by the generator. The whole system is started from rest by means of torque applied to the generator shaft, the motor being accelerated to normal speed by the synchronous torque developed. The performance of such a system was determined both by tests on several machines and by calculations with the aid of the differential analyzer at the University of Pennsylvania, and limiting conditions and criteria for successful starting were found in terms of the various pertinent system parameters. In a few cases a system consisting of a generator and two motors was studied.

Synchronous motors and condensers ordinarily are started by means of the induction-motor torque developed by their amortisseur windings. In some cases, however, for example when a generator is used as a condenser, it becomes necessary to start the machine in some other way. A salient-pole generator may have no amortisseur winding or one of insufficient torque and thermal capacity. A turbine generator will, in general, develop a large starting torque but will have insufficient thermal capacity. In many such cases the machine has been started by electrically connecting the motor and generator with both machines at rest and excited, then starting the generator by applied turbine torque and pulling up the motor by the synchronous torque developed at very low speeds. This method has been used for many years in the field testing of water-wheel generators and in miscellaneous shop tests. In case of extremely large motors with high starting torques, it may be convenient to arrange the electric-system connections so that a generator may be assigned the job of starting the motor on its synchronizing torque, thus permitting a reduction in the required capacity of the amortisseur winding.

The conditions required for successful starting in this manner have been studied in an approximate way by several investigators[1,2] and certain rules have been developed. However, it is believed that the problem is sufficiently complex and of sufficient general interest to justify the presentation of a more exact treatment, particularly since this more exact treatment has been found necessary to predict starting performance correctly in the range studied.

The problem is very similar to that of synchronizing or pulling-into-step of synchronous motors at normal speed[3,4] except that now the performance of two or three machines must be considered simultaneously, and the speed range is relatively very much greater.

Conclusions

From the results of the tests and calculations discussed in this paper, the following conclusions can be drawn regarding starting performance in the range covered:

1. The optimum ratio of motor excitation to generator excitation is such that the ratio of the normal speed, open circuit voltages is about 0.8. That is, at this ratio the maximum motor friction torque can be started (Figure 1). In general, the gener-

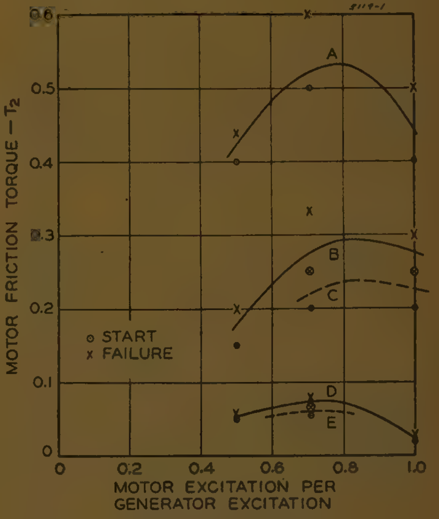

Figure 1. Motor and generator starting from standstill with applied generator shaft torque

Maximum motor friction torque T_2 which can be started, as function of excitation ratio E_2/E_1. $H_2=2.53$, $E_1=1.3$

Curve	r	x	H_1
A	0.01	1.0	5.05
B	0.025	1.0	5.05
C	0.025	1.0	1.26
D	0.05	1.0	5.05
E	0.05	0.5	5.05

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In the low load-factor range the coincidence factor increases rapidly with the increase in the individual consumers' load factors. In the medium range of load factors its magnitude remains substantially fixed at a level of around 84 per cent until very high load-factor values are reached when it approaches the lower limit of the possible relationship.

5. The general level in the degree of coincidence in the maximum requirements of electric service are higher for groups of consumers homogeneous as to load type than for heterogeneous groups, and large deviations between homogeneous and nonhomogeneous groups are found at the lower load factors than at the higher ones.

6. Intergroup coincidence factors for homogeneous-type loads may be expected to be higher than those for diverse types of load.

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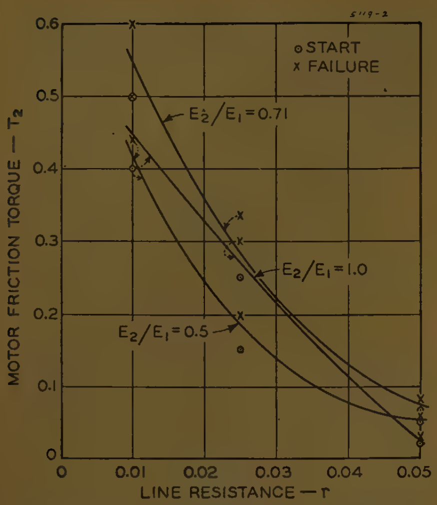


Figure 2. Motor and generator starting from standstill with applied generator shaft torque

Maximum motor friction torque T_2 which can be started, as function of circuit resistance r.
$H_1=5.05$, $H_2=2.53$, $x=1.0$

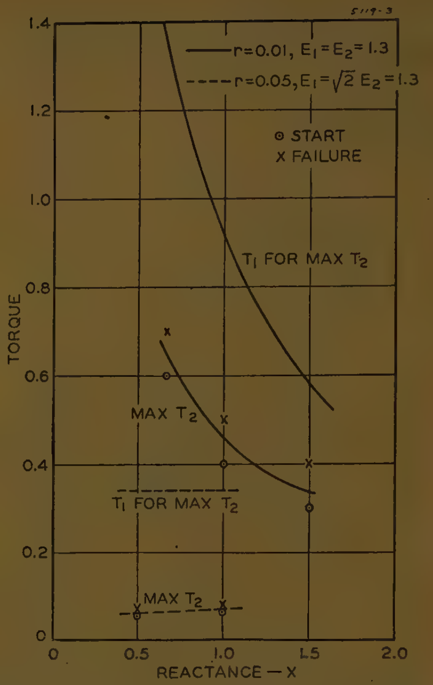


Figure 3. Motor and generator starting from standstill with applied generator shaft torque

Maximum motor friction torque T_2 which can be started, as function of circuit reactance X.
$H_1=5.0$, $H_2=2.55$

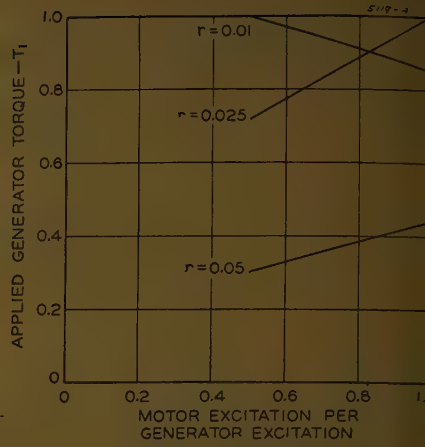


Figure 4. Motor and generator starting fron standstill

Required generator shaft torque T_1 to obtai maximum motor friction torque T_2, as functio of excitation ratio E_2/E_1
$H_1=5.05$, $H_2=2.53$, $x=1.0$, $E_1=1.3$

ator excitation should be as great as possible.

2. Considerable motor torque can be developed by synchronous starting when the total armature-circuit resistance is low, but high resistance reduces the maximum allowable motor friction torque (T_2) very much. With line resistances greater than five per cent, T_2 becomes less than seven per cent (Figure 2).

3. With small line resistance r ($r=0.01$) the maximum motor friction torque T_2 which can be started is practically inversely proportional to line reactance x, while with high r ($r=0.05$) T_2 is nearly independent of x (Figure 3).

4. The initial angle between motor and generator field axes has little effect on the maximum motor friction torque T_2 which can be started, but has a very great effect on the maximum torque T_1 which can be applied to the generator for successful starting with small T_2.

5. Neither motor nor generator inertia have very much effect on the starting performance. In general, low motor inertia and high generator inertia are slightly beneficial.

6. For each circuit condition there is an optimum value of applied generator torque which will start the maximum motor friction (Figures 3 and 4). For torques smaller than the optimum value, T_2 is approximately proportional to T_1; it is, in general, somewhat greater than the proportional value. For torques T_1 greater than the optimum value, T_2 decreases rapidly to zero, and if T_1 is too large the generator will run away leaving the motor oscillating about zero speed.

7. It is desirable to have a fairly low-resistance amortisseur winding on both motor and generator if they are not of solid-pole-face construction.

8. When motor and generator are directly connected electrically, turbomachines should, in general, be easier to start than salient-pole machines.

Analysis

The systems considered are shown in Figure 5. In order to reduce the performance equations of these systems to practically soluble forms, it was assumed that:

1. All machines have symmetric rotors.

2. The effective machine excitations and reactances are constant during the starting period. This may be interpreted as meaning that the rotor flux is constant or that some average value is taken as representative.

3. Amortisseur effects are negligible.

The system equations for the case of two machines, in an integral form suitable for solution on the differential analyzer, are then (see appendix)

$$\left.\begin{aligned} xi_\alpha &= +E_1\cos\theta_1 - E_2\cos\theta_2 - \int ri_\alpha dt \\ xi_\beta &= +E_1\sin\theta_1 - E_2\sin\theta_2 - \int ri_\beta dt \\ M_1p\theta_1 &= -E_1\int \cos\theta_1 d\int i_\beta dt + E_1\times \\ &\qquad \int \sin\theta_1 d\int i_\alpha dt - \int T_1 dt \\ M_2p\theta_2 &= +E_2\int \cos\theta_2 d\int i_\beta dt - E_2\times \\ &\qquad \int \sin\theta_2 d\int i_\alpha dt - \int T_2 dt \end{aligned}\right\} \quad (1)$$

where

t = per-unit time
x = per-unit armature-circuit reactance
r = per-unit armature-circuit resistance
E = per-unit excitation
θ = rotor angle
M = per-unit rotor inertia
$M=4\pi fH$, where H is the inertia constant as conventionally defined in stability studies

$$H=\frac{0.231\times WR^2\times(\text{rpm})^2}{10^6 \text{ kva (base)}}$$

f = rated circuit frequency in cycles pe second ($f=60$ in all numerical examples
i_α, i_β = per-unit equivalent two-phase armature currents of machine one
T = per-unit load torque

Subscripts 1 and 2 refer to generato and motor, respectively. A schemati diagram of the required analyzer connections is shown in Figure 6.

With the motor (machine two) a standstill θ_2 is constant and the performance is determined by the first thre parts of equation 1. The motor torque i

$$\text{torque}=i_\beta E_2\cos\theta_2 - i_\alpha E_2\sin\theta_2 \quad (2$$

so the motor may be started (that is, th fourth equation may be connected in th circuit) when its torque becomes equal t the friction torque selected.

The procedure followed on the differential analyzer was as follows. For eac value of resistance, reactance, motor inertia, generator inertia, motor excitation generator excitation, and applied generator torque, the motions of the two machines were determined up to a poin where it was decided that the motor coul or could not be started successfully. Thi was done for several values of moto friction and initial motor–generator angl until the maximum friction torque whic could be started at any angle was found Occasionally this procedure was varie in detail by finding the limiting generato torque for a given motor torque. Figur 7 shows a typical limiting curve an illustrates the fact that the allowabl motor friction torque increases to a maximum and then decreases to zero as th applied (constant) generator torque i increased.

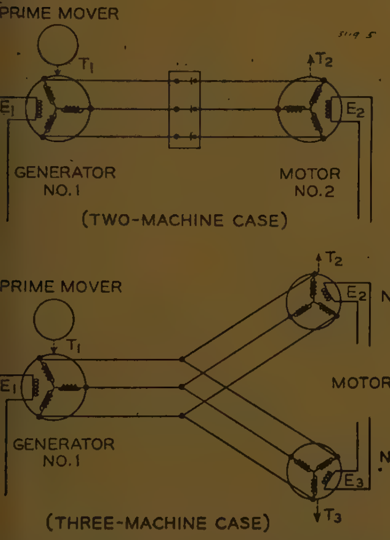
In most of the work done, the moto

iction torque was maintained at its atic breakaway value throughout the arting period. The friction torque ctually will decrease somewhat, but, for enerators with large numbers of poles, ie total mechanical angle through which ie motor has turned by the time it ither definitely has pulled in or failed to ull in is very small, since the total elec- rical angle is only two or three revolu- ions. For machines with two or four oles the results may be somewhat pessi- iistic in the sense that the calculations iay indicate a failure to start when a tart actually might be accomplished. n investigation of this point in a par- icular case showed about a 40-per-cent icrease in the allowable motor static riction, if it were dropped to zero at the istant of motor starting.

The generator excitation E_1 was 1.3 in early all cases. This was considered to orrespond to about the highest value of eld flux which could be maintained in a ormal machine without exceeding the apacity of the field winding. The motor xcitation E_2 was varied from 1.3 down to .65 since one of the principal objects of he study was to determine the optimum xcitation ratio as a function of the other ystem parameters. In order to inter- ret the results in terms of other gener- tor excitations, and, in general, to obtain he maximum amount of information rom the results, one may note that the ystem behavior may be expressed in erms of the six groups:

$$\frac{M_1}{E_1^2},\ \frac{xM_2}{E_1^2},\ \frac{xT_1}{E_1^2},\ \frac{xT_2}{E_1^2},\ \frac{r}{x},\ \frac{E_2}{E_1}$$

The attempt to start was taken as suc- essful if the motor speed exceeded the enerator speed at least twice. That is, n a successful start the motor usually vould start up and oscillate about the enerator. Since all rotor damping was neglected it sometimes was observed that these oscillations tended to increase in magnitude. A calculation for the condition of small oscillations about a constant average speed shows that in the speed range considered the system treated here is damped negatively. At very low speeds the damping is positive, but at between one per cent and two per cent speed the damping becomes negative, reaching a negative maximum at about two per cent speed and remaining negative but becoming continually smaller in magnitude thereafter. Calculations have shown that an amortisseur winding of normal design on both motor and generator is adequate to keep the damping positive at all speeds. Moreover, even if damping is negative over a certain speed range, the machines may not drop out of step unless the period of operation in the range is sufficiently long.



igure 5. Motor and generator starting from andstill with applied generator shaft torque

System diagrams

Calculations were made to find the effects of line resistance and reactance, excitation ratio, motor–generator initial electrical angle, inertias of motor and generator, and applied torque on the starting characteristics, that is, on the permissible motor starting friction. The total reactance was taken as 1.0 per unit, which should correspond approximately to the total transient reactance of the usual two-machine system. This reactance was varied in a few cases between 0.5 and 1.5. The total line resistance was varied from 0.01, which could correspond to that for two machines tied directly together, to 0.05, which allows for some tie-line resistance.

Depending on the initial displacement angle, the motor starts either forward or backward and may or may not synchronize if it is a border-line case. The worst initial angle of the motor relative to the generator was usually between −90 degrees and −180 degrees. In most cases only 90-degree intervals were checked, but in a few cases 45-degree intervals were considered. The effect of angle was not sufficiently great to make it necessary to consider smaller intervals.

Test Checks

In order to check the validity of the simplified equations used to represent the system, several tests were made. A 9,000-horsepower 18-pole vertical synchronous motor having a static friction of 17 per cent was started from a 20-pole horizontal machine of about equal size. The prime mover was a d-c motor and the system was started by closing the motor

Figure 6. Motor and generator starting from standstill with applied generator shaft torque

Differential-analyzer connection diagram

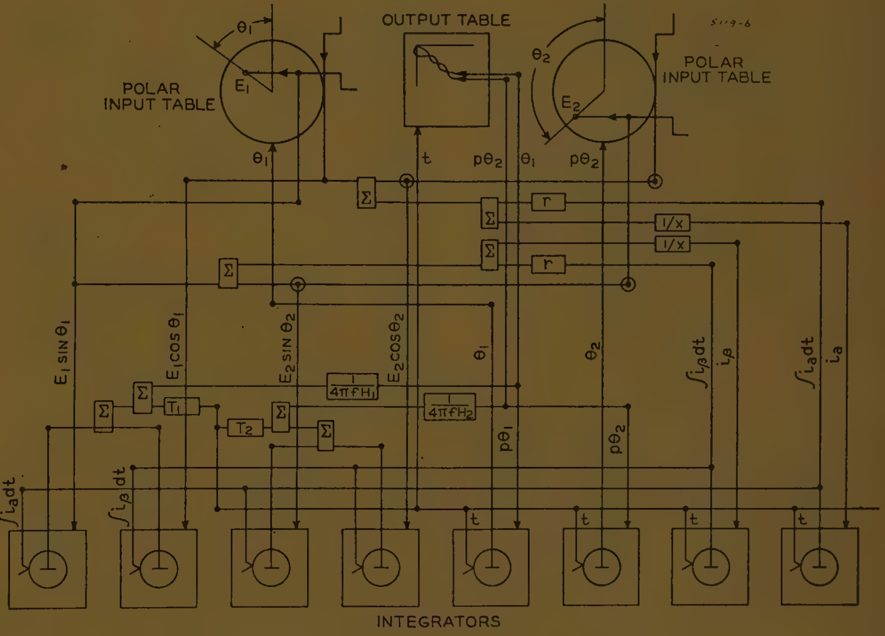
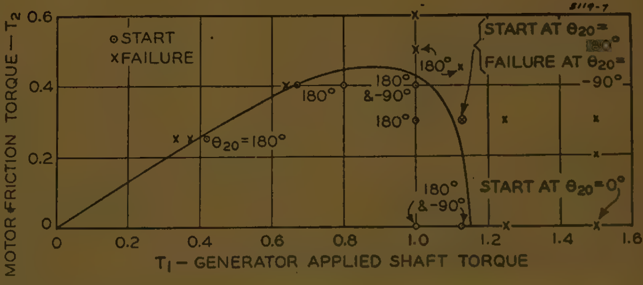
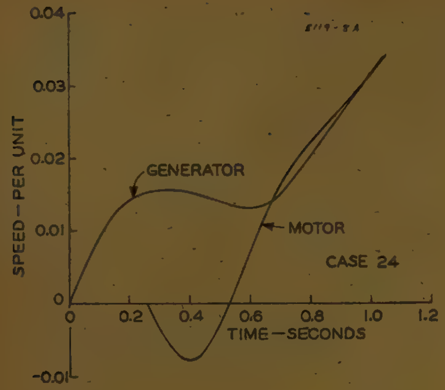


Figure 7. Motor and generator starting from standstill with applied generator shaft torque

Motor friction torque T_2 which can be started, as function of generator applied torque T_1

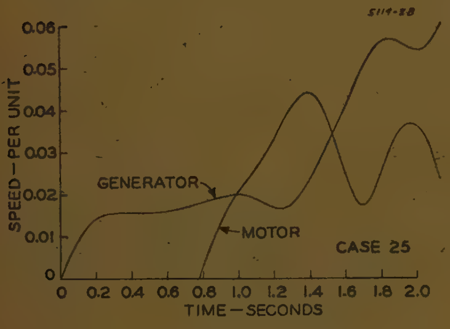
$H_1=5.0$, $H_2=2.55$, $r=0.01$, $x=1.0$, $E_1=E_2=1.3$





(A)

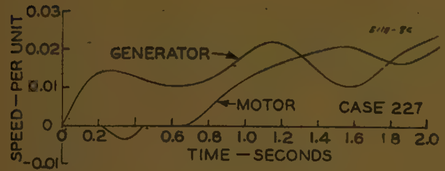
$T_1=1.0$ $T_2=0.41$ (Reduced to zero
$E_1=1.3$ $E_2=0.65$ when motor
$H_1=5.0$ $H_2=2.55$ starts)
$x=1.0$ $r=0.015$
$\theta_{20}=180$ degrees



(B)

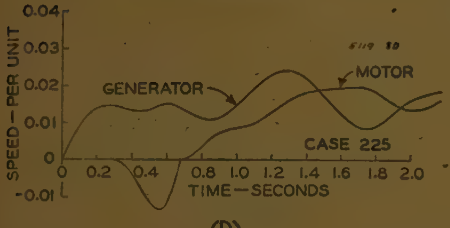
$T_1=1.0$ $T_2=0.648$ (Reduced to zero
$E_1=1.3$ $E_2=0.65$ when motor
$H_1=5.0$ $H_2=2.55$ starts)
$x=1.0$ $r=0.015$
$\theta_{20}=180$ degrees

(Compare with Figure 8A for effect of T_2)



(C)

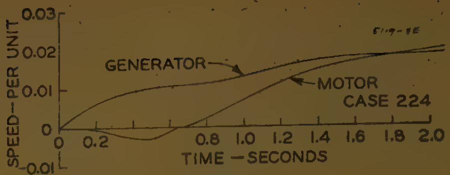
$T_1=1.0$ $T_2=0.5$
$E_1=1.3$ $E_2=0.92$
$H_1=5.0$ $H_2=2.53$
$x=1.0$ $\theta_{20}=180$ degrees $r=0.01$



(D)

$T_1=1.0$ $T_2=0.5$
$E_1=1.3$ $E_2=0.92$
$H_1=5.0$ $H_2=2.53$
$x=1.0$ $\theta_{20}=-90$ degrees $r=0.01$

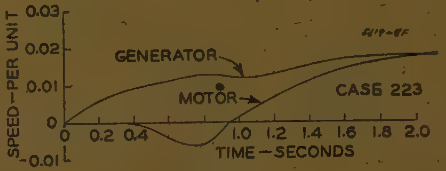
(Compare with Figure 8C for effect of θ_{20})



(E)

$T_1=0.375$ $T_2=0.0667$
$E_1=1.3$ $E_2=0.92$
$H_1=5.0$ $H_2=2.53$
$x=1.0$ $\theta_{20}=180$ degrees $r=0.05$

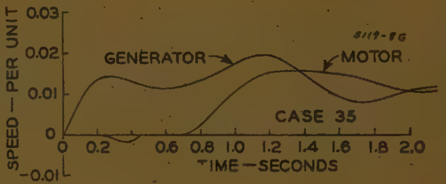
(Compare with Figure 8C for effect of r, and with Figure 8F for effect of θ_{20})



(F)

$T_1=0.375$ $T_2=0.0667$
$E_1=1.3$ $E_2=0.92$
$H_1=5.0$ $H_2=2.53$
$x=1.0$ $\theta_{20}=-90$ degrees $r=0.05$

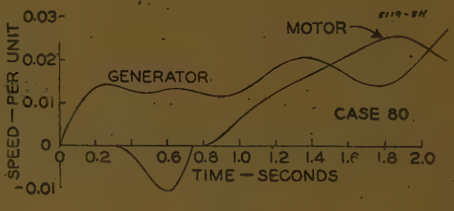
(Compare with Figure 8E for effect of θ_{20}, and with Figure 8D for effect of r)



(G)

$T_1=1.0$ $T_2=0.4$
$E_1=1.3$ $E_2=0.65$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $\theta_{20}=180$ degrees $r=0.01$

(Compare with Figure 8K for effect of E_2/E_1)



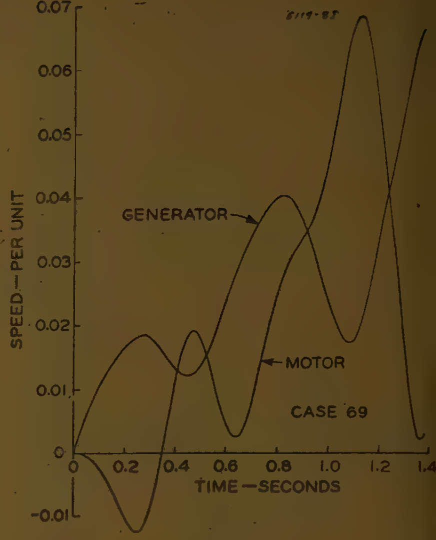
(H)

$T_1=1.0$ $T_2=0.4$
$E_1=1.3$ $E_2=0.65$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $\theta_{20}=-90$ degrees $r=0.01$

(Compare with Figure 8G for effect of θ_{20})

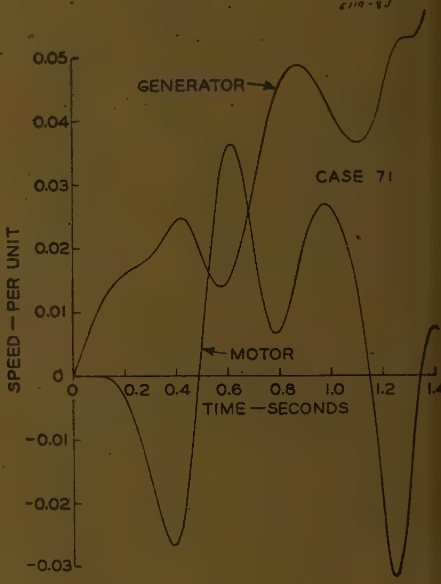
Figure 8. Motor and generator starting from standstill with applied generator shaft torque

Curves of speed versus time



(I)

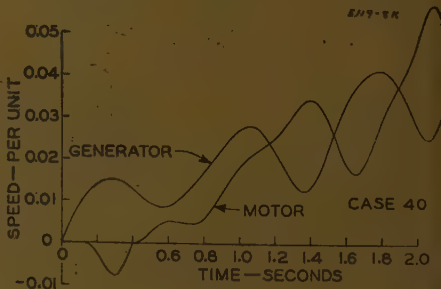
$T_1=1.125$ $T_2=0$
$E_1=1.3$ $E_2=1.3$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $\theta_{20}=180$ degrees $r=0.01$



(J)

$T_1=1.125$ $T_2=0$
$E_1=1.3$ $E_2=1.3$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $\theta_{20}=-90$ degrees $r=0.01$

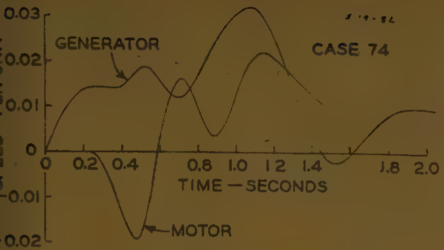
(Compare with Figure 8I for effect of θ_{20})



(K)

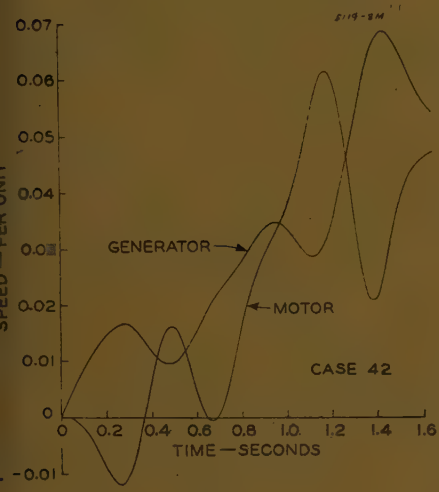
$T_1=1.0$ $T_2=0.4$
$E_1=1.3$ $E_2=1.3$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $\theta_{20}=180$ degrees $r=0.01$

(Compare with Figure 8G for effect of E_2/E_1



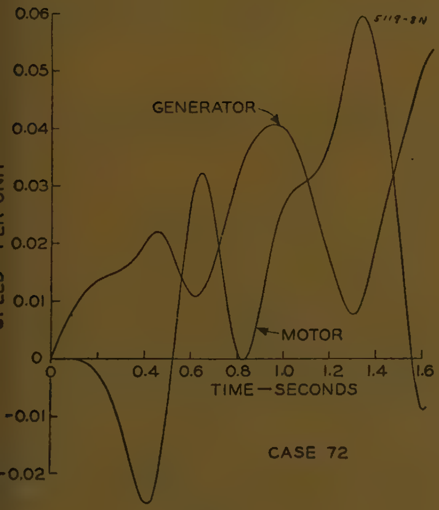
(L)

$T_1=1.0$ $T_2=0.4$
$E_1=1.3$ $E_2=1.3$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $r=0.01$
$\theta_{20}=-90$ degrees



(M)

$T_1=1.0$ $T_2=0$
$E_1=1.3$ $E_2=1.3$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $r=0.01$
$\theta_{20}=180$ degrees
(Compare with Figure 8K for effect of T_2)



(N)

$T_1=1.0$ $T_2=0$
$E_1=1.3$ $E_2=1.3$
$H_1=5.0$ $H_2=2.55$
$x=1.0$ $r=0.01$
$\theta_{20}=-90$ degrees
(Compare with Figure 8L for effect of T_2)

armature switch with field already applied. The torque thus was built up very rapidly and decayed slowly as the d-c motor accelerated. With excitation ratios of from 0.5 to 1.0 the lowest generator excitation (about 65 per cent) at which the a-c motor could be started was found.

The critical excitations for this system then were calculated on the differential analyzer and checked very closely. (See Table I.) There were, however, some differences between the test and analyzer conditions which are enumerated in the following.

In the calculation the motor friction torque was reduced to zero as soon as the motor started, while the friction of the test motor may be presumed to require some little time to fall to a completely negligible value. This indicates that the calculation may be really pessimistic since the consideration of an additional motor friction would have increased the critical excitations obtained. An increase of about five to ten per cent in excitation would be expected.

On the other hand, an armature resistance of $r=0.015$ was used in some of the calculations, while the test d-c resistance was closer to $r=0.01$. This indicates a somewhat optimistic calculation. The resultant decrease in critical excitation is shown by Figure 2 to be not more than about five per cent (torque varies as the square of excitation). Thus the two differences between calculation and test offset each other to a considerable degree. Moreover, the calculations at $E_2/E_1=0.5$ were made with $r=0.01$ and showed little deviation.

Other factors which should be considered are that the test machines had rather small field time constants, which would indicate that the calculations would be more pessimistic for larger machines, and rather effective amortisseurs, which would indicate that the calculations might be somewhat optimistic for machines with little or no amortisseur effect. The machines were represented by their transient reactances in the calculations.

Results and Discussion

The results obtained directly from the differential analyzer are in the form of curves of motor and generator speeds, and of the two components of armature current. The motor and generator angles also were determined in most cases. Figure 8 shows a few typical curves of motor and generator speed.

The information taken from these curves is summarized in Figures 1–4. Figure 1 shows the effect of excitation ratio and of line resistance. It is seen that regardless of line resistance the optimum excitation ratio is about 0.8, and that for the highest resistance ($r=0.05$) the permissible motor friction at a ratio of 1.0 is relatively much lower. For this high-resistance case an optimum ratio of 0.7 might apply, since it is possible to draw the curve in several ways through the check points.

Figure 2 is a replot of some of the curves of Figure 1 to show the effect of line resistance more directly. This curve brings out very clearly the importance of a low tie-line resistance if this method of starting is to be used on a motor having any very large breakaway torque, and also shows that if the tie resistance is kept down considerable torque may be developed.

Figures 3 and 4 show the generator torque T_1 which should be applied in order to start the maximum possible motor torque. These T_1 curves are only approximate, as the optimum value of T_1 is not very critical (see Figure 7). From Figure 7 it is seen that the optimum T_1 should not be greatly exceeded. Each point of the summary curves (Figures 1–4) was determined from curves similar to Figure 7, and each point of Figure 7 was, in turn, determined from curves similar to those of Figure 8.

Figure 9 shows the results of a calculation for the case of one generator starting two motors simultaneously, each of half the generator rating. One motor is at an initial angle of 90 degrees, the other at 180 degrees. If both motors are initially at 180 degrees, they may be considered as one. The starting performance is then the same as that of the two-machine case of Figure 10A, which is slightly worse than that of Figure 9. In general, it would seem that the two-machine case, with the motor at the worst angle, adequately should represent also the worst three-machine case. The case of Figure 10B differs only slightly from that of Figure 10A. It was worked out on the differential analyzer as an approximate check of the step-by-step calculations required for Figures 9 and 10A.

Appendix

In this appendix the performance equations for three electrically connected symmetric rotor machines will be derived first, since they may be of interest, not only to study, for example, the case of a single generator starting two motors simultaneously or of two generators starting a single motor, but also as an indication of the phenomena to be expected when several machines are so connected together. The equations for two machines studied in detail in this paper then will be derived as a special case.

Table I

E_2/E_1	E_1 (Calculated)	E_1 (Test)
0.5	0.65	0.64
0.67	0.64	0.63
1.0	0.60	0.58

The voltage equations for machine one are[5]

$$\begin{aligned} e_{\alpha 1} &= p\psi_{\alpha 1} - r_1 i_{\alpha 1} \\ e_{\beta 1} &= p\psi_{\beta 1} - r_1 i_{\beta 1} \end{aligned} \qquad (3)$$

Under the assumptions, the flux linkages are[5]

$$\begin{aligned} \psi_{d1} &= E_1 - x_1 i_{d1} \\ \psi_{q1} &= -x_1 i_{q1} \end{aligned} \qquad (4)$$

The *d*-, *q*-axis quantities are related to the α-, β-axis quantities by the relations[6]

$$\left.\begin{aligned} i_d &= i_\alpha \cos\theta + i_\beta \sin\theta \\ i_q &= -i_\alpha \sin\theta + i_\beta \cos\theta \end{aligned}\right\} \qquad (5)$$

with similar relations for ψ and e. Substituting equations 5 in equations 4, yields

$$\left.\begin{aligned} \psi_{\alpha 1} &= E_1 \cos\theta_1 - x_1 i_{\alpha 1} \\ \psi_{\beta 1} &= E_1 \sin\theta_1 - x_1 i_{\beta 1} \end{aligned}\right\} \qquad (6)$$

Now by substituting equation 6 in equation 3, the voltage equations become

$$\left.\begin{aligned} e_{\alpha 1} &= E_1 p \cos\theta_1 - (px_1 + r_1) i_{\alpha 1} \\ e_{\beta 1} &= E_1 p \sin\theta_1 - (px_1 + r_1) i_{\beta 1} \end{aligned}\right\} \qquad (7)$$

Similarly, for machines two and three

$$\left.\begin{aligned} e_{\alpha 2} &= E_2 p \cos\theta_2 - (px_2 + r_2) i_{\alpha 2} \\ e_{\beta 2} &= E_2 p \sin\theta_2 - (px_2 + r_2) i_{\beta 2} \end{aligned}\right\} \qquad (8)$$

$$\left.\begin{aligned} e_{\alpha 3} &= E_3 p \cos\theta_3 - (px_3 + r_3) i_{\alpha 3} \\ e_{\beta 3} &= E_3 p \sin\theta_3 - (px_3 + r_3) i_{\beta 3} \end{aligned}\right\} \qquad (9)$$

To connect the three machines together, one puts

$$\left.\begin{aligned} & e_{\alpha 1} = e_{\alpha 2} = e_{\alpha 3} \\ & e_{\beta 1} = e_{\beta 2} = e_{\beta 3} \\ & i_{\alpha 3} = -i_{\alpha 1} - i_{\alpha 2} \\ & i_{\beta 3} = -i_{\beta 1} - i_{\beta 2} \end{aligned}\right\} \qquad (10)$$

whence equations 7, 8, and 9 become

$$\left.\begin{aligned} (Ap^2+Bp+C)i_{\alpha 1} &= [(x_3+x_2)p + (r_3+r_2)]pE_1 \cos\theta_1 - (x_3p+r_3)\times \\ &\quad pE_2 \cos\theta_2 - (x_2p+r_2)pE_3 \cos\theta_3 \\ (Ap^2+Bp+C)i_{\beta 1} &= [(x_3+x_2)p + (r_3+r_2)]pE_1 \sin\theta_1 - (x_3p+r_3)\times \\ &\quad pE_2 \sin\theta_2 - (x_2p+r_2)pE_3 \sin\theta_3 \\ (Ap^2+Bp+C)i_{\alpha 2} &= [(x_3+x_1)p + (r_3+r_1)]pE_2 \cos\theta_2 - (x_3p+r_3)\times \\ &\quad pE_1 \cos\theta_1 - (x_1p+r_1)pE_3 \cos\theta_3 \\ (Ap^2+Bp+C)i_{\beta 2} &= [(x_3+x_1)p + (r_3+r_1)]pE_2 \sin\theta_2 - (x_3p+r_3)\times \\ &\quad pE_1 \sin\theta_1 - (x_1p+r_1)pE_3 \sin\theta_3 \end{aligned}\right\} \qquad (11)$$

where

$$\left.\begin{aligned} A &= x_1x_2 + x_2x_3 + x_3x_1 \\ B &= r_1(x_2+x_3) + r_2(x_3+x_1) + r_3(x_1+x_2) \\ C &= r_1r_2 + r_2r_3 + r_3r_1 \end{aligned}\right\} \qquad (12)$$

The torque equation of each machine is of the form

$$Mp^2\theta + Ei_q + T = 0 \qquad (13)$$

where T is the load torque. From equations 5 and 10, the three torque equations are

$$\left.\begin{aligned} & M_1p^2\theta_1 + E_1(i_{\beta 1}\cos\theta_1 - i_{\alpha 1}\sin\theta_1) + T_1 = 0 \\ & M_2p^2\theta_2 + E_2(i_{\beta 2}\cos\theta_2 - i_{\alpha 2}\sin\theta_2) + T_2 = 0 \\ & M_3p^2\theta_3 + E_3[(i_{\alpha 1}+i_{\alpha 2})\sin\theta_3 - (i_{\beta 1}+i_{\beta 2})\cos\theta_3] + T_3 = 0 \end{aligned}\right\} \qquad (14)$$

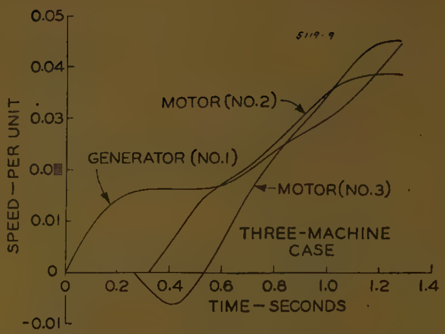


Figure 9. Two motors and generator starting from standstill with applied generator shaft torque

Speed versus time

$T_1 = 0.985$ $T_2 = T_3 = 0.20$*
$E_1 = 1.3$ $E_2 = E_3 = 0.65$
$H_1 = 5.0$ $H_2 = H_3 = 1.65$
$x_1 = 0.5$ $x_2 = x_3 = 1.0$
$r_1 = 0.008$ $r_2 = r_3 = 0.016$
$\theta_{10} = 0$ degrees, $\theta_{20} = 90$ degrees, $\theta_{30} = 180$ degrees

*T_2 and T_3 reduced to 0.008 when motors start

(Compare with Figure 10A for effect of combining the motors)

Although equations 11 and 14 may be put directly in an integral form for solution on the differential analyzer, the number of integrations required is large. They may be simplified with little loss in generality by making the ratios of resistance to reactance the same in all three machines. If

$$\frac{r_1}{x_1} = \frac{r_2}{x_2} = \frac{r_3}{x_3} = \rho \qquad (15)$$

then

$$B = 2\rho A \text{ and } C = \rho^2 A$$

and

$$Ap^2 + Bp + C = A(p+\rho)^2$$

The current equations 11 are reduced to

$$\left.\begin{aligned} A(p+\rho)i_{\alpha 1} &= (x_3+x_2)pE_1\cos\theta_1 - x_3pE_2\cos\theta_2 - x_2pE_3\cos\theta_3 \\ A(p+\rho)i_{\beta 1} &= (x_3+x_2)pE_1\sin\theta_1 - x_3pE_2\sin\theta_2 - x_2pE_3\sin\theta_3 \\ A(p+\rho)i_{\alpha 2} &= (x_3+x_1)pE_2\cos\theta_2 - x_3pE_1\cos\theta_1 - x_1pE_3\cos\theta_3 \\ A(p+\rho)i_{\beta 2} &= (x_3+x_1)pE_2\sin\theta_2 - x_3pE_1\sin\theta_1 - x_1pE_3\sin\theta_3 \end{aligned}\right\} \qquad (16)$$

Machine number three may be eliminated by letting $x_3 = \infty$, $i_{\alpha 1} = -i_{\alpha 2} = i_\alpha$, and $i_{\beta 1} = -i_{\beta 2} = i_\beta$. The first and last pair of equations 16 become identical and are represented by

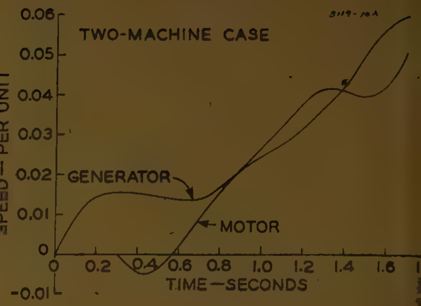
$$\left.\begin{aligned} (xp+r)i_\alpha &= p(E_1\cos\theta_1 - E_2\cos\theta_2) \\ (xp+r)i_\beta &= p(E_1\sin\theta_1 - E_2\sin\theta_2) \end{aligned}\right\} \qquad (17)$$

where

$$x = x_1 + x_2 \text{ and } r = r_1 + r_2$$

The torque equations 14 become

$$\left.\begin{aligned} & M_1p^2\theta_1 + E_1(i_\beta\cos\theta_1 - i_\alpha\sin\theta_1) + T_1 = 0 \\ & M_2p^2\theta_2 + E_2(i_\alpha\sin\theta_2 - i_\beta\cos\theta_2) + T_2 = 0 \end{aligned}\right\} \qquad (18)$$

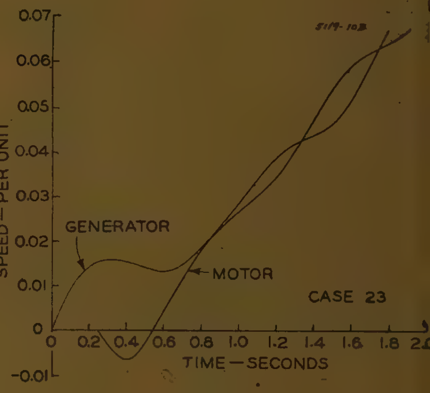


(A)

$T_1 = 0.985$ $T_2 = 0.4$*
$E_1 = 1.3$ $E_2 = 0.65$
$H_1 = 5.0$ $H_2 = 3.3$
$x = 1.0$ $r = 0.016$
$\theta_{20} = 180$ degrees

*T_2 reduced to 0.015 when motor starts

(Compare with Figure 9 for effect of two half size motors)



(B)

$T_1 = 1.0$ $T_2 = 0.409$ (Reduced to zero when motor starts)
$E_1 = 1.3$ $E_2 = 0.65$
$H_1 = 5.0$ $H_2 = 3.4$
$x = 1.0$ $r = 0.015$
$\theta_{20} = 180$ degrees

Figure 10. Motor and generator starting from standstill with applied generator shaft torque

Speed versus time

Equations 17 and 18 are integrated once to obtain equations 1.

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Biased-Core-Current-Transformer Design Method

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THAT peculiar improvement in performance often occurred when a current-transformer core was divided into two parallel magnetic circuits, with different turns on each, has been well known for years, and the reasons for it have been understood in a general way. The calculation of performance of the transformer, however, has been a most baffling problem—one that could be worked out only on a tedious trial-and-error basis. The lack of a practical and straightforward method has been a serious handicap to full use of the method of construction, despite strong evidence that this method of construction, which is both practical and simple, would produce real improvement in performance.

The difficulty of calculation lies in the fact that the flux densities in the cores are different by an amount depending on the ratio and phase angle between the primary and secondary currents and that the ratio and phase angle depend in turn on the two flux densities.

Solution depends on the conception of series magnetizing impedances for the two magnetic circuits, representing the impedance curve functions in such a way that simultaneous solution is not tedious. The principal purpose of the paper is to develop a method of calculation which is sufficiently simple, yet sufficiently accurate, to be used in engineering design.

The conventional current-transformer core operates at a relatively low flux density (less than 1,000 gausses), because only in this way can the ampere turns required to circulate the flux in the core be kept to a sufficiently low value for high accuracy of current transformation. Unfortunately, however, magnetic materials generally have a low permeability at small values of flux; thus, if the flux could be greater, the permeability would be higher. The idea of supplying auxiliary biasing ampere turns from an independent source to raise the total flux density to a value where the core would have good permeability was conceived long ago, but has been little used in practice, partly because of the complications involved in supplying the bias, but mostly because it is by no means clear that the effective permeability will be raised to any easily determined value.

A method invented by Wilson[1] in 1929 supplies bias to a very small element of the core simply by passing the secondary conductor through a hole in the core. The improvement in performance is difficult to calculate with any degree of exactness, and is not so great as that obtained when the whole core can be biased. A method described by Boyajian and Camilli[2] has the disadvantage of requiring auxiliary apparatus for excitation.

The general method to be described in this paper is illustrated by Figure 1. The core is divided into two separate magnetic circuits. The primary winding links both, but (for the general case) the secondary main windings and the secondary biasing windings are wound separately on each core. Then the main windings on the two cores are connected series aiding and the biasing windings series opposing—an arrangement that will give the minimum of coupling between the total main secondary winding and the total biasing winding. This can be seen more clearly from Figure 1.

It is not obvious that the biasing winding can produce a flux density corresponding to maximum permeability; nor is it obvious that this maximum permeability would be the effective permeability which would govern the performance of the transformer; nor is it clear how the bias can be practically obtained. These difficulties have prevented wide practical application of the idea, although many patents have been issued on various schemes for using the idea.

A practical and almost obvious method for using the biasing idea shown generally in Figure 1 is the double core with a single secondary winding shown in Figure 2, in which the bias is obtained by bringing the secondary conductor through between cores. The biasing ampere turns are then proportional to the secondary current. This construction is obviously practical, but the calculation of flux density in the two cores, the effective permeabilities, and the resulting performance of the current transformer, present a most tedious problem. The calculation process is not fundamentally obscure, but, like any problem in parallel magnetic circuits, solution by conventional methods is a cut-and-try procedure, is incredibly tedious, and is not practical from an engineering design standpoint. Thus, although a structure has been devised which is practical, economical, and promises real improvements in performance, it has been impossible to apply it to advantage, because no method of calculation has existed whereby performance could be calculated. It has been impossible to determine analytically the proper proportions of the cores and the best degree of bias.

This paper will develop the theory of a method for calculating performance. Since the arrangements of Figure 1 with the main and biasing windings in series, and of Figure 2 and Figure 4, to be used later for the actual development of theory, are all identical fundamentally, the theory will be equally applicable to any one of them.

Theory of Operation

On closer examination of Figure 1, the following facts are evident, which will assist analysis of the operation. The

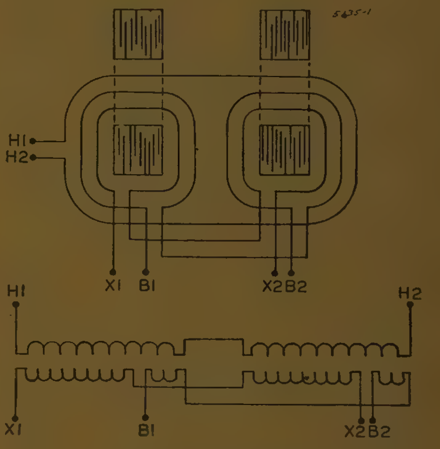

Figure 1. General construction of a biased-core current transformer

The primary winding *H1–H2* links both the cores. The main secondary windings are connected series aiding and are labeled *X1–X2*. The biasing secondary windings are connected series bucking and are labeled *B1–B2*

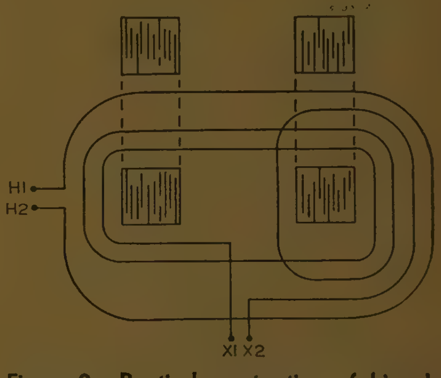

Figure 2. Practical construction of biased-core current transformer

Bias is secured from secondary current by having a few additional secondary turns around one core

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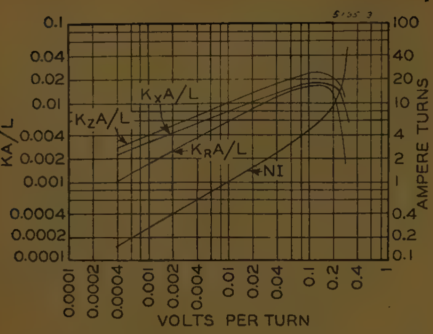

Figure 3. Nominal 60-cycle characteristic curves for the cores used in the tests and calculations

The ampere turns and the coefficients for the formulas for the resistance, reactance, and impedance of the equivalent series circuit for the core are plotted as functions of the volts per turn

main secondary windings in Figure 1 are connected in series aiding, and the secondary biasing windings are in series bucking; that is, with a voltage applied to the primary winding the sum of the voltages of the main secondary windings appears across the two in series, and the difference of the voltages of the biasing windings appears across the two in series.

At first it will be assumed that the main secondary turns on each of the cores are equal and that the biasing turns are so arranged that, when voltage is applied to the main secondary winding of the transformer, no voltage appears across the biasing winding; or conversely, when voltage is applied to the biasing-winding terminals, no voltage appears across the main secondary terminals. In practice this restriction may or may not be necessary, depending on the source for the biasing winding. If the coupling between the main and biasing windings is not zero, an error current will flow in the main secondary windings proportional to the current in the biasing winding. This error current can be used to compensate the transformer. If the biasing current does not vary with the main secondary current, this error current will become an increasingly large percentage of the main secondary current as the main secondary current decreases. If, however, the biasing current is made proportional to the main secondary current, the error current will remain a constant percentage of the main secondary current. Therefore, for the former condition it is important that the coupling be zero, whereas for the latter it is not so important.

If it is assumed that the coupling between the main and biasing windings is zero, the current in the biasing winding can be adjusted until the cores are biased at their point of maximum permeability without affecting the main secondary current. Var'ous methods of supplying the biasing winding are possible, but the most usual one is simply to put it in series with the main secondary winding. This puts an additional burden on the main secondary winding, but for a small current transformer this will be less than five volt-amperes. Inasmuch as this burden is largely reactive, it usually will reduce the phase angle of the current transformer.

In an actual transformer, the windings will be arranged in a different manner from that just described. The two cores are put end to end with a spacer between them. The larger portion of the winding is wound about both cores, and then a few additional turns are wound about one core only. See Figure 2. This arrangement then can, by properly proportioning the turns, be made exactly equivalent to the arrangement in Figure 1.

The Derivation of the Mathematical Theory

The double-core current transformer is difficult to analyze, because simultaneous solution of the flux densities in the two cores must be made, and, unless a simple mathematical expression for the exciting current in terms of the induction is available, the equations cannot be solved. Any number of trial-and-error methods have been devised, all of which have been found to be impractical on account of the amount of numerical work involved. The method given in the following analysis is based on the equivalent circuit of Figure 4 (see Table I for explanation of the symbols). The arrangement of the windings in Figure 4 is fundamentally the same as in Figure 2, since the turns linking each of the cores are the same in the two figures. With the arrangement shown in Figure 4 the equivalent circuit for the transformer is much more apparent, as Figure 4 is essentially two current transformers in series. In the equivalent circuit the primary as well as the secondary windings are connected in series. The exciting impedance for each core is shown external to its transformer. In this analysis it was found more convenient to use an equivalent series circuit consisting of a resistance and reactance to represent the exciting-current branch. The fundamental procedure based on Figure 4 is as follows:

(*a*). Estimate the approximate value of the volts per turn per square inch in the core as the sum of the zero-burden biasing volts per turn per square inch (obtained from the biasing ampere turns per inch and a curve) plus the volts per turn per square inch required for the burden.

(*b*). Determine the resistance and reactance of the equivalent circuit for the cores from a curve, and, assuming the resistance and reactance to be constant, recalculate the volts per turn per square inch in each core. The use of a curve is the key to a rapid solution of the problem.

(*c*). Continue recalculating until the volts per turn per square inch calculated is that first assumed. This then represents the actual operating condition of the transformer. Usually the second calculation is sufficiently accurate.

(*d*). Calculate the transformer ratio and phase angle at the final values of volts per turn per square inch.

The principal reason this method has merit is that an initial value quite close to the actual value of volts per turn per square inch can be determined and that the resistance and reactance of the equivalent circuit for the core do not change too rapidly with changes in volts per turn per square inch. Usually the second estimate is sufficiently close to the final value.

An equivalent series circuit for the core was found to be the most convenient for this analysis. To obtain the curve we must secure a general expression for the resistance, reactance, and impedance of the equivalent series circuit for the core in terms of the volts per turn per square inch at 60 cycles as follows.

The flux density in the core is given by

$$B=\frac{E}{NA}\cdot\frac{1}{4.44f\,10^{-8}}$$

and, if $f=60$ cycles,

$$B=3.75\times10^{5}\left(\frac{E}{NA}\right)\text{ lines per square inch} \tag{1}$$

The watts, volt-amperes reactive, and volt-amperes necessary for the core are

$$W=\left(\frac{W}{\#}\right)\delta lA \tag{2}$$

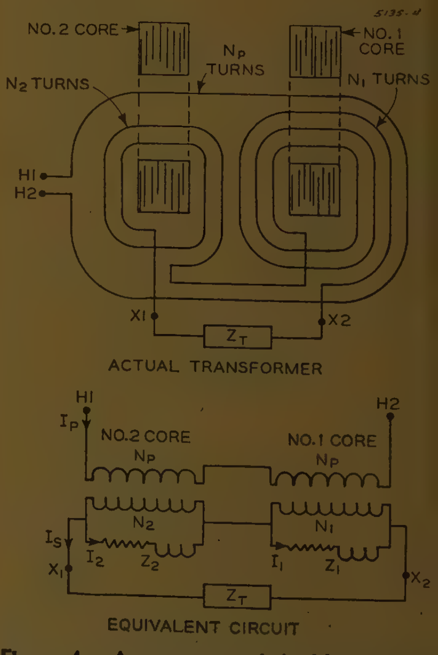

Figure 4. Arrangement of double-core current transformers used for development of the theory of calculation

The transformers are considered ideal, and the exciting branches, Z_1 and Z_2, are placed across the secondary windings of the transformers

$$VAR=\left(\frac{VAR}{\#}\right)\delta lA \qquad (3)$$

$$VA=\left(\frac{VA}{\#}\right)\delta lA \qquad (4)$$

The impedance of the equivalent circuit for the core is

$$Z=\frac{E^2}{(VA)}=\frac{E^2}{\left(\frac{VA}{\#}\right)\delta lA}\cdot\frac{N^2A}{N^2A}$$

$$Z=\frac{N^2A}{l}\left[\frac{\left(\frac{E}{NA}\right)^2}{\left(\frac{VA}{\#}\right)_\delta}\right]\text{ohms} \qquad (5)$$

The resistance component of this impedance is

$$R=Z\,\frac{W}{VA}$$

Substituting in this the values for Z, W, and VA from equations 5, 2, and 4, we find

$$R=\frac{N^2A}{l}\left[\frac{\left(\frac{E}{NA}\right)^2}{\left(\frac{VA}{\#}\right)_\delta}\right]\frac{\left(\frac{W}{\#}\right)}{\left(\frac{VA}{\#}\right)}$$

$$=\frac{N^2A}{l}\cdot\left[\frac{\left(\frac{E}{NA}\right)^2\left(\frac{W}{\#}\right)}{\left(\frac{VA}{\#}\right)^2_\delta}\right] \qquad (6)$$

The equation for the reactance component of Z can be developed in a similar manner and is

$$X=\frac{N^2A}{l}\cdot\left[\frac{\left(\frac{E}{NA}\right)^2\frac{VAR}{\#}}{\left(\frac{VA}{\#}\right)^2_\delta}\right] \qquad (7)$$

The quantities in the brackets in equations 5, 6, and 7 are only functions of the flux density in the core. For convenience they will be called K_Z, K_R, and K_X as follows:

$$K_R=\frac{\left(\frac{E}{NA}\right)^2\left(\frac{W}{\#}\right)}{\left(\frac{VA}{\#}\right)^2_\delta} \qquad (8)$$

$$K_X=\frac{\left(\frac{E}{NA}\right)^2\left(\frac{VAR}{\#}\right)}{\left(\frac{VA}{\#}\right)^2_\delta} \qquad (9)$$

$$K_Z=\frac{\left(\frac{E}{NA}\right)^2}{\left(\frac{VA}{\#}\right)_\delta} \qquad (10)$$

These constants, K_R, K_X, and K_Z then can be plotted in curve form. The equation for the impedance can be written

$$Z=\frac{N^2A}{l}\quad K_Z=\frac{N^2A}{l}\,(K_R+jK_X) \qquad (11)$$

$$K_Z=K_R+jK_X \qquad (12)$$

It will also be necessary to know the volts per turn per square inch induced as a function of the ampere turns per inch. This is developed as follows.

The exciting current for the core is

$$I=\frac{VA}{E}=\frac{\left(\frac{VA}{\#}\right)\delta lA}{E}$$

Solving equation 10 for $VA/\#$, substituting it in the above equation, and rearranging the terms gives

$$\frac{E}{NA}=K_Z\left(\frac{NI}{l}\right) \qquad (13)$$

It is also possible then to plot as a function of E/NA, the value of NI/l which, as will be explained, leads to solution of the problem.

In the equivalent circuit of the double-core current transformer as shown in Figure 4, it is assumed that the mean lengths of flux paths are equal. N_1 is greater than or equal to N_2, but A_1 can be greater than, equal to, or less than A_2. The transformers also are assumed to be ideal and their exciting impedances represented by shunt impedances external to the transformers. It is desired first to develop three sets of equations:

(*a*). For calculating the volts per turn per square inch in each core for values of K_Z corresponding to estimated values of volts per turn per square inch.

(*b*). For estimating the approximate values of volts per turn per square inch in each core for the first try in (*a*).

(*c*). For calculating the ratio and phase angle when the actual values of volts per turn per square inch have been determined.

From Figure 4, the following equations may be written

$$N_1(I_S+I_1)=N_2(I_S+I_2) \qquad (14)$$

Table I. 200-to-5-Ampere Current Transformer, Biased-Core Through-Type Construction

Burden: 2.5 *VA*, 90 Per-Cent Power Factor
Frequency: 60 Cycles

Data	Per-Cent Current	Per-Cent Ratio	Phase Angle, Minutes
*	100	100.17	+ 3
*	10	100.34	+13
**	100	100.06	+ 3
**	10	100.23	+13
†	100	100.07	− 1
†	10	100.17	+16
††	100	100.22	+ 9
††	10	100.45	+26

* Calculated values from nominal core data using method of this paper.

** Calculated values corrected for ratio instability, −0.11 per cent, due to cores with different characteristics.

† Test values of actual transformer as a biased current transformer.

†† Test values of actual transformer as an unbiased current transformer.

$$I_SZ_T=I_1Z_1+I_2Z_2 \qquad (15)$$

Solving equations 14 and 15 simultaneously for I_1 and I_2 yields

$$I_1=I_S\frac{N_2Z_T-(N_1-N_2)Z_2}{N_1Z_2+N_2Z_1} \qquad (16)$$

$$I_2=I_S\frac{N_1Z_T+(N_1-N_2)Z_1}{N_1Z_2+N_2Z_1} \qquad (17)$$

$E_1=I_1Z_1$. Substituting in this equation the value of I_1 from equation 16 and the values of Z_1 and Z_2 from equation 11, yields

$$\frac{E_1}{N_1A_1}=\frac{I_SK_{Z1}}{l}\left[\frac{lZ_T-(N_1-N_2)N_2A_2K_{Z2}}{N_2A_2K_{Z2}+N_1A_1K_{Z1}}\right] \qquad (18)$$

Similarly,

$$\frac{E_2}{N_2A_2}=\frac{I_SK_{Z2}}{l}\left[\frac{lZ_T+(N_1-N_2)N_1A_1K_{Z1}}{N_2A_2K_{Z2}+N_1A_1K_{Z1}}\right] \qquad (19)$$

With equations 18 and 19, if the values of K_{Z1} and K_{Z2} for the cores are known, the values of volts per turn per square inch can be calculated. It is necessary to know initially an approximate value for the volts per turn per square inch in each core so as to have some place from which to start. Equations 18 and 19 now will be modified, so that approximate values can be calculated.

If equations 18 and 19 now are rewritten, assuming that $K_{Z1}=K_{Z2}=K_Z$,

$$\frac{E_1}{N_1A_1}=\frac{I_SZ_T}{N_2A_2+N_1A_1}-\left[\frac{I_S(N_1-N_2)N_2A_2}{l(N_2A_2+N_1A_1)}\right]K_Z \qquad (20)$$

$$\frac{E_2}{N_2A_2}=\frac{I_SZ_T}{N_2A_2+N_1A_1}+\left[\frac{I_S(N_1-N_2)N_1A_1}{l(N_2A_2+N_1A_1)}\right]K_Z \qquad (21)$$

The first terms in each equation may be calculated now, but the last terms still contain K_Z. The last term in each equation is a function of the secondary current and the difference in turns between the two cores; or in other words, it is proportional to the biasing ampere turns. This can be shown as follows: rewriting equations 16 and 17, substituting in the values of Z_1 and Z_2 from equation 11, letting $K_{Z1}=K_{Z2}$, and multiplying the equations respectively by N_1/l and N_2/l gives:

$$\frac{N_1I_1}{l}=\frac{I_SZ_T}{(N_2A_2+N_1A_1)K_Z}-\left[\frac{I_S(N_1-N_2)N_2A_2}{l(N_2A_2+N_1A_1)}\right] \qquad (22)$$

$$\frac{N_2I_2}{l}=\frac{I_SZ_T}{(N_2A_2+N_1A_1)K_Z}+\left[\frac{I_S(N_1-N_2)N_1A_1}{l(N_2A_2+N_1A_1)}\right] \qquad (23)$$

If equations 22 and 23 are multiplied through by K_Z, they are identical to equations 20 and 21. That the left-hand terms also should be equal was shown by equation 13. The last terms of equations 22 and 23 then represent the approximate biasing ampere turns per inch for each of the cores. From equation 13 it is seen that the last terms of equations 20 and 21 then can be read from the curve where the volts per turn per square inch are a function of the ampere turns per inch. Equations 20 and 21 then may be rewritten in functional notation where the last term of each equation is read from the curve as a function of the ampere turns per inch.

$$\frac{E_1}{N_1A_1}=\frac{I_SZ_T}{N_2A_2+N_1A_1}-f\left[\frac{I_S(N_1-N_2)N_2A_2}{l(N_2A_2+N_1A_1)}\right] \qquad (24)$$

$$\frac{E_2}{N_2A_2}=\frac{I_SZ_T}{N_2A_2+N_1A_1}+f\left[\frac{I_S(N_1-N_2)N_1A_1}{l(N_2A_2+N_1A_1)}\right] \qquad (25)$$

For the first approximation it is sufficiently accurate if only the arithmetic sum of the two terms is taken.

With the values of E/NA, calculated by equations 24 and 25, the values of K_Z may be read from the curve and new values of voltage determined from equations 18 and 19. Sufficient calculations are made until no further significant changes in the voltages result. Usually only two calculations are required.

The actual transformer ratio then can be calculated when the correct values of voltage have been determined. The reversed primary current then is

$$N_pI_p=N_2(I_S+I_2) \qquad (26)$$

Substituting in the value of I_2 from equation 17 and using the values for Z_1 and Z_2 from equation 11 yields

$$N_pI_p=I_S\left[N_2+\left(\frac{lZ_T+(N_1-N_2)N_1A_1K_{Z1}}{N_2A_2K_{Z2}+N_1A_1K_{Z1}}\right)\right] \qquad (27)$$

This equation will reduce to the form:

$$N_pI_p=I_S(N_2+a+jb) \qquad (28)$$

where a is the real part of the last term of equation 27, and b is the imaginary part. Both a and b are usually small compared to N_2. If the name-plate ratio is npR, the transformer ratio is approximately

$$\text{per cent } R=\frac{N_2+a}{(npR)N_p}\times 100 \qquad (29)$$

and the transformer phase angle is

$$\text{phase angle}=-\frac{b}{N_2+a}\,3{,}438 \text{ minutes} \qquad (30)$$

The negative sign is introduced by virtue of the definition of phase angle. That is, the transformer phase angle is positive if the secondary current leads the reversed primary current.

The nominal per-cent ratio at zero burden and cores of equal characteristics is a useful equation for estimating the ratio. In equation 27 let $Z_T=0$ and $K_{Z1}=K_{Z2}$

$$N_pI_p=I_S\left[N_2+\frac{(N_1-N_2)N_1A_1}{N_2A_2+N_1A_1}\right] \qquad (31)$$

Hence, the nominal per-cent ratio is

$$\frac{N_2+\dfrac{(N_1-N_2)N_1A_1}{N_2A_2+N_1A_1}}{(npR)N_p}\,100 \qquad (32)$$

If the two cores in the transformer are better or worse than they are supposed to be nominally, they have a considerable effect on the ratio and produce a phenomenon we might call "ratio instability." With reference to equation 27 the per-cent ratio is

$$\text{per cent } R=\frac{100}{N_p(npR)}\times\left[N_2+\frac{lZ_T}{N_2A_2K_{Z2}+N_1A_1K_{Z1}}+\frac{(N_1-N_2)N_1A_1K_{Z1}}{N_2A_2K_{Z2}+N_1A_1K_{Z1}}\right]$$

The total derivative with respect to K_{Z1} and K_{Z2} is

$$\Delta \text{ per-cent } R=\frac{100}{N_p(npR)}\left[-\frac{lZ_T(N_2A_2\,\Delta K_{Z2}+N_1A_1\,\Delta K_{Z1})}{(N_2A_2K_{Z2}+N_1A_1K_{Z1})^2}+\frac{(N_1-N_2)N_1N_2A_1A_2(K_{Z2}\,\Delta K_{Z1}-K_{Z1}\,\Delta K_{Z2})}{(N_2A_2K_{Z2}+N_1A_1K_{Z1})^2}\right]$$

The only case considered will be that in which the first quantity in the brackets is small, which is so if either the burden Z_T or $N_2A_2\,\Delta K_{Z2}+N_1A_1\,\Delta K_{Z1}$ is small. Simplifying the last term of the foregoing expression by letting N_1, N_2, $N_p(npR)$ be about equal to each other, and K_{Z1} and K_{Z2} about equal to each other:

$$\Delta \text{ per cent } R=\frac{N_1-N_2}{N_1\left(2+\dfrac{A_1}{A_2}+\dfrac{A_2}{A_1}\right)}\times\left[\frac{\Delta K_{Z1}}{K_{Z1}}-\frac{\Delta K_{Z2}}{K_{Z2}}\right]100 \qquad (33)$$

From equation 10 at constant induction

$$\frac{\Delta K_Z}{K_Z}=-\frac{\Delta\left(\dfrac{VA}{\#}\right)}{\left(\dfrac{VA}{\#}\right)}.$$

$$\Delta \text{ per cent } R=\frac{N_1-N_2}{N_1\left(2+\dfrac{A_1}{A_2}+\dfrac{A_2}{A_1}\right)}\times\left[\frac{\Delta\left(\dfrac{VA}{\#}\right)_2}{\left(\dfrac{VA}{\#}\right)_2}-\frac{\Delta\left(\dfrac{VA}{\#}\right)_1}{\left(\dfrac{VA}{\#}\right)_1}\right]100 \qquad (34)$$

$\Delta\left(\frac{VA}{\#}\right)$ is the difference between the actual and the nominal volt-amperes per pound at some average point. $\Delta\left(\frac{VA}{\#}\right)\div\left(\frac{VA}{\#}\right)$ is not constant at all inductions. Equation 34 is not an exact equation but is meant only as a guide to how accurately the cores must be matched. It is also valid only for low secondary burdens or where $\Delta\left(\frac{VA}{\#}\right)/\left(\frac{VA}{\#}\right)$ for the two cores vary in opposite directions.

Another difficulty is immediately perceived; that is, as the diameter of the transformer is increased, N_1-N_2 must increase in order to keep the biasing flux density constant. This means that the nominal no-load ratio becomes even more erratic. The possibility of adjusting this no-load ratio by means of short-circuited turns of resistance wire on one core is entirely practical; and, if the unit is a bushing-type current transformer competing with a wound-type high-voltage current transformer, the expense in adjustment can be justified.

It has been postulated previously that, if the effective coupling between the main secondary winding and the biasing winding be zero, the biasing winding will act as an additional burden in series with the total secondary burden. If the following values are substituted in equation 27, they will fulfill the requirement for zero coupling:

$$N_1=N+n;\quad N_2=N-n;\quad K_{Z1}=K_{Z2}=K_Z;\quad A_1=A_2=A$$

where N is the main secondary turns per core and n the biasing turns per core. Figure 1 shows the arrangement assumed for this. The biasing winding is in series with the secondary.

$$N_pI_p=I_SN\left[1+\frac{Z_T+\dfrac{2n^2AK_Z}{l}}{\dfrac{2N^2AK_Z}{l}}\right] \qquad (35)$$

From equation 11 it is seen that n^2AK_Z/l would be the impedance of one of the biasing windings and twice that would be the impedance of the two in series. It is

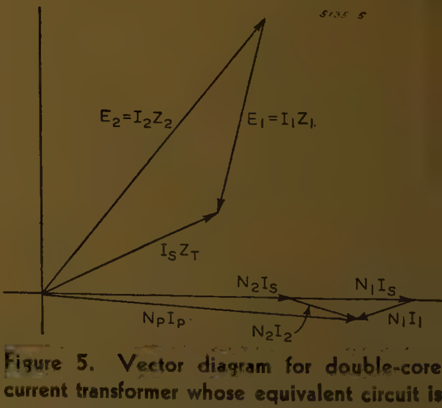

Figure 5. Vector diagram for double-core current transformer whose equivalent circuit is shown in Figure 3

E_1 and E_2 are the induced voltages of the current transformer secondary windings

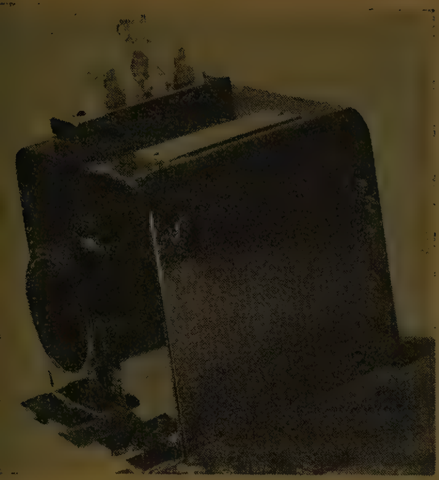

Figure 6. Type *FW* current transformer, 200-to-5-amperes ratio with double-core construction

This transformer has an accuracy class of 0.3 at a *B*-0.1 burden

to be noted that this impedance appears in equation 35 in series with the total secondary burden, Z_T. Also from equation 11, N^2AK_Z/l would be the impedance of one of the secondary main windings and twice that would be the impedance of the two in series.

For an ordinary current transformer with a series burden of Z_T' ohms, and a shunt exciting impedance of Z_X ohms, the ratio is

$$1+\frac{Z_T'}{Z_X} \tag{36}$$

If equation 35 is compared with equation 36, it is seen that the numerator of the last term of equation 35 corresponds to Z_T' of equation 36, the total series burden, and the denominator corresponds to Z_X, the shunt exciting impedance. It becomes apparent that the full permeability of the core at the point of bias is utilized, for K_Z is calculated on the basis of the a-c permeability at the point of actual operation. Also, it is apparent that the least variation in errors with secondary load or current will occur if K_Z is a maximum, or if the transformer is biased at the point of maximum permeability. Of course, instability of the nominal ratio according to equation 34 may be a factor in determining whether it is desirable to bias the transformer at so high an induction.

A vector diagram of the equivalent circuit of Figure 4 is shown in Figure 5. In general E_1 and E_2 are both large and about 180 degrees out of phase, the vector sum of the two voltages being equal to the voltage of the total burden. For high-power-factor burdens, the reversed primary current usually lags the secondary current, whereas for low-power-factor burdens it will lead.

In Figure 6 is a typical low-voltage through-type current transformer using the biased-core type of construction. This type of transformer has found much application in metering boxes where the through-type construction is convenient for the three-wire metering connection and makes possible changing transformer ratio by taking the primary conductor through the opening a number of times.

Experimental and Calculated Results

The following data are for the sample transformer used:

npR: 40 to 1; 200 to 5 amperes
$N_1=41$ turns
$N_2=39$ turns
$A_1=A_2=A$
$\Delta(VA/\#)_2/(VA/\#)_2=-0.18$
$\Delta(VA/\#)_1/(VA/\#)_1=-0.09$
Secondary resistance = 0.035 ohm

Figure 3 is the nominal curves for both cores. For this problem, the curves are plotted as follows:

Ampere turns as a function of volts per turn

$\frac{KA}{l}$ as a function of volts per turn

To use these curves, the equations are modified as follows:

$$\text{Equation 13:}\quad \frac{E}{N}=\left(\frac{K_ZA}{l}\right)(NI) \tag{37}$$

If this equation is transposed, its significance is more apparent:

$$\frac{K_ZA}{l}=\frac{E}{IN^2}$$

which means that K_ZA/l is the exciting impedance measured at the terminals of N turns divided by the turns squared. K_RA/l is K_ZA/l multiplied by the power factor and K_XA/l is K_ZA/l multiplied by the reactive factor. This shows how KA/l can be obtained from measurements on an actual winding on one core.

$$\text{Equation 18:}\quad \frac{E_1}{N_1}=I_S\left(\frac{K_{Z1}A}{l}\right)\times \left[\frac{Z_T-(N_1-N_2)N_2\left(\frac{K_{Z2}A}{l}\right)}{N_2\left(\frac{K_{Z2}A}{l}\right)+N_1\left(\frac{K_{Z1}A}{l}\right)}\right] \tag{38}$$

$$\text{Equation 19:}\quad \frac{E_2}{N_2}=I_s\left(\frac{K_{Z2}A}{l}\right)\times \left[\frac{Z_T+(N_1-N_2)N_1\left(\frac{K_{Z1}A}{l}\right)}{N_2\left(\frac{K_{Z2}A}{l}\right)+N_1\left(\frac{K_{Z1}A}{l}\right)}\right] \tag{39}$$

$$\text{Equation 20:}\quad \frac{E_1}{N_1}=\frac{I_sZ_T}{N_2+N_1}-\left[\frac{I_s(N_1-N_2)N_2}{N_2+N_1}\right]\left(\frac{K_ZA}{l}\right) \tag{40}$$

$$\text{Equation 21:}\quad \frac{E_2}{N_2}=\frac{I_sZ_T}{N_2+N_1}+\left[\frac{I_s(N_1-N_2)N_1}{N_2+N_1}\right]\left(\frac{K_ZA}{l}\right) \tag{41}$$

$$\text{Equation 27:}\quad N_pI_p= I_s\left[N_2+\frac{Z_T+(N_1-N_2)N_1\left(\frac{K_{Z1}A_1}{l}\right)}{N_2\left(\frac{K_{Z2}A_2}{l}\right)+N_1\left(\frac{K_{Z1}A_1}{l}\right)}\right] \tag{42}$$

Equations 40 and 41 are the equations for the first estimation of the volts per turn in each core. The bracketed term of each of these equations is the biasing ampere turns for each of the cores. From equation 37 it is seen that when this is multiplied by K_ZA/l the result is the biasing volts per turn. This quantity may be read directly from Figure 3, where the ampere turns are plotted as a function of the volts per turn. The actual values of the volts per turn are found by repeated substitution in equations 38 and 39 and then the true primary ampere turns from equation 42. The per-cent ratio and the phase angle then may be determined by equations 28, 29, and 30. The results of these calculations are shown in the first two lines of Table I. The ratio must be corrected though, for the difference in the core characteristics according to equation 34. This correction is −0.11 per cent and the corrected values are shown in lines 3 and 4 of the table. The actual test values on the biased transformer also are given as well as tests as a straight unbiased transformer. For this latter condition, 40 turns linked each core.

Conclusions

A method of calculation for the double-core current transformer has been developed that is practical and has sufficient accuracy that the designer can determine the proper proportions of core and the best degree of bias to get the best possible performance, as well as the performance obtainable. It shows that much can be gained in reducing transformer ratio errors and in correcting phase-angle errors in bushing-type current transformers by using the double-core construction. This method of calculation also makes it possible to determine how closely the cores must be matched, and whether adjustment by test will be necessary. This will make possible the construction of high-accuracy bushing-type current transformers to smaller dimensions and lower primary-current ratings than was heretofore possible.

List of Symbols

Subscripts 1 and 2 refer to values for cores 1 and 2.

B—Induction in lines per square inch
E—Voltage across a given winding, volts

Impulse-Failure-Detection Methods as Applied to Distribution Transformers

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Synopsis: Investigation of several methods for the detection of impulse failures in distribution transformers has strengthened our belief in the inadequacy of the methods described in American Standard *C*57.2. All of the methods are discussed in detail, and the most hopeful one is compared with the present method of analyzing the voltage wave and making the physical observations as outlined in the Standards. Data are presented on transformers utilizing present and proposed methods.

DEMONSTRATION of the ability of electric apparatus to withstand impulse tests is of little value unless adequate failure-detection methods are used. Everyone who has had occasion to do impulse testing at one time or another has experienced some uneasiness concerning the certainty of detecting small failures when utilizing present testing technique as covered in the Standards. Many transformers have received impulse tests as a condition of specification and purchase to demonstrate the ability of their windings to withstand such tests. Examination of the windings after testing has disclosed failures which had not been suspected from any known failure indications. Since such a procedure is impractical and often impossible because of economics, it becomes necessary to revise impulse-testing technique relative to failure detection so as to acquire an accurate knowledge of the true condition of windings after impulse voltages have been applied.

Present Impulse-Failure-Detection Methods

For many years now it has been the practice to follow the testing procedure covered in American Standard *C*57.2, 10.122. This reference is quoted here for convenience.

> "*Detection of Failure During Impulse Test.* Because of the nature of impulse-test failures, one of the most important matters is the detection of failure. There are a number of indications of insulation failure. Some of these are: noise within the transformer; presence of smoke; excessive current or drop in voltage in the excitation circuit as indicated by the magnetic oscillograph; failure of the gap or bushing to flash over, although the oscillogram indicates a chopped wave; presence of oscillations or other variations from the expected wave shape as indicated by cathode-ray oscillograms."

It may be seen that certain electrical observations are required, such as the wave shapes of the applied impulse by means of cathode-ray oscillograph by comparison between reduced-full-wave and full-wave oscillograms, and also inspection of the normal-frequency disturbance occasioned by normal-frequency follow current which may be initiated by an impulse failure. In addition to this there are such physical observations to be made as the notation of unusual sound and the presence of smoke or bubbles occasioned by arcing within the apparatus.

Experience With Present Impulse-Failure-Detection Methods

The experience cited here is the result of the test and examination of a great many small distribution transformers which were tested to destruction with impulse voltages. With smaller transformers, the ratio of capacitive impedance to inductive impedance is much smaller than is the case with larger power transformers. Therefore, the problems involved in impulse-failure detection are different. Superimposed high-frequency oscillations[1] rarely occur on the crest of the applied voltage wave as they do in the case of power transformers, so that meticulous examination of the voltage wave near its front is of little help in failure detection in small transformers. Likewise, the examination of the applied voltage wave on its tail yields unpredictable results, in some cases actually showing a shortening of the tail due to a layer failure, while in other instances revealing a considerable portion of the coil to be short-circuited out in the failure

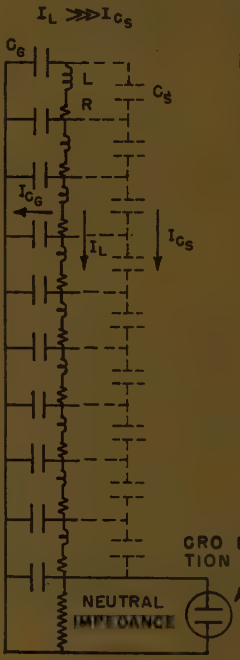

Figure 1. Equivalent circuit of idealized power-transformer winding

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N_p—Primary turns
N—Secondary turn in a given winding
A—Cross sectional area of a given core, square inches
W—Watts, core loss
VAR—Volt-amperes reactive, core loss
VA—Volt-amperes, core loss
δ—Density of core, pounds per cubic inch
l—Mean length of flux path, inches
R—Equivalent series resistance of core, ohms
X—Equivalent series reactance of core, ohms
Z—Equivalent series impedance of core, ohms
$W/\#$—Watts per pound core loss
$(VAR)/\#$—Volt-amperes reactive per pound core loss
$(VA)/\#$—Volt-amperes per pound core loss
K_R—See equation 8
K_X—See equation 9
K_Z—See equation 10
I—Exciting current in a given winding, amperes
I_s—Secondary current of current transformer, amperes
I_p—Primary current of current transformer, amperes
Z_T—Total secondary burden of current transformer winding resistance plus external burden ohms
npR—Transformer name-plate ratio
Per-cent R—Transformer per-cent ratio

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without the tail of the voltage wave being altered.

Smoke and bubbles cannot always get out of buried layers with sufficient velocity to be noticeable to the observer and therefore are not a dependable failure-detection method. The presence of normal-frequency excitation confuses the oscillography with disturbances caused by arcing of isolating gaps and should not be considered as a dependable indicator of failure, since many transformers known to have failed on impulse have successfully withstood normal voltage.

The usual sequence of evidence secured during tests deliberately increased in kilovolts until failure occurs, with the present American Standards Association methods, is as follows:

1. Failure may occur without any of the indicators showing the occurrence of failure.

2. Failure is shown by slight changes in wave shape of the applied wave without bubbles, or

3. Bubbles become noticeable.

4. Normal-frequency current will follow through the impulse path as the applied impulse voltage is increased above normal test levels.

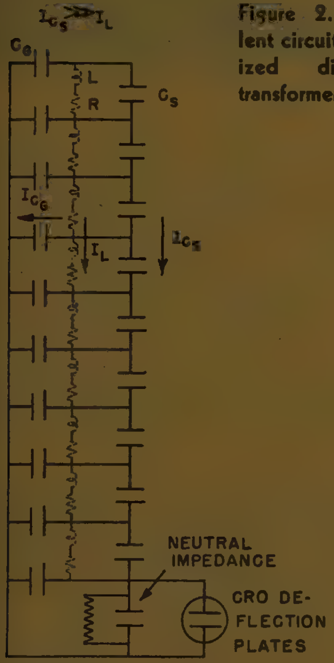


Figure 2. Equivalent circuit of idealized distribution-transformer winding

As a result of this experience, it is known that these methods of failure detection often will not show turn or layer failures or even the failure of major insulation to ground.

In view of such experience, we believe that the present procedures for fault detection are not satisfactory.

Investigation of Impulse-Failure-Detection Methods

In an effort to improve this situation, several impulse-failure-detection methods were investigated as applying to distribution transformers, with the aid of the electric transient analyzer, and the most promising were checked with the usual design of impulse generator. The work done with the transient analyzer is described in methods 1 to 8 inclusive, Table I, and the relative efficiency of the methods is tabulated in Table II. Methods 1, 2, 3, 4, and 5 consist of measurement of a voltage wave at some point in the transformer structure, whereas methods 6, 7, and 8 constitute current-wave measurements in the transformer neutral.

Table I. Description of Test Method

	Record Impulse Voltage Wave					Record Impulse Current Wave in Neutral Impedance Consisting of		
Method	Applied Wave C_1	C_2	Core	On Winding Tap	Low-Voltage Winding	Resistance	Capacitance	Shunt Capacitance and Resistance
1	x							
2		x						
3			x					
4				x				
[illegible]					x			
6						x		
7							x	
8								x

C_1 Impulse-generator capacitance—0.025 microfarad.

C_2 Impulse-generator capacitance—0.006 microfarad.

Table II

Efficiency of Testing—Method								Rating of Test Piece		
1	2	3	4	5	6	7	8	Kva	Kv	Per Cent of Winding Failed
U	*F*	*E*	*P*	*E*	*F*	*E*	*E*	1.5	7.2	4.0
			P					1.5	7.2	8.0
G	*E*	*G*	*E*	*G*	*E*	*E*	*E*	5.0	2.4	12.0
			E					5.0	2.4	17.5
P	*E*	*F*	*G*	*F*	*E*	*E*	*E*	5.0	7.2	7.0
U	*P*	*F*	*G*	*P*	*F*	*E*	*E*	10.0	13.8	1.0
								10.0	13.8	3.0
E	*E*	*E*	*E*	*E*	*E*	*E*	*E*	25.0	2.4	16.0

Definition of method efficiency:
E—Excellent *G*—Good *F*—Fair *P*—Poor *U*—Unsatisfactory

Methods 1 and 2

Methods 1 and 2 consist of the observation of the applied voltage wave which is the present recommended procedure, except for the omission of normal-frequency voltage, for the determination of the electrical condition of a winding when subjected to impulse voltages. Method 2 employs a loosely coupled impulse generator; that is, a generator of small capacitance so as to have an inherently high regulation (voltage drop) as a result of a change in the impedance of the test piece brought about by the failure of its windings.

It was found that anything less than a generator capacitance of 0.006 microfarad was impractical, because it could not deliver a $1^1/_2$ x 40 microsecond wave to low-impedance high-voltage windings. When the results of method 2 are compared with the results of method 1 (in Table II), in which the generator capacitance of 0.025 microfarad more nearly represents present testing practice, the advantage as far as failure detection is concerned lies with the low-capacitance generator (method 2).

Method 3

Oscillations within the winding of a transformer can be recorded to some extent by measuring the voltage produced on its core. In general, this voltage is due to its capacitive coupling with the winding and is a reproduction of the applied wave plus some high-frequency oscillations. The magnitude of this voltage is a function of capacitance coupling and may be sufficiently high to stress unduly the insulation between core and coil. Although the method works very well in some cases, it should not be considered as a universal method of failure detection, because the utilization of this method involves the isolation of the core from ground which in itself produces voltage stresses unnatural to those experienced in conventional circuits.

Method 4

A modification of method 1 consists of the observation of the voltage at some other point in the same winding than that to which the wave is applied. This can be done where taps have been provided in the winding. The impulse voltage from tap to ground, when measured on the tap nearest the ground end of the

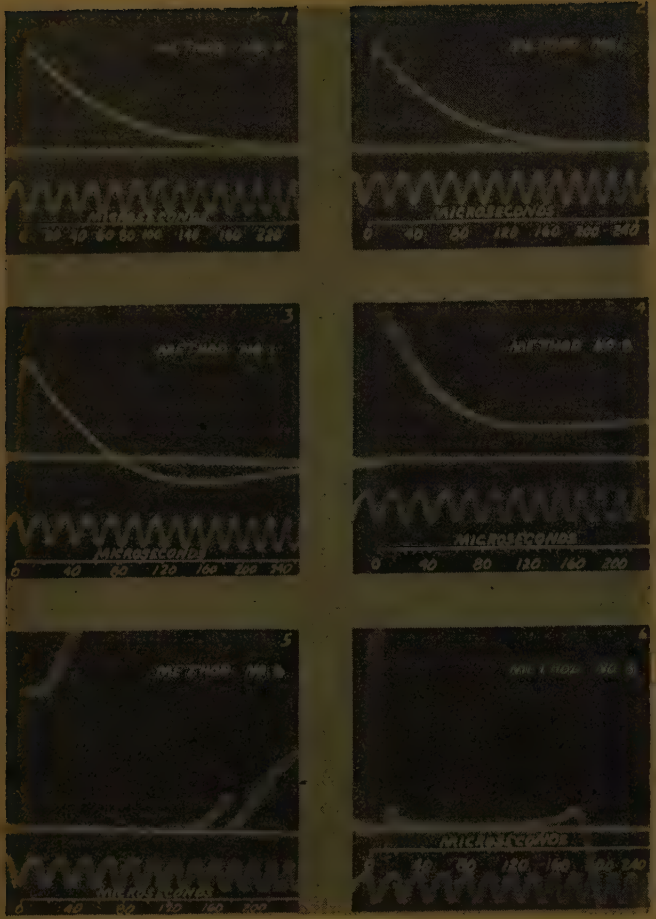


Figure 3 (left). 10-kva 2,400-volt transformer

Applied voltage wave

1. No failure
2. Failure of one turn (0.1 per cent)
3. Failure of ten per cent of winding

Impulse current

4. No failure
5. Failure of one turn (0.1 per cent)
6. Failure of ten per cent of winding

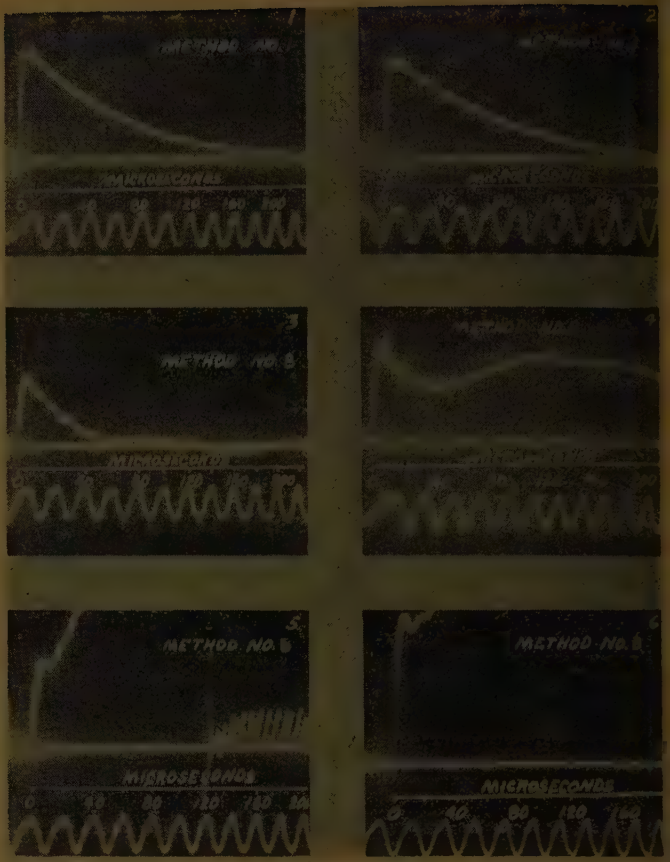


Figure 4 (right). 1½-kva 7,200-volt transformer

1. Applied voltage wave—no failure
2. Applied voltage wave—failure of $13^{1}/_{2}$ per cent of winding (ground end)
3. Impulse current—no failure
4. Impulse current—failure of 0.02 per cent of winding (one turn)
5. Impulse current—failure of $13^{1}/_{2}$ per cent of winding (ground end)
6. Impulse current—$4^{1}/_{2}$ per cent of coil short-circuited through 300.0 ohms

winding, in many cases changes substantially in both wave shape and magnitude at the occurrence of a failure. The utilization of this method is contingent on the presence of suitable taps within a given winding and the problem of bringing out of the tank an additional bushing not provided for in the design.

METHOD 5

This method involves the observation of voltage on a winding other than the one under test. This is equivalent to loose coupling in the impulse generator in that observation is made on a winding which is loosely coupled both electrostatically and magnetically with the winding to which the impulse is applied. In many cases, a layer failure in the high-voltage coil will change substantially the shape of the wave which is reflected in the low-voltage coil. Since one end of the low-voltage coil may be conveniently grounded during test, this method can be more readily adapted to impulse-failure detection than methods 3 and 4.

Measurement of Neutral-Impedance Current

METHODS 6, 7, AND 8

The dielectric condition of a winding during impulse testing can be determined by the measurement of the impulse current[1] as measured in a neutral impedance connected outside the transformer. Changes in the impedance of the transformer winding due to impulse failure will bring about a change in the impulse current.

The use of a neutral impedance during impulse testing results in a circuit not encountered in actual transmission service. However, it should be stated that such a procedure does not alter perceptibly the impulse-voltage distribution within the transformer windings.

As there are three different types of impedance, namely, resistance, capacitance, and combination resistance and capacitance, used in the circuits described in methods 6, 7, and 8, it will be well to review the factors which should be considered in the proper selection of the neutral impedance for a particular transformer rating. To do this, it is necessary to consider the controlling characteristics of the transformer winding. This depends upon the size of the transformer and its voltage rating.

Impulse current that flows through a distribution-transformer winding during the time interval in which it is desirable to detect failures will have a changed distribution from that of power transformers.

In the case of the power transformers,[1] the equivalent circuit is represented by Figure 1, where a major part of the impulse current flows through the inductance–resistance circuit, as indicated by the bold lines of the circuit. Although no dividing lines can be drawn, distribution transformers in general would be represented more nearly by the equivalent circuit of Figure 2, where a major part of the impulse current flows through the capacitance circuit, as indicated by the bold lines of the figure.

As the measurement of impulse neutral current is accomplished by measuring the voltage drop across a neutral impedance which is placed in series with the transformer under test, and as this circuit is a voltage-divider arrangement, it is ad-

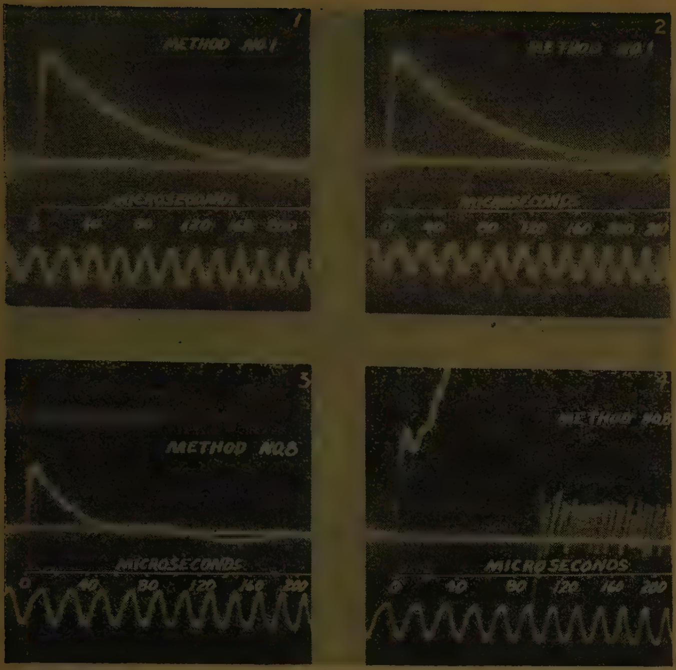


Figure 5. 1½-kva 7,200-volt transformer

Applied voltage wave

1. No failure
2. Failure of 2.0 per cent of winding

Impulse current

3. No failure
4. Failure of 2.0 per cent of winding

Figure 6 (upper right). 3-kva 7,200-volt transformer

Applied voltage wave

1. No failure
2. Failure of 50 per cent of winding (ground end)

Impulse current

3. No failure
4. Same failure as in oscillogram 2

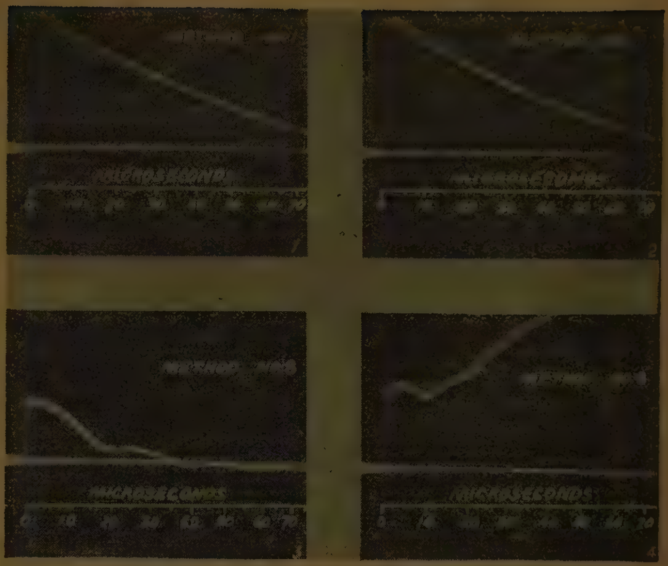


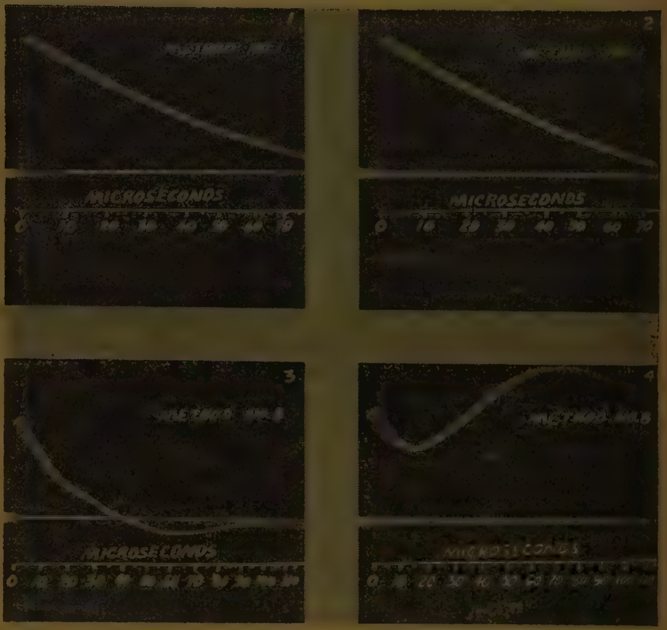
Figure 7 (right). 37½-kva 13,200-volt transformer

Applied voltage wave

1. No failure
2. Failure of 33⅓ per cent of winding (line end)

Impulse current

3. No failure
4. Same failure as in oscillogram 2



vantageous to match the neutral impedance with the controlling impedance of the transformer network. In other words, the desirable neutral impedance consists of a resistance[1] or an inductance in the case of large transformers, Figure 1, and a capacitor in the case of small distribution transformers, Figure 2. When a capacitor is used for a neutral impedance, the charge increases as long as current flows, and there is a tendency to increase the amplitude of the trace on the oscillogram. The use of a resistor in shunt with the capacitor helps to drain off the charge, thereby improving the oscillography and at the same time providing a discharge circuit for the capacitor.

Method 6 evaluates this current by measuring the voltage across an impedance consisting of a resistor placed in the transformer-winding neutral.[1] As most of the current is charging current, a pure resistance shunt will give a very large initial deflection. This peak current is of little value in failure detection, and so some means such as gaps has to be devised to protect the oscillographic equipment.

Method 7 utilizes a capacitor as the neutral impedance. This gives a very sensitive failure-detection method. An impedance change, caused by the short-circuiting of one layer (seven per cent) of a high-voltage coil, caused the impulse current to increase 3,300 per cent. A disadvantage of this method is that the capacitor charge continues to increase for a considerable time, particularly if isolating gaps are used between the test piece and the testing circuit. If isolating gaps are used, the capacitor cannot discharge except by leakage.

In method 8 it was found that a resistor could be used in shunt with the capacitor without appreciably reducing the sensitivity of the scheme. This method is an improvement of methods 6 and 7, and in the work done, a *time constant* of this capacitor–resistor combination of 100 microseconds or less was found to be satisfactory.

Experience in the Application of Methods 1 and 8

Tests on Transformers With Controlled Faults

In order to study impulse-failure detection where turn and layer failures are involved, it was necessary to provide full-scale transformers with a single turn and some portion of the layers brought out so that turn or layer failures could be arranged at will; otherwise a great many transformers might have to be tested to get the type of failure desired.

The data presented on these transformers were obtained using the present ASA method 1, and the best of the methods, as outlined in Table II, method 8. Circuit diagrams for these methods are shown in Figures 8 and 9. The oscillograms shown in Figures 3, 4, and 5 were taken on transformers having such con-

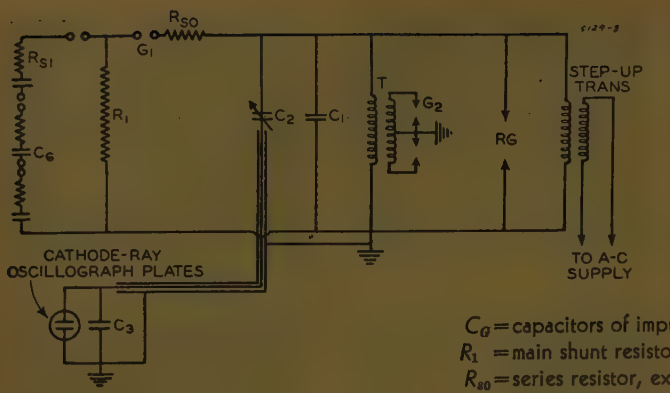


Figure 8. Conventional impulse test circuit for distribution transformers

C_G = capacitors of impulse generator
R_1 = main shunt resistor
R_{s0} = series resistor, external to generator
R_{s1} = series resistor, internal to generator
C_1 = auxiliary capacitor to control wave shape
C_2 = main capacitor of voltage divider
C_3 = auxiliary capacitor of voltage divider
G_1 = protecting gaps (high-vacuum)
T = transformer under test
G_2 = gaps to protect low-voltage winding
RG = test gap (rod gap)

trolled points of failure while using a conventional impulse generator of ASA recommended voltage level for the particular transformer voltage class. It was necessary to omit normal-frequency voltage because of the controlled faults.

10-Kva 2,400-Volt Transformer, Figure 3. When testing with method 1 turn failures could not be detected, but a layer failure is indicated by a shortening of the tail of the wave. The impulse current, method 8, readily detects both the turn and layer failures. In this instance, it was necessary to protect the oscillograph equipment by means of gaps. It would have served the same purpose to reduce the resistivity of the current-measuring impedance.

$1^1/_2$-Kva 7,200-Volt Transformer, Figures 4 and 5. Because of its higher voltage rating, this transformer has a considerably larger number of turns and layers than the ten-kilovolt-ampere 2,400-volt transformer shown in Figure 3. Oscillograms 1 and 2 in Figures 4 and 5 show that method 1 will not detect failures involving either 2 or $13^1/_2$ per cent of the winding by observing the applied voltage wave. The impulse-current detection, method 8, readily detects either defect, as shown in Figures 4 and 5. It is also to be observed in this case that this method shows a one-turn failure very effectively (see Figure 4, oscillogram 4). As the detection of failure using the impulse-current method is contingent on being able to observe current changes due to impedance changes of the circuit brought about by failures within the winding, it is important to demonstrate that this method will detect failures even though this current is relatively small. To do this, a $4^1/_2$-per-cent winding fault was closed through a 300.0-ohm resistance (see Figure 4, oscillogram 6).

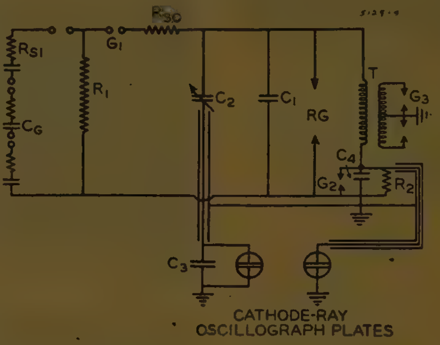


Figure 9. Recommended impulse test circuit for distribution transformers

C_G = capacitors of impulse generator
R_1 = main shunt resistor
R_{s0} = series resistor, external to generator
R_{s1} = series resistor, internal to generator
C_1 = auxiliary capacitor to control wave shape
C_2 = main capacitor of voltage divider
C_3 = auxiliary capacitor of voltage divider
G_1 = protecting gaps (high-vacuum)
RG = test gap (rod gap)
C_4 = capacitor of impulse-current-measuring shunt
R_2 = auxiliary resistor of impulse-current-measuring shunt
G_2 = gap to protect oscillograph
T = transformer under test
G_3 = low-voltage protective gap

Tests on Transformers in Which the Faults Developed During Testing

Oscillograms shown in Figures 6 and 7 are actual breakdown tests on developmental transformers built for that purpose. In these cases, the transformers were excited at normal-frequency voltage. However, the voltage wave shapes recorded on oscillograms 1 and 2 of Figures 6 and 7 are tests made in addition to the ASA method with normal-frequency voltage purposely omitted to avoid confusing the oscillography by the arcing of isolating gaps.[1]

3-Kva 7,200-Volt Transformer, Figure 6. In this case, a failure of 50 per cent of the winding occurred at the ground end of the high-voltage winding. Oscillogram 4 definitely indicates a failure by the neutral-current method. Oscillogram 2 does not indicate a failure although this oscillogram was recorded after the winding had failed, as shown in oscillogram 4. This was a consistent failure, and this condition was observed on many repeat tests.

$37^1/_2$-Kva 13,200-Volt Transformer, Figure 7. In this case, a failure of $33^1/_3$ per cent of the winding occurred at the line end of the transformer. Here again, the current method, film 4, shows the failure, but oscillogram 2 for the potential method 1 cannot be interpreted to indicate a failure, although this test followed the current test. Here also, many checks were made to verify this finding.

Conclusions

1. The observation of changes in the impulse-voltage wave cannot be depended upon to indicate a fault in transformers.

2. The use of normal-frequency excitation in conjunction with impulse testing as a means of fault detection cannot be justified, on the basis of either the data submitted here or experience obtained in the past.

3. Smoke or bubbles are difficult to detect in small transformers with compact and buried layers and therefore cannot be depended upon to indicate impulse failure.

4. The utilization of the present ASA methods of fault detection will not detect a very large percentage of failures.

5. The observation of voltage across a neutral impedance in which impulse neutral current flows in a relatively simple and effective method of impulse-failure detection and therefore is recommended for inclusion in the test code.

Reference

1. Progress in Impulse Testing of Transformers, J. H. Hagenguth. AIEE Transactions, volume 63, 1944, pages 999–1005.

When the Welding Load Outgrows the Distribution Power Supply

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Synopsis: The ability of a 2,300–4,000-volt distribution system to supply single-phase a-c welders is limited not only by current-carrying capacity, but also by voltage regulation. The voltage limitations are reviewed and methods suggested for increasing the limits by changes in the power system and the customers' installations. Three types of subtransmission connections are described for use when the welding load outgrows the distribution power supply.

SINCE THE BEGINNING of World War II there has been a very rapid increase in both the size and number of electric welders in operation. Not only are they used in large numbers and in many sizes for large-scale production in industrial plants, but individual units and small groups of welders are being used for light manufacturing and for repair work in the smaller shops. A large portion of these welders are of the single-phase a-c resistance or arc types. The power factors are relatively low, and the loads are highly fluctuating in character.

In specifying the type of power supply for these welders, the power-company engineers have encountered certain system limitations which would not apply in serving an equivalent steady three-phase load. Service to single-phase welding loads from the usual 2,300–4,000-volt distribution system can be permitted only under favorable combinations of locations, welder sizes, and operating sequences.

The expense of a connection from a subtransmission system frequently may be avoided by modifications to the existing distribution supply or the customer's installation. In other cases, the size, characteristics, and location of the welder load are such that it is necessary to make a connection from the subtransmission system.

This paper will review some of the more commonly encountered limitations of the 2,300–4,000-volt distribution system and modifications of existing power systems and customer installations to permit serving welding loads and will describe some of the more common types of higher-voltage power supply for the larger welding loads.

Limitations of the Existing 2,300–4,000-Volt Distribution System

The two principal system limitations are those due to

A. Lighting flicker at adjacent customers caused by the fluctuating characteristics of welding loads.

B. Delivery of sufficient voltage at the welder to permit satisfactory operations.

Both of these limitations are due to the inherent impedances of the power system, including the customers wiring for each location. Attention is called to the fact that all references in this paper to the power-system impedance include the entire electric system to the welder input terminals.

The annoyance from lighting flicker is dependent upon both the magnitude of the voltage fluctuation and the frequency with which it occurs, or at which the welding load is applied. The most critical frequency is of the order of six times per second, and the annoyance is progressively less for slower operations. The Duquesne Light Company uses the following limits for voltage fluctuation where lighting service is affected, based upon tests and operating experience over a number of years.

Frequency of Load Application	Volts (115-Volt Terms)	Per Cent
Over 1 per second	0.50	0.44
1 per second to 15 per minute	1.00	0.87
15 per minute to 3 per hour	2.00	1.74

The foregoing three frequency classifications correspond approximately to the frequencies of load and voltage fluctuations commonly encountered in electric welding. For example, seam and pulsation welders operate in the first frequency band. The second group includes many spot welders used for regular production work. The lower-frequency classification usually is applied to manually controlled welders, such as spot and a-c arc welders used for repair work or for very limited production. In order to insure the proper frequency classification, the customer is asked to furnish an estimate showing the proposed frequency of operation and welding load for each new installation.

In determining what voltage will be delivered at the welder, account must be taken of the total impedance from the welder to the sources of power. For the smaller welders, under 20 kva, the regulation of the substation, transmission, and power stations ahead of the local 2,300- or 4,000-volt circuit usually can be ignored.

Representative values of voltage regulation in 230-volt terms are shown in Figure 1 for typical 2,300-volt single-phase circuits, and also for several combinations of transformer and secondary wire sizes. The transformer regulation is based upon impedances of 2.6 per cent to 3 per cent for the sizes shown. The assumed spacing of conductors is 16 inches for 2,300-volt circuits and eight inches for 230-volt circuits. In this chart, a welding load of 50 amperes at 60-per-cent power factor has been assumed at the 220-volt welder input terminals. For practical purposes the voltage drops for other small low-power-factor welding loads would be approximately proportional to the currents assumed.

Primary and secondary distances are shown in feet, and the voltage drops for the primary circuit must be added to that for the transformer and secondary circuit to obtain the approximate voltage drop at the customer's service. An additional small allowance may be made, if desired, for the regulation ahead of the primary circuit and in the customer's wiring. Figure 1 is particularly useful in determining the power supply for small welders. The calculation time is reduced by the early elimination of high regulation methods.

In selecting a suitable power supply for the larger single-phase a-c resistance and arc-type welders, the usual approach is to calculate the voltage regulation at the welder for a number of supply methods. This procedure is simplified and the calculation time reduced through the use of the curves in Figure 2 to obtain the allowable power-system impedance for the proposed welder load.

These curves assume 200 volts at the welder input terminals with the welder in operation and 220, 230, and 240 volts at the welder input terminals with the welder out of service; 200 volts is the approximate lower limit of voltage usually supplied for 220-volt power service and is within the plus and minus ten-per-cent voltage range usually tolerated by the manufacturers.

The impedance values used in the curves are expressed in volts drop per ampere at 60-per-cent power factor in 230-volt terms. The impedance value from the curve then is compared with corresponding values, calculated by simple addition from tables or curves, for each supply method being considered. All high regulation methods can be

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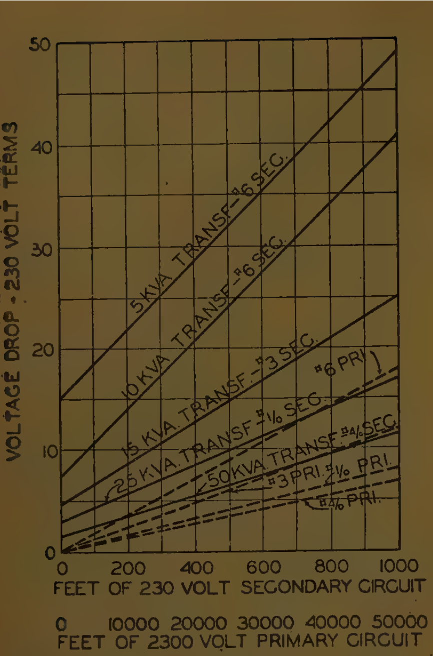

Figure 1. Voltage regulation due to supplying a load of 50 amperes at 60-per-cent power factor from a 230-volt single-phase service

eliminated easily in the early stages, and full attention can be concentrated on the low regulation methods. Separate calculations are necessary to determine if lighting flicker will result.

In working out welder problems, it is helpful to consider the welder as a load of variable impedance which can be adjusted within the limitation of the equipment design to draw a required kilovolt-ampere load from the line under a rather wide range of voltages. The relation between voltage at the welder and the impedance of the power system for a constant kilovolt-ampere load is illustrated in reference 3.

Corrective Methods to Permit Service From the Local 2,300–4,000-Volt Distribution Circuit

When a preliminary investigation shows that a given welder with specified duty cannot be tolerated at a proposed location due to excessive lighting flicker, or would not make satisfactory welds due to abnormal voltage regulation at the welder terminals, there are a number of changes which can be made either to the electric-supply system or to the customer's installation which often will eliminate the necessity for bringing in a service from a higher-voltage source. Some of these methods are described here.

SUBSTITUTE 4,000–230-VOLT TRANSFORMER FOR 2,300–230-VOLT TRANSFORMER AND CONNECT BETWEEN PHASES

On the 2,300–4,000-volt system, a change in the welder supply from 2,300 volts to 4,000 volts doubles the substation impedance but does not increase the impedance of the primary circuit. For locations near the substation, the allowable load fluctuation is increased by only 50 per cent. At the end of long primary circuits, the substation impedance becomes negligible, and the allowable load fluctuation is increased by nearly 200 per cent.

SUBSTITUTE A THREE-PHASE STAR–DELTA TRANSFORMER BANK FOR A SINGLE-PHASE 2,300–230-VOLT TRANSFORMER

When a star–delta bank with a floating primary neutral is used, two thirds of the load is supplied from one phase of the primary circuit, and the drop in the neutral conductor has been eliminated. This again reduces the regulation to the distribution transformer. The allowable load fluctuation is increased by 50 per cent near the substation and by 200 per cent at the end of a long primary circuit. To obtain the same distribution-transformer regulation, 1–75-kva transformer would be replaced with 1–50-kva and 2–25-kva transformers, as two thirds of the load is supplied by one transformer.

REPLACE PRIMARY CONDUCTORS

The replacement of conductors with those of larger size generally will be found more expensive than other methods, and the possible improvement is rather small due to the small change in regulation for low-power-factor loads. This method is seldom recommended to eliminate lighting flicker.

REPLACE OR RELOCATE DISTRIBUTION TRANSFORMERS

A considerable improvement in regulation often can be obtained by replacing the distribution transformer with a larger size, also by relocation of the distribution transformer to reduce the length of the 230-volt secondary winding.

INSTALL INDEPENDENT PRIMARY CIRCUITS

In those cases where the principal problem is lighting flicker, this may be reduced considerably by installing independent primary conductors from the industrial plant back to some point on the electric supply system where the voltage fluctuation will be within the required limits.

INSTALL SERIES CAPACITOR

A limited improvement may be obtained by installing a series capacitor in the primary circuit to compensate for part or all of the increase in primary-circuit regulation due to the welder load, but the protective equipment is expensive, and the space for mounting on poles in urban areas usually is limited. Improvement also can be obtained through the use of welders equipped with suitable series capacitors to reduce the kilovolt-ampere input for a given weld.

INSTALL A TRANSLATOR AND NECESSARY COPPER

Under certain conditions the translator described by R. O. Askey and E. R. Hendrickson,[4] is a very economical method of eliminating lighting flicker for other customers supplied from the same distribution circuit. The authors claim that 100-per-cent voltage correction results from the use of this device.

DISTRIBUTE THE WELDERS OVER THE THREE PHASES

The regulation of the primary circuit due to the operation of a single welder is the same as described for the star–delta bank. With a number of welders balanced between phases and operating at the same time, the primary-circuit regulation near the substation would be one third of that obtained when the same total load is served from one phase. At the end of a long primary circuit the regulation for the balanced welding load would be only one sixth of the single-phase value, due to division of load between phases and elimination of the neutral drop.

The transformer regulation due to a given single-phase welder supplied from a three-phase bank is greater than when it is served from a single-phase transformer of the same total capacity. With only one third of the welders connected to each phase, less advantage can be taken of diversity between welders than when all are supplied from one phase. Therefore, the three-phase service to a few welders may require more transformer capacity than the single-phase service. The size of transformers to supply a large number of welders usually is determined by the equivalent thermal loading.

USE THREE-PHASE WELDERS

A few installations of three-phase resistance-type welders have been made. This development should be watched carefully, since the three-phase balanced load which can be supplied from a three-phase power system is much larger than the single-phase value from the same system for the same frequency of load fluctuations. If it should operate in the cyclic flicker range, however, the permissible kilovolt-ampere demand might be less than for many of the conventional types of welders.

USE STORAGE-TYPE WELDERS

The instantaneous kilovolt-amperes drawn from the line are much less for the storage-type welders than for the conventional resistance welder. It is particularly applicable for certain types of welding, but to date the storage type has been costlier for many operations.

INSTALL A THREE-PHASE MOTOR-GENERATOR SET

The voltage fluctuations on the supply circuit can be reduced greatly by supplying single-phase welding loads through a three-phase motor-generator set. The regulation of the generator and the cost of the added equipment limit the applications of this method.

Higher-Voltage Supply for the Larger Welding Loads

When it is necessary to serve a large number of small welders, or a small number of large welders, the possibilities for stretching the local 2,300–4,000-volt system often are exhausted. When this occurs it is necessary to provide some other system possessing greater capacity and a lower impedance in terms of the service voltage. Since the higher-voltage circuits are not found on nearly every street like the 2,300–4,000-volt distribution circuits, the connection from the higher-voltage circuit usually involves quite large expenditures.

The relatively low load factor of many welding loads makes it imperative that the most economical use be made of existing facilities. In this connection, there is a quite general feeling that welders should pay a fair share of the costs involved in serving them, but that they should not be penalized by requiring a more expensive installation than is necessary to protect the service to other customers.

A description follows of some typical 11-kv and 22-kv connections for welder loads which are beyond the modified 2,300–4,000-volt distribution-circuit limits.

Exclusive 2.3-Kv Circuit From Distant 22–2.3-Kv Station

There are two of these cases and both are supplied by single-phase circuits. In general if a three-phase exclusive distribution circuit is required for load or voltage reasons, then it might be as well to consider a single-phase transmission extension instead.

The first installation utilizes the spare 667-kva 22,000-to-2,300-volt transformer at one of the distribution substations, a 5,700-foot extension of two number-three wires, and a 75-kva 2,300–460-volt transformer adjacent to the customer's plant. This load was supplied initially over a regular circuit from the distribution substation, but changes in the customer's process resulted in complaints of lighting flicker. These were eliminated by connections as just described.

This method of correction is a wartime expedient and would not be utilized for a new installation in peacetime. Due to very large voltage regulation on this supply circuit, it is necessary to connect the electronic controls for the 200-kva welder to the lighting circuit, which is separate from the welding circuit.

The second installation consists of one 100-kva 22,000–2,300-volt transformer adjacent to a through transmission line, a 3,200-foot extension of two number-three wires, and one 100-kva 2,300–230-volt transformer adjacent to the customer's plant.

The main objection to this type of installation is the fact that it does not lend itself very well to growth. In general the impedance of the two transformers in series is a very large part of the total, and, if regulation is a problem, and it usually is, then this is not a satisfactory solution.

One analysis made prior to an expected change in welders resulted in the following comparison of voltage drops for four methods of reinforcement:

(*a*). 2,300-volt, single-phase—doubling the size of one transformer, 82 per cent drop as compared with present.

(*b*). 2,300-volt, single-phase—doubling the size of both transformers, 68.3 per cent drop as compared with present.

(*c*). 4,000-volt, single-phase—doubling the size of both transformers, 54.5 per cent drop as compared with present.

(*d*). 22,000-volt, single-phase—doubling the size of one transformer, 21.3 per cent as compared with present.

22-Kv or 11-Kv Customer's Substations for Welders Only

Six installations of this type are served from the 11,000- or 22,000-volt cable or overhead transmission system. These installations in general are single phase but may be three phase. Service is provided by a tap to the transmission system, using either cable or overhead wire, depending upon the district, and an air-break switch, fuses, transformer, and metering in the customer's substation.

This type of installation is very satisfactory and lends itself very nicely to growth merely by changing the size of the transformer and possibly replacing the fuses and metering current transformers with larger ones.

22-Kv or 11-Kv Customer's Substation for Power and Welders

These installations are three-phase by definition, since they involve three-phase power loads as well as single-phase welders. They may or may not require the same voltage for the welders and for power.

One installation consists of a three-phase cable tap and an air-break switch, fuses, 11,000–440–220-volt transformer bank and metering for both 440- and 220-volt services. Welders are supplied at 440 volts and motor load is supplied at 220 volts.

Another installation is similar except that fortunately the welders and power load are of the same voltage. Separate feeders are operated by the customer in his plant for service to the various power and welding loads.

Conclusions

The use of electric welding has been greatly accelerated during World War II. More general use of present types of welders and new developments in welding equipment and applications in the post-war period may necessitate changes in power-system design.

With the concentration of greater numbers of larger welders with automatic controls for high-speed production in individual plants, the voltage variation at the welder input terminals becomes very important. The regulation must be kept within small limits in order that the rejections will be small and that a uniformly high quality can be maintained in the finished product.

The increasing use of electronic controls and pulsation welding will require greater ingenuity by power-system engineers in the prevention of objectionable flicker at adjacent lighting customers. The annoyance from lighting flicker is dependent upon both the magnitude and frequency of the voltage fluctuations.

The important key to providing adequate voltage at the welder and prevention of lighting flicker is the design of the electric-supply system for relatively low impedance. The regulation at the welder is affected by the design of the power system and customer's wiring to the welder, but the lighting flicker is affected only by the design of that portion of the power system which is common to both the welding and lighting loads.

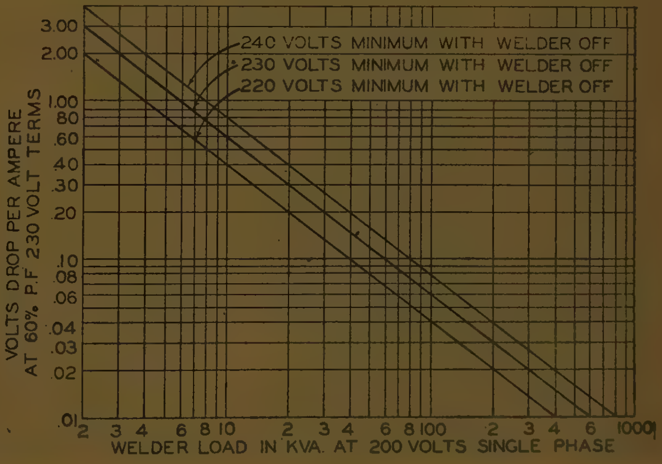

Figure 2. Limiting volts drop per ampere of power system to deliver 200 volts minimum to various 60-per-cent power-factor welder loads

Much calculation time can be saved in the solution of both general and specific welder problems through the use of charts similar to those shown in Figures 1 and 2. High regulation methods can be eliminated in the early stages, and attention then can be given to a choice between adequate supply methods.

Larger and faster welding loads can be supplied from 2,300–4,000-volt distribution circuits by making certain modifications in the power system and customer's equipment, as outlined in this paper. When these possibilities have been exhausted, a connection must be provided from a higher-voltage circuit.

Welders should pay a fair share of the costs involved in serving them, but they should not be burdened with a more expensive installation than is necessary to insure satisfactory operations and to avoid complaints from lighting flicker. In choosing between several methods of power supply, consideration should be given to the over-all cost of welding equipment and power-system facilities.

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An Interval and Dwell Tester for Bomb-Release Interval Controls

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Synopsis: The device described is essentially a frequency meter with a scale proportional to the logarithm of 1/frequency, and a dwell meter suitable for use on inductively loaded contacts in d-c circuits. It has been found useful in several jobs in military aviation, one of which is described in some detail. An appendix gives a simple analysis of the shape of the frequency scale distribution obtained.

IN MILITARY AVIATION the device that originates the release impulses when a train (or series) of bombs is to be dropped is known as a bomb-release interval control. This bomb-release interval control (or interval control) receives its starting impulse either manually or automatically from the bomb-sight mechanism.

The usual interval control is adjustable as to speed at which the bombs are to be dropped; that is, the interval between the dropping of the bombs is preset on a dial graduated in feet of interval, opposite a ground speed scale graduated in miles per hour (or knots). The number of bombs to be dropped is preset on another dial, and after that the procedure can be automatic.

The interval range usually covered is from 7.3 to 147 feet at 100 miles per hour, corresponding to 1 to 20 release impulses per second. In the high-speed part of the range the duration of the release impulses becomes quite important, as each impulse must last long enough to operate the bomb-release mechanism, and the space between impulses must be long enough to permit a switching operation that automatically sends the next release impulse to the next bomb release.

This situation results in rather narrow tolerances on the operation of the interval control at short intervals, and, even though these tolerances are met at the factory, there has been no positive assurance that they still would be met when the device went into service. There has been still less assurance that they would be met after several months of service. In fact the effectiveness of bombing missions was being reduced by failure of bomb-release interval controls, mainly because there was no practicable method available for preflight tests.

Requirements for Instrument for Preflight Tests

It was believed that an instrument for adequate preflight testing should include an indicator that would give a direct reading of the interval between bombs, and another indicator that would give direct information as to the duration of contact and the "off time" between contacts. For short-interval operation the impulses from certain interval controls are originated alternately by two relays; so it is necessary to measure not only the average contact duration and spacing but also the average performance of each of two series of alternating contacts; that is, the average of the even-numbered impulses must be measured independently of the average of the odd-numbered impulses.

As the interval calibration must extend over a range of approximately 25/1 and should be reliable within at least five per cent throughout, it was essential that the scale be at least approximately logarithmic; and the divisions should correspond to the interval dial of the bomb-release interval control. Other requirements were that the instrument must be portable, small enough to be used in restricted space, durable, and essentially independent of ambient temperature.

As the performance of interval controls is influenced by applied voltage and by the nature of the load, it was necessary to include a voltmeter and an internal dummy load closely approximating the usual bomb release and average wiring.

It was expected that the instrument frequently would be used while the bomb racks of the airplanes were being loaded, and so it must be impossible under any circumstances for impulses from an interval control under test to reach the bomb releases.

A further requirement was that the equipment should operate with either 100-microfarad electrolytic or 1-microfarad paper snubbing capacitor across the load contacts of the interval control.

Interval and Dwell Tester

The basic design of the instrument to be described was developed as a direct-reading frequency and dwell meter in 1942 and demonstrated at Wright Field early in 1943. ("Dwell" as used here is the contact duration expressed as a per cent of the contact period.) The final instrument was almost identical to the original with the addition of a sorting circuit to make possible independent dwell measurements on the even- and odd-numbered impulses. The complete instrument is shown in Figure 1 in actual operation as a bench testing device, testing the 50-foot interval setting of the interval control in the foreground. Figures 2 and 3 are sketches illustrating the use of the interval and dwell tester to test an interval control mounted in a bombing plane.

The compartments shown on either side of the instrument panel hold the connection cords and spare fuses.

Interval Circuit

With reference to the circuit diagram of Figure 4, the interval circuit includes a small 45-volt radio *B* battery, low-temperature-coefficient resistors R_1, R_2, and R_c, damping capacitor C_1, and interval relay I, arranged alternately to charge and

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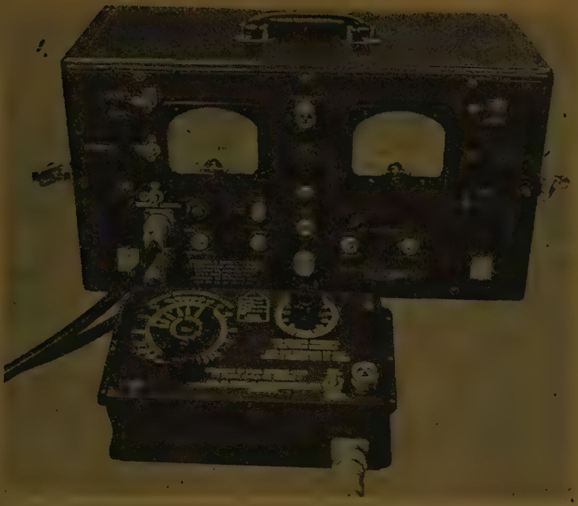

Figure 1. Interval and dwell tester in use as bench-instrument testing bomb-release interval control in foreground at 50-foot interval setting

discharge metering capacitors C_2 and C_3. The dummy load is connected between the lower point of S_1 and ground, with provision for the alternative use of an external load. Relay 1 is in parallel with either load so that each impulse takes capacitors C_2 and C_3 through a charge and discharge cycle. The dummy load is a five-ohm 200-watt resistance in series with an inductance. The inductance is a single-layer winding on a three-eighths-inch-diameter soft-iron rod ten inches long.

It is obvious that if C_1 is large enough that the cyclic variations in its voltage are negligible when relay I is operating, the discharge current i through metering capacitors C_2 and C_3 will be equal to fCe where

f = the impulse frequency
$C = C_2 + C_3$ in farads
e = voltage across C_1

If we ignore for the moment the current through the microammeter circuit which is in fact quite small, and assume that e is a small part of the battery voltage, then i will be approximately constant as it is limited by R_1. Then $e=i/fC$ and the scale is reciprocal with frequency, as the instrument deflection is proportional to e.

A reciprocal frequency scale is a linear-interval scale, and the lower part of the interval scale is substantially linear. For increasing intervals the scale becomes more condensed and is in fact a good approximation of a logarithmic scale, since i is reduced and e is increased until at full scale e is roughly 40 per cent of battery voltage. A more detailed analysis is given in the appendix.

Field calibration of the interval circuit is made by adjusting the instrument zero adjuster to a zero calibration mark, turning power on (S_3), opening calibrate switch (S_2), and setting the interval meter by an adjuster under the instrument (this is the potentiometer across the instrument) to a full-scale calibration mark. Switch S_2 then is released, the pointer swings off scale to the right, and the instrument is in calibration. This calibration eliminates any influence on interval readings that otherwise would result from changes in battery voltage or from the temperature coefficients of the instrument winding and its shunted potentiometer.

Temperature will affect the accuracy of the interval circuit by its effect on R_1, R_2, R_e, C_2, and C_3. A one-per-cent increase in capacitance or in resistance will reduce the interval reading by one per cent. The variation of capacitance within the operating-temperature range is somewhat irregular but on the order of one per cent. The temperature change in the resistance is of course below one per cent.

Dwell Circuit

The dwell circuit comprises an input potentiometer, a resistance-loaded selenium rectifier, an averaging circuit, and a microammeter calibrated in per-cent dwell. Switch S_5 and an external resistance make it convenient to use this instrument to check battery voltage so the instrument is also calibrated in volts.

At the usual operating voltage of 27 volts the input to the dwell circuit measures on the order of 500 ohms resistance so that it will have a negligible damping effect on the inductive kick from the six-ohm load circuit. This inductive kick is blocked out of the instrument circuit by the two disk rectifier. Dwell full-scale calibration is made by setting the input potentiometer for full-scale reading when a continuous forward voltage is applied from the interval-control power supply through switch S_4. The instrument then will read in proportion the average forward voltage of a series of impulses which is by definition the dwell in per cent. Of course, if the nature of the load were such that current interruption caused oscillations that become appreciably positive, this method of measuring dwell would be subject to some error, but the usual bomb release is definitely not of this type.

Under operating conditions the maximum reverse voltage that appears across the rectifier output is from 0.3 per cent to 4 per cent of the impressed reverse volt-

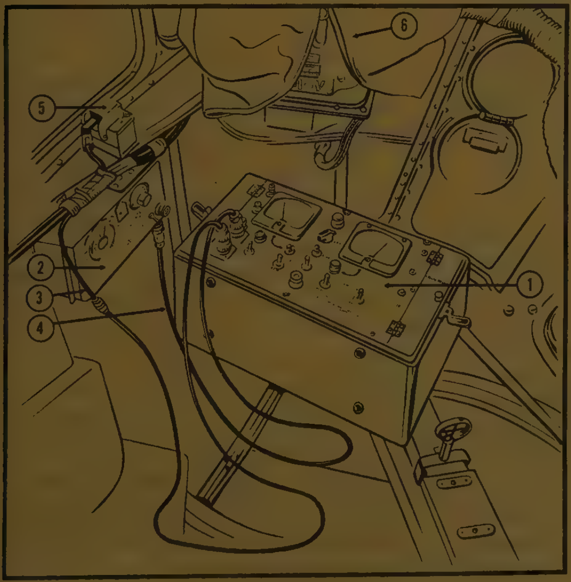

Figure 2 (left). Interval and dwell tester connected inside bomber

1—Tester
2—Interval control
3—Connection to power cable
4—Connection to interval control
5—Bomb-release switch
6—Bomb sight

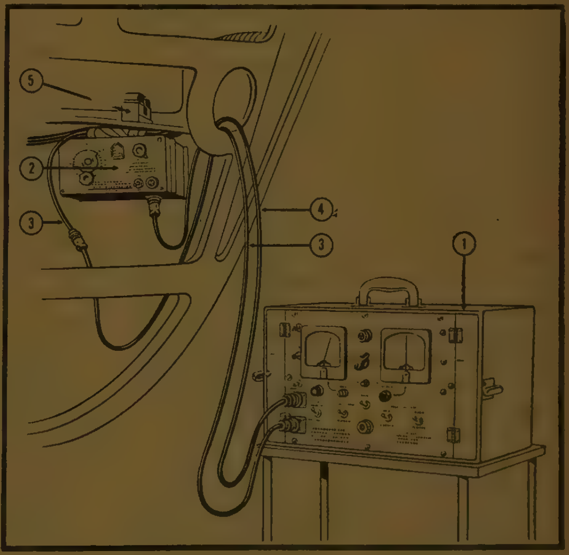

Figure 3 (right). Interval and dwell tester connected outside of bomber

1—Tester
2—Interval control
3—Connection to power cable
4—Connection to interval control
5—Bomb-release switch

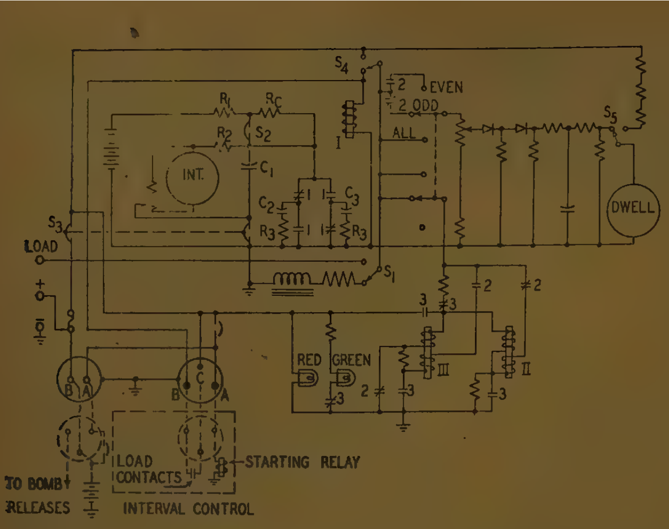

Figure 4. Circuit diagram of interval and dwell tester

The circuits associated with the interval and dwell tester are shown in broken lines

The impulse selector is shown in position to measure the dwell of odd-numbered impulses

age. With the internal load in use the error introduced by the rectifier is negligible, but if a 15-ampere solenoid is used there may be an error of about one per cent attributable to the rectifier.

As the dwell circuit only makes a comparison measurement it has no temperature error.

The dwell full scale must be recalibrated whenever the interval-control voltage changes. This is particularly important at short intervals since a one-volt increase can cause the dwell to change from 50 per cent to 53 per cent. If the dwell circuit is not recalibrated the dwell reading would change from 50 per cent to 55 per cent.

The sorting circuit alternately connects and disconnects the dwell circuit and the load in the idle time between impulses. This makes it possible to measure the dwell of even-numbered and odd-numbered impulses independently. This is accomplished by relays II and III in a modification of a conventional circuit in which the operation of relay III during an impulse changes the circuit to reduce the operating time of relay II at the end of that impulse. Then the operation of relay II sets up the circuit to reduce the release time of relay III at the beginning of the next impulse, and so on. When ordinary telephone-type relays are used, this circuit operates reliably at 30 impulses per second in parallel with the dummy load and 100-microfarad capacitor. It is required to follow 20 impulses per second with any dwell from 35 per cent to 65 per cent at 27 volts supply.

As the operation of the sorting circuit keeps half of the impulses out of the dwell circuit, the effective impulse period is doubled and the actual dwell reading halved. This simply means that the individual readings must be multiplied by two, or that the two readings must be added together.

The impulse selector switch energizes the sorting circuit only in the "odd" and "even" positions. If a train containing an odd number of impulses is run off, the sorting circuit will be left with both relays energized, and unless the circuit is "cleared" by momentarily operating S_4 (the dwell calibrate switch) the odd impulses in the next train will be read at the "even" switch position, and vice versa. Necessity for clearing is indicated when the green "clearance-for-sorting" light is out.

The impulse selector switch is of the short-circuiting type so that it is possible to switch back and forth from "odd" to "even" during a series of impulses without losing step.

Discussion

It has been found possible to keep the interval meters within three per cent of a printed scale calibration throughout the range in spite of instrument variations. This is done by setting the instrument zero adjuster and the potentiometer adjustment so the calibration is exact at intervals of 7.5 feet and 150 feet. Then two or three points in the middle of the scale are checked. Occasionally one will be more than three per cent low. To correct this the resistance of either R_1 or R_2 is reduced about 15 per cent by shunting with another resistor, and the process of line-up is repeated. This 15-per-cent reduction in resistance increases the mid-scale readings about four per cent when the ends are lined up, so that the new mid-scale readings will be within the three-per-cent limit. After checking this the full-scale and zero calibration points are established and marked. The assembly is then completed and a deviation card run on the interval circuit in two positions (panel vertical and horizontal) and the dwell and sorting circuits are tested. The deviation card is shipped with the instrument.

In wet climates the internal load makes it simple to dry out the interior of the device. It is only necessary to connect to a 20-volt storage battery, turn the power switch on, and tie down the "dwell-calibrate" switch (S_4).

Other uses of a similar nature are being found for this equipment. A specific example of this is in the measurement of the performance of the "peck" relay in the *C-1* pilot.

It should be understood in connection with possible other uses that the interval-meter circuit is restricted to a maximum frequency on the order of 30 cycles unless vacuum tubes are to be used. The present dwell circuit is not limited as to frequency, but for some applications a vacuum-tube circuit would eliminate frequent adjustments and errors caused by oscillations.

Appendix

In the following notation capacitors and resistors are identified by the same subscripts used in Figure 4:

f = contacts per second of any one contact of relay *I*
S = interval in feet at 100 miles per hour ground speed = $146.7/f$
C = capacitance of C_2+C_3 in farads
E = battery electromotive force, volts
e = volts across C_1
e_0 = volts across C_1 during calibration ($f=0$)
i = average current to C_2+C_3
i_1 = average current through R_1
i_2 = average current through R_2
R_1 includes battery resistance
R_2 includes instrument and shunt resistance
C_1 is assumed large in comparison to C_2+C_3

$$i=fCe \tag{1}$$

$$E=e+R_1i_1=e+R_1(fCe+e/R_2) \tag{2}$$

$$\begin{aligned} E/e&=1+R_1(fC+1/R_2)\\ &=\{R_2+R_1(fCR_2+1)\}/R_2 \end{aligned} \tag{3}$$

$$e/E=R_2/(R_1+R_2+fCR_1R_2) \tag{4}$$

$$e_0/E=R_2/(R_1+R_2+R_c) \tag{5}$$

$$\begin{aligned} e/e_0&=(R_1+R_2+R_c)/(R_1+R_2+fCR_1R_2)\\ &=(R_1+R_2+R_c)/(R_1+R_2+\\ &\qquad 146.7CR_1R_2/S) \end{aligned} \tag{6}$$

if $R_1=R_2$ $(=R)$

$$e/e_0=(2+R_c/R)/(2+146.7CR/S) \tag{7}$$

The voltage variation across C_1 that is ignored in equation 1 introduces an error in the calculated scale distribution of equation 7 that amounts to about one per cent of full-scale deflection at mid-scale; this is for a practical case where $R_1=R_2=R=139{,}000$ ohms, $C_1=31.5$ microfarads, $C_2=C_3=2.16$ microfarads, $R_3=560$ ohms, and $f=1$ to 30 cycles per second.

Voltage versus time for C_1 can be assumed linear while increasing and approximately, rate of change

$$=\frac{E/2-e}{(C_1+C/2)R/2}=fe/n \text{ volts per second} \tag{8}$$

if $C_1+C/2=nC$. The balance of a half period may be called the partial-discharge time T, and during this period voltage versus time is exponential. This exponential begins when C_2 or C_3 is connected to C_1 and may be assumed to continue until the rate of voltage

Symmetrical Components as Applied to the Single-Phase Induction Motor

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THIS PAPER is intended to show in detail the application of the method of symmetrical components to the solution of single-phase-induction-motor performance characteristics. By the use of this method single-phase-motor theory follows very closely the simpler case of the balanced polyphase-motor theory. The author of this paper is of the opinion that a clearer concept of the physical theory can be obtained as contrasted to other methods—either from the revolving-field or the cross-field concepts.

In their AIEE paper of May 1936, Lyon and Kingsley[1] ably presented this subject in a more general manner and also with particular emphasis on the motor having main and start phases not in space quadrature. Their work and the work of Mr. Morrill[2,3] form the basis of the thesis now to be considered.

Assumptions

1. The motor is magnetically symmetrical:

   (a). Sinusoidal distribution of conductors in the stator slots.

   (b). Uniform air gap.

   (c). Uniform iron permeance.

2. Sinusoidal voltage applied.

3. Linear constants—saturation of the iron flux paths is negligible.

4. Friction and windage, iron loss,[5] and load losses are subtractive from the rotor output.

5. Main and start phases are in space quadrature (90 electrical degrees between axes of these windings).

General Considerations

The solution of power or current flow in polyphase circuits when the applied voltages are not balanced, or when the load impedances are not equal can be accomplished best by the application of the method of symmetrical components. We will show presently that the case of the single-phase motor is a special case of an unbalanced two-phase motor.

Let us assume the general two-phase circuit with phase voltages (V_M and V_S) unequal in magnitude and not in time quadrature. It has been shown[4] that two balanced systems can be formed: one system, called the "positive sequence" (+) consists of components of V_M and V_S which are equal in magnitude and in time quadrature, and have positive (+) phase rotation or the same rotation as the V_M and V_S system; the other system, called the "negative sequence" (−) consists of other components of V_M and V_S equal in magnitude and in time quadrature, *but* phase rotation is opposite that of the positive-sequence components. No "uniphase" or "zero-sequence" components exist.

Part I

By definition we then have

$$V_M=V_M^{+}+V_M^{-} \tag{1}$$

$$V_S=V_S^{+}+V_S^{-} \tag{2}$$

$$V_S^{+}=V_M^{+}\underline{|90^\circ}=+jV_M^{+} \tag{3}$$

$$V_S^{-}=V_M^{-}\underline{|\overline{90^\circ}}=-jV_M^{-} \tag{4}$$

Phase S leads phase M in time angle. From equations 2, 3, and 4 we then have

$$V_S=j(V_M^{+}-V_M^{-}) \tag{2a}$$

which defines V_S in terms of the components of V_M. When the magnitude and time phase of V_M and V_S are each known, the symmetrical components of each can be calculated by adding equation 1 and $-j$ times equation 2a and clearing

$$V_M^{+}=\frac{V_M-jV_S}{2} \tag{5}$$

By subtracting $-j$ times equation 2a from equation 1 and clearing we have

$$V_M^{-}=\frac{V_M+jV_S}{2} \tag{6}$$

All the foregoing quantities are vectors representing rms magnitude and time phase voltages.

By the same procedure we arrive at current values:

$$I_M=I_M^{+}+I_M^{-} \tag{7}$$

and

$$I_S=I_S^{+}+I_S^{-} \tag{8}$$

$$I_S^{+}=jI_M^{+} \tag{9}$$

$$I_S^{-}=-jI_M^{-} \tag{10}$$

$$I_M^{+}=\frac{I_M-jI_S}{2} \tag{11}$$

and

$$I_M^{-}=\frac{I_M+jI_S}{2} \tag{12}$$

In a general two-phase system as shown in Figure 1, Z_M and Z_S are the load impedances of phases M and S, respectively. Now applying Ohm's law, we have

$$V_M=I_MZ_M \tag{13}$$

and

$$V_S=I_SZ_S \tag{14}$$

Then from equations 13, 7, and 1,

$$V_M^{+}+V_M^{-}=(I_M^{+}+I_M^{-})Z_M \tag{15}$$

and from equations 14, 8, and 2,

$$V_S^{+}+V_S^{-}=(I_S^{+}+I_S^{-})Z_S \tag{16}$$

or from equations 3 and 4 equation 16 becomes

$$jV_M^{+}-jV_M^{-}=(+jI_M^{+}-jI_M^{-})Z_S \tag{16a}$$

or

$$V_M^{+}-V_M^{-}=(I_M^{+}-I_M^{-})Z_S \tag{16b}$$

Adding equations 15 and 16b we have

$$V_M^{+}={}^1/_2[I_M^{+}(Z_M+Z_S)+I_M^{-}(Z_M-Z_S)] \tag{17}$$

Subtracting equation 16b from equation 15 and clearing, we have

$$V_M^{-}={}^1/_2[I_M^{+}(Z_M-Z_S)+I_M^{-}(Z_M+Z_S)] \tag{18}$$

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drop across C_1 (ignoring currents through R_1 and R_2) $= fe/n$.
Then

$$T=(R_3C/2)\log_e(1/fR_3C) \tag{9}$$

This varies from 0.0073 to 0.0032 second for f from 1 to 30 cycles per second. This makes the magnitude e_v of the voltage variation across C_1

$$e_v=(1-2fT)e/2n \tag{10}$$

Then from equation 1

$$i=fC(e+e_v/2) \tag{11}$$

and a more correct value of

$$e/e_0=\frac{2+R_c/R}{2+146.7CR[1+(1-2fT)/4n]/S} \tag{12}$$

This reduced the one-per-cent error in scale distribution to about 0.25 per cent. The error was determined by establishing 13 frequencies to ±0.05 per cent by a tuning fork method and measuring the average current in the microammeter circuit by a resistance and potentiometer.

It will be seen that the interval-meter deflection is independent of the dwell, provided that the minimum duration of contact is greater than about $2T$.

In the case of a transformer or single-phase motor where there are not the same turns in each of the windings or phases then,

$$|I_S^+| = \frac{M}{S}\,|I_M^+| = \frac{|I_M^+|}{a} \tag{19}$$

and

$$|I_S^-| = \frac{|I_M^-|}{a} \tag{20}$$

$$V_S^+ = \frac{S}{M}|V_M^+| = a|V_M^+| \tag{21}$$

and

$$V_S^- = a|V_M^-| \tag{22}$$

where

M = effective conductors of the M phase
S = effective conductors of the S phase
$a = S/M$

We can now write

$$V_S^+ = jaV_M^+ \tag{3a}$$

$$V_S^- = -jaV_M^- \tag{4a}$$

$$V_S = ja(V_M^+ - V_M^-) \tag{2b}$$

similarly

$$I_S = j/a(I_M^+ - I_M^-) \tag{8a}$$

In a single-phase motor it is obvious that

$$V_S \equiv V_M \tag{23}$$

Now equation 2b becomes

$$V_M = ja(V_M^+ - V_M^-) \tag{2c}$$

Furthermore, we will find that the impedance to positive-sequence currents is not the same as for negative-sequence currents in rotating apparatus.

Let

Z_M^+ = impedance to I_M^+
Z_M^- = impedance to I_M^-
Z_S^+ = impedance to I_S^+
Z_S^- = impedance to I_S^-

Multiplying equation 2c by $-j/a$ and adding to equation 1, we obtain

$$V_M^+ = \frac{V_M}{2}(1 - j/a) \tag{24}$$

or by subtraction we obtain

$$V_M^- = \frac{V_M}{2}(1 + j/a) \tag{25}$$

From equations 3a and 24 we have

$$V_S^+ = \frac{V_M}{2}(1 + ja) \tag{26}$$

and from equations 4a and 25 we have

$$V_S^- = \frac{V_M}{2}(1 - ja) \tag{27}$$

Equations 13 and 14 now can be reconsidered together with equations 9, 10, 8a, and 23 and at the same time the sequence impedances Z_M^+, Z_M^-, Z_S^+ and Z_S^- can be included.

$$V_M = I_M^+Z_M^+ + I_M^-Z_M^- \tag{13a}$$

$$V_M = j/a(I_M^+Z_S^+ - I_M^-Z_S^-) \tag{14a}$$

To solve for I_M^- multiply equation 13a by Z_S^+/a^2 and equation 14a by $-j/aZ_M^+$

$$V_M\frac{Z_S^+}{a^2} = I_M^+Z_M^+\frac{Z_S^+}{a^2} + I_M^-Z_M^-\frac{Z_S^+}{a^2} \tag{13b}$$

$$-jV_M\frac{Z_M^+}{a} = I_M^+\frac{Z_S^+}{a^2}Z_M^+ - I_M^-Z_M^+\frac{Z_S^-}{a^2} \tag{14b}$$

If we subtract equation 14b from equation 13b

$$V_M\left(\frac{Z_S^+}{a^2} + j\frac{Z_M^+}{a}\right) = I_M^- \times \left(\frac{Z_M^-Z_S^+}{a^2} + \frac{Z_M^+Z_S^-}{a^2}\right)$$

or

$$I_M^- = V_M\left[\frac{Z_S^+ + jaZ_M^+}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right] \tag{28}$$

By applying equations 10 and 20

$$I_S^- = V_M\left[\frac{Z_M^+ - j\dfrac{Z_S^+}{a}}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right] \tag{29}$$

To solve for I_M^+ multiply equation 13a by Z_S^-/a^2 and equation 14a by j/aZ_M^-.

$$V_M\frac{Z_S^-}{a^2} = I_M^+Z_M^+\frac{Z_S^-}{a^2} + I_M^-Z_M^-\frac{Z_S^-}{a^2} \tag{13c}$$

$$jV_M\frac{Z_M^-}{a} = -I_M^+Z_S^+\frac{Z_M^-}{a^2} + I_M^-Z_S^-\frac{Z_M^-}{a^2} \tag{14c}$$

If we subtract equation 14c from equation 13c,

$$V_M\left(\frac{Z_S^-}{a^2} - j\frac{Z_M^-}{a}\right) = I_M^+ \times \left(Z_M^+\frac{Z_S^-}{a^2} + Z_M^-\frac{Z_S^+}{a^2}\right)$$

or

$$I_M^+ = V_M\left(\frac{Z_S^- - jaZ_M^-}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right) \tag{30}$$

By applying equations 9 and 19

$$I_S^+ = V_M\left[\frac{Z_M^- + j\dfrac{Z_S^-}{a}}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right] \tag{31}$$

From equations 7, 28, and 30 we then obtain the general equation:

$$I_M = V_M\left[\frac{(Z_S^+ + Z_S^-) + ja(Z_M^+ - Z_M^-)}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right] \tag{32}$$

and from equations 8, 29 and 31

$$I_S = V_M\left[\frac{(Z_M^+ + Z_M^-) - j/a(Z_S^+ - Z_S^-)}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right] \tag{33}$$

The line current is the combined current of main and start phases.

$$I_L = I_M + I_S =$$

$$V_M\left[\frac{(Z_M^+ + Z_M^- + Z_S^+ + Z_S^-) + j\left(\quad + - aZ_M^- + \dfrac{Z_S^-}{a} - \dfrac{Z_S^+}{a}\right)}{Z_M^+Z_S^- + Z_M^-Z_S^+}\right] \tag{34}$$

Part II

These general equations will now be applied specifically to the single-phase motor.

The equivalent circuits of the M phase are shown in Figure 2. (See end of paper for definition of symbols.)

$$Z_{2M}^+ = \frac{R_2}{S} + jX_2 \tag{35}$$

secondary impedance to positive-sequence current.

By combining the magnetizing impedance jX_ϕ with the secondary impedance Z_{2M}^+ the "apparent secondary impedance" can be written

$$Z_2^{+\prime} = \frac{1}{\dfrac{1}{R_2/s + jX_2} + \dfrac{1}{jX_\phi}} = \frac{X_\phi^2R_2/s}{(R_2/s)^2 + (X_\phi + X_2)^2} + j\frac{X_\phi\left(\dfrac{R_2}{S}\right)^2 + X_\phi X_2(X_\phi + X_2)}{(R_2/s)^2 + (X_\phi + X_2)^2} \tag{36}$$

or

$$Z_2^{+\prime} = R_2^{+\prime} + jX_2^{+\prime} \tag{36a}$$

where $R_2^{+\prime}$ and $jX_2^{+\prime}$ are apparent secondary positive-sequence resistance and apparent secondary positive-sequence reactance, respectively.

Similarly,

$$Z_{2M}^- = \frac{R_2}{2-S} + jX_2 \tag{37}$$

secondary impedance to negative-sequence current and

$$Z_2^{-\prime} = \frac{X_\phi^2\dfrac{R_2}{2-S}}{\left(\dfrac{R_2}{2-S}\right)^2 + (X_\phi + X_2)^2} + j\frac{X_\phi\left(\dfrac{R_2}{2-S}\right)^2 + X_\phi X_2(X_\phi + X_2)}{\left(\dfrac{R_2}{2-S}\right)^2 + (X_\phi + X_2)^2} \tag{38}$$

$$Z_2^{-\prime} = R_2^{-\prime} + jX_2^{-\prime} \tag{38a}$$

where $R_2^{-\prime}$ and $X_2^{-\prime}$ are apparent secondary negative-sequence resistance and apparent secondary negative-sequence reactance, respectively.

The primary impedance of the M phase is

$$Z_{1M}^+ = Z_{1M}^- = R_{1M} + jX_{1M} \tag{39}$$

and of the S phase is

$$Z_{1S}^{+}=Z_{1S}^{-}=(R_{1S}+R_C)+j(X_{1S}-X_C) \quad (40)$$

in the case of a capacitor motor or

$$Z_{1S}^{+}=Z_{1S}^{-}=(R_{1S}+R_C)+jX_{1S} \quad (40a)$$

in the case of a resistance-split-phase motor having external resistance R_C.

The generalized impedances of the single-phase motor can now be written:

$$Z_M^{+}=(R_{1M}+R_2^{+\prime})+j(X_{1M}+X_2^{+\prime}) \quad (41)$$

$$Z_M^{-}=(R_{1M}+R_2^{-\prime})+j(X_{1M}+X_2^{-\prime}) \quad (42)$$

$$Z_S^{+}=(R_{1S}+R_C+a^2R_2^{+\prime})+j(X_{1S}-X_C+a^2X_2^{+\prime}) \quad (43)$$

$$Z_S^{-}=(R_{1S}+R_C+a^2R_2^{-\prime})+j(X_{1S}-X_C+a^2X_2^{-\prime}) \quad (44)$$

If we combine equation 30 with equations 41, 42, 43, 44,

$$I_M^{+}=V_M\times\left[\frac{(R_{1S}+R_C+a^2R_2^{-\prime})+j(X_{1S}-X_C+a^2X_2^{-\prime})-ja(R_{1M}+R_2^{-\prime})+a(X_{1M}+X_2^{-\prime})}{\{(R_{1M}R_2^{+\prime})+j(X_{1M}+X_2^{+\prime})\}\{(R_{1S}+R_C+a^2R_2^{-\prime})+j(X_{1S}-X_C+a^2X_2^{-\prime})\}+\{(R_{1M}+R_2^{-\prime})+j(X_{1M}+X_2^{-\prime})\}\{(R_{1S}+R_C+a^2R_2^{+\prime})+j(X_{1S}-X_C+a^2X_2^{+\prime})\}}\right] \quad (45)$$

$$=V_M\times\left[\frac{\{(R_{1S}+R_C+a^2R_2^{-\prime})+a(X_{1M}+X_2^{-\prime})\}+j\{(X_{1S}-X_C+a^2X_2^{-\prime})-a(R_{1M}+R_2^{+\prime})\}}{\{\text{denominator of equation 45}\}}\right]$$

Likewise from equation 28,

$$I_M^{-}=V_M\times\left[\frac{\{(R_{1S}+R_C+a^2R_2^{+\prime})-a(X_{1M}+X_2^{+\prime})\}+j\{(X_{1S}-X_C+a^2X_2^{+\prime})+a(R_{M}+R_2^{+\prime})\}}{\{\text{denominator of equation 45}\}}\right] \quad (46)$$

and from equation 32,

$$I_M=V_M\times\left[\frac{\{2R_{1S}+2R_C+a^2(R_2^{+\prime}+R_2^{-\prime})\}+j\{2X_{1S}-2X_C+a^2(X_2^{+\prime}+X_2^{-\prime})\}+ja\{(R_2^{+\prime}-R_2^{-\prime})+j(X_2^{+\prime}-X_2^{-\prime})\}}{\{\text{denominator of equation 45}\}}\right] \quad (47)$$

From equation 31,

$$I_S^{+}=V_M\times\left[\frac{\left\{(R_{1M}+R_2^{-\prime})-\frac{1}{a}(X_{1S}-X_C+a^2X_2^{-\prime})\right\}+j\left\{(X_{1M}+X_2^{-\prime})+\frac{1}{a}(R_{1S}+R_C+a^2R_2^{-\prime})\right\}}{\{\text{denominator of equation 45}\}}\right] \quad (48)$$

From equation 29,

$$I_S^{-}=V_M\times\left[\frac{\left\{(R_{1M}+R_2^{+\prime})+\frac{1}{a}(X_{1S}-X_C+a^2X_2^{+\prime})\right\}+j\left\{(X_{1M}+X_2^{+\prime})-\frac{1}{a}(R_{1S}+R_C+a^2R_2^{+\prime})\right\}}{\{\text{denominator of equation 45}\}}\right] \quad (49)$$

From equation 33,

$$I_S=V_M\times\left[\frac{\{(2R_{1M}+R_2^{+\prime}+R_2^{-\prime})+j(2X_{1M}+X_2^{+\prime}+X_2^{-\prime})\}-\frac{j}{a}\{a^2(R_2^{+\prime}-R_2^{-\prime})+ja^2(X_2^{+\prime}-X_2^{-\prime})\}}{\{\text{denominator of equation 45}\}}\right] \quad (50)$$

OUTPUT AND TORQUE

In the case of a balanced two-phase motor, the *gross* rotor output is

$$WO_{2\phi}=2|I_2|^2\frac{R_2}{S}\ (1-S)\ \text{watts} \quad (51)$$

Here only positive-sequence currents exist. The net rotor output is the gross output less friction and windage, iron loss, and load loss.

The motor torque in ounce feet is

$$T_{2\phi}=WO_{\text{net}}\times\frac{16\times33{,}000}{2\pi\times\text{rpm}\times746}$$

$$=\frac{112.7}{\text{rpm}}\times WO_{\text{net}} \quad (52)$$

The positive-sequence currents in the secondary winding produce torque in the direction of the phase rotation of these currents. When negative-sequence currents exist they produce a counter torque. As we are dealing with two balanced systems of two-phase currents we may consider the torques they produce as positive sequence and negative sequence.

Since

$$I_{2M}^{+}=\frac{jX\phi}{R_{2/S}+j(X_\phi+X_2)}I_M^{+} \quad (53)$$

or

$$I_{2M}^{+}=\frac{X_\phi(X_\phi+X_2)+jX_\phi\dfrac{R_2}{S}}{(R_{2/S})^2+(X_\phi+X_2)^2}I_M^{+} \quad (53a)$$

and

$$I_{2M}^{-}=\frac{jX_\phi}{\dfrac{R_2}{2-S}+j(X_\phi+X_2)}I_M^{-} \quad (54)$$

or

$$I_{2M}^{-}=\frac{X_\phi(X_\phi+X_2)+jX_\phi\dfrac{R_2}{2-S}}{(R_{2/2-S})^2+(X_\phi+X_2)^2}I_M^{-} \quad (54a)$$

We now can write the gross power output in terms of the component secondary currents.

$$WO_{\text{gross}}=\left[|I_{2M}^{+}|^2\left(\frac{R_2}{S}\right)+|I_{2S}^{+}|^2\left(a^2\frac{R_2}{S}\right)-|I_{2M}^{-}|^2\left(\frac{R_2}{2-S}\right)-|I_{2S}^{-}|^2\left(a^2\frac{R_2}{2-S}\right)\right](1-S) \quad (55)$$

From equations 9 and 10,

$$|I_{2S}^{+}|=\frac{|I_{2M}^{+}|}{a}\quad\text{and}\quad|I_{2S}^{-}|=\frac{|I_{2M}^{-}|}{a}$$

Substituting these in equation 55, we have

$$WO_{\text{gross}}=2\left[|I_{2M}^{+}|^2\frac{R_2}{S}-|I_{2M}^{-}|^2\frac{R_2}{2-S}\right]\times(1-S)\ \text{watts} \quad (55a)$$

Now observe that from equations 53a, 36, and 36a

$$|I_{2M}^{+}|^2\frac{R_2}{S}=\frac{X_\phi^2\dfrac{R_2}{S}}{(R_{2/S})^2+(X_\phi+X_2)^2}|I_M^{+}|^2$$

$$=R_2^{+1}|I_M^{+}|^2 \quad (56)$$

and from equations 54a, 38, and 38a,

$$|I_{2M}^{-}|^2\frac{R_2}{2-S}=\frac{X_\phi^2\dfrac{R_2}{2-S}}{\left(\dfrac{R_2}{2-S}\right)^2+(X_\phi+X_2)^2}|I_M^{-}|^2 \quad (57)$$

$$=R_2^{-\prime}|I_M^{-}|^2$$

so that equation 55a reduces to the more convenient form:

$$WO_{\text{gross}}=2[|I_M^{+}|^2R_2^{+\prime}-|I_M^{-}|^2R_2^{-\prime}](1-S)\ \text{watts} \quad (55b)$$

EFFICIENCY AND POWER FACTOR

By adding I_M (equation 47) and I_S (equation 50) the line current I_L can be obtained.

The "inphase" component of I_L multiplied by the line voltage (V_M) gives us the watts input, while $I_L\times V_M$ gives us the volt-ampere input.

$$\text{Power factor}=\frac{\text{watts input}}{\text{volt-amperes input}} \quad (58)$$

After subtracting the rotational losses (friction and windage, iron loss and load loss) from equation 55b, we have the net output watts, WO net

$$\text{Efficiency}=\frac{WO\ \text{net}}{\text{watts input}} \quad (59)$$

Part III. Some Cases of Particular Interest

EXACT TWO-PHASE MOTOR PERFORMANCE

A single-phase motor can be made to have phase balance and perform as a two-phase motor if the negative-sequence components of current can be made to vanish. From equations 46 and 49 we can write

$$O=\{(R_{1S}+R_C+a^2R_2^{+\prime})-a(X_{1M}+X_2^{+\prime})\}+j\{(X_{1S}-X_C+a^2X_2^{+\prime})+a(R_{1M}+R_2^{+\prime})\} \quad (46a)$$

$$O=\left\{(R_{1M}+R_2^{+\prime})+\frac{1}{a}(X_{1S}-X_C+a^2X_2^{+\prime})\right\}+j\left\{(X_{1M}+X_2^{+\prime})-\frac{1}{a}(R_{1S}+R_C+a^2R_2^{+\prime})\right\} \quad (49a)$$

By equating the real and imaginary components to zero separately, in each case

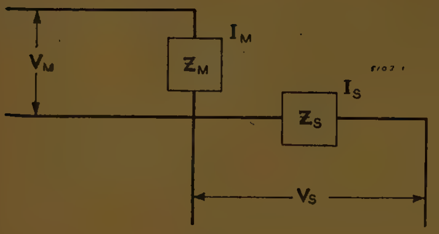

Figure 1

we can write the required condition for balance,

$$\begin{cases} R_C=a(X_{1M}+X_2^{+\prime})-(R_{1S}+a^2R_2^{+\prime}) & (60)\\ X_C=a(R_{1M}+R_2^{+\prime})+(X_{1S}+a^2X_2^{+\prime}) & (61)\end{cases}$$

This relationship holds for a particular value of speed as $X_2^{+\prime}$ and $R_2^{+\prime}$ are functions of slip. The condition cannot always be met as $(R_{1S}+a^2R_2^{+\prime})$ may be greater than $a(X_{1M}+X_2^{+\prime})$ which would require R_C to be negative. Completely eliminating the negative-sequence currents does not necessarily mean higher efficiency as additional losses may be incurred if R_C must be much greater than a small percentage of R_{1S}.

START PHASE DISCONNECTED

In the usual capacitor-start or resistance-split-phase motor the start phase is disconnected by centrifugal switch or relay action at one half to eight tenths of synchronous rpm. Of course, I_S becomes zero, but its symmetrical components still exist.

We can consider

$$R_C \xrightarrow{\lim} \infty$$

If ∞ is substituted for R_C in equations 45 and 46, they will take the indeterminate form ∞/∞. By differentiating numerators and denominators with respect to R_C and again substituting ∞ for R_C we have in the limit,

$$I_M^+=V_M\frac{1}{\{(R_{1M}+R_2^{+\prime})+j(X_{1M}+X_2^{+\prime})\}+\{(R_{1M}+R_2^{-\prime})+j(X_{1M}+X_2^{-\prime})\}} \quad (62)$$

$$=V_M\frac{1}{(2R_{1M}+R_2^{+\prime}+R_2^{-\prime})+j(2X_{1M}+X_2^{+\prime}+X_2^{-\prime})}$$

and

$$I_M^-=V_M\frac{1}{(2R_{1M}+R_2^{+\prime}+R_2^{-\prime})+j(2X_{1M}+X_2^{+\prime}+X_2^{-\prime})} \quad (63)$$

Note that $I_M^+\equiv I_M^-$

so that

$$I_M=I_M^++I_M^- \quad (7)$$

or

$$I_M=2I_M^+=2I_M^- \text{ for this case}$$

or

$$I_M^+=\frac{I_M}{2}=I_M^- \quad (64)$$

Therefore,

$$I_M=\frac{V_M}{\left(R_{1M}+\frac{R_2^{+\prime}+R_2^{-\prime}}{2}\right)+j\left(X_{1M}+\frac{X_2^{+\prime}+X_2^{-\prime}}{2}\right)} \quad (65)$$

or

$$I_M=V_M\frac{\left(R_{1M}+\frac{R_2^{+\prime}+R_2^{-\prime}}{2}\right)-j\left(X_{1M}+\frac{X_2^{+\prime}+X_2^{-\prime}}{2}\right)}{\left(R_{1M}+\frac{R_2^{+\prime}+R_2^{-\prime}}{2}\right)^2+\left(X_{1M}+\frac{X_2^{+\prime}+X_2^{-\prime}}{2}\right)^2} \quad (65a)$$

From equations 64 and 55b we can now obtain the output for this condition

$$WO_{\text{gross}}=2\left[\left|\frac{I_M}{2}\right|^2R_2^{+\prime}-\left|\frac{I_M}{2}\right|^2R_2^{-\prime}\right](1-S)$$

$$=|I_M|^2\left(\frac{R_2^{+\prime}-R_2^{-\prime}}{2}\right)(1-S) \text{ watts} \quad (66)$$

At standstill $(S=1)$ $R_2^{+\prime}=R_2^{-\prime}=R_2''$ since

$$\frac{R_2}{S}=\frac{R_2}{2-S}$$

Equation 66 shows that the output is zero in actual as well as synchronous watts. As the synchronous-watts output is zero there naturally can be no torque.

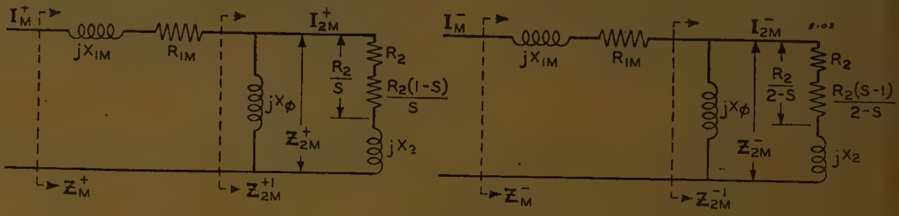

Figure 2

The standstill current input becomes

$$_{SS}I_M=\frac{V_M}{(R_{1M}+R_2'')+j(X_{1M}+X_2'')} \quad (67)$$

At synchronous speed $(S=0)$

Examining equation 36 we find that $Z_2^{+\prime}$ takes the indeterminate form $\infty/\infty + j\infty/\infty$. When numerators and denominators are differentiated with respect to R_2/S and ∞ is again substituted for R_2/S, the equation becomes:

$$_{S=0}Z_2^{+\prime}=jX_\phi \quad _{S=0}R_2^{+\prime}=0 \quad _{S=0}X_2^{+\prime}=jX_\phi \quad (68)$$

while

$$_{S=0}Z_2^{-\prime}=\frac{X_\phi^2\frac{R_2}{2}}{\frac{R_2^2}{4}+(X_\phi+X_2)^2}+j\frac{X_\phi\frac{R_2^2}{4}+X_\phi X_2(X_\phi+X_2)}{\frac{R_2^2}{4}+(X_\phi+X_2)^2} \quad (69)$$

From equation 65a we derive

$$I_M=V_M\frac{\left\{R_{1M}+\frac{X_\phi^2\frac{R_2}{4}}{R_2^2/4+(X_\phi+X_2)^2}\right\}-j\left\{X_{1M}+\frac{X_\phi}{2}\left[1+\frac{R_2^2/4+X_2(X_\phi+X_2)}{R_2^2/4+(X_\phi+X_2)^2}\right]\right\}}{\left\{R_{1M}+\frac{X_\phi{}^2R_2^2/4}{R_2^2/4+(X_\phi+X_2)^2}\right\}^2+\left\{X_{1M}+\frac{X_\phi}{2}\left[1+\frac{R_2^2/4+X_\phi(X_\phi+X_2)}{R_2^2/4+(X_\phi+X_2)^2}\right]\right\}} \quad (70)$$

VOLTAGE ACROSS START PHASE

Frequently there is need of knowing the voltage across the starting winding proper to make control applications either on the motor or for auxiliary apparatus.

Case of Capacitor Motor

$$V_{\phi S}=V_M-E_C \quad (71)$$

where

$$E_C=I_S(R_C-jX_C) \quad (72)$$

I_S is calculated from equation 50.

Case of Capacitor-Start or Resistance-Split-Phase Motor After the Starting Switch Opens the Start Phase

If

$$R_C \xrightarrow{\lim} \infty$$

equations 48and 49 take the indeterminate form ∞/∞, but by differentiation it will be found that

$$I_S^+=V_M\frac{j/a}{(R_{1M}+R_2^{+\prime})+j(X_{1M}+X_2^{+\prime})+(R_{1M}+R_2^{-\prime})+j(X_{1M}+X_2^{-\prime})} \quad (73)$$

$$I_S^+=V_M\frac{j/a}{(2R_{1M}+R_2^{+\prime}+R_2^{-\prime})+j(2X_{1M}+X_2^{+\prime}+X_2^{-\prime}} \quad (73a)$$

and

$$I_S^-=V_M\frac{-j/a}{(2R_{1M}+R_2^{+\prime}+R_2^{-\prime})+j(2X_{1M}+X_2^{+\prime}+X_2^{-\prime})} \quad (74)$$

Note that $I_S^+ = -I_S^-$ and $I_S = I_S^+ + I_S^- = 0$

$$V_{S\phi}^{+\prime}=I_S^+a^2Z_2^{+\prime} \quad (75)$$

$$V_{S\phi}^-=I_S^-a^2Z_2^{-\prime}=-I_S^+a^2Z_2^{-\prime} \quad (76)$$

If we add

$$V_{S\phi}=V_{S\phi}^++V_{S\phi}^-=I_S^+a^2(Z_2^{+\prime}-Z_2^{-\prime})$$

$$=V_M\frac{j/a\{a^2(R_2^{+\prime}-R_2^{-\prime})+ja^2\times(X_2^{+\prime}-X_2^{-\prime})\}}{\{(2R_{1M}+R_2^{+\prime}+R_2^{-\prime})+j(2X_{1M}+X_2^{+\prime}+X_2^{-\prime})\}} \quad (77)$$

$$V_{S\phi}=V_{M}a\frac{-(X_2{}^{+\prime}-X_2{}^{-\prime})+j(R_2{}^{+\prime}-R_2{}^{-\prime})}{(2R_{1M}+R_2{}^{+\prime}+R_2{}^{-\prime})+j(2X_{1M}+X_2{}^{+\prime}+X_2{}^{-\prime})} \quad (77a)$$

Observe that at standstill, when

$X_2{}^{+\prime}=X_2{}^{-\prime}=X_2''$ and $R_2{}^{+\prime}=R_2{}^{-\prime}=R_2''$

$V_{S\phi}$ becomes zero

STARTING TORQUE

In routine design work the calculation of starting torque, together with the capacitor voltage or starting current, is of particular importance as the running performance can often be proportioned accurately from existing similar tested designs.

$$\text{Let } I_M=A+jB=\frac{V_M}{Z_M} \quad (78)$$

$$\text{Let } I_S=C+jD=\frac{V_M}{Z_S} \quad (79)$$

Since

$R_2{}^{+\prime}=R_2{}^{-\prime}=R_2''$ and $X_2{}^{+\prime}=X_2{}^{-\prime}=X_2''$
$\mathbf{Z}_S{}^{+}=\mathbf{Z}_S{}^{-}=\mathbf{Z}_S$ and $\mathbf{Z}_M{}^{+}=\mathbf{Z}_M{}^{-}=\mathbf{Z}_M$

Equation 30 becomes

$$I_M{}^{+}=V_M\frac{Z_S-jaZ_M}{2Z_SZ_M}=\frac{V_M}{2Z_M}-ja\frac{V_M}{2Z_S} \quad (80)$$

or

$$I_M=\frac{I_M}{2}-ja\frac{I_S}{2} \quad (80a)$$

and equation 28 becomes

$$I_M{}^{-}=V_M\frac{Z_S+jaZ_M}{2Z_MZ_S}=\frac{V_M}{2Z_M}+ja\frac{V_M}{2Z_S} \quad (81)$$

or

$$I_M{}^{-}=\frac{I_M}{2}+ja\frac{I_S}{2} \quad (81a)$$

Now if we use equations 78 and 79 in equation 80a,

$$I_M{}^{+}=\left(\frac{A}{2}+j\frac{B}{2}\right)-ja\left(\frac{C}{2}+j\frac{D}{2}\right)$$
$$=\left(\frac{A}{2}+a\frac{D}{2}\right)+j\left(\frac{B}{2}-a\frac{C}{2}\right) \quad (82)$$

$$|I_M{}^{+}|^2=\frac{(A+aD)^2}{4}+\frac{(B-aC)^2}{4} \quad (83)$$

Similarly if we use equations 78 and 79 in equation 81a,

$$I_M{}^{-}=\left(\frac{A}{2}+j\frac{B}{2}\right)+ja\left(\frac{C}{2}+j\frac{D}{2}\right)$$
$$=\left(\frac{A}{2}-a\frac{D}{2}\right)+j\left(\frac{B}{2}+a\frac{C}{2}\right) \quad (84)$$

$$|I_M{}^{-}|^2=\frac{(A-aD)^2}{4}+\frac{(B+aC)^2}{4} \quad (85)$$

and

$$|I_M{}^{+}|^2-|I_M{}^{-}|^2=a(AD-BC) \quad (86)$$

to obtain the theoretical torque we will combine equations 52, 55b, and 85 and at the same time eliminate the motor speed by the following relationship

rpm $=(1-S)$ synchronous rpm

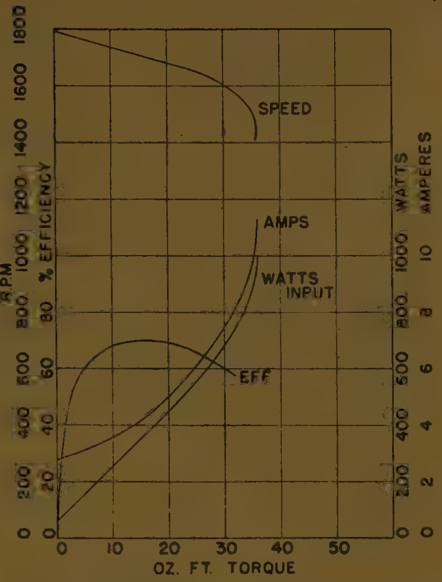

Figure 3. Calculated one-quarter-horsepower-motor performance data

so that

$$T=\frac{112.7}{(1-S)\text{synchronous}}\times 2R_2''a(AD-BC)(1-S) \quad (87)$$

This is an indeterminate of the form 0/0 for $S = 1$, but

$$\lim_{S\to 1}\frac{1-S}{1-S}=1$$

$$T=\frac{225.4}{\text{synchronous}}R_2''a(AD-BC)\text{ ounce-feet} \quad (87a)$$

For the usual capacitor motor:

A is positive.
B is negative.
C is positive.
D is positive.

For the usual resistance-split-phase motor:

A is positive.
B is negative.
C is positive.
D is negative.

This theoretical value of standstill torque must be modified by an empirical factor to take account of friction, iron loss, and tooth-locking effects in a practical design.

Conclusions

The author has endeavored to demonstrate the versatility of the symmetrical-component method in the solution of single-phase induction motors. The theory is "continuous" mathematically in that special cases are set up simply by allowing some parameter to be zero or infinity to fit the physical aspects of the problem.

The derivations are general enough so that, if R_2 is not the same for the low-frequency positive-sequence rotor currents as it is for the higher-frequency negative-sequence rotor currents, allowances can be made in the calculations.

Nomenclature

V_M = main-phase rms volts
V_S = start-phase rms volts
$V_M{}^{+}$ = positive-sequence component of V_M
$V_M{}^{-}$ = negative-sequence component of V_M
$V_S{}^{+}$ = positive-sequence component of V_S
$V_S{}^{-}$ = negative-sequence component of V_S
I_M = main-phase rms amperes
I_S = start-phase rms amperes
$I_M{}^{+}$ = positive-sequence component of I_M
$I_M{}^{-}$ = negative-sequence component of I_M
$I_S{}^{+}$ = positive-sequence component of I_S
$I_S{}^{-}$ = negative-sequence component of I_S
$a=\frac{\text{effective conductors of } S \text{ phase}}{\text{effective conductors of } M \text{ phase}}$
s = per-unit slip
$I_{2M}{}^{+}$ = positive-sequence component of M-phase secondary current referred to primary current
$I_{2M}{}^{-}$ = negative-sequence component of M-phase secondary current referred to primary current
$I_{2S}{}^{+}$ = positive-sequence component of S-phase secondary current referred to primary current
$I_{2S}{}^{-}$ = negative-sequence component of S-phase secondary current referred to primary current
$Z_M{}^{+}$ = total positive-sequence impedance of M phase
$Z_M{}^{-}$ = total negative-sequence impedance of M phase
$Z_S{}^{+}$ = total positive-sequence impedance of S phase
$Z_S{}^{-}$ = total negative-sequence impedance of S phase
X_{1M} = reactance of M-phase primary winding
R_{1M} = resistance of M-phase primary winding
R_2 = equivalent M-phase secondary resistance referred to primary winding
X_2 = equivalent M-phase secondary reactance referred to primary winding
X_ϕ = equivalent M-phase mutual or magnetizing reactance
$\mathbf{Z}_2{}^{+}=\frac{R_2}{S}+jX_2$ = positive-sequence secondary impedance of M phase referred to primary winding
$\mathbf{Z}_2{}^{-}=\frac{R_2}{2-S}+jX_2$ = negative-sequence secondary impedance of M phase referred to primary winding
$R_2{}^{+\prime}$ = apparent positive-sequence M-phase secondary resistance

$$=\frac{X_\phi{}^2R_2/s}{(R_2/s)^2+(X_\phi+X_2)^2}$$

$R_2{}^{-\prime}$ = apparent negative-sequence M-phase secondary resistance

$$=\frac{X_\phi{}^2\dfrac{R_2}{2-S}}{\left(\dfrac{R_2}{2-S}\right)^2+(X_\phi+X_2)^2}$$

$X_2^{+\prime}$ = apparent positive-sequence M-phase secondary reactance

$$= \frac{X_\phi(R_2/s)^2 + X_\phi X_2(X_\phi + X_2)}{(R_2/s)^2 + (X_\phi + X_2)^2}$$

$X_2^{-\prime}$ = apparent negative-sequence M-phase secondary reactance

$$= \frac{X_\phi(R_2/(2-S))^2 + X_\phi X_2(X_\phi + X_2)}{(R_2/(2-S))^2 + (X_\phi + X_2)^2}$$

$Z_2^{+\prime} = R_2^{+\prime} + jX_2^{+\prime}$ Z_2'' = standstill apparent secondary impedance

$Z_2^{-\prime} = R_2^{-\prime} + jX_2^{-\prime}$ R_2'' = standstill apparent secondary resistance

X_2'' = standstill apparent secondary reactance

$Z_{1M}^{+} = Z_{1M}^{-} = R_{1M} + jX_{1M}$ = primary impedance

R_{1S} = resistance of S-phase primary winding

X_{1S} = reactance of S-phase primary winding

X_C = capacitor reactance

R_C = capacitor resistance or other external resistance in S phase

$T_{2\phi}$ = two-phase-motor torque, ounce-feet

Synch = synchronous rpm

WO = gross watts output of rotor

T = single-phase-motor torque, ounce-feet

E_C = capacitor voltage

$V_{S\phi}$ = voltage across S-phase winding

Sample Calculation

One-quarter horsepower, four pole, 110 volts 60 cycle; capacitor-start motor

Constants

$R_2 = 2.48$ (25 degrees centigrade)

$R_1 = 1.63$ (25 degrees centigrade)

$X_1 = 2.47$, $X_2 = 2.44$, $X_\phi = 67.28$

Friction loss $= W_f = 16.5$ watts

Iron loss $= W_i = 23.5$ watts

$S = 0.038$

Rpm $= (1-0.038)\ 1{,}800 = 1{,}732$

$R_{2/S} = 65.25 \quad R_{2/2-S} = 1.264$

$$R_2^{+\prime} = \frac{67.28^2 \times 65.25}{65.25^2 + (67.28+2.44)^2} = 32.40$$

$$R_2^{-\prime} = \frac{67.28^2 \times 1.264}{1.264^2 + (67.28+2.44)^2} = 1.177$$

$$X_2^{+\prime} = \frac{67.28 \times 65.25^2 + 67.28 \times 2.44(67.28+2.44)}{65.25^2 + (67.28+2.44)^2} = 32.70$$

$$X_2^{-\prime} = \frac{67.28 \times 1.264^2 + 67.28 \times 2.44(67.28+2.44)}{1.264^2 + (67.28+2.44)^2} = 2.375$$

$$I_M = 110 \left[\frac{\left(1.63 + \frac{32.40+1.18}{2}\right) - j\left(2.47 + \frac{32.70+2.38}{2}\right)}{\left(1.63 + \frac{32.40+1.18}{2}\right)^2 + \left(2.47 + \frac{32.70+2.38}{2}\right)^2} \right]$$

$$= 110 \left[\frac{18.42 - j20.01}{339.5 + 441} \right]$$

$$= 2.595 - j2.962 \quad |I_M| = 3.94$$

$$|I_M|^2 = 15.51$$

$$WO_{\text{gross}} = 15.51(1-0.038)\left(\frac{32.40-1.18}{2}\right)$$

$$= 15.51 \times 0.962 \times 15.61 = 233.2 \text{ watts}$$

$$WO_{\text{net}} = 233.2 - 16.5 - 23.5 = 193.2$$

Torque $= (112.7/1732) \times 193.2 = 12.55$ ounce-feet

Watts input $= 110 \times 2.595 = 285.5$

Eff $= 193.2/285.5 = 0.695$

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Automatic Load Control for Turbine Generators

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Synopsis: This paper describes a load-control apparatus recently installed on two 20,000-kw turbines at the power plant of the Tennessee Coal Iron and Railroad Company at Fairfield, Ala., to reduce the magnitude of load swings imposed on a tie line by a hot-strip mill in the Tennessee Coal Iron Company plant. The power plant contains three 20,000-kw turbine-driven generators and is tied in with the Alabama Power Company to furnish the steel-mill load. Previous to the installation of the equipment herein described, the three 20,000-kw turbine generators have carried load under control of their speed governors and the variations in strip-mill load which are about 20,000 kw were imposed upon the Alabama Power Company tie line causing objectionable disturbances on the connecting power system. It was desired to develop a means of imposing all or part of the strip-mill load on the Tennessee Coal Iron and Railroad Company turbine generators to reduce these disturbances. Several methods were considered, and the one selected was based upon a static watt-measuring circuit, the output of which is an Amplidyne generator. The Amplidyne in turn operates the pilot motor of an hydraulic (oil) relay at the turbine to control the steam admission valves.

GIVEN a base load and a variable load and a variable supply of steam for either one of two prime movers, the regulator is to cause the prime movers to take

(*a*). A base load, adjustable from $^1/_{20}$th rated load to rated load.

(*b*). In addition to condition *a* any desired percentage of the variable load from 0 to 100 per cent.

(*c*). Both the base load and the variable load are to be reduced automatically as the steam pressure falls below a nominal pressure.

(*d*). The variable load is to be divided in any desired ratio between the two prime movers by simple adjustment.

(*e*). The variable load may have instantaneous rates of change as high as 5,000 kw per second with average rates of approximately 2,000 kw per second during load increase and decrease. The variable load peaks are approximately 20,000 kw in magnitude of 20 to 100 seconds duration with intervals of light load of about the same duration. It is desired to minimize the variation of load on the tie line insofar as practical.

In addition, the governing system is desired to have the following features:

(*a*). A definite limit on turbine speed must at all times be in opeation.

(*b*). It must be possible for operating either or both turbines on their speed governors.

(*c*). It must be possible to change easily from power to speed governing and vice versa.

(*d*). The power-governing system must be fast in response and be stable in operation.

Fundamentals of Electric-Power-Measuring Circuit

As balanced three-phase loads are to be governed a single-phase power-measuring circuit suffices to control the operation. Also, as the machines operate at constant voltage it is sufficient to control the unity power factor or kilowatt component of generator current.

The essentials of the power-measuring circuit are shown in Figure 1.

The Amplidyne generator has been described adequately elsewhere.[1–4] For pres-

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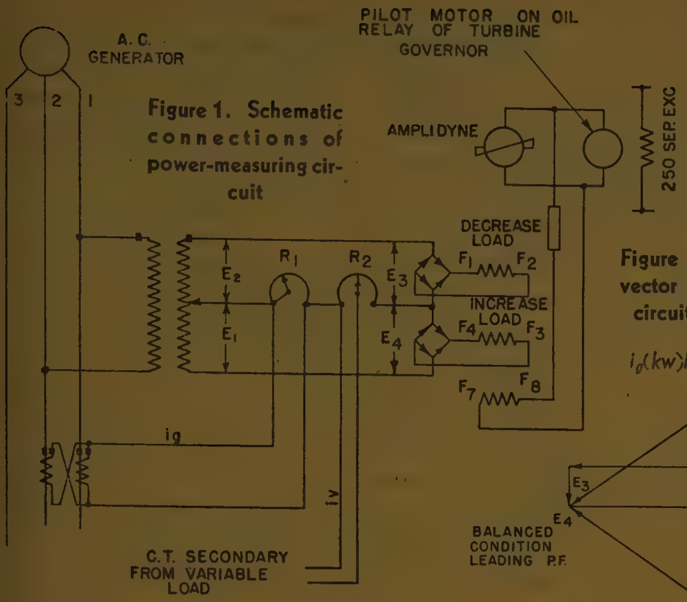


Figure 1. Schematic connections of power-measuring circuit

Figure 2 (right). Voltage vector diagram of measuring circuit at zero load setting

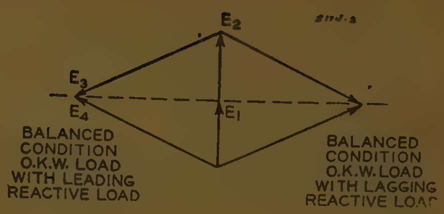
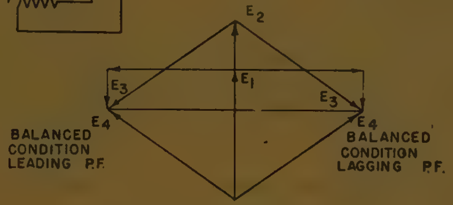

E_2
E_3
E_4
E_1
BALANCED CONDITION O.K.W. LOAD WITH LEADING REACTIVE LOAD
BALANCED CONDITION O.K.W. LOAD WITH LAGGING REACTIVE LOAD

Figure 3 (below). Voltage vector diagram of measuring circuit at fixed load setting

$$i_g(kw)R_1=\frac{E_1+E_2}{2}-E_2=\frac{E_1-E_2}{2}$$


E_2
E_3
E_1
E_3
E_4
E_4
BALANCED CONDITION LEADING P.F.
BALANCED CONDITION LAGGING P.F.

ent purposes it is sufficient to say that the Amplidyne output voltage is extremely sensitive to a difference between the ampere turns of the fields F_1F_2 and F_3F_4 which are excited in opposite directions (so as to oppose each other) from selenium rectifiers as shown. The Amplidyne output voltage will be zero, and the motor which operates the hydraulic governing mechanism will be at rest only when the currents in these two fields are equal and opposite. As the two fields have the same number of turns and the same resistance this balance occurs only when the alternating voltages at the rectifier inputs are equal in magnitude. Assume for the moment that the adjustable voltage tap is set so that $E_1=E_2$ and that $i_v=0$ in Figure 1. Then $|E_3|=|E_4|$ only when the kilowatts or unity-power-factor component of current i_g is zero regardless of the reactive component of current i_g. Figure 2 illustrates this.

If the adjustable tap is set so that E_1 is greater than E_2 while the sum of E_1 and E_2 remains constant, the balanced condition when $|E_3|=|E_4|$ will occur only when the kilowatt load on the turbine generator is constant so that the inphase component of $i_g\times R_1=(E_1-E_2)/2$.

Figure 3 illustrates this condition at various power factors on the generator.

From the foregoing it is evident that for a fixed value of R_1 and with R_2 set at zero the load on the turbine when the Amplidyne output voltage is zero is proportional to the voltage between the midtap of the potential-transformer secondary winding and the position of the adjustable tap. The adjustable tap then is used as the base-load setting of the generator.

If R_2 is not zero the load on the turbine when $|E_3|=|E_4|$ will be such that

$$i_g\times R_1=\frac{E_1-E_2}{2}+i_v\times R_2$$

where

i_g = unity-power-factor component of generator current

i_v = unity-power-factor component of variable load current

By setting the adjustable voltage tap the base load of the turbine is fixed at any desired value and by setting the value of R_2 any desired percentage of the variable load up to 100 per cent may be carried.

In order to decrease both the base-load setting and the percentage of the variable load taken by the turbine the value of R_1 is increased automatically as the steam pressure decreases below a set value.

In Figure 1 field F_7F_8 of the Amplidyne is connected as an inverse feedback of the Amplidyne output into its input. This results in a more linear relationship between the Amplidyne input and output than would be the case without this connection and gives a means of adjusting rate of operation of the governing mechanism as a function of the deviation between the load on the turbine and the load setting of the regulator

Electrical Connections

A schematic diagram of the electric-system connections and of the power regulators is shown in Figure 4. In order to make the kilowatt component of turbogenerator current in phase with the voltage to the regulator circuit it was necessary to add the currents of phases 1 and 2 when using potential 1–2. This was done with current transformers in the secondary circuits of existing current transformers. Owing to existence of a wye–delta transformer bank between the generator and the strip-mill circuit a single current transformer in phase 1 gave the required phase relationship of strip-mill current with respect to the regulator voltage. This latter current is transmitted approximately 4,000 feet to the control room of the power plant. As it was desired to shift the variable load between the two power-regulated generators without disturbing the percentage of strip-mill load carried an adjustable-ratio current transformer was used to divide the strip-mill current between the two regulators in any ratio so that the sum of the currents to the two regulators remains in constant ratio to the strip-mill current. A "variac" was used as a means of adjusting the relationship of voltages E_1 and E_2, and its setting determined the base load. The transformers used between

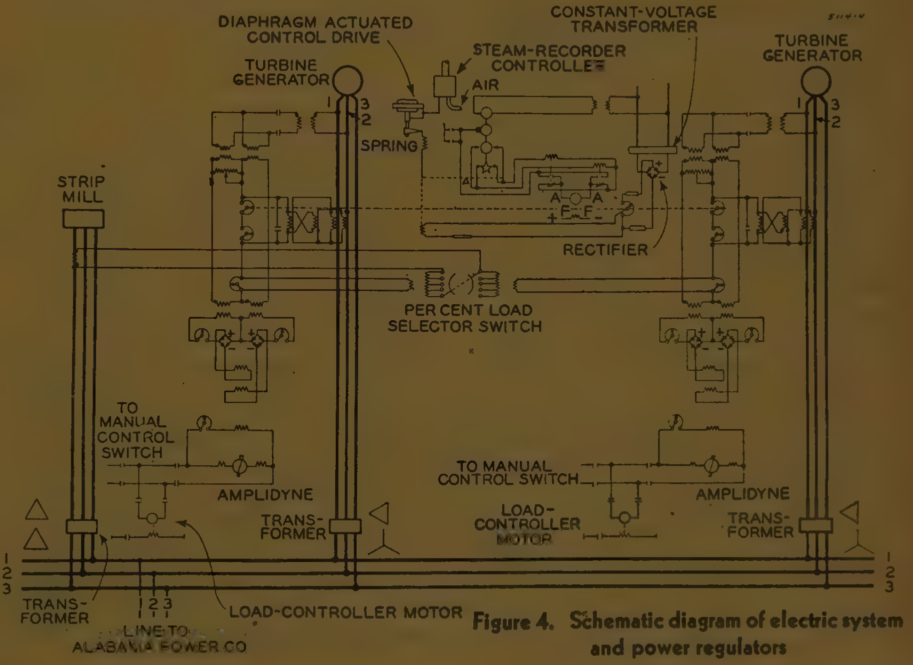


Figure 4. Schematic diagram of electric system and power regulators

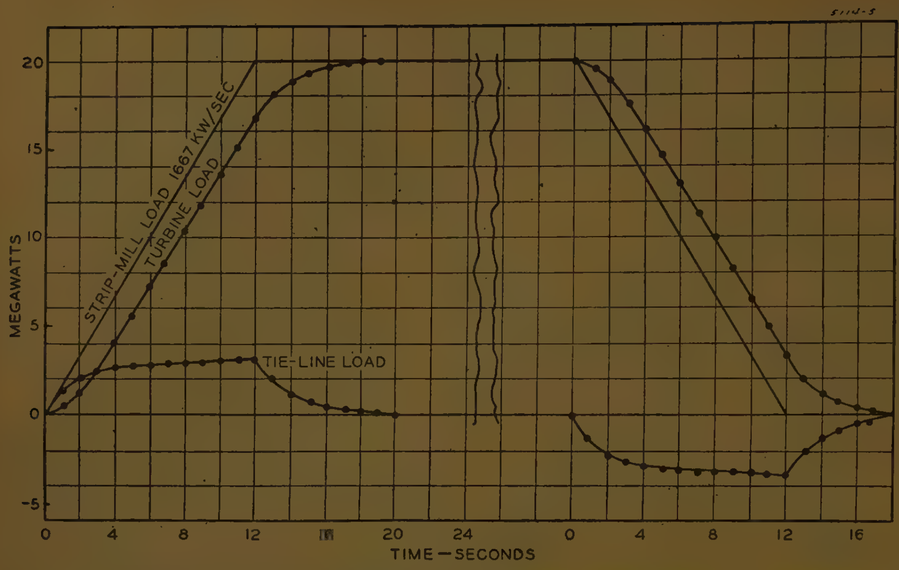


Figure 5. Calculated response to assumed load cycle having constant rate of load change of 1,667 kw per second

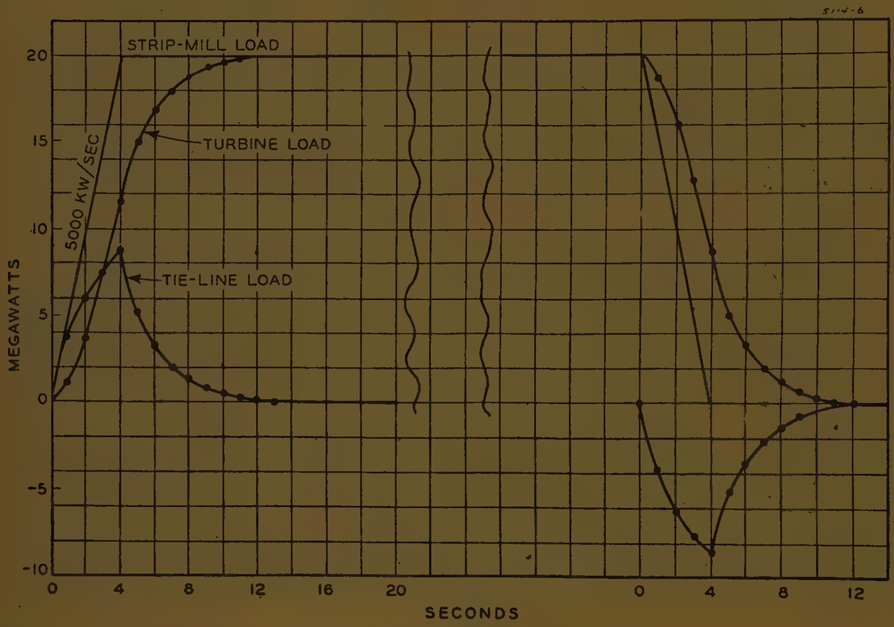


Figure 6. Calculated response to assumed load cycle having constant rate of load change of 5,000 kw per second

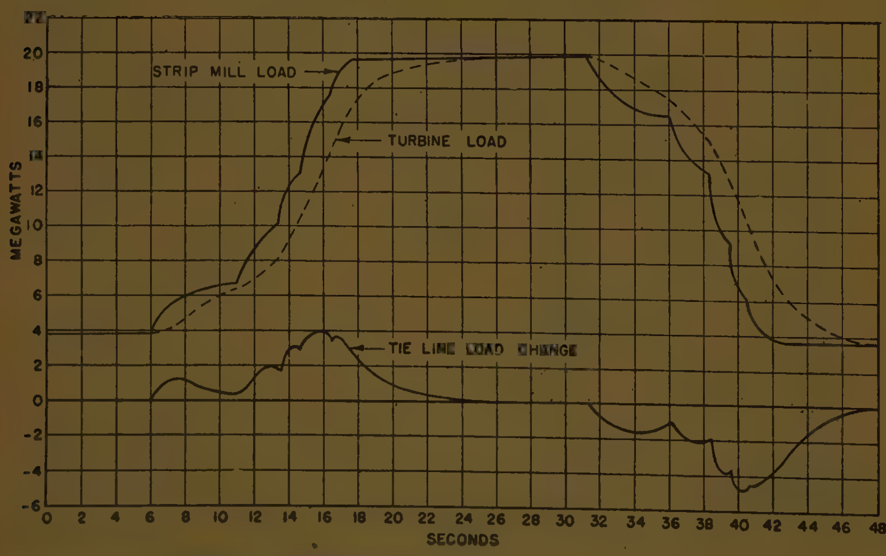


Figure 7. Actual load cycle and calculated response

the power-measuring circuit and the rectifiers supplying the opposing Amplidyne fields were for the purpose of reducing the voltages to the rectifiers, though it would have been possible to rectify these voltages directly without the transformers. An adjustable resistor is connected in series with each Amplidyne field to match exactly the currents to the two fields with the base-load setting at zero and with zero current from the generator current transformers. Resistors in the current-transformer circuits were chosen to obtain the desired range of adjustment of base load and percentage of strip-mill load, with both the potential and the current transformer ratios taken into account.

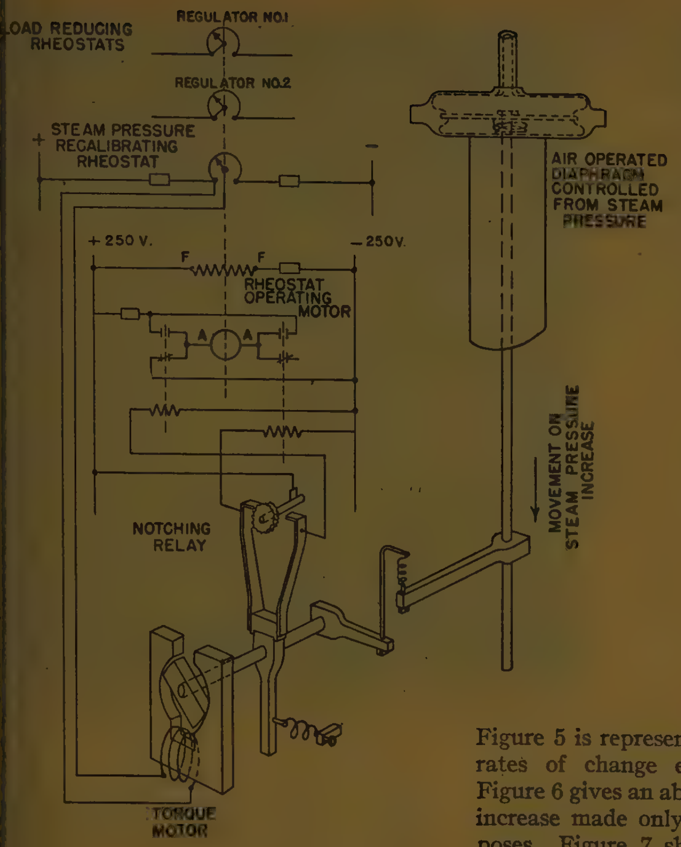
Owing to the high amplification of the Amplidyne, it was easily possible to obtain fields which would withstand the highest currents to which they are subjected (which vary with power factor) and at the same time to obtain more than adequate sensitivity to power variation.

Sensitivity

The Amplidyne voltage output per kilowatt difference between regulator setting and turbogenerator load may be adjusted by means of the variable resistor in series with field F_7F_8 of the Amplidyne. Without this feedback it requires approximately 2,000 kw deviation between regulator setting and generator load to run the pilot motor through full travel in four seconds. In practice it was necessary to adjust the feedback to obtain stable operation so that the deviation required to cause the generator to increase its load at a rate of 5,000 kw per second was approximately 6,500 kw. Final adjustment with which the operation data were taken was with 10,000 kw deviation required to give a rate of 5,000 kw per second. With this adjustment the kilowatt deviation to cause the Amplidyne voltage necessary just to rotate the pilot motor against friction was approximately 200 kw, so that with a setting of 20,000 kw the power is held within plus or minus one per cent during steady state.

Response

All of the regulating elements are relatively fast in response in comparison with the time required to operate the pilot motor of the governing mechanism. This time together with the kilowatts deviation required to obtain a given pilot-motor speed determines the response of the system. Also, it will be shown that these factors, together with the rate of change of the variable load and the percentage of the variable load carried by the Tennessee Coal Iron and Railroad Company turbines, determine the effectiveness of the regulating system


LOAD REDUCING RHEOSTATS
REGULATOR NO.1
REGULATOR NO.2
STEAM PRESSURE RECALIBRATING RHEOSTAT
+
−
+ 250 V.
− 250V.
F
F
RHEOSTAT OPERATING MOTOR
A
A
NOTCHING RELAY
TORQUE MOTOR
AIR OPERATED DIAPHRAGM CONTROLLED FROM STEAM PRESSURE
MOVEMENT ON STEAM PRESSURE INCREASE

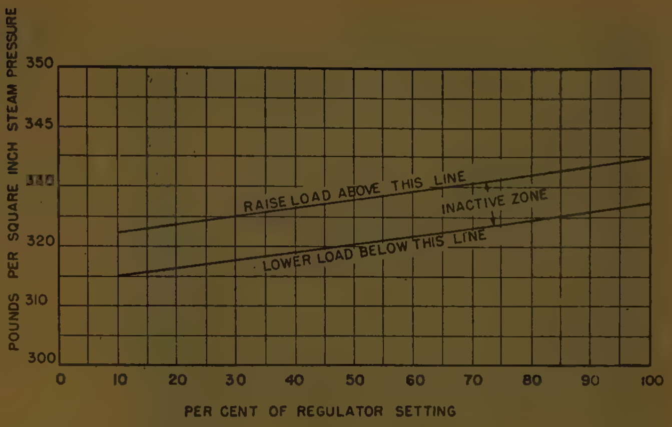


Figure 9 (above). Characteristics of steam-pressure regulator

Figure 8 (left). Schematic diagram of steam-pressure-regulating system

in minimizing the variation in tie-line load.

Let

T_{kw} = kilowatt load on turbine generator
L_{kw} = kilowatt setting of load regulator
$P=\frac{d}{dt}$
K = proportionality constant between rate of change of turbine load and deviation of load from regulator setting

Then

$$KP\,T_{kw}=(L_{kw}-T_{kw})$$

$$T_{kw}=\frac{L_{kw}}{KP+1}$$

With

T_{kw} = 5,000 kw per second
$L_{kw}-T_{kw}$ = 10,000 kw
$K=2$

The solution for unit change of L_{kw} (caused by a change of strip-mill load) is

$$T_{kw}=L_{kw}(1-\epsilon^{-1/2t})$$

If now L_{kw} is varying at a constant rate R it can be shown by the superposition theorem that (if we assume $L_{kw}=0$ when $T=0$)

$$T_{kw}=R(t-2+2\epsilon^{-1/2t})$$

With an assumed load cycle as in Figure 5 the response and the variation in tie-line load were calculated as shown in this same figure from this formula and the preceding one. A similar curve is shown in Figure 6 for a rate of strip-mill-load increase of 5,000 kw per second. Figure 5 is representative of the average rates of change encountered, whereas Figure 6 gives an abnormally high rate of increase made only for comparison purposes. Figure 7 shows an actual strip-mill-load peak taken from a load chart. The calculated turbine load based upon 100-per-cent pickup of strip-mill load and the load on the tie line are calculated and shown in this same figure.

Means of Reducing Loads Upon Decrease in Steam Pressure

The means of reducing both the base load and the variable load in equal percentages upon reduction in steam pressure was to increase resistance R_1, Figure 1. A Bailey Meter Company pilot valve was used to give a definite motion of a spring opposing the action of a torque motor to control a motor-operated rheostat to vary the value of R_1. Figure 8 shows a schematic diagram of this equipment.

For full normal steam pressure or above the torque-motor notching system is balanced with the rheostat all cut out. For any steam pressure within a range of 30 pounds per square inch, that is, from 340 pounds per square inch to 310 pounds per square inch the rheostat is notched until the value of load is decreased to a point where the torque motor is balanced. The notching contacts have adjustable dead band, and the action is to notch to the center of the dead band so that the load will be reduced to hold an average steam pressure. Figure 9 shows the characteristics of the steam-pressure regulator. This steam-pressure regulator is intended primarily as a means of reducing the loads to keep the steam pressure above a certain minimum. It can operate continuously in range to regulate steam pressure. Normally, however, it was intended that the load settings would be such that full steam pressure could be maintained, with the steam pressure above the range of the pressure regulator. Figure 10 shows the control panels for the two turbine generators. The panel at the left is for one generator. The middle panel contains the variable-ratio current transformer by means of which the strip-mill load may be divided between the two machines. The third panel from the left duplicates the first panel for the other turbine. On the extreme right is the steam-pressure controller. The base-load adjustment, regulator on-and-off switch, and the percentage of strip-mill load all are operated from the front of these panels.

Turbine Valve Control

The schematic arrangement of the turbine control is shown on Figure 11. The load-control relay unit and the initial pressure regulator were added to the original governor unit and connected with a telescope joint in the governor connection so that, while the load control is used, the speed governor can be run to its full-load position and act as a pre-emergency governor. In this condition the telescope joint is pulled open and the the load-control piston rod controls the main pilot valve through connection F and E. The initial pressure-regulator piston rod can pick up through connection D. These three connections (F, D, and E) will transmit motion only in one direction, so that any one of them may close the control valves regardless of the setting of the others. This allows the speed governor to close the valves in case of loss of load and the pressure regulator to close the valves in case of loss of pressure.

The load-control unit has a shunt-wound d-c motor with constant excitation from a 250-volt source. The armature current comes from the Amplidyne generator. The motor operates a lead screw raising and lowering the relay pilot valve and so raises and lowers the relay piston. A friction clutch is provided in the drive so that no limit

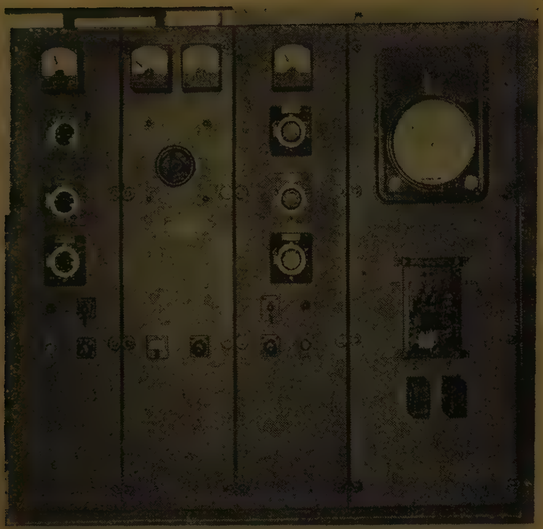


Figure 10 (left). Load-control panel installation

Figure 12 (right). Load-control unit of turbine-valve control

switches are necessary and limit-switch dead band is eliminated. An oil spray lubricates the clutch to prevent wear in case the motor runs long periods of time at the end of the travel. Motion of the relay piston is transmitted to the main pilot valve and so to the main hydraulic piston, opening and closing the control valves through the rack and pinion and the cams.

The pressure regulator consists of a spring-loaded bellows which operates a relay pilot valve and controls an oil piston. The spring plug may be adjusted by handwheel *C* so that the regulator will close the valves at pressures from 275 to 350 pounds per square inch. A stop is provided on both the load control and the pressure regulator so that they cannot close the turbine valves beyond the no-load steam-flow point and cause overheating of the turbine by motoring. The speed governor always can close the valves completely on a rise in speed to the pre-emergency setting. However, an adjustment is provided on the load-control piston rod so that when the turbine is shut down the control valves can be closed completely, and then handwheel *A* on the pilot-motor shaft can be used to bring the turbine to speed. There is a switch on the load-control unit to open the motor circuit so that the handwheel can be used without danger of the motor starting up. The load-control and initial pressure regulator are enclosed in a small case mounted on the turbine base adjacent to the front pedestal. Figure 12 is a view of the installation.

The rates of load change imposed by this regulator will exceed what normally is considered good practice from the standpoint of economy and maintenance, but the 20,000-kw turbines at Tennessee Coal Iron and Railroad Company are low pressure and temperature and of a rugged design with a relatively short span between bearings; they should be able to withstand the quick load changes without undue distress. There most likely will be some increased maintenance necessary on machines subjected to continuous quick load changes, especially on the control mechanism, valves, cams, rack, and pinion, and so on.

The actual performance of the load-control unit was as good as anticipated. Load changes were made quickly, positively, and without hunting.

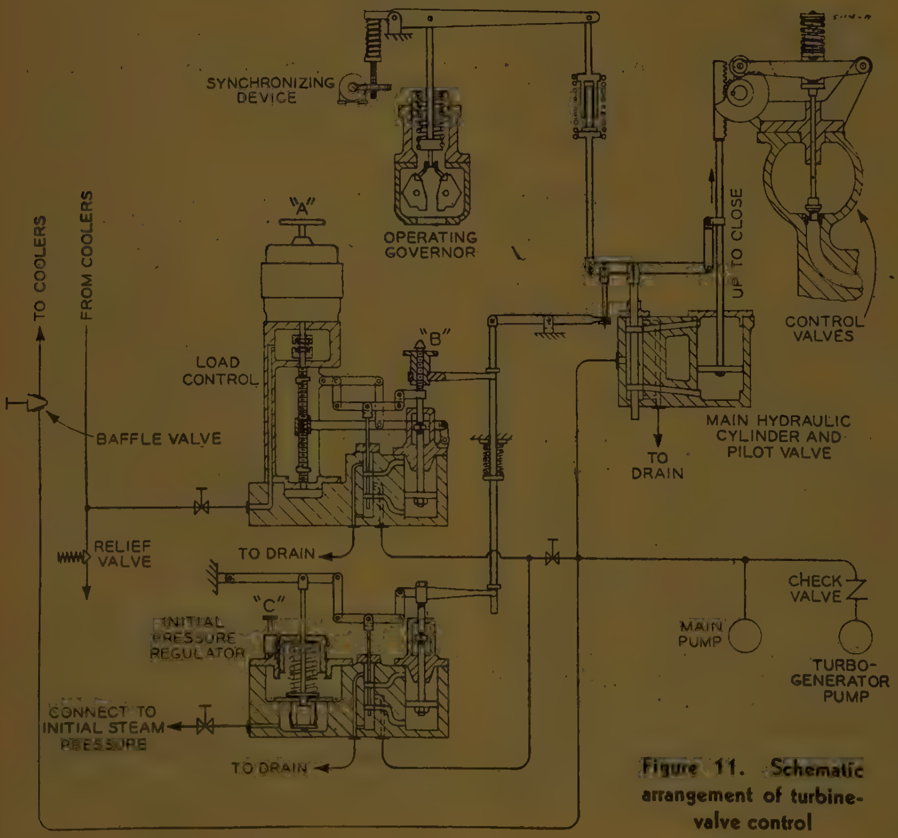


Figure 11. Schematic arrangement of turbine-valve control

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Tapered-Thickness Bimetal

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Synopsis: The most important properties of thermostatic bimetal strips of uniformly tapered width or thickness or both have been analyzed mathematically, and the findings are described in this paper. A tapered-thickness form is found to be the best practical one from the standpoint of economy of material. Conveniently usable charts are included which are applicable in the form of correction coefficients to the standard formulas for thermostatic deflection and mechanical deflection of strips of uniform thickness and width. The dual-tapered form is especially useful in reducing vibration troubles in slow-acting thermostats, and formulas are given for determining the fundamental frequency of single- or dual-tapered strips of any taper ratio.

THERMOSTATIC BIMETALS of the "low-temperature" or brass–Invar type have been used for many years in the construction of innumerable thermostatic-control devices. The substitution of stronger and more active metals for the brass component by Spencer and others during the '20's led to an impressive extension of the use of bimetals. The announcement of the manganese–Invar bimetal in 1942 gave the industry a further remarkable fundamental improvement.

Bimetals now are used almost universally in sheet form, and extensive work has been done in development of various outlines and bent shapes for which well-known simplified deflection and force formulas are available. In addition, a limited amount of progress has been made toward the useful development of bimetal rods, cones, and tapered-thickness sheets. The last-mentioned form gives promise of a substantially increased effectiveness in utilization of the material, and this, coupled with discoveries of inexpensive fabrication methods, has led to the present exploration of its characteristics. The results of these studies have been interpreted into suitable form for convenient engineering use.

Although it is well-known that unloaded bimetal strips deflect into circular curvatures in response to temperature changes, the temperature-deflection formulas and constants in common use are based upon the assumption of parabolic curvatures. Such an approximation is reasonably acceptable, as the errors are very small for the short arcs involved and the parabolic formulas are vastly simpler than the circular ones. The assumption of parabolic curvatures will be retained throughout the present studies.

Deflection Characteristics

The deflection characteristic of an initially flat bimetal strip of uniform thickness, supported at one end and subject to uniform temperature variations, is given approximately by the parabolic expression:

$$\delta = \frac{K\Delta l^2}{t_0} \tag{1}$$

where

K = deflection constant governed by composition of strip
l = distance along strip from support
t_0 = thickness of strip
Δ = temperature change of strip
δ = deflection of strip at distance l from support, due to temperature change Δ

If we differentiate equation 1, the slope of the strip at any point is

$$\frac{d\delta}{dl} = \frac{2K\Delta l}{t_0} \tag{2}$$

and if we differentiate again the rate of change of slope is

$$\frac{d^2\delta}{dl^2} = \frac{2K\Delta}{t_0} \tag{3}$$

If we consider now a bimetal strip of variable thickness t, the rate of change of slope will be

$$\frac{d^2\delta}{dl^2} = \frac{2K\Delta}{t} \tag{4}$$

If the strip tapers uniformly, it will appear as shown in Figure 1, where

L = length of strip
t_0 = thickness of strip at base or support
t_1 = thickness of strip at free end
t = thickness of strip at any point
l = distance along strip from t_0 to t_1
b = width
R_t = taper ratio = $\dfrac{t_1}{t_0}$

It will be seen from Figure 1 that

$$\frac{t_0 - t_1}{L} = \frac{t - t_1}{L - l}$$

and

$$t = t_0\left[1 - \left(\frac{1 - R_t}{L}\right)l\right] \tag{5}$$

If we substitute equation 5 in equation 4, the rate of change of slope of the tapered strip of Figure 1 is

$$\frac{d^2\delta}{dl^2} = \frac{2K\Delta}{t_0\left[1 - \left(\frac{1 - R_t}{L}\right)l\right]} \tag{6}$$

If we integrate twice, the deflection characteristic of a uniformly tapering bimetal strip is found to be

$$\delta = \frac{K\Delta l^2}{t_0} \cdot \frac{2L}{l(1 - R_t)} \times \left\{\left[1 - \frac{L}{l(1 - R_t)}\right] \ln \frac{1}{1 - \frac{(1 - R_t)l}{L}} + 1\right\} \tag{7}$$

The deflection of the free end ($l = L$) is

$$\delta = \frac{K\Delta L^2}{t_0} \cdot \frac{2}{(1 - R_t)}\left\{1 - \frac{R_t}{1 - R_t} \ln \frac{1}{R_t}\right\} \tag{8}$$

With reference to equation 1, the ratio of deflection of a tapered-thickness strip to that of a uniform strip of identical thickness at the support is expressible in terms of the taper ratio alone, as follows:

$$\frac{\delta \text{ tapered strip}}{\delta \text{ uniform strip}} = \frac{2}{(1 - R_t)}\left\{1 - \frac{R_t}{1 - R_t} \ln \frac{1}{R_t}\right\} \tag{9}$$

and this expression may be treated as a correction to be applied to the conven-

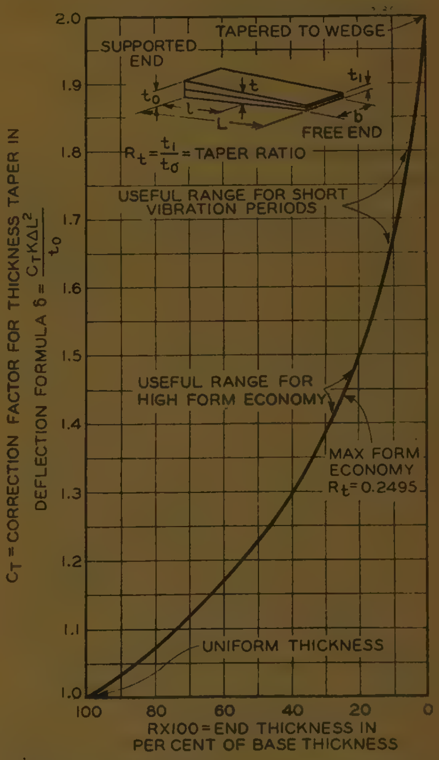

Figure 1. Deflection characteristics of flat bimetal strips having uniformly tapered thickness

Supported rigidly at thick end

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tional formula for bimetal deflections when the deflection of any tapered-thickness bimetal strip is to be found. This correction will be designated C_T and may be conveniently represented graphically, as in Figure 1. The conventional formula for the deflection of a bimetal strip may now be generalized as follows:

$$\delta=\frac{C_TK\Delta L^2}{t_0} \tag{10}$$

where C_T is a form factor for the strip and is obtainable directly from Figure 1. From this figure it is found that the deflection of any flat bimetal strip is exactly doubled if the strip is uniformly tapered to zero thickness at the free end. The width evidently does not enter into this relation.

Force Characteristics

Consider the general case of a flat bimetal strip rigidly supported at one end, of a negligible weight, and loaded as a cantilever beam, at the free end by a concentrated load. If the strip is nontapered as well as flat and of uniform width, the following conventional formulas are readily available from handbooks.

Uniform-Thickness Strips:

$$\text{Deflection}=\delta=\frac{4FL^3}{Eb_0t_0^3}=\frac{FL^3}{K^1b_0t_0^3} \tag{11}$$

$$\text{Force}=F=\frac{Eb_0t_0^3\delta}{4L^3}=\frac{K^1b_0t_0^3\delta}{L^3} \tag{12}$$

$$\text{Stiffness}=\frac{F}{\delta}=\frac{Eb_0t_0^3}{4L^3}=\frac{K^1b_0t_0^3}{L^3} \tag{13}$$

where

F = concentrated load at free end
b_0 = width
E = modulus of elasticity
K^1 = force constant for bimetal composition

If we now consider the tapered-thickness strip of Figure 1, a similar group of formulas may be derived by ordinary calculus as follows:

Tapered-Thickness Strips:

$$\delta=\frac{FL^3}{K^1b_0t_0^3}\cdot\frac{3}{(1-R_t)^2}\times\left[\frac{1}{(1-R_t)}\ln\frac{1}{R_t}-\frac{(3-R_t)}{2}\right] \tag{14}$$

$$F=\frac{K^1b_0t_0^3\delta}{L^3}\cdot\frac{(1-R_t)^2}{3\left[\frac{1}{(1-R_t)}\ln\frac{1}{R_t}-\frac{(3-R_t)}{2}\right]} \tag{15}$$

$$\frac{F}{\delta}=\frac{K^1b_0t_0^3}{L^3}\cdot\frac{(1-R_t)^2}{3\left[\frac{1}{(1-R_t)}\ln\frac{1}{R_t}-\frac{(3-R_t)}{2}\right]} \tag{16}$$

Here again the effect of tapering the thickness of the strip can be expressed as a correction C_{Ft} expressed in terms of R_t, which may be applied to equations 14, 15, and 16, resulting in generalized equations as follows:

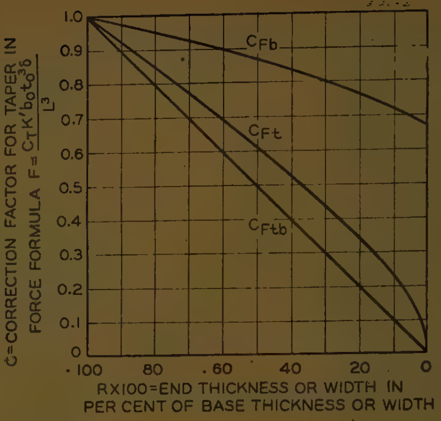

Figure 2. Force characteristics of flat bimetal strips having uniformly tapered dimensions and supported rigidly at thick end: cantilever load on free end

C_{Ft}—Thickness taper
C_{Fb}—Width taper
C_{Ftb}—Equal thickness and width taper combined

$$\delta=\frac{FL^3}{C_{Ft}K^1b_0t_0^3} \tag{17}$$

$$F=\frac{C_{Ft}K^1b_0t_0^3\delta}{L^3} \tag{18}$$

$$\frac{F}{\delta}=\frac{C_{Ft}K^1b_0t_0^3}{L^3} \tag{19}$$

where

$$C_{Ft}=\frac{(1-R_t)^2}{3\left[\frac{1}{(1-R_t)}\ln\frac{1}{R_t}-\frac{(3-R_t)}{2}\right]} \tag{20}$$

Specific values of C_{Ft} for any taper ratio may be obtained conveniently from a plot of this expression as given in Figure 2. The conventional bimetal force formulas therefore may be used for tapered bimetals provided the appropriate correction factors from Figure 2 are applied to the values.

For comparative purposes, similar expressions and curves have been derived for the cases of width-tapered bimetal strips of uniform thickness, and bimetal strips tapering in both width and thickness.

C_{Fb}(width tapered only)

$$=\frac{(1-R_b)^2}{3\left[\frac{R_b^2}{(1-R_b)}\ln\frac{1}{R_b}-\frac{3R_b-1}{2}\right]} \tag{21}$$

$$C_{Ftb}(\text{equal dual taper})=R_t=R_b \tag{22}$$

where

R_b = width-taper ratio $=\frac{b_1}{b_0}$
b_1 = width of strip at free end

Economy

In judging the relative merits of different tapered forms of thermostatic bimetal members, it will be useful to determine the amount of work performed per unit weight of each taper form under comparable conditions. From such values a criterion will be established and called the "form-economy" characteristic of the shape. This criterion will be defined as the ratio of the work obtainable from a bimetal piece of a specified form to the work obtainable from a piece of flat bimetal of uniform cross section and of equal total weight.

Form economy will be indicated by the symbol ϵ, and for the flat reference standard under this definition ϵ is taken as unity and may be expressed more formally as follows:

$\epsilon=1.00$ = constant × work required to restore free end of flat strip of uniform section to original position after deflection due to unit temperature change ÷ weight of bimetal strip
= (constant × $^1/_2$ × unloaded thermostatic deflection × force to restore bimetal end by amount of thermostatic deflection) ÷ (volume of bimetal × density of bimetal)

$$=\frac{k_f\times{}^1/_2\times\frac{K\Delta L^2}{t_0}\times\frac{K^1b_0t_0^3\delta}{L^3}}{b_0t_0L\times W} \tag{23}$$

Since by equation 1,

$$\delta=\frac{K\Delta L^2}{t_0}$$

$$\epsilon=1.00=\frac{k_f\times{}^1/_2\times\left(\frac{K\Delta L^2}{t_0}\right)^2\times\frac{K^1b_0t_0^3}{L^3}}{b_0t_0L\times W} \tag{24}$$

$$=\frac{k_fK^2K^1\Delta^2}{2w} \tag{24a}$$

and

$$k_f=\frac{2w}{K^2K^1\Delta^2} \tag{25}$$

For a tapered bimetal, the form economy will be

$$\epsilon=\frac{\frac{2w}{K^2K^1\Delta^2}\times{}^1/_2\times\left(\frac{C_TK\Delta L^2}{t_0}\right)^2\times\frac{C_FK^1b_0t_0^3}{L^3}}{\text{volume}\times W} \tag{26}$$

or

$$\epsilon=\frac{C_T^2C_FLb_0t_0}{\text{volume}} \tag{26a}$$

But volume can be expressed in terms of b_0, t_0, and L, and the taper ratio R, for example:

Volume (tapered thickness, parallel edges)

$$=b_0t_0L\left(\frac{1+R_t}{2}\right) \tag{27}$$

Volume (uniform thickness, tapered edges),

$$=b_0t_0L\left(\frac{1+R_b}{2}\right) \tag{28}$$

Volume (tapered thickness, tapered edges $[R_b=R_t=R_{tb}]$)

$$=b_0t_0L\left(\frac{1+R_{bt}+R_{bt}^2}{3}\right) \tag{29}$$

Consequently, from equation 26a,

ϵ_t(tapered thickness, parallel edges)

$$= \frac{2C_T^2 C_{Ft}}{(1+R_t)} \qquad (30)$$

ϵ_b(uniform thickness, tapered edges)

$$= \frac{2C_{Fb}}{(1+R_b)} \qquad (31)$$

ϵ_{tb}(tapered thickness, tapered edges $[R_b=R_t]$)

$$= \frac{3C_T^2}{1+R_{bt}+\frac{1}{R_{bt}}} \qquad (32)$$

The form economy of the foregoing shapes has been evaluated for various values of taper ratio and the results plotted in Figure 3. Maximum values of form economy and corresponding taper ratios were determined by calculus. These maxima occur as given in Table I. From an inspection of these values, it is evident that for flat bimetal strips of the taper forms studied, the most economical in point of work performed per unit weight is the conventional bimetal sheet of uniform thickness, and having the side edges tapering to a point at the free end. Unfortunately, however, this form seldom can be utilized in any practical thermostatic construction, as it is always necessary to make the free end of the bimetal strip physically wide enough to carry an electric contact or to actuate a co-operating working member. For any value of width–taper ratio of more than 0.1, which generally is below a usable minimum, the form economy is seen to have fallen off to approximately less than that for the thickness-tapered bimetal, and the latter form would take preference.

Maximum form economies for the thickness-tapered bimetals appear in the taper-ratio region between $R = 0.2$ to 0.3. In this region the metal will perform approximately one-third more work than it would if shaped as a conventional uniformly thick strip. If the bimetal can be produced at costs appreciably less than 130 per cent of the cost of flat bimetal stock, then the tapered-thickness bimetal construction becomes the most economical form for general flat-strip applications. Actual costs of the tapering process are said to be negligible compared to the gain in performance.

Fortunately, a thickness-tapered bimetal strip of maximum form economy will have a free-end thickness lying between one fourth to one fifth of the base thickness, which usually will be quite adequate mechanically for mounting of electric contacts. This form, therefore, can be recognized as a new and very useful embodiment of the bimetallic thermostatic strip.

Table I. Maximum Form Economy and Taper Ratio for Several Taper Forms

Form	ϵ_{Max}	Taper Ratio for ϵ_{Max}
Tapered thickness Uniform width	1.304	0.2495
Uniform thickness Tapered width	1.333	0.000
Tapered thickness Tapered width $(R_t = R_b)$	1.296	0.435

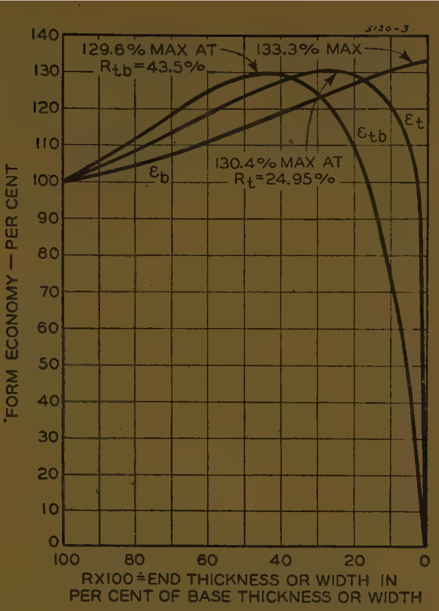

Figure 3. Form economy of flat bimetal strips of various taper forms

ϵ_t—Thickness taper
ϵ_b—Width taper
ϵ_{tb}—Equal thickness and width taper combined

It will be noticed that the dual-tapered form yields a form economy nearly equal to that for the tapered-thickness strip if a taper ratio of somewhat less than 0.5 is used for the former. This form is rendered uneconomical, however, by the practical consideration that cutting of width-tapered strips from thickness-tapered stock would produce an appreciable quantity of scrap, rendering the high economy unrealizable.

In order to visualize better the manner in which the tapered-thickness form improves the form economy of the bimetal, Figure 4 has been prepared. In this figure, a flat bimetal strip has been subjected to a progressive alteration of form consisting in a shift of the angle between faces over the range from parallelism to wedge form. The total quantity of metal, as well as all other factors, has been held constant during this metamorphosis in order to show precisely how the properties of the strip are modified by the change. Figure 4 reveals that the deflection decreases during the transformation until a taper ratio of 22.35 per cent is reached, at which point the deflection is 89.65 per cent that of the original flat strip. Beyond this point the deflection again increases until the full original value is regained at the point where the strip reaches wedge form. The base thickness evidently is doubled at this time. During this transformation the stiffness of the member increased much more rapidly, however, reaching 161.7 per cent of the stiffness of the flat strip at R = 23.92 per cent. The net result is an increase in form economy to a maximum of 130.4 per cent at R = 24.95 per cent, which confirms this finding from Figure 3.

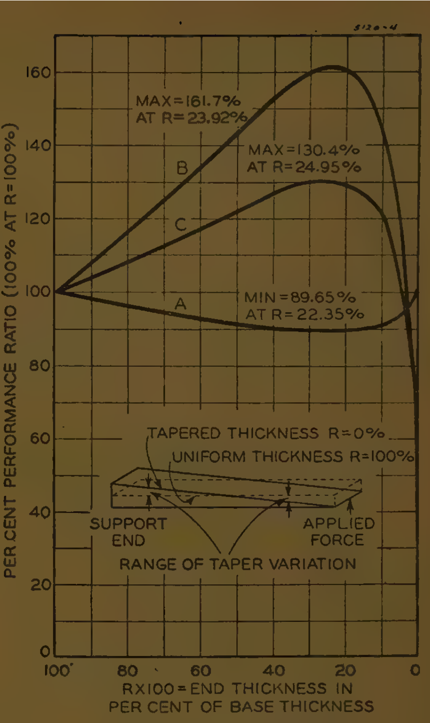

Figure 4. Variation in bimetal-strip characteristics with change in thickness taper

Volume, width, and length held constant

A—Deflection
B—Stiffness
C—Form economy

Vibration Characteristics

In slow-acting bimetal thermostatic controls considerable trouble may be encountered at the moment of make or break of the electric control circuit, especially if the rate of temperature change is very slow, because of vibrations of the bimetal strip. Such vibrations produce chattering of the contacts and consequent disturbances in the controlled system. It has been found that such disturbances can be prevented if the bimetal member be so designed that its natural frequency is higher than that of the components of the controlled system. Single or dual tapers of the bimetal strip have been found sufficient in some instances to eliminate completely such vibration troubles and to render the slow-acting thermostat construction entirely satisfactory for the stable control of relays. Small telephone-type relays which formerly chattered severely when controlled by conventional flat-strip thermostats with heavy silver contacts, were found to close slowly and remain closed when the bimetal strip was tapered in width and thickness and the contact reduced in weight. Elimination of thickness taper only restored the chattering condition.

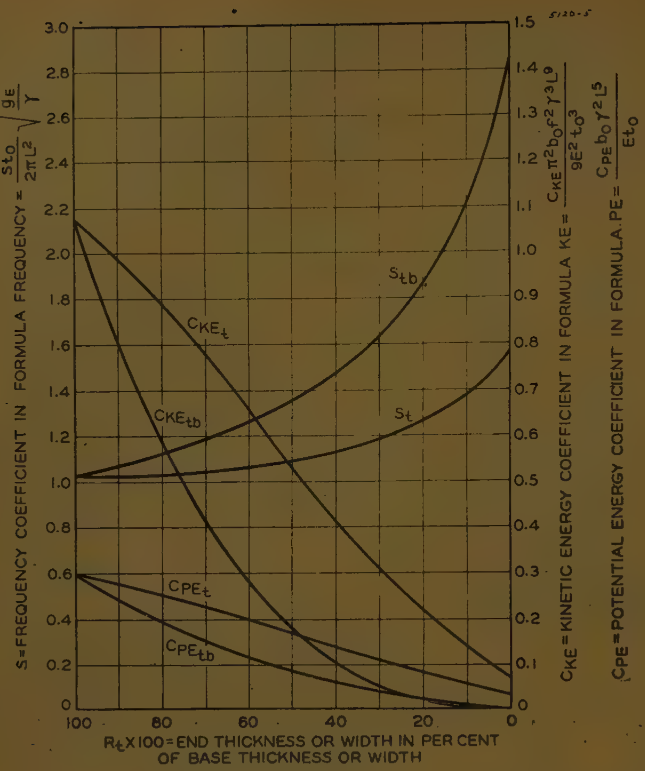

Figure 5. Energy and frequency characteristics of tapered bimetal strips

C_{KE_t}=kinetic - energy coefficient for tapered - thickness strip

$C_{KE_{tb}}$=kinetic - energy coefficient for strip of tapered thickness and width

C_{PE_t}=potential-energy coefficient for tapered - thickness strip

$C_{PE_{tb}}$=potential-energy coefficient for strip of tapered thickness and width

S_t=frequency coefficient for tapered thickness strip

S_{tb}=frequency coefficient for strip of tapered thickness and width

An analysis of the natural frequencies of tapered bimetal strips has been made, to determine the vibration characteristics as a function of form. The frequencies (fundamental only) were calculated for plain flat strips of various tapers, with rigid support at the base assumed and zero mass carried by the free vibrating end. If the contact actually carried by the end of such a tapered strip in practice is light in weight compared to the weight of the strip, the contact weight will have slight effect on the frequency of the strip, and the results of the present analysis will be applicable. If the contact is made large and heavy, its weight will predominate in the determination of frequency, and tapering of the supporting strip will be defeated.

The frequencies were here determined by the method of Rayleigh in which the potential and kinetic energies of the vibrating strip are equated. The results are believed accurate to within 0.5 to 1.5 per cent. An exact mathematical determination would be difficult and very laborious and will not be attempted as the accuracy of the present method is entirely adequate.

The steps in computation of the natural frequencies are given in Table II for three flat-strip forms; the conventional strip of uniform thickness, the thickness-tapered wedge and the pyramidal or dual taper.

Expressions similar to those given in Table II also were derived for the more general case of bimetal strips of taper R_t or R_{tb}. These expressions yield the values given in Table II, columns 1 and 2, when values of R_t = 100 per cent and R_t = 0 per cent, respectively, are substituted. The complete expressions are given in Appendix I. The coefficients for the expressions for natural frequency for all ratios are plotted in Figure 5.

The curves of frequency-versus-taper ratio show the wedge form to vibrate about 50 per cent faster than a strip of uniform thickness. For a usable and economical strip of taper ratio 0.25, the increase in frequency over that of a flat strip is only about 20 per cent. Consequently, where high frequency of the strip is the predominant design consideration, thickness tapering alone is only slightly effective.

Figure 5 similarly shows the dual wedge or pyramid to have a theoretical maximum increase in frequency to over $2^1/$ times the value for strips of uniform thickness and width. For practically usable strips, increases to 200 to 220 per-cent frequency probably cannot be exceeded. In those cases, however, in which suppression of vibration troubles is of much greater importance than economy of material, the dual taper is of considerable practical value.

It is interesting to note that thermostatic deflection and period of vibration are governed by the same dimensional parameters, L^2/t_0. Because of this fact, short vibration periods cannot be obtained by modifying the physical proportions of any type of strip when a specified thermostatic deflection is required. Changes of shape appears to be the most convenient means of obtaining independent frequency adjustment of the thermostatic strip.

Appendix I

Frequency of Tapered-Thickness Strips

$$\text{Thickness } t=t_0\left[(1-R_t)\frac{(L-l)}{L}+R_t\right] \tag{33}$$

$$\text{Bending moment}=\frac{\gamma b_0 t_0}{2}(L-l)^2\times\left[\frac{(1-R_t)(L-l)}{3L}+R_t\right] \tag{34}$$

Table II. Natural Frequencies of Flat Bimetal Strips

Fundamentals Only

Form	Uniform Thickness	Wedge	Pyramid
Moment of inertia	$\frac{b_0t_0^3}{12}$	$\frac{b_0t_0^3(L-l)^3}{12L^3}$	$\frac{b_0t_0^3(L-l)^4}{12L^4}$
Bending moment	$\frac{\gamma b_0t_0(L-l)^2}{2}$	$\frac{\gamma b_0t_0(L-l)^3}{6L}$	$\frac{\gamma b_0t_0(L-l)^4}{12L^2}$
Deflection of strip under its own weight	$\frac{6\gamma}{Et_0^2}\left(\frac{L^2l^2}{2}-\frac{Ll^3}{3}+\frac{l^4}{12}\right)$	$\frac{\gamma L^2l^2}{Et_0^2}$	$\frac{\gamma L^2l^2}{2Et_0^2}$
Maximum velocity at any point if free end is released from rest under its own weight	$\frac{12\pi f\gamma}{Et_0^2}\left(\frac{L^2l^2}{2}-\frac{Ll^3}{3}+\frac{l^4}{12}\right)$	$\frac{2\pi f\gamma L^2l^2}{Et_0^2}$	$\frac{\pi f\gamma L^2l^2}{Et_0^2}$
Kinetic energy under maximum velocity	$\frac{1.1556\pi^2b_0f^2\gamma^3L^9}{gE^2t_0^3}$	$\frac{0.0666\pi^2b_0f^2\gamma^3L^9}{gE^2t_0^3}$	$\frac{0.00476\pi^2b_0f^2\gamma^3L^9}{gE^2t_0^3}$
Potential-energy change	$\frac{0.3b_0\gamma^2L^5}{Et_0}$	$\frac{0.04166b_0\gamma^2L^5}{Et_0}$	$\frac{0.00833b_0\gamma^2L^5}{Et_0}$
Frequency from $KE=PE$	$\frac{1.02t_0}{2\pi L^2}\sqrt{\frac{gE}{\gamma}}$	$\frac{1.58t_0}{2\pi L^2}\sqrt{\frac{gE}{\gamma}}$	$\frac{2.646t_0}{2\pi L^2}\sqrt{\frac{gE}{\gamma}}$

where

γ=density of material
f=natural frequency of strip
g=acceleration of gravity

Moment of inertia

$$=\frac{b_0t_0^3}{12}\left[(1-R_t)\frac{(L-1)}{L}+R_t\right]^3 \quad (35)$$

Deflection of strip under its own weight

$$=\frac{2\gamma L^2}{Et_0^2}\left[\frac{l^2}{2(1-R_t)^2}+\frac{LR_t^2(3-R_t)l}{(1-R_t)^3}+\frac{3L^2R_t^2}{(1-R_t)^4}\ln\frac{L-(1-R_t)l}{L}+\frac{L^2R_t^3l}{(1-R_t)^3[L-(1-R_t)l]}\right] \quad (36)$$

Kinetic energy

$$=\frac{\pi^2b_0f^2\gamma^3L^9}{gE^2t_0^3(1-R_t)^9}\Big[(16R^8-96R^7+98R^6-16R^5+12R^4)\ln\frac{1}{R_t}-36R_t^6\left(\ln\frac{1}{R_t}\right)^2-2R_t^{10}+22.66R_t^9-86.266R_t^8+124.8R_t^7-67.5R_t^6+18.933R_t^5-13R_t^4+4.266R_t^3-1.966R_t^2+0.0666\Big] \quad (37)$$

Potential-energy change

$$=\frac{b_0\gamma^2L^5}{Et_0(1-R_t)^5}\left[\frac{3}{2}R_t^4\ln\frac{1}{R_t}-\frac{R_t^6}{3}+2R_t^5-\frac{15}{8}R_t^4+\frac{2}{3}R_t^3-\frac{R_t^2}{2}+\frac{1}{24}\right] \quad (38)$$

Frequency was found for values of R_t by equating kinetic and potential energies.

Frequency of Dual-Tapered Strips

$$\text{Thickness}=t_0\left[(1-R_{bt})\frac{(L-l)}{L}+R_{bt}\right] \quad (39)$$

$$\text{Width}=b_0\left[(1-R_{bt})\frac{(L-l)}{L}+R_{bt}\right] \quad (40)$$

Bending moment

$$=\frac{\gamma b_0t_0L^2}{12(1-R_{bt})^2}\left\{\left[(1-R_{bt})\frac{(L-l)}{L}+R_{bt}\right]^4-R_{bt}^3\left[\frac{4(1-R_{bt})(L-l)}{L}+R_{bt}\right]\right\} \quad (41)$$

Moment of inertia

$$=\frac{b_0t_0^3}{12}\left[(1-R_{bt})\frac{(L-l)}{L}+R_{bt}\right]^4 \quad (42)$$

Deflection of strip under its own weight

$$=\frac{\gamma L^4}{Et_0(1-R_{bt})^4}\left[\frac{u^2}{2}-\frac{2R_{bt}^3}{u}+\frac{R_{bt}^4}{2u^2}-(1+2R_{bt}^3-R_{bt}^4)u+\frac{1}{2}+4R_{bt}^3-\frac{3R_{bt}^4}{2}\right] \quad (43)$$

where

$$u=1-(1-R_{bt})\frac{1}{L}$$

Kinetic energy

$$=\frac{\pi^2b_0f^2\gamma^3L^9}{gE^2t_0^3(1-R_{bt})^9}\left[4R_{bt}^7\ln R_{bt}-\frac{2R_{bt}^{13}}{5}+\frac{31R_{bt}^{12}}{10}-\frac{101R_{bt}^{11}}{10}+\frac{52R_{bt}^{10}}{3}-\frac{224R_{bt}^9}{15}+\frac{7R_{bt}^8}{10}+\frac{204R_{bt}^7}{35}-\frac{12R_{bt}^6}{5}+\frac{2R_{bt}^5}{5}+\frac{23R_{bt}^4}{30}-\frac{3R_{bt}^3}{10}+\frac{1}{210}\right] \quad (44)$$

Potential-energy change

$$=\frac{b_0\gamma^2L^5}{Et_0(1-R_{bt})^5}\left[-\frac{R_{bt}^8}{8}+\frac{R_{bt}^7}{2}-\frac{2R_{bt}^6}{3}+\frac{R_{bt}^5}{5}+\frac{R_{bt}^4}{4}-\frac{R_{bt}^3}{6}+\frac{1}{120}\right] \quad (45)$$

Frequency was similarly determined from the energy expressions.

In all cases the foregoing general equations are reducible to the specific cases for $R=0$ and $R=1$ as given in Table II in the body of this paper. In order to accomplish such reductions it is usually necessary to apply L'Hopital's theorem for the evaluation of indeterminate forms. In the case of the expressions for kinetic energy it was necessary to perform nine successive differentiations.

Response Time and Lag of a Thermometer Element Mounted in a Protecting Case

W. N. GOODWIN, JR.
FELLOW AIEE

Synopsis: In this paper equations are derived and applied for determining the response time and temperature lag, in general, of a body which exchanges heat with a medium indirectly through a second body. Application is made to a thermometer element mounted in a protecting case, with specific reference to a resistance-type thermometer such as is used for aircraft and marine purposes. The temperature distribution along the stem and the errors resulting when the head of the bulb and medium differ in temperature also are derived.

IN THE DESIGN of many forms of thermometers it is frequently customary and often necessary to mount the active element within a protecting case. This is particularly true in the case of resistance and bimetal types and often true with the thermocouple and ordinary mercury types.

When protecting cases are used, the active element is not in direct contact with the surrounding medium, the temperature of which is to be measured, but its exchange of heat, in acquiring the temperature of the medium, must take place through the protecting case and through the thermal contact between the element and case.

Under the conditions referred to, there is an additional time lag in the indications of the thermometer element over that which would result if the element were in direct contact with the medium, when the temperature of the latter differs from it or changes with time.

The object of this analysis is to determine the equations of the cooling and heating curves of an element contained in a protecting case, from which design data and operating characteristics can be determined for specified conditions. Part of this analysis was done in 1921 and completed and applied practically during recent years.

Application to a Resistance-Type Thermometer

To make the analysis of this problem of more practical use, we will direct our attention to a specific device by considering a resistance-type thermometer bulb developed by an associate of the author, K. M. Lederer. This is illustrated in Figure 1, cut away to show the construction. The analysis, however, is perfectly general and applies to any object exchanging heat with a surrounding medium indirectly, through another body in direct contact with the medium.

In this part of the paper the final equations and their applications are given. The complete derivations, however, are given in the appendixes.

The thermometer element m in Figure 1 consists of a wire resistor wound on a thin-walled tube and electrically insulated from it. This tube is contained in a protecting tube M and is in good thermal contact with it.

The protecting tube containing the resistor element is secured to the head B through which the connections to the resistor element pass to the indicating instrument.

In the derivation of the equations the following quantities appear, MS/h_1, $ms\div h_2$, and ms/h_1; in which M and m are the masses, S and s the specific heats of the protecting tube and thermometer element, respectively, and h_1 and h_2 the rates of heat transfer per degree difference in temperature, from the protecting tube to the surrounding medium, and from the thermometer element to the protecting tube, respectively, all corresponding to the heat-exchange area. The definitions of these and other symbols are given in the list of symbols. These

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have the dimensions of time, which we will designate as t_1, t_2, and t_m, respectively, where $t_1=MS/h_1$ is the time constant of the protecting tube relative to the surrounding medium, $t_2=ms/h_2$ is the time constant of the resistor element relative to the protecting tube, and t_m is the mutual time constant ms/h_1 of the resistance element relative to the surrounding medium acting through the protecting case. That is, it is the time constant the inner tube would have if its heat were exchanged with the surrounding medium from the surface of the protecting tube.

Physically considered, the time constant is the time required for the body to change one degree in temperature, for the specified rate of heat exchange, if the temperature difference is maintained at one degree.

Thermometer Transferred From a Hotter to a Cooler Medium

If the thermometer bulb, having its resistor element and protecting tube both initially at the temperature θ_0 is suddenly immersed in the cooling medium at a temperature T_c, then the ratio of the temperature elevation $(\theta-T_c)$ of the element above that of the cooling medium at any time t, to the initial elevation (θ_0-T_c), is, as shown in Appendix I,

$$\frac{\theta-T_c}{\theta_0-T_c}=\frac{1}{P}\,e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times \sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}P\right] \qquad (1)$$

where

$$P=\sqrt{\frac{(t_1+t_2+t_m)^2}{4t_0^2}-1}$$

and

$$t_0=\sqrt{t_1t_2}$$

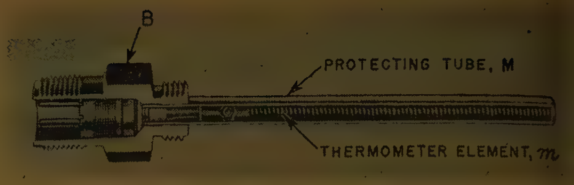
Thermometer Transferred From a Cooler to a Hotter Medium

If the initial temperature θ_0 of the thermometer element is lower than that of the surrounding medium T_c, then the resulting heating curve is exactly similar to the cooling curve previously given, except that it is reversed as shown in Figure 3 in accordance with the following equation, as shown in Appendix II,

$$\frac{\theta-\theta_0}{T_c-\theta_0}=1-\frac{1}{P}\,e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times \sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}P\right] \qquad (2)$$

To apply these equations to a practical device, let us consider the resistance-thermometer bulb cooled in various media for which the constants given in Table I have been determined experimentally. As the inner and protecting tubes are actually in contact in this case, it is convenient to use values for M, m, h_1, and h_2

Figure 1. Resistance-type-thermometer bulb, cut away to show internal construction



per square centimeter of contact area.

Cooling curves derived from this equation for the tabulated values for P and t_0 are shown in Figure 2. As an example, assume that a bulb is heated to 100 degrees centigrade and suddenly plunged into ice water at zero degrees centigrade, it will fall to ten degrees centigrade in 8.3 seconds.

The curves in Figure 2 are based upon the time t as the independent variable. If t/t_0 is made the independent variable, the curves become more general and will apply to devices of any time constant, consistent with the value of P. Such a family of curves is shown in Figure 3, which for the resistance-type thermometer, range from water to still-air cooling.

Mechanical Analogy

The temperature equations previously given have the same form as that for an aperiodic mechanical motion derived by the author in about the year 1908, namely:

$$\frac{\theta}{\phi}=\frac{1}{\sqrt{n^2-1}}\,e^{-2\pi n\left(\frac{t}{T_0}\right)}\times \sinh\left[2\pi\sqrt{n^2-1}\left(\frac{t}{T_0}\right)+\sinh^{-1}\sqrt{n^2-1}\right] \qquad (3)$$

in which, when applied to a spring-controlled damped instrument, for example, ϕ is the initial deflection, θ the deflection after a time t given in terms of a time constant T_0 which is the undamped period, and n the ratio of the actual to the critical damping coefficient, designated by the author as the specific damping coefficient.

When the constants in equations 1 and 2 are equated to those given in equation 3, it will be found that n corresponds to $\sqrt{P^2+1}$, and T_0 to $2\pi t_0$.

If, therefore, a permanent-magnet movable-coil-type instrument has an undamped period of $T_0=2\pi t_0$ seconds and is damped to an amount such that $n=\sqrt{P^2+1}$, then the motion of the pointer toward zero, as a function of time, will follow the same curve as the thermal system having the constants t_0 and P.

Temperature Lag

When a thermometer is used to measure the temperature of a medium, which is changing in temperature, the indication of the thermometer lags behind the temperature of the medium as a result of the heat capacity of the thermometer and the thermal resistance to the exchange of heat between the medium and the thermometer element. An example of this condition occurs when measurements are made of the surrounding air from ar airplane which is changing its altitude. Assume that a thermometer has initially the same temperature as the surrounding medium. Then let the temperature of the medium change at a constant rate of r degrees per second. The indication of the thermometer will start to follow, relatively slowly at first, but, as the difference in temperature or lag increases, the change in indication of the thermometer increases its rate until finally it approaches the same rate r as that of the change in temperature of the medium. This final difference in temperature then is maintained and is the maximum value of the lag.

The value of the lag in indication for a thermometer element in a protecting case, for an increasing temperature, relative tc the final lag, at any time t after the initial change, as shown in Appendix III, is

$$\frac{T_a-\theta}{L}=1-\frac{1}{2P\sqrt{P^2+1}}\,e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times \sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}2P\sqrt{P^2+1}\right] \qquad (4)$$

where L is the final and maximum value of the lag, and is

$$L=2rt_0\sqrt{P^2+1}=r(t_1+t_2+t_m)\text{ degrees} \qquad (5)$$

and r is the rate of change in temperature in degrees per second, which is positive for increasing and negative for decreasing temperatures.

This shows that the lag is a simple func- tion of the three time constants of the thermometer bulb or other device.

For a falling temperature, the lag equation is the same as equation 4 except that r becomes negative and therefore L is negative, and the left-hand membe becomes $(\theta-T_a)/L$, whereas the right hand member is unchanged.

Figure 4 gives curves showing the in crease of lag with time, for various value of time constants as represented by P, ir terms of time as a function of the tim constant t_0 of the thermometer bulb o other device.

The ranges in values of P illustrate are perfectly general, but as applied to resistance-type thermometer they cove ambient media from water to still air.

As a specific example, let us determin the temperature lag of the resistanc thermometer previously referred to, fo which $P=0.780$, and $t_0=3.32$ second when immersed in air moving at a spee of 50 miles per hour. Assume, for ex ample, that the air is changing in tem perature at a rate of five degrees centigrade per minute$=(^1/_{12})$ degree per second; then the maximum lag is, from equation 5,

$$L=2\times(^1/_{12})\times3.32\times\sqrt{(0.780)^2+1}$$
$$=0.702 \text{ degree centigrade}$$

The time for the lag to attain any fractional part of its final value can be determined from the curves in Figure 4 derived from equation 4. By interpolating between curves for $P=0.5$ and $P=1$, for $P=0.780$, we find that the time for the lag to reach, for example 90 per cent of its maximum value, is approximately $5.2\times t_0=5.2\times3.32=17.3$ seconds.

The same curves may be used for both increasing and decreasing temperatures.

Errors Resulting If Head of Bulb and Stem Differ in Temperature, Under Static Conditions

Let us assume that the bulb is screwed into the walls of a tank containing a material the temperature of which is to be measured and that the tank and therefore the head of the bulb have a temperature differing from that of the material. We shall determine the temperature at any point along the stem and also determine the error by computing the average deviation of the stem temperature from that of the material contained in the tank.

In Figure 5 the sketch in solid lines shows the bulb screwed into the walls of a tank. Assume that the head of the bulb is at a temperature T_h, and that of the material in the tank T_c. The dotted-line portion is referred to later.

In a previous paper[1] by the author, equations are given for the temperature distribution along a conductor connected at its ends to terminals, as a function of the terminal temperatures, the ambient temperature, and the heat generated in the conductor.

If the two terminals have the same temperature, then the conductor and its terminals are symmetrical with respect to the center of the conductor. The stem of the thermometer bulb, therefore, can be considered as one half of a symmetrical conductor with terminals, the other half being shown in dotted lines in Figure 5. From symmetry it is obvious that the two halves may be considered as entirely independent, as no heat passes the mid-point of the conductor, and exactly similar but opposite heat flows and temperature distribution occur in the two sides.

Table I. Thermal Constants of a Resistance-Type Thermometer

Object	Quantity*	Medium: Water	Still Air	Air at 50 Miles Per Hour
Protecting Case				
$M=0.295$ $S=0.529$	h_1	0.109	0.00151	0.0306
	t_1	1.432	100.0	5.10
Thermometer Element $M=0.0384$ $s=0.916$	h_2	0.0164	0.0164	0.0164
	t_2	2.16	2.16	2.16
	t_m	0.326	22.5	1.16
Derived Constants	P	0.491	4.12	0.780
	t_0	1.76	14.7	3.32

*h_1, h_2, watts per square centimeter per degree centigrade; t_1, t_2, t_0, t_m, seconds; M, m, grams per square centimeter; S, s, joules per gram per degree centigrade; P, numeric.

Therefore, the same equations apply to the half conductor that would apply if the conductor were completed by its image shown dotted.

After modifying the general equation given in the paper referred to, to meet the conditions of the present problem, assuming that no heat is generated in the bulb itself, we have, from the reference paper:

$$\frac{\theta_x-T_c}{T_h-T_c}=\frac{\sinh\,(2l-x)\sqrt{\dfrac{cp}{ak}}+\sinh x\sqrt{\dfrac{cp}{ak}}}{\sinh 2l\sqrt{\dfrac{cp}{ak}}} \qquad (6)$$

where

T_h, T_c, and θ_x=temperatures of the head, of the surrounding medium, and of the stem at any point x, respectively

cp=heat loss per second per unit length of stem per one degree difference in temperature between stem and medium

a=cross-sectional area of tube

k=thermal conductivity of tube material

l=the length of the stem

Expanding the numerator, simplifying, and for ease in handling the equation express x in terms of l, letting $x=yl$, and we have

$$\frac{\theta_y-T_c}{T_h-T_c}=\frac{\cosh\left[(1-y)l\sqrt{\dfrac{cp}{ak}}\right]}{\cosh l\sqrt{\dfrac{cp}{ak}}} \qquad (7)$$

This gives the temperature at any point along the stem above that of the surrounding medium relative to the corresponding elevation in temperature of the head of the bulb above the temperature of the medium.

At the end of the tube, where $y=1$, this reduces to

$$\frac{\theta_y-T_c}{T_h-T_c}=\frac{1}{\cosh\left[l\sqrt{\dfrac{cp}{ak}}\right]} \qquad (8)$$

In a resistance-thermometer bulb, for example, the constants, experimentally determined are as follows:

The convection rate in watts per centimeter length of tube per one degree centigrade, for still air, $cp=0.0032$; for air moving at 50 miles per hour $cp=0.0612$; and for water $cp=0.218$.

Then, $l\sqrt{cp/ak}$ for still air is 5.45; for air moving at 50 miles per hour, 16.75; and for water, 44.7; in which $a=0.0706$ square centimeter; $k=0.2$ watt per centimeter cube per degree centigrade, $l=11.4$ centimeters, and diameter of tube$=0.635$ centimeter.

If we substitute these constants in equation 7, and give y various values from 0 to 1, temperature-distribution curves for the resistance-type thermometer are obtained which are shown in Figure 6.

For the more general problem of any thermometer having a stem of uniform cross section, a family of curves is shown in Figure 7, giving the distribution of temperature along the stem, for various values of $l\sqrt{cp/ak}$, when the head of the thermometer and the medium differ in temperature. Values should be considered posi-

Figure 2. Cooling curves for resistance-type-thermometer bulb cooled in water, still air, and in air moving at 50 miles per hour

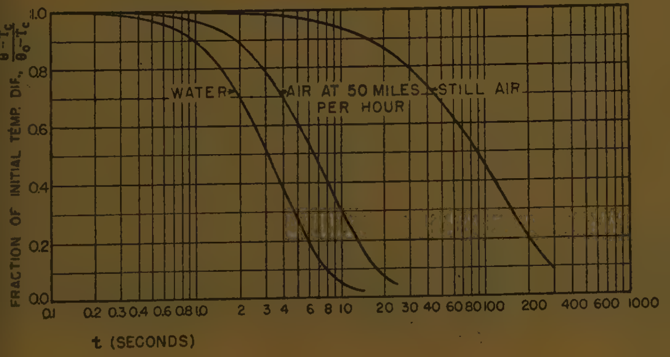
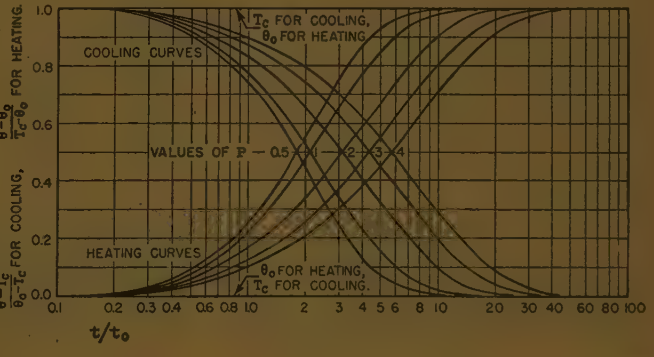


Figure 3. General cooling curves for a thermometer element in a protecting case in terms of its time constants



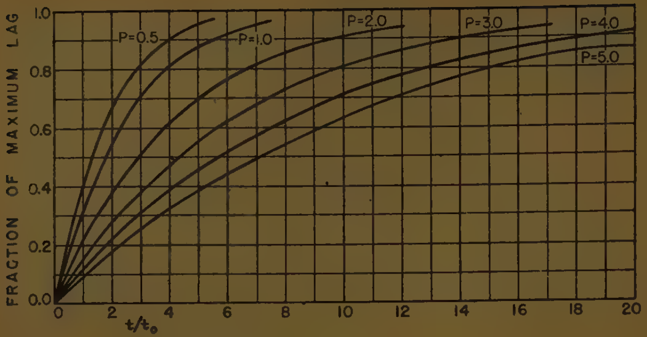


Figure 4. Temperature lag at any time, as a fraction of its maximum lag, of a thermometer in a protecting case, immersed in a medium changing in temperature at a constant rate, in terms of its time constants

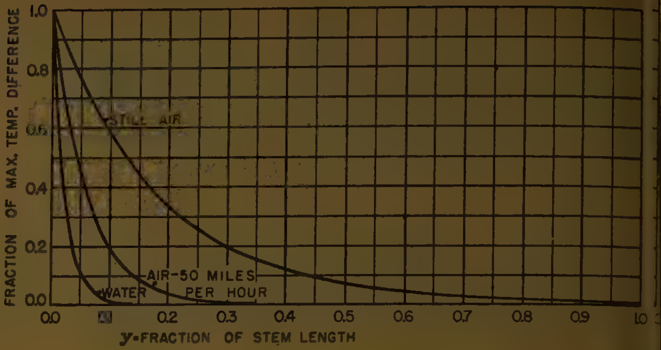


Figure 6. Distribution of temperature along a resistance-type-thermometer stem extending into a medium, as, for example, a thermometer inserted into a tank, when the head of the thermometer and the medium differ in temperature

Values are given as fractions of this difference in temperature

tive when the thermometer temperatures are higher than that of the medium, and negative when they are lower than that of the medium.

Errors

When the temperature of the head of the thermometer bulb differs from that of the medium being measured, then an error results. If a linear relationship between temperature and electrical resistance of the thermometer element throughout its length in the protecting tube is assumed, then the resistance change at any part of the tube is proportional to the change in temperature at that part of the tube. The total effect or error then, in terms of the difference in temperature (T_h-T_c) between the head and the medium, is the ratio of the average deviation in temperature throughout the active part of the stem from that of the medium, to the maximum temperature difference (T_h-T_c). Equation 7 gives the temperature at any point y, relative to the maximum temperature difference (T_h-T_c). Assume that the resistance element extends from the end of the stem to a point y. The average temperature then can be computed by dividing the area under the curve of temperature distribution for the active part of the stem by the length of this part of the stem as a function of y, that is by $(1-y)$. The average value is then

$$\left[\frac{\theta_y-T_c}{T_h-T_c}\right]_{\text{average}} = \frac{1}{1-y}\int_y^1 \left(\frac{\theta_y-T_c}{T_h-T_c}\right)dy \quad (9)$$

Substituting equation 7 for $(\theta_y-T_c)\div(T_h-T_c)$ in equation 9 we have

$$\left[\frac{\theta_y-T_c}{T_h-T_c}\right]_{\text{average}} = \frac{1}{1-y}\int_y^1 \frac{\cosh\left[(1-y)l\sqrt{\dfrac{cp}{ak}}\right]dy}{\cosh l\sqrt{\dfrac{cp}{ak}}} \quad (10)$$

Integrating this between the limits 1 and y and simplifying, we have

$$\left[\frac{\theta_y-T_c}{T_h-T_c}\right]_{\text{average}} = \frac{\sinh\left[(1-y)l\sqrt{\dfrac{cp}{ak}}\right]}{(1-y)l\sqrt{\dfrac{cp}{ak}}\cosh l\sqrt{\dfrac{cp}{ak}}} \quad (11)$$

For example we shall use this equation to determine the errors in the resistance-thermometer bulb in which the resistance element extends from $y=0.17$ to $y=1$, and for which constants are already given.

For still air, by substituting $l\sqrt{cp/ak}=5.45$ in equation 11, we have

$$\left[\frac{\theta_y-T_c}{T_h-T_c}\right]_{\text{average}} = \frac{\sinh\left[(1-0.17)\times 5.45\right]}{(1-0.17\times 5.45)\times\cosh 5.45} = 0.0878$$

Then the error in degrees is $0.0878\times(T_h-T_c)$.

For air moving at 50 miles per hour, for which $l\sqrt{cp/ak}=16.75$ we find

$$\left[\frac{\theta_y-T_c}{T_h-T_c}\right]_{\text{average}} = \frac{\sinh\left[(1-0.17)\times 16.75\right]}{(1-0.17)\times 16.75\times\cosh 16.75} = 0.00418$$

Then the error in degrees is $0.00418\times(T_h-T_c)$.

For water immersion the error is negligible as is evident by inspection of Figure 6.

Figure 8 shows a family of curves giving errors as a fraction of the difference in temperature (T_h-T_c) between the head of the bulb and the medium, for any length of resistor element relative to the length of the stem, assuming that the resistor element is placed in the stem between its end and the point y. These curves are computed from equation 11, and to make them general and applicable to any length or other dimensional or thermal constants they are given as a function of $l\sqrt{cp/ak}$.

List of Symbols

M = mass of the part of the protecting case which transfers heat from the thermometer element to the outside medium, grams
m = mass of thermometer element, grams
S = specific heat of material of protecting tube, watt-seconds per gram per degree centigrade
s = specific heat of material of thermometer element, watt-seconds per gram per degree centigrade
h_1 = rate of heat exchange of protecting case with outside medium, watts per degree centigrade
h_2 = rate of heat exchange of thermometer element with protecting case, watts per degree centigrade
$t_1 = MS/h_1$ = time constant of protecting case relative to outside medium, seconds
$t_2 = ms/h_2$ = time constant of thermometer element relative to protecting case, seconds
$t_m = ms/h_1$ = mutual time constant, seconds, the thermometer element would have if its exchange of heat with the external medium were effected from the surface of the protecting case
t = time, independent variable
P = for brevity $= \sqrt{\dfrac{(t_1+t_2+t_m)^2}{4t_0^2}-1}$, a numeric
$t_0 = \sqrt{t_1t_2}$ seconds
θ = temperature of the thermometer element at any time t

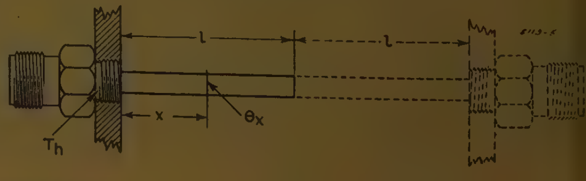


Figure 5. A thermometer passing through the wall of a tank and attached to it, for the purpose of measuring the temperature of its contents

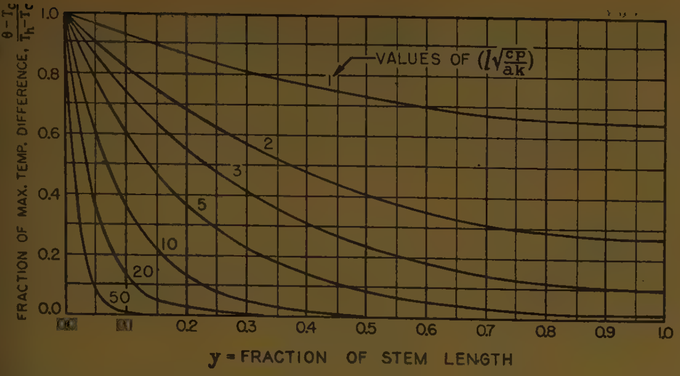


Figure 7. Temperature distribution along any thermometer stem of uniform cross section, immersed in a medium, when the temperature T_h of the thermometer head is higher than that of the medium T_c, given as a function of $l\sqrt{cp/ak}$

When T_h is lower than T_c, values are negative

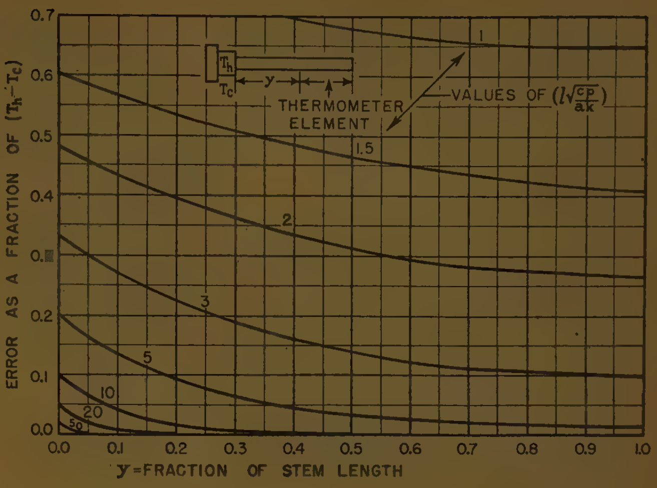


Figure 8. Errors of a resistance-type thermometer, with head at a temperature T_h and stem immersed in a medium at temperature T_c, when the resistance element extends inside the stem from its end to any position y, for various values of $l\sqrt{cp/ak}$, given as a function of y

θ_0 = initial temperature of the thermometer element and protecting tube

T = temperature of the protecting case at any time t

T_c = temperature of the surrounding medium when constant

T_a = temperature of the surrounding medium at any time t, when changing

θ_x = temperature of the thermometer stem above that of medium at any distance x from head

l = length of stem of thermometer, centimeters

a = cross-sectional area of thermometer stem, square centimeters

k = thermal conductivity in watts per centimeter cube per degree centigrade

cp = heat convection from tube to surrounding medium per second per unit length of tube, watts per degree centigrade

e = base of natural logarithms

Appendix I

This analysis considers the problem of the cooling of a body indirectly by a lower constant-temperature medium through a second body in direct contact with that medium, as for example, the cooling of a thermometer element contained in a protecting case.

With reference to Figure 2, and the list of symbols, the values M and m refer in general to the masses of the outer and inner bodies, respectively, and h_1 and h_2 to the rates of heat exchanges per degree difference in temperature between the outer body and the medium, and the corresponding values between the inner and outer bodies, respectively. When the specific case of a thermometer element in a protecting case is being considered, it is convenient to consider these values per square centimeter of contact area between the element and protecting tube, since the two are practically in direct contact.

The rate at which heat is dissipated from the outer body to the cooling medium is made up of the heat loss in this body together with that lost by the inner body. In the time dt, the outer body drops dT and the inner body drops $d\theta$, in temperature. Then in the time dt

$$(T-T_c)h_1dt=-(MSdT+msd\theta) \qquad (12)$$

where $MSdT$ and $msd\theta$ are the heat losses per dT and $d\theta$ for the outer and inner bodies, respectively, having masses of M and m, and specific heats of S and s, respectively.

The transmission of heat from the inner body to the outer body is equal to that lost by the inner body, and, therefore, in time dt,

$$(\theta-T)h_2dt=-msd\theta \qquad (13)$$

The negative signs are used as these are decreasing functions with time.

From equation 13

$$T=\theta+\frac{ms}{h_2}\frac{d\theta}{dt}$$

Differentiating this equation for T with respect to time and substituting in equation 12 for T and dT/dt, we have after collecting terms

$$\left(\frac{MSms}{h_1h_2}\right)\frac{d^2\theta}{dt^2}+\left(\frac{MS}{h_1}+\frac{ms}{h_2}+\frac{ms}{h_1}\right)\frac{d\theta}{dt}+(\theta-T_c)=0 \qquad (14)$$

Now MS/h_1, ms/h_2 and ms/h_1 have the dimensions of time, and so for brevity let them be t_1, t_2, and t_m, respectively, as the time constants of the thermometer. Substituting these time constants in equation 14 there results,

$$\frac{d^2\theta}{dt^2}+\left(\frac{t_1+t_2+t_m}{t_1t_2}\right)\frac{d\theta}{dt}+\frac{\theta-T_c}{t_1t_2}=0 \qquad (15)$$

Integrating this in the usual manner by letting

$$(\theta-T_c)=e^{vt} \qquad (16)$$

and substituting this in equation 15 and solving for v, we have

$$v=-\alpha\pm\sqrt{\alpha^2-\beta^2} \qquad (17)$$

where for brevity

$$\alpha=\frac{1}{2}\left(\frac{t_1+t_2+t_m}{t_1t_2}\right)$$

and

$$\beta=\frac{1}{\sqrt{t_1t_2}}$$

Substituting the value for v in equation 16 we have

$$(\theta-T_c)=e^{-\alpha t}\left[ae^{t\sqrt{\alpha^2-\beta^2}}+be^{-t\sqrt{\alpha^2-\beta^2}}\right] \qquad (18)$$

where a and b are constants of integration. Let $a=(^1/_2)(A+B)$, and $b=(^1/_2)(A-B)$, where A and B are constants to be determined. Then, substituting these values in equation 18, multiplying, and dividing by $\sqrt{B^2-A^2}$, expanding, and collecting terms, we have,

$$(\theta-T_c)=\left(\sqrt{B^2-A^2}\right)e^{-\alpha t}\times \sinh\left[t\sqrt{\alpha^2-\beta^2}+\sinh^{-1}\frac{A}{\sqrt{B^2-A^2}}\right] \qquad (19)$$

The constants A and B are determined by applying the known boundary conditions, $\theta=\theta_0$, and $d\theta/dt=0$, at $t=0$, which, when introduced into equation 19, there results,

$$\frac{\theta-T_c}{\theta_0-T_c}=\sqrt{\frac{\beta^2}{\alpha^2-\beta^2}}e^{-\alpha t}\times \sinh\left[t\sqrt{\alpha^2-\beta^2}+\sinh^{-1}\sqrt{\frac{\alpha^2-\beta^2}{\beta^2}}\right] \qquad (20)$$

Substitute in this equation the values of α and β in terms of t_1, t_2, and t_m, and for brevity let

$$P=\sqrt{\frac{\alpha^2-\beta^2}{\beta^2}}=\sqrt{\frac{(t_1+t_2+t_m)^2}{4t_0^2}-1}$$

where $t_0=\sqrt{t_1t_2}$, and we have

$$\frac{\theta-T_c}{\theta_0-T_c}=\frac{1}{P}e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times \sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}P\right] \qquad (21)$$

which is equation 1 used in the descriptive part of this paper.

Appendix II

This analysis considers the problem of heating of a body indirectly, by a higher constant-temperature medium through a second body in direct contact with that medium; for example, the heating of a thermometer element contained in a protecting case.

Let

θ = temperature of the inner body at any time t

T = temperature of the outer body at any time t

T_e = temperature of surrounding medium (constant)

θ_0 = initial temperature of both inner and outer bodies at time $t=0$

Then the heat transmitted to the outside body in dt from the medium is $(T_e-T)\,h_1dt$. This heat increases the temperature of the outer body by dT, and the inner body by $d\theta$. The heat absorbed by these bodies in dt is then $MSdT+msd\theta$.

Therefore

$$(T_e-T)h_1dt=MSdT+msd\theta \qquad (22)$$

The heat transmitted from the outside body to the inside body in dt is $(T-\theta)h_2dt$, and this equals the heat absorbed by the inner body, $msd\theta$.
Therefore

$$(T-\theta)h_2dt=msd\theta \qquad (23)$$

It will be noted that these are the same equations as those found in the problem of the cooling bodies, namely, equations 12 and 13, and the same final equation follows:

$$\frac{d^2\theta}{dt^2}+\left(\frac{t_1+t_2+t_m}{t_1t_2}\right)\frac{d\theta}{dt}+\frac{\theta-T_e}{t_1t_2}=0 \qquad (24)$$

This reduces in form, therefore, to the same equation as for the cooling problem, namely:

$$\frac{\theta-T_e}{\theta_0-T_e}=\frac{1}{P}e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times \sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}P\right] \qquad (25)$$

However, in this heating problem, what we wish to know is the elevation at any time t of the temperature θ of the inside body above the initial temperature θ_0; that is $(\theta-\theta_0)$, and not $(\theta-T_e)$ as given in equation 25.

Now if we subtract unity from each side of this equation, and multiply both sides by -1, we get

$$\frac{\theta-\theta_0}{T_e-\theta_0}=1-\frac{1}{P}e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times \sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}P\right] \qquad (26)$$

which is in the form desired for the problem of the heating body.

Appendix III

The following analysis considers the time–temperature lag in a thermometer element enclosed in a protecting case immersed in a medium changing in temperature at a constant rate.

Let T_a = temperature of the medium at any time t, and r = the rate of change in the temperature of the medium in degrees per second.

Then the same differential equations apply as were found for a constant-temperature medium as given in Appendix I except that the varying temperature T_a is substituted for the constant temperature T_e, and we have from equation 15, Appendix I,

$$\frac{d^2\theta}{dt^2}+\left(\frac{t_1+t_2+t_m}{t_0^2}\right)\frac{d\theta}{dt}+\frac{\theta-T_a}{t_0^2}=0 \qquad (27)$$

Assume that the thermometer and medium both start at the same temperature θ_0; then the temperature of the medium at any time t will be

$$T_a=\theta_0+rt \qquad (28)$$

The rate r will be positive for increasing temperatures and negative for decreasing temperatures.

Substituting equation 28 in equation 27 we obtain,

$$\frac{d^2x}{dt^2}+\frac{a}{b}\frac{dx}{dt}+\frac{x}{b}=\frac{rt}{b}=\frac{f(t)}{b} \qquad (29)$$

where for brevity, $a/b=(t_1+t_2+t_m)/t_0^2$, $b=t_0^2$, and $x=(\theta-\theta_0)$. The integral of an equation of the form of equation 29 in general, as given in any book on differential equations, is

$$x=\frac{1}{\sqrt{a^2-4b}}\left[e^{-\frac{t}{T_1}}\int e^{\frac{t}{T_1}}f(t)dt-e^{-\frac{t}{T_2}}\int e^{\frac{t}{T_2}}f(t)dt\right]+c_1e^{-\frac{t}{T_1}}+c_2e^{-\frac{t}{T_2}} \qquad (30)$$

where c_1 and c_2 are constants to be determined, and

$$T_1=\frac{2b}{a-\sqrt{a^2-4b}}=\frac{t_0}{\sqrt{P^2+1}-P}, \text{ and}$$

$$T_2=\frac{2b}{a+\sqrt{a^2-4b}}=\frac{t_0}{\sqrt{P^2+1}+P}$$

In this problem $f(t)=rt$. Substituting the thermal constants corresponding to a and b, and rt for $f(t)$, we have

$$(\theta-\theta_0)=\frac{1}{2t_0P}\left[e^{-\frac{t}{T_1}}\int rte^{\frac{t}{T_1}}dt-e^{-\frac{t}{T_2}}\int rte^{\frac{t}{T_2}}dt\right]+c_1e^{-\frac{t}{T_1}}+c_2e^{-\frac{t}{T_2}} \qquad (31)$$

Integrating, collecting terms, and substituting values of T_1 and T_2, we have

$$(\theta-\theta_0)=r\left[t-2t_0\sqrt{P^2+1}\right]+c_1e^{-\frac{t}{T_1}}+c_2e^{-\frac{t}{T_2}} \qquad (32)$$

To determine the constants of integration, substitute for T_1 and T_2 and apply the boundary conditions, $\theta=\theta_0$ at $t=0$, and $d\theta/dt=0$ at $t=0$, and we obtain

$$(\theta-\theta_0)=r\left[t-2t_0\sqrt{P^2+1}\right]+\frac{rt_0}{P}e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times\sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}2P\sqrt{P^2+1}\right] \qquad (33)$$

Now instead of the deviation $(\theta-\theta_0)$, of the temperature of the element at time t from the initial temperature as given directly by equation 33, we wish to know the lag, that is $(T_a-\theta)$. We know from equation 28 that $T_a=\theta_0+rt$, and if we substitute in equation 33 the value of θ_0 in terms of T_a and r we obtain the lag at any time,

$$(T_a-\theta)=2rt_0\sqrt{P^2+1}-\frac{rt_0}{P}e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times\sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}2P\sqrt{P^2+1}\right] \qquad (34)$$

The maximum lag results when $t=\infty$, and by substituting this value in equation 34 and evaluating we find that the maximum lag is

$$L=[T_a-\theta]_{\max}=2rt_0\sqrt{P^2+1}=r(t_1+t_2+t_m) \qquad (35)$$

Dividing equation 34 through by $L=2rt_0\sqrt{P^2+1}$, we have for the equation for increasing temperatures,

$$\frac{T_a-\theta}{L}=1-\frac{1}{2P\sqrt{P^2+1}}e^{-\left[\left(\frac{t}{t_0}\right)\sqrt{P^2+1}\right]}\times\sinh\left[P\left(\frac{t}{t_0}\right)+\sinh^{-1}2P\sqrt{P^2+1}\right] \qquad (36)$$

For decreasing temperatures, r becomes negative, and the resulting equation is the same as equation 36 except that $(T_a-\theta)/L$ becomes $(\theta-T_a)/L$. Since r does not appear in the right-hand member of the equations, the form of the lag curve with time is independent of the rate of change in temperature.

The maximum lag can be determined directly from the fundamental differential equation 27, by remembering that at $t=\infty$, $d^2\theta/dt^2$ becomes zero, and $d\theta/dt$ becomes r. Then

$$\left(\frac{t_1+t_2+t_m}{t_0^2}\right)r=\frac{[T_a-\theta]}{t_0^2}\max$$

which results in the same equation 35 as found by the complete analysis, namely:

$$L=[T_a-\theta]_{\max}=(t_1+t_2+t_m)r$$

Reference

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TRANSACTIONS SECTION

Preprint of Corresponding Pages From the Current Annual AIEE Transactions Volume

Ferroinductance as a Variable Electric-Circuit Element

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IT HAS LONG been known that the inductance of iron-cored coils is not a constant, but varies with the value of current flowing, and that this is not a linear relation, owing to the shape of the magnetization curve of the iron used.

Mathematical analysis of such circuits leads to differential equations with variable coefficients which are very difficult to solve. To obtain usable results the engineer is accustomed to assume the inductance of the iron-cored reactor as constant, thereby obtaining equations with constant coefficients and an easy mathematical solution for the circuit.

However, for this analysis, in terms of complex algebra or differential equations, it is assumed that the value of the inductance is not a function of current or time. In many present-day problems this assumption cannot be supported, and a new method of analysis is needed.

Definitions

Inductance is the proportionality factor between instantaneous induced voltage and the rate of change of current in an electric circuit. That is

$$L=\frac{-e}{\dfrac{di}{dt}} \qquad (1)$$

is the basic defining relation for inductance.

If it is remembered that equation 1 is a relationship between instantaneous quantities, then equation 1 becomes fundamental, and other definitions such as summarized by Raner and Litscher[1] become only special cases, based on weighted averages.

Ferroinductance or ferroreactance are terms used to designate the special properties of an inductance when iron is introduced into the magnetic circuit. Defining equation 1 still holds, but the value of L, the proportionality factor, is no longer a constant but is a function of i, the current.

Historical Background

Since the time the first magnetization curve was measured for a steel, there have been attempts to develop equations to fit the curve. These attempts have followed mathematical or physical paths, but it cannot be said that either method has been more successful than the other.

One of the oldest, and certainly the most famous, of the equations developed for the purpose is Frölich's equation: $B=H/(a+bH)$, which gives a satisfactory fit up to approximately the knee of the curve. Zenneck,[6] in his 1920 paper on the analysis of magnetic frequency doublers, used the relation: $B=sH-s'H^3$. It should be noted that this equation cannot be used much beyond the knee of the magnetization curve because of the negative cubic term. Zenneck succeeded in obtaining an expression for ferroinductance in terms of current, but did not carry the matter further and submitted no experimental proof.

Ryan,[7] also for the analysis of magnetic frequency doublers, used the relation: $B=A\tan^{-1}ax+Cx$, which is related to that of Zenneck through the arc tangent series. Keller[11] also used a similar series. Boyajian,[14] in order to simplify the mathematics, used an approximation to the magnetization curve consisting of two straight lines, one for use below the knee, the other above. The results, while easily obtained, are not too satisfactory for a design basis.

In 1926, Gokhale,[5] using physical methods, developed an exponential form which fitted well above saturation, leaving much to be desired below the knee. Rader and Litscher[1] have used the equation: $H=K'B+K''B^{10}$, and while this fits the magnetization curves fairly well, it is impossible for use for circuit analysis, as will be discussed in the next section.

One of the earliest applications of the nonlinear voltage-current characteristics of ferroinductors was made by Zenneck[13] in 1899. This involved the use of transformers, with d-c excitation to accentuate the nonlinearity, connected to produce frequency doubling. The first report of the phenomena present in R-L-C series circuits, in which the L was ferroinductive, was made by Martienssen[8] in 1910. This is the type of circuit which we now call ferroresonant. Martienssen, while obtaining what appear to be correct qualitative results, invalidated them for quantitative use by a serious mathematical error.

More recent investigators ordinarily have used the graphic approach for a solution, but this involves the construction of a reactor before the design of the other components of the ferroresonant circuit can be completed.

A New Equation for the Magnetization Curve

The shape of the magnetization curve is obviously the cause of the phenomena observed in ferroinductive circuits. If an empirical equation for the magnetization curve is to be useful in the analysis of a-c circuits it must be a function which behaves in every way exactly as the true magnetization curve. Since actual magnetization curves are "odd" functions, and display equal and opposite effects for positive and negative currents, then the function chosen to represent the magnetization curve must likewise be an "odd" function.

The magnetization curve affects our

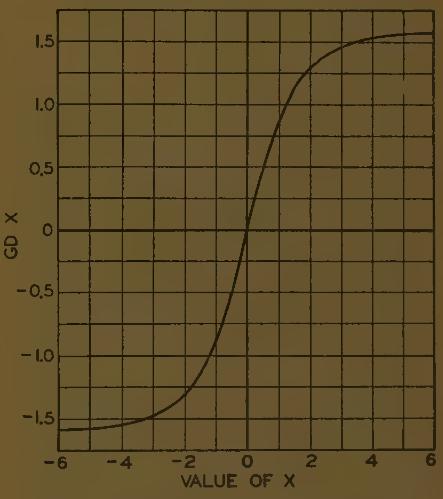

Figure 1. Variation of the gudermannian

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The material presented in this paper is abstracted from a dissertation presented for the doctor-of-philosophy degree at Iowa State College. The complete dissertation is on file at the Iowa State College Library, Ames, Iowa.

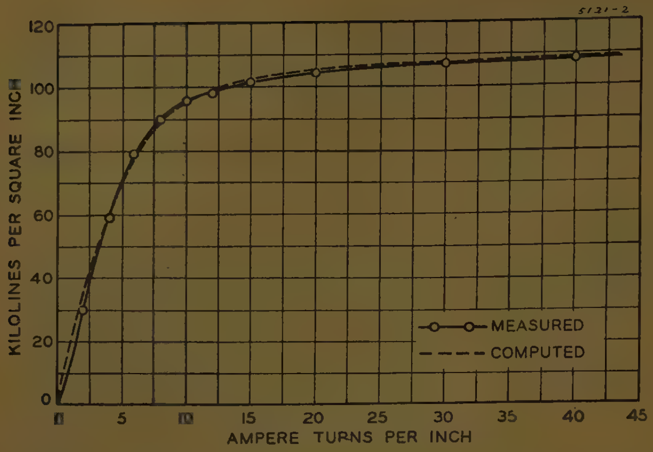

Figure 2. Measured and calculated magnetization curves for Hipersil steel

electric circuits indirectly, through the phenomena of flux-linkage changes or inductance. Since inductance is one of our basic circuit parameters, it is obvious that, for convenience alone, the equation chosen to represent the magnetization curve should lead to a simple expression for inductance.

Another equation for inductance, derived from equation 1 is

$$L=N\frac{d\phi}{di}\times10^{-8} \qquad (2)$$

Thus, inductance is proportional to the slope of the magnetization curve.

It is then possible to set up two criteria which a function, to be useful as an approximation for the magnetization curve of a steel, must fulfill:

1. The function must be "odd."
2. The current derivative or slope of the function must be mathematically simple.

By applying this test to the functions used by other investigators it is found that none of the empirical relationships considered are entirely suitable for the analysis of a-c circuits. However, there exists a function, the "gudermannian",[2] which is suitable for the purpose. This function is defined as

$$gd\,x=\tan^{-1}\sinh x \qquad (3)$$

It is an odd function and has as its derivative sech x, which is of simple form. It therefore fulfills, in satisfactory fashion, the two requirements laid down, providing a fit to the curves for various steels can be obtained. Values of $gd\,x$ are tabulated in the Smithsonian Tables.[3] A curve $gd\,x$ is presented in Figure 1, and it can be seen that the variation is in the right direction for the fitting of a magnetization curve.

The empirical form adopted for the magnetization curve was

$$B=B_n gd\frac{aNi}{l}+\frac{cNi}{l} \qquad (4)$$

This equation, being a transcendental relation, does not lend itself to direct solution for the values of the constants B_n, a, and c. The method developed was that of a trial process for a, after which B_n and c were obtained directly, as outlined in Appendix III.

Results for Typical Steels

As an indication of the ability of equation 4 to approximate the magnetization curves for various commercial steels, two types were selected. The first, Hipersil, was selected as typical of modern transformer steel. As no magnetization curves were obtainable from the manufacturer, the curve was measured in the laboratory. The second steel was Nicaloi (Allegheny 4750), a high permeability alloy. The curve was available from manufacturer's data.

Using English units, the equations determined for these steels were

Hipersil:

$$B=65{,}000gd\left(0.273\frac{Ni}{l}\right)+180\frac{Ni}{l} \qquad (5)$$

Nicaloi:

$$B=36{,}600gd\left(4.58\frac{Ni}{l}\right)+6{,}200\frac{Ni}{l} \qquad (6)$$

Curves of equation 5 compared with the measured data are shown in Figure 2, with excellent agreement indicated. Equally good results have been secured

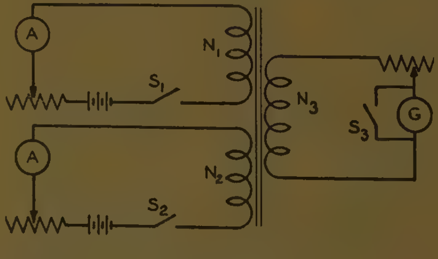

Figure 3. Circuit used for the measurement of ferroinductance

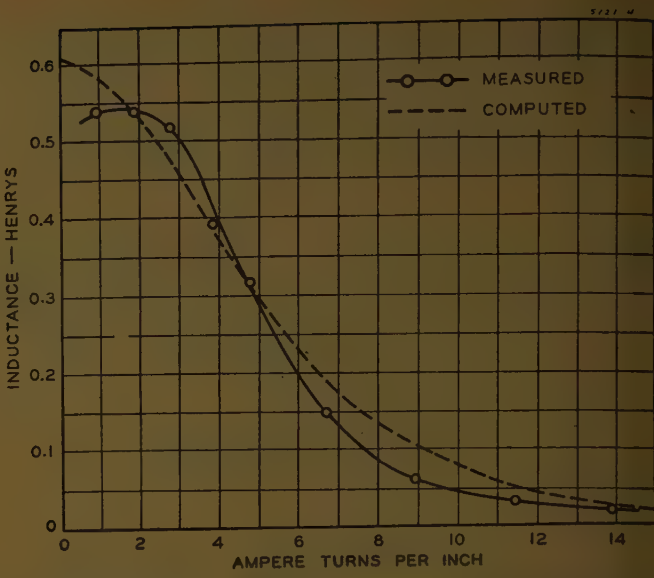

Figure 4. Measured and calculated inductance of a 156-turn reactor on Hipersil core

for many steels other than the two described.

Applications of the Equation

In most modern magnetic steels the hysteresis-loop area has been much reduced and has been here neglected to simplify the mathematics. It was assumed that magnetization of the steel throughout an a-c cycle took place along the normal magnetization curve, rather than around the hysteresis loop. Leakage flux also has been neglected as it was small for the core structures used.

Normal permeability is the term obtained from the relation

$$\mu=K\frac{B}{H} \qquad (7$$

and setting K = 0.3125 for English units normal permeability can be calculated from equation 4 giving

$$\mu=0.3125\left[\frac{B_n gd\frac{aNi}{l}}{\frac{Ni}{l}}+c\right] \qquad (8$$

Equation 8 has been used to calculate the permeability of Hipersil and other steel with accurate results.

Differential permeability is defined a the slope of the magnetization curve, o the permeability to small changes of magnetizing force. That is

$$\mu_d=K\frac{dB}{dH} \qquad (9$$

and, setting K = 0.3125 for English units

$$\mu_d=0.3125\left[aB_n\operatorname{sech}\frac{aNi}{l}+c\right] \qquad (10$$

Calculations for various steels using equation 10 yield results which closely chec values of permeability obtained from the slopes of the magnetization curves.

Since

$$\phi = AB_n \operatorname{gd}\frac{aNi}{l} + \frac{cANi}{l} \qquad (11)$$

$$\frac{d\phi}{di} = \frac{aAB_nN}{l} \operatorname{sech} \frac{aNi}{l} + \frac{cAN}{l} \qquad (12)$$

Then, from equation 2

$$L = \frac{aAB_nN^2}{10^8 l} \operatorname{sech} \frac{aNi}{l} + \frac{cAN^2}{10^8 l} \qquad (13)$$

Equation 13 gives the value, in henrys of the inductance of a ferroinductor with any value of current flowing. The first term on the right is the contribution resulting from the presence of iron, the second term is the value if the iron is removed, leaving an air core.

As the maximum value of sech x is unity and occurs for x zero, then the inductance is maximum at zero current. Actually, because of the small reverse curvature of the magnetization curve near zero, the measured inductance maximum occurs, not at, but near zero. By noting that sech ∞ equals zero, the value of inductance is seen never less than cAN^2/l. High values of current can reduce the effectiveness of the iron but never can reduce the inductance below that of the coil on an air core.

Equation 13 is a simple expression for the calculation of ferroinductance. By its use an inductor can be designed in advance of construction, to have a given inductance at a given value of current, the only data required being the constants B_n, a, and c of the steel.

Measurement of Ferroinductance

Ordinary a-c bridge methods of inductance measurement are not satisfactory. The voltage applied to the inductor varies as the bridge is balanced, changing the inductance, and a sliding balance results. If balance is obtained, knowledge of the actual current flowing through the reactor is not obtainable, and the value of inductance means nothing.

A bridge especially designed for this,[4] and including means for measuring alternating and direct currents still overlooks a fundamental point. It may give information on the inductance under the particular operating conditions, but it gives no clue to a means for determining the inductance under different conditions.

It has been necessary to develop a new method by which ferroinductance can be measured at any value of current or ampere turns. This new method is based on the relation

$$L = N\frac{d\phi}{di} \times 10^{-8} \qquad (14)$$

From equation 14, it can be seen that an inductance can be measured at any value of steady current, if a differential change in current be made, the resulting differential change in flux measured, and the ratio taken and multiplied by the number of turns in the coil.

The method is based on the fact that, after demagnetization, an iron core is magnetized along the normal curve, and if the differential change is made additive, then the iron is taken further along the normal curve and the effect of a hysteresis loop is eliminated.

For accurate measurement of the differential current change it is most convenient to use a three-coil arrangement, as shown in Figure 3. The steady d-c ampere turns are applied in N_1, the differential-current change is made by closing the circuit of N_2 whose inductance is to be measured, and N_3 is for the ballistic measurement of the flux change with the galvanometer. The ampere turns of the first and second coils must be connected to aid.

The size of the differential change can be made too small for the sensitivity of the apparatus used. Values of the change used were as low as 0.001-ampere turns per inch, but it was found that values of 0.1- to 0.5-ampere turns per inch gave satisfactory results. The values of inductance so obtained were checked independently by calculations made from values of permeability obtained at the operating points. Measurements made with the Hipersil core having an area of 1.87 square inches, length of 13.2 inches, and a coil of 156 turns, are shown in Figure 4, compared with the dashed-line curve secured by computation from equation 13.

Close agreement between measured and computed values is secured over most of the range, resulting in a check of the theory and measuring method developed.

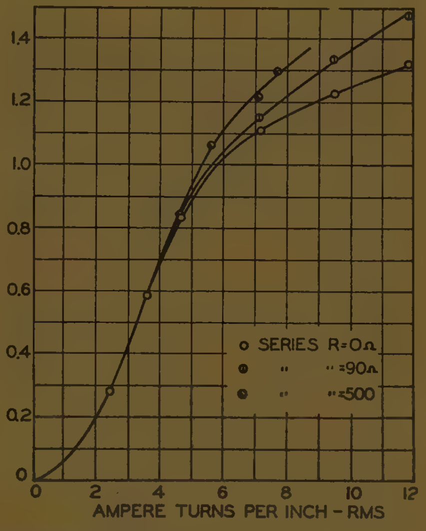

Figure 5. Wattage iron loss as a function of effective current

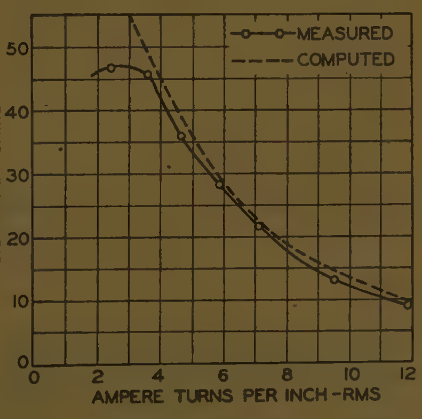

Figure 6. Effective resistance of a 156-turn reactor on Hipersil core

Value of Inductance for Applied Sine Electromotive Force

Consider a ferroreactor with sinusoidal electromotive force applied, neglecting the resistance of the circuit. As shown in Appendix I, an expression for the inductance as a function of time can be obtained

$$L = \frac{aAB_nN^2}{10^8 l}[J_0(G) + 2J_2(G)\cos 2\omega t + 2J_4(G)\cos 4\omega t + 2J_6(G)\cos 6\omega t + \ldots\ldots] \qquad (15)$$

where

$$G = \frac{E_m 10^8}{\omega a B_n N} \qquad (16)$$

This shows that, for a sine voltage, the inductance of a ferroinductor consists of a constant term plus sinusoidally varying terms in even harmonics of the applied frequency. This provides an explanation of the source of the harmonic components appearing in the current that flows through a ferroreactor.

Likewise, an expression for the current flowing can be secured in terms of the harmonic components (Appendix II)

$$i = \frac{l}{aN}\left[\left(G + \frac{G^3}{8} + \frac{5G^5}{192} + \frac{2{,}135G^7}{322{,}000} + \ldots\ldots\right) \times \sin \omega t - \left(\frac{G^3}{24} + \frac{5G^5}{384} + \frac{1{,}280G^7}{322{,}000} + \ldots\ldots\right) \times \sin 3\omega t + \left(\frac{G^5}{384} + \frac{427G^7}{322{,}000} + \ldots\ldots\right) \sin 5\omega t - \left(\frac{61G^7}{322{,}000} + \ldots\ldots\right) \sin 7\omega t + \ldots\ldots\right] \qquad (17)$$

Equation 17 shows that the frequencies are odd harmonics, and alternate in algebraic sign, as is well known for this case. In Table I is presented an analysis, made with a General Radio Company wave analyzer, of an input-current wave form for sine voltage. For comparison also presented is an analysis using equation 17.

One important source of error is represented. Equation 17 was developed under an assumption of negligible circuit resistance. The effective resistance of the Hipersil core reactor was not negligible, however, and consequently the currents flowing were somewhat less than had been predicted.

Impedance of a Circuit Containing a Ferroinductor

For a series circuit of resistance, ferroinductance, and capacity, the circuit-differential equation is

$$D \text{ sech } \frac{aNi}{l} \frac{di}{dt} + Ri + \frac{\int i dt}{c} = e \tag{18}$$

where

$$D = \frac{aAB_nN^2}{10^8 l} \tag{19}$$

This so far has been insoluble for sinusoidal electromotive force, but if $i = I_m \times \sin \omega t$ is assumed, then the equation becomes

$$RI_m \sin \omega t + I_m\left(\omega D \text{ sech } \frac{aNI_m \sin \omega t}{l} - \frac{1}{\omega c}\right)\cos \omega t = e \tag{20}$$

For equation 20 to be fulfilled, e must be nonsinusoidal. The equation is so similar in form to the conventional circuit equation

$$RI + jI\left(\omega L - \frac{1}{\omega c}\right) = E \tag{21}$$

involving rms values of current and voltage, that it should be possible in some manner to bridge the gap between the instantaneous quantities of equation 20 and the rms quantities of equation 21.

Inductance as determined by the ferroinductance equation 13 is an instantaneous quantity, dependent on the instantaneous value of a varying current. Over a cycle of current variation the inductance changes through a considerable range of values. Its effect in the circuit will be some sort of a weighted average of the values taken during the current cycle, or a weighted function of the current or ampere turns.

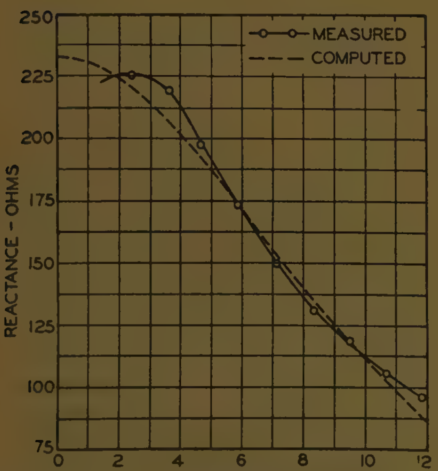

Figure 7. Measured and calculated reactance of the 156-turn reactor on Hipersil core

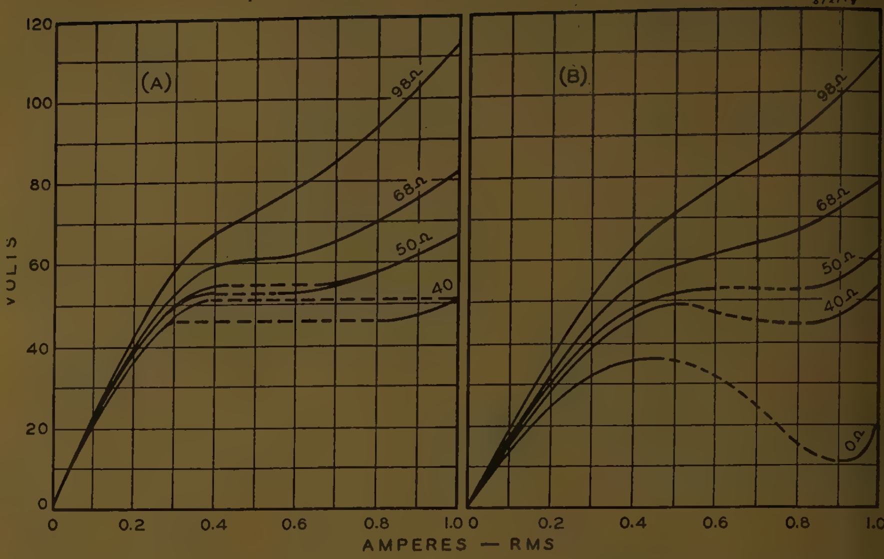

Figure 8. Voltage across a ferroresonant circuit, for various values of circuit resistanc exclusive of the effective resistance

A—Measured
B—Calculated

The magnitude of the weighting factor will depend on the manner in which the inductance varies with the current and this is fixed by the magnetization curve. The factor in the inductance relation which reflects the shape of the magnetization curve is sech (aNi/l). Consequently, if a weighting factor is present, it will appear in the angle of the hyperbolic secant as some factor k. The effective value of inductance L_e then may be written as

$$L_e = D \text{ sech } \frac{kaNI}{l} \tag{22}$$

where I is the rms value of current. Based on this argument and the similarity between equations 20 and 21 it seems reasonable to assume

$$I = \frac{E}{\sqrt{R^2 + \left(\omega D \text{ sech } \frac{kaNI}{l} - \frac{1}{\omega c}\right)^2}} \tag{23}$$

where E and I are effective values and k is the weighting factor for a particular steel.

The impedance of a ferroinductive circuit then becomes

$$Z = \sqrt{R^2 + \left(\omega D \text{ sech } \frac{kaNI}{l} - \frac{1}{\omega c}\right)^2} \tag{24}$$

for a particular value of current I. The resistance R includes the effective resistance of the reactor as well as other circuit resistance. Since equation 23 is an assumption, experimental verification must be obtained; the proof is supplied in these pages.

Determination of Effective Resistance

It is customary to calculate the effective resistance R_e by definition

$$R_e = \frac{W}{I^2} \tag{25}$$

where W is the wattage dissipated in the reactor iron and I the effective value of the fundamental component of current.

When a ferroinductor is in series with another impedance across a sinusoidal voltage, neither the current nor the voltage of the ferroinductor is sinusoidal, i general. Therefore the flux is not sinusoidal but is made up of harmonic frequencies. The total iron losses vary a some power of the frequency and are als a function of peak flux density. For convenience of measurement, the rms current, voltage, and power most readily ar obtained. Consequently the relation between rms current, or ampere turns, and watts loss for such distorted wave form would be interesting as it would reflect not only the variation of iron losses wit wave form, but also the variation of th losses with peak flux.

Data have been taken showing that th variation of losses with wave form is a much a function of rms current as of pea flux density. Evidently the change of th losses with frequency is somewhat compensated for by the accompanying chang in peak factor of the current.

Figure 5 shows the losses of a reactor o Hipersil steel for various wave forms induced by the use of resistance in serie with the reactor. The curves of Figure can be approximated to a fair degree o accuracy by a power series of the form

$$W/\text{lb} = K_1 \frac{NI}{l} + K_2\left(\frac{NI}{l}\right)^2 + \ldots \ldots \quad (26$$

which then gives as an expression for R_e, by use of equation 25, the following equation, in which I is in rms amperes

$$R_e/\text{lb} = \frac{K_1 N}{lI} + \frac{K_2 N^2}{l^2} + \ldots \ldots \quad (27$$

An experimental check of equation 2 was obtained by measuring the iron losse of the 156-turn coil on Hipersil core, com

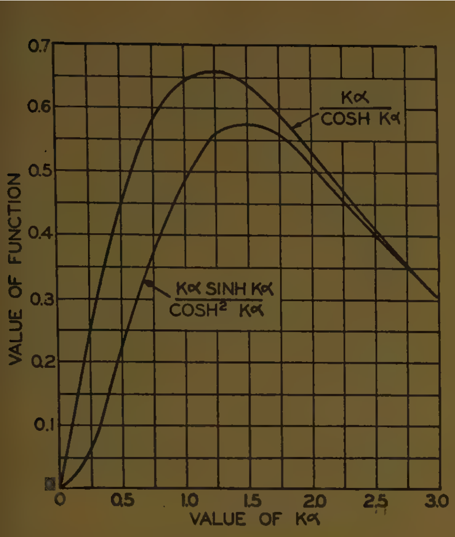

Figure 9. Values of the functions

$$\frac{k\alpha \sinh k\alpha}{\cosh^2 k\alpha} \text{ and } \frac{k\alpha}{\cosh^2 k\alpha}$$

puting the effective resistance by equation 25 and plotting as the solid curve of Figure 6. The dashed curve was computed from equation 27 using the weight of 6.9 pounds and $K_1 = 0.213$ and $K_2 = -0.00784$. The accuracy is good except over the unimportant low portion of the current values.

Calculation of Reactance

A discussion of reactance and of the design of units to give a specified value really hinges on the proof of existence of the weighting factor k of equation 22. The reactance then would be

$$X = \omega D \operatorname{sech} \frac{k a N I}{l} \tag{28}$$

The reactance can be obtained by measurement of impedance and effective resistance and using

$$X = \sqrt{Z^2 - R_e^2} \tag{29}$$

Such measurements were made on the reactor previously used and are plotted as the solid curve of Figure 7.

From Figure 7, k now can be calculated. Arbitrarily choose a point well out on the reactance curve, say at ten ampere turns per inch, at which point the reactance is 112 ohms. If a value k exists, then this point, and all other values on the curve, should be predicted by equation 28. Solving for k, a value of $k = 0.5$, is obtained. By using this value of k the calculated curve of Figure 7 was obtained, showing good agreement with the measured value. Similar good agreement has been obtained using other reactors and a wide range of steels, with values of k ranging from 0.3 to 0.5.

Table I. Harmonic Analysis of Input Current to a Reactor

	Voltage Applied = 60 Rms		Voltage Applied = 83 Rms	
	Measured	Calculated	Measured	Calculated
Total I rms	0.27	0.31	0.40	0.53
Fundamental (per cent)	100.0	100.0	100.0	100.0
Third harmonic (per cent)	7.8	7.5	20.4	17.5
Fifth harmonic (percent)	1.8	0.7	6.7	2.7
Seventh harmonic (per cent)	0.4	0.4	3.1	2.1

The experimental evidence definitely seems to establish that a factor k does exist as a weighting factor. It fully substantiates the assumptions of equation 28, and permits its use to calculate the reactance of a coil for any value of current, or to design a coil to have a particular reactance at a definite effective current value.

The Ferroresonant Circuit

Referring to equation 23, the possibility exists that the second term under the radical may go to zero at some value of I. At this point the denominator is small and the current rises. A circuit, including a ferroreactor, in which this happens is said to be ferroresonant, because of the similarity of the properties to frequency resonance.

One of the important circuit values, useful either for design or operation, is the voltage at which the resonant current jump takes place. Since the usual definition places the reactive terms equal to zero, then at resonance

$$\omega D \operatorname{sech} \frac{k a N I}{l} = \frac{1}{\omega C} \tag{30}$$

and then

$$I_R = \frac{l}{k a N} \cosh^{-1} \omega^2 D C \tag{31}$$

where I_R is the current at resonance. It is not readily possible to measure this current, since before it is reached the current rises to a higher value. The voltage is, however, practically constant through this range and can be measured as E_R, the resonant voltage.

At resonance, as the reactance is zero, the total voltage appears across the circuit resistance. Therefore

$$E_R = \frac{R l}{k a N} \cosh^{-1} \omega^2 D C \tag{32}$$

where R is made up of the circuit resistance plus the effective resistance of the reactor, or

$$R = R_c + R_e \tag{33}$$

Reversing equation 32 a value for the capacity required for resonance at a particular voltage E_R is

$$C = \frac{1}{\omega^2 D} \cosh \frac{k a N E_R}{R l} \text{ farads} \tag{34}$$

Table II presents data taken to check equation 32.

Assume a series circuit using the reactor with Hipersil core and 156 turns, a capacity C of 25 microfarads, and external resistance R_x variable. Figure 8 is a plot of the voltage across the circuit, computed from equation 23, for various values of external circuit resistance R_x. The dashed portions of the curves having a negative slope cannot be realized experimentally, as the current jumps directly across. Apparently this jump starts at the point at which the negative rate of change of impedance exceeds the rate of change of current.

Measured values of voltage for various values of R_x also are plotted in Figure 8. There is a very marked agreement, especially in values of voltage at which the current jumps occur, between the measured and totally calculated values.

For values of small R_x, the jump does not occur at the same value of voltage for decreasing as for increasing voltage. The current in this region is multivalued. The amount of resistance which just permits resonance without multivalued current,

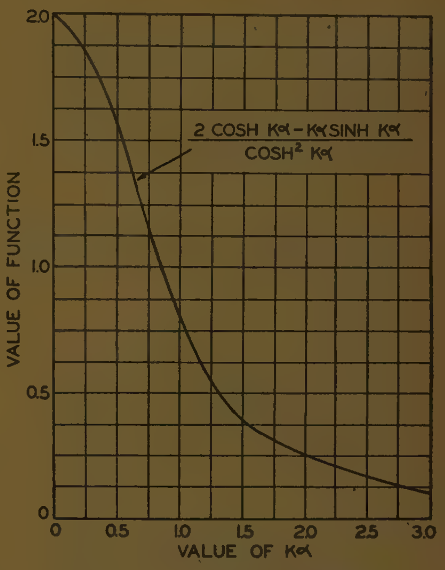

Figure 10. Value of the function

$$\frac{2 \cosh k\alpha - k\alpha \sinh k\alpha}{\cosh^2 k\alpha}$$

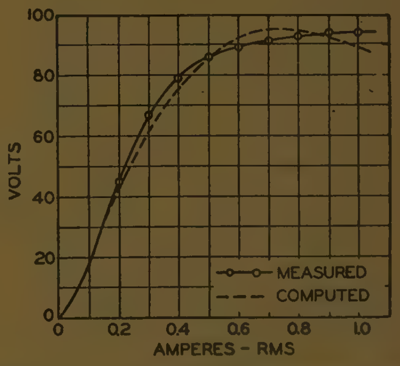

Figure 11. Variation of voltage across a pure reactance, measured and calculated

or the value producing the curve with a single point having a tangent of zero slope, is called the critical resistance R_k of the circuit. For many control applications the possibility of two current values is undesirable, so that the value of the critical resistance is of importance.

The value of R_k can be determined (see Appendix II). Briefly, the voltage can be expressed as

$$E=I\sqrt{R^2+\left(\omega D \operatorname{sech}\frac{kaNI}{l}-\frac{1}{\omega c}\right)^2} \tag{35}$$

and the point of zero slope will have

$$\frac{dE}{dI}=0 \tag{36}$$

Before taking the derivative it is necessary to write R as a function of I, as R includes the effective resistance. From equation 27 the effective resistance may be written, and adding R_x and the copper resistance of the reactor, with w signifying the weight in pounds,

$$R=R_c+R_x+\frac{K_1Nw}{lI}+\frac{K_2N^2w}{l^2} \tag{37}$$

Taking the three constant terms as R_0, then the voltage equation 35 becomes

$$E=I\times\sqrt{\left(R_0+\frac{K_1Nw}{lI}\right)^2+\left(\omega D \operatorname{sech}\frac{kaNI}{l}-\frac{1}{\omega c}\right)^2} \tag{38}$$

After letting $kaNI/l = k\alpha$ and applying equation 36, the equation reduces to a quadratic in the circuit reactance which has a solution,

$$\omega D \operatorname{sech} k\alpha-\frac{1}{\omega c}=\frac{\omega Dk\alpha \sinh k\alpha}{2\cosh^2 k\alpha}\pm\frac{\sqrt{\left(\frac{\omega Dk\alpha\sinh k\alpha}{\cosh^2 k\alpha}\right)^2-4R_0\left(R_0+\frac{K_1Nw}{lI}\right)}}{2} \tag{39}$$

This indicates, in general, two values of the reactance for which $dE/dI = 0$, unless the radical is zero. The curves of Figure 8 support this. For large R_0 the value of the radical is imaginary, hence no real values of reactance for $dE/dI = 0$ are possible. This is illustrated by the curves for 98- and 68-ohms resistance.

However, for the curve with only one point having $dE/dI = 0$, the value of the radical must be zero. Applying this condition leads to two equations, from one of which may be obtained

$$\omega D \operatorname{sech} k\alpha-\frac{1}{\omega c}=R_0\sqrt{1+\frac{K_1Nw}{lIR_0}} \tag{40}$$

Equation 40 states that at the point of inflection for $dE/dI = 0$, the circuit reactance is practically equal to the circuit resistance.

By the use of curves for the functions of $k\alpha$ involved, the value of the current at the inflection point may be found, and with further use of Figures 9 and 10 values may be obtained for substitution in

Table II. Resonant Voltage in a Ferroresonant Circuit

C, µf	Total R, Ohms	E_R Measured, Volts	E_R Computed, Volts
16.7	78	43	45
25.0	66	54	57
41.7	53	68	64
50.0	47	70	66

$$R_0=\frac{\omega D}{2}\frac{k\alpha \sinh k\alpha}{\cosh^2 k\alpha} \tag{41}$$

and

$$R_x=R_0-R_c-\frac{K_2N^2w}{l^2} \tag{42}$$

Values larger than this must be placed in the circuit to insure single-valued current at all times.

The value of R_k has been calculated for the circuit used for the curves of Figure 8. This results in a value of 52 ohms, which, by consideration of both measured and computed curves, is a satisfactory check.

Variation of Ferroreactive Voltage Drop

As mentioned, it has been customary for many years to assume ferroinductance a circuit constant, and the ferroreactive-voltage drop in a-c circuits as proportional to the current, as in

$$E_m \sin(\omega t+\theta)=\omega LI_m \sin \omega t \tag{43}$$

This is an obvious impossibility if L is considered a variable with either time or current, as actually occurs in an a-c circuit. It is now possible to determine just why and to what extent the assumption can be supported.

If

$$X=\omega D \operatorname{sech} k\alpha \tag{44}$$

then the ferroreactive voltage drop may be written as

$$E_L=\frac{\omega AB_nN}{10^8k}\,\frac{k\alpha}{\cosh k\alpha} \tag{45}$$

The ferroreactive voltage drop is seen to be a function of $k\alpha/\cosh k\alpha$. This function is plotted against $k\alpha$ in Figure 9.

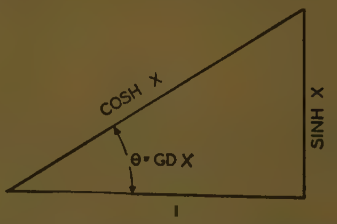

Figure 12. Property of the gudermannian

It is seen that it has an approximately linear rise to a maximum, followed by a gradual fall. If an inductor is operated on the rising portion, then the reactive voltage drop would be an almost linear function of the current, as assumed.

The reactive voltage drop for the reactor previously used was obtained from measurements of impedance and effective resistance drops, and plotted as the solid line in Figure 11. The voltage drop also was calculated from equation 45 by use of the curve of Figure 9 and is plotted as the dashed curve of Figure 11. Good agreement with theory is secured over most of the range. As the measurement of X, independent of R_e, is impossible, a better check is difficult.

Over most of the range, up to about five ampere turns per inch, the assumption of linear reactive drop is not difficult to support. This means that the linear region extends almost to the point at which the peak of the ampere-turns wave reaches the knee of the magnetization curve (see Figure 2).

Summary

This investigation was instituted in an attempt to develop methods which would make ferroinductive circuits as susceptible to analysis as circuits with constant parameters.

The work has led to the development of an accurate empirical equation for the magnetization curve of a steel. This equation has been applied in predicting the inductance of a ferroinductor at any value of current, and methods have been developed for measuring this inductance, which check the theory closely.

The equation for ferroinductance has been generalized into a relation for effective ferroreactance by an assumption, and experimental evidence has been obtained to support the validity of the assumption. Methods also were developed for calculating the effective resistance of an iron-cored reactor. Application then was made of both of these developments in calculating the impedance of circuits containing ferroreactors, and the results were again well supported by experiment.

Another application of the theory was made to the ferroresonant circuit, and simple relations for resonant voltage, capacity required, and the value of the critical resistance secured. All were checked satisfactorily by experiment.

Appendix I

Value of Inductance for Applied Sine Electromotive Force

With $e=E_m \cos \omega t$ and neglecting the resistance of the circuit, an equation for the flux present in the iron can be written

$$\phi=\frac{10^8}{N}\int e dt=\frac{E_m10^8}{\omega N}\sin \omega t \tag{46}$$

By the expression for the magnetization curve

$$\phi = AB_n gd\frac{aNi}{l} + \frac{cA\,Ni}{l} \tag{47}$$

and ordinarily the last term on the right can be neglected as very small. Then

$$gd\frac{aNi}{l} = \frac{\phi}{AB_n} \tag{48}$$

From the properties of the gudermannian[2] a right triangle can be drawn (Figure 12) where

$$\theta = gd\frac{aNi}{l} = \frac{\phi}{AB_n} \tag{49}$$

and from which it can be seen that

$$\text{sech}\,\frac{aNi}{l} = \cos\frac{\phi}{AB_n} \tag{50}$$

Substituting equation 46 into 50

$$\text{sech}\,\frac{aNi}{l} = \cos\left(\frac{E_m 10^8}{\omega AB_n N}\sin\omega t\right) = \cos\,(G\sin\omega t) \tag{51}$$

Then equation 13 for ferroinductance, neglecting the term containing c as small, becomes

$$L = \frac{aAB_nN^2}{10^8 l}\cos\,(G\sin\omega t) \tag{52}$$

Now it is known that

$$\cos\,(x\sin r) = J_0(x) + 2J_2(x)\cos 2r + 2J_4(x)\cos 4r + \ldots\ldots \tag{53}$$

where $J_0(x)$, $J_2(x)$, $J_4(x)$ ..... are Bessel-function coefficients which can be evaluated from tables of the function. Consequently the expression for inductance becomes

$$L = \frac{aAB_nN^2}{10^8 l}\,[J_0(G) + 2J_2(G)\cos 2\omega t + 2J_4(G)\cos 4\omega t + 2J_6(G)\cos 6\omega t + \ldots\ldots] \tag{54}$$

Current Through a Ferroinductor, Sine Electromotive Force Applied

From equations 46 and 48 the following can be written

$$gd\frac{aNi}{l} = \frac{E_m 10^8}{\omega AB_nN}\sin\omega t \tag{55}$$

and then

$$i = \frac{l}{aN}gd^{-1}\left(\frac{E_m 10^8}{\omega AB_nN}\sin\omega t\right) = \frac{l}{aN}gd^{-1}(G\sin\omega t) \tag{56}$$

The series expression for the antigudermannian is

$$gd^{-1}\mu = \mu + \frac{\mu^3}{6} + \frac{\mu^5}{24} + \frac{61\mu^7}{5{,}040} + \ldots\ldots \tag{57}$$

and equation 56 becomes

$$i = \frac{l}{aN}\left[G\sin\omega t + \frac{G^3}{6}\sin^3\omega t + \frac{G^5}{24}\sin^5\omega t + \ldots\ldots\right] \tag{58}$$

By substituting for the trigonometric relations their identities in terms of multiple angles and collecting terms,

$$i = \frac{l}{aN}\left[\left(G + \frac{G^3}{8} + \frac{5G^5}{192} + \frac{2{,}135G^7}{322{,}000} + \ldots\ldots\right)\times \sin\omega t - \left(\frac{G^3}{24} + \frac{5G^5}{384} + \frac{1{,}280G^7}{322{,}000} + \ldots\ldots\right)\times \sin 3\omega t + \left(\frac{G^5}{384} + \frac{427G^7}{322{,}000} + \ldots\ldots\right)\sin 5\omega t - \left(\frac{61G^7}{322{,}000} + \ldots\ldots\right)\sin 7\omega t + \ldots\ldots\right] \tag{59}$$

Appendix II

The Value of the Critical Resistance R_k

The circuit voltage, after inserting the resistance as a function of current, and setting $kaNI/l$ equal to $k\alpha$, is

$$E = I\sqrt{\left(R_0 + \frac{K_1Nw}{lI}\right)^2 + \left(\omega D\,\text{sech}\,k\alpha - \frac{1}{\omega c}\right)^2} \tag{60}$$

Then applying the condition of equation 36 and collecting terms

$$0 = \frac{dE}{dI} = \left(\omega D\,\text{sech}\,k\alpha - \frac{1}{\omega c}\right)^2 - \omega D\frac{k\alpha\sinh k\alpha}{\cosh^2 k\alpha}\left(\omega D\,\text{sech}\,k\alpha - \frac{1}{\omega c}\right) + \left(R_0 + \frac{K_1Nw}{lI}\right)^2 - I\left(\frac{R_0K_1Nw}{lI^2} + \frac{K_1{}^2N^2w^2}{l^2I^3}\right) \tag{61}$$

The last two terms reduce to

$$R_0\left(R_0 + \frac{K_1Nw}{lI}\right) \tag{62}$$

and equation 61 is seen to be a quadratic in the net circuit reactance and has a solution,

$$\left(\omega D\,\text{sech}\,k\alpha - \frac{1}{\omega c}\right) = \frac{\dfrac{\omega Dk\alpha\sinh k\alpha}{\cosh^2 k\alpha} \pm \sqrt{\left(\dfrac{\omega Dk\alpha\sinh k\alpha}{\cosh^2 k\alpha}\right)^2 - 4R_0\left(R_0 + \dfrac{K_1Nw}{lI}\right)}}{2} \tag{63}$$

For the curve for the critical resistance only one value of reactance is possible, so that the radical is zero, leading to

$$\frac{\omega Dk\alpha\sinh k\alpha}{\cosh^2 k\alpha} = 2R_0\sqrt{1 + \frac{K_1Nw}{lIR_0}} \tag{64}$$

$$\omega D\,\text{sech}\,k\alpha - \frac{1}{\omega c} = \frac{\omega D}{2}\,\frac{k\alpha\sinh k\alpha}{\cosh^2 k\alpha} \tag{65}$$

Substitute equation 64 into 65:

$$\omega D\,\text{sech}\,k\alpha - \frac{1}{\omega c} = R_0\sqrt{1 + \frac{K_1Nw}{lIR_0}} \tag{66}$$

Return to equation 65 and rewrite it as

$$\frac{2}{\omega^2DC} = \frac{2\cosh k\alpha - k\alpha\sinh k\alpha}{\cosh^2 k\alpha} \tag{67}$$

The left-hand side of this equation is calculable from circuit constants. Figure 10 is a plot of the right-hand side against values of $k\alpha$. By computing the left side of equation 67, entering the curve at the value obtained for the right side, the value of $k\alpha$ may be read from the curve.

The value of I may be secured from

$$I = \frac{(k\alpha)l}{aNk} \tag{68}$$

and from equation 64 may be written

$$R_0{}^2 + \frac{K_1NwR_0}{lI} - \left(\frac{\omega Dk\alpha\sinh k\alpha}{2\cosh^2 k\alpha}\right)^2 = 0 \tag{69}$$

and this may be solved as a quadratic,

$$R_0 = -\frac{K_1Nw}{2lI} \pm \frac{\sqrt{\left(\dfrac{K_1Nw}{lI}\right)^2 + \left(\dfrac{\omega Dk\alpha\sinh k\alpha}{\cosh^2 k\alpha}\right)^2}}{2} \tag{70}$$

Figure 9 is a plot of the function,

$$\frac{k\alpha\sinh k\alpha}{\cosh^2 k\alpha} \tag{71}$$

against $k\alpha$. From the value of $k\alpha$ obtained in the foregoing, and the value of I from equation 68, the value of R_0 may be determined. However, for most practical conditions, equation 70 reduces to

$$R_0 = \frac{\omega D}{2}\,\frac{k\alpha\sinh k\alpha}{\cosh^2 k\alpha} \tag{72}$$

and then R_x is found from

$$R_x = R_0 - R_c - \frac{K_2N^2w}{l^2} \tag{73}$$

Appendix III

Determination of the Constants B_n, a, and c

Select three points on the magnetization curve having co-ordinates x_1, y_1; x_2, y_2; x_3, y_3; where x is on the H axis and y on the B axis. Point x_1, y_1, should be selected approximately at the point at which a line drawn from the origin with the greatest possible slope is tangent to the magnetization curve. Point x_2, y_2 should be chosen near or above the middle of the curved portion of the knee, and point x_3, y_3, at a position above the knee and having x_3 equal to $2x_2$ or greater.

It is possible, if x_3 or x_2 are chosen too low, to have c become a negative number and this is undesirable as the curve will then drop at high values.

Then

$$y_1 = B_ngdax_1 + cx_1 \tag{74}$$

$$y_2 = B_ngdax_2 + cx_2 \tag{75}$$

$$y_3 = B_ngdax_3 + cx_3 \tag{76}$$

Multiply equation 74 by x_2, equation 75 by x_1 and after subtracting

$$x_2y_1 - x_1y_2 = B_n[x_2gdax_1 - x_1gdax_2] \tag{77}$$

Likewise multiply equation 75 by x_3 and equation 76 by x_2 and subtract

$$x_3y_2 - x_2y_3 = B_n[x_3gdax_2 - x_2gdax_3] \tag{78}$$

Heat-Flow Effects in a Resistance-Type Thermometer

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IT IS a well-established fact that temperature can be measured by observing the change in resistance of a given material whose resistivity is a function of temperature. This principle of temperature detection is utilized in present-day resistance-type thermometers which commonly are called "aircraft thermometer bulbs." When used in conjunction with the proper indicating instrument, the resistance-type thermometer provides an accurate system for furnishing information to the instrument panel on temperatures at remote points in an aircraft. This information is transmitted in the form of changes in electric current that are caused by variations in the electrical resistance of the temperature-sensitive element.

There are several kinds of resistance-type thermometers in use today. Probably the most common is the two-lead type used in aircraft for the measurement of engine-oil, engine-coolant, cabin-air, and free-air temperature. Figure 1 shows this type of thermometer. For these applications, the specified accuracy requirement is a resistance change of approximately 0.5 per cent throughout the temperature range of −70 degrees centigrade to +150 degrees centigrade. Although this accuracy generally is tabulated for several conditions of temperature, in terms of resistance limits, it will be convenient to use per-cent accuracy or per-cent error, as the case may be, in terms of temperature change in degrees centigrade for the purpose of the discussion in this paper.

In addition to the accuracy requirement of temperature detection, this type of thermometer must have a fast response to temperature change. For a change of temperature from 100 degrees centigrade to 0 degrees centigrade in water, the thermometer must reach the 10 degree-centigrade point within eight seconds.

Since specifications for the resistance-type thermometer for aircraft applications are necessarily severe with respect to accuracy and response, a theoretical investigation of some of the factors affecting the accuracy and time of response will be of value from the viewpoint of both test and design. Although there are several factors involved in an analysis of this sort, the discussion in this paper will be limited to the effects of variations in the speed of agitation of the medium being measured on:

1. The accuracy of temperature detection for several conditions of ambient temperature affecting the screw-base head.

2. The response of the temperature-sensitive element independent of head effects.

An analysis of the first condition will be useful from a test and calibration viewpoint as it is proposed to show how errors may be introduced when ambient-temperature conditions are not controlled adequately. In the second condition, the factors affecting response as analyzed will be of interest primarily to the designer. The equations derived for response aside from emphasizing the importance of controlled conditions in test should serve as an example of a method of attack in the solution of response problems.

The fundamental equations for heat conduction between bulb and head and resulting temperature gradient expressions are completely general and may be used where two cylinders or bars of different cross sections are joined. The response expression is sufficiently general in character that it can be applied with slight modifications to other makes of re-

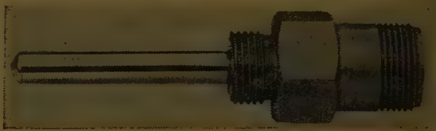

Figure 1. Typical resistance-type temperature transmitter

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Divide equation 77 by equation 78 and after clearing

$$(x_3y_1-x_1y_3)gdax_2=(x_2y_1-x_1y_2)gdax_3+(x_3y_2-x_2y_3)gdax_1 \quad (79)$$

The coefficients of each term in equation 79 can be evaluated and the equation divided by the coefficient of the left-hand term, giving

$$gdax_2=F_1gdax_3+F_2gdax_1 \quad (80)$$

Values of a then are assumed and inserted in equation 80 until the equation is satisfied, thereby fixing a for the steel used. After determining a, B_n may be found from either equation 77 or 78, and c from either equation 74, 75, or 76.

Special Symbols

a = constant of a steel
A = area
B_n = constant of a steel
c = constant of a steel
$D = aAB_nN^2/10^8l$
$G = E_m10^8/aB_nN$
k = weighting constant
K_1 = iron-loss constant
K_2 = iron-loss constant
R_e = effective resistance
R_k = critical resistance
R_o = constant circuit resistance
R_c = reactor copper resistance
R_x = resistance outside reactor
w = weight
W = watts
$\alpha = ani/l$ or anI/l

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sistance-type thermometers whose construction is similar to that shown in Figures 1 and 2.

The specific problems to which these equations are applied in this paper were encountered primarily in the development of the bulb which is illustrated. The constants used in the calculations refer to developmental samples of this type of temperature bulb.

Longitudinal Temperature Distribution of Bulb and Average Temperature Error

CONDITION WHERE BASE TEMPERATURE IS CONSTANT

In the application of the bulb to a normal measurement problem, the heat flow between the threads of the screw-base head and the wall into which the head is screwed is likely to be large enough to maintain the head at substantially the temperature of the wall. For this condition the temperature gradients that exist in the outer portion of the head become unimportant with respect to the temperature distribution in the bulb. In order that the temperature distribution of the bulb for the above condition may be obtained, it becomes necessary to make the following assumptions:

1. The temperature of the head is constant.

2. The bulb cross-sectional area is constant throughout its length.

3. The temperature at any one cross section of the bulb is constant, and the flow of heat through the lead supports inside the bulb is negligible.

4. At steady state the temperature distribution in the element inside the bulb is essentially that of the bulb.

5. The heat lost from the tip can be accounted for by imagining the length of the bulb extended to such a distance that the additional surface area is equivalent to the area of the tip. For this condition no heat is lost from the end; hence, the temperature gradient becomes zero.

6. The agitated medium (gas or liquid) flows uniformly at right angles to the bulb.

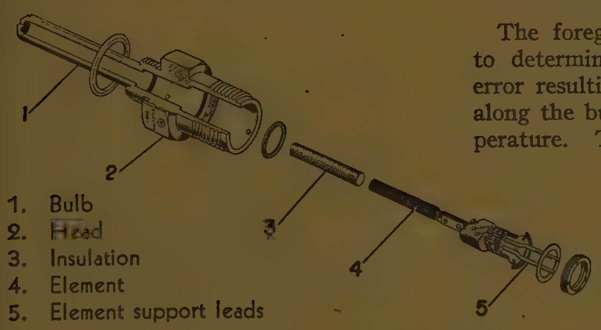

Figure 2. Exploded view of a typical resistance-type temerature transmitter

Assumption 5 can be explained best by referring to Figure 3, which is a schematic sketch of the bulb and extended length and a representative temperature-distribution curve. From Figure 3 (refer to Appendix I for list of symbols employed) let

$$\theta = T_l \pm \theta_x \qquad (1)$$

In the foregoing expression θ_x is positive if θ_0 is a rise above T_l and negative if θ_0 is a drop from T_l.

From assumption 5

$$\frac{\pi D_0^2}{4} = \pi D_0 \delta$$

or solving for δ

$$\delta = D_0/4 \qquad (2)$$

Therefore, the effective bulb length is

$$L' = L + D_0/4 \qquad (3)$$

As the solution of this type of heat-conduction problem has been worked out completely elsewhere,[1] the derivation will not be repeated here. The temperature distribution along the bulb becomes

$$\theta_x = \theta_0 \frac{\cosh (x - L')N}{\cosh L'N} \qquad (4)$$

If we assume that in equation 1 θ_x is positive, as shown in Figure 3, the temperature rise then becomes

$$\theta = T_l + \theta_0 \frac{\cosh (x - L')N}{\cosh L'N} \qquad (5)$$

Once the expression for the temperature distribution is determined, it becomes useful in evaluating the average temperature error of the detector element. Referring again to Figure 3, the cross-hatched area under the curve represents the temperature error of the element because of the conduction of heat along the length of the bulb. An evaluation of this error can be determined from the average obtained by integrating equation 5.

The average temperature over the effective length of the element inside the bulb becomes

$$\theta_{av} = T_l + \frac{\theta_0 \sinh b'N}{b'N \cosh L'N} \qquad (6)$$

Finally, the per-cent error is

$$\text{Per-cent error} = 100(\theta_{av} - T_l)/\theta_0 \qquad (7)$$

The foregoing equation enables one to determine the average temperature error resulting from conduction of heat along the bulb for a constant base temperature. The accuracy of results depends on how accurately the base temperature and ambient temperature of the medium are known. In practice very little error will be experienced, since the base temperature generally will be very near the value of the medium temperature.

CONDITION WHERE TEMPERATURE GRADIENTS EXIST IN THE SCREW-BASE HEAD

In the previous section the expression for the average temperature of the bulb is sufficient for evaluating the error for most applications, when it can be assumed reasonably that the temperature of the screw-base head is constant. However, for isolated cases where temperature gradients exist in the head, the foregoing equation will not be adequate. This condition can be realized to some extent when the bulb is supported by a material which is a relatively poor conductor of heat. It then can be assumed that negligible heat is conducted from the threads of the screw-base head into the supporting member and that the average temperature of the head is not constant. For the sake of completeness, it is proposed to record in this section the equations that may be used for evaluating the error of the bulb when ambient-temperature conditions control the average temperature of the screw-base head. No attempt will be made to derive these equations as the solution to this type of problem is described in the literature.[2]

Figure 4 shows a sketch of the bulb-head combination in which we have assumed that the head can be represented as having an average uniform cross section throughout its length. If we apply assumption 5 to the head, then

$$l' = l + D_1/4 \qquad (8)$$

For the condition assumed in Figure 4

$$\theta = T_l - \theta_x \qquad (9)$$

for the bulb, and

$$\theta_1 = T_a + \theta_x' \qquad (10)$$

for the head.

Once the expression for the temperature rise of the bulb (θ_x) and the head (θ_x') have been derived, equations 9 and 10 can be integrated to obtain the average temperature of the bulb and head respectively. For the bulb the average temperature becomes

$$\theta_{av} = T_l - \frac{(T_l - T_a)}{b'N} \left[\frac{1}{Z \dfrac{\tanh L'N}{\tanh l'N_1} + 1} \right] \times \frac{\sinh b'N}{\cosh L'N} \qquad (11)$$

Similarly for the head

$$\theta_{av}' = T_a + \frac{(T_l - T_a)}{l'N_1} \left[\frac{1}{1 + \dfrac{\tanh l'N_1}{Z \tanh L'N}} \right] \times \frac{\sinh l'N_1}{\cosh l'N_1} \qquad (12)$$

Finally, using the average temperature of the bulb (equation 11), the per-cent error becomes

$$\text{Per-cent error} = 100(T_l - \theta_{av})/(T_l - T_a) \quad (13)$$

Heat-Transfer Coefficients

At this point it might be well to consider a few expressions for the heat-transfer coefficient of gases and liquids. An expression for the heat-transfer coefficient between a solid and a gas or a solid and a liquid is extremely complicated and does not lend itself readily to rigorous mathematical treatment. However, empirical relations have been worked out for bodies of simple geometry. The equations herein listed are for liquids or gases flowing at right angles to single cylinders[3].

COEFFICIENT FOR LIQUIDS OR GASES FLOWING AT RIGHT ANGLES TO SINGLE CYLINDERS (FORCED CONVECTION)

$$h = 0.86k_f/D_0(D_0V\rho_f/\mu_f)^{0.43}(c_p\mu_f/k_f)^{0.3} \quad (14)$$

$$h = k_f/D_0[0.35 + 0.47(D_0V\rho_f/\mu_f)^{0.52}](c_p\mu_f/k_f)^{0.3} \quad (15)$$

For gases use equation 14 where $(D_0V\rho_f/\mu_f)$ varies between 0.1 and 1,000. For liquids use equation 14 when $(D_0V\rho_f/\mu_f)$ lies between 0.1 and 200 and equation 15 above 200.

COEFFICIENT FOR HEAT TRANSFER BY NATURAL CONVECTION (SINGLE CYLINDERS)

$$h_c = 0.53k_f/D_0\left[\left(\frac{D_0^3\rho_f^2\beta g\,\Delta t}{\mu_f^2}\right)\left(\frac{c_p\mu_f}{k_f}\right)\right]^{0.25} \quad (16)$$

This expression assumes that the cylinder is placed horizontally in the liquid or gas. The writer has been unable to find a relation for vertical single cylinders. However, it is felt that for short cylinders there would be very little difference in the coefficient for the two conditions, and therefore for approximations of the coefficient equation 16 can be used for either liquids or gases. Although in some cases radiation effects may be of the same order of magnitude as convection effects, radiation will not be considered in this article.

Any consistent set of units may be used for equations 14, 15, and 16. However, in the illustrative example of computations in Appendix III, English units were used in these expressions, as these are the generally preferred units. Consequently, the heat-transfer coefficients are in English units and a conversion factor must be applied to use these coefficients in the foregoing equations for average temperature. For conversion factors refer to Appendix II and for symbols used in equations 14, 15, and 16 see Appendix I.

Effect of Agitation Velocity on Average Temperature Distribution

In the preceding paragraphs for the heat-transfer coefficient we can see that among other variables the velocity of agitation is one that might considerably affect the value of the heat-transfer coefficient. Also, the error which a change in the velocity of agitation may cause in temperature detection certainly will be to a large extent dependent upon the ambient-temperature conditions affecting the head. For example, the head may be held in an ice bath or in air and cooled by natural convection, or again in air and cooled or heated by forced convection. To show the magnitude of these effects we will assume several test conditions where we arbitrarily have selected constants from a given design for illustrative purposes only.

CONSTANT HEAD TEMPERATURE

Suppose it is required to compare the temperature measured by the resistance bulb with some other means of an oil bath where the oil-bath temperature T_l = 100 degrees centigrade and the head temperature is controlled by an ice bath where T_a = 0 degrees centigrade. It also will be assumed that the container for the ice surrounding the head is insulated sufficiently from the oil bath that the head temperature is essentially constant at 0 degree centigrade. This condition corresponds to the required specification for test of thermal conductivity or the effect the head has on the accuracy of temperature detection. Figure 5 shows three separate plots of equation 5 of 0.5, and 2.0 feet per second, and for natural convection.

Referring again to Figure 5, the cross-hatched section represents the average temperature error over the limits of x = 0 to $x = b'$ where b' is the effective length of the detector element and for this particular case is equal to 4.25 centimeters. For the several conditions of agitation assumed, the errors represented by the cross-hatch sections using equations 6 and 7 are

(*a*). Natural convection per-cent error = −7.10 per cent

(*b*). V = 0.5 foot per second per-cent error = −1.39 per cent

(*c*). V = 2.0 feet per second per-cent error = −0.55 per cent

In the following table are a few average temperature errors taken from experiment on early sample production bulbs where the head has been packed in ice, the bulb inserted in oil at 100 degrees centigrade, and the oil agitation approximately 2.0 to 2.5 feet per second:

Bulb	Per-Cent Error
Number 1	−0.281
Number 2	−0.384
Number 3	−0.247
Number 4	−0.262
Number 5	−0.372
Number 6	−0.315

These errors are in good agreement with the calculated value of −0.55 per cent for an agitation of 2.0 feet per second on the same design bulb.

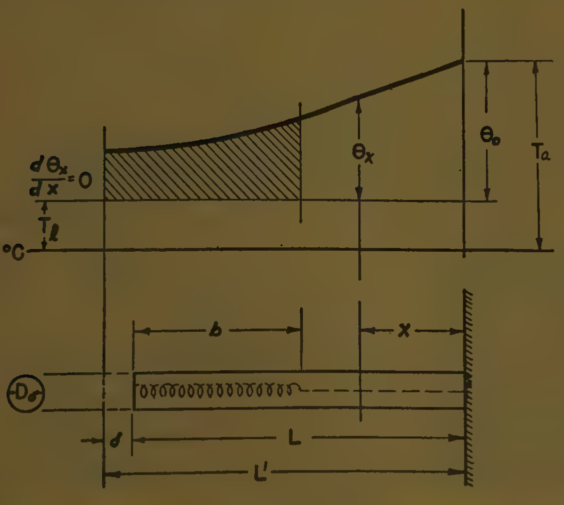

Figure 3 (left). Sketch of temperature distribution of bulb for constant base temperature

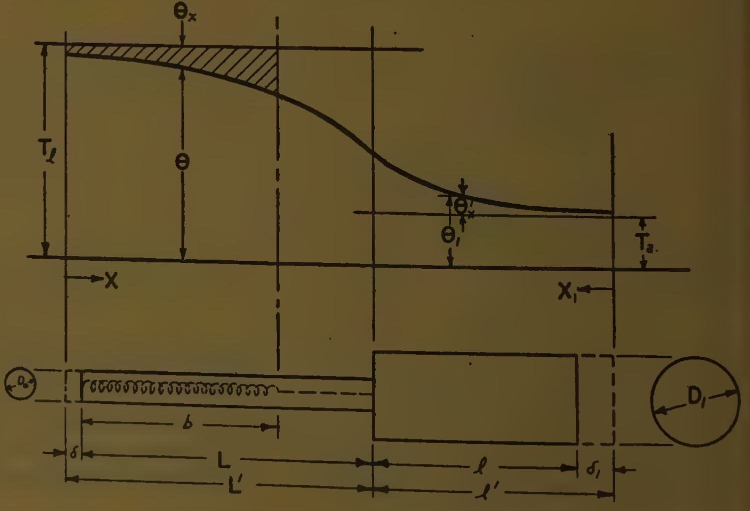

Figure 4 (right). Sketch of temperature distribution for bulb–head combination

VARIABLE AMBIENT TEMPERATURE CONTROLLING HEAD TEMPERATURE

In the previous section we have shown how the velocity of liquid agitation of the medium being measured affected the accuracy of temperature detection for constant head temperature. In this section we shall reverse the process and hold the bath conditions constant and determine the errors resulting from variable ambient-temperature head conditions.

Assume that the bath is oil at constant temperature $T_i = 100$ degrees centigrade and the velocity of agitation is also constant at $V = 0.5$ foot per second. For the temperature of the air, $T_a = 25$ degrees centigrade. Figure 6 shows a plot of the temperature distribution where the head is in air and cooled by natural convection and again by air flowing at right angles at a velocity of 15 feet per second. The errors as represented by the cross-hatched section for $b' = 4.25$ centimeters using equation 13 are

(*a*). Natural convection per-cent error = 1.09

(*b*). Velocity of air = 15 feet per second per-cent error = 1.32

From this analysis it is seen how variable ambient-temperature conditions controlling both the bulb and the head temperature distributions can affect materially the accuracy of temperature measurement. In an actual measurement problem in practice the ambient temperature generally will be closer to the temperature of the medium measured; hence, any errors because of conduction will tend to be negligible. However, for test purposes, in order to obtain measurable errors, the ambient temperatures often are held at extremes, and for this setup the importance of maintaining constant ambient-temperature conditions such as velocity of bath agitation, temperature, and so forth, cannot be overemphasized.

Equation for Response Time of Element

In the resistance-type temperature bulb, the time of response of the detector element is an important factor in the bulb's performance. It is desirable that the detector reach a steady-state condition in the shortest time possible. The ideal performance would be instantaneous response, but this is practically impossible to obtain as component parts of the bulb structure have certain heat capacities that introduce lagging-time characteristics. It is the designer's aim to reduce this time lag to a minimum. Consequently, from a design point of view, it would be helpful to have an expression for the response time of a typical temperature bulb that ties together all the variables of dimension and heat constants of material. Considerable testing time could be saved in the development of a design by using such an expression, as the engineer would be able to predict what would happen to the response time if changes in the variables of dimension or material were made.

Most temperature bulbs exhibit comparable component parts as viewed cross sectionally from the detector support to the outside casing. Figure 7 shows the essential components in cross section of the bulb pictured in Figure 2.

Before an analysis can be made, the following assumptions will be necessary:

1. For temperature bulbs of circular cross section, heat flow is radial to or from the detector.
2. Heat flowing along the bulb casing has negligible effect on the response time. It is recognized here that this is not a valid assumption but is made to simplify the problem. It is felt that in a properly designed bulb envelope this effect would be minimized.
3. Negligible heat flows along element support to the head.

The component parts of Figure 7 can be grouped to form bodies having definite heat capacities and resistances which impede the flow of heat. Schematically, these groups can be represented as a series-parallel circuit as shown in Figure 8. This is analogous to an electric circuit with capacitance and resistance.

The differential equation for the series-parallel heat diagram as shown in Figure 8 is

$$\{ss_1R_1R_hD^2+[s_1R_h+s(R_1+R_h)]D+1\}\theta_y = (R_1+R_h)q+T_i=\theta_p \qquad (17)$$

The solution to the differential equation 17 becomes

$$\theta_y=\frac{\theta_i-\theta_p}{m_2-m_1}[m_2e^{m_1t}-m_1e^{m_2t}]+\theta_p \qquad (18)$$

(Refer to Appendix IV for the derivation of equation 17 and solution 18 where m_1 and m_2 also are defined.)

Effect of Agitation Velocity on the Response Time

Referring to the expression for θ_p in equation 17 we see that θ_p is a sum of two temperatures, T_i which is the temperature of the medium being measured, and $(R_1+R_h)q$, which possibly could be called the temperature of the detector element due to self-heat when current flows through it. In good designs the effect of self-heat is made very small, and if this is done the term $(R_1+R_h)q$ can be neglected for all practical purposes. Should it ever become necessary to conduct response-time tests where it is felt the self-heat of the element is a contributing factor, then the complete expression of equation 17 should be used.

In the representative response curves of Figure 9 this self-heat term has been omitted from the calculations. These curves show the response of the element for various velocities of bath agitation. The solid lines are calculated curves assuming essentially the same arbitrary constants as previously used in the analysis of the effect of conduction, and the dotted curve is taken from experiment. The curves represent a test condition where the bulb has been heated to 100 degrees centigrade in a water bath and then suddenly immersed into a zero-degree-centi-

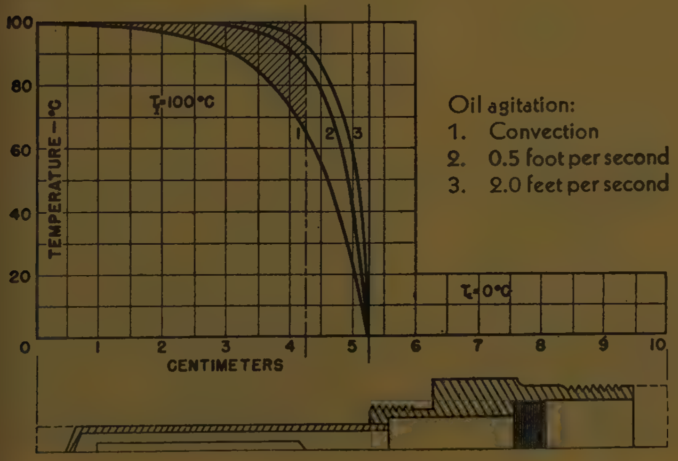

Figure 5 (left). Temperature distribution of bulb, head temperature constant

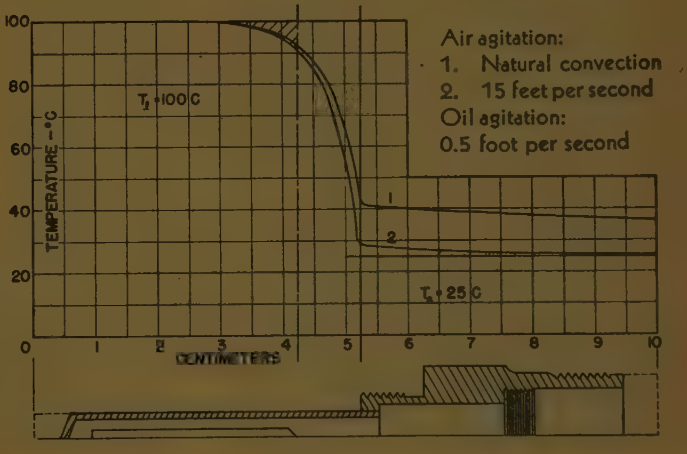

Figure 6 (right). Temperature distribution of bulb and head, bulb ambient controlled

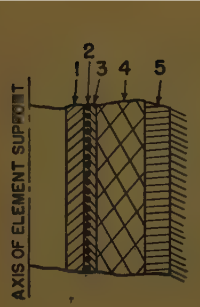

Figure 7. Sectional view of component parts for temperature bulb

1. Element support
2. Wire-wound temperature detector
3. Binder
4. Insulation
5. Wall of outside casing

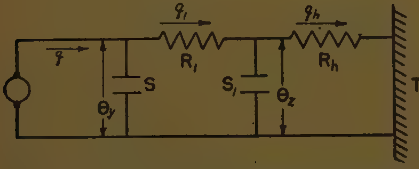

Figure 8. Diagram of grouped components for series-parallel heat flow

grade water bath. Equation 18 was used where

$\theta_i = 100$ degrees centigrade

and

$T_l = \theta_p = 0$ degrees centigrade

Again, it is seen how a change in the velocity of agitation affects the response time. The error in reponse readily can be visualized by noticing the differences in time at any particular temperature level; for example, the ten-degree-centigrade level.

The experimental curve tends to approach the zero-degree-centigrade level slower than the calculated curves. This is possibly due to the effect of the screw-base head cooling down slower than the element. It will be recalled that the effect of the head was neglected in the calculations, but the good agreement between experimental and calculated effects would seem to indicate that the assumption was justified.

The results and computations shown by the curves of Figure 9 are based on developmental samples. As a result of this analysis, critical parts were changed in the present design to improve appreciably the response time.

Conclusion

In the foregoing analyses on temperature distribution and response time, emphasis has been placed on the importance of controlled velocity agitation and ambient temperature in order that reliable test results may be obtained. The expressions for the temperature distribution of the bulb–head combination are general and can be applied to any similar problem where two bars or cylinders of different cross sections and materials are joined. Generality cannot be assigned to the response-time expression, as it must be modified and adapted to each make of temperature bulb. It was included in the discussion as an example of a method of attack in the solution of a particular response problem. The use of the response expression is limited in its accuracy when it is required to determine time-lag errors under ten per cent of the initial temperature difference. Above ten per cent of the initial temperature difference, the expression is entirely satisfactory. The condition that exists as the indicated temperature approaches more closely the final temperature is discussed extensively in an article by Noyes.[4]

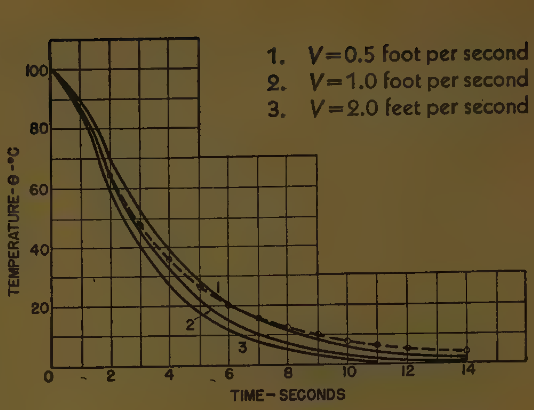

Figure 9. Response time of bulb

Response for temperature from 100 to 0 degrees centigrade in water

Solid line—Calculated

Dashed line—Experimental (V=0.75 foot per second approximately)

For reference convenience the equations applicable to the temperature sensitive element are grouped and listed again as follows:

A. Average temperature of element for constant base temperature:

$$\theta_{av} = T_l + \theta_0 \frac{\sinh b'N}{b'N \cosh L'N} \tag{6}$$

$$\text{Per-cent error} = 100(\theta_{av} - T_l)/\theta_0 \tag{7}$$

B. Average temperature of element where temperature gradients exist in the screw-base head:

$$\theta_{av} = T_l - \frac{(T_l - T_a)}{b'N} \left[\frac{1}{Z \dfrac{\tanh L'N}{\tanh l'N_1} + 1} \right] \times \frac{\sinh b'N}{\cosh L'N} \tag{11}$$

$$\text{Per-cent error} = 100(T_l - \theta_{av})/(T_l - T_a) \tag{13}$$

C. Response-time expression for element:

$$\theta_y = \frac{\theta_i - \theta_p}{m_2 - m_1} [m_2 e^{m_1 t} - m_1 e^{m_2 t}] + \theta_p \tag{18}$$

Appendix I. Symbols Used for Conduction and Response Expressions

A_0 = outside surface area of bulb casing (square centimeters)

A_i = inside surface area of bulb casing (square centimeters)

A_e = outside surface area of element (square centimeters)

$A_m = \dfrac{A_i + A_e}{2}$ = mean surface area of insulation (part 4, Figure 7) (square centimeters)

A_s = mean surface area of bulb casing normal to radial flow of heat (square centimeters)

a, a_1 = average area of cross section of bulb casing, head (square centimeters)

b = length of element (centimeter)

$b' = (b + \delta)$ = effective length of element (centimeter)

c_p = thermal capacity; numerically equal to specific heat (calories per gram per degree centigrade)

D_0, D_1 = average outside diameter of bulb casing, head (centimeter)

δ, δ_1 = additional length required to make the extended surface area equal to the area at the end of the bulb, head (centimeter)

h, h_1 = heat-transfer coefficient (forced convection) between medium and bulb, head (calories per second per square centimeter per degree centigrade)

h_c, h_{c1} = heat-transfer coefficient (natural convection) between medium and bulb, head (calories per second per square centimeter per degree centigrade)

k, k_1 = thermal conductivity of bulb, head (calories per second per square centimeter per degree centigrade per centimeter)

k_m = thermal conductivity of insulation (part 4, Figure 7) (calories per second per square centimeter per degree centigrade per centimeter)

L = normal length of bulb (centimeter)

$L' = (L + \delta)$ = effective length of bulb (centimeter)

l = normal length of head (centimeter)

$l' = (l + \delta_1)$ = effective length of head (centimeter)

M = mass (grams)

m_1, m_2 = negative real roots of differential equation for response time (see Appendix IV)

$N, N_1 = \sqrt{hp/ak}, \quad \sqrt{h_1 p_1/a_1 k_1}$

p, p_1 = perimeter of bulb, head (average) (centimeter)

q = heat flow (watts generated by element) (calories per second)

q_1, q_h = heat flow through resistances R_1, R_h (calories per second)

$R_1 = y/k_m A_m$—insulation resistance (part 4, Figure 7)

$R_h = y_1/kA_s + 1/hA_0$ = total transfer resistance made up of two items

y_1/kA_s = the resistance of bulb casing to normal flow of heat

$1/hA_0$ = the transfer resistance to heat flow between bulb and medium

s, s_1 = total heat capacities of components (calories per degree centigrade)

$= \sum_{1}^{n} (Mc_p)_i$ (see Appendix IV)

T_a = ambient temperature around head above 0 degrees centigrade (degrees centigrade)

T_l = ambient temperature around bulb above 0 degrees centigrade (degrees centigrade)

t = time (seconds)

θ, θ_1 = temperature above 0 degrees centigrade reference for any value x, x_1 for the bulb, head (degrees centigrade)

θ_x, θ_x' = temperature change for any value x, x_1 for the bulb, head (degrees centigrade)

θ_i = initial temperature of bulb at $t=0$ (degrees centigrade)

θ_0 = temperature at junction of bulb and head above medium temperature (degrees centigrade)

$\theta_p = (R_1+R_h)q+T_l$

θ_y = temperature of detector at time t (degrees centigrade)

θ_z = temperature of grouped component s_1 (degrees centigrade)

x, x_1 = distance along bulb, head (centimeter)

y = width of space occupied by insulation (centimeter)

y_1 = thickness of bulb wall (centimeter)

$Z = akN/a_1k_1N_1$

Symbols used in heat-transfer coefficient expressions, equations 14, 15, and 16

β = coefficient of expansion

c_p = thermal capacity at constant pressure (British thermal units per pound per degree Fahrenheit)

D_0 = outside diameter of cylinder (feet)

g = acceleration due to gravity = 4.17×10^8 (feet per square hour)

h, h_c = coefficient of heat transfer, forced and natural convection respectively (British thermal units per hour per square foot per degree Fahrenheit)

k_f = thermal conductivity (British thermal units per hour per square foot per degree Fahrenheit per foot)

ρ_f = density (pounds per cubic foot)

μ_f = absolute viscosity; 2.42×centipoises (pounds per hour per foot)

V = linear velocity (feet per hour)

Δt = temperature differential between the cylinder and medium (degrees Fahrenheit)

Appendix II. Conversion Factors[5]

h = heat-transfer coefficient; (British thermal units per hour per square foot per degree Fahrenheit)

Divide h by

7,373 to get calories per second per square centimeter per degree centigrade

0.2048 to get kilogram calories per hour per square meter per degree centigrade

1,763 to get watts per square centimeter per degree centigrade

k = thermal conductivity; (British thermal units per hour per square foot per degree Fahrenheit per inch)

Divide k by

12.0 to get British thermal units per hour per square foot per degree Fahrenheit per foot

8.064 to get kilogram calories per hour per square meter per degree centigrade per meter

694.0 to get watts per square centimeter per degree centigrade per centimeter

2,903.0 to get calories per second per square centimeter per degree centigrade per centimeter

Appendix III. Example of Computations

For the purpose of illustrating how the conduction equations can be applied, one example of the computations will be given. The per-cent error will be computed from constants of a given design for one condition of medium agitation. For this condition it will be assumed that the head is placed in water at 0 degrees centigrade and that heat is transferred from the head to the water by natural convection only. The temperature of the medium being measured will be held constant at 100 degree centigrade and the medium will be a light grade of machine oil.

Constants (see Appendix I for list of symbols)

T_l = 100 degrees centigrade

T_a = 0 degrees centigrade

D_0 = 0.711 centimeter = 0.0233 foot

D_1 = 1.53 centimeter = 0.0502 foot

a = 0.139 square centimeter

a_1 = 1.84 square centimeters

p = 2.24 centimeters

p_1 = 4.81 centimeters

k = 0.036 calorie per second per square centimeter per degree centigrade per centimeter

k_1 = 0.160 calorie per second per square centimeter per degree centigrade per centimeter

$k_f = 5.68 \times 10^{-2}$ Btu per hour per square foot per degree Fahrenheit per foot

k_{f1} = 0.29 Btu per hour per square foot per degree Fahrenheit per foot

ρ_f = 57.40 pounds per cubic foot

ρ_{f1} = 62.40 pounds per cubic foot

$\beta = 9.5 \times 10^{-4}$

$\beta_1 = 2.1 \times 10^{-4}$

c_p = 0.51 Btu per pound per degree Fahrenheit

c_{p1} = 1.0087 Btu per pound per degree Fahrenheit

μ_f = 43.5 pounds per hour per foot

μ_{f1} = 4.33 pounds per hour per foot

L = 5.08 centimeters

l = 4.13 centimeters

Let V = one foot per second (velocity of agitated oil bath)

From equations 3 and 8

L' = 5.26 centimeters

l' = 4.51 centimeters

In determining which equation to use for the heat-transfer coefficient for the agitated oil bath, we must evaluate the expression $(D_0V\rho_f/\mu_f)$, remembering that all the variables in all the expressions for the heat-transfer coefficients are in English units and V is in feet per hour.

$$\left(\frac{D_0V\rho_f}{\mu_f}\right)=\frac{0.0233\times 3{,}600\times 1\times 57.40}{43.5}=111$$

This is less than 200; therefore, we must use equation 14.

$$h=\frac{0.86k_f}{D_0}\left(\frac{D_0V\rho_f}{\mu_f}\right)^{0.43}(c_p\mu_f/k_f)^{0.3} \qquad (14)$$

$$h=\frac{0.86\times 5.68\times 10^{-2}}{2.33\times 10^{-2}}\times \left(\frac{0.0233\times 3{,}600\times 57.40}{43.5}\right)^{0.43}\left(\frac{0.51\times 43.5}{5.68\times 10^{-2}}\right)^{0.3}$$

$h=2.1(111)^{0.43}(390)^{0.3}=95$ Btu per hour per square foot per degree Fahrenheit or in the metric system (see Appendix II for conversion factors)

$h=\frac{95}{7{,}373}=0.013$ calorie per second per square centimeter per degree centigrade

Now

$$N=\sqrt{\frac{hp}{ak}}=\sqrt{\frac{0.013\times 2.24}{0.139\times 0.036}}=2.41$$

$L'N=5.26\times 2.41=12.68$

Before we can proceed any further with calculations on the bulb, we now will have to evaluate the heat-transfer coefficient for the head. For this determination we must use equation 16 for natural convection as the water bath was assumed unagitated.

$$h_{c1}=\frac{0.53k_{f1}}{D_1}\left[\left(\frac{D_1^3\rho_{f1}^2\beta_1 g\,\Delta t}{\mu_{f1}^2}\right)\left(\frac{c_{p1}\mu_{f1}}{k_{f1}}\right)\right]^{0.25} \qquad (16)$$

It will be noticed that this expression involves the term Δt which is the temperature difference between the water and the head. Therefore, it becomes necessary by trial and error to assign values to Δt until the value assigned is approximately equal to the temperature difference between the ice water and the average temperature of the head.

Let us assume for the first approximation that

Δt = 4 degrees centigrade = 7.2 degrees Fahrenheit

$$h_{c1}=\frac{0.53\times 0.29}{0.0502}\times \left[\left(\frac{\overline{0.0502}^3\times\overline{62.4}^2\times 2.1\times 10^{-4}\times 4.17\times 10^8\times 7.2}{\overline{4.33}^2}\right)\times \left(\frac{1.0087\times 4.33}{0.29}\right)\right]^{0.25}$$

$$h_{cl}=3.07[(16.5\times10^3)(15.05)]^{0.25}$$
$$=3.07\times22.3=68.5$$ Btu per hour per square foot per degree Fahrenheit

or

$$h_{cl}=\frac{68.5}{7{,}373}=0.0093$$ calorie per second per square centimeter per degree centigrade

Now

$$N_1=\sqrt{\frac{h_{cl}p_1}{a_1k_1}}=\sqrt{\frac{0.0093\times4.81}{1.84\times0.160}}=0.389$$

and

$$l'N_1=4.51\times0.389=1.76$$

$$Z=\frac{akN}{a_1k_1N_1}=\frac{0.139\times0.036\times2.41}{1.84\times0.160\times0.389}$$
$$=10.54\times10^{-2}$$

We can now calculate the average temperature of the head from equation 12.

$$\theta_{av}'=T_a+\frac{(T_l-T_a)}{l'N_1}\times\left[\frac{1}{1+\dfrac{\tanh l'N_1}{Z\tanh L'N}}\right]\frac{\sinh l'N_1}{\cosh l'N_1}\qquad(12)$$

$\tanh L'N=1$
$\tanh l'N_1=0.943$
$\sinh l'N_1=2.820$
$\cosh l'N_1=2.992$

Therefore

$$\theta_{av}'=\frac{100}{1.76}\left[\frac{1}{1+\dfrac{0.943}{10.54\times10^{-2}}}\right]\frac{2.820}{2.992}$$

$$\theta_{av}'=56.8\left(\frac{1}{9.95}\right)\times0.943=5.4$$ degrees centigrade

Now, since $T_a=0$ degree centigrade, this value represents the temperature change between the ice water and the head. Comparing this value of 5.4 degrees centigrade with the originally assumed value of $\Delta t=4$ degrees centigrade, we see that we must assume another value and go through the same procedure as previously outlined until the two values check within reasonable limits. The value of Δt obtained after two more trials is

$\Delta t=4.9$ degrees centigrade

If we use this value, then

$h_{cl}=0.00979$ calorie per second per square centimeter per degree centigrade
$N_1=0.399$
$l'N_1=1.80$
$Z'=10.28\times10^{-2}$

Having once found the above variables for the correct Δt, we now can proceed with the calculation of the average temperature of the bulb and the consequent error.

From equation 11 we have

$$\theta_{av}=T_l-\frac{(T_l-T_a)}{b'N}\times\left[\frac{1}{\dfrac{Z\tanh L'N}{\tanh l'N_1}+1}\right]\frac{\sinh b'N}{\cosh L'N}\qquad(11)$$

which is the expression for the average temperature distribution between the limits $x=0$, $x=b'$ for the element inside the bulb. (Refer to Figure 4)

$b'=4.25$ centimeters
$b'N=4.25\times2.41=10.25$

$\tanh L'N=1$
$\tanh l'N_1=0.947$
$\sinh b'N={}^1/_2e^{b'N}=1.41\times10^4$
$\cosh L'N={}^1/_2e^{L'N}=15.9\times10^4$

For $(L'N)$ or any other value of the hyperbolic radian >6.0,

$$\sinh L'N=\cosh L'N={}^1/_2e^{L'N}$$

Therefore,

$$\theta_{av}=100-\frac{(100-0)}{10.25}\times\left[\frac{1}{\dfrac{10.28\times10^{-2}}{0.947}+1}\right]\times\frac{1.41\times10^4}{15.9\times10^4}$$

$$\theta_{av}=100-9.75\left(\frac{1}{1.109}\right)\times0.0887=100-0.78$$

$\theta_{av}=99.22$ degrees centigrade

Finally from equation 13

$$\text{Per-cent error}=\frac{(T_l-\theta_{av})}{T_l-T_a}\times100\qquad(13)$$

$$\text{Per-centerror}=\frac{(100-99.22)}{100}\times100=0.78$$

Appendix IV. Derivation of Equation for Response Time

Referring to Figure 7, the summation of the heat capacities of parts 1, 2, 3, and one half of part 4 can be expressed mathematically as follows:

$$s=(Mc_p)_1+(Mc_p)_2+(Mc_p)_3+{}^1/_2(Mc_p)_4\qquad(19)$$

where, for example, $(Mc_p)_1$ is the heat capacity of part 1 or its mass M times its specific heat c_p.

Similarly

$$s_1={}^1/_2(Mc_p)_4+(Mc_p)_5\qquad(20)$$

Now referring to Figure 8, the heat gained in calories per second by the body having a heat capacity s is

$$s\frac{d\theta_y}{dt}=q-q_1$$

or, solving for $\frac{d\theta_y}{dt}$,

$$\frac{d\theta_y}{dt}=\frac{q-q_1}{s}\qquad(21)$$

Similarly

$$\frac{d\theta_z}{dt}=\frac{q_1-q_h}{s_1}\qquad(22)$$

Now the flow of heat from one body to another is equal to the difference in temperature of the two bodies divided by the resistance of the path; therefore

$$q_1=\frac{(\theta_y-\theta_z)}{R_1}\qquad(23)$$

and

$$q_h=\frac{(\theta_z-T_l)}{R_h}\qquad(24)$$

Substituting equations 23 and 24 in 21 and 22 and simplifying

$$\frac{d\theta_y}{dt}=1/sR_1[qR_1-(\theta_y-\theta_z)]\qquad(25)$$

and

$$\frac{d\theta_z}{dt}=1/s_1R_1R_h[R_h\theta_y-(R_1+R_h)\theta_z+R_1T_l]\qquad(26)$$

Making use of the differential operator $D=\frac{d}{dt}$ equations 25 and 26 can be written

$$(D+1/sR_1)\theta_y-\theta_z/sR_1-q/s=0\qquad(27)$$

$$[D+(R_1+R_h)/s_1R_1R_h]\theta_z-\theta_y/s_1R_1-T_l/s_1R_h=0\qquad(28)$$

If equation 28 is solved for θ_z and substituted in equation 27 and remembering that $Dq=0$, that is, for any one condition the flow of heat is constant, thus making the rate of change of heat flow equal to zero, one obtains after simplifying, the differential equation for the response time

$$\{ss_1R_1R_hD^2+[s_1R_h+s(R_1+R_h)]D+1\}\theta_y=(R_1+R_h)q+T_l=\theta_p\qquad(17)$$

Now the solution of the differential equation

$$(\alpha D^2+\eta D+1)\theta_y=0\qquad(29)$$

will be of the form

$$\theta_y=c_1e^{m_1t}+c_2e^{m_2t}\qquad(30)$$

where c_1 and c_2 are constants to be evaluated for the boundary conditions and m_1 and m_2 are roots of the auxiliary equation

$$\alpha m^2+\eta m+1=0\qquad(31)$$

Also

$$\alpha=ss_1R_1R_h\qquad(32)$$

$$\eta=s_1R_h+s(R_1+R_h)\qquad(33)$$

The auxiliary equation yields two negative real roots,

$$m_1=\frac{-\eta+\sqrt{\eta^2-4\alpha}}{2\alpha}\qquad(34)$$

and

$$m_2=\frac{-\eta-\sqrt{\eta^2-4\alpha}}{2\alpha}\qquad(35)$$

By actual substitution for the quantities under the radical sign it can be shown that m_1 and m_2 are negative real roots.

A particular solution of the differential equation 17 will be

$$\theta_y=(R_1+R_h)q+T_l\qquad(36)$$

or

$$\theta_y=\theta_p\qquad(37)$$

Sag and Tension Calculations for Cable and Wire Spans Using Catenary Formulas

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IT IS generally agreed that, in order to obtain accurate solutions of sag and tension problems involved in stringing electric power line conductors, it is necessary to use catenary formulas. The basic formulas are, of course, well known In order to utilize them, however, a great amount of laborious calculating is required. Certain short cuts in these calculations have been evolved, notably by James S. Martin whose published tables and method have proved of great help to engineers in simplifying sag calculations.

In connection with the design and construction of transmission lines in the mountainous Appalachian region the writers have evolved a method of making mathematically-exact sag and tension calculations based on catenary formulas which is quite different from the Martin method, and which eliminates the trial-and-error assumptions involved in a portion of the Martin method. The working formulas used by the writers require only simple arithmetical calculations which can be handled by clerical personnel usually available in the engineering office. The calculations are not laborious, and can be easily checked.

The writers' method not only develops the sags, tensions, and conductor lengths for the various final and initial conditions of the design span (basic or ruling span), but also similar data for span lengths other than the ruling span.

The procedure outlined is applicable to any cable or wire whose modulus of elasticity and coefficient of expansion are uniform throughout the cross section, such as is the case with copper, copper-coated-steel, or steel cables. Sag and tension problems for composite cables such as aluminum cables steel reinforced, are usually solved by graphic methods based on stress–strain curves developed from laboratory tests on the particular cable under consideration.

The writers' method does, however, develop previously unpublished relationships between sags, tensions, and cable lengths which are applicable not only to copper, copper-coated-steel, and steel cables, but also to problems involving composite cables, which information cannot be secured by graphic methods alone.

The following text gives the basic catenary formulas, the derivations thereof, and an explanation of the procedure followed in actual sag calculations.

The basic formulas of the catenary are shown on Figure 1. The nomenclature used in this figure is as follows:

X and Y = Cartesian co-ordinates of any point in the catenary curve, designated point B

Z = length of catenary from the vertex A to point B

S = the deflection of the vertex A below any point B which may be under consideration

V = vertical distance from the axis of abscissas of the catenary to the vertex A

A = vertex of the catenary curve

ϵ = base of the Naperian or natural system of logarithms equals 2.71828

These formulas are well known and need no proof of derivation. Formula A determines the shape of the catenary curve which shape depends on the V function. Formula B determines the length of the curve which depends on the same V function. It will be shown later that the value of V depends on the cable tension at the span vertex, and the value of Y depends on the total tension in the cable. The sag is $Y-V$ (formula C) which can be expressed in terms of the V value and span length (formula D).

Referring to Figure 1 note that point B is a cable support, X the horizontal distance from the vertex (low point of sag) to support, and Z the cable length from vertex to support. S is the sag of the cable below its support, or the span sag in problems with equal elevation supports.

Unit Values of Catenary Functions

In catenary spans the values of V, Y, S, and Z bear definite relationships to each other expressed by the basic formulas, A, B, and C. When these formulas are divided by X, unit value formulas result as follows:

$Y/X = V/X \cosh X/V$
$Z/X = V/X \sinh X/V$
$S/X = (Y/X)-(V/X)$

Related unit values of V/X, Y/X, S/X, and Z/X are tabulated in Table I, which values can be applied to any length span, or any type cable. When V, Y, Z, or S is known for any span the other values are quickly determined by interpolation. These table values are derived from ten-place hyperbolic sine and cosine tables and these values have the same accuracy as the sinh and cosh tables. Table I values

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where

$$\theta_p=(R_1+R_h)q+T_i \qquad (38)$$

therefore the general solution to equation 17 is

$$\theta_y=c_1e^{m_1t}+c_2e+^{m_2t}+\theta_p \qquad (39)$$

The boundary conditions that must be satisfied are

$\theta_y=\theta_i$

and

$\dfrac{d\theta_y}{dt}=0$

when $t=0$,

and

$\theta_y=\theta_p$

when $t=\infty$.

If these boundary conditions are substituted in equation 39, the value of c_1 and c_2 can be determined

$$c_1=(\theta_i-\theta_p)m_2/(m_2-m_1) \qquad (40)$$

$$c_2=-(\theta_i-\theta_p)m_1/(m_2-m_1) \qquad (41)$$

Finally putting c_1 and c_2 back into equation 39 the solution to the differential equation 17 becomes

$$\theta_y=\frac{\theta_i-\theta_p}{m_2-m_1}\,[m_2e^{m_1t}-m_1e^{m_2t}]+\theta_p \qquad (18)$$

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1. MATHEMATICAL EQUATIONS FOR HEAT CONDUCTION IN THE FINS OF AIR-COOLED ENGINES, D. R. Harper, W. B. Brown. National Advisory Committee for Aeronautics Report 158, 1922.

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3. HEAT TRANSMISSION (second edition), W. H. McAdams. McGraw-Hill Book Company, New York, N. Y., 1942. Pages 220–44.

4. STEM LAG OF INDUSTRIAL THERMOMETERS, Bradford Noyes. *Instruments*, volume 15, November 1942, pages 449–52, 454, and so forth.

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eliminate laborious calculations and provide speed and accuracy in solving sag and tension problems.

Additional Formulas and Nomenclature

The following text explains the additional formulas, derived from the basic catenary characteristics, which are used in this procedure.

Let the following nomenclature be used:

l = horizontal span length between supports

L = catenary arc length of the span

T = tension (total) in cable

F = horizontal longitudinal component of tension in cable. It will be seen from formulas given hereinafter that F equals T at the span vertex

H = horizontal loading per foot of cable due to wind pressure on cable (cable only or cable with ice loading)

W = vertical loading per foot of cable due to weight of cable (cable only or cable with ice loading)

R = resultant loading per foot of cable due to horizontal and vertical loading

ZE = elongation of Z (not permanent stretch) due to loading

ZT = change in length of Z due to temperature change

a = cross-section area of cable

M = modulus of elasticity of cable

C = coefficient of linear expansion per degree temperature change

d = temperature change in degrees

ku = unit permanent set of cable due to loading

kt = total permanent set of cable due to loading

The additional formulas are set out and explained in the following:

$l=2X$ (when both supports are at the same elevation) (E)

Refer to appendix for the relationship of l to x where span supports are at unequal elevations.

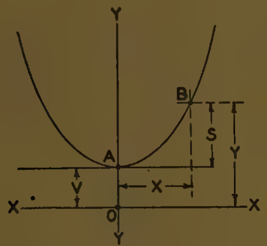


Figure 1. The catenary and its basic formulas

$$Y=(V/2)(\epsilon^{X/V}+\epsilon^{-X/V}) \qquad \text{(AA)}$$

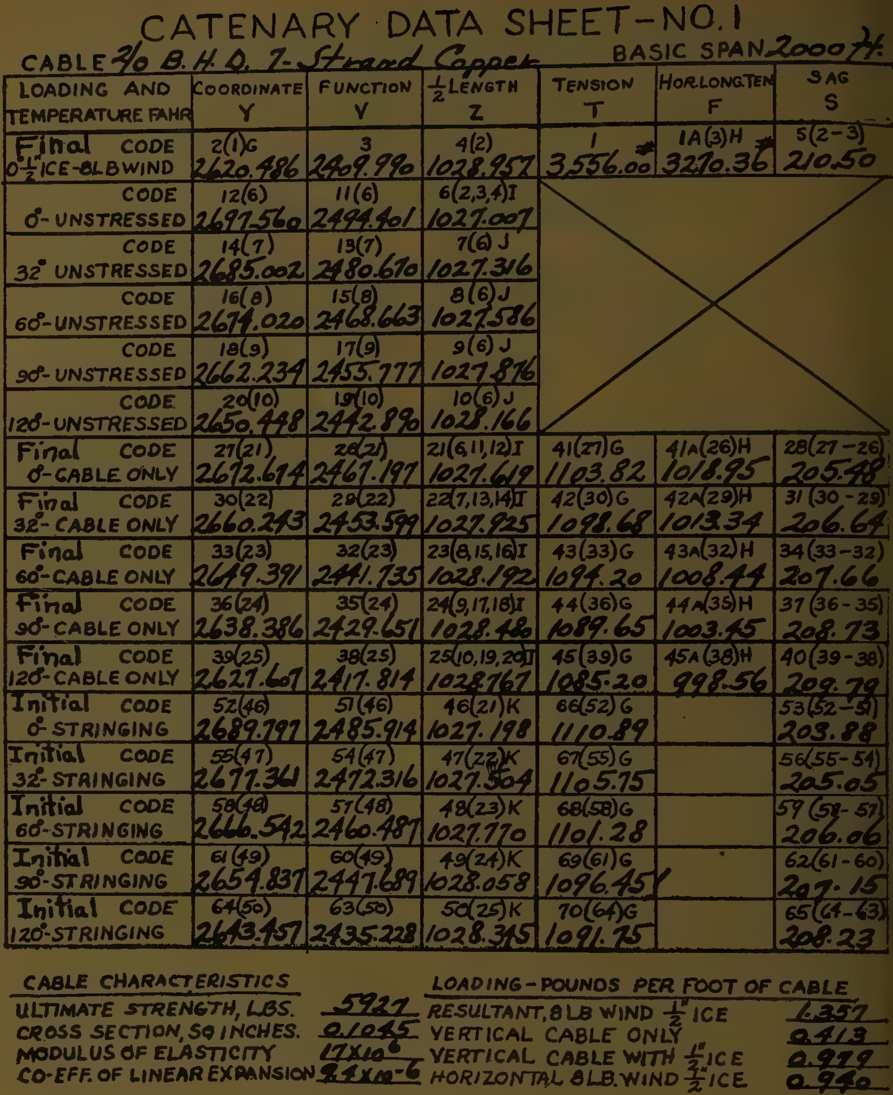
$$Y=V\cosh X/V \qquad \text{(A)}$$

$$Z=(V/2)(\epsilon^{X/V}-\epsilon^{-X/V}) \qquad \text{(BB)}$$

$$Z=V\sinh X/V \qquad \text{(B)}$$

$$S=Y-V \qquad \text{(C)}$$

$$S=V(\cosh X/V-1) \qquad \text{(D)}$$



CATENARY DATA SHEET—NO. 1

CABLE 2/0 B.H.D. 7-Strand Copper BASIC SPAN 2000 ft.

LOADING AND TEMPERATURE FAHR	COORDINATE Y	FUNCTION V	½ LENGTH Z	TENSION T	HOR.LONG.TEN F	SAG S
Final CODE 0° ½" ICE-8LB WIND	2(1)G 2620.486	3 2409.990	4(2) 1028.957	1 3,556.00#	1A(3)H 3270.36#	5(2-3) 210.50
CODE 0°-UNSTRESSED	12(6) 2697.560	11(6) 2494.401	6(2,3,4)I 1027.007			
CODE 32° UNSTRESSED	14(7) 2685.002	13(7) 2480.670	7(6)J 1027.316			
CODE 60°-UNSTRESSED	16(8) 2674.020	15(8) 2468.663	8(6)J 1027.586			
CODE 90°-UNSTRESSED	18(9) 2662.234	17(9) 2455.777	9(6)J 1027.876			
CODE 120°-UNSTRESSED	20(10) 2650.448	19(10) 2442.890	10(6)J 1028.166			
Final CODE 0°-CABLE ONLY	27(21) 2672.614	26(21) 2467.197	21(6,11,12)I 1027.619	41(27)G 1103.82	41A(26)H 1018.95	28(27-26) 205.48
Final CODE 32°-CABLE ONLY	30(22) 2660.243	29(22) 2453.599	22(7,13,14)I 1027.925	42(30)G 1098.68	42A(29)H 1013.34	31(30-29) 206.64
Final CODE 60°-CABLE ONLY	33(23) 2649.391	32(23) 2441.735	23(8,15,16)I 1028.192	43(33)G 1094.20	43A(32)H 1008.44	34(33-32) 207.66
Final CODE 90°-CABLE ONLY	36(24) 2638.386	35(24) 2429.651	24(9,17,18)I 1028.480	44(36)G 1089.65	44A(35)H 1003.45	37(36-35) 208.73
Final CODE 120°-CABLE ONLY	39(25) 2627.607	38(25) 2417.814	25(10,19,20)I 1028.767	45(39)G 1085.20	45A(38)H 998.56	40(39-38) 209.79
Initial CODE 0°-STRINGING	52(46) 2689.797	51(46) 2485.914	46(21)K 1027.198	66(52)G 1110.89		53(52-51) 203.88
Initial CODE 32°-STRINGING	55(47) 2677.361	54(47) 2472.316	47(22)K 1027.504	67(55)G 1105.75		56(55-54) 205.05
Initial CODE 60°-STRINGING	58(48) 2666.542	57(48) 2460.487	48(23)K 1027.770	68(58)G 1101.28		59(58-57) 206.06
Initial CODE 90°-STRINGING	61(49) 2654.837	60(49) 2447.689	49(24)K 1028.058	69(61)G 1096.45		62(61-60) 207.15
Initial CODE 120°-STRINGING	64(50) 2643.457	63(50) 2435.228	50(25)K 1028.345	70(64)G 1091.75		65(64-63) 208.23

CABLE CHARACTERISTICS	
ULTIMATE STRENGTH, LBS.	5927
CROSS SECTION, SQ INCHES.	0.1045
MODULUS OF ELASTICITY	17×10^6
CO-EFF. OF LINEAR EXPANSION	9.4×10^{-6}

LOADING—POUNDS PER FOOT OF CABLE	
RESULTANT, 8 LB WIND ½" ICE	1.357
VERTICAL CABLE ONLY	0.413
VERTICAL CABLE WITH ½" ICE	0.979
HORIZONTAL 8 LB. WIND ½" ICE	0.940

Figure 2

$L=2Z$ (supports at the same elevation) (F)

Refer to appendix for relationship of L to Z where span supports are at unequal elevations.

$T=YR$ (G)

The tension in a cable at any point is equal to the Y co-ordinate at that point times the per foot loading of the cable. In problems without wind pressure, $T=YW$.

This formula is due to a certain peculiarity of the catenary. The tension in a cable at any point in a span is equal to the loading on a piece of cable of the length of the function Y at that point in the span. If the cable, assumed to be perfectly flexible, were hung in a frictionless pulley at any point in the span with a length of cable hanging down just sufficient to reach the axis of the Y co-ordinate, the curve of the catenary would be undisturbed. (Reference: Martin's tables.)

The formula shows that the Y function used in the catenary formulas is proportional to the total tension in the cable.

At the low point of sag (vertex) $Y=V$ (refer to formula A) and $T=F$ so that substitutions in formula just given produces the one which follows.

$F=VR$ (H)

The horizontal longitudinal component of tension in a cable is equal to the V function times the per-foot cable loading. In problems without wind pressure, $F=VW$.

This formula shows that the V function used in the catenary function is proportional to the tension at the cable vertex.

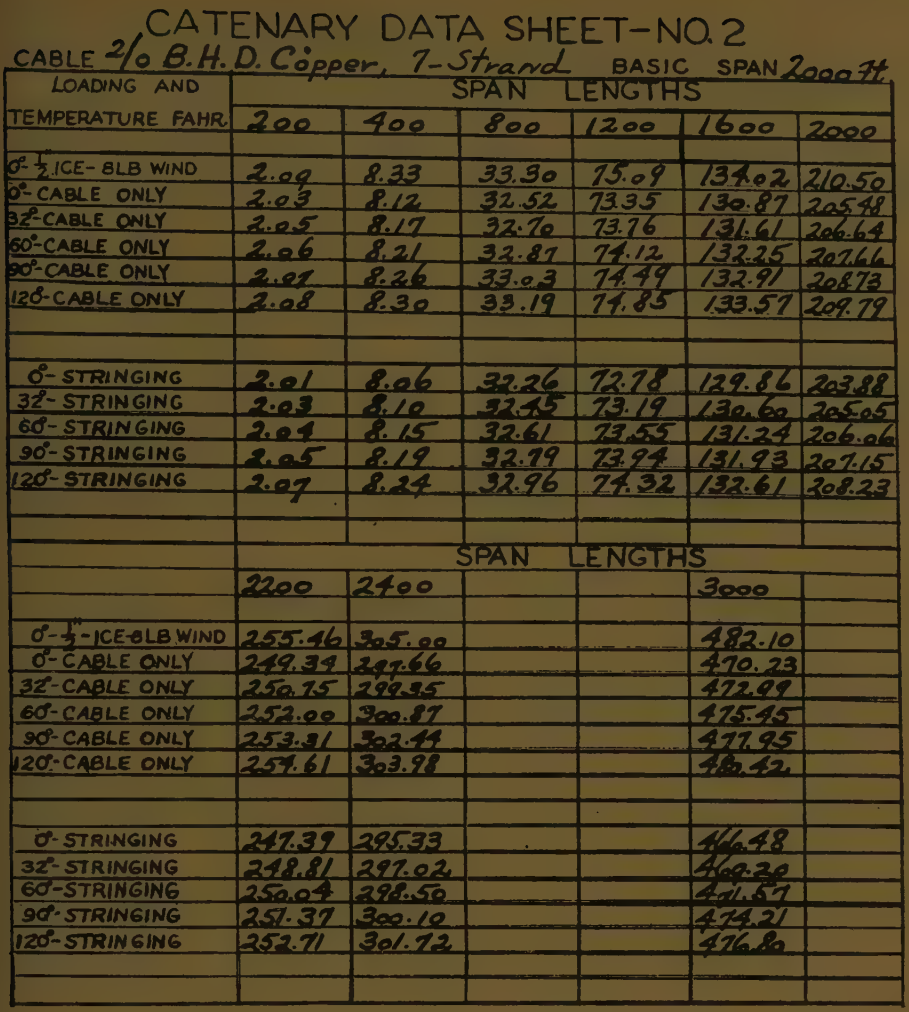
Since $Y=V+S$ (from formula C) formula G may be written:

$T=(V+S)R$

Therefore:

$T=F+SR$

This indicates that the tension in a cable at a support is equal to the horizontal longitudinal component of tension plus the product of the sag times the per foot cable loading. This relationship holds with spans which have supports at the same or at unequal elevations provided S is taken as the deflection of the vertex below the support in question. This formula provides a quick determination of tension increase at sup-



CATENARY DATA SHEET—NO. 2

CABLE 2/0 B.H.D. Copper, 7-Strand BASIC SPAN 2000 ft.

LOADING AND TEMPERATURE FAHR.	SPAN LENGTHS					
	200	400	800	1200	1600	2000
0°-½" ICE-8LB WIND	2.09	8.33	33.30	75.09	134.02	210.50
0°-CABLE ONLY	2.03	8.12	32.52	73.35	130.87	205.48
32°-CABLE ONLY	2.05	8.17	32.70	73.76	131.61	206.64
60°-CABLE ONLY	2.06	8.21	32.87	74.12	132.25	207.66
90°-CABLE ONLY	2.07	8.26	33.03	74.49	132.91	208.73
120°-CABLE ONLY	2.08	8.30	33.19	74.85	133.57	209.79
0°-STRINGING	2.01	8.06	32.26	72.78	129.86	203.88
32°-STRINGING	2.03	8.10	32.45	73.19	130.60	205.05
60°-STRINGING	2.04	8.15	32.61	73.55	131.24	206.06
90°-STRINGING	2.05	8.19	32.79	73.94	131.93	207.15
120°-STRINGING	2.07	8.24	32.96	74.32	132.61	208.23
	SPAN LENGTHS					
	2200	2400			3000	
0°-½"-ICE-8LB WIND	255.46	305.00			482.10	
0°-CABLE ONLY	249.34	297.66			470.23	
32°-CABLE ONLY	250.75	299.35			472.99	
60°-CABLE ONLY	252.00	300.87			475.45	
90°-CABLE ONLY	253.31	302.44			477.95	
120°-CABLE ONLY	254.61	303.98			480.42	
0°-STRINGING	247.39	295.33			466.48	
32°-STRINGING	248.81	297.02			469.20	
60°-STRINGING	250.04	298.50			471.67	
90°-STRINGING	251.37	300.10			474.21	
120°-STRINGING	252.71	301.72			476.80	

Figure 3

Solve for sag by using Table I, remembering that the V in V/X is the same V value occurring for the basic span under similar loading conditions. The related S/X value multiplied by one-half the span produces the sag value

ports over tension at the vertex. In long-span construction this increase is an appreciable item.

Effect in Change of Loading

Elongation of Z (not permanent stretch) due to cable loading is determined by formula I

$$ZE = (R/2aM)(XV+YZ) \qquad \text{(I)}$$

This formula is ably developed in the Martin method of sag and tension calculations Under conditions of no wind W replaced R in this formula.

Note the following variations in formula I.

$$ZE = (RVX+RYZ)/2aM$$

Therefore:

$$ZE = (FX+TZ)/2aM \qquad \text{(I-1)}$$

Effect of Temperature Change

Changes in cable length due to temperature changes are found by using formula J.

$$Z1 = CdZ \qquad \text{(J)}$$

The change in conductor length due to variation in temperature depends on the original length, the coefficient of expansion, and the degrees temperature variation.

Determination of Permanent Stretch

The following procedure is suggested for securing the permanent set (in Z) due to loading previously unstressed cables.

First determine loaded stress in cable as follows:

Stress (at support) $= T/a$
Stress (at vertex) $= F/a$
Stress (span average) $= (T+F)/2a$

This value is a close value to use with permanent-set charts for the cable under consideration From the chart determine the unit permanent set, ku, caused by design loading, $(T+F)/2a$, and the unit permanent set, ku, for the probable stringing stress. The difference between these k values will be the net unit permanent set to be used in calculating stringing sags and tensions.

$$KT = KuZ \qquad \text{(K)}$$

Problem Solution Procedure

The authors recently designed and built an 88-kv transmission line located in mountain country. The sag and tension design data for this line are contained in data sheets 1 and 2 as shown in Figures 2 and 3, and it is suggested that these be studied to determine the solution procedure of this paper. These data sheets are coded to show the sequence of calculations The left-hand number gives the calculation sequence, the numbers in parentheses the previous values used to derive the present value, and the right-hand key letter the formula used. Where no key letter is shown the value is obtained by using Table I. S values result from subtracting V from Y.

Data sheet number 1, shown in Figure 2, covers calculations for the basic (ruling) span where X is constant. Data sheet number 2, shown in Figure 3, covers calculations for other length spans related to the basic span where X varies with the span length, but the V value for each final or initial condition remains constant throughout varying span lengths.

While the data sheets are largely self-explanatory the steps in problem solution are further explained in the following paragraphs

1. Tabulate the cable characteristics, the per-foot cable loading values, the basic design span, and design final tension, item 1.

2. Calculate Y, item 2, for design load conditions using formula G. Use Table I to secure related values of V, item 3, and Z, item 4. Design sag item 5 is simply $Y-V$. Horizontal longitudinal component of tension F (item 1A) is secured by formula H. The calculating steps just outlined are based on the tension at support in the basic design span. If the design is to be based on a desired F value for the basic span first tabulate item 1A and calculate V, item 3, using formula H. Use Table I to secure related design values of Y, item 2, and Z, item 4. Sag is $Y-V$, and total tension, item 1, is solved by using formula G.

3. Solve for unstressed Z, item 6, all loads removed, but with the cable still at design temperature, using formula I.

4. Solve for unstressed Z at other temperatures, items 7, 8, 9, and 10, using the item 6 value of Z and formula J.

5. Determine related V and Y values for each unstressed Z value from Table I.

6. Calculate Z (cable only) at various temperatures, items 21 to 25 inclusive, using formula I.

7. Determine related V and Y values for each cable only load condition from Table I. Sags for each condition are $Y-V$.

8. Solve for Z values (initial conditions, not prestretched) at various temperatures, items 46 to 50 inclusive by the use of for-

Table I. Unit Values of *V*, *Y*, *Z*, and *S*, Catenary Functions

Function X/V	Function V/X	Difference in X/V	Function Y/X	Difference in Y/X	Function S/X	Difference in S/X	Function Z/X	Difference in Z/X
0.00	Infinity	Infinity	Infinity	Infinity	Infinity	Infinity	0.0000000	1.0000167
0.01	100.00000	50.000000	100.00500	49.995000	.00500000	.00500000	1.0000167	.0000500
0.02	50.000000	16.666667	50.01000	16.661667	.01000000	.00500000	1.0000667	.0000833
0.03	33.333333	8.333333	33.34833	8.328330	.01500000	.00500300	1.0001500	.0001167
0.04	25.000000	5.000000	25.020003	4.994997	.02000300	.00500300	1.0002667	.0001500
0.05	20.000000	3.333333	20.025006	3.328331	.02500600	.00500200	1.0004167	.0001834
0.06	16.666667	2.380953	16.696675	2.375947	.03000800	.00500600	1.0006001	.0002168
0.07	14.285714	1.785714	14.320728	1.780707	.03501400	.00500700	1.0008169	.0002501
0.08	12.500000	1.388889	12.540021	1.383880	.04002100	.00500900	1.0010670	.0002835
0.09	11.111111	1.111111	11.156141	1.106099	.04503000	.00501200	1.0013505	.0003170
0.10	10.000000	.909091	10.050042	.904078	.05004200	.00501300	1.0016675	.0003504
0.11	9.090909	.757576	9.145964	.752559	.05505500	.00501700	1.0020179	.0003838
0.12	8.333333	.641025	8.393405	.636005	.06007200	.00502000	1.0024017	.0004174
0.13	7.692308	.549451	7.757400	.544429	.06509200	.00502200	1.0028191	.0004507
0.14	7.142857	.476190	7.212971	.471163	.07011400	.00502700	1.0032698	.0004845
0.15	6.666667	.416667	6.741808	.411637	.07514100	.00503000	1.0037543	.0005178
0.16	6.250000	.367647	6.330171	.362613	.08017100	.00503400	1.0042721	.0005516
0.17	5.882353	.326797	5.967558	.321759	.08520500	.00503800	1.0048237	.0005852
0.18	5.555556	.292398	5.645799	.287355	.09024300	.00504300	1.0054089	.0006186
0.19	5.263158	.263158	5.358444	.258110	.09528600	.00504800	1.0060275	.0006525
0.20	5.000000	.238095	5.100334	.233042	.10033400	.00505300	1.0066800	.0006863
0.21	4.761905	.216450	4.867292	.211392	.10538700	.00505800	1.0073663	.0007200
0.22	4.545455	.197629	4.655900	.192567	.11044500	.00506200	1.0080863	.0007537
0.23	4.347826	.181159	4.463333	.176089	.11550700	.00507000	1.0088400	.0007878
0.24	4.166667	.166667	4.287244	.161592	.12057700	.00507500	1.0096278	.0008215
0.25	4.000000	.153846	4.125652	.148764	.12565200	.00508200	1.0104493	.0008555
0.26	3.846154	.142450	3.976888	.137362	.13073400	.00508800	1.0113048	.0008897
0.27	3.703704	.132275	3.839526	.127180	.13582200	.00509500	1.0121945	.0009236
0.28	3.571429	.123153	3.712346	.118051	.14091700	.00510200	1.0131181	.0009577
0.29	3.448276	.114943	3.594295	.109834	.14601900	.00510900	1.0140758	.0009917
0.30	3.333333	.107527	3.484461	.102410	.15112800	.00511700	1.0150675	.0010262
0.31	3.225806	.100806	3.382051	.095681	.15624500	.00512500	1.0160937	.0010606
0.32	3.125000	.094697	3.286370	.089564	.16137000	.00513300	1.0171543	.0010948
0.33	3.030303	.089126	3.196806	.083985	.16650300	.00514100	1.0182491	.0011294
0.34	2.941177	.084034	3.112821	.078884	.17164400	.00515000	1.0193785	.0011636
0.35	2.857143	.079365	3.033937	.074206	.17679400	.00515900	1.0205421	.0011984
0.36	2.777778	.075075	2.959731	.069908	.18195300	.00516700	1.0217405	.0012330
0.37	2.702703	.071124	2.889823	.065947	.18712000	.00517700	1.0229735	.0012675
0.38	2.631579	.067476	2.823876	.062289	.19229700	.00518700	1.0242410	.0013027
0.39	2.564103	.064103	2.761587	.058906	.19748400	.00519700	1.0255437	.0013371
0.40	2.500000	.060976	2.702681	.055769	.20268100	.00520700	1.0268808	.0013722
0.41	2.439024	.058072	2.646912	.052855	.20788800	.00521700	1.0282530	.0014073
0.42	2.380952	.055371	2.594057	.050143	.21310500	.00522800	1.0296603	.0014423
0.43	2.325581	.052854	2.543914	.047615	.21833300	.00523900	1.0311026	.0014777
0.44	2.272727	.050505	2.496299	.045254	.22357200	.00525100	1.0325803	.0015130
0.45	2.222222	.048309	2.451045	.043048	.22882300	.00526100	1.0340933	.0015484
0.46	2.173913	.046253	2.407997	.040979	.23408400	.00527400	1.0356417	.0015840
0.47	2.127660	.044327	2.367018	.039041	.23935800	.00528600	1.0372257	.0016189
0.48	2.083333	.042517	2.327977	.037220	.24464400	.00529700	1.0388446	.0016550
0.49	2.040816	.040816	2.290757	.035505	.24994100	.00531100	1.0404996	.0016910
0.50	2.000000	.039216	2.255252	.033893	.25525200	.00532300	1.0421906	.0017265
0.51	1.960784	.037707	2.221359	.032370	.26057500	.00533700	1.0439171	.0017628
0.52	1.923077	.036285	2.188989	.030936	.26591200	.00534900	1.0456799	.0017985
0.53	1.886792	.034940	2.158053	.029576	.27126100	.00536400	1.0474784	.0018352
0.54	1.851852	.033671	2.128477	.028294	.27662500	.00537700	1.0493136	.0018707
0.55	1.818181	.032467	2.100183	.027075	.28200200	.00539200	1.0511843	.0019079
0.56	1.785714	.031328	2.073108	.025922	.28739400	.00540600	1.0530922	.0019443
0.57	1.754386	.030248	2.047186	.024827	.29280000	.00542100	1.0550365	.0019808
0.58	1.724138	.029223	2.022359	.023787	.29822100	.00543600	1.0570173	.0020174
0.59	1.694915	.028248	1.998572	.022796	.30365700	.00545200	1.0590347	.0020548
0.60	1.666667	.027323	1.975776	.021856	.30910900	.00546700	1.0610895	.0020911
0.61	1.639344	.026441	1.953920	.020959	.31457600	.00548200	1.0631806	.0021286
0.62	1.612903	.025601	1.932961	.020101	.32005800	.00550000	1.0653092	.0021663
0.63	1.587302	.024802	1.912860	.019287	.32555800	.00551500	1.0674755	.0022030
0.64	1.562500	.024038	1.893573	.018506	.33107300	.00553200	1.0696785	.0022411
0.65	1.538462	.023310	1.875067	.017761	.33660500	.00555000	1.0719196	.0022785
0.66	1.515152	.022615	1.857306	.017048	.34215500	.00556600	1.0741981	.0023157
0.67	1.492537	.021949	1.840258	.016365	.34772100	.00558400	1.0765138	.0023542
0.68	1.470588	.021313	1.823893	.015711	.35330500	.00560200	1.0788680	.0023922
0.69	1.449275	.020704	1.808182	.015084	.35890700	.00562000	1.0812602	.0024305
0.70	1.428571	.020120	1.793098	.014482	.36452700	.00563800	1.0836907	.0024694
0.71	1.408451	.019562	1.778616	.013904	.37016500	.005658	1.0861601	.0025073
0.72	1.388889	.019026	1.764712	.013349	.37582300	.00567700	1.0886674	.0025460
0.73	1.369863	.018512	1.751363	.012817	.38150000	.00569500	1.0912134	.0025847
0.74	1.351351	.018018	1.738546	.012302	.38719500	.00571600	1.0937981	.0026239
0.75	1.333333	.017543	1.726244	.011807	.39291100	.00573600	1.0964220	.0026638

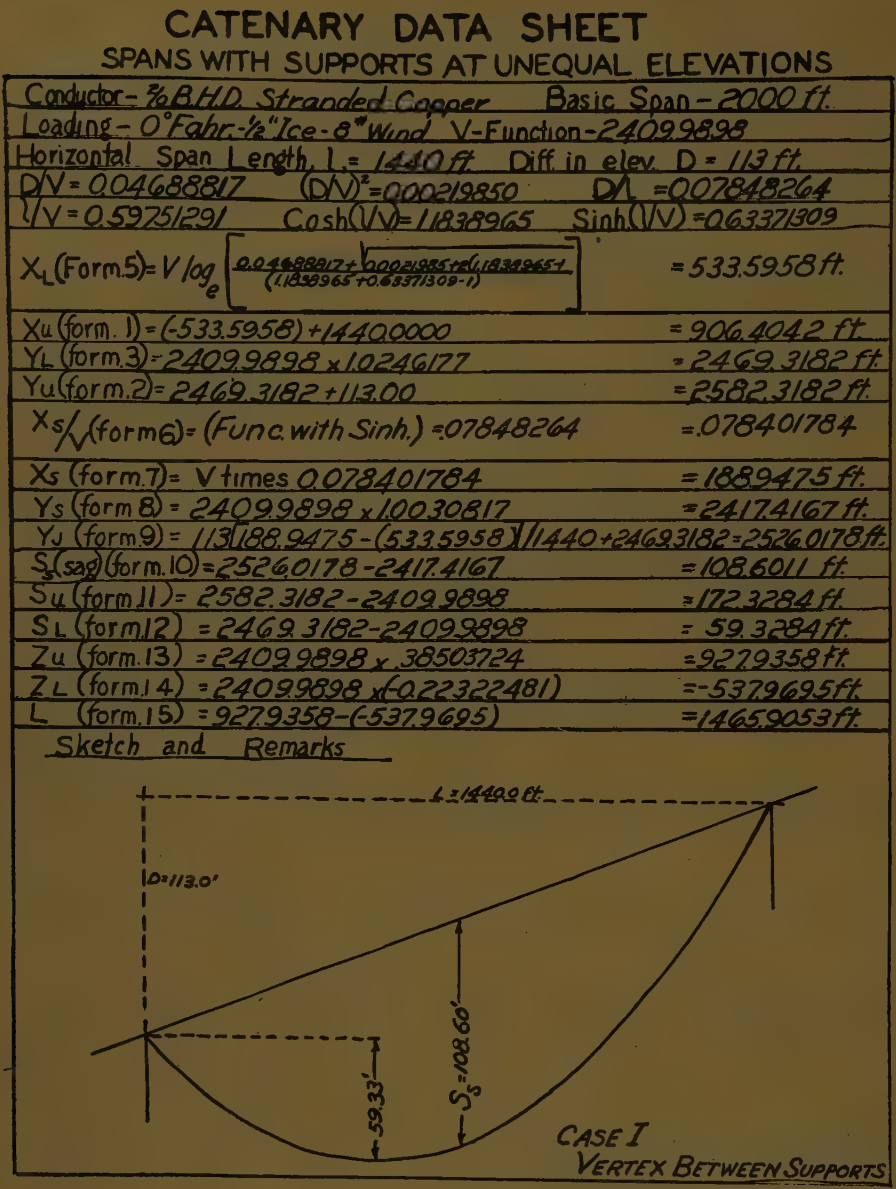
Table I. Unit Values of V, Y, Z, and S Catenary Functions (continued)

Function X/V	Function V/X	Difference in V/X	Function Y/X	Difference in Y/X	Function S/X	Difference in S/X	Function Z/X	Difference in Z/X
0.76	1.315790	.017089	1.714437	.011334	.39864700	.00575500	1.0990858	.0027017
0.77	1.298701	.016650	1.703103	.010874	.40440200	.00577600	1.1017875	.0027419
0.78	1.282051	.016228	1.692229	.010430	.41017800	.00579800	1.1045294	.0027820
0.79	1.265823	.015823	1.681799	.010005	.41597600	.00581800	1.1073114	.0028211
0.80	1.250000	.015432	1.671794	.009593	.42179400	.00583900	1.1101325	.0028614
0.81	1.234568	.015056	1.662201	.009194	.42763300	.00586200	1.1129939	.0029012
0.82	1.219512	.014693	1.653007	.008810	.43349500	.00588300	1.1158951	.0029417
0.83	1.204819	.014343	1.644197	.008437	.43937800	.00590600	1.1188368	.0029823
0.84	1.190476	.014005	1.635760	.008076	.44528400	.00592900	1.1218191	.0030236
0.85	1.176471	.013680	1.627684	.007728	.45121300	.00595200	1.1248427	.0030638
0.86	1.162791	.013366	1.619956	.007392	.45716500	.00597400	1.1279065	.0031043
0.87	1.149425	.013061	1.612564	.007062	.46313900	.00599900	1.1310108	.0031468
0.88	1.136364	.012768	1.605502	.006745	.46913800	.00602300	1.1341576	.0031877
0.89	1.123596	.012485	1.598757	.006439	.47516100	.00604600	1.1373453	.0032287
0.90	1.111111	.012210	1.592318	.006139	.48120700	.00607100	1.1405740	.0032712
0.91	1.098901	.011944	1.586179	.005847	.48727800	.00609700	1.1438452	.0033137
0.92	1.086957	.011688	1.580332	.005567	.49337500	.00612100	1.1471589	.0033550
0.93	1.075269	.011439	1.574765	.005292	.49949600	.00614700	1.1505139	.0033978
0.94	1.063830	.011198	1.569473	.005025	.50564300	.00617300	1.1539117	.0034407
0.95	1.052632	.010965	1.564448	.004766	.51181600	.00619900	1.1573524	.0034832
0.96	1.041667	.010739	1.559682	.004513	.51801500	.00622600	1.1608356	.0035261
0.97	1.030928	.010520	1.555169	.004268	.52424100	.00625200	1.1643617	.0035693
0.98	1.020408	.010307	1.550901	.004027	.53049300	.00628000	1.1679310	.0036133
0.99	1.010101	.010101	1.546874	.003793	.53677300	.00630800	1.1715443	.0036569
1.00	1.00000	.009901	1.543081	.003566	.54308100	.00633500	1.1752012	.0037008
1.01	.990 99	.009707	1.539515	.003344	.54941600	.00636300	1.1789020	.0037448
1.02	.9 0392	.009518	1.536171	.003126	.55577900	.00639200	1.1826468	.0037898
1.03	970874	.009336	1.533045	.002915	.56217100	.00642100	1.1864366	.0038332
1.04	.961538	.009157	1.530130	.002707	.56859200	.00645000	1.1902698	.0038795
1.05	.952381	.008984	1.527423	.002505	.57504200	.00648000	1.1941493	.0039236
1.06	.943396	.008817	1.524918	.002308	.58152200	.00650900	1.1980729	.0039691
1.07	.934579	.008653	1.522610	.002112	.58803100	.00654100	1.2020420	.0040155
1.08	.925926	.008495	1.520498	.001925	.59457200	.00657000	1.2060575	.0040604
1.09	.917431	.008341	1.518573	.001738	.60114200	.00660200	1.2101179	.0041072
1.10	.909091	.008190	1.516835	.001557	.60774400	.00663300	1.2142251	.0041531
1.11	.900901	.008044	1.515278	.001379	.61437700	.00666500	1.2183782	.0041995
1.12	.892857	.007901	1.513899	.001204	.62104200	.00669700	1.2225777	.0042471
1.13	.884956	.007763	1.512695	.001034	.62773900	.00672900	1.2268248	.0042934
1.14	.877193	.007628	1.511661	.000866	.63446800	.00676200	1.2311182	.0043407
1.15	.869565	.007496	1.510795	.000700	.64123000	.00679600	1.2354589	.0043890
1.16	.862069	.007368	1.510095	.000539	.64802600	.00682900	1.2398479	.0044368
1.17	.854701	.007243	1.509556	.000380	.65485500	.00686300	1.2442847	.0044852
1.18	.847458	.007122	1.509176	.000226	.66171800	.00689600	1.2487699	.0045324
1.19	.840336	.007003	1.508950	.000071	.66861400	.00693200	1.2533023	.0045817
1.20	.833333	.006887	1.508879	.000080	.67554600	.00696700	1.2578840	.0046310
1.21	.826446	.006774	1.508959	.000228	.68251300	.00700200	1.2625150	.0046805
1.22	.819672	.006664	1.509187	.000375	.68951500	.00703900	1.2671955	.0047299
1.23	.813008	.006556	1.509562	.000518	.69655400	.00707400	1.2719254	.0047805
1.24	.806452	.006452	1.510080	.000659	.70362800	.00711100	1.2767059	.0048294
1.25	.800000	.006349	1.510739	.000799	.71073900	.00714800	1.2815353	.0048807
1.26	.793651	.006249	1.511538	.000937	.71788700	.00718600	1.2864160	.0049315
1.27	.787402	.006152	1.512475	.001071	.72507300	.00722300	1.2913475	.0049816
1.28	.781250	.006056	1.513546	.001205	.73229600	.00726100	1.2963291	.0050340
1.29	.775194	.005963	1.514751	.001337	.73955700	.00730000	1.3013631	.0050853
1.30	.769231	.005872	1.516088	.001467	.74685700	.00733900	1.3064484	.0051373
1.31	.763359	.005783	1.517555	.001596	.75419600	.00737900	1.3115857	.0051897
1.32	.757576	.005696	1.519151	.001722	.76157500	.00741800	1.3167754	.0052425
1.33	.751880	.005611	1.520873	.001848	.76899300	.00745900	1.3220179	.0052953
1.34	.746269	.005528	1.522721	.001971	.77645200	.00749900	1.3273132	.0053485
1.35	.740741	.005447	1.524692	.002092	.78395100	.00753900	1.3326617	.0054015
1.36	.735294	.005367	1.526784	.002215	.79149000	.00758200	1.3380632	.0054564
1.37	.729927	.005289	1.528999	.002335	.79907200	.00762400	1.3435196	.0055111
1.38	.724638	.005214	1.531334	.002451	.80669600	.00766500	1.3490307	.0055637
1.39	.719424	.005138	1.533785	.002572	.81436100	.00771000	1.3545944	.0056215
1.40	.714286	.005066	1.536357	.002685	.82207100	.00775100	1.3602159	.0056759
1.41	.709220	.004995	1.539042	.002800	.82982200	.00779500	1.3658918	.0057295
1.42	.704225	.004924	1.541842	.002924	.83761700	.00784800	1.3716213	.0057887
1.43	.699301	.004857	1.544766	.003019	.84546500	.00787600	1.3774100	.0058419
1.44	.694444		1.547785		.85334100		1.3832519	

In a given span V, Y, S, and Z bear definite interrelationships expressed by the formulas, $Y = V \cosh (X/V)$, $Z = V \sinh (X/V)$, and $S = Y - V$. When these formulas are divided by X, unit values, $(Y/X) = (V/X) \cosh (X/V)$, $(Z/X) = (V/X) \sinh (X/V)$, and $(S/X) = (Y/X) - (V/X)$ result which apply to any length span. Related unit values are tabulated in Table I. If V, Y, Z, or S is known for any span the other values can be quickly determined by the use of this table.

Differences for interpolations speed these calculations.

These tables also will supply cosh and sinh values for use in the formulas $Y = V \cosh (X/V)$ and $Z = V \sinh (X/V)$. The Y/X value of the table when multiplied by X/V becomes the cosh (X/V) of X/V, and Z/X value when multiplied by X/V becomes the sinh (X/V) of X/V.



CATENARY DATA SHEET
SPANS WITH SUPPORTS AT UNEQUAL ELEVATIONS

Conductor - 2/0 B.H.D. Stranded Copper   Basic Span - 2000 ft.
Loading - 0°Fahr.-½" Ice-8# Wind   V-Function - 2409.9898
Horizontal Span Length, l = 1440 ft.   Diff. in elev. D = 113 ft.
$D/V = 0.04688817$ $(D/V)^2 = 0.00219850$ $D/l = 0.07848264$
$l/V = 0.59751291$ $\cosh(l/V) = 1.1838965$ $\sinh(l/V) = 0.63371309$
X_L (Form. 5) $= V \log_e \left[\frac{0.04688817+\sqrt{0.0021985+2(1.1838965-1}}{(1.1838965+0.63371309-1)}\right]$ $= 533.5958$ ft.
X_u (form. 1) $= (-533.5958)+1440.0000$ $= 906.4042$ ft.
Y_L (form. 3) $= 2409.9898 \times 1.0246177$ $= 2469.3182$ ft.
Y_u (form. 2) $= 2469.3182+113.00$ $= 2582.3182$ ft.
X_s/V (form 6) = (Func. with Sinh.) $= .07848264$ $= .078401784$
X_s (form. 7) = V times 0.078401784 $= 188.9475$ ft.
Y_s (form 8) $= 2409.9898 \times 1.0030817$ $= 2417.4167$ ft.
Y_J (form 9) $= 113[188.9475-(533.5958)]/1440+2469.3182 = 2526.0178$ ft.
S_s (sag) (form. 10) $= 2526.0178-2417.4167$ $= 108.6011$ ft.
S_u (form 11) $= 2582.3182-2409.9898$ $= 172.3284$ ft.
S_L (form 12) $= 2469.3182-2409.9898$ $= 59.3284$ ft.
Z_u (form. 13) $= 2409.9898 \times .38503724$ $= 927.9358$ ft.
Z_L (form. 14) $= 2409.9898 \times (-0.22322481)$ $= -537.9695$ ft.
L (form. 15) $= 927.9358-(-537.9695)$ $= 1465.9053$ ft.

Sketch and Remarks

CASE I
VERTEX BETWEEN SUPPORTS

Figure 4

mula K. For this problem the following *Ku* values were used:

Ku—design loading = 0.00049
Ku—stringing = 0.00008
Ku (for formula K) = 0.00041

The stress–strain chart from which these values were secured is reproduced in Figure 4. It should be noted that the *Ku* values for stringing (initial) conditions varies very little over the range of stringing stresses encountered. This indicates that even with a variation between assumed stringing tensions used to secure *Ku* (stringing) and the final determined stringing tension, close stringing sag and tension values are secured. Stringing temperature variations have practically no effect on the *Ku* value taken from the stress–strain chart.

9. Determine related V and Y values (initial) from Table I. Solve for stringing sags, $Y-V$.

10. Solve for T and F (if desired) under various conditions. The cable loading per foot, and the Y and V values for each condition give the T and F values.

11. Solve for sags under various final and initial conditions for other span lengths. Remembering that the V value for a given final or initial condition is the same for all span lengths (related to the design span), use these V values and Table I to secure these sags. Tabulate these sags on data sheet number 2 (Figure 3). F values for the related spans are the same as the value for the design span under like final or initial conditions. If desired, the T value can be quickly determined for any related span by means of Table I and formula G.

The procedure outlined here gives full sag and tension data for final and initial conditions, not only for the design span, but also for similar conditions in other related span lengths.

The calculation procedure can be reversed, if it is desired to base the calculations on other conditions such as final, cable only at 60 degrees Fahrenheit.

Hillside Span Calculations

Several approximate methods of determining sags in hillside spans are in use. One of the best of these methods is published in Martin's tables. Martin has also developed another method of calculating hillside spans which has an extremely high degree of mathematical accuracy.

The appendix outlines a mathematically exact method, developed by the writers and their associates, for determining sag, deflection of vertex below supports, and horizontal distances from the vertex to supports in hillside spans. This method is recommended where precision results are desired.

Appendix. Hillside Span Calculations

This paper outlines a mathematically accurate method, developed by the authors, of determining sag, deflection of vertex below supports, and distance (horizontally) from the vertex to supports in spans with supports at unequal elevations. Its use is recommended whenever precision results are necessary, such as with certain span designs crossing railroads, highways, or other lines.

The method is based on maintaining the same horizontal longitudinal component of tension, and therefore the same V value, as in the ruling span to which the hillside span is related.

The procedure outlined is applicable to any type cable such as copper, copper-coated steel, steel, or aluminium cable steel reinforced, where the ruling span sag (for a level span) is known.

There are two general types of spans with supports at unequal elevations:

Case I. Catenary vertex located between supports.
Case II. Catenary vertex located below the lower support.

Figure 5 illustrates these two conditions.

Nomenclature (Refer to Figure 5)

D = difference in elevation of cable supports
l = span length (horizontally)
X_u, Y_u = co-ordinates of upper support
X_L, Y_L = co-ordinates of lower support
S_s = sag of cable, which is the maximum deflection of the cable from the straight (slope) line between supports
X_s, Y_s = co-ordinates of catenary curve at the point of maximum deflection, S_s
Y_J = Y co-ordinate of a point on the slope line (between supports) directly above Y_s
S_u = deflection of vertex below upper support
S_L = deflection of vertex below lower support
Z_u = catenary length from vertex to upper support
Z_L = catenary length from vertex to lower support
L = catenary length of span (hillside span)

V = the ruling (basic) span V value for the cable under the assumed loading conditions

If this V value is not known, it may be quickly determined as follows:

Divide ruling span sag (level span) by X (1/2 span), to secure S/X. From Table I determine by interpolation the corresponding V/X value. This multiplied by X (1/2 span), gives the required V value.

This procedure may be used with any type cable.

The various formulas and their derivation are given in the following. In developing these formulas the writers have assumed that the upper support is to the right of the lower support. Under case I conditions the lower-support X and Z values should be set in the formulas as minus quantities since they are to the left-hand (minus) side of the vertex.

$$l=X_u-X_L \tag{1}$$

or

$$X_u=X_L+l$$

Case II—As X_u and X_L are on the same side of the vertex, both represent positive values.

Case I—As X_u and X_L are on opposite sides of the vertex, X_L is taken as a negative value.

$$Y_u=Y_L+D \tag{2}$$

$$Y_u=V\cosh(X_u/V)$$

$$Y_u=V\cosh\left(\frac{X_L+l}{V}\right)$$

$$Y_L=V\cosh(X_L/V) \tag{3}$$

$$D=Y_u-Y_L$$

Therefore:

$$D/V=\cosh\left(\frac{X_L+l}{V}\right)-\cosh\left(\frac{X_L}{V}\right)$$

$$\frac{D}{V}=\frac{\epsilon^{\left(\frac{X_L+l}{V}\right)}+\epsilon^{-\left(\frac{X_L+l}{V}\right)}}{2}-\frac{\epsilon^{(X_L/V)}+\epsilon^{-(X_L/V)}}{2}$$

Transpose and rearrange:

$$\frac{2D}{V}=\epsilon^{(X_L/V)}[\epsilon^{(l/V)}-1]+\epsilon^{-(X_L/V)}[\epsilon^{-(l/V)}-1]$$

Multiply by $\epsilon^{(X_L/V)}$:

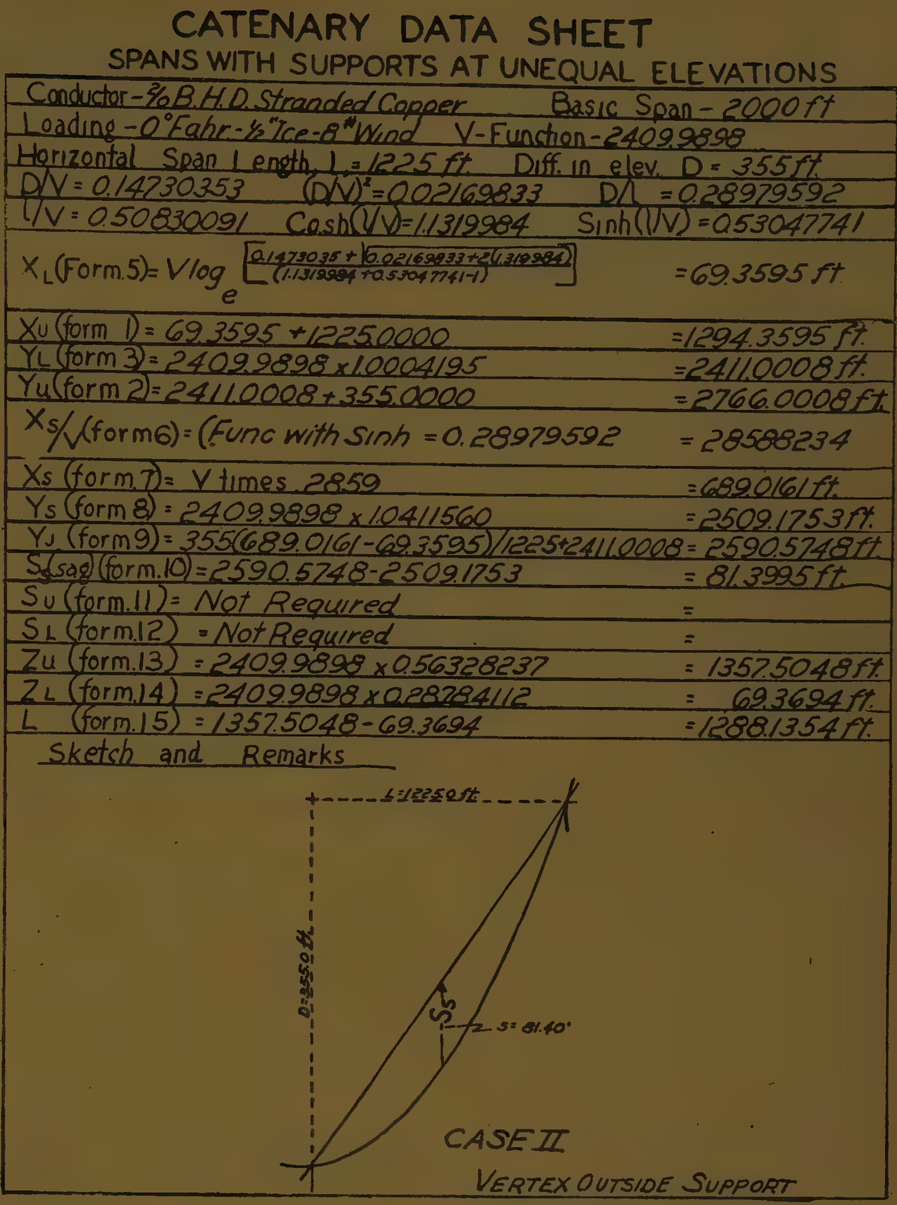
$$\frac{2D\epsilon^{(X_L/V)}}{V}=[\epsilon^{(X_L/V)}]^2[\epsilon^{(l/V)}-1]+[\epsilon^{-(l/V)}-1]$$

Transpose:

$$[\epsilon^{(X_L/V)}]^2[\epsilon^{(l/V)}-1]-\frac{2D}{V}\epsilon^{(X_L/V)}=-\epsilon^{-(l/V)}-1]$$

Multiply by $[\epsilon^{(l/V)}-1]$ and add $(D/V)^2$:

$$[\epsilon^{(X_L/V)}]^2[\epsilon^{(l/V)}-1]^2-\frac{2D}{V}\epsilon^{(X_L/V)}[\epsilon^{(l/V)}-1]+\left(\frac{D}{V}\right)^2=\epsilon^{(l/V)}+\epsilon^{-(l/V)}-2+\left(\frac{D}{V}\right)^2$$



CATENARY DATA SHEET

SPANS WITH SUPPORTS AT UNEQUAL ELEVATIONS

Conductor - 2/0 B.H.D. Stranded Copper Basic Span - 2000 ft

Loading - 0°Fahr. - ½" Ice - 8# Wind V-Function - 2409.9898

Horizontal Span Length, l = 1225 ft. Diff. in elev. D = 355 ft.

D/V = 0.14730353 (D/V)² = 0.02169833 D/l = 0.28979592

l/V = 0.50830091 Cosh(l/V) = 1.1319984 Sinh(l/V) = 0.53047741

X_L (Form.5) = $V\log_e\left[\frac{0.1473035+\sqrt{0.02169833+2(1.319984)}}{(1.1319984+0.53047741-1)}\right]$ = 69.3595 ft

X_U (form 1) = 69.3595 + 1225.0000 = 1294.3595 ft.

Y_L (form 3) = 2409.9898 x 1.0004195 = 2411.0008 ft.

Y_U (form 2) = 2411.0008 + 355.0000 = 2766.0008 ft.

X_S/V (form 6) = (Func with Sinh = 0.28979592 = 28588234

X_S (form 7) = V times .2859 = 689.0161 ft.

Y_S (form 8) = 2409.9898 x 1.0411560 = 2509.1753 ft.

Y_J (form 9) = 355(689.0161 - 69.3595)/1225 + 2411.0008 = 2590.5748 ft

S (sag) (form. 10) = 2590.5748 - 2509.1753 = 81.3995 ft.

S_U (form. 11) = Not Required =

S_L (form. 12) = Not Required =

Z_U (form. 13) = 2409.9898 x 0.56328237 = 1357.5048 ft.

Z_L (form. 14) = 2409.9898 x 0.28784112 = 69.3694 ft.

L (form. 15) = 1357.5048 - 69.3694 = 1288.1354 ft.

Sketch and Remarks

Figure 5

Extract square root:

$$[\epsilon^{(X_L/V)}][\epsilon^{(l/V)}-1]-\frac{D}{V}=\sqrt{\epsilon^{(l/V)}+\epsilon^{-(l/V)}-2+\left(\frac{D}{V}\right)^2}$$

$$\epsilon^{(X_L/V)}=\frac{\frac{D}{V}+\sqrt{2\cosh(l/V)-2+\left(\frac{D}{V}\right)^2}}{[\epsilon^{(l/V)}-1]}$$

$$\epsilon^{(X_L/V)}=\frac{\frac{D}{V}+\sqrt{\left(\frac{D}{V}\right)^2+2[\cosh(l/)V-1]}}{[\cosh(l/V)+\sinh(l/V)-1]} \tag{4}$$

$$X_L=V\times\log_\epsilon\left[\frac{\frac{D}{V}+\sqrt{\left(\frac{D}{V}\right)^2+2[\cosh(l/V)-1]}}{\cosh(l/V)+\sinh(l/V)-1}\right] \tag{5}$$

After X_L is determined, Y_L is found by equation 3.

$$\frac{D}{l}=\sinh\left(\frac{X_s}{V}\right)$$

The slope of the catenary curve at point X_s is the sinh at that point, since the sinh is the derivative of cosh (X_s/V)

$$\frac{X_s}{V}=\sinh^{-1}\left(\frac{D}{l}\right) \tag{6}$$

X_s/V = function which has its sinh $=D/l$

X_s = formula 6 value multiplied by V (7)

$$Y_s=V\cosh(X_s/V) \tag{8}$$

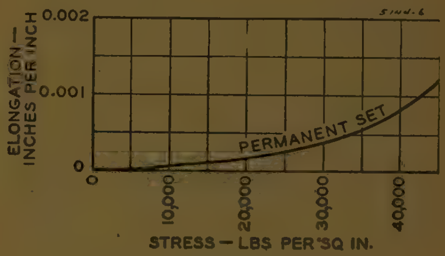
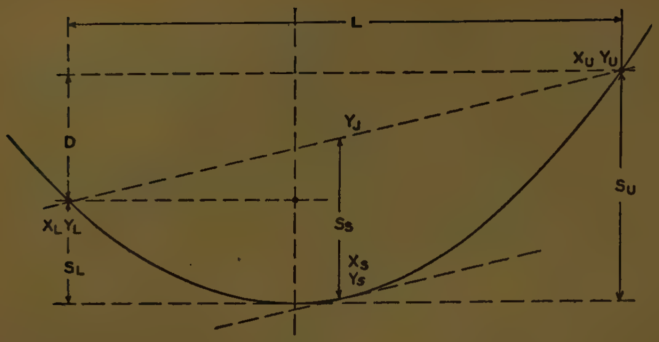


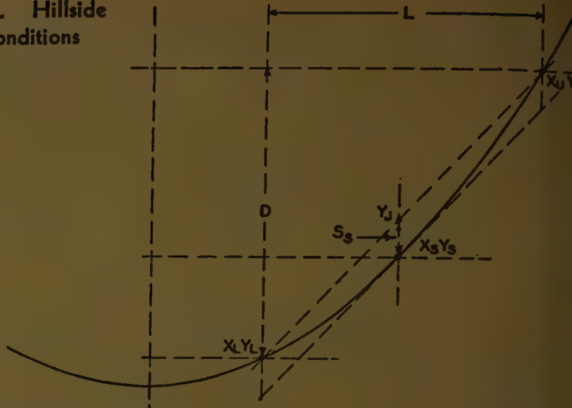
Figure 6. Stress–strain chart

Seven-strand hard-drawn copper cable

Figure 7. Hillside sag conditions



CASE I



CASE II

By proportion:

$$(Y_J - Y_L)/(X_s - X_L) = D/l$$

$$Y_J = D(X_s - X_L)/l + Y_L \tag{9}$$

$$S_s = Y_J - Y_s \tag{10}$$

$$S_u = Y_u - V \tag{11}$$

$$S_L = Y_L - V \tag{12}$$

$$Z_u = V \sinh (X_u/V) \tag{13}$$

$$Z_L = V \sinh (X_L/V) \tag{14}$$

$$L = Z_u - Z_L \tag{15}$$

Case II—Since Z_u and Z_L are both on the same side of the vertex, and originate at the vertex, both represent positive values.

Case I—Since Z_u and Z_L are on opposite sides of the vertex, and L is equal arithmetically to $Z_u + Z_L$, Z_L represents a negative value (algebraic).

Sample problems, case I and case II, are shown in Figures 6 and 7 which explain the procedure used in solving hillside problems. Tension values at supports in case-I problems are found as follows:

Horizontal longitudinal component:
$F = VR$

Tension in cable at upper support:
$T_u = Y_u R$

Tension in cable at lower support:
$T_L = Y_L R$

References

1. Sag Calculations by the Use of Martin Tables (book), James S. Martin. 1931.

2. Mechanical Characteristics of Tran mission Lines (book), L. E. Imlay. 1928.

Auxiliary-Power Supply for Generating Stations

V. E. McCALLUM
MEMBER AIEE

Synopsis: Industry, in general, has been adopting the electric motor drive and its control to all types of machines. In the central station, where sequential and correlated operation is a necessity, the use of electric-motor-driven auxiliaries also has shown a marked increase. All recent stations have been built with a major portion of their auxiliaries driven by electric motors. The author suggests the use of the station generator bus as the most reliable source of auxiliary-power supply in electric stations of a closely interconnected system similar to that in the Chicago area. Separate auxiliary service units, not exceeding in size the requirements of the largest turbogenerator unit, also are recommended.

THE PRESENT ELECTRIC generating station is dependent at all times on the integrity of the auxiliary-power system. Therefore, this system must be designed for maximum reliability. Other desirable characteristics of the auxiliary-power system are: safety of personnel, flexibility, simplicity of operation, ease of maintenance, and reasonable cost.

The auxiliaries of the modern electric station consume approximately five per cent to seven per cent of the gross output. Therefore, when designing an auxiliary-power system, the engineer should bear in mind that he is designing the equivalent of approximately 1/20th of the entire conversion and distribution system fed from the station He also should bear in mind that on the integrity of this 1/20th rests to a large degree the usefulness of the remaining 19/20ths.

An ideal auxiliary-power-supply system, regardless of its size, always should remain a simple system. The tendency to cross-connect and interconnect should be strongly curbed. Dependency should be placed on sturdy, rugged, and well-insulated equipment rather than on complicated switching and relay arrangements designed to remedy faulted conditions.

Trend to Electric Drive

In electric stations on a closely interconnected system similar to that in the Chicago area there has been a steady trend from individual steam drive to electric drive for all station auxiliaries.

The modern induction motor has become the simplest and most reliable of rotating machines, and it is being adopted by the industry as a universal drive. New insulations have been developed to withstand boiler-room temperature with continuous operation. In those applications where variable speed is a necessity, the wound-rotor induction motor is available, but with some sacrifice of reliability and simplicity and somewhat higher first cost.

Early Sources of Auxiliary-Power Supply

Several sources of electric power have been used to provide service for the auxiliary motors. The most common was the turbine connected to an auxiliary generator. Other sources were the shaft alternator on the main turbine and the transformer directly connected to either the generator leads or to the main station busses.

When separate turbogenerators are used for supplying power for the auxiliary-power centers, these units must be installed in duplicate in order to provide complete segregation and reliability. Where the station has a number of power centers, at least one spare turbogenerator is necessary with the connections provided for the relief of all other centers. The cross-connecting of these auxiliary-power units necessitates an excess capacity in each unit, and all the associated switching and relay protection should be of the same quality as that used on the main station busses.

The installation of the auxiliary-power generator on the shaft on the main unit has been supplanted in some cases with a transformer connected to the generator leads. There are some disadvantages of the shaft alternator or transformer on the generator leads, as these units are potential sources of outage of the main unit and they introduce the problem of changing the power source for the auxiliaries in the starting and stopping periods. If the auxiliary-power transformer is disconnected from the generator leads and reconnected to the main bus to which the unit itself is connected, the principle has not been changed, but there is greater flexibility, and there is no need for switching operations during the starting and stopping periods.

Station Bus as the Source of Auxiliary-Power Supply

The main bus of a station to which the generators are connected is the simplest type of electrical equipment, and it lends itself readily to a high order of mechanical and electrical protection. When the busses are divided into sections with each section in a separate bus chamber and equipped with either balanced relays or fault bus protection, it is difficult to conceive any major fault which would shut down more than one or two adjacent sections of a station bus. As each section of the bus is supported by the adjoining sections and as the entire station bus is supported by all generators in the station and also by transmission lines from other stations, during disturbances, these transmission lines become, in effect, auxiliary-power-supply lines for the station in distress.

This supporting function of the transmission lines makes it unnecessary to provide self-starting auxiliary-power generation equipment in more than one or two stations on the entire system.

Where the power system is so constructed that it will sectionalize automatically during trouble, a complete shutdown of any section will not prove disastrous to the system as a whole.

In view of the foregoing and because of the reliability of the station generator bus as proved by years of operating experience, there appears to be ample justification for full electric drive of all station auxiliaries and also for the use of the station generator bus as the source of auxiliary-power supply.

Auxiliary-Power Service Units

It generally is recognized that trouble on the auxiliary-power system may cause temporary outage of the generating units served. Where the auxiliary-power system is so interconnected that it embraces the power supply for the entire station, any trouble on this system also may affect the entire station. It follows, therefore, that by designing the auxiliary-power system in noninterconnected units, it is possible to confine the effects of failure of any power unit to the generating capacity which it serves.

Because on any interconnected system there is a possibility of the loss of any of the larger generating units and the system is so set up as to sustain such loss without undue disturbance, it follows that the size of the auxiliary-power service unit should be such that the generating capacity which it serves should not exceed the capacity of the largest generating unit on the system. Therefore, the generating capacity of the largest unit on the system should be used as the basis of design of a reliable auxiliary-power supply.

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In spite of the inherently high reliability of power lines and transformers, two supply lines are still considered necessary for each auxiliary-power unit. These auxiliary-power supply lines should be equal in reliability and capacity so that either can be considered as a reserve for the other, and each should be capable of carrying the entire auxiliary service unit

This principle is illustrated in Figure 1. Here the station busses are shown in their simplest form consisting of two bus sections connected by a tie circuit breaker. One of the auxiliary-power supply lines is connected to the bus supported by the generating capacity served by the auxiliary-power unit, and the other supply line is connected to a bus supported by a transmission line. It will be noted that each auxiliary-power center bus is fed from two transformers, one on each of the auxiliary-power supply lines. Section circuit breakers in these busses can be used for automatic power restoration in case of loss of either of the supply lines or to divide the busses for maintenance purposes.

To secure the maximum utilization of the equipment employed and to take advantage of its short-time overload capacity, the transformers supplying each bus could be selected to have a definite overload during the outage of the opposite transformer or of the power line which feeds it. When both transformers are in service, the total load of each bus will be divided between the two transformers, allowing a safe loading margin for this type of equipment.

Auxiliary-Power Centers

In designing the auxiliary-power center primary consideration must be given to features having a direct bearing on the reliability, ease of maintenance, and equipment life. Auxiliary-power centers should be located as near the load center as possible, and particular attention should be paid to the protection from external haz- ards such as water, smoke, and so forth. The room should be ventilated with filtered air and all auxiliary power switching equipment, relays, and associated control apparatus should be treated as any other vital station equipment. The switchgear should be of the draw-out type and easily interchangeable. The manufacturers are ready to furnish such equipment completely assembled which can be compactly and easily installed.

Separate auxiliary-power centers serving auxiliaries common to the entire station such as crib house and coal-handling equipment may be connected to the first pair of auxiliary-power lines to be installed to the station. These centers should be housed in the individual rooms with the same precautions taken as with the larger centers.

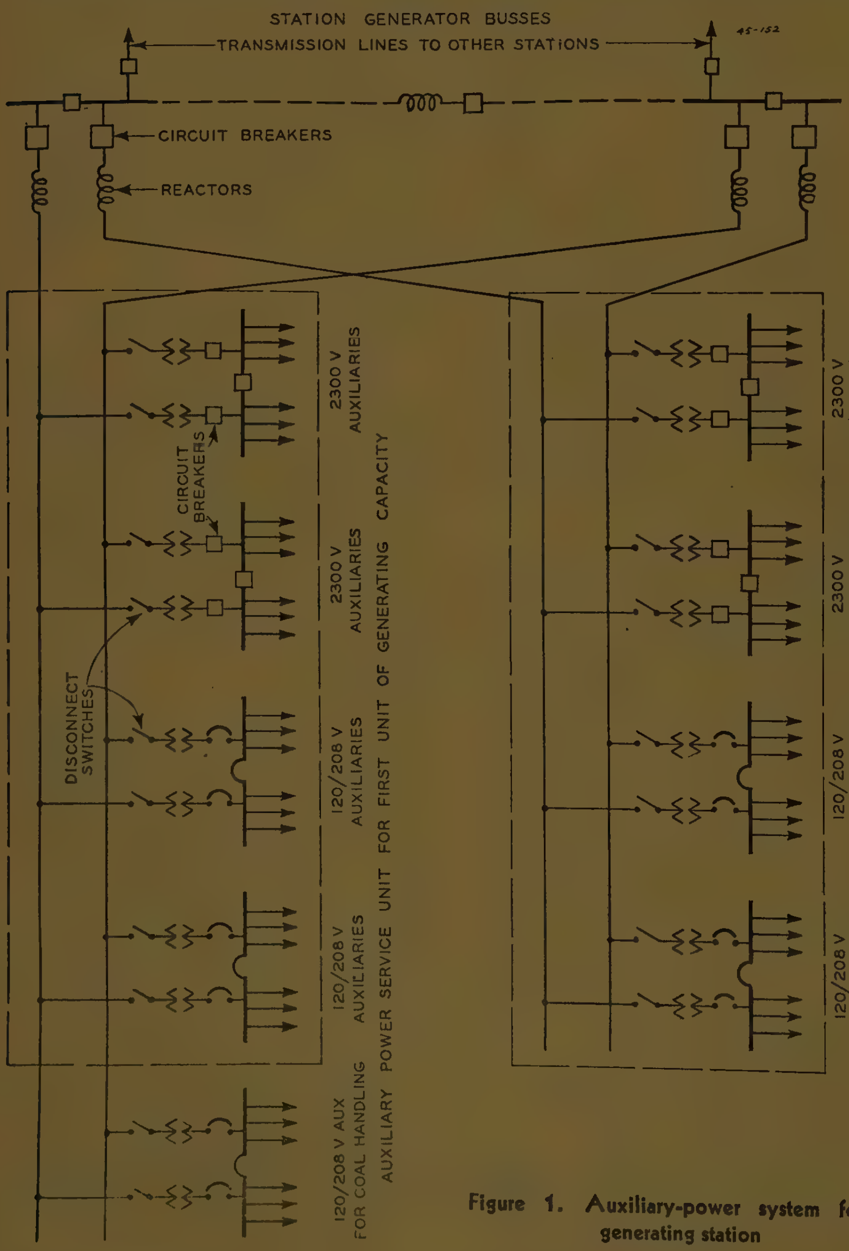

Figure 1. Auxiliary-power system for a generating station

Auxiliary-Power Voltages

In the interest of simplification and the reduction of kinds of equipment and voltages, it appears desirable to limit the number of voltages on two For large motors, that is, above 100 horsepower, the voltage of 2,300 is universally used, and, as the manufacturers are now ready to supply motors at this voltage in sizes as low as 60 horsepower, it would be difficult to justify the change of this voltage to a higher or lower value. Therefore, it is recommended as a standard voltage.

On the other hand, for the smaller motors and lighting requirements the 120–208-volt system has many advantages. Unless there are some other special reasons, this voltage should be used as the second standard voltage.

Conclusions

The essential characteristics of the power supply to the auxiliary motors of a generating station may be summarized as follows:

1. In an electric power station on an interconnected system the station generator busses should be used as the source of auxiliary-power supply.

2. A separate and independent auxiliary-power service unit should be provided to serve the maximum amount of generating capacity, the loss of which could be sustained by the system without undue disturbance.

3. Each auxiliary-power center should be supplied by two lines operated in parallel, one line connected to the bus supported by the generator and one line to the bus supported by a transmission line.

4. There can be as many auxiliary-power center busses connected to the supply lines through transformers as are required by the design of the station and by the limitations of the switching equipment involved.

5. All contactors, circuit breakers, relays, and control apparatus should be installed in separate well-ventilated rooms.

Industrial Control: Heating of Shunt-Motor Starting Resistors

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MEMBER AIEE

THE USUAL METHOD of starting a shunt motor from a constant-potential source is to insert one or more steps of armature resistance which are short-circuited by contactors operated after a definite time. The required capacity of a resistor for this type of duty is directly related to load conditions and time setting of relays as well as duty-cycle requirements. If time settings are made arbitrarily without some co-ordination with the kind of load, the energy dissipated in the resistor may be unduly high, because the load may be up to speed in a time much less than the setting. On the other hand if the load has a high inertia, the definite time may be too short, causing excessively high peaks on switching points after the first one. Although d-c motors have been used extensively in this way, equations governing such operation do not appear to have been published.

In many industrial applications load data have not been well known to the electric-system designer. He has had to depend upon a systematic grouping into a single classification of various loads, based only on initial inrush current and duty cycle. Such a classification in one case includes conveyors, line shafting, boring mills, punch presses, ball mills, and coal crushers. A better approach to starting-resistor design requires more information on load characteristics as well as the effect of these characteristics on starting-resistor capacity.

Scope of Paper

This paper presents an analysis of conditions which exist when a d-c shunt motor with constant applied field is started by inserting one step of armature resistance. Two cases are treated:

(*a*). Constant-load torque.

(*b*). Load torque varying directly as the speed.

It is the purpose of this work to obtain theoretical expressions for the total energy dissipated in a resistor as a function of load inertia, torque, final speed, initial inrush current, and accelerating time. Relations existing between speed, current, and second current peak as a function of time and circuit parameters are examined.

Assumptions

1. The starting resistor is assumed to hold constant resistance independent of temperature.
2. Armature reaction is neglected. This is a fairly good assumption for currents of 200 per cent or less.
3. In this paper all curves are plotted for a motor internal resistance of ten per cent. In practice this internal resistance may vary from 12 per cent at five horsepower to 4.5 per cent at 150 horsepower.

Results

The co-ordinates of all curves shown are in dimensionless form, so that the data are applicable to any size motor or load.

Time Constant. The developed equation of speed for both torque cases is similar to the equation for current build-up in a *RL* circuit, which allows the concept of time constant to be used. For the constant-torque case the term $\Omega J/T_o$ is the time constant of the system, and for the variable-torque case the equivalent term is $\Omega J/(T_o+\Omega k)$. At one time constant the speed is 63.2 per cent of its ultimate speed, and after two-time constants the speed is 86.5 per cent of the final. All results obtained in this paper are presented as a function of the number of time constants rather than time in seconds. In general two time constants has been selected as a good switching time, and most curves are presented for this value.

Speed and Current Curves. The curves in Figures 4a, 4b, 5a, and 5b show the current and angular velocity during the accelerating time for various ratios of load torque to full-load torque. The parameter, *a*, in Figures 4a and 4b is the ratio of load torque to full-load torque and, *g*, in Figures 5a and 5b is the ratio of the load torque at the no-load angular velocity Ω, to the full-load torque.

Resistor Watt-Seconds. The curves in Figure 6 show the factor by which the $J\Omega^2$ of the system can be multiplied to give the watt-seconds generated per start in the starting resistor. They are plotted for the case where the resistor is short-circuited after two time constants. These curves show clearly that the energy dissipated in the resistor is decreased as the initial inrush is increased. For example for a fully loaded motor the watt-seconds decrease in the ratio 1.37 to 0.83 as the initial current is raised from 125 per cent to 200 per cent.

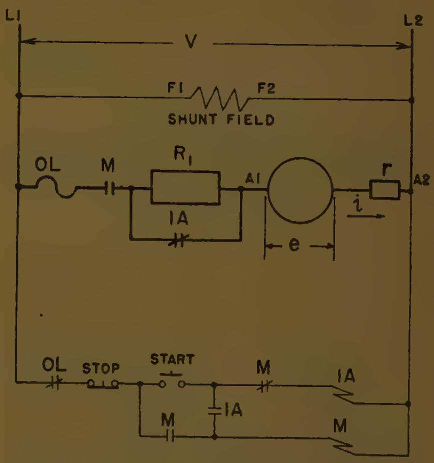

Figure 1. Elementary wiring diagram showing typical definite-time starting circuit

M = line contactor
IA = time accelerating contactor
OL = overload relay
R = total circuit resistance = R_1+r
V = line voltage
e = counterelectromotive force
i = armature current
R_1 = external resistance
r = internal resistance

Figure 7 represents the same data as Figure 6 for the case where the load torque varies directly as the speed.

Although the curves in Figure 7 show that the watt-seconds in the resistor decrease as the load is increased, it should be noted that the switching speed also has decreased because of a decrease in $d=T_o/(T_o+\Omega k)$, and $t_1=2\Omega J/(T_o+\Omega k)$ as Ωk is increased.

Second Current Peak. Figure 8 shows how the second current peak varies in terms of the first current peak for differ-

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6. For simplicity it is recommended that only two voltages be employed for the auxiliaries and that the service to all motors be uniform and have the same method of operation.

7. The entire auxiliary-power system for any unit should be so protected by relays that in case of a fault on any part of the auxiliary service unit the faulted equipment would be automatically disconnected.

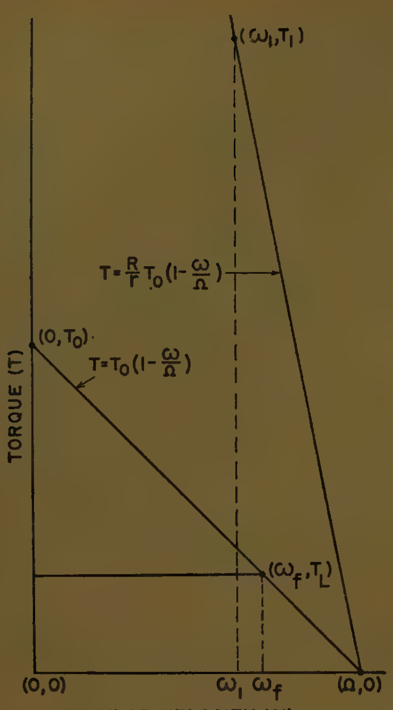

Figure 2. Relations existing between motor torque and angular velocity when the load torque is a constant

ent values of initial torque and load torque. These curves indicate that for a starter with only one section of resistance the motor must be very lightly loaded in order to keep the second peak down to reasonable values. A corollary to this statement is that, if the motor has an appreciable load, it will require more than one resistance step for good commutation.

Switching Time. Figure 9 shows how the watt-seconds generated in the resistor vary with the time allowed for acceleration. These curves indicate that, if a motor has an appreciable load (over 25-per-cent full load), it is important to short-circuit out the resistor as soon as possible consistent with a safe second current peak. For a switching time longer than two time constants Figure 9 shows how the heating severity on the resistor increases with only a slight gain in angular velocity. For example, if $a = T_L/T_{FL} = 0.75$ and $T_o = 2.0\ T_{FL}$ and switching occurs at two time constants, the speed is 86.5 per cent, $\omega_f = 62.5$ per cent Ω, and M is equal to 0.7. If switching occurs at three time constants, the speed is 95 per cent ω_f, and $M = 0.87$. This shows an increase of 24.3 per cent in watt-seconds in the resistor for a gain of only 9.8 per cent in speed. If the load is full load, then $a = 1.0$, and there is an increase of 31 per cent in watt-seconds for the same 9.8 per cent gain in speed.

Circuit Diagram

Figure 1 shows a typical definite time-starter connection diagram. When the start button is pressed the coil of the accelerating contactor 1*A* is energized, thus opening its normally closed power contact and closing its normally open control contact. When the control contact on 1*A* closes, it energizes the line contactor coil *M* which seals itself in through its own interlock. The closing of contactor *M* connects the motor armature to the line through the resistor R_1 and allows the initial current I_o to flow. Simultaneously, the coil of 1*A* is de-energized, so that after a definite time 1*A* drops out short-circuiting the resistor R_1 and allowing the second current peak I_1 to flow.

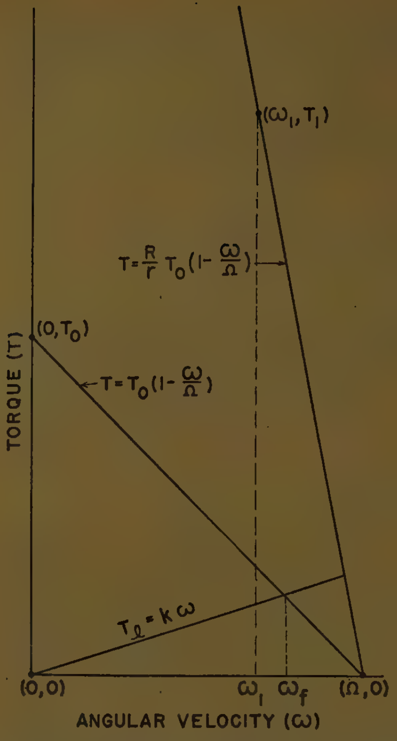

Figure 3. Relations existing between motor torque and angular velocity when the load torque varies directly as the speed

Speed–Torque Diagram

Figures 2 and 3 show relations between torque and angular velocity for the constant- and variable-torque case respectively. The developed motor torque is given by equation 8 and is shown as a straight line going through the points (*O*, T_o) and (Ω, *O*). The point of intersection of this line, with the load torque line determines the steady-state operating speed (ω_f) when resistance is in the armature circuit. The point ω_1 represents the speed at switching. The torque after switching rises to T_1 which is on the line given by equation 8a for a motor with internal resistance of *r*.

Analysis

For nomenclature used in this analysis see the appendix.

In a d-c motor the well-known equations for torque and back electromotive force are

$$T=\frac{c\phi i}{2\pi 10^8} \tag{1}$$

and

$$e=\frac{c\phi\omega}{2\pi 10^8} \tag{2}$$

Since $V = e + iR$, then

$$i=\frac{V-e}{R}=\frac{V}{R}-\frac{c\phi\omega}{R2\pi 10^8} \tag{3}$$

Putting equation 3 into equation 1 gives

$$\begin{aligned}T&=\frac{c\phi V}{2\pi 10^8 R}-\left[\frac{c\phi}{2\pi 10^8}\right]^2\frac{\omega}{R}\\&=\frac{c\phi}{2\pi 10^8}\frac{V}{R}\left[1-\frac{c\phi\omega}{2\pi 10^8 V}\right]\end{aligned} \tag{4}$$

With no load on the machine *e* becomes *V*, and $\omega = \Omega$, so that equation 2 resolves to

$$V=\frac{c\phi\Omega}{2\pi 10^8} \text{ and}$$

$$\Omega=\frac{2\pi 10^8 V}{c\phi} \tag{5}$$

Substituting equation 5 into equation 4 gives

$$T=\frac{c\phi}{2\pi 10^8}\frac{V}{R}\left[1-\frac{\omega}{\Omega}\right] \tag{6}$$

At $t = o$, $\omega = o$, and

$$T=T_o=\frac{c\phi}{2\pi 10^8}\frac{V}{R}=\frac{c\phi}{2\pi 10^8}I_o \tag{7}$$

Then equation 6 may be written

$$T=T_o\left(1-\frac{\omega}{\Omega}\right) \tag{8}$$

The torque equation for internal resistance *r* is

$$T=T_o\frac{R}{r}\left(1-\frac{\omega}{\Omega}\right) \tag{8a}$$

as initial torque is inversely proportional to circuit resistance. Dividing equation 7 by equation 5 gives the useful relationship,

$$\frac{T_o}{V}=\frac{V}{R\Omega}=\frac{I_o}{\Omega} \tag{9}$$

so that equating 6 and equation 1 gives the following equation for current

$$i=\frac{V}{R\Omega}(\Omega-\omega)=\frac{T_o}{V}(\Omega-\omega)=I_o\left(1-\frac{\omega}{\Omega}\right) \tag{10}$$

CASE 1: CONSTANT-TORQUE LOAD

The graphical representation for this case is shown in Figure 2.

The difference between generated motor torque, *T*, and load torque, T_L, causes acceleration, so that

$$T-T_L=\frac{Jd\omega}{dt} \tag{11}$$

Substituting for T from equation 8 gives

$$\frac{Jd\omega}{dt}+T_o\frac{\omega}{\Omega}=T_o-T_L \qquad (12)$$

whose solution is

$$\omega=\left(\frac{T_o-T_L}{T_o}\right)\Omega+A_e-\frac{T_ot}{\Omega J} \qquad (13)$$

At $t = 0, \omega = 0$, so that

$$A = -\frac{(T_o - T_L)}{T_o}\Omega$$

and equation 13 becomes

$$\omega=\left(\frac{T_o-T_L}{T_o}\right)\Omega\left[1-e^{-\frac{T_ot}{\Omega J}}\right] \qquad (14)$$

At $t = \infty$,

$$\omega=\omega_f=\left(\frac{T_o-T_L}{T_o}\right)\Omega \qquad (15)$$

so that

$$\omega=\omega_f\left[1-e^{-\frac{T_ot}{\Omega J}}\right] \qquad (16)$$

Putting the expression for ω from equation 14 into equation 10 yields

$$i=I_o\left[\frac{T_L}{T_o}+\left(\frac{T_o-T_L}{T_o}\right)e^{-\frac{T_ot}{\Omega J}}\right] \qquad (17)$$

Let

$$\left(\frac{T_o-T_L}{T_o}\right)=b\text{, so that }\frac{T_L}{T_o}=1-b$$

and the equation for current becomes

$$i=I_o\left[(1-b)+be^{-\frac{T_ot}{\Omega J}}\right] \qquad (18)$$

The energy dissipated in the resistor in time t_1 is

$$E=\int_0^{t_1}i^2R_1dt=I_o^2R_1\times$$
$$\int_0^{t_1}\left(1-b+be^{-\frac{T_ot}{\Omega J}}\right)^2dt \qquad (19)$$

$$=I_o^2R_1\left[(1-b)^2t_1+2b(1-b)\frac{\Omega J}{T_o}\times\right.$$
$$\left.\left(1-e^{-\frac{T_ot_1}{\Omega J}}\right)+\frac{b^2\Omega J}{2T_o}\times\left(1-e^{-\frac{2T_ot_1}{\Omega J}}\right)\right] \qquad (20)$$

Let $t_1 = \frac{2\Omega J}{T_o}$ so that equation 20 reduces to

$$E=\frac{I_o^2R_1\Omega J}{T_o}[2-2.27b+0.761b^2] \qquad (21)$$

When the value for I_o from equation 9 is put in, equation 21 reduces to

$$E=J\Omega^2\left[\frac{R_1}{R}\right][2-2.27b+0.761b^2] \qquad (22a)$$

or

$$E=J\Omega^2\left[\frac{R_1}{R}\right]\left[0.491+0.748\frac{T_L}{T_o}+\right.$$
$$\left.0.761\left(\frac{T_L}{T_o}\right)^2\right] \qquad (22b)$$

At $t_1=\frac{2\Omega J}{T_o}$ equation 14 becomes

$$\omega_1=\omega_f(1-e^{-2})=0.865\omega_f \qquad (23)$$

Second Current Peak, I_1

At $t = t_1$ the timing relay 1A (Figure 1) drops out short-circuiting out the external resistance R_1, so that the resistance of the circuit changes from R to r.

Just before switching

$$i=I_o\left(1-\frac{\omega_1}{\Omega}\right)\text{ and }e_1=\frac{c\phi\omega_1}{2\pi10^8}$$

Now

$$\frac{e_1}{V}=\frac{\omega_1}{\Omega}=\frac{0.865\omega_f}{\Omega} \qquad (24)$$

The new current peak is given by

$$I_1=\frac{V-e_1}{r}=\frac{V}{r}\left[1-\frac{0.865\omega_f}{\Omega}\right] \qquad (25)$$

The same relationship can be obtained by considering similar triangles in Figure 2. Equation 25 also can be written using the relationship of equation 15.

$$I_1=\frac{V}{r}\left[1-0.865\;\frac{T_o-T_L}{T_o}\right] \qquad (26a)$$

$$=\frac{V}{r}\left[0.135+0.865\frac{T_L}{T_o}\right] \qquad (26b)$$

$$=\frac{V}{r}\left[0.135+0.865\frac{I_L}{I_o}\right] \qquad (26c)$$

since $\frac{T_L}{T_o}=\frac{I_L}{I_o}$ from equation 1.

If switching takes place at $t = \infty$ so that

$$i=I_o\left(1-\frac{\omega_f}{\Omega}\right)$$

then from equation 15 and equation 1

$$I_1=\frac{V}{r}\left[1-\left(1-\frac{I_L}{I_o}\right)\right]=\frac{V}{r}\frac{I_L}{I_o} \qquad (27)$$

CASE 2: LOAD TORQUE DIRECTLY PROPORTIONAL TO SPEED

Let

$$T_l=k\omega \qquad (28)$$

and

$$T_\Omega=k\Omega$$

As before

$$T=T_o\left(1-\frac{\omega}{\Omega}\right)$$

These relationships are shown in Figure 3. Then

$$T-T_l=J\frac{d\omega}{dt}$$

so that

$$J\frac{d\omega}{dt}+\omega\left(\frac{T_o+k\Omega}{\Omega}\right)=T_o \qquad (29)$$

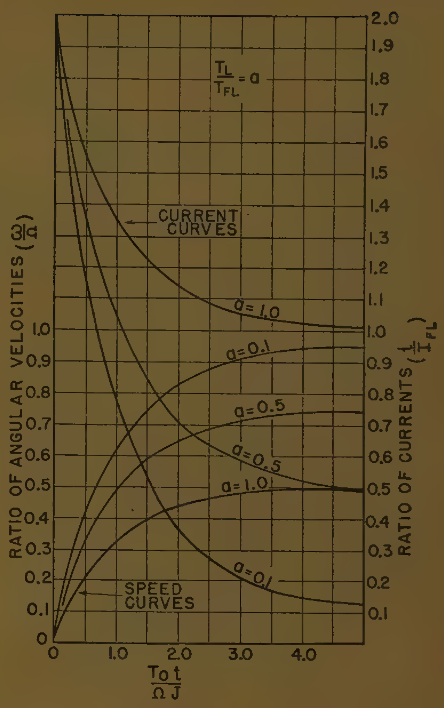

Figure 4a. Speed and current versus time for 200-per-cent initial torque and various ratios of a constant-load torque to full-load torque

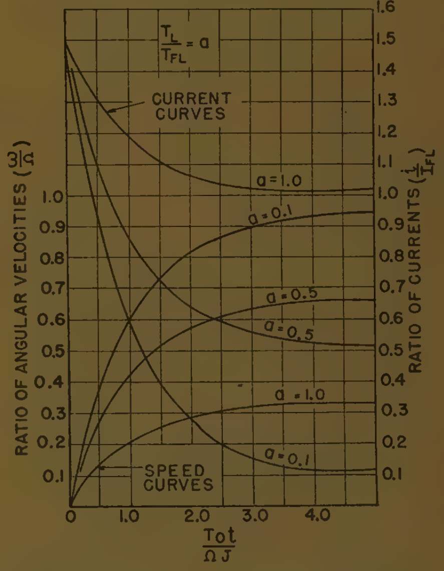

Figure 4b. Speed and current versus time for 150-per-cent initial torque and various ratios of a constant-load torque to full-load torque

and

$$\omega=\frac{T_o\Omega}{T_o+\Omega k}+Ce^{-\left(\frac{T_o+\Omega k}{\Omega J}\right)t}$$

At $t = 0$, $\omega = 0$, so that C is determined and

$$\omega=\frac{T_o\Omega}{T_o+\Omega k}\left[1-e^{-\left(\frac{T_o+\Omega k}{\Omega J}\right)t}\right] \qquad (30)$$

At

$$T=\infty,\quad\omega=\omega_f=\frac{T_o\Omega}{T_o+\Omega k} \qquad (31)$$

Let

$$B=\frac{\Omega J}{T_o+\Omega k} \tag{32}$$

so that equation 30 can be written

$$\omega=\omega_f\left(1-e^{-\frac{t}{B}}\right) \tag{33}$$

Now as in equation 10

$$i=I_o\left(1-\frac{\omega}{\Omega}\right)$$

which becomes from equation 33

$$i=\frac{I_o}{\Omega}\left[\Omega-\omega_f\left(1-e^{-\frac{t}{B}}\right)\right] \tag{34}$$

The watt-seconds generated in the resistor R_1 in the time t_1 is

$$E=\int_0^{t_1} i^2R_1dt=\frac{I_o{}^2R_1}{\Omega_2}\times \int_0^{t_1}\left[\Omega-\omega_f\left(1-e^{-\frac{t}{B}}\right)\right]^2dt \tag{35}$$

which becomes

$$E=\frac{I_o{}^2R_1}{\Omega^2}\left[(\Omega-\omega_f)^2t_1+B(2\omega_f)(\Omega-\omega_f)\times \left(1-e^{-\frac{t_1}{B}}\right)+\frac{\omega_f{}^2}{2}\left(1-e^{-\frac{2t_1}{B}}\right)\right] \tag{36}$$

Let

$$t_1=\frac{2\Omega J}{T_o+\Omega k}=2B \tag{37}$$

so that

$$\omega_1=\omega_f(1-e^{-2})=0.865\omega_f$$

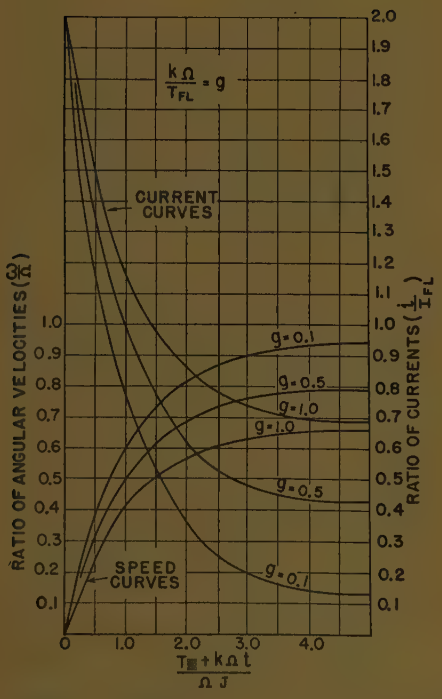

Figure 5a. Speed and current versus time for 200-per-cent initial torque and various ratios of load torque to full-load torque when the load torque varies directly as the speed

Let

$$d=\frac{\omega_f}{\Omega} \tag{38}$$

so that from equation 31

$$\frac{\omega_f}{\Omega}=d=\frac{T_o}{T_o+\Omega k} \tag{39}$$

Then from equation 37

$$B=\frac{\Omega J}{T_o+\Omega k}=\frac{d\Omega J}{T_o} \tag{40}$$

Substituting equation 38 and equation 40 into equation 36 gives

$$E=I_o{}^2R_1[2(1-d)^2+2d(1-d)(0.865)+0.491d^2]\frac{d\Omega J}{T_o}$$

which, using equation 9, reduces to

$$E=J\Omega^2\frac{R_1}{R}[2-2.27d+0.761d^2]d \tag{41}$$

Example 1

A centrifugal casting machine used for spinning molten metal to desired shapes is driven by a five-horsepower 230-volt 1,200-rpm d-c shunt motor with a full-load current of 20 amperes. The inertia referred to the motor shaft is 20 pounds feet squared. The load during acceleration is practically all friction and is ten per cent of rated full-load torque. A two-point starter with one section of starting resistance is to be used. In order to make the accelerating time short 200-per-cent starting torque is indicated.

The accelerating time setting for the timing relay and the watt-seconds lost per start in the starting resistor are required.

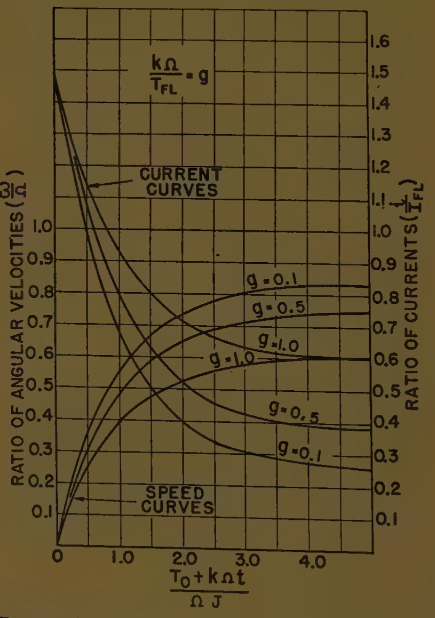

Figure 5b. Speed and current versus time for 150-per-cent initial torque and various ratios of load torque to full-load torque when the load torque varies directly with the speed

The rated full-load torque is 5(5,250) ÷ 1,200 = 21.9 foot-pounds, so that the initial torque and load torque are 43.8 and 2.19 foot-pounds respectively. Since the rated motor speed is 1,200 rpm, and the speed regulation or internal resistance drop is assumed to be ten per cent, the no-load speed is 1,333 rpm. From Figure 4a where T_L = 0.1 T_{FL} the switching speed is 0.82 × 1,333 or 1,093 rpm, and the steady-state speed, ω_f is 1,266 rpm. The time constant $\Omega J/T_o$, in the practical system, is given by the relation (WR^2N) ÷ $(307.8T_o)$ where WR^2 is in pounds foot squared, N is no-load revolutions per minute, and T_o is initial torque in foot-pounds. Inserting the values WR^2, N, and T_o given in this example gives the time constant of the system as 2.0 seconds. In a time of 1, 2, 3, and 4 time constants the speed will be 63.2, 86.5, 95, and 98 per cent of ω_f and the watt-seconds lost in the resistor will be 5,920, 6,950, 7,160, and 7,220.

If switching occurs at two time constants, the second current peak is given by equation 25 or can be obtained graphically from a construction similar to Figure 2. This current peak equals

$$I_1=\frac{V}{r}\left[1-0.865\frac{\omega_f}{\Omega}\right]$$

$$=\frac{230}{1.15}\left[1-0.865\left(\frac{1,266}{1,333}\right)\right]=36.0 \text{ amperes}$$

This inrush is allowable, being less than 200-per-cent full-load current, so that

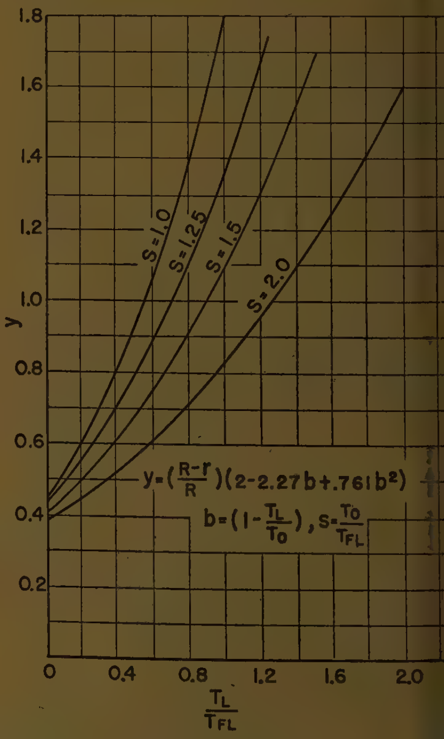

Figure 6. Multiplying factors for $J\Omega^2$ to obtain watt-seconds dissipated in resistor per start for various ratios of initial torque to full-load torque when the load torque is constant and switching occurs at a time equal to two time constants

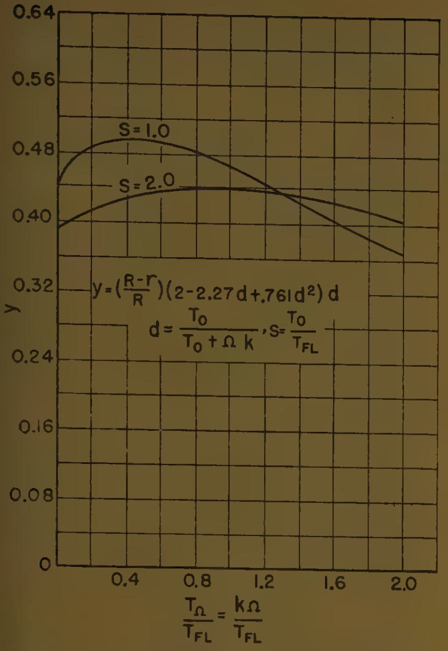

Figure 7. Multiplying factor for $J\Omega^2$ to obtain watt-seconds dissipated in resistor per start for various ratios of initial torque to full-load torque, when the load torque varies directly with the speed and switching occurs at a time equal to two time constants

switching at two time constants or four seconds is satisfactory.

Example 2

A five-horsepower planer motor rated 500 rpm, 230 volts with a full-load current of 20 amperes drives a friction load of 25-per-cent full-load torque. If a two-point starter is used, the switching time and watt-seconds in the starting resistor are to be determined.

The WR^2 of the system referred to the motor shaft is ten pounds feet squared. The full-load torque is 52.5 foot-pounds. If 200-per-cent starting torque is allowed T_o = 105 pounds. The no-load speed is 556 rpm. Then the time constant becomes $\frac{10\ (556)}{307.8\ (105)}=0.172$ second.

The $J\Omega^2$ of the system is

$$\frac{4.62(WR^2)(N^2)}{10^4}=\frac{4.62(10)(556)^2}{10^4}$$
$$=1{,}430 \text{ watt-seconds}$$

At two time constants the second current peak from equation 26 is

$$I_1=\frac{230}{1.15}\left[0.135+0.865\left(\frac{0.25}{2}\right)\right]$$
$$=48.6 \text{ amperes}$$

The multiplying factor for $J\Omega^2$ obtained from Figure 6 is 0.475. Then the watt-seconds become 0.475 × 1,430 = 685 watt-seconds.

It should be noted that this value is approximately ten per cent of the value found for the motor of example one, which has the same applied voltage, horsepower, internal resistance, and inrush current.

The watt-seconds per start are not much different for two, three, or four time constant because of the low-load torque. If a high-load torque is present, a two-point starter is not satisfactory because of the high resultant second peak.

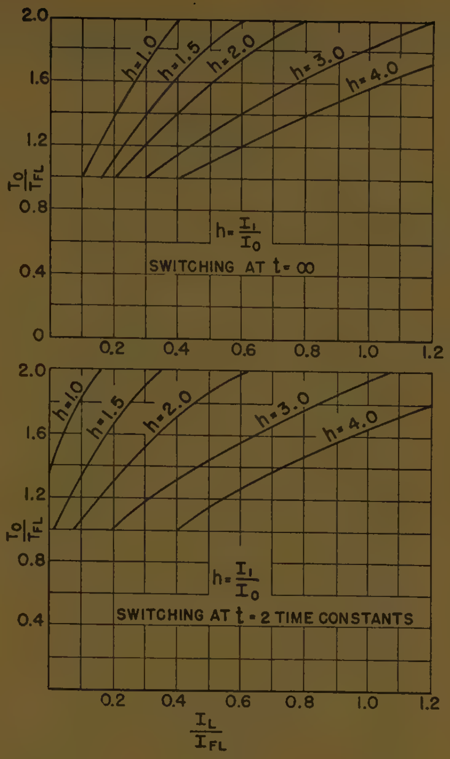

Figure 8. Curves showing the relation between first and second current peaks as a function of load torque, initial torque, and switching time

Appendix. Nomenclature and Units

e = counter electromotive force at any instant
e_1 = value of e at time t_1
V = line voltage
i = armature current at any instant
I_o = armature current at $t=0$
I_1 = armature current after switching time $t=t_1$
I_L = current corresponding to torque T_L
R = total resistance in armature circuit
R_1 = external (starting) resistance
r = internal resistance of motor
T = torque developed by motor in watt-seconds
T_o = value of T at $t=0$
T_L = constant load torque in watt-seconds
T_l = variable load torque at any speed in watt-seconds
T_Ω = value of T_l at speed of Ω
KE = kinetic energy of rotating system in centimeter-gram-second units
J = polar moment of inertia of rotating system in centimeter-gram-second units
WR^2 = inertia in pounds feet squared
E = energy in watt-seconds
ω = angular velocity at any instant in radians per second
ω_1 = value of ω at $t=t_1$
ω_f = value of ω at $t=\infty$
Ω = no-load angular velocity in radians per second
N = speed in revolutions per minute
α = angular acceleration at any instant in radians per second squared
t = time in seconds
t_1 = value at t at $\omega=\omega_1$
ϕ = useful flux per pole crossing the air gap
k = constant
A, C = constants of integration
$b=(T_o-T_L)/T_o$
$d=T_o/(T_o+\Omega k)=\omega_f/\Omega$

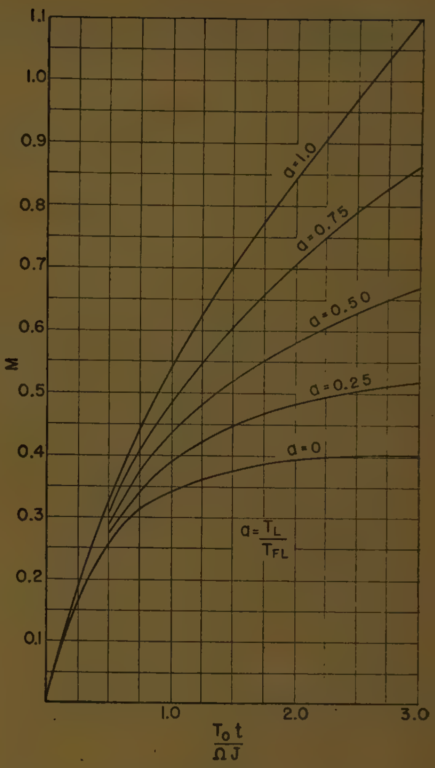

Figure 9. Multiplying factor for $J\Omega^2$ to obtain watt-seconds dissipated in resistor per start as a function of switching time for various ratios of load torque to full-load torque for a constant-torque load

Plotted for 200-per-cent initial torque

In order to change from metric units to the practical system the following factors can be used.

$$\frac{WR^2}{32.2}\times 1.358 = J \text{ watt-second}^2$$

$$4.62\,WR^2N^2 10^{-4} = J\Omega^2 \text{ watt-seconds}$$

$$\frac{2\pi N}{60}=\Omega \text{ radians per second}$$

Statistical Methods in the Development of Apparatus Life Quality

ENOCH B. FERRELL
NONMEMBER AIEE

Synopsis: Life tests on a new design of relay were so planned that the methods of statistical quality control could be used to separate assignable causes for variation from random causes for variation during life.

The relays were inspected frequently at the beginning of the test and less frequently later. The results of these inspections were plotted in the form of control charts. From a study of points outside control limits, improvements were made in the design, the method of manufacture, and the method of test. From a study of the control limits and the number of points within them, predictions were made as to the useful life to be expected.

Simplified methods used in recording and analyzing the data and in establishing trends are described.

DEVELOPMENT PEOPLE, manufacturers, even consumers, make a great many life tests. A common way of making these life tests is something like this: Go to the end of the assembly line, get a number of whatever kind of gadget is being made, put them in a life-test station, and run them to destruction. Results: A nice table of numbers and a lot of dead gadgets. Nothing more.

Now, what is the purpose of a life test? The consumer wants to know: How often will I have to replace this piece of equipment? The manufacturer or his development man wants to know: How long a life can we guarantee to our customer?

The very nature of these questions shows that they are not at all interested in how long a particular gadget that was made last month did run before failure. They are interested only in predictions about how long many gadgets that will be made next month will run before failure. To restate the questions to be answered: First, can reliable predictions be made about the life of the gadget? Second, if predictions can be made, what are the expected average and dispersion of the life, or, if reliable predictions cannot be made, why not, and what can be done about it?

There is another question: Can a design change be made that will increase the life or reduce the cost? That question was given little consideration in setting up the tests to be described, because we had a design that we intended to produce in the existing form to meet certain uses. We wanted to know whether this same design might be useful elsewhere. We did, of course, learn things from the investigations that led us to both design improvement and process improvement. If they had been our main objectives, we would have made little, if any, change in our method of study. But our original objective was simply an evaluation of a design.

Consider the first question, reliability. One of the fundamental requirements for reliable predictions of this type is that they be based on measurements of representative samples from a uniform product. In the tests to be described here the product to be studied was a new type of relay which was in the midst of development and preparation for manufacture. We had made a good many small lots. We had studied the more important characteristics of these by the control-chart technique.[1,2] We had some confidence in a process standard that had been based on our control limits. We were getting a good many outages in our manufacturing process. That is, we were getting a good many points outside our control limits. But we were identifying and eliminating the troubles. Another way to say this is that we were making a good many relays that were not what we intended, but we knew what we wanted to make, and we knew a process that would do it.

For the life test we took samples from those lots which were in statistical control. We believed our samples were representative of the stable manufacturing process that would exist after we had eliminated the troubles. And we knew what the characteristics of the relays made by that process would be, knew that they would be useful to our customers.

We believed that, if we were to put much faith in an ultimate life figure, it must be arrived at by a more or less continuous and understandable process. That is, we were not satisfied merely to let these relays die; we wanted to know how they aged and what was the nature of their final failure. We arranged to operate and release each relay in a work circuit 60 times per second in an effort to wear it out. At stated intervals we would remove it from this work circuit and inspect it. This inspection would consist of accurate measurements of its two principal characteristics: the current at which it would operate or pull up, and the current at which it would release or drop out. We would plot a control chart for each relay. We wanted several points on each control chart before its relay failed.

We arranged to make about 20 inspections during the first ten days. If the relays failed in less than ten days and we had not enough data, we could start over without too much loss. But to continue two inspections per day was too expensive. Moreover, it would give us more data than we had time to study. So, after an initial period, we would inspect the relays whenever their age had increased about ten per cent. Table I shows the inspection schedule. It will be noticed that, if we plot control charts using subgroups of two or four, the time intervals within a subgroup show some uniformity. Beyond the schedule shown, the ten-per-cent rule was applied. Throughout there was some adjustment to fit convenient working hours.

In measuring the operate and release current at the inspections, we have used a measuring set that minimizes the influence of the operator or the surroundings. The relay is connected to the measuring set, and the operating switch is thrown. The measuring set automatically increases the current in the relay slowly until it operates and then displays the value of the operate current on a meter for several seconds. The operator merely throws the switch, reads the meter, and writes down the reading. The meter has a six-inch scale and is read to one or two parts in 1,500.

Table II shows the method of recording the data. This is one half of a double page. Release current is recorded on the other half. There is a separate double page for each relay. At each inspection four observations of operate current are made and recorded on one line. After them are recorded the range and midrange of these four repetitive observations. They may be called repetition range and repetition midrange. When two inspections have been made, the two

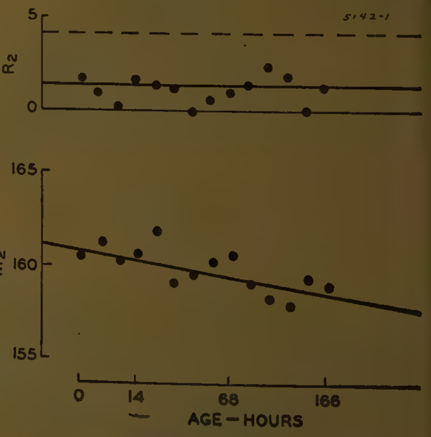

Figure 1. Operate current: relay 740

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repetition midranges are treated as new data, and the range and midrange are computed for the group of two.

In this work and in most of our work we have used ranges and midranges in preference to standard deviations and averages. The ranges and midranges are easier to compute, and the method of computing them is exactly the same in all details, whether the sample size is two or six. This simplifies the instructions to the technical assistants and speeds up their work. As these statistics depend on only the extreme observations, they may be a little more useful in detecting trouble. Control chart coefficients are published for ranges. For a sample size of two, the midrange and average are, of course, the same. For small samples, in general, it may be that the midranges and the averages, with their statistical fluctuations, are enough alike to permit their interchange in an analysis of this type. We have assumed this to be so, and up to sample sizes of six, we have used the midrange with the published control-chart coefficients for the average.

Repetition ranges represent variations in the relay, and also variations due to meter reading, building vibration, and the like. We would expect this dispersion to be constant, both within and among the relays. The average repetition range and control limits based on it were computed by lumping together several inspections on several relays. They are shown at the bottom of Table II. The repetition ranges here are badly out of control. This was at the beginning of the tests. We convinced ourselves that the trouble was due to operator inexperience and was not seriously affecting the experiment. The repetition ranges later did come into fairly good control.

The group ranges represent the aforementioned variations plus the variations caused by handling the relays, drifts in the measuring set and the like, plus any variation due to real aging in the relay. They show, on the whole, very good control. They indicate that the effect of aging between inspections is small as compared with what we might call the effect of handling.

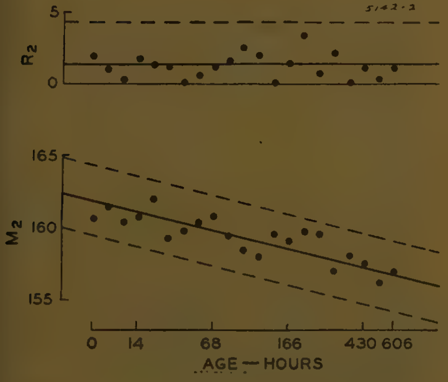

Figure 2. Operate current: relay 740

We do not expect the repetition midranges to stay within limits based on repetition ranges, because an additional cause for variation, the handling, is interposed between inspections.

We might have expected the group midranges to be in control within limits based on group ranges. They were not. They showed definite trends. Here was a real indication of aging. This aging trend was evaluated in the following manner.

After some 14 group midranges had been plotted for a relay, a median line was drawn as in Figure 1. It was a sloping median line. It was placed at such a height that half the points were above it and half below. It was placed at such a slope that the number of runs, above and below, was a maximum.

The use of this line and the method of placing it may need some explanation. If we believed that there existed some functional relation between operate current and age, we could compute the current for each age on the basis of that functional relation, subtract the computed current from the observed current, and get a set of remainders. These remainders would represent the departure of our experience from the assumed functional relation. We could study these remainders statistically. If we found, in the remainders, evidence of cycles or trends, then we would say there are functional relations other than that already considered. If we found what the statistician calls "no evidence of lack of control," then we would think there is no functional relation between the remainders and the age. Our confidence in this belief would depend on the thoroughness of our search for lack of control.

We have done just this, but have done it graphically. We have used, in effect, a graphical transformation of co-ordinates. The functional relation that we have used is defined by the time scale, the position, the slope, and the straightness of the line. Because we have an unusual time scale, we are testing an unusual functional relation. There may be a better or more logical functional relation. The best of statistical control in the remainders would not prove that this functional relation is "the right one" but merely that empirically it is a good approximation in the region studied.

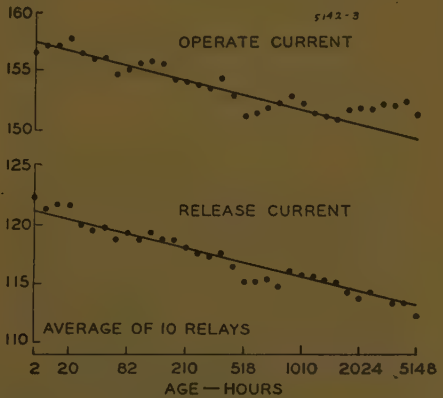

Figure 3. Operate and release currents: average of ten relays

Table I. Life-Test Inspection Schedule

Number of Inspection	Age in Hours	Added Age	Number of Days	Number of Groups (2)	Number of Groups (4)
1*	0				
2**	0	0		1	
3	1	1			
4	2	1	1	2	1
5	5	3			
6	8	3	2	3	
7	11	3			
8	14	3	3	4	2
9	17	3			
10	20	3	4	5	
11	23	3			
12	26	3	5	6	3
13	33	7			
14	40	7	6	7	
15	47	7			
16	54	7	7	8	4
17	61	7			
18	68	7	8	9	
19	75	7			
20	82	7	9	10	5
21	89	7			
22	103	14	10	11	
23	110	7			
24	124	14	11	12	6
25	131	7			
26	145	14	12	13	
27	152	7			
28	166	14	13	14	7
29	188	22	14		
30	210	22	15	15	
31	232	22	16		
32	254	22	17	16	8
33	276	22	18		
34	298	22	19	17	
35	320	22	20		
36	342	22	21	18	9

*At completion of manufacture.
**At start of life test.

In our case the time scale and straightness of the line were purely matters of convenience. They fixed the form of a two-parameter function whose two parameters, the height and slope of the line, could be easily chosen. The choice of height rests on our desire to have the central value of the remainders after transformation equal to zero. It is somewhat easier to use the median as the measure of central value than some of the others. So, we placed the line with half the points above and half below.

We have chosen the slope of the line in an effort to make one of the simplest tests for control show no assignable causes. This test is the study of runs[3] above and below the median. A run is a sequence of consecutive points on the same side of the line. The expected number of runs is roughly half the number of points, actually $(n+2)/2$. This makes the expected average length of run about two. The presence of longer and fewer runs tends to indicate the presence of trends. We would like to include any such trends in our functional relation, and that would eliminate those long runs. The presence of shorter and more numerous runs, which

Table II. Life-Test Data

NI_{op} Switch 740

Date	Age	Observations				Repetition		Group	
						R_4	M_4	R_2	M_2
713	0	159.8	160.1	159.9	160.1	0.3	160.0	1.9	160.9
806	0	160.3	160.2	163.8	160.0	3.8	161.9		
	1	162.0	161.9	162.0	162.0	0.1	162.0	1.0	161.5
807	2	161.9	160.2	161.8	160.0	1.9	161.0		
	5	160.1	160.0	160.8	160.1	0.8	160.4	0.2	160.3
	8	160.4	159.8	160.5	159.9	0.7	160.2		
809	11	160.2	159.7	160.2	159.8	0.5	160.0	1.9	160.9
	14	162.0	161.7	162.0	161.9	0.3	161.9		
810	17	161.9	161.0	161.9	161.5	0.9	161.5	1.2	162.1
	20	162.5	162.4	163.1	162.4	0.7	162.7		
811	23	159.9	160.0	159.9	159.8	0.2	159.8	1.1	159.2
	26	158.3	159.0	158.8	159.0	0.7	158.7		
812	33	158.0	160.0	160.5	160.0	1.5	159.8	0.0	159.8
813	40	159.8	159.9	159.8	159.9	0.1	159.8		
	47	160.0	160.0	160.0	160.0	0.0	160.0	0.6	160.3
814	54	161.0	160.2	161.0	160.2	0.8	160.6		
816	61	160.0	160.2	160.5	160.5	0.5	160.3	1.1	160.8
	68	161.0	161.8	161.5	161.5	0.8	161.4		
817	75	160.5	159.8	161.0	159.9	1.2	160.4	1.7	159.6
818	82	158.5	158.5	159.0	158.5	0.5	158.7		
		Process standard			Average	0.24		1.33	
					Limits	0 0.55		0 4.35	

means many runs of length one, indicates a cyclic variation with a period of two. That would not affect our choice of slope to represent a long trend. So, we have used that slope which gave the greatest number of runs.

These ideas explain, whether or not they justify, our method of drawing this line, which because of its characteristics we call a median line. Because this is an extension of the original concept of the median of a lumped or unordered distribution to the concept of a median of an ordered array, we have qualified it and called it a sloping median. Perhaps trend median would be a better name.

These sloping median lines were drawn for eight relays with encouraging results. The number of runs was fairly close to the expected number. The slopes were in good agreement. To show this we measured and averaged these slopes and redrew the plotted medians, so that all had this average slope. Since the manufacturing variations among relays is greater than any of the variations discussed here, each line was drawn at a height to make it the median for that one relay.

To study further the singleness of this assignable cause for variation, control limits were placed above and below these median lines at spacings derived in the usual manner from the average group range. Figure 2 shows the complete control chart for group range and midrange.

Note that this chart goes up to 600 hours age. The determination of slopes and control limits was made on eight relays at an age of 166 hours. The control chart is holding good through an extrapolation by a factor of nearly four in age.

Figure 3 shows the aging trend up to 5,000 hours. The upper curve is for operate current, the lower for release current. Each point is the average for ten relays. It represents four repetitive readings, at each of two inspections of each of ten relays—a total of 80 observations, averaged in each point. The slope of the median line for operate current is the same as that determined at an age of 166 hours. The last few points are significantly above the line. It may be that the whole aging trend is caused by change in a repeatedly overstressed spring and that this flattening of the curve shows approach to a new equilibrium. The continued slope of the release current seems to disagree with this. We do not know.

Table III. Summary of Outages

Number of Relays	Identification	Action
All	General trend	New spring
9	Initial	Preage product
10	Measuring set	Repair set
14	Spring stabilization	New spring
6	Temperature	Redesign measuring set
7	Faulty test	Start over
	Faulty manufacture	Modify process
3	Heavy load	Study load circuits
10*	Unexplained	No action

*Number of individual points outside control limits

The median line for release current as drawn at 166 hours was steeper. Only 14 points were available then, so it may have been a "small sample" effect. It is more likely to be related to erratic behavior sometimes encountered in the first 20 hours. This will be considered later. The slope of the release line shown in Figure 3 is the same as that of the operate line.

Control limits have not been drawn on the ten relay average curves of Figure 3. If they were, they would be spaced less than a third as far apart as in Figure 2, and we would observe many outages. We think the explanation is this: In addition to the dispersion due to what we called "handling," there is an effect of temperature on the measuring set. This is less than the "handling" effect, and, in single relays, is concealed by it. When we average ten relays, the dispersion due to handling is reduced. But, because the relays were started into their tests in a few groups and measured together later as a group on a cold day or a hot day, averaging does not reduce the temperature effect. This now shows up to give outages on limits set by the "handling" dispersion. This also will be mentioned again. This same temperature effect probably accounts for some of the long runs observed in the individual curves.

At the beginning of this discussion two questions were asked: What prediction? How reliable? A prediction has been obtained: A life of over 5,000 hours with an aging in that life of nine parts in 150. For reliability it is necessary to return to our control charts.

Table III is a summary of outages observed in tests on 28 relays. At the time of preparing this summary, some had been run to destruction, others had been run a few hundred hours without failure, and others a few thousand hours.

Nine relays showed outages in the first 20 hours, mainly high values. The time scale used was that of the schedule of Table I. If the chart is replotted on other time scales, some of these outages disappear, and others appear. We do not know what time scale should be used. It may not be important. We think these early outages indicate a real trouble and contributed to our bad guess on the aging rate for release current at 166 hours. There is still more to be said about this.

Ten relays showed outages all at about the same time when the measuring set went bad. That trouble was corrected. It was found that the calibration of the set had been in error by about two per cent. In the plotting of these charts, the early points have been corrected by that amount.

Fourteen relays showed outages by having high operating current after about 2,000 hours. This may be the aging of the spring as mentioned before.

There were six outages that were traced to the temperature effects in the measuring set already mentioned.

Seven relays were removed from the life test without giving information about life. For one pair the circuit had been wired wrong, and the relays were worn out in 600 hours. The fault was discovered at about 40 hours, but the test was continued just to see what would happen. For another pair the circuit broke down and damaged the relays. Another pair aged very rapidly and was found to have been

Selsyn Design and Application

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IN the last few years, new developments of Selsyn principles and new applications of modified designs have opened a vast field of use, the emphasis of which is on the system, within which the Selsyns exert control. The many advantages of these new uses are not yet fully realized. Therefore an explanation of the fundamentals of design and application of Selsyns will give a firmer basis for present applications and increase the future usefulness of this highly versatile means of control.

In certain forms Selsyns have had a reasonably long and distinguished service in industry in the past in such applications as remote indication of water level, controller position, and so forth. Many improvements have been made, and much has been learned about the possibilities and limitations of such systems. This paper presents a discussion of design and application fundamentals as distinguished from the mathematical analysis.

What Is a Selsyn

Selsyns, as discussed in this paper, are electric machines used as control devices which in systems involving two or more units can indicate the relative position of their shafts to a high degree of accuracy, even though a considerable distance may intervene between the units (the name is a contraction of self-synchronous). A servo mechanism, or servo system, is an arrangement of power and power-control units within which the Selsyn system operates to guide the action of the power units.

The different kinds of Selsyns can be classified roughly by size and accuracy as:

1. Power Selsyns.
2. Instrument Selsyns.
3. Indicator and control Selsyns.

Power Selsyns are similar in construction to wound-rotor induction motors and have an equivalent size. They might be used, for example, to maintain synchronism of the ends of a lift bridge. Instrument Selsyns are usually extremely small and light and used only to carry a light-weight pointer to indicate position, for example, of the trim tabs or wheels in an airplane. Extreme accuracy is not required of either of the foregoing kinds of Selsyns. Indicator and control Selsyns are midway in size and have errors counted only in tenths of degrees. Because of their high accuracy as control devices they enjoy a much wider field of use than either of the other two types.

Basic Selsyn Systems

An explanation of elementary Selsyn systems is needed as a basis for design specifications. Selsyn systems may be divided into two basic groups, depending on whether their principle of operation is

1. Torque transmission

or

2. Voltage indication.

Each type of system has its own unique advantages, depending on the needs of

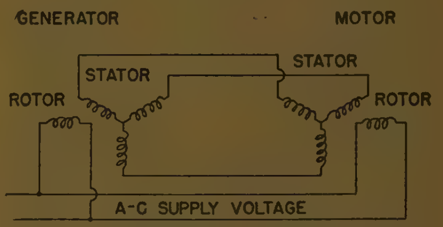

Figure 1. Selsyn generator and motor connection diagram

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improperly made. The seventh failed suddenly at 2,690 hours and was found to have a foreign particle in it that had worked into the hinge. This also was faulty manufacture. These seven told us nothing about normal relays.

Three relays operated at heavier loads have failed. The ten relays used to plot the average aging curve were worked into loads of 20 watts or less. One of these three relays was worked at 200 watts. It shows a normal aging curve but failed at 2,645 hours. The other two were worked at 300 watts. They showed abnormal aging and failed at 490 hours. There was some question as to the manufacture of these two as they were taken from a lot with a large number of known defectives. From this meager information we are tempted to guess that at these heavy loads the dependable life is about 500 hours.

In addition to these outages for which causes have been assigned, there were ten outages in over 2,000 points for which the cause is unknown. With three sigma limits, the expected number of outages is six, and the sampling fluctuations might take this up to 13. It is possible that these ten outages are really due to random fluctuations.

From these tests we have two sets of conclusions to draw. For the consumer, we can promise a useful life of over 5,000 hours at 60 operations per second with a load of 20 watts or less in a properly designed circuit. At 200 or 300 watts, we have some indication of a life of a few hundred hours. This applies to relays made by the standard process as it existed at the time of the test.

For the development man we have several conclusions. First, we need a measuring set that is easier to operate and free from temperature effects. We are now building a set that we think will be better. Second, something should be done about the initial aging. We now think this is caused by a combination of chemical action and impact polishing. It seems to stabilize after a few hours. We have introduced a stabilization aging run as part of the manufacturing procedure. Third, the aging trend, though small as relay aging usually goes, is undesirable. We should note that the very fact that we had to draw our control charts on sloping medians indicates the presence of an assignable cause. We traced this to the over-stressed spring; we changed the spring material, and new tests indicate that we have almost eliminated this slope. Fourth, the relays were damaged by heavy loads in the circuit used. This may be because of the load and hence an inherent limitation, or it may be because of the circuit which is now being studied.

This report may be summarized this way: The life tests were so designed that random causes and assignable causes could be separated. From the data related to the random causes we have made predictions about the useful life of our standard product in which we have much confidence. From the data related to the assignable causes we have been led to improvements in our design, our manufacturing process, and our testing process, and we may be led to improvements in our method of use.

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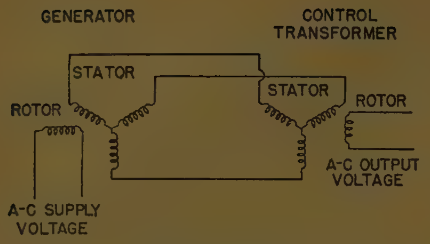

Figure 2. Selsyn generator and control-transformer connection diagram

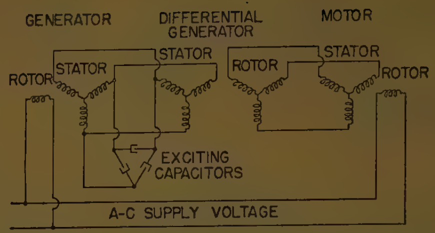

Figure 3. Selsyn-system connection diagram including differential Selsyn and exciting capacitors

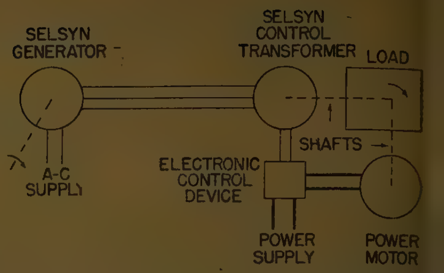

Figure 5. Connection diagram for typical servo system

the application. In some cases a group of interconnected Selsyns forms a system of its own, but more generally it is only part of a larger servo system which it controls.

Selsyn Generator and Motor System (Torque Transmission)

The first basic type of system comprises two Selsyns between which torque is transmitted electrically to rotate the shaft of the motor, or receiver Selsyn, in synchronous correspondence with the shaft of the generator, or transmitter Selsyn. The motor shaft may carry a dial for indication purposes or may rotate some other mechanical device; the principle is the same.

The Selsyn generator and motor are electrically and mechanically identical except for the addition of a damper flywheel to the motor rotor to reduce undesirable mechanical oscillations. In each excitation power is applied to a single-phase primary (usually rotor) winding which induces voltages by transformer action in the three secondary (usually stator) windings which are connected in the conventional Y with 120-degree mechanical spacing.

When the stator windings are interconnected as in Figure 1, any lack of correspondence between generator and motor rotors causes the voltages in the stator windings to become unequal, thus circulating currents through the windings and producing restoring torques to bring the motor into correspondence with the generator. When both rotors are in correspondence, the stator voltages are equal, and no current flows.

The voltages induced in the stator windings are all in time phase, since only single-phase power is applied to the rotor and the Selsyn revolves only slowly. The magnitudes of the various voltages depend on the relative postion of stator and rotor windings, and thus a unique set of voltages is produced in the stator for each position of the rotor. The fact that these sets of voltages are a very accurate measure of rotor position is responsible for the widespread use of Selsyn systems.

Selsyn-Generator and Control-Transformer System (Voltage Indication)

The second basic type of system comprises two Selsyns connected electrically, in the second of which (the control transformer) a voltage is induced whose polarity and magnitude are functions of the relative direction and amount of displacement between the shafts of the two units. This voltage can be used for indication purposes but generally acts as control voltage for a servo system.

The Selsyn generator of this system is identical in all respects to that of the first basic system described. The Selsyn control transformer is similar to the generator, except that it has a high-impedance winding on the stator and the rotor winding is not connected to the power supply.

When stator windings of generator and control transformer are interconnected as in Figure 2, the currents in the control-transformer winding are proportional to the voltages in the generator windings, thus producing a magnetic field in the control transformer corresponding to that in the generator. The voltage induced in the single-phase winding of the control transformer is thus a sine function of the relative displacement of the rotors.

Modification of Basic Systems

A differential Selsyn may be included in either basic system. It functions either as a differential generator to transmit a set of Selsyn voltages, while modifying them by adding or subtracting the amount of rotation of its shaft, or as a differential motor to receive voltages from two generators and rotate the sum or difference of their rotations. The voltages applied to the three-phase primary (usually stator) windings of a differential generator from the Selsyn generator induce a new set of voltages in the secondary three-phase windings, which appear as if the generator has been turned additionally by the amount of shaft rotation of the differential. An exciting capacity in parallel with the primary winding of a differential Selsyn often is used to reduce the heating of the rest of the system. Figure 3 illustrates a typical connection diagram for a Selsyn system, including a differential generator.

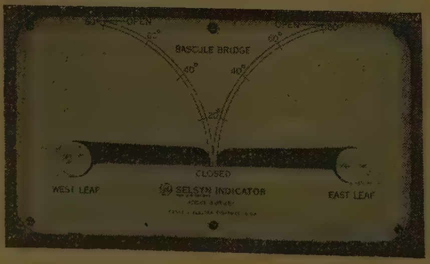

Figure 4. View of indicator panel for bascule bridge showing Selsyn-operated indicators

Typical Selsyn Applications

To illustrate the basic systems just described, some typical applications of Selsyn systems are helpful.

Selsyn Generator–Motor System AS Position Indicator

The leaves of a bascule bridge must be kept closely synchronized in position to insure correct operation. Automatic equipment is provided to accomplish this, but Selsyn systems often are used to provide visual proof to the operator that all is well.[1] A Selsyn generator is geared to each leaf of the bridge, and the shafts of corresponding Selsyn motors are attached to pointers on a dial to represent the bridge. The pointers correspond accurately to the position of the leaves and thus quickly check operations. Figure 4 shows the dial of such an installation. Figure 1 shows the connection diagram for the installation.

Similar applications of Selsyn systems were made as early as 1914 to show the position of lock gates, safety chains, and water levels in the Panama Canal.[2]

Selsyn Generator–Motor System Used for Transmitting Torque

Continued rotation of a Selsyn generator will cause rotation of the connected Selsyn motor, whose shaft will rotate any attached load up to the limit of the allowable transmitted torque. This flexible-shaft operation of Selsyns is similar in all respects to the previous example, except that an average angular difference between generator and motor is required to supply load torque.

This use of Selsyns is obviously very convenient in many cases but has definite limitations in the amount of torque that can be transmitted without overheating the system and in the mechanical oscillations that may occur with changes in load. A better system is described in the following paragraphs.

SELSYN-GENERATOR–CONTROL-TRANSFORMER SYSTEM CONTROLLING TYPICAL SERVO SYSTEM

Torque amplification, or the control of a large or high-power object by means of a small or low-power one is often necessary in industry (for example, in the machine-tool trade). Many new developments, and therefore the greatest possibilities, are in a system utilizing the high accuracy of control of a servo system possible with a Selsyn-generator–control-transformer system. A typical servo system is diagrammed in Figure 5.

The output voltage of the control transformer (which is proportional to the relative position of generator and control transformer) is fed to an electronic control device, which controls the flow of power to the driving motor. The driving motor turns the load (and control transformer) until the output voltage of the control transformer becomes zero. Any motion of the generator results in a corresponding motion of the load with an accuracy independent of the size of the load, as the driving motor is chosen accordingly. This is a new type of application for Selsyns and bids fair to eclipse all others in versatility of usefulness. A great amount of development has been done designing electronic control devices for maximum performance for each application. Their design is highly important to successful system operation but is outside the scope of this paper.

Characteristics, Design, and Construction of Selsyns

The use specifies the operating characteristics of a machine, and these in turn determine the design and method of construction. Selsyn systems of the first basic type have been in operation for some time, although the use of both types has been rapidly broadening in the last few years. Consequently, many present standard types of construction and methods of design have evolved from other machines such as small a-c synchronous motors. From a study of the requirements of each kind of Selsyn the most suitable design may be evolved.

SELSYN GENERATOR

For a generator used in either of the basic Selsyn systems, the general specifications are about the same.

1. Secondary voltages are such functions of rotor position that a highly accurate indication of position can be obtained when operated with a complementary Selsyn.

2. Sufficient torque developed when coupled with a suitable Selsyn motor, so that motor friction, or the resistance of a specified connected load, may be overcome sufficiently for required accuracy.

3. Minimum drain of current and power from supply lines when operated at rated voltage and frequency.

Table I. Torque Gradients

Values Are for 60 Cycles

Model*	Inch-Ounces Per Degree	Model	Inch-Ounces Per Degree
1	0.07	2	0.16
3	0.45	4	0.40
5	1.2	6	1.75
7	3.4		

* See Figure 9 for relative sizes of units.

4. Adequately large torque capacity and low-voltage regulation to give proper performance of the rated number of connected Selsyns in specified systems.

5. A temperature rise under operating conditions adequate to insure satisfactory life.

6. Bearing design and lubrication suitable for high position accuracy and adequate life at specified rotation speeds and temperatures. Allowance must be made for possible limitations in available torque from the driving device.

7. Construction must include convenient means of mounting and resistance to normal hazards of operation such as vibration, shock, moisture, dust, fungus, and so forth.

The design of a Selsyn generator is like that of any small a-c machine, and the construction is often similar. A standard type of construction of stator and rotor for a generator is shown at the left in Figure 6. All circuit quantities and methods necessary to make standard analyses of Selsyns and their circuits are contained in the usual analysis of synchronous a-c machines.[4] Unfortunately, since Selsyn errors are caused by deviations from the approximate theory commonly used, there is as yet no precise method of predicting error or designing for a given controlled amount of it. The usual design formulas for larger synchronous machines can be used for excitation and torque calculations on all types of Selsyns, if correction is made for the changes in relative dimensions. Ordinary precautions regarding distribution of the coils of the winding, number and skew of slots, and so forth must be taken, but special precautions are needed on machining tolerances, if accuracy is to be high.

Most Selsyns have a sinusoidal variation of stator line voltage with rotor position. Figure 7 shows the variation of the magnitude of the three line voltages. All voltages are in time phase. It has been shown that any space harmonics in the flux distribution, caused by the location of coils or dissymmetry of the magnetic structure, and time-phase differences between current in the various windings, caused by any circuit unbalance, can produce errors in the Selsyn system. The collector rings and brushes must be of durable materials with low stable voltage drop to avoid erratic errors. Gold or silver alloys are commonly used.

The torque developed by a pair of Selsyns (except control transformers) when their rotors are out of correspondence varies with the sine of the relative angular difference as shown in Figure 8. The amount of this torque for a given design may be calculated in terms of the usual electric-machine-circuit constants.[3] Representative generators develop torque gradients with electrically identical motors as shown in Table I.

The models 1, 3, 5, 7 generally are used for position indication only, whereas models 2, 4, 6 are used alternatively for torque transmission as electrical flexible shafts.

Since for normal operation Selsyns are at or near correspondence position, the circulating currents in the stator are relatively small; consequently, the excitation power is the principle factor in determining heating. A stiffer torque gradient therefore can be built into Selsyns which are intended for position-indication use only, resulting in increased torque to overcome bearing and brush friction and provide better accuracy.

Recommended system loadings of various sized generators are given in the next section. Accuracy and heating are the limiting factors.

SELSYN MOTOR

A motor is used only in the first of the basic systems—that utilizing torque transmission. As previously mentioned there are two possible uses of that system;[5] the motor shaft may carry a lightweight dial

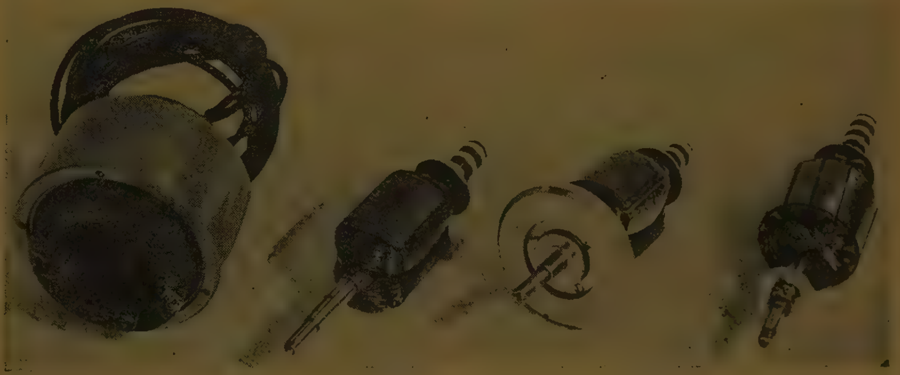

Figure 6. Standard Selsyn construction showing stator, generator rotor, motor rotor with damper, and differential generator rotor

which, once set properly, records the position of the generator shaft very accurately, or the motor shaft may drive some device or load in synchronism with the generator shaft. The requirements are the same as for generators, but, in addition

1. The motor must come quickly into synchronism with the generator and not oscillate unduly when power is applied.

2. Bearing and brush friction must be kept low to insure a high accuracy of correspondence with the generator.

The design of all motor parts is identical with that of the generator in order to satisfy the general requirements of torque and accuracy, but in addition a damper is required. Oscillation of the motor is reduced by a mechanical inertia-friction damper fastened to the shaft, as shown in the middle rotor in Figure 6.

Special precautions must be taken with bearings for indicator Selsyns to insure low friction and high accuracy. All parts are cleaned carefully and aligned, and in critical cases special bearings are used. On the other hand Selsyns for torque transmission are inherently less accurate and in some cases may be built entirely from standard small induction-motor parts without special design.

SELSYN CONTROL TRANSFORMER

A control transformer is used only in the second of the two basic systems—that using voltage indication. Its special requirements are:

1. The output voltage should be an accurate measure of the relative angle between generator and control transformer, and the power available should be sufficient to operate specified electronic devices.

2. The excitation power and current must be a minimum, as they must flow through the generator and therefore limit the number of control transformers that can be connected.

3. The reaction of a control transformer on the generator must not affect seriously the accuracy of transmission of position information to other control transformers that may be connected to the generator.

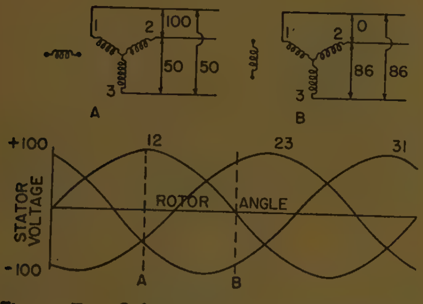

Figure 7. Selsyn-stator line voltages as a function of rotor angle

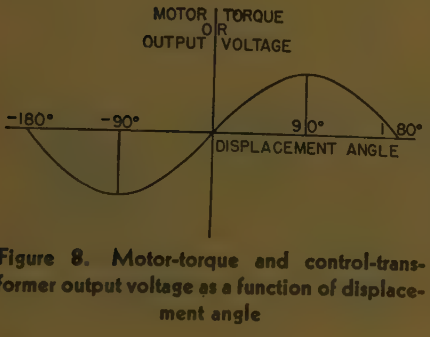

Figure 8. Motor-torque and control-transformer output voltage as a function of displacement angle

The mechanical design of a control transformer is similar to that of a generator. The variation of the output voltage with rotor position is sinuscidal as shown in Figure 8. To minimize the reaction of the control transformer on the generator the magnetic circuit is kept symmetrical by carefully shaping the rotor as shown in Figure 10 and using a three-phase stator winding similar to that on the generator but of much higher impedance. Thus only an equalized three-phase load is presented, the generator signal is not unbalanced, and the excitation power required is kept low.

The volt-ampere burden of electronic devices operated by the control transformer is commonly low, making easy the fulfillment of the requirements. However, if the load consists of peaking transformers for operating thyratron circuits, the volt-ampere burden is high, and a compromise is necessary with the requirement for low power drain from the generator, or excessive voltage regulation will be present.

Because the control transformer produces practically no torque and its shaft always will be positioned by some external driving motor, no special precautions need be taken with the bearings to keep friction to a minimum.

DIFFERENTIAL SELSYN

A differential Selsyn may be used in either type of system and may be either a generator or motor with similar general requirements, but in addition

1. The series impedance introduced by the differential must be low enough, so that the torque or voltage stiffness of the system to which it is added will not be seriously reduced.

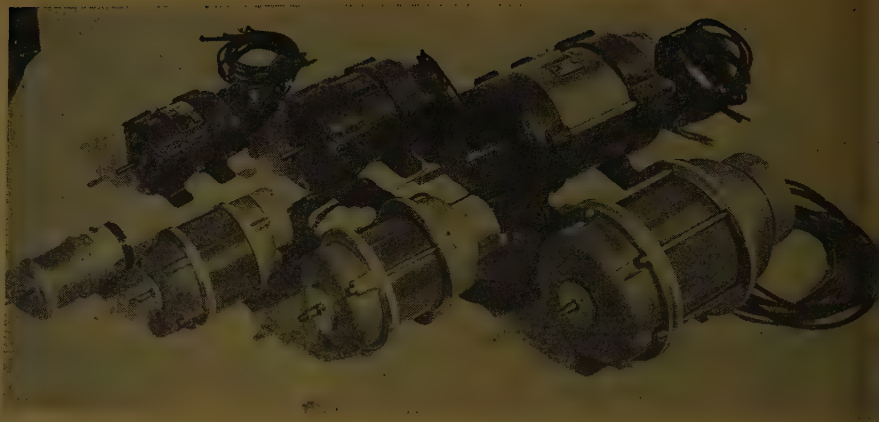

Figure 9. Standard Selsyn sizes

Left to right, front row: models 1, 3, 5,
back row: models 2, 4, 6

2. The excitation power and current r quirements must be a minimum to reduc the heating of the generator.

The foregoing requirements require compromise for best design. Usually low-impedance design is aimed at, an then exciting capacity is placed in shun with the primary winding to reduce th current drain load from the generato The rotor at the right in Figure 6 is tha for a Selsyn differential generator. Th corresponding stator winding is prac tically identical with that of a generator

Design Modification for Special Applications

The standard Selsyn designs just described can be built in all the variation of mounting, voltage, frequency, numbe of phases, and so forth of equivalent-size standard induction motors. The fundamental Selsyn principle, however, can b applied in two special designs which ar worthy of note here because of the flexibility of Selsyn application which the indicate.

Extremely small size may be obtaine by a radical change in design to a Gramm ring type as shown in Figure 11. Her the stator winding is in the form of toroid, while the rotor is of salient-pol design. This design lends itself to automatic machine winding in core and wit sizes impractical with conventional slotte stators. The principal disadvantages ar that the impedance is inherently high an the design of a differential is quite awkward. The accuracy possible is quit comparable to that for larger standar units of the generator and control-transformer varieties.

For applications of limited rotatio (±30 degrees), very high accuracy i sometimes made possible by winding th stators with only one phase, thus utilizin the winding volume and distribution t best advantage. The physical appearanc of such Selsyns may be identical with those previously shown, or stator and rotor may be mounted independently to occupy minimum space. The circuit in Figure 12 illustrates such an application.

The single-phase Selsyn units are identical and are operated as generators. The secondary voltages are connected opposing (for like rotation), and the difference voltage operates a servo system with one unit rotated by the sending mechanism and the other turned into synchronism by the servo-system power drive. Note the differences between Figures 12 and 5.

Possible Selsyn and Servo Systems

Selsyn systems of generators and motors for remote indication or generators and control transformers for controlling servo systems may consist of from 2 to 40 Selsyns or more. The problems of such systems are largely those of heating limitations in the generators and differential generators, since the accuracy is determined readily and the stability of the system is controlled by proper design of the electronic control device. Table II shows the number of motors and control transformers that can be handled by different sizes of generators without overheating. The model numbers refer to standard sizes and are not proportional to dimensions.

In general each proposed Selsyn system must be checked for heating experimentally as this method is much simpler and less fallible than usual calculations for small machines.

Accurate control of large amounts of power by simple and low-power means is the object of the design of a servo system.[6] A vast fund of knowledge has been acquired on the design of these systems during the last few years, and much of this data is yet to be applied industrially. An understanding of the principles of Selsyn control should hasten such applications. Servo systems controlled by Selsyns have been developed in a range from $^1/_{200}$ horsepower to 65 horsepower (there is no apparent upper limit).

Table II. Motor and Control Transformers Handled by Generators

Values Are for 60-Cycle Machines

Generator Model Number	Number of Motors Model 1	Number of Motors Model 3	Number Current Transformers Model 1 or 3	
			No Cap.*	With Cap.*
1	1	0	0	1
3	4	2	2	12
5	18	9	6	24
7	36	18	10	40

* Cap. refers to the exciting capacity needed for power-factor correction to reduce current drain.

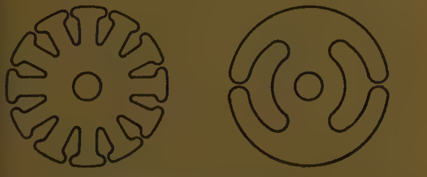

Figure 10. Possible shapes of rotor punchings for control transformer

Figure 11. Miniature Selsyn using Gramme ring construction

The typical servo system of Figure 5 described previously used Selsyn generator and control transformers whose output controlled a power motor with the aid of an electronic device such as an amplifier. This is the most accurate Selsyn system and allows plenty of latitude in the choice of power control equipment. Any size of load motor can be controlled equally accurately by simple changes in the amplifier design.

Selsyn motors also may be used to control servo systems, however. If a suitable contact-making arrangement is fastened to its shaft, a Selsyn motor can control a split-series d-c motor which drives the load and at the same time repositions the stator of the Selsyn motor (ball-bearing mounted and called a control motor), until a null position is reached by the contacts. Figure 13 shows the circuit arrangement for such a servo system. Figure 14 shows a control motor disassembled. The cage in the upper part of the figure carries the stator on ball bearings, so that it may be turned by means of a gear (not shown). Alternatively, a Selsyn system similar to the foregoing might be connected to a pilot valve in a hydraulic pump and motor system thus obtaining delicate and accurate control over large forces.

All applications of Selsyn systems which use Selsyn motors as the control element suffer from the vagaries of friction in the Selsyn and the load it drives.

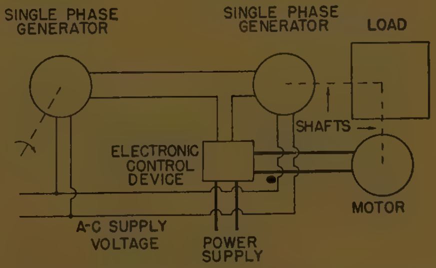

Figure 12. Selsyn servo system using single-phase Selsyns for limited-rotation application

Possible instability is thereby increased, and many manufacturing difficulties may crop up. It is best where possible to drive the shafts of Selsyns positively and employ the excellent and predictable accuracy possible with the generator–control-transformer system.

Selsyn Errors

Errors in Selsyn systems are caused by friction or electrical discrepancies such as impedance unbalance. The reasons for the various errors and the probable magnitudes will be discussed.

Selsyn Systems Involving Selsyn Motors. These always have friction in the motor bearings and sometimes friction in the load to contend with. There is also some electrical error of the magnetic field, but usually the error due to friction is larger. Representative values of guaranteed maximum static error for pairs of like units used as indicators are given in Table III.

The insertion of another Selsyn, such as a differential, or the presence of appreciable line impedance reduces the torque that can be developed by the Selsyn motor, and therefore increases its error. For example, a Selsyn generator and motor system of model-3 series has a guaranteed static error of not more than ±0.6 degree maximum; the insertion of a differential generator of the same size increases the system static error by from 50 per cent to 75 per cent. The effect of line impedance on torque can be seen in Figure 15. Error presumably will change inversely with torque. The addition of several Selsyn motors in parallel will reduce the available torque per motor and therefore increase the error of each.

Selsyn motors have dampers to reduce unwanted oscillations, but during changing conditions or continual motion a certain amount of oscillation is likely to be present. These dynamic errors vary considerably, but it is easily possible to have errors twice or three times those for static conditions. It is important to note that this oscillation is inherent in generator–motor systems and is almost impossible to remove. Variations in torque caused by line-impedance unbalance will of course accentuate this trouble.

Selsyn-Generator–Control-Transformer Systems. These have only electrical errors which under static conditions involve inherent errors due to the design of the units and manufacturing errors due to the various tolerances between parts. When a curve was drawn of error versus angle of rotation from some reference point for a certain design, the inherent error was found to be principally a sixth harmonic of rotation, whereas the manufacturing errors tend to be second harmonics. Reduction of the second harmonic may be possible by better control of manufacture; the reduction of the

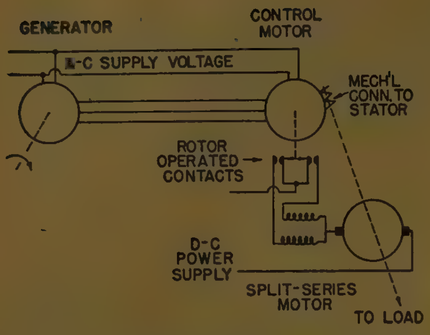

Figure 13. Selsyn-generator and control-motor contact follow-up system

sixth harmonic is possible only through a change in design.

The maximum error of a reasonably good generator–control transformer is not more than ±0.3 degree. Insertion of a differential Selsyn will increase this maximum error about 75 per cent. It will be noted that among other advantages the generator–control-transformer system has approximately one half the error of a generator–motor system. Improved design will better these figures.

One of the sources of errors in Selsyn systems is unbalance of the line impedances. It has been found necessary to keep the sections of the exciting capacity used for differential Selsyns balanced within 1 per cent. In the case of one control transformer it was found that a 1 per cent unbalance in resistance in the stator windings caused nearly a ten per-cent increase in maximum error. Balanced loads may be connected safely, but unbalanced loads change impedance and cause time-phase differences between the currents of the various circuits, resulting in an extraneous voltage or error appearing.

Each Selsyn generator–control-transformer system has an inherent limit of accuracy, as the output-voltage signal of the control transformer becomes too small for convenient use at very small angles from correspondence. By paralleling the original Selsyn system with another geared up by 16–1 to 36–1 it is possible to multiply the accuracy of any given system and obtain very much smaller system errors. The two systems can be synchronized automatically electrically, so that the low-speed system prevents the high-speed system from pulling-in on a false zero point.

One operational error in generator–control-transformers system is worthy of note. Rotation of the system produces generated voltages in the control-transformer output in addition to those induced by transformer action. The magnitude of this effect is dependent on the Selsyn impedances and usually may be compensated for at a given speed only. For 60-cycle units the control transformer will lag the generator by 0.5 degree to one degree when rotating at 200 rpm.

Table III. Guaranteed Maximum Static Error

Model Number	Maximum Static Error, Degrees	Model Number	Maximum Static Error, Degrees
1	1.5	2	1
3	0.6	4	1
		6	1

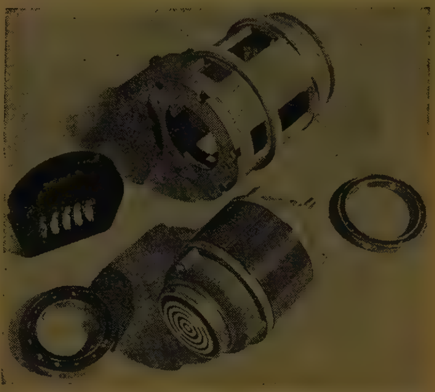

Figure 14. Selsyn generator with ball-bearing-mounted stator showing stator support and brush rigging

Selsyn Application Suggestions

Selsyns are high-grade accurate instruments and should be treated as such. Ordinary tests in the field can check only the continuity of the wiring and do not give any indication of the accuracy of a unit. Selsyns are built to withstand all reasonable conditions of operation, but careful handling is a requisite during installation. Manufacturer's suggestions on mounting should be followed if the best use is to be made of them.

ZEROING

In order to provide a reference position from which to measure rotation, for each Selsyn type there has been arbitrarily designated a combination of stator and rotor voltages and corresponding relative positions known as zero positions. The procedure of locating these positions is known as zeroing. Standard instructions are available.

APPLICATION SPECIFICATIONS FOR SELSYN SYSTEMS

When planning a Selsyn system for a new application the following points should be considered.

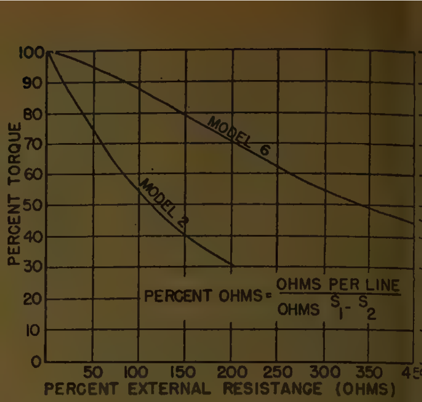

Figure 15. Variation of motor torque wit line impedance for models 2 and 6

1. Is a generator-motor or a generator control-transformer system indicated?

2. Check supply voltage and frequency Determine allowable drain in watts an amperes and regulation of supply.

3. What are requirements of load i torque, speed of rotation, duty cycle, ac curacy? Will any tendency of a generator motor system to oscillate be objectionable

4. Determine limiting conditions at appl cation location: temperature, humidity dirt, vibration, shock, and so forth.

5. Does the generator have a stiff drivin mechanism, or is the friction of bearing and brush rigging important? If a genera tor-motor system is used, can the drivin mechanism handle the friction of bot machines as well as the load?

6. Are the line impedances balanced, in cluding any exciting capacities?

7. Are the method of mounting, the gear ing, and so forth accurate enough to mak the best use of the Selsyn system?

An understanding of the general prin ciples of Selsyn system operation is th best guide to satisfactory applications with it the possibilities for greatly in creased use of Selsyn-controlled system will come into use rapidly in industry.

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Optimum Air Gap for Various Magnetic Materials in Cores of Coils Subject to Superposed Direct Current

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Synopsis: A method is developed for calculating optimum air gap and core size for choke coils whose windings must carry direct current in addition to alternating current. It is employed with data on typical magnetic materials to give curves from which design dimensions can be obtained readily to fulfill specified requirements for inductance, resistance, and d-c burden.

THE PROBLEM of proper proportioning of air-gap and metallic-path length of the magnetic core of choke coils and transformers frequently arises in the design of coils whose windings must carry direct current along with the desired alternating current. This problem has been approached in the past along lines laid down by Hanna.[1] This involved calculations based on known characteristics of the magnetic materials and taking into account the effects of air gaps of various lengths inserted in the magnetic circuit. Taking an estimatedly correct core volume, V, the optimum value of applied field then was computed, from which the number of turns in the coil was obtained. The resulting coil resistance then was calculated to see if it was satisfactory. If not, a new core volume was assumed, and the calculations were repeated until the desired winding resistance was attained.

It has seemed that a more direct approach to optimum dimensions could be devised, which would give an immediate answer as to the suitability of any given core material for a specified inductance, resistance, and d-c burden. An analysis of the problem has been made, and curves have been plotted for several important magnetic materials from which design dimensions can be readily obtained.

General Problem: L, R, and I Specified

The general method is based on the known fact that prerequisites of inductance L, resistance R, and d-c burden I, can be met by proper expansion or contraction of the size of a core, leaving the ratios of dimensions fixed. Thus, for example, all dimensions can be expressed as fixed multiples of the metallic core length. The essential equations then can be set up and solved for minimum core length. This gives relationships between the (μ, B) curve, (μ_r, B) curve, and the specified L, R, and I of the coil, which must be met for the optimum conditions. The necessary metallic path length, air-gap percentage, and number of turns in the coil follow directly.

Considerable study has been given by others to determine the ratios of dimensions in various types of coils to apportion space properly to windings and magnetic materials. The present analysis assumes that technically or commercially approved dimension ratios are known for whatever coil and core configuration may be considered and confines itself to determination of core and air-gap relationships. If the metallic-path length is l, the average length of a turn of wire in the winding will be ul, the area available for copper winding will be vl^2, and the cross-sectional area of the magnetic core will be wl^2, where u, v, and w are known constants. The length of the air gap is similarly αl, where the factor α is to be adjusted to make l as small as possible. Air gaps will be initially assumed so short as to avoid fringing, so that the flux density in the air is as great as that in the metal. Necessary adjustments to take account of fringing with longer air gaps will be described.

With these assumptions the specified resistance R, inductance L, and d-c burden I enter into the equations as follows.

The resistance of the copper winding will be proportional to the total length of wire (N × average turn length, ul) and inversely proportional to the cross-sectional area per turn, vl^2/N, or

$$R = \frac{\rho N u l}{v l^2 / N} = \frac{u \rho N^2}{v l} \text{ ohm} \qquad (1)$$

where N is the number of turns, l is in centimeters, ρ is the resistivity in ohm-centimeters, and the copper area constant v takes into account the packing factor and insulation of the winding.

The inductance, as measured with alternating current of small amplitude, depends upon the permeability of the metal and the relative length of air gap. The permeability to be used is the a-c permeability at the value of superposed field set up by the direct current in the winding. For simplicity only the reversible permeability μ_r will be considered. The results for values of alternating current differing from zero will approximate quite closely those obtained for zero current, namely, μ_r. This follows from the fact that the a-c permeability changes less with changing current in the presence of superposed field than without. The inductance is derived from the basic equation

$$L = \frac{N \Phi_m}{i_m} \times 10^{-8} \text{ henry}$$

where Φ_m is the peak flux in maxwells corresponding to the a-c peak i_m, in amperes. The flux can be written as the ratio of magnetomotive force, $0.4\pi N i_m$, to the reluctance, $l/wl^2\mu_r + \alpha l/wl^2$, whence

$$L = \frac{4\pi N^2 wl}{l \div \mu_r + \alpha} \times 10^{-9} = 4\pi N^2 \mu_e wl \times 10^{-9} \qquad (2)$$

where μ_e is the effective permeability, that is, the permeability of a uniform core material occupying the same space as the

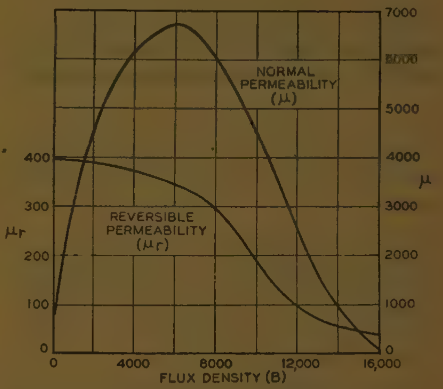

Figure 1. Normal and reversible permeability of four-per-cent silicon iron

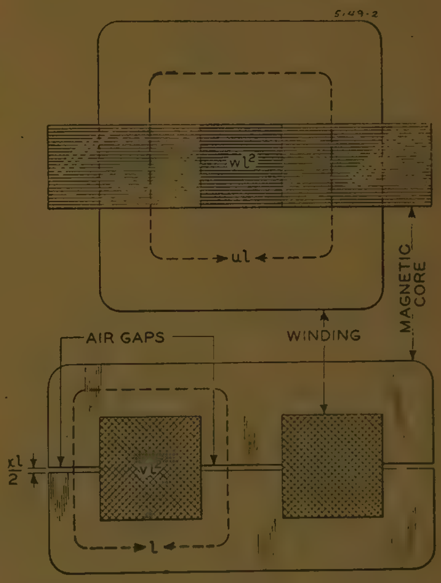

Figure 2. Typical shell-type core with relative dimensions indicated

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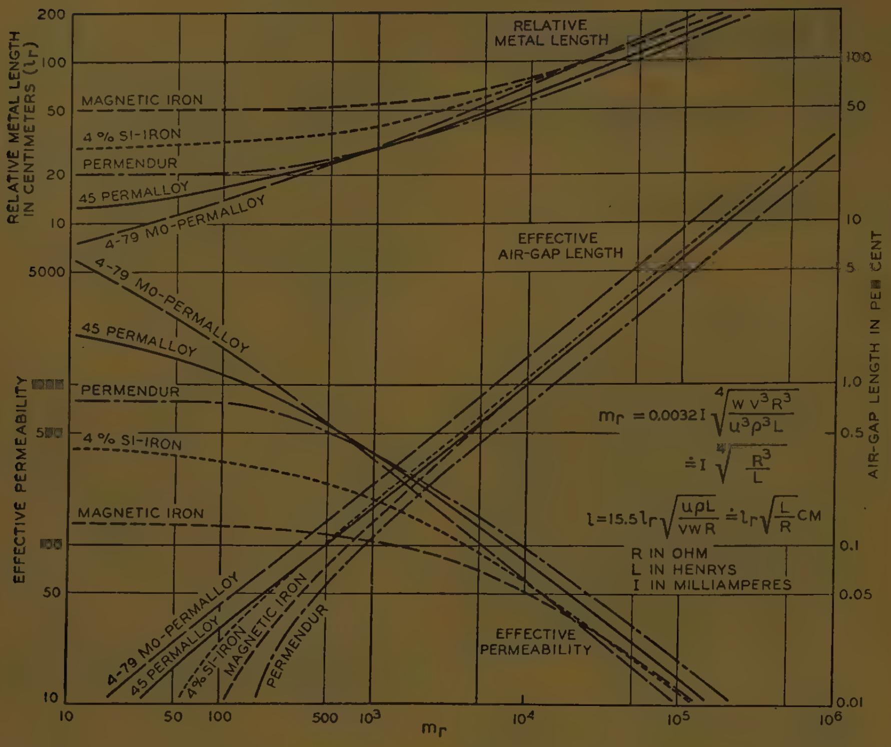

Figure 3. Optimum air gap, relative metal-path length, and effective permeability for magnetic cores subject to superposed d-c field

core in question and yielding the same inductance. Substituting N^2 from equation 1 gives

$$1/\mu_r+\alpha=\frac{4\pi vwRl^2}{u\rho L}\times 10^{-9}=kl^2 \tag{3}$$

where

$$k=(4\pi vwR/u\rho L)\times 10^{-9}$$

The magnetomotive force due to direct current in the winding is distributed between the magnetic core and the air gap so as to maintain the steady flux density B in the core by means of field strength $H=B/\mu$, and in the air gap by field strength $H_a \doteq B$, neglecting fringing in the gap. The magnetomotive force is thus

$$lB/\mu+\alpha lB=0.4\pi NI$$

Substituting N from equation 1 gives

$$\frac{1}{\mu}+\alpha=\frac{0.4\pi I}{B}\sqrt{\frac{Rv}{u\rho l}}=\frac{m}{B\sqrt[4]{kl^2}} \tag{4}$$

where

$$m=4\pi I\sqrt[4]{(4\pi R^3v^3w\div[u^3\rho^3L])}\times 10^{-13}$$

Equations 3 and 4 contain essentially three unknowns—B, α, and l (μ and μ_r are functions of B). The final equation needed to determine these unknowns and to satisfy the condition for smallest possible coil is $dl/dB=0$, all other dimensions having been assumed proportional to powers of l. Eliminating α between equations 3 and 4 gives

$$B\sqrt{kl^3}(1/\mu-1/\mu_r+kl^2)=m \tag{5}$$

Differentiating with respect to B and setting $dl/dB=0$ gives

$$kl^2=\frac{1}{\mu_r}-\frac{1}{\mu}+B\left[\frac{1}{\mu^2}\frac{d\mu}{dB}-\frac{1}{\mu_r^2}\frac{d\mu_r}{dB}\right]=\frac{1}{\mu_r}-\frac{1}{\mu}+B\Delta \tag{6}$$

where

$$\Delta=\left[\frac{1}{\mu^2}\frac{d\mu}{dB}-\frac{1}{\mu_r^2}\frac{d\mu_r}{dB}\right]$$

Substituting this value of kl^2 back into equation 5 gives

$$B^2\Delta\sqrt[4]{B\Delta+1/\mu_r-1/\mu}=m \tag{7}$$

This equation gives the optimum value of B for the desired value of m, but it also implies selection of corresponding values of μ, μ_r, $d\mu/dB$, and $d\mu_r/dB$, to satisfy the equation. These functions must be determined from (μ, B) and (μ_r, B) curves such as shown in Figure 1 for four per cent silicon iron. On account of the involved nature of the data, equation 7 is handled most readily in reverse direction, by inserting known values of B, μ, μ_r, and so forth, from the magnetic data and solving for the constant m. Such calculatio from the curves of Figure 1 are given i Table I.

The value of the air-gap ratio is ob tained from equations 3 and 6 as

$$\alpha=B\Delta-1/\mu \quad (8$$

The metallic-path length from equatio 5 is

$$l=\sqrt{(B\Delta+1/\mu_r-1/\mu)/k} \quad (9$$

A function which reveals the influenc of the air gap is the effective permeability μ_e. Thus, from equations 2 and 8

$$\mu_e=\frac{1}{1/\mu_r+\alpha}=\frac{1}{B\Delta+1/\mu_r-1/\mu} \quad (10$$

The necessary number of turns in th winding follows from equations 1 and 9.

It is now feasible to plot pertinent dat of air-gap and metal length for any desired magnetic material against values o m. From such curves essential dimension of any coil can be read off immediatel when m has been computed. For practical purposes it is convenient to us values for m corresponding to typicall proportioned cores. The parameters i the foregoing equations are

$$m=4\pi I\times 10^{-6}\sqrt[4]{4\pi R^3v^3w/(10u^3\rho^3L)} \text{ and}$$

$$k=4\pi vwR\times 10^{-9}\div(u\rho L)$$

where I is in milliamperes, R in ohms, in ohm-centimeters, L in henrys, and l is in centimeters. The constants u, v, an w will have various values, depending o the shape of the core, whether annular shell type, or otherwise. Under the assumption that shell-type cores as illustrated in Figure 2 are most common fo use with air gaps, typical cores have bee measured for values of the dimensio ratios. Thus $u=1.8$, $v=0.02$, and $w=$ 0.04, approximately. Inserting these values in the above equations gives

$$\left.\begin{aligned}&m=4.17\times 10^{-3}IR\div\sqrt[4]{RL}\\&\text{and}\\&k=3\times 10^{-6}R/L\end{aligned}\right\} \quad (11$$

For cores having such typical dimensio ratios, it is convenient to express results i terms of

$$m_r=IR\div\sqrt[4]{RL}=m\div(4.17\times 10^{-3})$$

Table I. Data and Computations for Four-Per-Cent Silicon Iron

B(gauss)	2,000	4,000	6,000	8,000	10,000	12,000	15,000
μ	4,520	6,150	6,740	5,950	4,550	2,600	430
$d\mu/dB$	1.03	0.45	0	−0.56	−0.85	−1.00	−0.42
μ_r	386	369	343	294	188	95	20
$d\mu_r/dB$	−0.007	−0.010	−0.017	−0.034	−0.068	−0.031	−0.008
$\Delta\times 10^9$	98	85	144	377	1,889	3,292	17,720
$kl^2\times 10^3$	2.56	2.89	3.63	6.25	24.0	49.5	313.5
m	0.0882	0.315	1.27	6.77	74.2	2,230	29,900
m_r	21.15	75.6	305	1,625	17,800	535,000	7,170,000
α (per cent)		0.018	0.072	0.285	1.87	3.90	26.3
l_r	29.2	31.0	34.7	45.6	89.5	128.5	
μ_e	386	346	276	160	41.6	20.2	3.2
LI^2/V	0.478	5.09	59.4	740	11,780	3,120,000	
$(N/l)\sqrt{V/L}$	451	479	536	704	1,383	1,985	
$NI/l\times 10^{-3}$	0.312	1.082	4.13	19.18	150.0	3,760	

Values of m_r have been computed as given in Table I for four per cent silicon iron. They have been used as abscissas in Figure 3, for plotting optimum air gap and effective permeability of four per cent silicon iron. Computations similar to those in Table I based on published magnetic data[2] have been made for magnetic iron, Permendur, 45 Permalloy, and 4–79 molybdenum-Permalloy, and the results plotted in Figure 3.

An additional family of curves is shown in Figure 3, labeled relative metal length, l_r. The necessary metal length from equation 9 involves k, for which an expression is given in equations 11 for typical core dimension ratios. If the ratio R/L is taken as unity, the typical value of k is 3×10^{-4}. With this value "relative metal-path lengths" have been computed from equation 9. The actual core length in any given case then would be $l_r\sqrt{L/R}$.

If other values of u, v, and w are to be used than the assumed typical values the curves of Figure 3 apply if the abscissas are calculated from the equation

$$m_r=(4/4.17)\pi I\times10^{-3}\sqrt[4]{4\pi R^3v^3w/(10u^3\rho^3L)} \qquad (12)$$

Similarly the correct metal length l will be obtained from equation 9 in terms of the relative length l_r given in the curves by the equation

$$l=l_r\sqrt{3{,}000u\rho L/4\pi vwR} \qquad (13)$$

Special Case I. L, I, and l (or V) Specified

The problem thus far has assumed complete freedom in the choice of dimensions. Frequently, however, the problem may begin with a certain size of core and inquire after the optimum air-gap length to provide a specified inductance in the presence of a given amount of direct current. In this case it will be necessary to adjust the coil resistance along with the air-gap length to satisfy the optimum conditions. The problem becomes that of ascertaining the values of l_r and m which correspond to the optimum conditions as defined by the above equations and which yield the specified inductance and metallic-path length for the specified current.

The metallic-path length l is known in this case, but l_r and R in equation 13 are not known. Substituting the value of R from equation 13 in equation 12 gives

$$\frac{26.6m_r^2}{l_r^3}=\frac{LI^2}{wl^3}=\frac{LI^2}{V} \qquad (14)$$

where V is the volume of the core. This resembles the fundamental equation derived by Hanna,[1] but it has the advantage that m_r and l_r can be computed for any given flux density and material by the methods here described. If the first member of equation 14 is computed using associated values of m_r and l_r which already satisfy the optimum conditions, it yields the necessary parameter to insure that the corresponding l_r and α are optimum under the specified conditions of L, I, and l.

With optimum values of l_r and α, the required number of turns in the winding from equations 1 and 13 becomes

$$N=l_r\sqrt{\frac{3{,}000L}{4\pi lw}}=ll_r\sqrt{\frac{3{,}000L}{4\pi V}} \qquad (15)$$

The resistance in the winding then follows from equation 1.

The data used for Figure 3 have been analyzed further to find the values of LI^2/V which correspond to the optimum values of m_r, l_r, and α. The results have been plotted in Figure 4, showing values of

$$l_r\sqrt{3{,}000/4\pi}=(N/l)\sqrt{V/L}$$

and α against LI^2/V.

Case II. L, N, (or R), and l (or V) Specified

If inductance is specified, and it is desired to know the percentage air gap required to permit the use of the largest possible direct current in the winding of a coil which is already constructed, the curves in Figure 4 can again be used. All of the factors required to compute $N\div l\sqrt{V/L}$ now are specified. It remains only to locate the indicated point on the $N\div l\sqrt{V/L}$ curve for the material of which the core is constructed, and read the corresponding value of LI^2/V. From the latter value can be calculated the maximum current which can be employed without understepping the specified inductance.

The necessary air-gap length can be read off directly from Figure 4.

Case III. N (or R), I, and l (or V) Specified

If the d-c burden is specified, and the coil is already constructed (that is, l and N or R are fixed) it may be desired to adjust the air-gap length to yield the maximum inductance. Proceeding as in case I, the inductance is solved for in equation 13, and substituted in equation 12, which, together with equation 1, gives as the necessary relation

$$79.6m_r/\sqrt{l_r}=NI/l \qquad (16)$$

Proceeding as in case I, the first member of equation 16 is computed for each flux density, using associated values of m_r and l_r. This insures that the specified conditions of N, I, and l are satisfied when

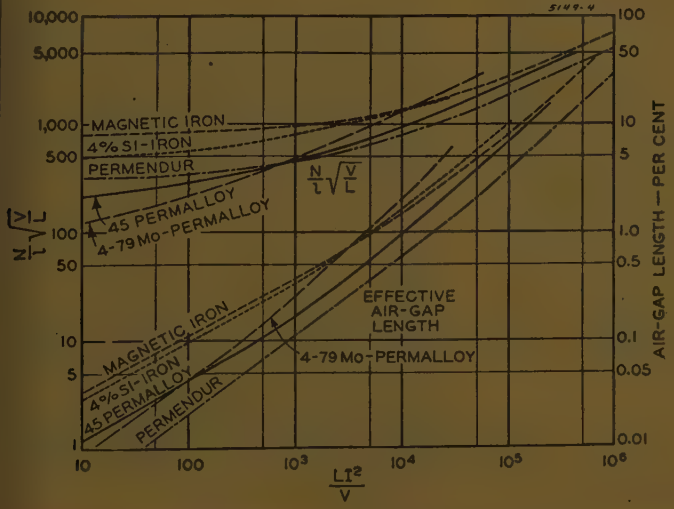

Figure 4. Optimum air gap and number of turns for a coil having specified inductance, core size, and d-c burden

L in henrys
l in centimeters
I in milliamperes

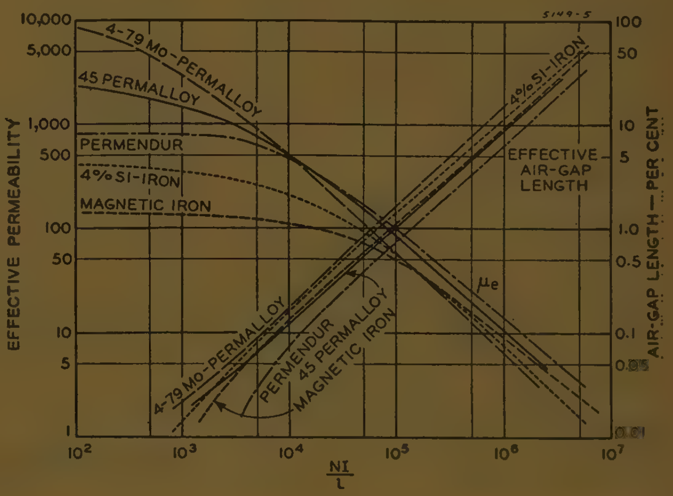

Figure 5. Optimum air gap and permeability for a coil having specified inductance, core size, and d-c burden

L in henrys
l in centimeters
I in milliamperes

the optimum conditions of l_r and α are employed. Curves in Figure 5 give the effective permeability and air gap plotted against NI/l. The inductance attained with the values of μ_e and α so chosen will be the maximum possible. The inductance in henrys is computed from equation 2.

Case IV. N (or R), L, and l (or V) Specified

If the inductance is specified and the coil is already constructed, the curves of Figure 5 can be read backwards to find the allowable direct current and the necessary air gap.

Case V. N (or R), l (or V), and α Specified

If a finished coil is at hand, the question may arise as to the direct current which should be used to employ the core and coil most efficiently. Curves on Figure 5 may be used in this case to locate the value of NI/l which corresponds to the specified value of α. With this value of NI/l, optimum current is computed. Similarly the corresponding effective permeability is read from the curves, from which the optimum inductance is found.

The Air Gap

The foregoing analysis has been concerned with determining the optimum air-gap length to satisfy certain conditions. When the practical question arises as to the number of millimeters spacing to insert, still further analysis is required, on account of magnetic fringing at the gap. In general a longer physical gap must be inserted into the magnetic circuit than indicated by the optimum percentage. The correction is less for very short gap lengths, or if the total amount of gap is divided into two or more short sections with interspersed core material. The correct gap length can be found empirically by building the coil as designed and then adjusting the air gap to give the calculated inductance with the specified direct current in the windings. An approach to the calculation of leakage around an air gap has been given by Partridge.[3]

Conclusion

Consideration has been given to the general problem of adjusting the size of magnetic core, air-gap length, and number of turns in the winding to fulfill specifications as to inductance, resistance, and d-c carrying capacity. Mathematical means have been worked out for fulfilling specified conditions with a minimum quantity of core materials whose permeability and reversible permeability curves are given. Calculations have been made for several typical materials, to illustrate the method of utilizing this type of analysis.

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2. Survey of Magnetic Materials and Applications in the Telephone System, V. E. Legg. *Bell System Technical Journal*, volume 18, 1939, page 438.

3. The Inductance of Iron Cored Coils Having an Air Gap, G. F. Partridge. *Philosophical Magazine* series 7, volume 22, 1936, page 665.

Eddy-Current Resistance of Multilayer Coils

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Synopsis: Most of the earlier studies of eddy-current resistance have been made by those interested primarily in the effect of compromise transpositions on the losses of rotating equipment. Their results have included formulas for the eddy-current resistance for perfect transpositions, but these have gotten into handbooks in difficult form or have been ignored entirely. Such formulas are presented here in more usable form, with more information on their limitations, and further formulas are derived for minimum losses for a number of different cases. Included is a set of curves showing eddy-current resistance as a function of the number of layers and conductor thickness. Formulas also are included that take into account the differing lengths of conductor in successive layers and that approximate the end losses in a spiral coil on account of radial flux.

THE CUSTOMARY approach to the problem of eddy-current-loss calculations for multilayer coils has been to assume a coil side surrounded on three sides by a medium of infinite permeability and no loss and the use of rectangular conductors. There has been one exception to this that is discussed later.

The principal aim of most of these investigations has been to arrive at a loss factor that might be applied to the d-c resistance of slot windings of various types to find the effective or a-c resistance. This has included evaluating the loss factor for the individual layers, which is all that needs to be applied if the layers are in series or if a perfect transposition system is used, and a different loss factor that includes the circulating current caused by compromise transpositions in certain practical windings.

Formulas so derived for slot windings have been applied with reasonable accuracy but some awkwardness to such ends as calculating the loss in transformer coils. It is the purpose of the present work to derive a formula that can be modified readily to account for the difference in layer lengths, and that will lead to simple and general approximate formulas that can be used for a very rapid survey of design possibilities in many cases.

A paper by Butterworth[7] considers an inductance without the usual partial iron circuit and wound with round conductors. It is shown that the ratio of reactance to resistance is a maximum for a coil so proportioned that the ratio of diameter to length is from 2.5 for low frequencies to 3.2 for high frequencies, the distinction between low and high frequencies being made according to the degree to which skin effect is evident in an isolated conductor. At high frequencies the reactance-to-resistance ratio is improved by a spaced winding. Formulas are derived for losses in such coils and shown to correspond to measured results. It also is shown that for the coils in question the losses are almost entirely due to the radial component of flux rather than the axial component. As these results are limited in their application to single-circuit systems (also end-to-end coupling) of rather limited proportions, they are not considered further here.

Returning to the more usual analysis of eddy-current problems, it has been customary to assume further that:

1. The conductors could be represented by rectangular bars of the type shown in Figure 1A, so disposed that two of their flat surfaces are parallel to the general direction of the leakage flux.

2. That component of leakage flux parallel to the other two flat surfaces could be neglected.

3. That the defect in the length l of net copper (along leakage) compared to the total length of the leakage path l_1 could be accounted for by a fictitious permeability $\mu=l/l_1$ and is constant throughout the coil.

4. That the temperature and resistance of the conductors do not vary through the winding.

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Coil Loss and Effective Resistance

In the derivation outlined in the appendix additional assumptions have been made to the effect that the eddy-current losses in question are due only to the current in the coil being examined and that this current is in phase through all of the layers comprising the coil. Appropriate modifications for other cases are reasonably obvious and have been given by Lyon, among others.

It is shown in equation 33 that the loss in the nth layer is, in watts

$$wA = B_{\Delta}^2 C_1 A[(2n^2-2n+1)F_1+F_2] \qquad (1)$$

n which

B_{Δ} = increment in leakage-flux density through layer in lines per square inch
$C_1 = 1.935 m\rho 10^{-8}/\mu^2$
A = area in square inches of one surface of conductor parallel to leakage flux
$x = 1.01 r_1\sqrt{\mu f/\rho}$ (r_1 is in inches, see Figure 1B)
f = frequency in cycles per second
$F_1 \approx x^3/48 - x^7/18{,}660$
$F_2 \approx 2/x - 14x^3/1{,}440$
$\mu = l/l_1$
$m = 0.505\sqrt{\mu f/\rho}$
ρ = microhms per centimeter cube
w = watts loss per square inch of A

Precise definitions of F_1 and F_2 are given in the appendix.

From this result it follows that the sum of these losses from $n = 1$ to $n = N$ is, in watts

$$B_N^2 C_1 A\{(2N^2+1)F_1/3+F_2\}/N \qquad (2)$$

where B_N is the highest flux density associated with the Nth layer under the original assumptions. For comparison it is shown in equation 41 that the loss neglecting eddy current is $2B_N^2C_1A/Nx$. Therefore it follows that

$$R_{ac}/R_{dc} = (x/2)\{(2N^2+1)F_1/3+F_2\} \qquad (3)$$

When $x < 2$ the error involved in approximating F_1 and F_2 by writing their series to the first terms in x with positive exponents amounts to $+4.5$ per cent and -0.4 per cent, respectively. Making these substitutions we find that

$$W = B_N^2 C_1 A\{x^3(5N^2-1)/360+2/x\}/N \qquad (4)$$

$$R_{ac}/R_{dc} = x^4(5N^2-1)/720+1 \qquad (5)$$

So the proportionate increase in resistance due to eddy current is

$$R_e/R_{dc} = x^4(5N^2-1)/720 \qquad (6)$$

The restrictions on this formula are that $x < 2$, and those of the stated assumptions. For most practical designs in which eddy-current losses are on the order of ten per cent, this approximation gives results of slide-rule accuracy.

Accounting for differences in layer circumferences, it is shown in equation 38 that if a_0 and a_1 are, respectively, the radii (or circumferences) of the layers at zero and maximum leakage-flux densities, the loss for the coil is

$$W = (B_N^2 C_1 A/N)\left\{F_1\left[\frac{2N^2+1}{3} - \frac{3N^2+N+1}{6}\,\frac{a_0-a_1}{a_0}\right] + F_2\,\frac{(a_0+a_1)}{2a_0}\right\} \qquad (7)$$

From this and from equation 41 it follows that, for $x < 2$,

$$R_e/R_{dc} \approx \frac{x^4}{720}\left\{5N^2-1-\frac{15N^2+5N+5}{4}\,\frac{a_0-a_1}{a_0}\right\}\frac{2a_0}{a_0+a_1} \qquad (8)$$

Derivations for Minimum Loss

The minimum practicable loss in an a-c coil is usually desirable, and in order to investigate the relation between parameters for this condition it will be convenient to use equation 4. This may be treated as a variable in either N or x and the condition for minimum loss determined by setting the appropriate differential equal to zero.

This shows that when N is variable the losses are a minimum when $N \approx 12/x^2$, and that when x is variable the condition for minimum loss is that $x = \sqrt[4]{240/(5N^2-1)}$.

Substituting these relations successively in equation 5, we find that if N is variable minimum loss corresponds to $R_e/R_{dc} = 1$, and that if x is the variable minimum loss corresponds to $R_e/R_{dc} = 0.33$. This may seem a little surprising at first glance, but the first relation simply represents a less efficient use of conductor. It is necessary that the relationships be different in order that they be valid. For any particular frequency either relation can be shown to be a condition for a maximum in the ratio of reactance to resistance.

These two cases are of more academic than practical interest at standard frequencies except in the case of air-core reactors, since they assume that economy is unimportant as applied to conducting material and space. At radio frequencies, and even at lower frequencies, economy of conducting material and space is not important as very little of either can be used from considerations of energy economy.

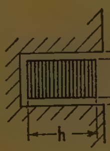

Figure 1A (left). Geometrically simple coil side assumed in derivation

Coil is surrounded on three sides by iron

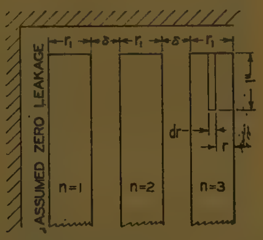

Figure 1B (right). Enlarged view of one corner of Figure 1A

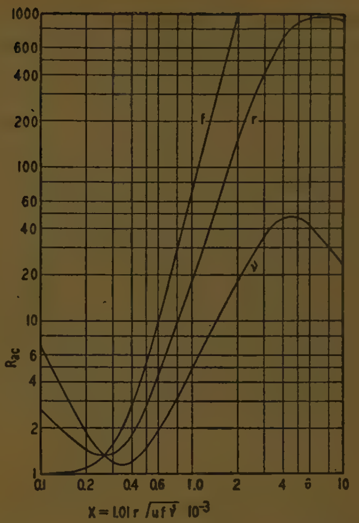

Figure 2. Effective resistance of 100-layer coil plotted against x

Three cases are shown to illustrate the effect of varying f, r_1, and γ independently

Equation 4 can be rewritten as a function of ρ by writing out the various terms included in C_1 and x. Then by setting the differential with respect to ρ equal to zero it can be shown that the losses are a minimum when ρ is such that $N \approx 12/x^2$. This is the same as the condition for minimum losses with N variable.

It can be shown that where both direct and alternating currents are present in a coil the total loss is

$$W = (2B_N^2 C_1 A/N)\{x^3(5N^2-1)/720+k_5/x\} \qquad (9)$$

where $k_5 = 1 + 2I_{dc}/I_{ac}$ in which the ratio is taken for the peak value of the a-c wave, and B_N refers only to the alternating component of flux. From this it can be shown that if x is taken as the variable the losses are a minimum when

$$x = \sqrt[4]{240k_5/(5N^2-1)} \qquad (10)$$

A somewhat more practical case is for square or round wire and for which the

Table I. Data on Two Small Coils Wound From 0.022-Inch Round Enameled Wire

Inside Diameter $^{13}/_{16}$ Inch; Outside Diameter $1^3/_{16}$ Inches, Eight Layers

	Long Coil	Short Coil
Turns	686	332
Length, inches	2	1
D-c resistance, ohms	3.88	1.93
Axial space factor for equivalent rectangular wire	0.83	0.83
Nagaoka's constant	0.828	0.703
Millihenrys at 1,000 cycles per second	3.97	1.93
μ	0.687	0.583
$x/f^{1/2}$	0.01273	0.01178

frequency, conductivity, coil length, and number of turns are fixed. The number of layers is now proportional to the wire diameter and may be substituted for in equation 7. By approximating F_1 and F_2, dropping certain terms that are not significant for $N>3$, and collecting terms, it is possible to arrive at a manageable expression for W as in equation 53; possible d-c loss also has been included. Then the square-wire diameter for minimum loss is determined to be approximately $2.76/(tf/l_2)^{1/3}$. The corresponding round-wire diameter is 12 per cent greater.

These simple approximations for the wire size for least loss are based on the assumptions that:

1. There is no external magnetic circuit or coupled coil.

2. There is no unidirectional component of current.

3. Coil length ≈ effective coil diameter and $a_0 \approx 2a_1$.

If these conditions are not all fulfilled a more exact solution for the wire diameter for minimum loss from equation 56 should be used. Any one of the foregoing items readily can influence the wire diameter by one gauge number (12 per cent).

Figure 7 shows the relation between frequency, turns per inch of coil length, and round-wire diameter for minimum loss based on the approximations in equation 57 outlined in the foregoing.

For comparison with the minimum-loss, cases in which N, X, or ρ are independent variables, the value of x from equation 55 can be substituted in equation 6, assuming that $k_5=1$ and that $a_0=a_1$ so that $k_6=0.5$. Then $R_e/R_{dc}=0.5$.

In this last case for least loss it has been assumed tacitly that in the absence of an external magnetic circuit (see Figure 1) the radial field could be ignored. Some justification is given for this later.

A situation that is more frequently of practical importance is the case in which dimension h of Figure 1A, that is, the radial thickness of the coil, is fixed, and the insulation required between laminations is of fixed thickness. Ordinarily extra insulation will be required between conductors, and still more extra insulation between coil sides, but these extras may be lumped together as a constant before determining h. In this case N is a function of x, or $N = 2mh/(x + 2m\delta)$; this value for N can be substituted in equation 7, and, after approximations that are reasonably good for $N > 3$, the value of r_1 for minimum loss can be found from

$$r_1 q/\delta = \sqrt[3]{\frac{9}{2\delta^2 h^2 m^4}\,\frac{2k_4}{3k_4-1}} \tag{11}$$

where $k_4 = (a_0 + a_1)/2a_0$, and $q = \{(1 + 3\delta/2r_1)/(1 + \delta/r_1)^2\}^{1/2}$; r_1/δ may be plotted as a function of $r_1 q/\delta$ and then this may be used as a step in determining r_1. As an approximation when $r_1/\delta > 1$, and to slide-rule accuracy when $r_1/\delta > 3$,

$$r_1 = \delta\left(0.17 + \sqrt[3]{\frac{9}{2\delta^2 h^2 m^4}\,\frac{2k_4}{3k_4-1}}\right) \tag{12}$$

An instructive approximation can be made from equation 12 when $r_1/\delta > 4$, and if the influence of k_4 is neglected, the value of r_1 for minimum losses becomes

$$r_1 \approx \sqrt[3]{\frac{9\delta}{2h^2 m^4}} \tag{13}$$

This can be substituted back into equation 6 along with the definition of N in terms of r_1, and the eddy-current loss for that value of r_1 that makes the total loss the least can be determined as

$$R_e/R_{dc} \approx (1/36)(m^4 h^2 \delta^2)^{1/3}(1-\delta/r_1)^2 \tag{14}$$

To take a numerical example assume that at 60 cycles

$m=2$
$h=6$ inches
$\delta=0.01$ inch

Then, for minimum-loss equation 12 determines r_1 to be 0.0444 inch and N to be 110 layers, and as $x = 0.1776$ equation 6 evaluates the eddy-current loss at 8.35 per cent. Equation 14 states that the eddy-current loss is $0.117\ (1 - \delta/r_1)^2$, and if the difference between this last factor and one is ignored the determination of eddy-current loss is rather rough. However, this factor involving δ/r_1 will vary only slowly as the other parameters of equation 14 vary, so that equation gives an accurate indication of the effect of these variables when $r_1/\delta>4$.

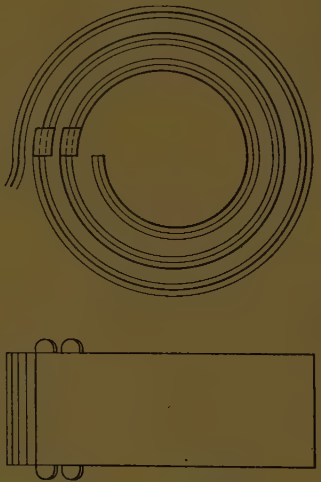

Figure 3. Three-turn spiral with three parallels showing simple method of transposing

Effect of Radial Component of Field

The work up to this point has assumed that the winding is surrounded on three sides by a medium of infinite permeability and no loss so that the loss for rectangular conductors could be determined quite precisely under the assumption of no radial component of field. If the external circuit be removed the situation becomes quite different, whether the conductors are round or rectangular.

By measurement on an eight-layer coil (described later) in which the length was about equal to the diameter, the field strength along the axis of the coil varied about 50 per cent from the center to the ends, and the radial component at the ends was not much less than the axial component at the ends. It would seem that in such a case, as far as losses are concerned, the presence of the radial field would just about make up for the deficit in the axial field. Experiment shows this to be the case, and, when the ratio of coil-length to diameter is greater than one, eddy-current losses can be approximated reasonably by using equation 6 with r_1 determined as the diameter of a square wire having the same cross section as the actual round wire. In this case μ should be determined as the space factor of the assumed square wire in the axial direction multiplied by a factor giving the ratio between the actual inductance and the inductance calculated from the long-coil formula. Such a factor can be approximated from Nagaoka's constant by using for coil diameter the actual diameter plus two thirds of the radial depth.

When $l_2/D>0.3$, Nagaoka's constant can be approximated to within two per cent by $1.02-1/(1+2l_2/D)$ in which l_2 is the axial length of the coil and D is the effective diameter of the coil.

It should be pointed out that, if the radial thickness of a winding is less than about one quarter of the effective coil diameter and also less than one quarter of the coil length, the eddy-current loss may be mainly the result of the radial component of field, and the formulas derived here cannot be used correctly.

These formulas are intended for concentrically coupled coils having equal and opposed ampere turns, and for other coils in which the coil geometry indicates that most of the eddy-current losses are due to the axial field. For concentrically coupled coils, the effective diameter can be taken as the radial distance between coils plus one third of the sum of their radial thicknesses.

In the case of coils wound with rectangular conductor with the flat of the conductor parallel to the principal magnetic field, the situation is not so clear, as a given radial field can produce much higher losses on account of having a wider conductor to work on.

It is quite apparent that the losses will be reduced by any steps that will reduce the radial component of field, and this dictates the use of coils that are long as compared to their diameter, and what amounts to the same thing, the close coupling of load coils either inside or outside the coil being studied.

As a one-turn-per-layer coil has no radial field except at the ends when energized by alternating current the use of such a coil may represent a step in the right direction when a considerable conductor area per turn is required. From a practical point of view such a coil has the additional recommendation that voltage stresses from steep-wave-front transients are distributed rather equally between turns, because the turn-to-turn capacity is large and quite uniform, and all except the first and last turns are quite well shielded from ground. Figure 3 is an illustration of such a coil, and it also shows a transposition system that can be used when the number of turns must be less than the number of layers.

It should be noted that if other conditions are appropriate the end losses for a one-turn-per-layer coil will be the same as for a two-turn-per-layer coil formed by assembling two spirals each half as long and suitably connecting them in series. It would seem that spiral coils would represent the most efficient construction possible since they have radial field losses only at the ends; a possible exception would be those cases in which conductors can be used for which the x measurement, determined for the radial field, can be less than one or two. This follows as otherwise the losses at the ends will be about the same in any case, and the spiral construction eliminates radial field losses through the rest of the coil.

In practical constructions the conductor width of such spirals will be limited to perhaps five inches or less by considerations of heat transfer, but a number of such spirals can be assembled end to end and connected in parallel to make up any desired conductor width, with spaces between the spirals for coolant circulation.

The end losses of such a spiral coil can be approximated from equation 1 for both ends as

$$\frac{R_a}{R_{dc}} \approx \left(\frac{B_a}{B_N}\right)^2 \frac{\mu^{3/2}}{(\mu')^{1/2}} \frac{2\pi a_0|a_0-a_1|}{A} Nx \qquad (15)$$

in which B_a is the radial component of field strength and μ' is the radial space factor. This is not a particularly satisfactory approximation as it assumes the radial field to be constant over the end of the coil, but it at least will indicate the magnitude of the end loss to be expected.

Experimental Measurements

ROUND-WIRE COILS

In order to test the application of formulas derived for rectangular conductors and a partial magnetic circuit to coils from round wire and surrounded by air, two small coils were wound from 0.022-inch round enameled wire, each coil having an inside diameter of 0.81 inch, an outside diameter of 1.19 inches, and eight layers. Other information is given in Table I.

Table II. Observations on 60-Turn Spiral Coil of Electrolytic-Copper Ribbon

One Inch Wide by 0.0035 Inch Thick

a_0, inches radius	0.97
a_1, inches radius	0.437
D-c resistance, ohms	0.0715
Effective ribbon thickness, inches	0.0025
μ	0.642
μ'	0.281
$m/f^{1/2}$	0.3075
Millihenrys at 10,380 cycles per second	0.0706

Table III

x	0.157	0.174	0.206	0.226
R_a/R_{dc}	2.33	2.95	3.43	3.87
R_a/R_{dc} corrected	2.45	2.82	3.43	3.87
B_a/B_N calculated	0.71	0.724	0.731	0.743

The effective resistance was measured by providing a mica decade capacitor for parallel resonance and driving from the 5,000-ohm tap of an oscillator through a 33,000-ohm decoupling resistance that was noninductive. The voltage across the parallel-resonant circuit was measured by a vacuum-tube voltmeter, and then a decade resistance was substituted for the parallel circuit and so adjusted that the voltage across the resistance was the same as that which had appeared across the parallel circuit. If the capacitor loss is charged to the inductance the effective series resistance of the inductance is given by the formula: inductance÷(capacitance times substitution resistance).

This method is not precise but for frequencies such that $x < 2$ the change in inductance with frequency will be very small and the method is reasonably satisfactory. The reactance–resistance ratio of the coils used was roughly from 10 to 30 for frequencies from 2,500 cycles to 20,000 cycles. At the upper limit of frequency the method begins to introduce appreciable inaccuracies on account of the inductance in the resistance decade, but a check by using a carbonized resistance indicated that the error was only two or three per cent.

The results are shown in Figure 6 as two sets of points plotted and the theoretical curve of equation 8 drawn in.

EQUATION 56

Another test of the same nature, but intended specifically as a check on equation 56, was made on a series of five 2,000-turn coils and at the single frequency of 10,000 cycles. These coils were each one inch long, they were layer wound but not layer insulated, and they were wound on wooden bobbins of 0.75-inch diameter.

A series resonance and substitution method was used to determine the effective resistance, using either 0.003 or 0.004 microfarad of a mica decade plus a variable air capacitor.

The following observations were made at 10,000 cycles:

Nominal wire diameter, mils	8.9	10.0	11.3	12.6	15.9
Outside coil diameter, inches	1.12	1.19	1.39	1.52	1.92
D-c resistance, ohms	65.5	54.1	44.8	39.3	29.2
R_{ac} by equation 8, ohms	71.6	63.9	59.8	63.8	93.0
R_{ac} measured, ohms	74.5	66.5	64.3	68.0	91.0

The measured value of R_{ac} includes a capacitor resistance correction arbitrarily figured for 0.001 power factor. This amounts to five ohms for the first three columns and seven ohms for the last two columns. Correction is necessary since the reactance/resistance ratio of some of the coils was above 60.

The round-wire diameter for minimum loss as determined from equation 56 is 0.0107 inch which is confirmed by the foregoing experimental result of 0.0110 inch.

EFFECT OF SMALL RADIAL DEPTH OF WINDING

By way of emphasizing the inherent defects of the approximations used, in the absence of an external magnetic circuit and for the case in which the radial depth of the winding is small in comparison to the effective diameter of the coil, three coils of 2.375 inch inside diameter were wound from 0.010 round wire.

Data on the coils and measurements at 10,000 cycles follow:

Total turns	600	1,000	1,250
Number of layers	12	8	5
Coil length, inches	0.58	1.45	2.90
Radial thickness, inches	0.180	0.12	0.075
D-c resistance, ohms	41.8	69.0	84.7
R_{ac} by equation 8, ohms	42.4	70.2	85.2
R_{ac} measured, ohms	49.2	92.0	155

The same correction for capacitor power factor was made in the foregoing case as was made for the set of five coils.

It is obvious from the geometry of these coils that most of their eddy-current loss will result from the radial field when there is no external magnetic circuit, and so any method based on axial field losses only must be seriously in error.

END LOSS IN SPIRALS

Experimental work on small-scale spiral coils is complicated by the low impedances involved, and the following account is not presented as being particularly conclusive, but rather as being the only experimental evidence available relative to equation 15.

A 60-turn spiral coil of electrolytic-copper ribbon 1 inch wide by 0.0035 inch thick was wound and the observations listed in Table II were made.

Measurements were made with parallel capacitances from 0.768 to 3.33 microfarads corresponding to frequencies from 10,380 to 21,660 cycles and substitution resistances from 89 to 203 ohms. The circuit was driven from the 125-ohm tap of the oscillator through a 500-ohm resistance. Results were calculated as shown in Table III.

The corrected values were arrived at by plotting on logarithm paper and drawing the best possible straight line to fit the points. Values of B_a/B_N were calculated from equation 15 as the curves of Figure 4 indicate that the maximum value of eddy-current loss would be about six per cent; these values substituted into equation 15 give the corrected end-loss factors. The value of B_a/B_N was measured with a 20-turn one-quarter-inch-diameter search coil, and the maximum that could be measured at 10,000 cycles was just under 40 per cent. This makes it seem questionable as to whether the actual value could have been as large as the 70-per-cent tabulated indication. The measuring method would be expected to give results that are substantially too small, and the corrected end-loss factor is too large as it includes both the axial-field loss and the capacitor loss.

Comparison With Literature

Equation 1 is equivalent to equation 12 of Field's[1] paper and to equation 27 of Bennett and Larson's.[6]

Equation 6 is a refinement of an equation given in the appendix of Summers' paper[5] and is equivalent to Taylor's[2] equation 3.

Equation 3 is exactly equivalent to an expression given by Lyon[3] for the loss factor in a one-coil-side-per-slot winding of N layers. To demonstrate this it is necessary only to expand the hyperbolic functions of complex arguments given there into the corresponding combination of hyperbolic and circular functions of real arguments. The latter form seems to be much easier to look up in tables, but the functions of complex arguments in this case are much easier to expand into series if the series is to be carried beyond the first positive exponent with a real coefficient.

Conclusions

The formulas presented here to measure the eddy-current resistance of coils with partial magnetic circuits (but excluding the losses in the iron) are equivalent to earlier formulas that have been reasonably well tested by a number of years' experience.

The extension of these formulas to coils wound from round wire and without an external magnetic circuit by the device of correcting for the end effect with a fictitious permeability and for the wire shape by using an equivalent square section seems to give results of usable accuracy, at least up to frequencies for which the eddy-current resistance is 50 per cent of the d-c resistance. This may be subject to a further limitation that the coil length should be at least as great as half the diameter. It will take more experimental work to establish such limitations definitely.

The extension of the original formulas to coils from ribbon conductors wound spirally to give a net effect of one or two turns per layer seems to be justified by the addition of another formula to calculate the end-effect loss as a separate item. While the end-effect formula cannot be regarded as satisfactory, in most practical constructions it will be possible to keep the end losses quite low simply by keeping B_a/B_N low, and it seems reasonable to expect that the end losses will be substantially smaller than the axial-field loss.

Another generally practicable method of keeping down the loss due to radial field is the construction shown in Figure 1; as long as the iron does not saturate, the

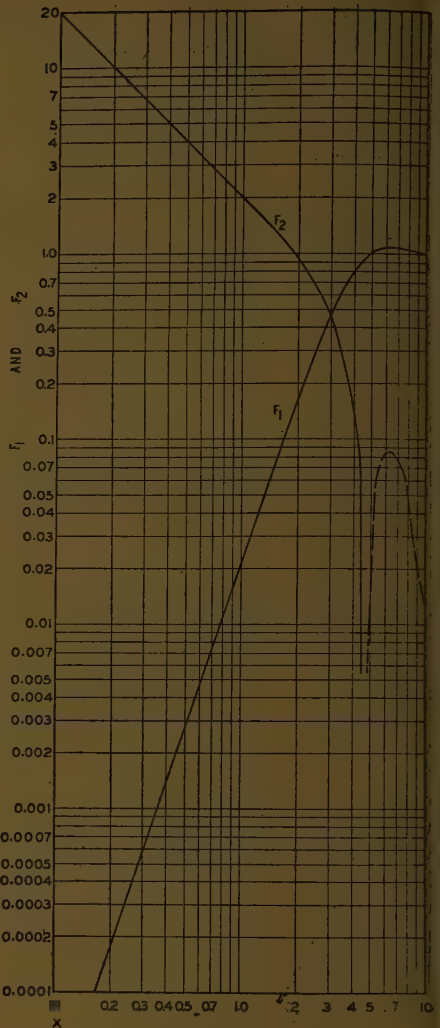

Figure 5. F_1 and F_2 plotted against x

Broken line indicates negative values

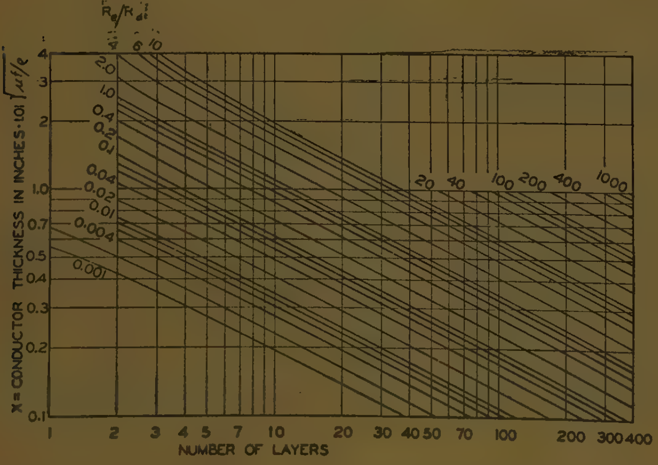

Figure 4. R_e/R_{dc} as a function of x and N, for the case that neglects the difference in length of layers and assumes arrangement of Figure 1A

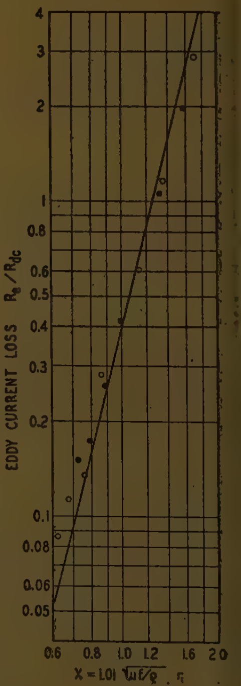

Figure 6. Eddy-current resistances plotted against x for round-wire coils

Circles are for long coil; dots are for short coil. Theoretical curve of equation 8 is solid line

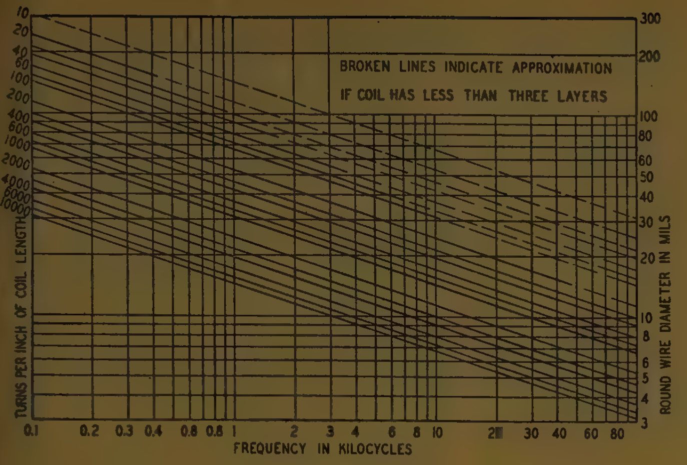

Figure 7. Relation of frequency, turns per inch of coil length, and round-wire diameter for minimum loss

Taken from equation 57 with $\rho=1.8$

radial field will approach zero as l_1 approaches l.

Certain formulas have been derived for determining conditions for minimum losses, and as these have been arrived at by conventional mathematical methods they seem certainly valid. The end losses of spiral coils have been ignored in these derivations, because the end losses are determined for the most part by coil proportions and not by layer thickness. These formulas are presented, not as requirements of logical design, but as a formulation of simple standards against which logical designs can be measured, and as exploring tools.

It should be noted that when the layer thickness of spiral coils is selected for minimum loss the total loss varies as $N^{-1/2}$. This relation is quite precise for $N>4$, and for $N=1$ the theoretical minimum of loss occurs for $x=3.14$ and is about eight per cent less than that determined by $N^{-1/2}$.

As a speculative item, the possibility is indicated of increasing the output of such equipment as transformers, while decreasing the amount of active material and increasing the efficiency, by the simple expedient of increasing the winding section in a direction perpendicular to the principal leakage flux. There will be of course cases in which such improvement is difficult or impossible.

Appendix

Practical units and inch dimensions are used throughout this paper except that ρ is in microhms per centimeter cube, and γ is in mhos per centimeter cube to correspond to customary usage in tables.

Notation is defined where first used, but for convenience those terms used more than once are defined below;

a_0 = radius of turn at zero leakage-flux density
a_1 = radius of turn at maximum leakage-flux density
A = area of layer at zero flux density, measured parallel to leakage
B = flux density at plane of dr in nth layer in lines per square inch
B_N = highest flux density associated with Nth layer in lines per square inch
B_Δ = increase in flux density per layer in lines per square inch
$C_1=1.935m\rho 10^{-8}/\mu^2$
e = resistance drop per inch of periphery at plane of dr
f = frequency in cycles per second

$$F_1=\left\{\sinh x+\sin x-2\left(\cosh\frac{x}{2}\sin\frac{x}{2}+\sinh\frac{x}{2}\cos\frac{x}{2}\right)\right\}/(\cosh x-\cos x)$$

$$F_2=2\left(\cosh\frac{x}{2}\sin\frac{x}{2}+\sinh\frac{x}{2}\cos\frac{x}{2}\right)\div(\cosh x-\cos x)$$

i = current density in amperes per square inch at dr
$k_1=(n-1)/n$
$k_2=(a_0-a_1)/(N-1)a_0$
$k_3=(a_0-a_1)/2a_0$
$k_4=(a_0+a_1)/2a_0$
l = net conductor dimension in inches measured along leakage flux
l_1 = length of leakage-flux path in inches
l_2 = axial coil length in inches
$m=0.505\,(\mu f/\rho)^{1/2}$
n = layer number counting from zero leakage-flux density
N = total number of layers in winding being studied
r = distance of plane of dr from high flux-density side of layer in inches
r_1 = thickness of layer in inches as measured across leakage flux
w = total loss in watts for one square inch of the flat surface of layer n
W = total loss in watts for N layers
$x=2mr_1$
γ = conductivity in mhos per centimeter cube
δ = insulation thickness in inches
ρ = resistivity in microhms per centimeter cube
μ = fictitious permeability l/l_1 or lL/l_2L_1
R = resistance as denoted by subscripts; R_a/R_{dc} is proportionate increase in resistance due to end loss in both ends of spiral
L = actual inductance
L_1 = inductance calculated by long-coil formula (that is, without correction for end effect)

Derivation for Loss in Layer n

This part of the derivation is not new and so is presented in outline only. The method is simular to that of Bennett and Larson, in that the differential equation is set up for the flux density. This permits a direct substitution for the boundary conditions and a differentiation to determine current density.

It will be assumed that one side of a spiral coil is shown in section in Figure 1, and that the conditions are as set forth in the introduction. It will be convenient to take as the differential element of the conductor a piece one inch by one inch in the directions of current flow and leakage flux, with a thickness of dr inches measured across the leakage flux.

Let the voltage impressed on any particular layer of this spiral coil be E volts per inch of periphery, and let the induced voltage be e' per inch at the position of dr, a thin section parallel to the flat of the conductor and located r inches from that side of the conductor that is at the highest flux density. Then, if e is the resistance drop in volts per inch,

$$E+e'=e \tag{16}$$

If γ is the conductivity in mhos per centimeter cube, and i is the current density in amperes per square inch of conductor cross section,

$$2.54e\gamma=i \tag{17}$$

Due to idr, there is, in lines per square inch,

$$dB=0.4\pi 2.54^2e\gamma\mu dr=8.1e\gamma\mu dr \tag{18}$$

in which μ is the ratio of the leakage path to the net conductor along that path. If the difference in e' on the two sides of dr per inch of periphery is called de', then

$$de'=-j2\pi fBdr10^{-8}=de \tag{19}$$

and from equation 18,

$$\frac{d^2B}{dr^2}=8.1\gamma\mu\frac{de}{dr} \tag{20}$$

If we substitute from equation 19,

$$\frac{d^2B}{dr}=-j50.8f\gamma\mu B10^{-8}=-2jm^2B \tag{21}$$

A solution of equation 21 is

$$B=A_1\epsilon^{(1-j)mr}+A_2\epsilon^{-(1-j)mr} \tag{22}$$

If n is the layer number counting from the region of zero leakage flux, the boundary conditions can be stated thus: when $r=0$, $B=B_n$; when $r=r_1$, $B=B_{n-1}$;

$$B_{n-1}=[(n-1)/n]B_n=k_1B_n \tag{23}$$

If we eliminate A_1 and A_2:

$$B=B_n\frac{\left\{\begin{matrix}k_1(\epsilon^{(1-j)mr}-\epsilon^{-(1-j)mr})+\\ \epsilon^{(1-j)m(r_1-r)}-\epsilon^{-(1-j)m(r_1-r)}\end{matrix}\right\}}{\epsilon^{(1-j)mr_1}-\epsilon^{-(1-j)mr_1}} \tag{24}$$

$$\frac{dB}{dr}=\frac{B_n(1-j)m\left\{\begin{matrix}k_1\cosh mr\cos mr-\\ \cosh m(r_1-r)\cos m(r_1-r)\\ -j[k_1\sinh mr\sin mr-\\ \sinh m(r_1-r)\sin m(r_1-r)]\end{matrix}\right\}}{\sinh mr_1\cos mr_1-j\cosh mr_1\sin mr_1} \tag{25}$$

The rate of power loss per cubic inch of conductor in watts is

$$2.54e^2\gamma/2 \tag{26}$$

If the conductor is measured on one of its flat sides (parallel to principal flux) the loss per square inch of conductor is in watts:

$$\int_0^{r_1}\frac{2.54e^2\gamma}{2}dr=\int_0^{r_1}\left|\frac{dB}{dr}\right|^2\frac{dr}{51.6\gamma\mu^2}=w \tag{27}$$

Let

$$C_1=m/51.6\gamma\mu^2 \tag{28}$$

$$w=\frac{2mB_n^2C_1\times\int_0^{r_1}\left\{\begin{matrix}k_1^2(\cosh 2mr+\cos 2mr)+\\ \cosh 2m(r_1-r)+\\ \cos 2m(r_1-r)\\ -2k_1[\cosh mr_1\times\\ \cos m(r_1-2r)+\\ \cosh m(r_1-2r)\cos mr_1]\end{matrix}\right\}dr}{\cosh 2mr_1-\cos 2mr_1} \tag{29}$$

$$=\frac{B_n^2C_1\left\{\begin{matrix}(k_1^2+1)(\sinh 2mr_1+\sin 2mr_1)-\\ 4k_1(\cosh mr_1\sin mr_1+\\ \sinh mr_1\cos mr_1)\end{matrix}\right\}}{\cosh 2mr_1-\cos 2mr_1} \tag{30}$$

$$=B_n^2C_1\{(k_1^2+1)F_1+F_2/n^2\} \tag{31}$$

in which F_1 and F_2 have the meanings defined under notation.

Now let the increment of flux density per layer

$$B_n/n=B_\Delta \tag{32}$$

$$wA=B_\Delta^2C_1A\{(2n^2-2n+1)F_1+F_2\} \tag{33}$$

Total Loss for N Layers

$$\Sigma wA=W$$

$$=B_\Delta^2C_1A\{F_1\sum_1^N(2n^2-2n+1)+NF_2\} \tag{34}$$

$$W=B_N^2C_1A\{F_1(2N^2+1)/3+F_2\}/N \tag{35}$$

It will be found that for $x<2$ the foregoing expression can be closely approximated by writing the series expansions for F_1 and F_2 to include the first term in x with a positive exponent. If we make this substitution,

$$W=B_N^2C_1A\{x^3(5N^2-1)/360+2/x\}/N \tag{36}$$

Correction for Differences in Layer Lengths. In the foregoing two equations no account has been taken of the difference in periphery of the various layers of a spiral coil. As the area of the layer at zero leakage-flux density has been taken as A, the area of any other layer will be $A\ \{1-(n-1)k_2\}$, where $k_2=(a_0-a_1)/(N-1)a_0$; a_0 is the length of the layer at zero flux density, and a_1 is the length of the layer at maximum flux density. Then:

$$W=B_\Delta^2C_1A\{F_1\sum_1^N(2n^2-2n+1)\times$$

$$[1-(n-1)k_2]+F_2\sum_1^N[1-(n-1)k_2]\} \tag{37}$$

$$=\{B_N^2C_1A/N\}\left\{F_1\left(\frac{2N^2+1}{3}-\frac{3N^2+N+1}{6}\frac{a_0-a_1}{a_0}\right)+F_2\frac{(a_0+a_1)}{2a_0}\right\} \tag{38}$$

Effective Resistance and Resistance Ratio. The most useful form in which to have information on eddy-current losses is frequently an expression for R_{ac}/R_{dc} or a related expression $(R_{ac}-R_{dc})/R_{dc}=R_e/R_{dc}$. To obtain such an expression it will be necessary to state the d-c loss in terms conveniently comparable to equation 36. If l is net conductor width measured along leakage flux in inches,

$$R_{dc}=NA(a_0+a_1)/5.08r_1l^2\gamma a_0 \tag{39}$$

$$\text{D-c loss}=\frac{B_N^2A}{25.8\gamma\mu^2Nr_1}\frac{a_0+a_1}{2a_0}$$

$$=\frac{4B_N^2C_1A}{Nx}\frac{a_0+a_1}{2a_0} \tag{40}$$

Since for a given value of B_N and ignoring eddy-current losses the a-c loss will be one half of the d-c loss, this a-c loss may be called W' and then $R_{ac}/R_{dc}=W/W'$. If layer length differences are neglected,

$$W'=2B_N^2C_1A/Nx \tag{41}$$

$$R_{ac}/R_{dc}=x^4(5N^2-1)/720+1 \tag{42}$$

Determination for Minimum Loss With Dimension h Restricted

To determine the layer thickness for minimum loss when dimension h is restricted, it is expedient to make the usual series approximations for F_1 and F_2 in equation 38 that are quite accurate for $x<2$, and to assume that $N>3$, so that the loss equation becomes

$$W\approx B_N^2C_1A\left\{\frac{x^3N}{72}\left(1-\frac{3}{2}k_3\right)+\frac{2}{Nx}(1-k_3)\right\} \tag{43}$$

Since

$$N=2mh/(x+2m\delta)$$

$$W\approx B_N^2C_1A\left\{\frac{hm}{36}\left(1-\frac{3}{2}k_3\right)\left(x^2-2mx\delta+4m^2\delta^2-\frac{8m^3\delta^3}{x+2m\delta}\right)+\left(\frac{1}{mh}+\frac{2\delta}{hx}\right)(1-k_3)\right\} \tag{44}$$

$$\frac{dW}{dx}=B_N^2C_1A\left\{\frac{hm}{36}\left(1-\frac{3}{2}k_3\right)\left(2x-2m\delta+\frac{8m^3\delta^3}{(x+2m\delta)^2}\right)-\frac{2\delta}{hx^2}(1-k_3)\right\}=0 \tag{45}$$

$$\frac{1-\frac{3}{2}k_3}{1-k_3}\ \frac{h^2mx^4}{18}\ \frac{x+3m\delta}{(x+2m\delta)^2}=2\delta \tag{46}$$

$$\frac{3k_4-1}{2k_4}h^2m^4r_1^3\frac{(1+3\delta/2r_1)}{(1+\delta/r_1)^2}=\frac{9\delta}{2} \tag{47}$$

$$\frac{r_1^3}{\delta^3}\frac{1+3\delta/2r_1}{(1+\delta/r_1)^2}=\frac{9}{2\delta^2h^2m^4}\ \frac{2k_4}{3k_4-1} \tag{48}$$

$$\frac{r_1}{\delta}q=\sqrt[3]{\frac{9}{2\delta^2h^2m^4}\ \frac{2k_4}{3k_4-1}} \tag{49}$$

$$r_1\approx\delta\left(0.17+\sqrt[3]{\frac{9}{2\delta^2h^2m^4}\ \frac{2k_4}{3k_4-1}}\right) \tag{50}$$

when $r_1/\delta>1$

Minimum Loss for Round- (or Square-) Wire Coils

When the axial length l_2 and the number of turns t are fixed, the number of layers is a function of the diameter d of round wire, so that

$$N=tr_1/l=tx/2ml \tag{51}$$

in which the only variable is $r_1=0.89d$, and the axial space factor of the winding l/l_2 is assumed independent of wire size.

For the present purpose equation 43 can be revised to include d-c loss if any, and slightly rearranged, so that

$$W\approx U(x^3N/72+4k_5k_6/NX) \tag{52}$$

where

$$k_6=1-2a_1/(a_0+3a_1)$$

and

$$U=B_N^2C_1A(1-3k_3/2)$$

Substituting from equation 51

$$W\approx U(x^4t/144ml+8k_5k_6ml/tx^2) \tag{53}$$

$$dW/dx=U(x^3t/36ml-16k_5k_6ml/tx^3)=0 \tag{54}$$

$$x=2.88(k_5k_6)^{1/6}(ml/t^{\ 1/3} \tag{55}$$

$$r_1=1.44(k_5k_6)^{1/6}(l/m^2t)^{1/3}$$
$$=2.27(k_5k_6/K^2)^{1/6}(\rho l_2/ft)^{1/3} \tag{56}$$

since

$$m^2=0.254fKl/l_2\rho$$

in which K = Nagaoka's constant.

If there is no d-c loss $k_5=1$; $(k_6/K^2)^{1/6}$ usually will exceed unity by not more than a few per cent. Then if $\rho=1.8$ for copper,

$$d=r_1/0.89=3.11/(ft/l_2)^{1/3} \tag{57}$$

In this equation d is the round-wire diameter for minimum loss, and r_1 is the square-wire diameter for minimum loss, provided that $N>3$. The approximation is fair for $N=2$.

It is interesting to note that the wire diameter for minimum loss is independent of the axial space factor (assumed constant) except as this influences K and k_6.

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Lightning Arresters for Distribution Apparatus

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THERE are today two general types of lightning arresters in use for the protection of distribution apparatus. They are the valve type covered by AIEE Standard 28, and the expulsion type, for which AIEE Standards are being formulated. Both have been used widely for many years. Typical specimens are illustrated in Figure 1. The valve type is the older. The first distribution-valve arrester was an Autovalve produced in 1922. The first expulsion-type distribution arrester was the deion arrester (United States Patent, 2,050,397) used on surge-protected distribution transformers in 1931. By far the largest number of expulsion-type arresters manufactured have been applied as part of completely self-protected transformers. Many of these have used a somewhat different physical construction and mounting as shown in Figure 2 than that used for the arresters installed separately from the transformers. There exist today several variations of both the expulsion and valve types.

With so many years of service behind the two types, the question arises why both still persist. Why has not the industry decided that one is better and discarded the other? Why do some manufacturers make both types, and some only one? Affiliated with a company that manufactures both types, the authors frequently are asked, not only by users but also by associates in the company, why the company does not eliminate one of them and concentrate on the other. From a manufacturing standpoint this would be a highly desirable simplification, but the reason why both are made is that there are demands for both. Apparently the industry is not ready to accept only one in lieu of both.

A brief review of the history that stimulated the two types is of passing interest. There were two major reasons for the development of the expulsion type to parallel the valve type. These were radio broadcasting and the growth of rural electrification.

Valve-type arresters in the 1920's were contemporary with the growth of radio broadcasting into a public service. In those days the valve arresters were frequent causes of radio interference as a result of moisture collecting inside of them. Methods of testing for radio interference were not developed, and arresters were not designed to avoid it as it was not a known requirement. It soon became evident, however, that it was a very important requirement. Many engineers believed that it was impossible to seal an arrester against moisture entrance and that an arrester should be thoroughly ventilated. The expulsion arrester is such a device, and this was then greatly in its favor. At the present time radio interference is hardly a worry in either valve or expulsion arresters since methods of sealing and testing valve arresters have been devised that effectively prevent moisture collection. Autovalve arresters manufactured since 1932 have been signally free from moisture or radio interference.

Most expulsion-type arresters, because of the requirement of clearing appreciable power-follow currents, inherently possess a high degree of ability to discharge severe lightning current without damage to themselves. In this respect they surpass the early valve types. The importance of surge capacity became more apparent as rural electrification proceeded. It might be said that the appearance and the successful operation of the expulsion-type arrester threw a challenge to the valve type which resulted in a general increase in quality of both types of arresters. This is a healthy condition, consequently, it is well that the industry has not yet settled on one type to the exclusion of the other.

The Functions of Lightning Arresters

Fundamentally lightning arresters, regardless of type, must perform the same function, to provide by-passes around insulation. When lightning strikes a circuit, transient currents flow in the system. With these currents are associated potentials that are functions of the current and the impedance to the current flow. If the flow of lightning current is not obstructed by impedance or insulation, the voltages between points that are electrically adjacent in the circuit are small, but, if the flow is obstructed, very high voltages may appear, because the lightning current puts forth all its effort to flow. If the obstruction consists of insulation, the voltage developed across it may be sufficient to puncture or flash around it, thereby giving the current a means of continuing by way of damage or outage. A lightning arrester provides an easy path for the lightning current to get around the insulation so that high voltages will not develop. The by-pass, however, must be such that it does not interfere with the supply of power to users. This means that, when operating conditions are normal with no surges on the system, the arrester must be an insulator, so that it prevents flow of power current. When a transient voltage appears which might exceed the insulation strength of the apparatus to be protected, the arrester must become as quickly as possible a good by-pass for lightning current. After the release of this current, the by-pass must transform itself back to its insulation condition so quickly that no circuit breakers or fuses in the circuits are called upon to open in order to interrupt flow of system current through the arrester.

A lightning arrester may be looked upon as a very fast circuit breaker connected from line to ground, normally open, but

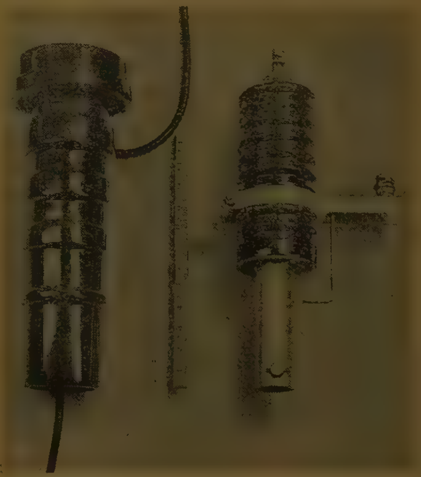

Figure 1. Typical distribution lightning arresters of the valve and expulsion types

Rating, nine kilovolts

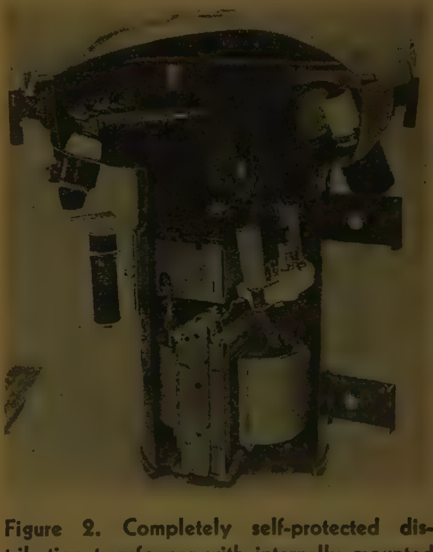

Figure 2. Completely self-protected distribution transformer with internally mounted expulsion-type arresters

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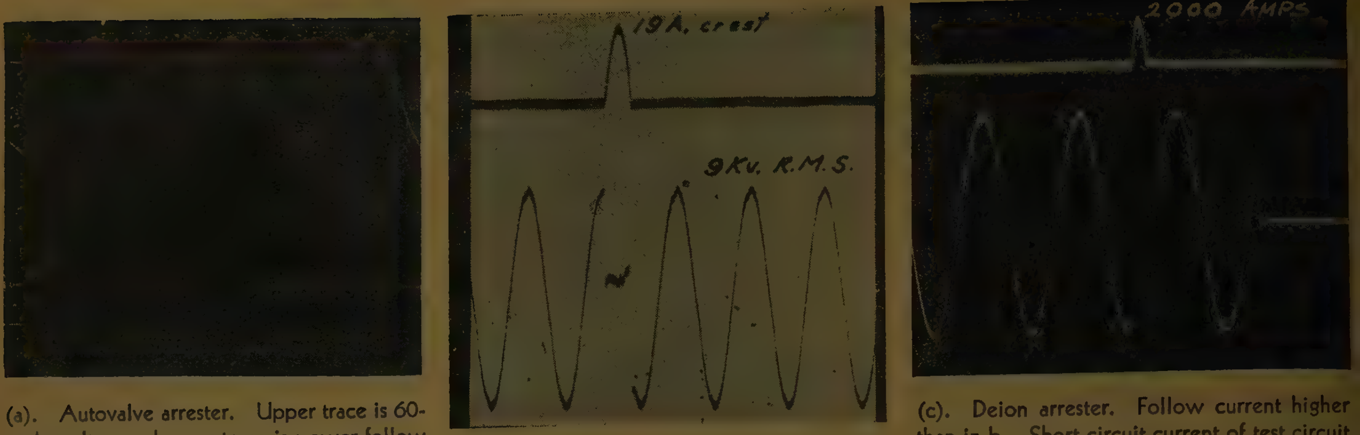


(a). Autovalve arrester. Upper trace is 60-cycle voltage. Lower trace is power-follow current. Crest of follow current is 36 amperes. Current limited by valve element only as apparent from uniformity of voltage wave

(b). Deion arrester. Lower trace is 60-cycle voltage. Upper trace is follow current limited by circuit

(c). Deion arrester. Follow current higher than in b. Short-circuit current of test circuit was 10,000 amperes rms. Arrester limits follow current

Figure 3. Typical oscillograms of power-follow currents and their interruption for valve and expulsion arresters

Arrester ratings, nine kilovolts
Power voltage used in tests, nine kilovolts rms

one which closes immediately at a predetermined voltage and reopens quickly after the disturbances have passed. The closing mechanism in the lightning arrester is a spark gap of some form which becomes conducting when a transient voltage appears. Thus, the circuit for the lightning current is established, but this circuit is then also a path to ground for system fault current. Arresters therefore must incorporate a contrivance that will open this power current path or aid the spark gap to reopen it. Simple gaps, such as rod gaps, are not classed as arresters, because, although they will provide more or less effective by-passes for lightning currents, they are unable to interrupt the path. Their power-follow currents must be cleared from the circuit by the operation of a circuit breaker or a fuse. The lightning arrester takes care of this by itself. Both types of distribution arresters accomplish the closing and reopening of the relief circuit in essentially the same fashion. The differences lie principally in the process involved in re-establishing the insulation of the arrester and in the magnitudes of the impulse voltages that appear across the terminals of the devices.

The simplest analogy to the expulsion type of arrester is that it is like a fuse with the fuse link omitted. It consists essentially of a tube, usually of fiber, with electrodes in each end. In series with this is a spark gap. When a high voltage occurs, the series gap and the gap in the tube spark. By this process a circuit is made through the tube just as if a fuse link had been present. The arrester thus becomes a path of low impedance, and the surge voltage across the terminals drops to a low voltage after spark-over. This voltage drop consists only of the arc drop in the extinction chamber. Therefore, as far as power-follow current is concerned, the device may become nearly a short circuit. However, like the fuse, it interrupts the system fault current when its current wave passes through zero because of the gas evolved by the heat of the arc from the walls to the extinction chamber. Unlike the fuse, it does not have to be refilled, because it has no fuse link. The expulsion-type arrester operates repeatedly without attention. The devices are so designed that their spark-over is sufficiently low to provide protection for the apparatus or insulation they are intended to protect. In some recent constructions for use on distribution circuits, there are incorporated features that produce appreciable arc voltage while power current flows, so that the power current follow is determined partly by the device and partly by the system characteristics. This arc voltage may be an appreciable part of the normal circuit voltage, but compared to the spark-over voltage it is small.

The expulsion arrester thus performs the cycle required of a lightning arrester. It is normally an insulator, it becomes a conductor of low impedance when necessary to prevent high voltage, and it interrupts the system current quickly and reverts to an insulator. Its operation does not interfere with the flow of power in the system.

A familiar analogy for the valve type of arrester is not so easily found, unless it be likened to an automatic valve that opens and closes as circumstances require. The valve arrester, too, has a spark gap which normally provides insulation and operates as a switch to close by sparking when a surge voltage appears. Unlike the expulsion arrester, the series gap in the valve arrester also must reopen the circuit after the necessity for limiting the voltage has passed. Simple air gaps are not capable of interrupting high currents such as might be involved in system faults. Gaps such as are used in valve arresters can interrupt reasonably high alternating currents, of 50 to 100 amperes, but not much more. Therefore, to insure successful operation of the device, the power-follow current that flows after the lightning discharge must be limited to a magnitude that the gap can interrupt. This can be accomplished by series resistance. However, if this is the common type of constant resistance and of such a magnitude that it will limit the power-follow current to the afore-mentioned value when system voltage only is impressed across its terminals, then when high lightning currents flow the *IR* drop will be large. This would impress high voltage on the insulation supposedly protected. An example, would be a nine-kilovolt arrester with a constant series resistance that will limit the current at nine kilovolts to 100 amperes. If a surge current of 2,000 amperes flows through it, it will develop 20 times normal voltage or 180 kv. Such a device would be inadequate for purposes of lightning protection. For this reason the series impedance used in valve arresters is of a special kind that has a high effective resistance at low voltage and a low effective resistance when high currents flow. It is for this reason that they are called valve elements. The impedance or the valve opening regulates itself to the flow of current and limits the voltage. The regulation in most valve elements is a property of the electrical contacts between the particles of which it consists. If we choose to think of the valve element as a resistor, then in a 3,000-volt valve-type arrester its apparent resistance at 3,000 volts may be 100 or more ohms, whereas when it is passing 50,000 amperes of surge current its apparent resistance may be less than half an ohm.

By the co-operative functioning of the series gap and the valve element, the valve type also performs the functions expected of an arrester. It is normally an insulator, it becomes a path of relatively low impedance, and it re-establishes itself as an insulator after the lightning discharge has passed. It does not interfere with the flow of power in the system.

The principal difference in the operation of the two types of arresters is in the way in which they dispose of the system follow current. In the valve arrester the follow current is limited by the device itself regardless of system capacity. Valve arresters, therefore, are rated only on the basis of voltage. The rating defines the maximum voltage applied across the arrester terminals against which the arrester can return itself to an insulator subsequent to a surge discharge. In the expulsion arrester the follow current may be determined principally by the system and these devices generally are rated in both voltage and current somewhat like fuses. As far as service is concerned, the difference between the two is not significant because either device, as shown by Figure 3, operates without causing what is generally termed a fault.

Protective Characteristics

The most important quality that an arrester should possess is the ability to protect insulation against high voltage. The magnitudes to which it limits surge voltages are related in various ways to the power voltage which the arrester is able to clear.

Valve and expulsion arresters both provide surge protection for distribution apparatus. This can be seen by comparison of the impulse characteristics of the arresters with the standard impulse levels of the equipment. The major use of distribution lightning arresters is to protect distribution transformers against damage to their insulation or flashover with consequent outage. A ready means of comparison is found in the relation of the lightning-arrester characteristics to the impulse withstand tests required of transformers. The transformer manufacturers have, in the National Electrical Manufacturers Association Transformer Standards stated certain impulse voltages which the transformers covered by these standards are able to withstand without damage. The test voltages are described and coordinated with the insulation class designations of the transformers as in Table I They provide a definition of the impulse strength of transformers and thus a means of relating the lightning protection to them directly. Present AIEE Standards for valve arresters specify a number of tests which are complete and possibly more voluminous than necessary to furnish the industry with adequate information of the characteristics of these protective devices. The tests described in lightning-arrester standards are not related directly to the withstand impulse-voltage tests specified for transformers, although their relation can be determined by reference to curves. A simple and complete comparison between transformer-insulation strengths and the degree of protection provided by arresters can be made by applying to the arrester the withstand test voltages specified for those transformers with which the arresters may be used. If the arrester, when subjected to this test voltage, prevents voltages of the magnitude and character of the withstand voltages from developing, it is obvious that the transformer is protected by the arrester. The relation between transformer strength and arrester performance for a typical application is shown by the oscillograms of Figure 4. The 15-kv insulation class transformer and the nine-kilovolt arresters, a common combination, are illustrated. In order to show both the arrester impulse spark-over characteristics and the effect of voltag drop while the arrester carries current two sets of oscillograms are shown wit different time scales.

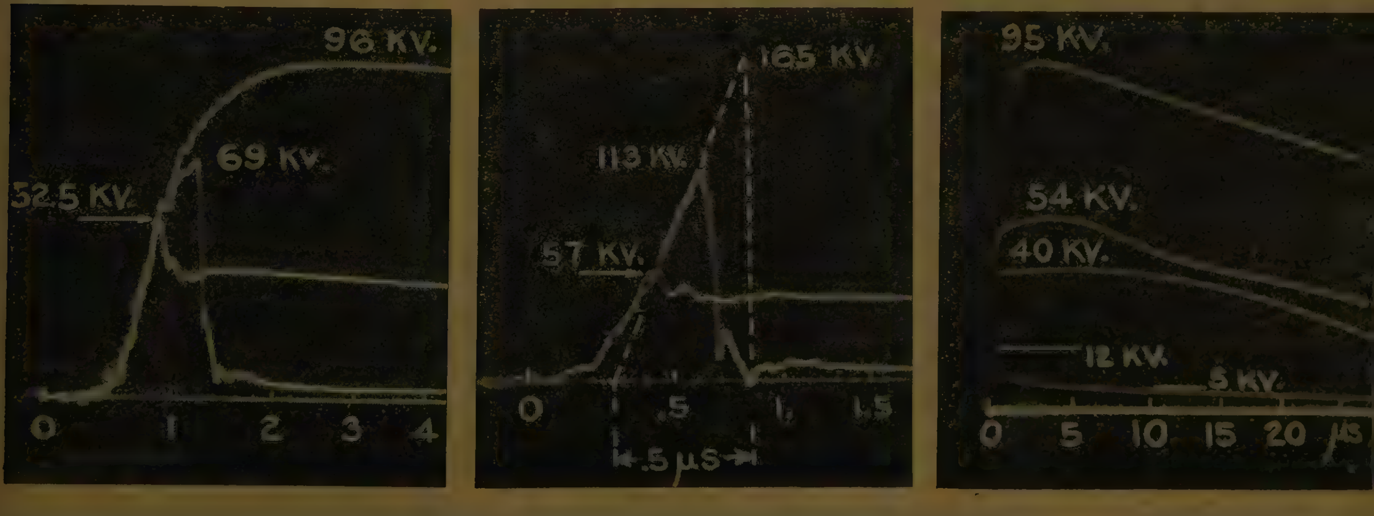
Impulse Spark-Over Voltages

Figures 4a and b show the arreste spark-over characteristics in relation t the transformer withstand voltages. I Figure 4a the trace marked 96 kv is the 1.5x40 wave which the transformer withstands if it meets NEMA Standards. I a nine-kilovolt expulsion arrester of th type illustrated in Figure 1 is connecte in the test circuit and the transforme withstand voltage applied, spark-over o the arrester occurs at 69 kv. The nine-kilovolt valve arrester shown in Figure 1 sparks at 52.5 kv. Both of these are well below the full-wave voltage that the transformer will withstand.

The response of arresters to the chopped wave used for transformer testing is practically the same as in the case of the full wave. Therefore, it has not been shown. Of interest, however, would be the arrester behavior on a front of wave test fo the transformer, because this would have an appreciably faster rate of voltage rise. Such front of wave tests have not been standardized for distribution transformers. An approximate value, which might in the authors' opinion be used, is shown in Figure 4b by the broken-line trace marked 165 kv. Such a test wave applied to a nine-kilovolt deion arrester causes spark-over at 113 kv and on the nine-kilovolt Autovalve arrester at 57 kv. Again, both of these values are well below the impulse voltage that can be impressed on a transformer without damage. It should be

Figure 4. Relation between transformer impulse withstand voltage (see Table I) and arrester voltages

Transformer insulation class, 15 kv
Arrester ratings, nine kilovolts



(a). Transformer full wave, 96 kv. Deion arrester spark-over, 69 kv. Autovalve spark-over, 52.5 kv

(b). Transformer assumed front of wave test, 165 kv. Deion arrester spark-over, 113 kv. Autovalve spark-over, 57 kv

(c). Transformer full wave, 95 kv. Voltage across arresters during discharge of specified surge currents:

Deion arrester, 20,000 amperes, 12–5 kv
Autovalve arrester, 5,000 amperes, 40 kv, 20,000 amperes, 54 kv

These oscillograms do not show the spark-over voltages. They are shown in Figures 4a and b noted from the comparison of Figures 4a and b that the transformer will withstand a higher voltage, if applied for only a short time, than the full 1.5x40 wave test voltage.

Figure 4 indicates that the impulse spark-over voltage of the expulsion arrester is higher than that of the valve arrester. In the expulsion arrester, spark-over takes place over two air gaps, the series gap, and the gap in the extinction chamber. The gap in the extinction chamber is relatively long to insure clearing of the power-follow current, and the time lag curves of spark-over, Figure 5, indicate that the spark-over voltage increases with decreasing time to spark-over. This is not serious since insulation time-lag curves have somewhat the same shape. Although the spark-over characteristics can be modified by means such as the electrostatic shielding used in the arrester shown in Figure 1, the foregoing factors in general fix the order of impulse spark-over voltage.

The gap in the valve arrester is not called upon to interrupt high power-follow currents, as their magnitude is limited by the valve element. Therefore, it can be made with low spark-over voltage and with time-lag characteristics that resemble those of sphere gaps which are more or less flat and begin to turn up only at very short times to spark-over as illustrated in Figure 5. Although there is this difference between the spark-over voltages of the valve and expulsion types, Figure 4 shows that there is ample margin between the transformer impulse strength and the impulse spark-over of either arrester.

Voltage During Discharge

More than the impulse spark-over should be taken into account in the correlation between transformer strength and protective characteristics of the arrester, because, while the arrester is discharging surge current, some voltage exists across its terminals as the result of arc or impedance drops. In Figure 4c are shown these voltage characteristics of both types of arresters for several discharge currents of magnitudes recommended in AIEE Standards. These again are related to the transformer full-wave withstand voltage shown again in Figure 4c by the trace marked 95 kv. The spark-over voltages of the arresters are not apparent in Figure 4c because of the larger time scale. The spark-over voltages are shown in Figures 4a and b.

Consider the valve arrester characteristics. The trace marked 54 kv is the voltage developed across the arrester during discharge of a 20,000-ampere test surge. The trace marked 40 kv is the voltage across the arrester with a test current of 5,000-amperes crest. It should be noted that the voltage across the valve arrester drops from the crest reached after a few microseconds. It is sometimes thought that the voltage drop of the valve arrester persists at its crest magnitude for the duration of the discharge. This is not the case, especially when high currents are involved.

The voltage that exists across the expulsion arrester while discharging 20,000 amperes is shown by the lowest trace in Figure 4c. Immediately after discharge, the voltage collapses to 12 kv and then drops further, until after several microseconds it has decayed to five kilovolts. For the expulsion arrester, the voltage for a 5,000-ampere discharge is not shown. It is less than that for 20,000 amperes. It is apparent that the discharge voltages of the expulsion arrester are appreciably lower than those for the valve arresters. However, in either case, there is considerable margin between the withstand voltage of the transformer and the voltages developed across either arrester during the discharge of surge current. These differences are thus not of practical importance, although they sometimes arouse controversy. The valve arrester becomes conducting on steep impulse voltages at lower values than the expulsion type. One might say that this is an advantage. On the other hand, the voltage across the expulsion arrester collapses to a lower value once discharge has begun. This is an advantage in favor of the expulsion arrester.

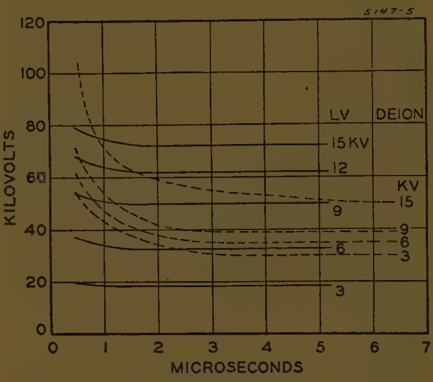


Figure 5. Impulse spark-over time-lag characteristics of distribution arresters of the valve and expulsion types

Full lines—Autovalve arresters
Broken lines—Deion arresters
Numbers on curves indicate voltage rating

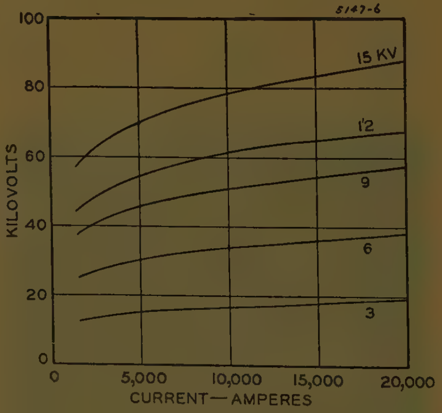


Figure 6. Relation between crest of 10x20-microsecond discharge current and crest of voltage developed by the current for Autovalve arresters

Numbers on curves are arrester ratings

The important impulse characteristics of both arresters are shown in curve form in Figures 5 and 6. As far as transformers manufactured during the last decade or so are concerned, either device is adequate. This is particularly true on completely self-protected transformers where the manufacturer has a known insulation strength with which it is relatively easy to co-ordinate the arresters. On very old transformers the impulse strength may be lower, as they were not built to meet present standards. For the protection of such transformers, the valve arrester may be preferable because of its lower sparking voltage. It is true that with very heavy current discharges the voltage developed across the valve arrester during the discharge may exceed the impulse strength of the old transformer, so that the transformer will not be protected under these conditions. For example, when discharging a surge current with a crest of 65,000 amperes, the peak voltage across a nine-kilovolt valve arrester may be 70 kv. Though this is well below the full-wave strength of a recent transformer that meets Table I, it may well exceed the strength of a very old transformer. However, the occurrence of such heavy currents is rare. Field measurements of the surge currents discharged by arresters in rural service indicate that 50 per cent of the operations involve currents of less than 1,500-amperes crest and 96 to 98 per cent less than 20,000 amperes. By and large, therefore, valve arresters should give better protection to the old transformers being considered. On the other hand it is probably desirable that transformers with such low-impulse strengths be replaced by modern ones.

Surge-Current Capacity and Life

An arrester fundamentally is intended to prevent service outages during thunder storms. Therefore, it should not only prevent insulation failures as far as possible but also should be rugged to prevent damage to itself. Electrical damage may result from very severe lightning—after all a distribution arrester is a device of only modest size—or it may result from a temporary system voltage condition in excess of the arrester rating or because of a defect in the arrester. Any of these is possible. Considering the effects of lightning, it is desirable that the arrester have surge discharge capacity sufficient to

Table I. Impulse Insulation Withstand Tests for Distribution Transformers (Oil Immersed)

Insulation Class, Kilovolt Rms	Transformer* Voltage Rating	Chopped Wave		Full-Wave Voltage, Kilovolt Crest	Arrester** Voltage Rating, Rms
		Voltage, Kilovolt Crest	Minimum Time to Flashover, Microseconds		
2.5		54	1.25	45	
5.0	2,400	69	1.5	60	3,000
8.66	4,800	88	1.6	75	6,000
15.0	7,200	110	1.8	95	9,000
15.0	13,200	110	1.8	95	15,000

*These are usual standard ratings and usually fall into the insulation classes shown. The insulation class indicates the insulation strength.

**These are the arrester ratings usually used with the transformers listed in column 2. In some cases because of circuit conditions, other arrester ratings may be used with a given transformer rating.

Test voltage wave shape is 1.5x40 microseconds (1.5 microseconds from 0 to crest, 40 microseconds to half of crest).

Full-wave test is an application of the full unmodified 1.5x40 wave to the test specimen. (See Figure 4a.) In the chopped-wave test, a 1.5x40 wave shape is applied, but the test voltage is removed or chopped by flashover of a properly set parallel gap in the time specified in the table.

In front of wave tests, the voltage applied to the test specimen is removed or chopped by a gap in a specified time, but while the test voltage is still rising. (See Figure 4b.) Front of wave tests for distribution transformers have not yet been determined.

This table is based on Table V of NEMA Transformer standards, publication 42-73. For more complete data, refer to those standards.

Table II. Comparison of Deion and Autovalve Lightning Arresters for Distribution Apparatus

Characteristic	Type of Arrester	
	Deion	Autovalve
Impulse spark-over, 0.5 microsecond		Lower
Voltage during discharge	Lower	
Power-follow current		Less
Surge-current capacity:		
65,000 amperes	Same	Same
Long low currents	Better	
Service life	Comparable	Comparable
Construction	Open	Sealed
Series gap	Integral part of arrester	
Is series gap setting critical	No	Sealed
Is application affected by system recovery voltage	No*	No*
Is application limited by system fault currents	No**	No
Is co-ordination of feeder relaying or fusing and arrester required	Usually not	
Does operation depend on expenditure of material	Refer to text	
Does system frequency affect arrester application	***	****
Is arrester applicable to delta systems	Yes	Yes
Radio interference	No	No
Line lockouts	No	No

*Theoretically yes, practically no. Experience shows that either type operates satisfactorily on existing distribution circuits.

**Holds for the arrester described in text.

***Ratings are based on 60 cycles.

****Ratings are based on 25–60 cycles.

minimize the chances of arrester damage by lightning in the field. Experience has developed a basis of test for the distribution arrester. This is the 65,000-ampere test recommended in the AIEE Standards. Years of experience with arresters of various surge current capacities indicate that capacities of less than 65,000 amperes will engender unsatisfactory rates of failure from lightning in the field. The 65,000-ampere test is thus a reasonable index to the ability of arresters to weather severe storms. Both of the arresters shown in Figure 1 meet this criterion, and field experience with both has been excellent.

The dangers of system overvoltage are well known. A valve arrester is sensitive to the system voltage impressed on its terminals during the clearing period. In fact excessive system voltage may be called its chief enemy. It is not in general sensitive to rate of system recovery voltage, because it itself determines the rate of recovery voltage across its series gap as a result of the effective resistance of the valve element. However, if the system voltage exceeds the arrester rating, the gap may not be able to clear and the arrester may be damaged.

The ability of the expulsion arrester to interrupt depends on several factors besides voltage, such as the rate of recovery voltage, overshoot of recovery voltage, current to be interrupted, and on the power factor of the circuit. When any of these conditions are not as severe as those for which the arrester is designed, an additional margin in interrupting ability results. It is not recommended, however, that a lower voltage arrester be used because of any margin of this type. The voltage rating should be considered a maximum rating.

A very live question to which a direct answer is difficult involves the life of an arrester. How long should it last, and how long may the various types be expected to last? Laboratory tests and records of discharges through arresters in actual service give clues on the basis of which estimates may be made, but the best indication is experience. This may appear unfair to the user, because he is the one that has to acquire the experience. Fortunately, the art is now in a phase where considerable experience has been accumulated.

Discharges of appreciable magnitude combined with the arrester's normal power follow will have little effect on the ability of the valve arrester to function for many operations. The impulse protective characteristics of the valve arresters of Figure 1 are not affected by high discharge currents. Repeated discharges of currents of 20,000 amperes or more exert some effect on the valve material which ultimately will cause it to puncture. With the 65,000-ampere surge-capacity test, a few repeated discharges may damage the arrester. The test current that will destroy a distribution arrester with the first shot may be considerably higher. The duration of the discharges also has a bearing on life. There is no doubt a relation between crest magnitude of current and shape and duration of wave tail on the one hand and the number of discharges an arrester will withstand on the other. This relation is not well known. Something is known of the high-crest short-duration effects and something is known of the low-current long-duration effect as shown by the 1/2 to several cycles of normal power follow current that the arrester may withstand. Between these is a range that is not determined because of the tedious labor and the expense that would be involved in establishing it. The lack of this information is not serious as indicated by the low rate of damage in the field to modern distribution arresters of either type.

The expulsion arrester wears. Expenditure of material is the principle upon which it operates to interrupt power-follow current. The wear caused by surge current is on the whole small. An excessively high surge current may burst the device or crack it, but this is the exception. The principal cause of wear in an expulsion arrester is the follow current that it must interrupt. The degree of erosion is a function of the magnitude and the duration of the follow current, and this again depends to some degree on the system characteristics and the relation of the timing of the surge discharge with respect to the phase of the system voltage. The factors that affect erosion are so variable that it is difficult to evaluate wear in terms of years of service. The expulsion arrester has been in use for some 13 years as part of the *SP* and *CSP* transformers. Neglecting causes of damage other than lightning, the life, measured in annual percentage of arresters damaged, obtained from the deion arresters used on those transformers as well as from deion arresters used as separately mounted devices actually has been better than secured from the comparable Autovalve arrester. This would indicate that the erosion effect of power-follow current, which can be accentuated by specific test conditions, is not the complete determinant of service in years. There is further corroboration of this by the fact that lightning field studies have indicated that a lightning arrester on any given distribution transformer in rural service discharges on an average of about once a year, but that over a period of years the number of discharges in any year may vary between zero and eight. Further, as already mentioned, power follow in expulsion arresters depends on how the surge and the 60-cycle power voltage overlap. In many cases there will be no power follow at all, and it is found in laboratory testing that the conditions of timing must be controlled carefully to obtain the worst power-follow conditions. Moreover, heavy discharge currents in themselves often cause the evolution of sufficient gas from the walls of the extinction chamber to prevent power follow. Although tests can be made which will show a longer life, in number of operations for valve arresters than for expulsion arresters and vice versa, it is logical to expect that both arresters will give comparable years of satisfactory service. Experience appears to bear this out. It may be that within certain ranges of lightning-discharge wave shape and magnitude the effect of the surge alone is less marked on the expulsion arrester than it is on the valve type.

Some weight should be given to the consideration that practically all makers of self-protecting distribution transformers employ expulsion arresters for protection. They have had excellent experience with their choice. On the other hand for separately mounted arresters the activity on valve-type distribution arresters exceeds that of expulsion arresters

Physical Differences

As the operation of the expulsion arrester generates gas which must be vented, it must be more or less open in construction. This has advantages in the eyes of many users and designers and is something not readily feasible in valve arresters.

Most valve arresters do not generate gas, nor do they have to handle high power-follow currents. Therefore, total enclosure is feasible, thereby protecting the parts from the weather and avoiding any exposed line or ground terminals or leads. This in turn is regarded as an advantage by many users, and it is something not readily done with expulsion arresters.

Effect on Operation of Primary Fuses

Recently some statements have been made that the collapsing voltage of the expulsion arrester has a distinct advantage in that it will permit fewer primary fuse operations. In brief, the reason given for this is that the expulsion arrester, because of this voltage collapse, exposes the transformer to less voltage for the duration of the discharge than the valve type, and, therefore, there is less tendency for lightning current to penetrate through the transformer winding and the fuse. There is clearly a difference in these voltages; however, if all of the factors that enter into the problems of fuse operation are considered, so broad a statement hardly is warranted. The subject is complex and beyond the scope of this paper. It should be explored further. If fuse operations can be decreased by the use of a certain type of lightning arrester, that is important, and it is to be hoped that experience of operating engineers in this connection may be presented to the Institute.

Special Application

There is one application in which there exists a definite preference. It is an application which requires both types of arresters to supplement each other and is, therefore, a good case. It is the protection of rotating machines. This subject is covered elsewhere.[1] In this application arresters must be used at the terminals of the machine and arresters also must be used at some point on the line removed from the machine terminals. The arresters at the machine terminals definitely must be of the valve type, because they spark at low voltages and because the voltage does not collapse after spark-over. For the arresters on the line, however, the expulsion type is to be preferred for the reason that its voltage does collapse and thereby aids the functioning of the capacitors and arresters at the machine.

Conclusion

This paper describes the operating characteristics of the two types of lightning arresters and discusses their advantages and disadvantages. A generalized comparison is shown by Table II. If there were conclusive evidence as to the superiority of one type over another, only one type would be in demand and only one type manufactured. Manufacturers who build both do so, because they believe that superiority of one type over the other cannot be demonstrated clearly and that operating engineers will insist that both be available. It is the authors' opinion that both devices serve the purpose for which lightning arresters are intended. There are certain advantages that can be cited for either one. Probably the selection of one or the other is influenced largely by the weight given by the user to those points in which the expulsion and valve arresters for distribution service differ.

Reference

1. Lightning Protection for Rotating Machines, G. D. McCann, E. Beck, L. A. Finzi. AIEE Transactions, volume 63, 1944, pages 319–33.

Resistance and Capacitance Relations Between Short Cylindrical Conductors

F. L. ReQua
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The conventional formula for computing capacitance or conductance between parallel cylindrical conductors is derived on the assumption that the electrostatic or current flux between conductor surfaces is confined to paths lying in planes perpendicular to the axes of the conductors.

Actually this condition obtains only at the mid-point between the ends of the conductors. At all other points along the length the lines of flux curve outwardly toward the ends. However, for conductors of length which is very great compared to the distance of separation the flux paths are nearly planar except near the ends. Hence the conventional formula, which indicates direct proportionality between capacitance or conductance and length of the conductors, gives values which approach more and more closely to the true value as the length of the conductors increases.

For relatively short conductors however the effect just mentioned becomes of increasing importance. The true capacitance or conductance is not even approximately proportional to length of conductors and the values given by the conventional formula are no longer satisfactory.

In this paper formulas are developed in which the end effect is taken into account. This leads to an indicated integration which can be closely approximated by the summation of a small number of terms but is inconvenient for engineering computation. It is found that the func-

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tion to be integrated may be approximated by a parabolic function and by this empirical substitution simplified formulas are derived which give results of satisfactory precision for the requirements of engineering design and may be expressed in simple functional form. Tabulated values of this function are given in the paper.

Experimentally determined values obtained from tests on a large three-phase water rheostat at Bonneville are compared with the same quantities computed by the formulas derived in this paper, and the agreement is found to be satisfactory.

Relationship Between Formulas for Capacitance and Conductance

The electrical capacitance C between any two conductors, spatially related in a specified manner and immersed in a dielectric medium of permittivity p, bears a simple relation to the conductance G between the same conductors immersed in a medium of conductivity γ. It is

$$C/p \propto G/\gamma \tag{1}$$

The constant of proportionality will depend upon the system of units adopted. In this paper all derivations are based upon the meter-kilogram-second system, modified so that the unit of length is inches instead of meters. Hereby the unit of capacitance is the farad, of conductance the mho, of resistance R the ohm, of conductivity the mho per inch cube, of resistivity ρ the ohm per inch cube and

$$C/p = G/4\pi\gamma = \rho/4\pi R \tag{2}$$

The absolute permittivity p in equation 2 is the permittivity p_0 of free space multiplied by the dielectric constant k of the medium. In the system of units used herein p_0 has the value 2.8262×10^{-12}.

All formulas in this paper are derived initially for the capacitance between parallel cylindrical conductors of equal diameter. Corresponding formulas for resistance between the conductors then are obtained by use of the relations 2.

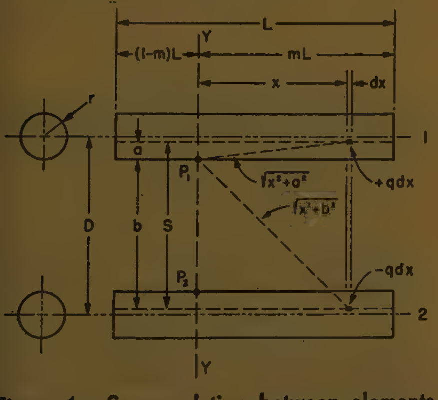

Figure 1. Space relation between elements of equivalent linear charges and surface potential points on two parallel cylindrical conductors

Derivation of Conventional Formula for Capacitance

The conventional formula for capacitance between two parallel cylindrical conductors 1 and 2 of length L, radius r, and distance D between centers, is derived on the assumption that L is so large compared to D that, for a given difference of potential between the conductors, the charge per unit of axial length may be considered to be the same at all points along the length of each conductor.

The surface charge per unit length so transferred can be shown to be the electrostatic equivalent, for points lying outside the conductor surfaces, of the same charge per unit length concentrated along a line inside each conductor in the plane of and parallel to the axes of the two conductors.

When L is very large compared to D the distance S between these two lines of charge is related to D and r by the expression:[1]

$$S = \sqrt{D^2 - 4r^2} \tag{3}$$

The shortest distance a from the line of charge inside conductor 1 to the surface of conductor 1 is readily deduced from equation 3 to be

$$a = \frac{\sqrt{D^2-4r^2}-(D-2r)}{2} \tag{4}$$

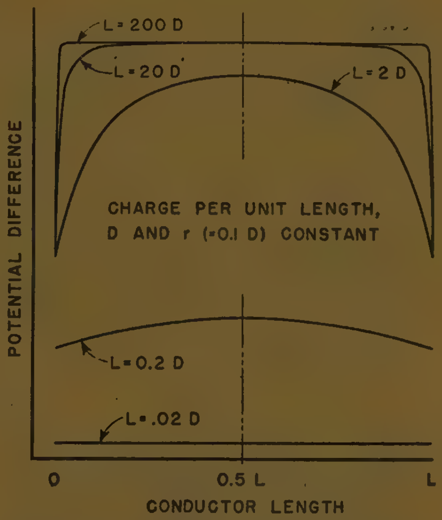

Figure 2. Variation of sectional potential difference with length along parallel cylindrical conductors with uniform linear charge

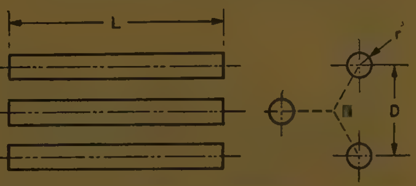

Figure 3. Three-phase arrangement of identical cylindrical conductors, equilaterally spaced in a homogeneous medium of infinite extent

Use formula 20 for capacitance
Use formula 23 for resistance

Similarly, the shortest distance b from the line of charge inside conductor 2 to the surface of conductor 1 is

$$b = \frac{\sqrt{D^2-4r^2}+(D-2r)}{2} \tag{5}$$

From equations 4 and 5 it follows that

$$\frac{b}{a} = \frac{D+\sqrt{D^2-4r^2}}{2r} \tag{6}$$

When L is very small compared to both D and r, the relations 3, 4, and 5 are shown in the appendix to change to

$$S = D - \frac{2r^3}{D^2} \tag{3'}$$

$$a = \frac{D^2 - r^2}{D^2} \cdot r \tag{4'}$$

$$b = D - \frac{D^2 + r^2}{D^2} \cdot r \tag{5'}$$

If a linear charge of $+q$ per unit length is assumed within one conductor and $-q$ per unit length within the other, the potential at a point on the surface of each conductor midway between the ends can be computed. The magnitude of the total charge transferred, qL, divided by the computed difference of potential between the mid-points of the two conductors gives the familiar formula[2] for capacitance:

$$C = \frac{0.30685 \times 10^{-12} kL}{\log_{10}\left(\frac{b}{a}\right)} = \frac{0.30685 \times 10^{-12} kL}{\log_{10}\left(\frac{D+\sqrt{D^2-4r^2}}{2r}\right)} \tag{7}$$

and thence its approximate equivalent,

$$C = \frac{0.30685 \times 10^{-12} kL}{\log_{10}\left(\frac{D}{r} - \frac{r}{D}\right)} \tag{7'}$$

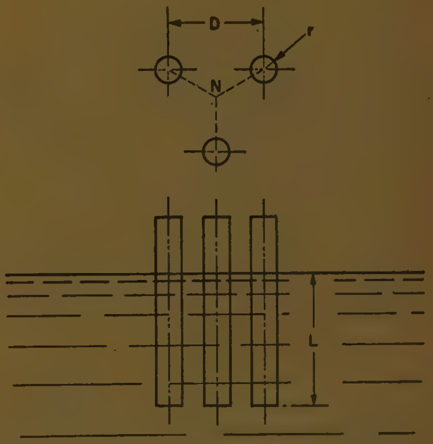

Figure 4. Three-phase arrangement of identical cylindrical conductors, equilaterally spaced and submerged in the surface of a homogeneous liquid of infinite area and depth

Use formula 23 for resistance

The approximate values obtained by use of formula 7′ differ by less than one eighth of one per cent from those of formula 7 for all cases where r does not exceed 20 per cent of D.

Formulas 7 and 7′ and corresponding expressions for resistance derived therefrom are applicable when the flux between conductors follows planes perpendicular to the conductor axes. This condition obtains

1. Approximately, when the length L of the conductors is very great compared to the distance of separation D.

2. Exactly, in the case of electric-current flow, when the conducting liquid of finite depth L is confined in a tank of infinite area with an insulated floor and the conductors are immersed vertically to the full depth of the liquid.

Derivation of More General Formula for Capacitance

Figure 1 represents two parallel tubular conductors of length L and radius r separated by a distance D between axes. It is postulated that each conductor is subdivided into a very large number of short tubular sections formed by cutting through the original conductors with a corresponding number of insulating planes of negligible thickness, each normal to the axes of the conductors. Hereby the necessity of assuming the same potential at all points along the length of the conductor is eliminated.

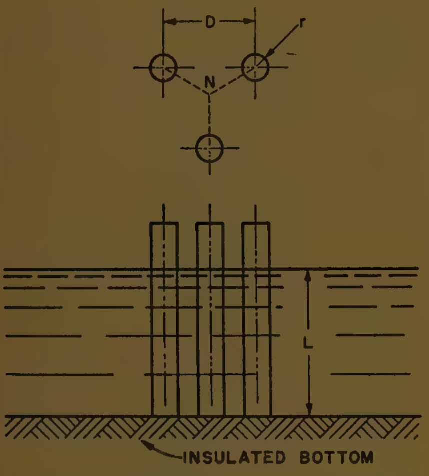

Figure 5. Three-phase arrangement of identical cylindrical conductors, equilaterally spaced and submerged in a homogeneous liquid of infinite area and of depth L

Use formula 25 for resistance

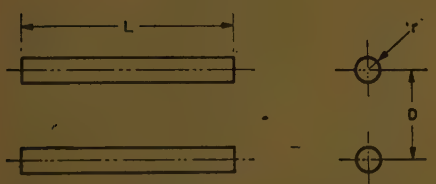

Figure 6. Single-phase arrangement of identical cylindrical conductors in a homogeneous medium of infinite extent

Use formula 18 for capacitance
Use formula 24 for resistance

It is assumed that a charge of q per unit length has been transferred from conductor 2 to conductor 1 and that the resulting deficit and surplus, respectively, of charge are located along two symmetrically disposed lines inside the conductors, separated by the distance S.

If the point P_1 is located at a distance mL from one end of the conductor, where m is a numeric equal to or less than unity, then the potential V_1 at this point is

$$V_1=\frac{q}{p}\left[\int_0^{mL}\frac{dx}{\sqrt{x^2+a^2}}+\int_0^{(1-m)L}\frac{dx}{\sqrt{x^2+a^2}}-\int_0^{mL}\frac{dx}{\sqrt{x^2+b^2}}-\int_0^{(1-m)L}\frac{dx}{\sqrt{x^2+b^2}}\right]$$

$$=\frac{q}{p}\log_e\left[\frac{1+\sqrt{1+a^2/(mL)^2}}{1+\sqrt{1+b^2/(mL)^2}}\times\frac{1+\sqrt{1+a^2/[(1-m)L]^2}}{1+\sqrt{1+b^2/[(1-m)L]^2}}\times\left(\frac{b}{a}\right)^2\right] \qquad (8)$$

This is also the potential at all other points on the periphery of the short cylindrical section of conductor 1 located at the plane Y–Y.

Since the potential V_2 at point P_2 is, by symmetry, equal to $-V_1$ it follows that the difference of potential between the two conductor sections located by the plane Y–Y is twice the value given by expression 8.

If the two conductors are divided into n insulated sections, each of length L/n, the charge transferred between homologous sections is $(Lq)/n$ and the sectional capacitance C_i is equal to this charge divided by the difference of potential between the sections,

$$C_i=\frac{pL}{\left\{2n\log_e\left[\frac{1+\sqrt{1+a^2/(mL)^2}}{1+\sqrt{1+b^2/(mL)^2}}\times\frac{1+\sqrt{1+a^2/[(1-m)L]^2}}{1+\sqrt{1+b^2/[(1-m)L]^2}}\times\left(\frac{b}{a}\right)^2\right]\right\}} \qquad (9)$$

and the total capacitance between conductors is

$$C=\sum_{i=1}^{n}C_i \qquad (10)$$

It follows that the total capacitance between conductors is also equal to the total charge transferred divided by V_m, the arithmetic mean of the potential differences between all pairs of homologous sections. If we refer to Figure 2, which shows sectional difference of potential along the length of pairs of conductors of given r and D but differing length L, it will be noted that for relatively long conductors the mean sectional difference of potential is very nearly the same as the difference of potential at the mid-point. However for shorter conductors this is not true.

A satisfactory approximation for engineering purposes to the mean difference of potential can be obtained by computing the difference of potential at the mid-point of the sectionalized conductor and at two points near to and equidistant from the ends, passing a parabolic curve through these three points and computing the mean difference of potential therefrom. It is found that, if the two end points arbitrarily are taken at nine tenths of the distance from mid-point to end, the approximate value will not differ by more than one half of one per cent from that obtained by subdividing the conductors into 20 sections and using equation 8 as the basis for computation.

The potential at the mid-point and that near either end of one of the sub-

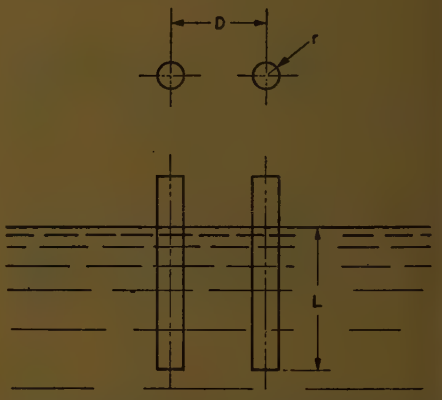

Figure 7. Single-phase arrangement of identical cylindrical conductors submerged in the surface of a homogeneous liquid of infinite area and depth

Use formula 24 for resistance

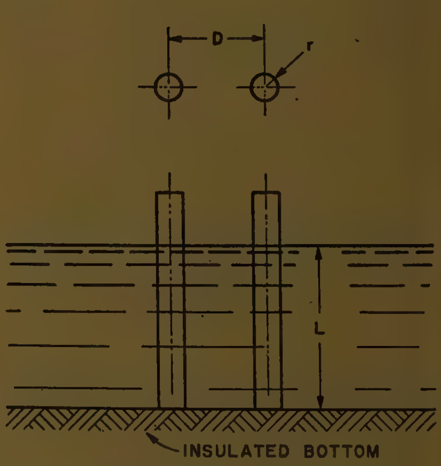

Figure 8. Single-phase arrangement of identical cylindrical conductors submerged in a homogeneous liquid of infinite area and of depth L

Use formula 26 for resistance

divided conductors can be determined from equation 8, taking $m = 0.5$ and $m = 0.05$ in turn. The values so obtained, multiplied by two, will give the difference of potential between homologous sections at each of these two locations. If we call the difference of potential at the midpoint V_C and that near the ends V_E,

$$V_C=\frac{4q}{p}\log_e\left[\frac{1+\sqrt{1+(4a^2)/L^2}}{1+\sqrt{1+(4b^2)/L^2}}\times\frac{b}{a}\right] \tag{11}$$

$$V_E=\frac{2q}{p}\log_e\left[\frac{1+\sqrt{1+(400a^2)/L^2}}{1+\sqrt{1+(400b^2)/L^2}}\times\frac{1+\sqrt{1+(1.108a^2)/L^2}}{1+\sqrt{1+(1.108b^2)/L^2}}\times\left(\frac{b}{a}\right)^2\right] \tag{12}$$

For parabolic distribution of section potential differences along the length of the subdivided conductors,

$$V_z=A+Bz^2 \tag{13}$$

where z is distance measured from midpoint of the conductor toward either end. The mean potential difference,

$$V_M=\frac{\int_0^{0.5L} V_z dz}{0.5L}=A+\frac{BL^2}{12} \tag{14}$$

If we evaluate the constants A and B from $V_z = V_C$ at $z = 0$ and $V_z = V_E$ at $z = 0.45L$ and substitute into equation 14,

$$V_M=\frac{143\,V_C+100\,V_E}{243} \tag{15}$$

If we insert the values of V_C and V_E from equations 11 and 12 into equation 15, the expression for the capacitance between two cylindrical conductors becomes

$$C=\frac{243pL}{\left\{572\log_e\left[\frac{1+\sqrt{1+4a^2/L^2}}{1+\sqrt{1+4b^2/L^2}}\times\frac{b}{a}\right]+200\log_e\left[\frac{1+\sqrt{1+400a^2/L^2}}{1+\sqrt{1+400b^2/L^2}}\times\frac{1+\sqrt{1+1.108a^2/L^2}}{1+\sqrt{1+1.108b^2/L^2}}\times\left(\frac{b}{a}\right)^2\right]\right\}} \tag{16}$$

If we replace p by its value $2.8262 \times 10^{-12}\,k$ and convert the logarithms to base ten,

$$C=\frac{0.52142\times10^{-12}kL}{\left\{\log_{10}\left[\frac{1+\sqrt{1+4a^2/L^2}}{1+\sqrt{1+4b^2/L^2}}\times\frac{b}{a}\right]+0.34965\log_{10}\left[\frac{1+\sqrt{1+400a^2/L^2}}{1+\sqrt{1+400b^2/L^2}}\times\frac{1+\sqrt{1+1.108a^2/L^2}}{1+\sqrt{1+1.108b^2/L^2}}\times\left(\frac{b}{a}\right)^2\right]\right\}} \tag{17}$$

By expressing the denominator of equation 17 in terms of sums and differences of logarithms, appropriately grouped, the formula can be written in the form:

$$C=\frac{0.30685\times10^{-12}kL}{\log_{10}(b/a)-[F(b/L)-F(a/L)]} \tag{18}$$

where F (—) represents a function such that

$$F(u)=0.58848\ [\log_{10}(1+\sqrt{1+4u^2})+0.34965\{(1+\sqrt{1+400u^2})\times(1+\sqrt{1+1.108u^2})\}] \tag{19}$$

Numerical values of the function 19 are given in Table I. It will be noted that as L becomes very large compared to a and b, $F(b/L)$ approaches $F(a/L)$ in value, and thus formula 18 approaches the conventional formula 7 as a limit.

All of the foregoing derivations are based on an implicit assumption that the flux terminates only on the outside surfaces of the conductors. Actually, in the electrostatic case, some flux always terminates on the planes forming the ends, or on the interior surfaces adjacent to the ends if the conductor is tubular. In many practical problems the proportion so terminating is small enough so that it may be neglected without introducing appreciable error. In any event the capacitance given by the formulas will be less than the true value in the ratio of peripheral to total flux.

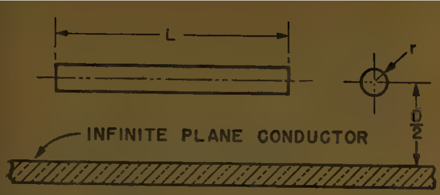

Figure 9. Single-phase arrangement of a cylindrical and a plane conductor in a homogeneous medium of infinite extent

Use formula 20 for capacitance
Use formula 23 for resistance

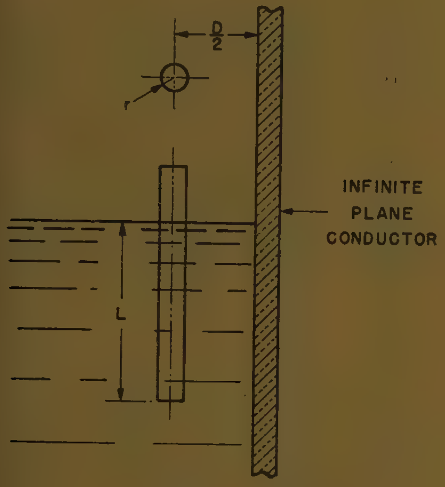

Figure 10. Single-phase arrangement of a cylindrical and a plane conductor submerged in the surface of a homogeneous liquid of infinite area and depth

Use formula 23 for resistance

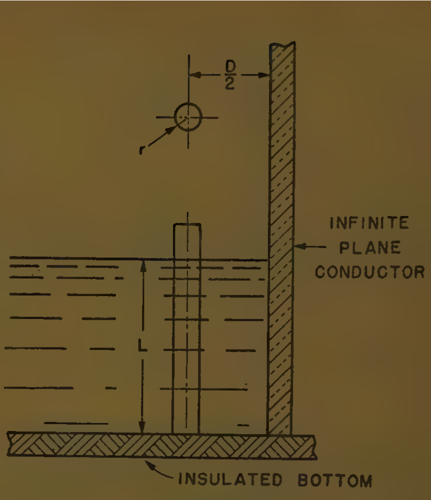

Figure 11. Single-phase arrangement of a cylindrical and a plane conductor submerged in a homogeneous liquid of infinite area and of depth L

Use formula 25 for resistance

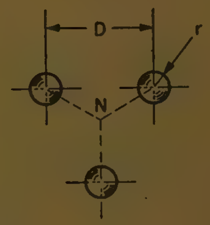

Figure 12. Three-phase arrangement of equal spherical conductors, equilaterally spaced in a homogeneous medium of infinite extent

Use formula 22 for capacitance
Use formula 27 for resistance

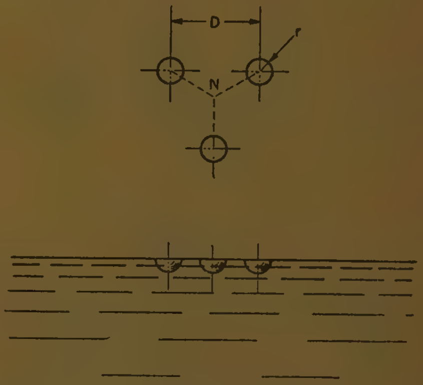

Figure 13. Three-phase arrangement of equal hemispherical conductors, equilaterally spaced and submerged in the surface of a homogeneous liquid of infinite area and depth

Use formula 28 for resistance

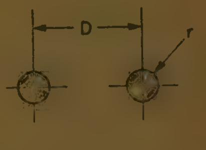

Figure 14. Single-phase arrangement of equal spherical conductors in a homogeneous medium of infinite extent

Use formula 21 for capacitance
Use formula 28 for resistance

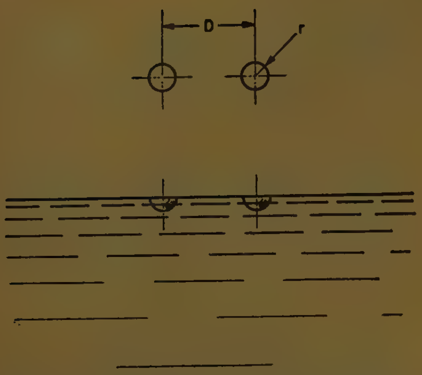

Figure 15. Single-phase arrangement of equal hemispherical conductors, submerged in the surface of a homogeneous liquid of infinite area and depth

Use formula 29 for resistance

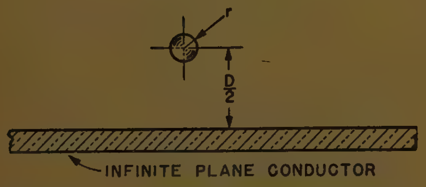

Figure 16. Single-phase arrangement of a spherical and a plane conductor in a homogeneous medium of infinite extent

Use formula 22 for capacitance
Use formula 27 for resistance

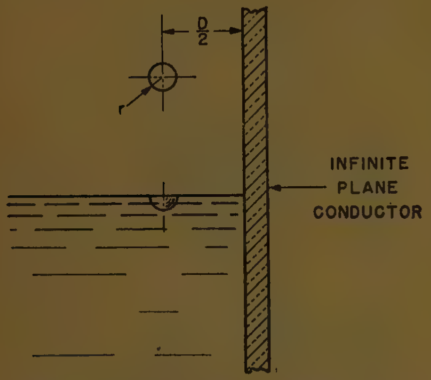

Figure 17. Single-phase arrangement of a hemispherical and a plane conductor submerged in a homogeneous liquid of infinite area and depth

Use formula 28 for resistance

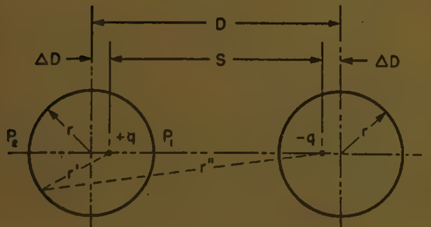

Figure 18. Position of equivalent internal charge in short cylindrical conductors

In the resistance case, it is possible for the plane ends of the conductors to be of insulating material, and in this event the fact accords with the assumption. Otherwise, the same comment applies as for the electrostatic case, except that the error is in the opposite sense so that the resistance given by the formulas will be greater than the true value in the ratio of total to peripheral flux.

Formulas for Capacitance and Resistance

In the following formulas, all linear dimensions are in inches, k is the dielectric constant of the medium (unity for air), and ρ is the resistivity of the medium in ohms per inch cube. C, the capacitance in farads and R, the resistance in ohms, are from line to line for single-phase circuits and from line to neutral for three-phase circuits. F (—) is the function of which tabulated values are given in Table I.

In addition to formulas based on the derivations given in this paper, approximation formulas for capacitance and resistance between spheres are included for convenience. The accuracy of these latter increases as D/r increases, capacitance formulas 21 and 22 giving values lower than actual, resistance formulas 27, 28, and 29 giving values higher than actual, by the percentages indicated in Table II.[3]

For $L \gg D$,

$$a=\frac{\sqrt{D^2-4r^2}}{2}-\frac{D-2r}{2} \tag{4}$$

$$b=\frac{\sqrt{D^2-4r^2}}{2}+\frac{D-2r}{2} \tag{5}$$

$$\frac{b}{a}=\frac{D+\sqrt{D^2-4r^2}}{2r} \tag{6}$$

For $L \ll D$,

$$a=\left[1-\left(\frac{r}{D}\right)^2\right]\times r \tag{4'}$$

$$b=D-\left[1+\left(\frac{r}{D}\right)^2\right]\times r \tag{5'}$$

Capacitance Formulas

Figure 6:

$$C=\frac{0.3068\,kL\times 10^{-12}}{\log_{10}(b/a)-[F(b/L)-F(a/L)]} \tag{18}$$

Figures 3, 9:

$$C=\frac{0.6137\,kL\times 10^{-12}}{\log_{10}(b/a)-[F(b/L)-F(a/L)]} \tag{20}$$

Figure 14:

$$C=\frac{1.411\,kDr\times 10^{-12}}{D-r} \tag{21}$$

Figures 12, 16:

$$C=\frac{2.822\,kDr\times 10^{-12}}{D-r} \tag{22}$$

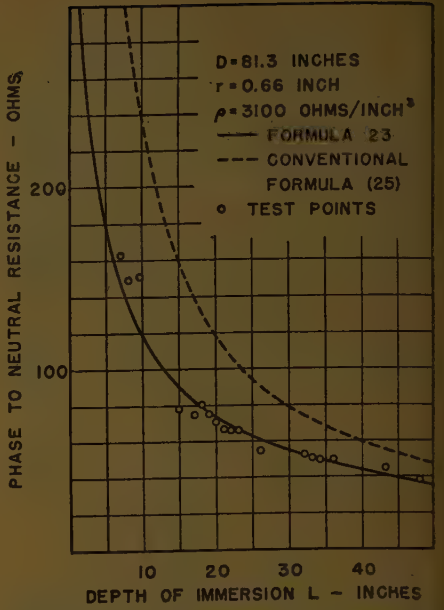

Figure 19. Comparison of computed and test values of resistance to neutral—Bonneville water rheostat

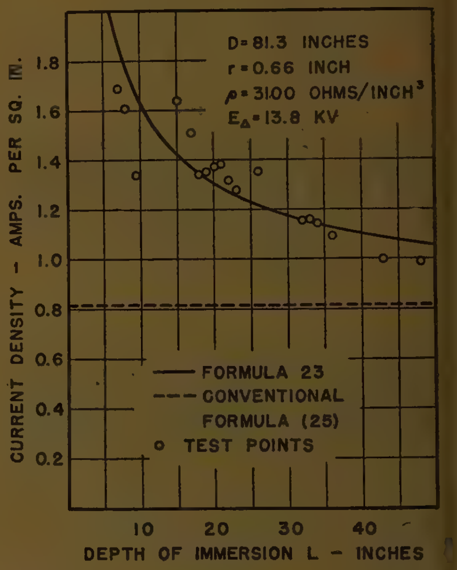

Figure 20. Comparison of computed and test values of electrode-surface current density—Bonneville water rheostat

Resistance Formulas

Figures 3, 4, 9, 10:

$$R=0.3665\,\frac{\log_{10}(b/a)-[F(b/L)-F(a/L)]}{L}\,\rho \tag{23}$$

Figures 6, 7:

$$R=0.7330\,\frac{\log_{10}(b/a)-[F(b/L)-F(a/L)]}{L}\,\rho \tag{24}$$

Figures 5, 11:

$$R=0.3665\,\frac{\log_{10}(b/a)}{L}\,\rho \tag{25}$$

Table I. Values of the Function $F(u)$

u	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.3010	0.3468	0.3957	0.4366	0.4733	0.5075	0.5397	0.5701	0.5990	0.6264
1.0	0.6524	0.6779	0.7019	0.7246	0.7461	0.7666	0.7861	0.8049	0.8229	0.8402
2.0	0.8569	0.8730	0.8886	0.9037	0.9184	0.9326	0.9464	0.9599	0.9730	0.9858
3.0	0.9983	1.0100	1.0214	1.0327	1.0437	1.0545	1.0651	1.0756	1.0857	1.0957
4.0	1.1055	1.1151	1.1244	1.1335	1.1425	1.1512	1.1597	1.1680	1.1761	1.1840
5.0	1.1917	1.1994	1.2069	1.2144	1.2217	1.2290	1.2361	1.2432	1.2501	1.2570
6.0	1.2637	1.2703	1.2769	1.2833	1.2896	1.2959	1.3020	1.3080	1.3140	1.3198
7.0	1.3255	1.3312	1.3368	1.3423	1.3478	1.3533	1.3587	1.3640	1.3692	1.3744
8.0	1.3796	1.3847	1.3897	1.3947	1.3996	1.4044	1.4092	1.4140	1.4186	1.4232
9.0	1.4278	1.4323	1.4368	1.4413	1.4457	1.4501	1.4544	1.4586	1.4628	1.4670

u	0	1	2	3	4	5	6	7	8	9
10	1.4711	1.5103	1.5463	1.5796	1.6106	1.6396	1.6668	1.6924	1.7166	1.7395
20	1.7613	1.7820	1.8019	1.8208	1.8369	1.8563	1.8730	1.8890	1.9045	1.9194
30	1.9338	1.9473	1.9606	1.9736	1.9863	1.9988	2.0110	2.0229	2.0345	2.0459
40	2.0569	2.0678	2.0783	2.0886	2.0986	2.1083	2.1177	2.1269	2.1358	2.1443
50	2.1527	2.1611	2.1694	2.1775	2.1856	2.1935	2.2012	2.2089	2.2165	2.2239
60	2.2312	2.2384	2.2454	2.2524	2.2592	2.2659	2.2725	2.2790	2.2853	2.2915
70	2.2976	2.3037	2.3097	2.3156	2.3214	2.3272	2.3330	2.3386	2.3442	2.3498
80	2.3552	2.3606	2.3659	2.3712	2.3764	2.3815	2.3865	2.3915	2.3965	2.4013
90	2.4061	2.4109	2.4156	2.4203	2.4250	2.4295	2.4340	2.4385	2.4429	2.4473

u	0	10	20	30	40	50	60	70	80	90
100	2.4516	2.4926	2.5302	2.5648	2.5969	2.6268	2.6548	2.6811	2.7059	2.7294
200	2.7516	2.7728	2.7930	2.8123	2.8307	2.8485	2.8655	2.8818	2.8975	2.9128
300	2.9273	2.9410	2.9545	2.9676	2.9805	2.9931	3.0055	3.0175	3.0293	3.0408
400	3.0520	3.0630	3.0736	3.0840	3.0941	3.1039	3.1134	3.1227	3.1317	3.1404
500	3.1489	3.1573	3.1656	3.1738	3.1819	3.1899	3.1977	2.2054	3.2130	3.2205
600	3.2279	3.2351	3.2422	3.2492	3.2561	3.2629	3.2695	3.2760	3.2824	3.2887
700	3.2948	3.3009	3.3069	3.3129	3.3188	3.3247	3.3304	3.3361	3.3418	3.3473
800	3.3528	3.3582	3.3636	3.3689	3.3741	3.3792	3.3843	3.3893	3.3942	3.3991
900	3.4039	3.4087	3.4135	3.4182	3.4228	3.4274	3.4320	3.4365	3.4409	3.4453

Figure 8:

$$R=0.7330\,\frac{\log_{10}(b/a)}{L}\,\rho \qquad (26)$$

Figures 12, 16:

$$R=0.7970\,\frac{D-r}{Dr}\,\rho \qquad (27)$$

Figures 13, 14, 17:

$$R=0.1594\,\frac{D-r}{Dr}\,\rho \qquad (28)$$

Figure 15:

$$R=0.3188\,\frac{D-r}{Dr}\,\rho \qquad (29)$$

Comparison Between Test Values and Those Computed by Formula

At the time acceptance tests were to be made on the generators at Bonneville power plant the need arose for a conveniently adjustable artificial load. Accordingly, a preliminary experimental water rheostat was constructed and tested by R. L. Earnheart of the Corps of Engineers, United States Army.[4] Mr. Earnheart is now on foreign duty with the Army and it is through the courtesy of F. M. Lewis, senior engineer, War Department, Portland, Oregon, District, that data from the original field notes have been made available to the author.

The tests were made on a three-phase rheostat immersed in the water of the tailrace. It consisted of three one-inch standard iron pipes parallel to one another and equilaterally spaced 81.3 inches between centers, Figure 4. Readings of current were taken at recorded line-to-line potentials between 12 and 14 kv and at immersion depths varying from 7 to 48 inches. The depth of tailrace water was 50 to 56 feet during the tests.

Table II. Error in Formulas 21, 22, 27, 28, and 29

$\frac{D}{r}$	Per-Cent Error
2.1	20.8
2.2	14.8
2.5	6.25
3.0	2.28
3.5	1.14
4.	0.60
5.	0.24
6.	0.17

In the reference article it is stated that the resistivity ρ of Columbia River water is 3,620 ohms per inch cube at 45 degrees Fahrenheit and 2,200 ohms per inch cube at 80 degrees Fahrenheit. Water temperature during tests, which would be expected to vary somewhat with load, was not given. An intermediate value of 3,100 ohms per inch cube, which seems to approximate the data best, therefore has been used for computations.

Computed values are compared graphically with test values in Figures 19 and 20. Figure 20, in particular, emphasizes the basic difference between the conventional formula 25 and that developed in this paper, 23, for relatively small depth of immersion. The former indicates no change in electrode-surface current density with depth of immersion; the latter reflects the observation made in this and other[5] tests that the current density increases as depth of immersion decreases.

Appendix I. Values of a and b When $L \ll r$

If the length L is small, compared to the radius r of the conductors, and if in addition the radius does not exceed 0.1 D, then at point 1, Figure 18,

$$r_1'=r-\Delta D$$
$$r_1''=D-\Delta D-r$$

and V_1 is proportional to

$$\frac{1}{r_1'}-\frac{1}{r_1''}=\frac{1}{r-\Delta D}-\frac{1}{D-\Delta D-r}\approx$$
$$\frac{1}{r}\left(1+\frac{\Delta D}{r}\right)-\frac{1}{D}\left(1+\frac{\Delta D}{D}+\frac{r}{D}\right)\approx$$
$$\frac{1}{r}+\frac{\Delta D}{r^2}-\frac{1}{D}-\frac{\Delta D}{D^2}-\frac{r}{D^2}$$

Similarly, at point 2,

$$r_2'=r+\Delta D$$
$$r_2''=D-\Delta D+r$$

Automatic Temperature Control for Aircraft

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AUTOMATIC temperature control for aircraft long has been neglected, and it is only during the war that adequate attention has been given to this important subject. The increased duties imposed upon flight personnel by the requirements of modern military aircraft, together with the necessity for reducing air drag to the minimum consistent with required cooling, have prompted aircraft designers to give consideration to automatic operation of cowl flaps, oil and coolant flaps, and intercooler doors. Automatic flap operation can be appreciated fully when it is considered that engine cowl flaps fully opened might represent an air drag as high as 40 miles per hour on a 400-mile-per-hour airplane.

A primitive form of automatic temperature control was used initially on oil or coolant and employed the by-pass valve principle. This method directed the oil or coolant through or around the cooling surface in response to its temperature. Serious problems of oil congealing were introduced. As this type of control provides no control of the air through the cooler, the drag on the airplane is dictated by the amount of air required to provide the necessary cooling under the most adverse conditions of heat release, cooling air temperature, air density, and air speed. Much has been done to reduce congealing difficulties, but many authorities believe that fast-acting controls, operating to modulate the cooling air supply, offer the most direct approach to the problem.

Since most cooling flaps or doors are subjected to air loads of considerable magnitude, an electric motor or hydraulic servo mechanism is used as the load actuating device. The operating characteristics of the load actuating device are allied closely to the over-all operation of the system and must be considered as a component part of the system. The control of engine-head, oil or coolant, and carburetor-air temperature requires the same fundamental equipment, and further discussions will be confined to engine-head control as a representative system.

Types of System

Before any detailed discussion of automatic temperature control is attempted, it is desirable to establish complete understanding of the various types of systems together with a clear appreciation of the limitations and advantages offered by each. The more common classifications are described briefly under the headings by which they are generally known. Only electric types of thermosensitive elements are considered, although it is understood that the same fundamentals apply if the thermosensitive element is

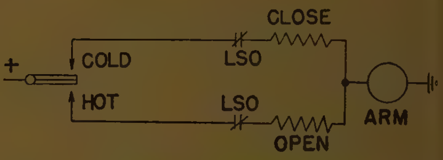

Figure 1. Schematic diagram of on–off system of temperature control

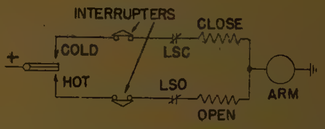

Figure 2a. Schematic diagram of point-modulating system of temperature control showing use of periodic interrupters

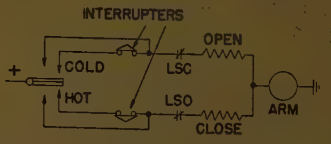

Figure 2b. Schematic diagram of point-modulating system of temperature control showing by-pass principle

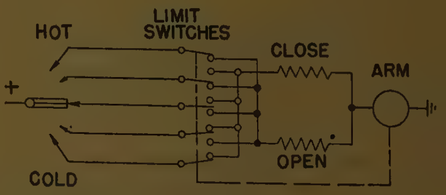

Figure 3. Schematic diagram of multipoint system of temperature control

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and V_2 is proportional to

$$\frac{1}{r_2'}-\frac{1}{r_2''}=\frac{1}{r+\Delta D}-\frac{1}{D-\Delta D+r}\approx$$

$$\frac{1}{r}\left(1-\frac{\Delta D}{r}\right)-\frac{1}{D}\left(1+\frac{\Delta D}{D}-\frac{r}{D}\right)\approx$$

$$\frac{1}{r}-\frac{\Delta D}{r^2}-\frac{1}{D}-\frac{\Delta D}{D^2}+\frac{r}{D^2}$$

In order that V_1 shall equal V_2

$$\frac{1}{r}+\frac{\Delta D}{r^2}-\frac{1}{D}-\frac{\Delta D}{D^2}-\frac{r}{D^2}$$

$$=\frac{1}{r}-\frac{\Delta D}{r^2}-\frac{1}{D}-\frac{\Delta D}{D^2}+\frac{r}{D_2}$$

from which $\Delta D=r^3/D^2$ and

$$S=\frac{D^3-2r^3}{D^2}$$

$$a=\frac{D^2-r^2}{D^2}\times r$$

$$b=D-\frac{D^2+r^2}{D^2}\times r$$

Appendix II. List of Symbols

A = constant, equation 13
B = constant, equation 13
C = capacitance, farads
D = conductor spacing, inches
$F(u)$ = a function of the numeric u (see Table I)
G = conductance, mhos
L = conductor length, inches
R = resistance, ohms
S = hypothetical charge spacing, inches
V = potential, volts
a = shortest distance from hypothetical line of charge to surface of the same conductor, inches
b = shortest distance from hypothetical line of charge to surface of the opposite conductor, inches
k = dielectric constant of the medium
m = distance from a point to the end of the conductor, in terms of L
n = number of hypothetical insulated sections into which the conductor is assumed to be divided
p_0 = permittivity of free space, 2.870×10^{-12} in the dimensional system used in this paper
p = permittivity of the medium, kp_0
q = charge, coulombs
r = radius of conductors, inches
γ = conductivity, mho per inch cube
ρ = resistivity, ohms per inch cube

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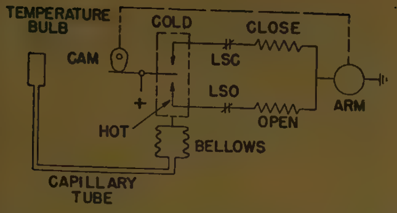


Figure 4a. Schematic diagram of the bulb-and-bellows type of band-modulating temperature-contol system

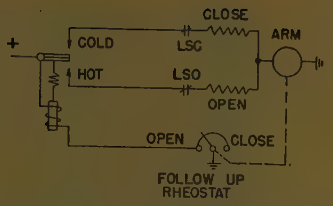


Figure 4b. Schematic diagram of an electric type of band-modulating temperature-control system

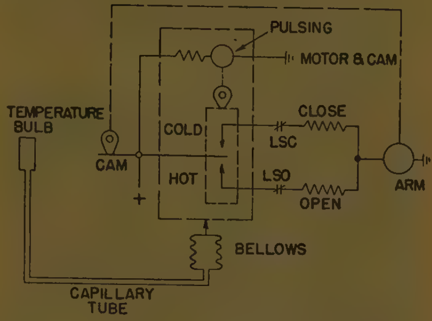


Figure 5. Schematic diagram of the sensitized band-modulating system of temperature control

used to actuate hydraulic or pneumatic valves instead of electric contacts.

On–Off System

The on–off system is the simplest of all temperature-control methods. It is well exemplified by the conventional household heating system. In this system a thermostat is calibrated to open or close contacts at some predetermined temperature. When the temperature is below this value, circuits are completed to start the heat-generating equipment. When the temperature rises to the calibrated value, the supply of heat is discontinued.

The on–off system is shown schematically in Figure 1. When the engine-head temperature reaches a predetermined value, the hot contact is closed, which energizes the flap motor in the open direction. When the temperature falls to a predetermined value, the cold contact is energized, and the flaps are closed. This system might tend to hold engine-head temperatures within narrow limits, but the resulting duty cycle would be quite impractical from the standpoint of the life of the equipment. Flying characteristics also would be affected violently, as the flaps suddenly would go wide open or completely closed.

Point-Modulating System

In an attempt to overcome the deficiencies of the on–off system, the point-modulating system was developed. This system must have the speed of flap opening or flap closing slow enough to permit stabilization before reaching the limits of travel. Various methods have been employed to reduce the speed of flap operation. One such method is shown in Figure 2a, where interrupters are interposed between the thermostat and the motor to permit opening and closing in a series of increments. This gives the cowl flaps more positions and permits the temperature of the engine to stabilize, which allows the thermostat to assume a neutral position. Modifications of this general idea range from thermal time relays to interrupters operated by centrifugal action from the flap motor. Another modification of this system is that shown in Figure 2b, wherein the interrupter is by-passed when the temperature is at wide variance but becomes operative as the temperature approaches the balance point.

This system introduces the element of time, and it is almost impossible to balance the time of flap operation with the time constant of the heating or cooling system. For instance the engine-head time constant varies over a wide range with the power output of the engine.

In many industrial applications where only a single variable exists, the point-modulation system has been successfully applied. However, in aircraft work where the cooling air temperature, air density, air speed, and heat release represent the variables, any attempt to fit the time of flap operation to the wide range of conditions encountered becomes an extremely difficult problem.

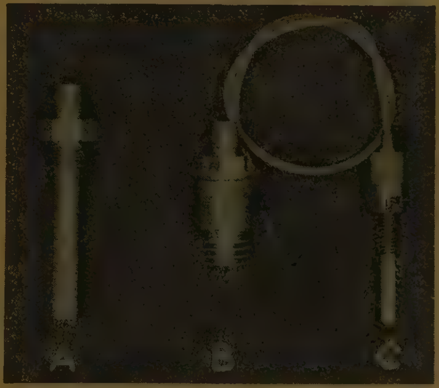


Figure 6. Electric thermosensitive elements

A—Oil or coolant service
B—Carburetor-air service
C—Engine-head service
Respective weights: 0.167 pound, 0.088 pound, 0.093 pound

Multipoint System

In this system the thermostat becomes a multicontact device with each contact representing a definite flap position as shown in Figure 3. As the temperature of the controlled medium varies, the flaps assume corrective positions until a balance is reached where the cooling supply approaches the cooling demand. To be successful this system requires a reasonable number of steps or points on the thermostat. If the thermostat has many points, then a large number of conductors between the thermostat and motor or limit switch assembly are required. If a small number of points are used, continuous hunting may result between the two steps nearest the required flap opening. This type of thermosensitive element is large and bulky and is very difficult to apply.

Band-Modulating System

Band-modulating temperature control may be defined as a control which recalibrates the temperature-sensing element as the flaps assume successive positions. As its name implies, the band-modulating system operates over a temperature range. The temperature range or band width is determined by two factors: system sensitivity, that is, the temperature change required to make the control operate and the number of flap positions required by the airplane's cooling system in order to obtain stable operation. Figure 4a is a typical aircraft control system of this type where the thermosensitive element is the bulb of a bulb-and-bellows assembly, and where the follow-up or

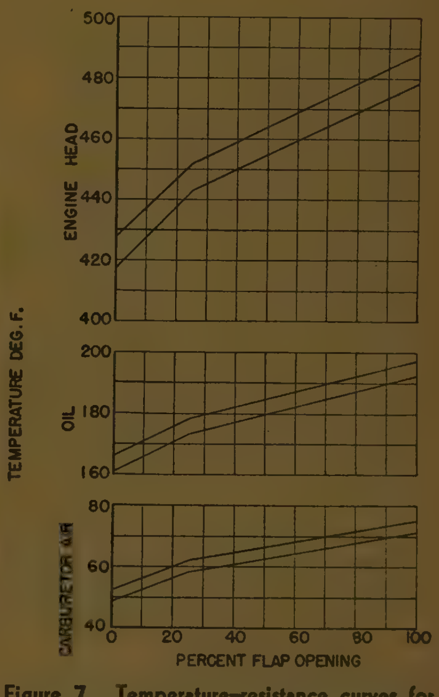


Figure 7. Temperature–resistance curves for engine-head, oil, and carburetor-air thermosensitive elements

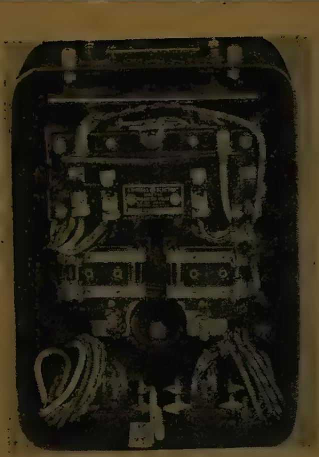


Figure 8. Open and enclosed views of the controller for automatic temperature control

Weight, 3.5 pounds

Figure 9 (below). Terminal identification of the polarized relay used in the automatic temperature-control system

recalibrating action is obtained by cam motion in the power unit. In this system one element of the contact mechanism is moved by the pressure in the bellows, while the recalibrating action is obtained through the cam which returns the other element of the contact mechanism to neutral. Many modifications of this system have been employed, differing chiefly in the type of follow-up mechanism used.

One of the difficulties of this system is in the relatively large size and low response rate of the bulb. In order to obtain the degree of sensitivity required, the amount of liquid in the bulb must be relatively large, and, since the ratio of surface to volume is low, the response rate is correspondingly low. In order to overcome this difficulty some systems employ multiple bulbs connected to a common bellows, thus giving a lower ratio of volume to surface. This, of course, takes more space. Another difficulty encountered in this system is the use of a capillary tube. If a long run is necessary, it becomes vulnerable to damage, and, unless compensating provisions are made, the system becomes ambient-temperature sensitive, because the capillary tube and bellows are subjected to different temperatures from the bulb.

Figure 4b is representative of an all-electric band-modulating system. In this type of control the thermostat controls the power unit as previously described, but, in addition, a biasing electromagnet is attached to the thermosensitive element and excited by a rheostat in such a way that, as the flap assumes corrective positions, the operating point of the thermostat is varied. Thus, if the thermostat is subjected to a colder temperature than desired, the cold contact is made. This operates the actuator in the close direction. Operation of the actuator in the close direction decreases the resistance of the follow-up rheostat which increases the pull on the biasing electromagnet, giving it greater pull which returns the thermostat to neutral. Full modulation of the cooling flap opening is thus obtained.

Sensitized Band-Modulating System

In an effort to obtain higher system sensitivity and, therefore, narrower band widths several schemes, all basically alike, have been devised. A mechanical pulse is imposed on the system such that in its neutral position the control almost operates in either direction; a slight change in temperature will cause a small correction to be made in the flap position. Figure 5 represents such a system. A small motor continuously operates a cam, which in turn moves the contacts up and down so that they almost make contact. If the bellows either expands or contracts slightly, a momentary contact is made, and the flap motor is jogged in the proper direction. If a large change should take place, the pulsing action will not break the contact until the flap is almost in position, at which time the flap will be jogged into position.

Very sensitive systems can be made in this manner, but the more sensitive the system becomes the greater the duty cycle of the flap actuator becomes. Also the pulsing motor must be a continuously rated motor with exceptionally long life.

Since the advent of the resistance-bulb temperature indicators, several electronic temperature control systems have been designed. Narrow band widths can be obtained because of the great amplification obtained with electronic tubes. However, an electronic system has a great weight disadvantage because of the extra equipment necessary such as inverters, transformers, and so forth.

An Electric System

The only systems which have had any success in controlling engine-head cooling flaps have been the band-modulating or sensitized-band-modulating systems.

The bulb-and-bellows type of control has been successfully applied to oil and coolant temperature control. However, the only successful simple lightweight control so far developed for engine-head service has been an all-electric band-modulating system which, with the use of different thermosensitive elements, also will control oil carburetor-air, or coolant temperatures.

Components of the System

The system consists of a thermosensitive element, an actuator, which may be either an electric motor or hydraulic servo mechanism, and a follow-up rheostat to recalibrate the system. The necessary cockpit switch and limit switches also must be included.

The thermosensitive element, see Figure 6 for illustrations, consists of a material which has a high negative temperature coefficient of resistance. Figure 7 shows the resistance temperature curves for three types of elements: engine head, oil, and carburetor air. The constants of the circuit are such that the thermosensitive element always has approximately constant wattage input over the systems operating band. This heat input is such that the thermosensitive elements themselves are always at a higher temperature than the body being cooled.

The controller, Figure 8, consists of a very sensitive polarized relay and power relays. The power relays control the operation of the flap motor; they are of conventional design.

The polarized relay works in conjunction with the thermosensitive element and the follow-up rheostat and controls the power relays. The relay, Figure 9, is a flux-balancing relay. It consists of a permanent magnet for polarization, two coils, and a balanced armature. Each

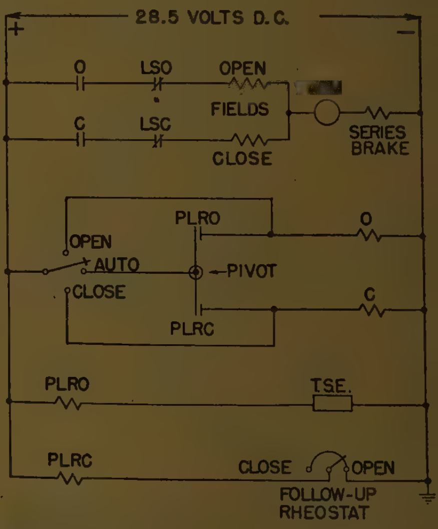


Figure 10. Elementary diagram of an automatic temperature-control circuit

coil has an inside winding, connected in series and to terminals 4 and 5, and an outside winding, connected in series and to terminals 4 and 6. The inside windings have twice as many turns as the outside, but both windings have equal resistance. The armature has a common connection at terminal 7 which may contact either terminal 8 or terminal 9.

Figure 10 is an elementary diagram of the basic circuit. The high-turn windings of the polarized relay are coil *PLRO* and the low-turn windings, coil *PLRC*. In order to balance the relay equal flux must be produced by each coil, therefore, the current through coil *PLRC* must be twice as great as the current through coil *PLRO*. This condition can be met only if the thermosensitive-element (*TSE*) resistance is equal to the resistance of coil *PLRC* plus twice the resistance of the follow-up rheostat.

Inasmuch as the thermosensitive element controls the position of the flap, the excitation of coil *PLRO* always is changed first. When the flap position is changed, it moves the follow-up rheostat until the relay is rebalanced. Figure 11 is a graphical representation of the operation of the polarized relay. Note that it takes a greater change in the flux produced by coil *PLRO* to reverse direction. This is done in order to prevent overshooting or hunting. Coil *PLRO* is energized only 0.8 of a milliwatt greater or less than its balancing excitation in order to operate the relay.

Figures 12 and 13 are typical examples of follow-up units. Figure 12 is a geared actuator with the follow-up unit mounted at the left on the assembly; Fig. 13 is a separately mounted follow-up unit which can be connected by a cable to any part of the flap actuating mechanism which has relatively straight line motion. Both of these units incorporate a rheostat and limit switches. The resistance of the rheostat controls the width of the temperature band as previously described.

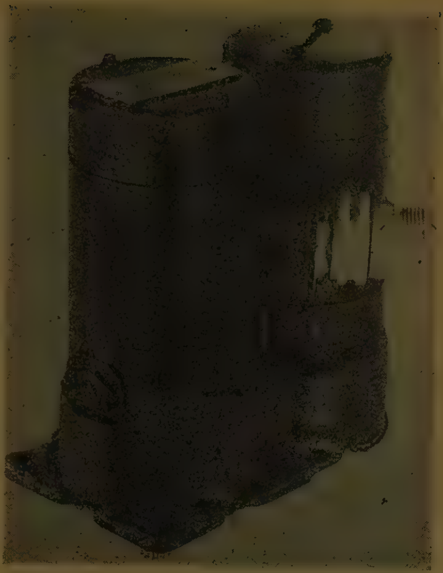
Since a given flap change near the flap-closed position produces more cooling effect than the same flap change near the flap-open position, the follow-up rheostat must compensate for this condition by being tapered.

The rheostat in the gear-driven follow-up unit has a tapered winding. The taper is produced by the cam in the separately mounted follow-up unit. Both tapers are shown in Figure 14.

Any conventional actuator, either electric or hydraulic, can be used with this system, so long as it can be stopped quickly.

System Operation

In the diagram, in Figure 10, when the medium to be cooled is at a low temperature, the flaps are closed. The follow-up rheostat then has full resistance in coil *PLRC* circuit. As the medium rises in temperature the temperature-sensing element (*TSE*) decreases in resistance until the excitation of coil *PLRO* is greater than that of coil *PLRC*. This causes contact *PLRO* to close, exciting coil *O*. When coil *O* is energized, it closes contact *O* which energizes the motor in the open direction. As the flaps are opened, the follow-up rheostat decreases in resistance until coil *PLRC* is energized the same amount as coil *PLRO*. This causes contact *PLRO* to open, de-energizing



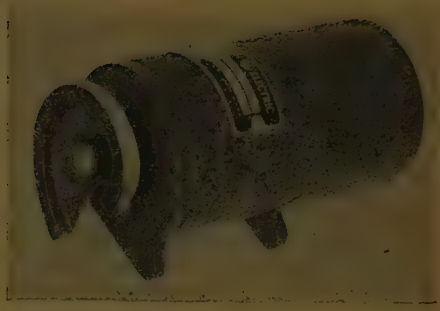


Figure 13. A separately mounted cam-driven follow-up unit

Weight, 1.32 pounds

Figure 12 (left). Electric actuator with follow-up unit forming a complete unit assembly

Weight of follow-up unit, 1.2 pounds

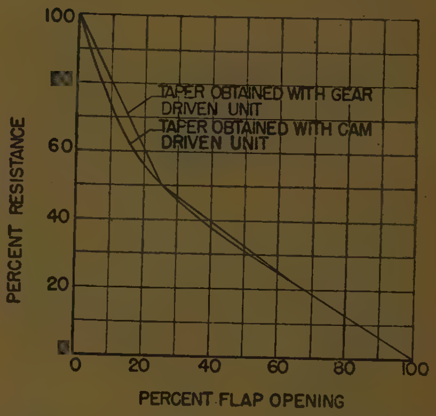


Figure 14. Typical per-cent resistance versus per-cent flap opening curves for follow-up units

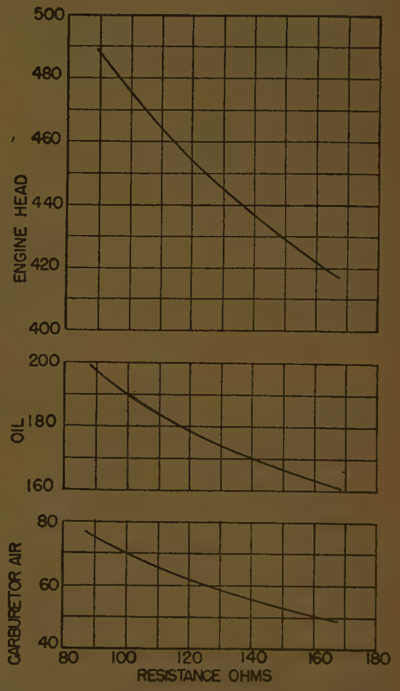


Figure 15. Calculated envelopes of temperature versus per-cent flap opening for engine-head, oil, and carburetor-air service

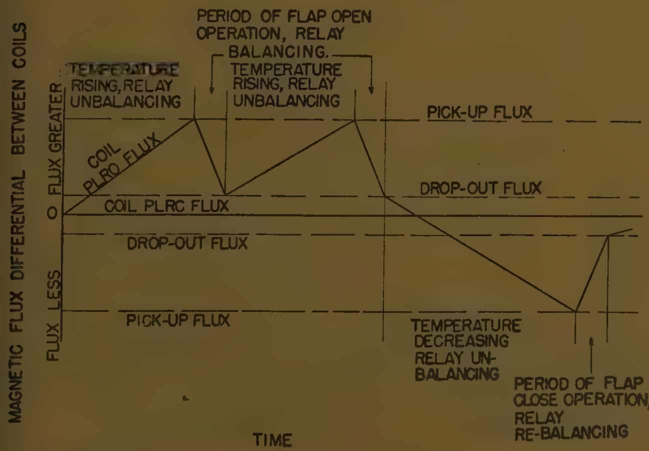


Figure 11. Graphical representation of the polarized-relay operating principle

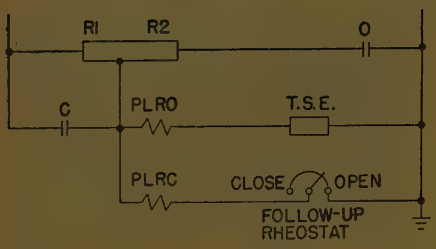


Figure 16 (left). Elementary diagram showing how circuit in Figure 10 can be sensitized

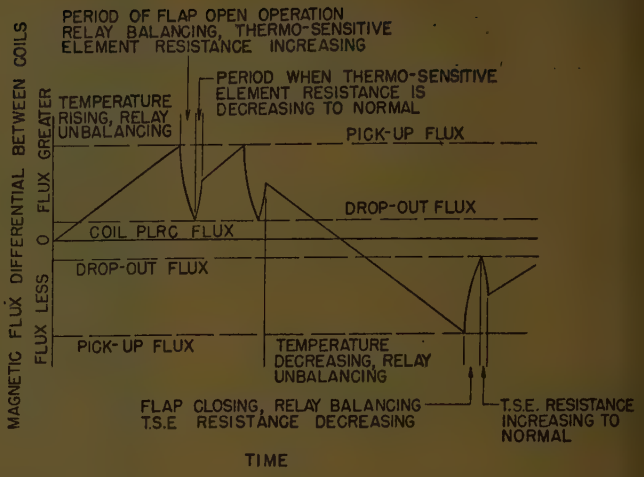


Figure 17. Graphical presentation of the operation of the polarized relay in the sensitized circuit

coil *O*, and opening contact *O*. This de-energizes the motor and sets the series brake, thus stopping the flaps at an open position. If the medium to be cooled is not yet receiving enough cooling air, the same procedure is followed. The flaps will stabilize at a point where the cooling air equals the heat release of the medium.

If the medium drops in temperature, then the temperature-sensing element increases in resistance until coil *PLRO* is energized less than coil *PLRC*. This causes contact *PLRC* to close energizing coil *C* and causing contact *C* to close which energizes the motor in the close direction. As the flaps are closed the follow-up rheostat increases in resistance until the excitation of coil *PLRC* is reduced to equal that of coil *PLRO*. When the coils are balanced, contact *PLRC* opens, de-energizing coil *C*, and opening contact *C*. This de-energizes the motor and sets the series brake to stop the flaps at a further closed position. Limit switches *LSO* and *LSC* limit the flap travel in the open-and-close direction.

Figure 15 shows the various envelopes of temperature versus flap opening for the engine-head, oil, and carburetor-air service. These curves are plotted assuming a 32-ohm follow-up rheostat, following the typical taper curve of Figure 14 and using the curves shown in Figure 7.

Unfortunately flight data cannot be included in this paper because of their restrictive nature.

The band-modulating system just described also can be changed to a sensitized band-modulating system. It must be remembered that the thermosensitive elements dissipate about two watts which causes them to be at a higher temperature than the medium being cooled. If it is possible to introduce less heat to the element when the flaps are opening or more heat to the element when the flaps are closing (preheating and precooling), the system can be sensitized. This is done by varying the voltage across the polarized relay coil circuits as shown in Figure 16.

In the balanced condition the voltage drop across *R*1 is the drop due to the current flowing through the polarized relay coils, the thermosensitive element, and the follow-up rheostat. When the flaps are driven in the open direction, contact *O* closes, which causes an additional voltage drop through *R*1 due to the current drawn through *R*2. Thus the power input to the thermosensitive element is decreased by reducing the voltage across it, which causes the resistance of the element to increase. This causes the polarized relay to rebalance sooner than it normally would.

When the flap is driven in the close direction, contact *C* closes, throwing the sensitive circuit across the line. This increases the power to the thermosensitive element by increasing the voltage across it, which causes the element to decrease in resistance. Again the polarized-relay rebalances sooner than it normally would. Figure 17 presents graphically the operation of the polarized relay under sensitized conditions. Note that a small amount of time is necessary for the relay to build up its flux differential after rebalancing. This prevents relay chatter which is present in every other type of sensitizing scheme so far devised for this system.

Conclusions

Automatic temperature control of cooling flaps increases the performance of the airplane, because it protects against destructive operating temperatures. It insures maximum efficiency and minimum drag by eliminating the human element.

Electric controls are more flexible than mechanical controls and are therefore easier to apply, especially in fighter aircraft where space in certain parts of the air frame is at a premium.

Judging Mica Quality Electrically

K. G. COUTLEE
NONMEMBER AIEE

MICA CAPACITORS by the millions
have been used in radio, radar, and
her important electronic equipment for
ır Armed Forces. The unprecedented
emand has strained all our resources, and
uch time and money have been spent in
forts to solve the mica supply problem.
ne of these was a research project con-
ucted by Bell Telephone Laboratories
r the War Production Board. It was
ncerned with the development of elec-
ical test methods to predetermine the
uality of raw mica in terms of end-use
quirements in various grades of capaci-
rs. After over two years of work cov-
ing development, design, laboratory
ial and an extensive commercial trial of
o types of test instruments, this project
s been brought to a successful conclu-
on. As the electrical method of classi-
cation makes possible the use of mica
ailable in large quantities, but previ-
usly considered unsuitable for capaci-
rs, the mica supply for capacitors is
eatly increased. An estimated saving
to 60 per cent of the total amount of
gher-quality mica used in capacitors is
dicated.

Mica possesses many useful properties
at make it particularly attractive as a
pacitor dielectric. In general, these
e high dielectric strength, high dielec-
c constant, low dielectric loss, chemical
ertness, low-temperature coefficient of
pansion, and good resistance to heat.
The important feature that makes it possible to utilize these properties in a capacitor is that mica can be readily split into very thin films of uniform thickness.

India is the chief source of supply of capacitor mica due to its extensive resources of high-quality mica, low labor cost, and a long established business in supplying world-wide markets. Other less developed sources are South America, the United States, and Canada. With the advent of war, the sharp increase in demand for mica, coupled with wartime shipping hazards, created a scarcity. The Government immediately took steps to insure that the supply from established sources would be increased, to find new sources, and to encourage the development and use of mica substitutes.

Visual Quality Classification of Mica

The Indian visual system of classifying natural ruby mica incorporated in American Society for Testing Materials method *D*351 lists seven different qualities (see Table I). It had become more or less standard practice with capacitor manufacturers to consider all qualities below fourth quality (ruby good stained) as unsuitable for capacitor use, and a great many preferred not to go beyond third-best quality (ruby fair stained). Under no conditions was mica used that contained black spots or stains. Capacitor manufacturers had found from experience that by employing these better qualities of ruby muscovite mica a satisfactory product would be obtained. Unfortunately, the better qualities constitute a relatively small portion of total mica production, and the expected wartime demand far exceeded the supply. There was no satisfactory evidence however, that some and perhaps much of the lower-quality ruby and other types of mica might not also have been suitable for capacitor use.

Table I. India* Visual System for Classifying Quality of Natural Mica

Clear
Clear and slightly stained
Fair stained
Good stained
Stained
Heavy stained
Black stained and spotted

*See reference 1.

More Positive Test Needed

The War Production Board sought the assistance of Bell Telephone Laboratories in the fall of 1942 to devise electrical test methods that could be used to select mica for capacitor use. Such tests, or methods of classification, while assuring that no defective mica could be put into finished capacitors, were also to insure that no good mica of any type was rejected. The problem was not only to select mica from new sources where appearance was a questionable guide, but, of perhaps even greater importance, to salvage satisfactory capacitor mica from the large available stocks of mica which had been rejected as unsuitable for capacitors by visual standards.

Mica-Capacitor Requirements

Generally speaking, mica capacitors for military equipment can be divided into two broad groups in respect to duty required. The first group comprises those

Table II. *Q* and Power-Factor Values for Electrical Quality Groups, *E*-1, *E*-2, and *E*-3

				Rapid-Method Meter Reading		
Q or Power-Factor Group	Form	Q Value	Power Factor	0.010 In.† (0.007 to 0.015 In.)	0.020 In.† (0.015 to 0.025 In.)	0.030 In.† (0.25 to 0.035 In.)
E-1	Block mica	2,500 minimum	0.0004 maximum	95 to 100	95 to 100	95 to 100
	Films*	2,500 minimum	0.0004 maximum			
E-2	Block mica	350 to 2,500	0.00285 to 0.0004	87 to 95	77 to 95	71 to 95
	Films*	1,500 minimum	0.00066 maximum			
E-3	Block mica	50 to 350	0.02 to 0.00285	50 to 87	32 to 77	24 to 71
	Films*	200 to 1,500	0.005 to 0.00066			

*Probable *Q* values of molded-type 1,000-micromicrofarad (American War Standard *C*75.3) stacked-foil and silvered capacitors. These will apply when all factors that would adversely influence the *Q* value are under control.

†Thickness of block mica or mica films stacked to this thickness.

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ies, Inc., New York, N. Y.

is work was carried out by the Bell Telephone
boratories, Inc., under a contract between
stern Electric Company and the War Production
ard under the supervision of the National
ademy of Sciences. The capacitor manufac-
ers that participated in the tests were Aerovox,
rnell-Dubilier, Electro Motive Manufacturing,
camold, Sangamo, Solar, Sprague, and Western
ctric Companies. The various types of mica
d in the tests were chosen by the mica-graphite
ision of the War Production Board and furnished
Colonial Mica Corporation as agent of Metals
serve Corporation. This project was carried
under difficult wartime conditions, and particu-
credit is due the capacitor manufacturers for
ir patience and skill in steering the test lots
ough the shop without losing track of their
ntity. Through their efforts, it is believed that
valuable contribution has been made to the
acitor art. It has afforded an opportunity to
erve, by actual tests, the effects of various de-
s in mica that generally have been considered
lesirable for capacitor use.

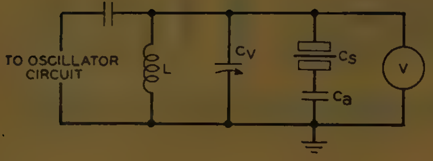


Figure 1 (left). Simplified circuit indicating the method of testing for Q or power factor

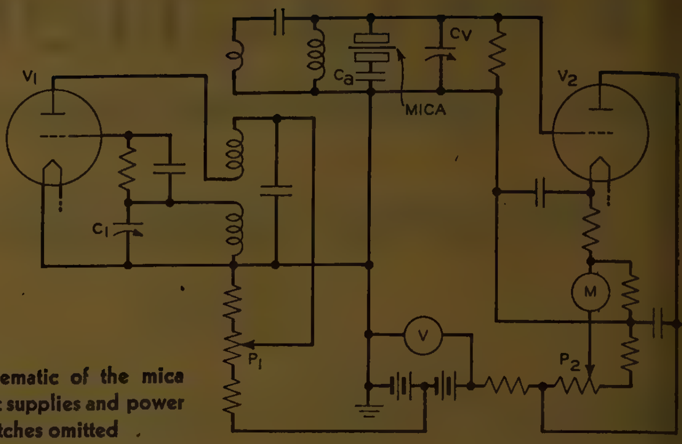


Figure 2. Circuit schematic of the mica Q test set with filament supplies and power and voltage switches omitted

used principally in high-stability tuned circuits, some of which must carry appreciable amounts of radio-frequency current. The second group embraces those used in less critical circuits for coupling, blocking, and by-pass purposes. Of equal importance to both groups is dielectric strength and long life; hence, all capacitor mica must be free of cracks, tears, pinholes and conducting foreign inclusions, which might adversely influence such properties. Mica for the first group must have low dielectric loss, and its characteristics must be such that wide temperature changes will cause only minor capacitance changes, and these always must repeat themselves. It is also preferable that mica be free of excessive waviness and air inclusions.

In the case of capacitors in the second group, the requirements are such that they can be met easily even with low-quality mica; a number of other materials, such as oil-impregnated paper, resin-treated paper, and resin-treated clay, also proved suitable. There also are certain other materials such as fused silica, titanium-dioxide mixtures, steatite, and glass, with characteristics that permit their use in some capacitors of the first group. However, the inferiority to mica in either dielectric strength or dielectric constant generally limited their use to smaller capacitance types.

Mica Properties Needing Control

As the dielectric constant of mica was found to show relatively little variation for different types and qualities from world-wide sources, no control of this property seemed necessary. Efforts were directed accordingly toward developing suitable test methods to determine the other important electrical properties of mica for capacitors: namely, dielectric loss and dielectric strength. As it seemed considerable testing might be carried out at mine locations, standard[7] power-factor methods for mica obviously were too cumbersome and slow. The ideal would be a rapid power-factor test method in a portable test-set form. Several methods were tried, and finally one was found that appeared suitable for quickly classifying the power factor of raw "block" mica (0.007 to 0.030 inch thick) as it appears in commerce.

Rapid Power-Factor Test Method

This method gives a reliable numerical indication of power factor or its approximate reciprocal equivalent Q as a single reading of a meter. Essentially it comprises a resonant circuit in conjunction with adjustable two-inch circular flat test electrodes mounted in a heavy metal frame. The choice of two-inch electrodes was based on the optimum size of block mica used for capacitors, and permitted the testing of mica somewhat smaller than the electrodes up to five inches in width. Forming an integral part of the moving upper electrode and connected to the grounded side of the test circuit is a fixed air capacitor of 20 micromicrofarads (20×10^{-12} farads). A schematic of the test circuit is shown in Figure 1, and in diagrammatic form in Figure 2. A vacuum-tube oscillator is capacitively coupled and tuned to the test circuit. Resonance is indicated by a vacuum-tube voltmeter connected in parallel with the tuned circuit. The inductive portion of the tuned circuit is a coil having a Q of approximately 300 at one megacycle. A test frequency of one megacycle was used at the request of the War Production Board, as the greatest number of capacitors were operating in the radio-frequency range. During initial adjustment, the test electrodes are closed, which connects the 20-micromicrofarads fixed capacitor in parallel with the coil. Oscillator input and tuning adjustments are made to bring the reading of the voltmeter to full-scale deflection. The electrodes then are opened by turning the knob that raises the upper fixed-capacitor electrode, and a test sample is clamped between electrodes. Only one adjustment is necessary to re-establish resonance. This is accomplished by adding to the circuit the small amount of capacitance that is lost when the relatively high-capacitance block-mica sample (150 to 450 micromicrofarads) is placed in series with the 20-micromicrofarad fixed capacitor. This additional capacitance is furnished by a 10-micro-

Table III. Electrical, Physical, and Visual Quality Requirements of Natural Block Mica and Mica Films for Use in Capacitors

		Class C-1	Class C-2	Class C-3
Electrical Properties				
Conductivity		None	None	None
Q value or power factor at 1 megacycle		E-1	E-2	E-3
Dielectric strength, volts per mil at 60 cycles per second	average	1,000	1,000	1,000
	single	850	850	850
Dielectric constant		*	*	*
Physical Properties				
Weight loss on heating (5 min at 600 C), maximum, per cent		0.2	0.2	
Thickness uniformity (mica films)		Best	Best	Intermediate
Temperature coefficient of capacitance and retrace		†	†	†
Visual Qualities				
Air inclusions	*A*	None to slight	None to slight	None to slight
	B	Medium	Medium	Medium
	C	Medium to heavy	Medium to heavy	Medium to heav
Waves	*A*	Flat to slight	Flat to slight	Flat to slight
	B	Medium	Medium	Medium
	C	Medium to heavy	Medium to heavy	Medium to heav
Cracks		None	None	None
Tears		None	None	None
Pinholes		None	None	None
Stones		None	None	None
Buckles		None	None	None
Ridges		None	None	None

*As the dielectric constant of natural block mica suitable for use in capacitors is fairly uniform, no specified requirement is needed.

†It has been found that the temperature coefficient of capacitance and retrace of capacitors made with classes C-1, C-2, and C-3 mica are more dependent on such factors as electrical and mechanical design and manufacturing technique than any differences that may be attributed to the mica itself.

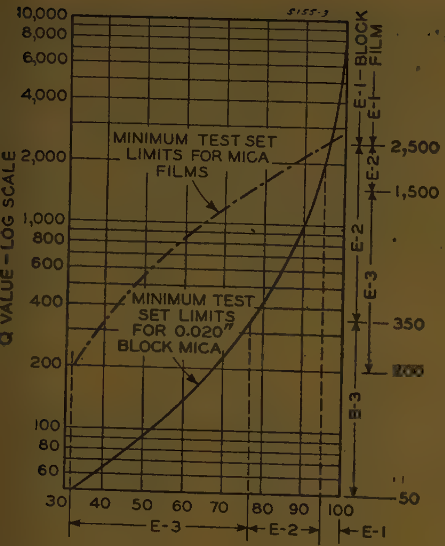
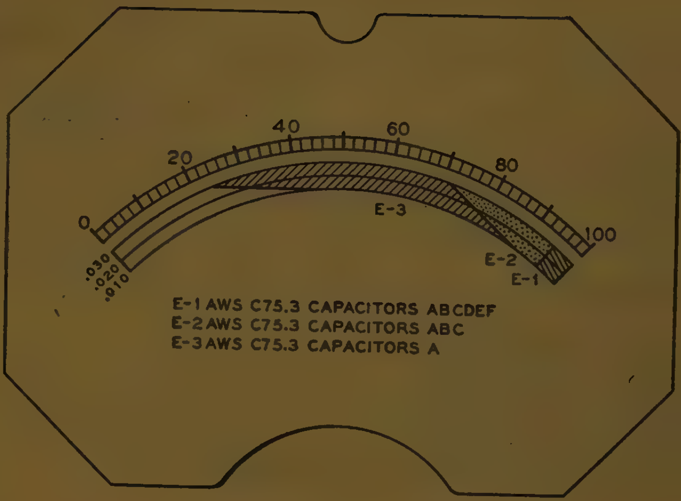


Figure 3 (left). Typical calibration curves for mica Q test set

Figure 4 (right). Meter scale of the mica Q test set


0
20
40
60
80
100
.030
.020
.010
E-3
E-2
E-1
E-1 AWS C75.3 CAPACITORS ABCDEF
E-2 AWS C75.3 CAPACITORS ABC
E-3 AWS C75.3 CAPACITORS A

nicrofarad vernier capacitor connected n parallel with the coil.

In following such a procedure the initial requency of the circuit is duplicated each ime a sample is tested. In other words, nce the initial setting is made, all that is ecessary to make a series of tests is to lamp the test specimen between the lectrodes each time, make one simple djustment of the vernier capacitor to btain maximum deflection of the meter nd read the meter scale corresponding to he thickness of the specimen as indicated y a scale on the clamping dial. It was ossible to test up to 15 samples per min-te, compared with about 15 minutes per ample.

If the Q of the test specimen of mica is f the order of 10,000 (power factor .0001), the needle of the meter will return o the initial full-scale deflection. If the) is lower, the meter will read less than ll scale, depending upon its magnitude.

The Q of block mica of any basic color r visual quality (based on minimum values) is shown in terms of rapid method Q test-set readings in Figure 3. The minimum equivalent Q of films split from such block mica (in the form of molded silver-electrode 1,000-micromicrofarad capacitors) also is plotted in terms of Q test-set readings originally obtained for the block. These data indicate that the Q at one megacycle of films (in capacitors) split from high Q block may not attain so high a value as the block. Probable reasons for this are dielectric loss in the molded capacitor housing, impregnants and moisture residuals, as well as series resistance in leads and electrodes. In the case of films split from low Q block such films most always show a higher Q at one megacycle than the block. Generally, this is believed to be due to the release of moisture during splitting, since the Q reading of a bundle of splits usually is higher than that of the block. However, in some cases this apparently is not so, and no improvement in Q is accomplished, even after baking at temperatures as high as 1,000 degrees Fahrenheit. Such behavior does not seem to be specifically confined to any particular type of mica or to any peculiar variety of foreign inclusions. Persistently low Q mica may be:

1. Clear with no visible foreign inclusions.
2. Clear with numerous small air inclusions.
3. Clear with minute black specks in the form of a gray cloudy haze.
4. Black spotted and stained mica.

It seems evident, based on Q-frequency behavior, wherein the Q drops to abnormally low values at low frequency, that such low Q as cases 1 and 2 is due to entrapped moisture that is not readily released. In cases 3 and 4 low Q apparently arises from dielectric loss in the spots and stains.

In setting limits for Q, it appeared reasonable to establish three ranges based on Q actually obtainable with mica rather than on Q requirements imposed by circuits. The upper range would cover the highest Q obtainable for any block mica, the middle range a very high order of Q but somewhat less than the best obtain-

Table IV. Experimental Laboratory Trial of Mica Test Sets

	Block						Capacitors Made With Films From Electrical Selected Block						
				Electrical Quality			Average† Q	Dielectric Strength‡	Insulation Resistance‡	Corona Voltage§	Life Test§		Aging¶
Lot	Source	Visual Quality	Color	Conducting Inclusions	1 Mc Q Range	Meter Reading	(1 Mc)	(Per Cent Correct)	(Per Cent Correct)	(Volts 60 Cps)	(Tested)	(Failed)	(Per Cent Change in Capacitance)
dia	India	Fair stained	Ruby	No	*E*-1	99	5,000	100	100	244 average	8	0	0.077 average
A	S. D.	Clear	Ruby	No	*E*-1	99	3,900	100	100	269 average			0.195 average
A	S. D.	Stained	Ruby	No	*E*-2	93	3,100	100	100	253 average	6	2	0.211 average
B	N. C.	Clear	Ruby	No	*E*-1	99	4,900	100	100	270 average			0.100 average
B	N. C.	Stained	Ruby	No	*E*-2	94	3,800	100	100	261 average	6	1	0.177 average
C	N. M.	Clear	Green	No	*E*-2	93	2,800	100	100	256 average			0.077 average
C	N. M.	Stained	Green	Yes	Below *E*-3	*	140	100	100	268 average	6	0	0.319 average
D	N. C.	Clear	Green	No	*E*-1	96	3,500	100	100	265 average			0.212 average
D	N. C.	Stained	Green	No	*E*-2	93	2,800	100	100	243 average	6	0	0.161 average
E	N. H.	Clear	Ruby	No	*E*-1	97	4,100	100	100	266 average			0.260 average
E	N. H.	Stained	Ruby	No	*E*-1	96	3,800	100	100	245 average	6	0	0.169 average
F	Conn.	Clear	Ruby	No	*E*-2	93	3,500	100	100	260 average			0.216 average
F	Conn.	Stained	Ruby	No	*E*-2	89	2,400	100	100	246 average	6	0	0.264 average
G	N. C.	Stained	Green	Yes	*E*-2	80	1,700	100	100		4	0	

atisfactory readings could not be obtained in most cases.

,000-micromicrofarad silver-molded capacitors.

,000-micromicrofarad silver and 4,000-micromicrofarad foil-molded capacitors.

,000-micromicrofarad foil-molded capacitors.

cycles, +72 F to −40 F to +72 F to +185 F. 1,000-micromicrofarad silver-molded capacitors.

Table V. Summary of Manufacturers' Data on Capacitors Made With Selected *E*-1,

Property	No. of Mfrs. I	No. of Mfrs. II	Capacitors Per American War Standard C75.3		Part I A	Part I B	Part I D	Part I E	Part I F
Block Mica									
Source					India	Brazil	United States	United States	United States
Basic color					Ruby	Ruby	Ruby	Green	Ruby
Commercial quality					Stained	Stained *B*	2d quality	2d quality	2d quality
Grade (size)					No. 4	No. 4	2x2	2x3	2x2
Chief defects					*Air	Air	Air		Black spots
Block Q range (original lot), per cent									
Block Q, *E*-1, 2,500 minimum					94.0	77.0	68.1	64.3	80.5
Block Q, *E*-2 350 minimum					6.0	20.6	31.0	34.5	19.1
Block Q, *E*-3 50 minimum					0.0	2.4	0.9	1.2	0.4
Per cent conducting (original lot)					0	0	0	0	5
Selected block Q range (used in capacitors)					*E*-1	*E*-1	*E*-1	*E*-1	*E*-1
Selected block conduction (used in capacitors)					None	None	None	None	None
Selected block quality class (based on Q air inclusions, waviness, etc.)					**C*-1 *BB*	*C*-1*CB*	*C*-1*C*+*B*	*C*-1*AB*	*C*-1*BB*
Splitting properties	8	2			Fair	Good	Very good	Good	Fair
Films, 1 to 4 mils (per cent yield)	9	2		Average	75.0	93.0	94.0	90.5	87.7
				Minimum	75.0	93.0	94.0	79.0	77.7
Laminations (per cent yield)	7	2		Average	35.5	37.1	34.8	39.3	35.4
				Minimum	22.8	28.9	27.3	31.2	21.3
Silvered laminations (per cent yield)	5	1		Average	89.9	86.8	81.7	90.3	88.5
				Minimum	82.5	82.0	70.7	84.6	79.3
Capacitors									
Capacitor Q (statistical distribution)	5	1	*CM*30—1,000-μμf, silver	Per cent>2,500	98.8	93.7	90.1	97.1	99.1
				Per cent 1,500–2,500	1.1	5.7	9.0	2.8	0.9
				Per cent 200–1,500	0.1	0.6	3.7	0.1	0.0
	6		*CM*30—1,000-μμf, foil	Per cent>2,500	94.5	95.6	97.2	94.7	96.8
				Per cent 1,500–2,500	5.4	4.1	2.4	5.2	2.2
				Per cent 200–1,500	0.1	0.3	0.4	0.1	1.0
	3	1	*CM*55—1,000-μμf, foil	Per cent>2,500	99.8	95.0	96.6	99.1	98.8
				Per cent 1,500–2,500	0.2	2.7	2.1	0.7	1.0
				Per cent 200–1,500	0.0	2.3	1.3	0.2	0.2
	3	1	*CM*70—510-μμf, foil	Per cent>2,500	76.5	87.0	51.3	86.0	78.1
				Per cent 1,500–2,500	18.5	13.0	44.2	7.0	21.9
				Per cent 200–1,500	5.0	0.0	4.5	7.0	0.0
American War Standard dielectric strength (per cent meeting requirement)	7	1	*CM*30—1,000-μμf, silver		95.5	98.1	98.7	97.0	96.7
	8		*CM*30—1,000-μμf, foil		95.5	95.6	96.8	96.6	96.7
	3	1	*CM*55—1,000-μμf, foil		95.9	98.2	95.8	96.5	96.1
	2	1	*CM*35—6,200-μμf, foil		92.2	94.8	99.3	96.8	99.7
	2	1	*CM*60—6,200-μμf, foil		96.1	99.5	98.9	93.2	91.0
	4	1	*CM*70—510-μμf, foil		100.0	100.0	100.0	100.0	100.0
American War Standard insulation resistance (per cent meeting requirement)	7	1	*CM*30—1,000-μμf, silver		99.9	99.6	99.7	99.8	99.6
	8		*CM*30—1,000-μμf, foil		99.8	99.1	99.3	99.8	99.4
	3	1	*CM*55—1,000-μμf, foil		99.7	99.7	99.9	99.9	99.9
	2	1	*CM*35—6,200-μμf, foil		98.4	99.2	99.2	96.0	99.3
	2	1	*CM*60—6,200-μμf, foil		100.0	100.0	100.0	100.0	100.0
	4	1	*CM*70—510-μμf, foil		100.0	100.0	100.0	100.0	100.0
American War Standard radio-frequency heating (per cent meeting requirement)	4	1	*CM*70—510-μμf, foil		100.0	100.0	100.0	100.0	100.0
Corona starting voltage—60 cycles per second—(no American War Standard requirement)	1		*CM*70—510-μμf, foil	(5 sections)	1,900	2,200	2,700	2,300	2,000
	1		*CM*70—510-μμf, foil	(8 sections)	3,600	3,500	3,600	3,600	3,400
	1		*CM*70—510-μμf, foil	(9 sections)	4,300	4,100	4,000	4,000	4,600
					†1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
American War Standard temperature—coefficient and drift (American War Standard characteristic)	5	1	*CM*30—1,000-μμf, silver		*E D B*	*D C B*	*D C B*	*E D B*	*E D C*
	6		*CM*30—1,000-μμf, foil		*C B B*	*B B B*	*B B B*	*B B B*	*C B B*
	3		*CM*55—1,000-μμf, foil		*B B B*	*B B B*	*B B B*	*C B B*	*C B B*
	2	1	*CM*70—510-μμf, foil		*D D D*	*D D D*	*D C B*	*D D C*	*D D D*
‡ American War Standard capacitor life (per cent meeting requirement)	5	1	*CM*30—1,000-μμf, silver		93.7	100.0	100.0	97.9	100.0
	5		*CM*30—1,000-μμf, foil		100.0	100.0	98.2	98.2	100.0
	3	1	*CM*55—1,000-μμf, foil		100.0	92.7	100.0	100.0	98.3
	2	1	*CM*35—6,200-μμf, foil		100.0	96.0	96.0	100.0	100.0
	1	1	*CM*60—6,200-μμf, foil		92.0	88.0	96.0	96.0	96.0
	3	1	*CM*70—510-μμf, foil		100.0	100.0	100.0	100.0	100.0
Grand Averages									
American War Standard dielectric strength (per cent meeting requirement)			All types and sizes of above capacitors		95.8	97.7	98.2	96.6	96.7
American War Standard capacitor life (per cent meeting requirement)			All types and sizes of above capacitors		97.6	96.1	98.4	98.7	99.1

able, and a third range of somewhat lower order of Q, but high as compared with other types of insulating materials in general. The top Q range simulated that normally assumed for mica and presumably used in capacitors in the past. While it became evident that such high Q actually is only needed in a relatively small percentage of capacitors, of the previously mentioned first group, it affords a margin of safety to which capacitor manufacturers apparently are accustomed. The middle range probably is high enough for all but the most critical types of capacitors of the first group, and the third Q range is suitable for by-pass, blocking, and coupling types of capacitors of the second group. These three ranges are designated *E*-1, *E*-2, and *E*-3, respectively, and the minimum Q limits assigned to each range are shown in Table II as well as equivalent test-set readings. This table also shows minimum Q limits fo mica films split from the block. Th actual calibration of the scale of the mete is shown in Figure 4.

Spark-Test Set

It was observed in studies of variou types of block mica that the Q of "blac stained and spotted" samples, in som cases, was just as high as that observe

E-2, and E-3 Block Mica Showing Over-All Performance From Block to Capacitors

Part I (Continued)					Part II								
H	I	J	L	M	N-1	N-2	N-3	N-4	N-5	O	P	Q	R
United States	United States	United States	Canada	India	United States	United States	United States	United States	United States	United States	United States	Canada	Canada
Ruby	Green	Ruby	Green	Ruby	Ruby	Ruby	Ruby	Ruby	Ruby	Green	Green	Green	Green
2d quality	2d quality	2d quality	2d quality	Good stained	3d quality	3d quality	3d qualify	3d quality	3d quality	3d quality	3d quality	2d quality	3d quality
2x3	2x2	2x3	2x3	No. 4	Mixed sizes, 1x1½ to 2x3					2x3	2x3	2x3	2x3
Waves	Black spots	Waves		Air	Black spots	Reeves	Smoke	Clay	Waves	Black spots	Reeves	Smoke	Spots
79.0	67.8	91.2	45.6	96.6						35.0	92.0	41.5	18.1
20.5	27.7	8.4	34.8	3.1						42.0	8.0	30.8	18.0
0.5	3.6	0.4	19.6	0.3						23.0	0.0	27.7	63.9
0	5	0	0	0	0	0	0	0	0	0	0	0	50
E-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-1	*E*-3
None	None	None	None	None	None	None	None	None	None	None	None	None	Slight
C-1*BC*	*C*-1*BC*	*C*-1*AB*	*C*-1*AA*	*C*-1*BB*	*C*-1*AB*	*C*-1*BC*	*C*-1*BB*	*C*-1*BB*	*C*-1*BC*	*C*-1*BB*	*C*-1*BC*+	*C*-2*AB*	*C*-3*AB*
Good	Fair	Good	Good	Very good	Very good	Poor	Good	Good	Poor	Fair	Poor	Good	Good
87.9	90.4	90.8	90.3	93.7	95.2	90.9	95.4	93.7	94.0	83.4	66.0	84.3	83.4
78.0	83.0	82.0	86.0	92.0									
36.8	35.7	35.7	38.6	38.5	42.3	42.6	41.6	43.4	43.1	34.2	26.8	36.1	38.6
23.5	22.6	27.0	28.2	21.8									
82.9	87.5	85.3	90.0	85.3						97.2	91.1	90.2	86.6
68.2	80.6	69.3	81.6	75.8									
99.5	94.9	98.8	95.0	98.2						87.1	98.5	79.2	16.2
0.3	4.7	1.1	3.0	1.4						11.6	1.5	17.9	37.3
0.2	0.4	0.1	2.0	0.4						1.3	0.0	2.9	46.5
95.3	92.1	97.9	92.0	96.2									
4.0	5.6	2.0	4.5	3.7									
0.7	2.3	0.1	3.5	0.1									
99.0	92.8	99.7	96.0	98.6						91.7	98.0	59.0	3.4
0.8	6.1	0.3	1.2	1.0						6.7	0.5	20.0	13.6
0.2	1.1	0.0	2.9	0.4						1.6	1.5	21.0	83.0
95.2	82.0	89.3	46.1	91.9	100.0	80.0	90.0	100.0	100.0				
4.8	10.2	8.3	47.7	8.1	0.0	10.0	10.0	0.0	0.0				
0.0	7.8	2.4	6.2	0.0	0.0	10.0	0.0	0.0	0.0				
97.7	96.1	96.7	97.9	99.0						95.9	98.8	98.8	98.3
96.6	95.2	97.5	97.4	96.3									
94.4	93.5	97.9	98.6	98.5						92.3	98.7	98.1	95.1
95.7	99.2	96.2	94.0	97.7						86.9	99.2	97.7	95.7
93.7	94.0	99.1	97.9	96.1						89.4	79.1	95.5	93.1
100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0				
99.9	99.5	99.4	98.4	99.7						100.0	100.0	100.0	100.0
99.6	99.8	99.7	99.8	99.7									
98.9	99.9	99.8	99.7	98.1						100.0	100.0	100.0	100.0
99.6	96.1	100.0	99.6	97.7						100.0	100.0	100.0	100.0
100.0	100.0	100.0	100.0	100.0						100.0	100.0	100.0	100.0
100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0				
100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0				
2,000	2,500	1,900	1,900	2,400									
3,500	3,800	3,500	3,800	3,600									
3,900	4,200	4,200	3,400	4,100									
1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
E D C	*E D B*	*D C B*	*E D B*	*E D C*						*F* — *D*	*F* — *D*	*F* — *D*	*F* — *D*
C B B	*C B B*	*C B B*	*C B B*	*C B B*									
B B B	*C B B*	*B B B*	*C B B*	*C B B*									
D D D	*D C B*	*D D D*	*E D D*	*D D D*	*D* — *B*	*E* — *D*	*E* — *B*	*D* — *B*	*D* — *B*				
100.0	97.9	100.0	100.0	100.0						100.0	100.0	100.0	100.0
100.0	96.6	100.0	100.0	98.2									
98.0	95.5	100.0	96.9	97.9						100.0	100.0	100.0	100.0
91.0	91.0	96.0	91.0	100.0						100.0	100.0	100.0	92.0
92.0	96.0	96.0	88.0	96.0						96.0	91.0	92.0	75.0
100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0				
96.3	96.3	97.9	97.6	97.9	100.0	100.0	100.0	100.0	100.0	91.1	93.9	97.5	95.5
96.8	96.2	98.7	96.0	98.7	100.0	100.0	100.0	100.0	100.0	99.0	97.7	98.0	91.7

Quality designation per ASTM 748. † 1 = average–averages. 2 = average–minimums. 3 = minimums.

Excluding breakdowns due to causes other than mica puncture.

or any of the samples including the clear nes, whereas other "black stained and potted" samples had very low Q values. t was found that low Q in "black stained nd spotted" samples usually occurred hen the stains were semiconducting in ature. Such conductivity, when pres- nt, could be detected by a high-voltage park furnished by a vibrator-type induc- on coil, connected to a probe electrode, when the probe was used to explore the surface of the mica. Conducting spots and stains are revealed by sparkling and glowing at or near the probe but not by puncture by the spark. Mechanical faults, such as pinholes, tears, and cracks also can be located with this test; such defects are revealed by puncture of the mica by the spark. A schematic of this test circuit is shown in Figure 5.

Laboratory Trial of Test Sets

First, an experimental laboratory trial of the two types of test instruments was conducted in which "clear" to "black stained and spotted," green and ruby block mica from seven domestic mines was classified for Q at one megacycle with the rapid-method Q test set and tested for conducting inclusions with the spark test set. This electrically selected block was split and punched into capacitor laminations 1.2 to 1.6 mils thick using standard commercial methods. These laminations were used indiscriminately, regardless of any visual inclusions and only were discarded if found to be mechanically unsound. Both 1,000-micromicrofarad and 4,000-micromicrofarad molded-silvered and foil "postage-stamp"-type capacitors were made by the Western Electric Company with this mica.

Thorough performance tests were made on these capacitors, and they satisfactorily met Q, insulation-resistance, dielectric-strength, corona-voltage and life requirements (see Table IV). As a result of these tests, it was concluded that block mica could be selected dependably for use in capacitors, some for the better grades and the rest for less critical types, and that the test methods appeared entirely adequate for the electric control of capacitor mica. Even the two lots of capacitors made with block mica in which conducting inclusions were detected by the spark test and low Q by the block-mica Q test gave excellent dielectric strength and life performance. It therefore appeared evident that such mica of *E*-2 and *E*-3 electrical quality might be used for less critical grades of capacitors with better over-all characteristics than can be obtained with some of the available alternate materials. However, as a precautionary measure, the use of mica with conducting inclusions was ruled out, in view of the apparent plentiful supply of types not possessing this defect.

Over-All Quality Specification

A specification[8] was developed containing over-all requirements for block mica for use in capacitors incorporating the two electrical test methods as well as other electrical, visual, and physical requirements. The electrical requirements covered Q, dielectric strength, and absence of conducting inclusions. The physical requirements covered splitting quality and weight loss on heating. Visual requirements covered air inclusions, waves, buckles, ridges, cracks, tears, pinholes, and stains. Three quality classes were set up for block mica suitable for use in capacitors, designated *C*-1, *C*-2, and *C*-3. Class *C*-1 is considered adequate for any and all of the various grades of capacitors in American War Standard *C*75.3. Classes *C*-2 and *C*-3 are considered usable for the less exacting grades. *C* classifications cover both electrical and physical properties and *E* classifications electrical properties only. Class *C*-1 covers *E*-1 mica with best electrical and physical properties, *C*-2 covers *E*-2 mica with intermediate electrical and physical properties and *C*-3 covers *E*-3 mica with fair electrical and physical properties. The stipulation is made, however, that mica shall meet all requirements of its respective class.

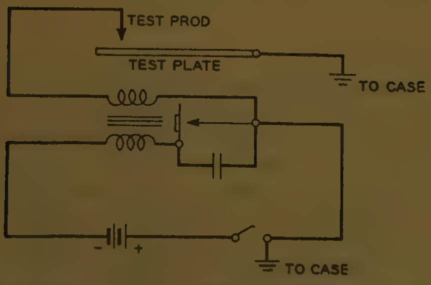


Figure 5. Circuit schematic of the spark-test set

Commercial Trial of Test Sets

The next step was to try out the proposed system on a commercial scale, using the testing equipment that had been developed. A commercial design of the block-mica Q test set which is portable and contains its own batteries is shown in Figure 5. The spark-coil conductivity test set, which also is battery-operated and portable, is shown in Figure 6.

The commercial trial covered the electrical classification of over 3,000 pounds of block mica of 22 different types from the United States, India, Canada, and Brazil, previously classified by visual standards as stained, stained *B*, United States second and United States third quality. All such classifications were previously considered unsuitable for capacitor use. After being split, gauged, and punched, this mica was used, regardless of visible inclusions, providing it did not contain cuts, tears, or other visible mechanical imperfections, in the manufacture of over 47,000 capacitors of the following types and sizes per American War Standard *C*75.3: molded *CM*30 and 55, 1,000-micromicrofarad silvered and stacked-foil; and *CM*35 and 60, 6,200-micromicrofarad stacked-foil capacitors; as well as potted *CM*70, 510-micromicrofarad transmitter-type capacitors. A total of eight capacitor manufacturers participated in the commercial trial, and the number that made specific types of capacitors and tests in each case is shown in Table V. This table contains a grand summary of test results and manufacturing experience in connection with the processing of electrically selected block mica into finished capacitors. It shows the number of capacitors that passed test requirements specified in American War Standard *C*75.3 and gives the results of other special tests.

Conclusions

Based on satisfactory Q, dielectric strength, insulation resistance, temperature coefficient, capacitance stability, radio-frequency heating, corona voltage, and life tests on these capacitors obtained by eight participating manufacturers, is concluded that electrically selected *E*- nonconducting block mica is suitable f use in the manufacture of all types ar sizes of fixed mica-dielectric capacito per American War Standard *C*75.3 up and including *CM*70. However, nothi has been revealed by these studies whi would indicate that such mica would n be equally as satisfactory for use in sizes and types of capacitors, per Ame can War Standard *C*75.3. Likewis based on somewhat less extensive tests electrically selected *E*-2 and *E*-3 no conducting block mica, such mica with the limits specified for class *C*-2 and *C* mica, per ASTM *D*-748–45*T*, is suitab for use in the manufacture of all typ and sizes of capacitors up to and includi *CM*60 per American War Standa *C*75.3.

Figure 6. The mica Q test set developed b Bell Telephone Laboratories

Figure 7. The spark test locates conducti regions in block mica and permits defecti material to be rejected before the applicati of the Q classification test

The splitting quality and yield of sat factory splittings and punchings of lots of selected *E*-1 nonconducting mica part 1 of the commercial trial, consider as a group, is estimated to be 10 to 20 p cent poorer than that reported for the lot-

ndia (good stained) sample. The poor-
t splitting quality and film yield was re-
orted for selected *E*-1 nonconducting
t-*P* (heavy-reeved and ribbed) light-
reen mica of part II. This lot did not con-
rm to the physical requirements of class
-3 mica per ASTM *D*-748–45*T*, but was
cluded in the tests to study the per-
rmance of such mica. It is important
note, however, that the electrical per-
rmance of capacitors made with
echanically sound punchings from lot *P*
ectrically selected mica approached that
btained for the control sample. The
oorest capacitor life performance was
perienced with lot *R*, *E*-3 mica which
d not pass the spark-test requirement
at was included to check the *E*-3 range
the *Q* test set.

The proposed system of classifying
ock-mica quality based on a combina-
on of dielectric and physical properties
s been shown to be commercially reliable
d practicable. This system differs
dically from past practices, and previ-
s concepts of mica quality considered
itable for use in capacitors. It does not
scriminate against the presence of spots
d stains in even first-quality electrically
ected block mica, providing the mica
nforms to specific electrical and physical
quirements. Air inclusions and wavi-
ss also are permitted in the lower-
ality classes, providing the mica meets
ecific electrical and physical require-
nts. Mica capable of meeting specific
ctrical and physical requirements also
y be of any basic color.

The experimental work in connection
h part I and II of the commercial trial on all lots of mica confirms that Bell Telephone Laboratories block-mica *Q*-test and spark-test sets or commercial equivalents* are satisfactory for classifying block mica in accordance with ASTM specification *D*-748–45*T*. Furthermore, the requirements for all classes of block mica, per this specification, have been found to be conservatively set, thereby providing a good factor of safety.

As the electrical system of classification qualifies mica previously considered unsuitable for capacitors, the mica supply for such use is greatly increased owing to the much more plentiful supply of lower visual-quality types of mica. An estimated saving up to 60 per cent of the total amount of good stained or better mica now used in capacitors is indicated.

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*Boonton Radio Corporation instruments.

Electric Drives for Steam-Electric Generating-Station Auxiliaries

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ynopsis: For several years there has been
n increasing trend toward all-electric drives
r steam-station auxiliaries on account of
e recognized advantages in flexibility,
onvenience, low operating cost, and com-
arable investment cost. Substantial econ-
mies in first cost also have been obtained
long with further improvements in flexi-
ility through the use of transformers as the
le means of station power supply. Grati-
ing reliability of performance has been
chieved by co-ordinating design, backed by
tensive tests, including momentary interruptions of station power supply, and has been incorporated in approximately 500,000 kw of modern plants of the 850-pound 900-degrees-Fahrenheit powdered-fuel-fired unit type. Operating experience has demonstrated the over-all reliability and economy of all-electric drives for station auxiliaries and of the use of transformers as the sole source of station power supply.

THE INCREASING RELIABILITY of modern steam-electric generating stations has been an important factor in the dependable supply of electric power in the face of decreasing reserves and promises future economies in system investment. One of the vital elements of this reliability is the treatment of station auxiliaries and station power-supply equipment.

Before the advent of the extraction turbine, the need of steam for feed water heating strongly favored small steam turbines as drives for most of the station auxiliaries. Multistage bleeding brought about a critical examination of operating costs of turbine-driven auxiliaries, and resulted in the application of electric motors to many of them, certain steam-turbine drives being retained either to obtain convenient variable-speed operation or with the objective of securing greater reliability for critical drives.[1] The advent of modern steam pressures such as 850 pounds and above has resulted in even lower efficiencies for small turbines, unless they are made so complex that their reliability is seriously reduced.

Many factors have contributed to the improved reliability of electrically driven auxiliaries.[2] Modern motors are highly reliable, and variable-speed couplings provide flexible and efficient operation where required. The growing interconnection of both stations and systems makes it feasible to secure emergency electric power for auxiliaries not only from several points within a station but from remote sources as well. Several years' experience has demonstrated that the proper co-ordination of driven equipment, motors, control, and power source insure an extremely high degree of reliability. Many stations now operate with all auxiliaries electrically driven, with the minor exception in some cases of the turbine-bearing oil pump, which is furnished by the manufacturer as a part of the turbine.

Basic Requirements of Auxiliary Drives

The requirement of reliability for many auxiliary drives is most important. Especially with powdered-fuel-fired boilers, the interruption for even a few seconds of certain critical drives may result in flame extinction, and the resulting necessity of purging the furnace system and relighting may cause a serious (though temporary) loss of capacity. Thus the over-all system must be made relatively immune to short-time disturbances, but without sacrificing the simplicity and flexibility which are so essential to successful operation. Adequate safety interlocking is required but must be scrutinized for possible hazards to reliability.

For some of the drives variable-speed operation or its equivalent is necessary. Where storage capacity can be secured readily, the automatic intermittent running of constant-speed motors provides convenient, flexible, and low-cost performance. For other auxiliaries the use of constant-speed motors with variable-speed couplings can provide the desired solution.

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Though consideration of cost is secondary to that of reliability, a careful consideration of all factors will facilitate low-first-cost design without any measurable sacrifice in reliability. The low operating cost of electric-driven auxiliaries is inherent when compared with steam-driven auxiliaries.

Motor Application

By far the largest portion of motors used for station auxiliary drives are three-phase squirrel-cage induction motors conforming to National Electrical Manufacturers Association standards regarding starting current, starting torque, and maximum torque. In spite of the requirements of high reliability, only a very few motors and auxiliaries, such as boiler feed pumps, pulverizers, and circulating water pumps, are installed in duplicate. For this reason particular attention is paid to the mechanical and electrical construction of the motors purchased. This does not mean that the motors need to be of special design or construction.

Voltage Rating and Starting Characteristics

A few of the small motors are rated 220 volts, but most drives use either 440- or 2,300-volt motors. An economic survey taking into consideration the cost of motors, control equipment, and the station distribution system has resulted in a decision to place the dividing line between the use of 440-volt and 2,300-volt motors at approximately 100 horsepower.

Full-voltage starting is used for both 440- and 2,300-volt motors. This permits the use of a low-cost control scheme which is simple and direct in its operation and lends itself to quick automatic transfer from one power source to another in an emergency. It also provides full starting and pull-in torque for normal voltage conditions. It may, however, be necessary at times to start an essential auxiliary when the voltage at the motor terminals is below normal as the result, for example, of the loss of a sizable amount of generation from the system. The speed-torque requirements of all auxiliaries are given careful consideration, and motors having adequate speed-torque characteristics at the minimum operating voltage are selected.

For some applications, such as certain types of pulverizers, the starting torque is the critical value, and, if a motor is selected having adequate torque to start the pulverizer during low-voltage conditions, no difficulty will be experienced in bringing the pulverizer up to speed. In other applications, such as boiler feed pumps the starting duty is relatively light, but the torque curve of the pump may rise very rapidly with speed, and in this case it is necessary to check the speed-torque curve of the motor under minimum-voltage conditions to make sure that it remains above the pump characteristic at all speeds.

Motors having normal starting current usually are provided, as light flicker due to motor starting is not usually a problem when transformers are used for station power supply. Co-ordinated unit design results in satisfactory operation from the standpoint of disturbances resulting from starting the largest motor or the transferring of a group of running motors from one bus to another.

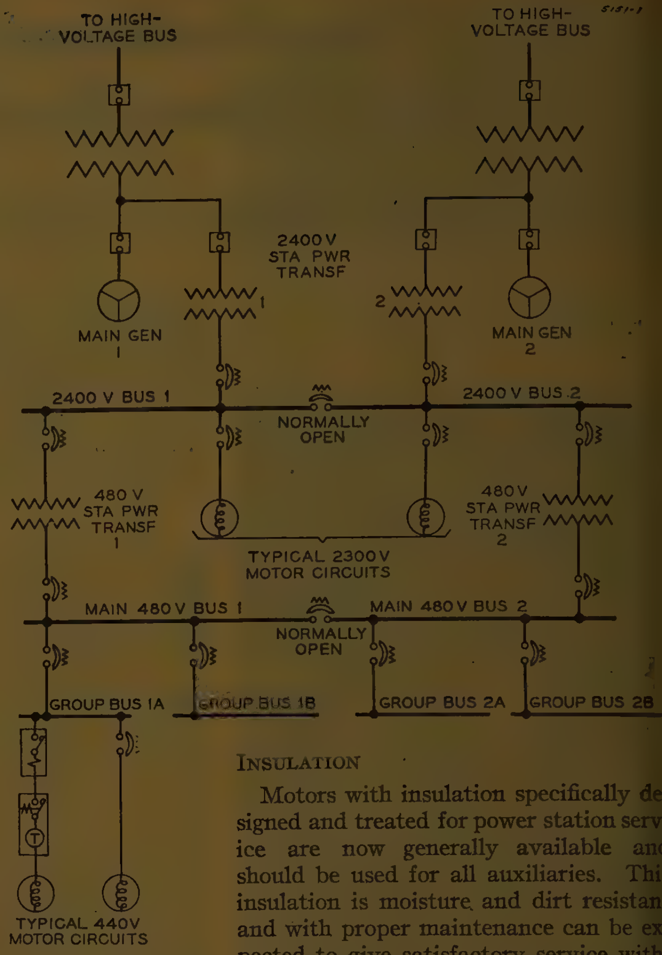

Figure 1. Typical power supply and motor connections for station auxiliaries

Mechanical Construction

In order to assure reliable continuous operation, careful consideration is given to mechanical construction. The motor windings and rotor bars must be braced to withstand the stresses due to full-voltage starting and must be thermally adequate for the current associated with it. Insofar as practicable the construction should be such as to provide ease in making periodic inspections and checks. This means, for example, that it should be easy to check the oil level in the bearings and to make sure that the oil rings are performing properly. Provision should be made for checking bearing temperatures, and suitable openings for checking air gaps are essential. Proper end play is important, especially in 3,600-rpm motors, and the motor should be so constructed that this can be checked readily.

Insulation

Motors with insulation specifically designed and treated for power station service are now generally available an should be used for all auxiliaries. Thi insulation is moisture and dirt resistan and with proper maintenance can be expected to give satisfactory service without the necessity of using splashproof o totally enclosed motors. Dripproof motors are provided where there is a possibility of water dripping on the motor.

Both class-*A* and class-*B* insulation ar used. In general it is easier to obtai satisfactory class-*A* insulation for powe station service, and class-*B* insulation i specified only for motors operating i ambients above 40 degrees centigrad such as draft fan motors.

Speed Control

For most station auxiliary applicatio constant-speed drives are used. Th induced- and forced-draft fans are an exception to this. The power requirement of these fans are substantial, and thei speed adjustment is controlled automatically by the combustion-contr equipment.

In many of the earlier plants the moto for these drives were of the wound-roto variable-speed type. However, equ efficiency and better control can be pr vided with constant-speed motors an variable-speed couplings of the hydraul or magnetic type.[3] These are in mo cases used to provide the entire range speed control and have proved qui satisfactory, responding very well to t combustion-control equipment. In som instances two separate motors are pr vided with different constant-speed ra ings, with dampers to provide inte mediate control.

Power-Supply Systems

The choice of a system for supplying power to electrically driven auxiliaries is a vital one. The primary objective of adequate reliability must be obtained, but this does not mean that the most elaborate and expensive scheme is necessary or even desirable. One of the important advantages of electric drive for station auxiliaries in an interconnected transmission system is that power may be secured from a large number of sources and transferred instantly from a central location. The extension and improved performance of interconnected power stations and systems have increased the popularity of the unit system of plant design; that is, one boiler, one turbine, and one step-up transformer operating as a unit, with electric connections with other units made at the high-voltage bus only. The interconnected system can compensate for the loss of any one unit, and accordingly the statement has been made that the over-all reliability of the station auxiliaries need be no greater than that of the main units. This is true with certain minor exceptions. In the case of an emergency outage of a main generator for a temporary cause, it may be necessary to provide emergency service within a short time to the boiler feed pump. If the interruption to the main unit is long enough for the turbine to stop turning, then the availability of emergency power for the auxiliary turbine-turning motor will save a delay of two or three hours in restoring the unit to service

House Turbine, Shaft Generator, Transformer

In early applications of electric drive to include essential auxiliaries, it was sometimes thought necessary to provide an entirely separate house turbogenerator, supplemented with an auxiliary generator on the shaft of the main unit and a transformer source. Such an installation was so expensive as to offset in part the $1^1/_2$ per cent to $2^1/_2$ per cent reduction in operating costs achieved with electric drive. Experience soon showed that the costly house turbogenerator was unnecessary and added very little to reliability. The upward trend in steam pressures and co-ordinated bleeding cycles was also a factor in the gradual abandonment of the house turbine.

The less costly auxiliary generator driven on the shaft of the main unit is still much more expensive than a transformer. The principal advantage of the shaft generator is that the auxiliary bus supplied from it is not exposed to voltage disturbances originating on the transmission system. On the other hand the line starting of large induction motors from a shaft generator may involve serious voltage disturbances of several seconds' duration which are not encountered with transformer supply.

Careful analysis of fundamental requirements has shown that the proper co-ordination of the characteristics of motors and control with those of the driven equipment provides adequate reliability using transformers for the sole supply of auxiliary power. Some of the factors contributing to the improved reliability of systems using transformers are:

1. Co-ordination of control methods to avoid unnecessary interruptions.

2. The more rapid clearing of transmission-system faults.

3. The use of squirrel-cage induction motors with full-voltage starting.

4. The choice of power sources to provide a natural cushion against external power-system faults.

Special tests were made to check the over-all operation of the systems and subsequent experience provided further confirmation

Transfer Scheme With Unit System

Figure 1 illustrates a typical scheme of connections with the station power transformers connected to their respective main units at generator voltage and with the latter grouped in pairs for emergency operation of both sets of auxiliaries from either main generator. Transfer of an entire group of auxiliaries may be made from the control center and normally is accomplished automatically in case a station power transformer separates by differential relay. This automatic transfer also may be initiated by operation of the relays protecting the main transformer bank. Reliability suitable for a unit system is obtained with single busses which are well protected electrically and mechanically. In the case of the 480-volt system, the relatively large number of motors makes it desirable to divide them among group busses as shown so as to limit the total exposure on the main 480-volt busses. Experience to date has not shown the desirability of providing voltage-chasing schemes.

Influence of Voltage Dips During External Faults

The main step-up transformer of a unit installation provides a natural cushion for the station auxiliary bus against transmission faults. Figure 2 shows the positive sequence voltage on the station power busses as a function of the type of fault and its duration for short circuits immediately adjacent to the high-voltage bus. The initial voltage drop does not lower the pull-out torque of standard squirrel-cage induction motors below the normal full-load torque in case of these most pessimistic phase-to-ground or phase-to-phase faults. Even during three-phase faults adjacent to the bus the motors develop appreciable torque. With control properly co-ordinated the motors and

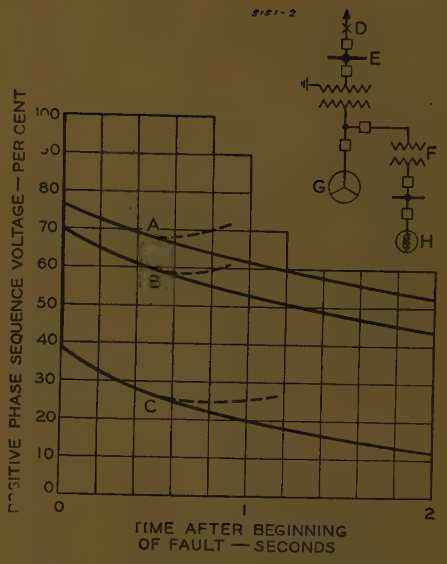

Figure 2. Influence of transmission faults on auxiliary-motor voltage

- - - - - - - - -Effect of voltage regulator
A—Single phase to ground
B—Single phase to phase
C—Three phase
D—Fault
E—High-voltage bus
F—Station power transformer
G—Main generator
H—Motor

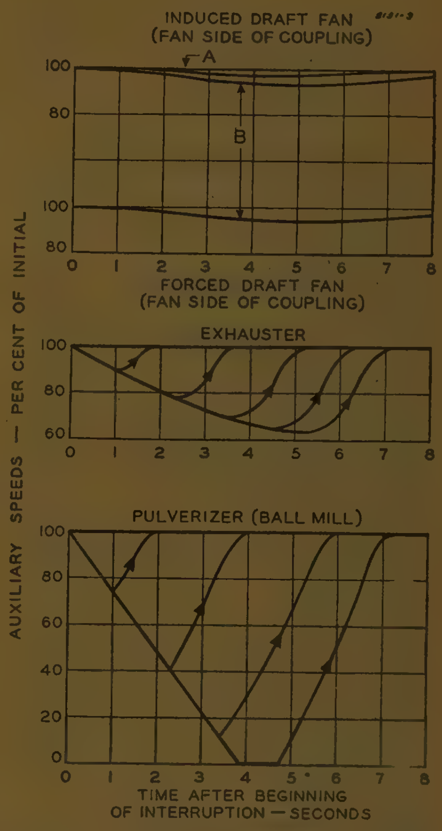

Figure 3. Speed variations caused by power interruptions

A—Power restored at 2.3 seconds
B—Power restored at 3.7 seconds

drives are remarkably immune to severe voltage disturbances, even if the station supply source is taken from a subtransmission bus.

TESTS TO DETERMINE OPERATION DURING DISTURBANCES

Early experience with a unit using a subtransmission system for auxiliary supply indicated the possibility of the powdered-fuel flame being extinguished by external voltage disturbances. The number of contributing elements hindered the proper interpretation of preliminary operating experience. Accordingly special tests were arranged with sufficient design and operating personnel and special instruments to provide isolation of the various factors under controlled conditions. These tests were made on a typical unit which utilized the ball-mill type of unit pulverizers (two per boiler) to determine the influence of surges on the boiler operation. With one pulverizer (and two burners) operating at its maximum output, the entire group of boiler auxiliaries was intentionally interrupted for varying periods of time; checking the speed reduction of the different drives with oscillographs and recording indications of the boiler draft gauges with a moving picture camera.

At first the automatic transfer scheme was used to provide an interruption of some 18 cycles to the boiler auxiliaries. There was no observable difference in the flame performance and the speed changes were imperceptible. Fluctuations in the induced draft, forced draft, mill pressure, and mill differential could not be distinguished from those experienced for normal operation. The power interruption period was extended on subsequent tests to as much as six seconds. Figure 3 illustrates the speed reduction and return to normal of the various drives for the different lengths of power interruption. In no case did the disturbances themselves extinguish the flame, but after some five seconds the pulverizer exhauster slowed down sufficiently that the pressure responsive safety devices operated to close the emergency dampers and thus shut down the boiler. The inertia of most of the drives carried them through the interruption. The ball mill pulverizer running at slow speed slows down rapidly and comes to a complete stop in $3^{1}/_{2}$ to 4 seconds. However, the coal carried in suspension provides the necessary carry-over and subsequent restarting of the motor and mill is rapid.

Figure 4 illustrates the ratio of the total inrush current to the total normal load current of the auxiliaries and the duration of this inrush current as a function of the length of the interruption. is seen that this automatic transfer of th auxiliaries involves no difficulties wi inrush current or voltage reduction.

Table I. Steam-Plant Auxiliaries—Two Units

Typical Motor Classification

Application	Horsepower	Revolutions Per Minute	Number of Motors
A. Essential Motors—Supplied at 2,400 Volts			
Induced-draft fan	700	900	2
Forced-draft fan	350	1,200	2
Boiler feed pump	800	3,600	3
Circulating water pump	200	400	4
Pulverizer mill	250	360	4
B. Nonessential Motors—Supplied at 2,400 Volts			
Ash jet pump	100	1,800	2
C. Essential Motors—Supplied at 480 Volts			
Pulverizer exhauster	75	1,200	4
Pulverizer feeder	1	1,200	4
House-service pump	50	1,800	3
Induced-draft fan—oil coupling pump	$^{3}/_{4}$	1,200	2
Forced-draft fan—oil coupling pump	$^{3}/_{4}$	1,200	2
Combustion-control air compressor	$7^{1}/_{2}$	1,800	2
Hydrogen-vacuum pump	2	360	2
Generator-cooling pump	10	1,800	2
Hydrogen-seal oil pump	$7^{1}/_{2}$	438	2
Condensate pump	60	1,200	4
D. Nonessential Motors—Supplied at 480 Volts			
Ash-disposal pump	60	600	2
Tempering-air fans	$7^{1}/_{2}$	1,800	2
House sump pump	10	1,800	2
Cinder eliminator	20	1,200	4
Fuel-oil pump	$^{3}/_{4}$	1,200	2
Screen drive	3	1,200	2
Drip pump	1	1,800	2
Turbine turning	$7^{1}/_{2}$	1,200	2
Auxiliary oil pump	3	1,200	2
Various valve motors	$2\text{–}7^{1}/_{2}$		12
Vapor extractor	$^{1}/_{2}$	1,800	2
Evaporator pump	3	3,600	2
Ash-pit sump	2	1,800	1
House air compressor	60	1,200	1
Pure-oil pump	5	1,800	1
Oil purifier	1	1,800	1
High-level tank pump	10	3,600	1
Battery-charging motor generator set	$7^{1}/_{2}$	1,800	1
Vacuum cleaner	20	3,600	1
Elevator	25		1

Control and Switching Schemes

As shown in Figure 1, 2,300-volt moto are connected to the bus through circu breakers which provide automatic pr tection against faults in the motor or ci cuit (and in some instances thermal pr tection) and also serve as motor starte Either air or oil circuit breakers of ad quate interrupting ability are used, b the trend is toward the use of air circu breakers because of their superior pe formance for starting duty. Air circu breakers also have an advantage from t standpoint of inspection and maintenan of contacts. This is of considerable in portance in view of the fact that these ci cuit breakers are operated much mo frequently than circuit breakers on di tribution circuits.

For 440-volt motors the control for ma of the individual motors consists of small air circuit breaker with instant neous trip attachment for fault protectio and a magnetic contactor for starti duty. For nonessential motors standa industrial control equipment is sat factory. For some applications, ho ever, latched-in contactors or air circu breakers suitable for motor-starting ser ice are required.

In a powdered-fuel plant it is necessa for safety and operating reasons that t sequence of starting and stopping certa motors be carefully interlocked. For e ample, the induced-draft fan is start first. This is followed in sequence the starting of the forced-draft fan, t exhauster, the pulverizer, and the feed In order to avoid clogging of the fu supply system and to avoid a possible e plosion if any piece of equipment in t afore-mentioned group is stopped, p vision is made to stop automatically all the motors ahead of it. In the abo group, if the exhauster stops for a reason, its circuit breaker or contactor opened, and an auxiliary switch trips t pulverizer, which in turn trips the feed It is essential that the device which i tiates the tripping sequence respond failure of the driven quantity and n simply to undervoltage, because stoppa may be the result of a sheared pin, broken shaft, or some cause other th failure of power supply to the motor. the example taken, the emergency dam ers are tripped by the differential-pressu relay, damped to give a slight time dela This in turn trips the exhauster motor a the remaining motors in the sequence.

Though undervoltage protection is n required for full-voltage starting moto the use of standard magnetic starters 440-volt motors would provide uninte tional low-voltage protection. Most these starters will drop out at around 50 per cent of normal voltage and remain open, unless a latched-in type of auxiliary relay is used. This is not serious for automatic-start intermittent running equipment with storage capacity such as the feeder, house-service pump, combustion-control air compressor, and condensate pump, provided they are not included in any interlocking scheme. Standard starters even with latched-in auxiliary relays are not suitable for motors in an interlocked scheme, because the auxiliary switch on the contactor would start the tripping sequence of the other motors and emergency safety devices in the series. For such motors either latched-in contactors or circuit breakers suitable for starting duty are used.

Latched-in contactors or circuit breakers also are provided for highly essential motors, even though they may not be part of an interlocked sequence. This will permit utilization of all available power, even at voltages below 50 per cent of normal.

Protection of Motors and Circuits

Reasonably adequate devices of the relay type are available for the protection of 2,300-volt motors and circuits. However, for smaller 440-volt motors, it has been customary to employ devices degned for industrial use. These are usually of the thermal variety, based on extra protection to the motor, allowing for wide manufacturing tolerances in inexpensive equipment, different ambient conditions at the motor and the controller, unusual operating conditions, and so orth. This is not a suitable basis for central-station practice where conditions are materially different from the typical ndustrial installation. Proper co-ordinaion with driven equipment and mehanical protection of the motors and circuits is inherent in central-station design. Operators are able to devote more careful ttention to the motors and drives, and maintenance is usually on a program basis. Conditions most likely to result n motor overloading usually will be deected through operating results.

Essential Versus Nonessential Motors

An essential motor is classed as one which, if interrupted, would cause a serious loss of unit output within a relatively hort time. Induced-draft fans, forced-draft fans, boiler feed pumps, circulating water pumps, and pulverizer mills are examples of the larger essential drives. Some of the smaller units are just as essential, such as pulverizer exhausters and condensate pumps and to a lesser extent such drives as pulverizer feeders, combustion control air compressors, and even small 3/4 horsepower motors used to pump oil into the hydraulic couplings for the forced draft and induced-draft fans. Typical nonessential motors are ash-jet pumps, cinder eliminators, fuel-oil pumps, screen drives, evaporator pumps, house air compressors, and so forth. A typical list of essential and nonessential motors is given in Table I.

Types of Protection

Conditions found in central-station practice, together with improved reliability of properly applied motors, have led to the use of less protection for the motors with emphasis on eliminating unnecessary interruptions. For both essential and nonessential motors, instantaneous overcurrent protection is provided which will cover the entire circuit but will be safely above locked-rotor current. For essential motors this is the only automatic protection used, but thermal devices are provided to light warning lamps, so that the operator can take necessary steps to shut down the motor after making alternate provision for its drive. In the case of nonessential motors thermal devices are provided which will disconnect the motor but at values of current somewhat higher than found in industrial practice.

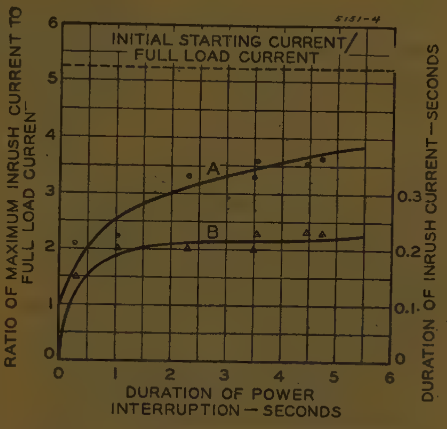

Figure 4. Effect of power interruptions on auxiliary-motor inrush current and its duration

A—Inrush current
B—Duration of inrush current

It is seldom practicable to provide positive locked-rotor protection for motors without risking unnecessary shutdown during normal starting. This is because under locked-rotor conditions, the motor does not have the inherent air cooling experienced in normal starting. Proper precautions will minimize the probability of locked-rotor conditions and alert operators usually can guard against the remaining ones.

There is a definite need for improved devices, especially for the protection of motors of the 440-volt class. Typical industrial devices are not sufficiently reliable, are subject to unsatisfactory tolerances, and are influenced by too many factors for the most reliable operation. There is also a need for a simple but positive method of locked-rotor protection.

Conclusions

1. Experience over a number of years, including that with 500,000 kw of modern 850-pound 900-degree steam plants, has demonstrated the reliability and economy (both first cost and operation) of a station power system supplied from transformers rather than from house turbines or shaft generators.

2. In the unit system of plant arrangement, it is advantageous to connect station power transformers at generator voltage, so as to provide a natural cushion against transmission-system faults. However, this is not so necessary, if faults on the transmission system are cleared with modern relay and circuit-breaker speeds and the control system is properly co-ordinated.

3. The use of intermittent-running automatic-starting constant-speed motors for certain applications and variable-speed couplings for others results in the use of line-starting squirrel-cage induction motors for practically all drives with important advantages in reliability, simplicity, flexibility, and convenience in operation.

4. Careful analysis of drive characteristics and comprehensive performance tests have resulted in the use of inexpensive control and switching equipment for the many noncritical auxiliaries, with more costly equipment limited to a few essential applications.

5. The problem of protecting motors and circuits in a central station differs from the typical industrial application, as in the former the proper co-ordination of motors with drive characteristics, maintenance on a program basis, and close surveillance by station operators of equipment and its performance eliminates many causes of motor failures. Accordingly, in central stations schemes of protection should emphasize continuity of service.

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Central-Station Auxiliary-Drive Motors for Constant and Adjustable Speed

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IN SELECTING MOTORS for central-station auxiliary drives it might be of interest to note that the range of motor sizes for any particular voltage from the standpoint of reliable operation is much wider than the range when considered from the standpoint of a distribution system and the over-all economic situation including transformers, distribution system, switchgear, and motors.

Table I indicates what would be considered good practice, for the central-station auxiliary drives, from the standpoint of *motor design only*. Completely satisfactory motors can be built, with a maximum reliability for central-station drives, through the range of horsepower indicated at the commonly accepted voltage levels used in central-station auxiliary drive systems, as indicated in the table. Motor manufacturers build motors for the various voltages over a wider range than indicated. However, the table shows roughly the essentials of what might be called good central-station auxiliary practice.

Type of Motors

The great majority of motors used in central-station auxiliary drives, in new modern power-plant practice, are the induction type, although recently in smaller power plants with the advent of adjustable-speed couplings synchronous motors have been used to some extent. The use of d-c motors, in the large modern stations, is at present practically extinct.

The induction motors which are used for central-station drives can be divided readily into three classifications, the most widely used being the normal starting-torque motor. This type of induction motor is used and recommended, for nearly all of the major auxiliary drives, such as draft fans, boiler-feed pumps, condensate pumps, most pulverizers, and service and miscellaneous pumping applications throughout the station. Usually the motor will have a locked-rotor full-voltage inrush current of approximately 600 to 700 per cent of normal full-load motor rating. This design is commensurate with high efficiency, and power factor, adequate starting, and maximum torque. It is well suited for accelerating loads involving high inertias, which frequently arise in central-station practice on the induced-draft fans whose wheels are built very heavy to resist erosion. With the values of inrush current indicated, and with normal WK^2 or inertias, this motor usually is built with a starting torque in the range of 100 to 125 per cent. However, in the case requiring the motor to accelerate exceedingly high WK^2 requiring long accelerating periods, these motors may be designed with lower starting torques in the range of 50 to 60 per cent, still maintaining 200 per cent maximum torque, and not exceeding the 600 to 700 per cent locked-rotor current. This motor with lower starting torque on full-voltage starting would have sufficient starting torque to break away from rest most of the auxiliaries, particularly fans, centrifugal pumps, some pulverizers (starting unloaded) and miscellaneous other applications.

Relatively few are the applications requiring the induction motor designed for high starting torque. This motor, as the description implies, is normally used on applications requiring high starting torque, such as compressors starting loaded, or on some types of pulverizers, such as a ball-bearing type, which requires a very high starting torque, owing to the large bearings and massive construction. Other applications might be small air compressors, used for station service. It might be well to point out that the high-starting-torque motor should be checked carefully for high-inertia drives before being used, inasmuch as it might have very low-heat storage capacity in the rotor, which might make it unsatisfactory for this type of drive. Hig starting torque is sometimes confuse with the ability to accelerate the inert rapidly, reducing the starting time, ar thus not requiring a motor which coul accelerate the high inertia over a lon accelerating period. The high-startin torque induction motor could be used c some types of stoker drives which al require high starting torque but have lo effective inertia.

The wound-rotor induction motor used for adjustable-speed drives and co stant or adjustable drives with especial difficult starting duties, which might r quire the starting of inertia beyond th ability of the normal-starting-torque i duction motor, or loads which require high starting torque beyond the abilit of high-starting-torque induction moto

The usual application of the woun rotor motor, in central-station auxiliar drives, is for adjustable-speed drives c draft fans and boiler-feed pumps.

As previously indicated, synchronou motors are not used very extensivel with the exception of the drive for lar low-speed air compressors, requiring cor pressor-shaft-mounted motors, where the low-speed synchronous motor, wit out base, bearing, or shaft, becomes t most economical drive for the machin Synchronous motors have been used on few occasions for driving special types pulverizers, and at other times for drivi large low-speed constant-speed conde sate pumps In recent years, the mo frequent application of synchronous m tors has been found in the smaller pow plants, for driving draft fans throu variable-speed couplings of either the h draulic or the electric type.

Usually, the small advantages, such better power factor and possibly bett efficiency, would not offset the disa vantages of increased costs and mo complicated control.

In the future, with the increased si and pressure of central stations, larg and higher-speed boiler-feed pumps m come into use. If such is the case, t large two-pole synchronous motor, simil

Figure 1. Drip-proof sleeve-bearing normal-starting-torque squirrel-cage induction motor coupled to forced-draft fan

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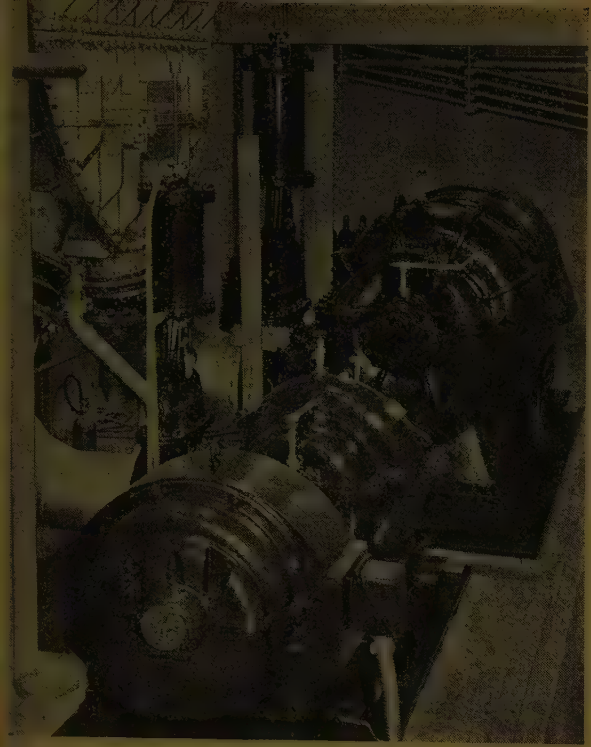


Figure 2. Splashproof sleeve-bearing normal-starting-torque squirrel-cage induction motor coupled to centrifugal circulating-water pump at end of steam condenser

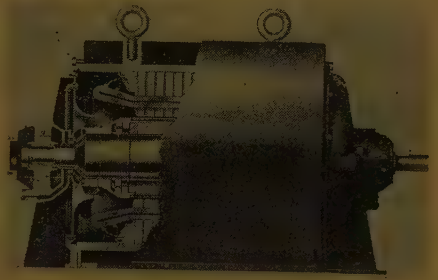
in construction to the turbine generator, may find application in central stations in the range of 2,000 horsepower and above. In this range and at 3,600 rpm, synchronous motors are less expensive than the induction motors of similar size and speed and should be equal in reliability at least to wound-rotor motors.

All the motors mentioned, both of the induction and synchronous types, are suitable for full-voltage starting. The inrush currents drawn, during the starting period are not excessive, particularly in view of the closeness of the motors to the source of generated power. The motor is satisfactory for withstanding the higher electric and mechanical shocks which occur on full-voltage starting even if they are repeated frequently. The motor shafts are liberally designed to have low tensile stress. The end turns are strongly supported by means of steel supporting rings, tie blocks, and binding. By this method the end turns are prevented from chafing and bending, which could cause mechanical failure of the insulation. Similarly by the proper use of insulation and slot wedges, the coils are prevented from making large movements in the slots, which might eventually cause failure owing to stresses encountered under starting conditions

Modern central-station auxiliary practice is to use full-voltage starting in nearly 100 per cent of the cases.

Figure 3 (below). Base-ventilated high-speed sleeve-bearing induction motor

Longitudinal semisection showing typical construction



High-Speed Motors and Gears

For high-speed drives, particularly above 1,800 rpm, lower-speed motors with speed-increasing gears are occasionally used.

This is sometimes an economical solution to the question of what type of motor to use for high-speed drives, since a two-pole motor of the synchronous, induction, or wound-rotor type is more expensive than a lower-speed motor at its highest speed. It may be found that the two-pole motor costs about 50 per cent more in smaller sizes encountered in a central-station auxiliary drive and up to 100 per cent more in the larger sizes. In some cases, it is possible to purchase a gear for less than the difference between the cost of the two-pole motor and the most economical speed motor.

A drive consisting of a motor and a gear is entirely reliable, particularly for application to fans and pumps of the centrifugal type, which do not have any pulsating or reverse-torque loads, and which are infrequently started. If a high-precision-type gear is used under the foregoing circumstances, long life and high reliability usually result. The high-precision-type gear is mentioned in contrast to the usual industrial-type gear so frequently found in many industrial applications where reliability is not so important as in a central-station auxiliary drive. High-precision-type gears are the type used by the United States Navy for marine-propulsion applications.

Enclosures

For most of the applications encountered in a modern clean central station, it has been found that the dripproof motor, so constructed or protected that its successful operation is not interfered with when it is subjected to falling moisture or dirt, is entirely satisfactory and reliable. This is especially true for the auxiliaries, such as the draft fan, boiler-feed pumps, pulverizers, and most service and miscellaneous drives. Such a motor is illustrated in Figure 1 which shows a dripproof (open) motor, driving a forced-draft fan. This motor is protected against falling water. The dripproof construction also shields against accidental contacts with windings or rotating parts, and thus the motor is protected against accidental damage. This type of motor is furnished with split-sleeve-bearing end shields on both ends from 125 to 2,000 horsepower at 1,800 rpm. This construction greatly facilitates the maintenance of the motor by making it very easy to inspect or to change a bearing without removing the motor rotor from the stator, or without the necessity of pressing off the motor half of the coupling.

Other types of motors, such as splashproof, totally enclosed fan-cooled, base-ventilated, and totally enclosed motors with surface air coolers should be used if the ambient conditions justify the additional expense.

For application to auxiliaries, subject to continually splashing water (such as those which might be located in the lower part of the station), or in the case of auxiliaries located where it is necessary quite frequently to hose down the surrounding area, splashproof motors should be used instead of dripproof. A splashproof motor is shown in Figure 2 driving a circulating pump, and it can be seen that the major difference between it and the dripproof motor, is a different type of end shield, which is more protected from splashing water than is the dripproof motor.

Totally enclosed fan-cooled motors are not frequently used for central-station auxiliary drives, with the possible exception of motors used for coal-handling equipment. A motor of the open type exposed to coal dust will require frequent cleaning in order to prevent the coal dust from blocking the ventilating paths in and around the windings. Obviously, if these paths become blocked, the motor will overheat and eventually fail.

Similarly, where particularly dirty conditions are encountered, requiring too frequent cleaning of an open or dripproof motor (and in sizes too large for totally enclosed fan-cooled motors), base-ventilated motors, as shown in Figure 3, may be used if access is available to clean air below the motor. If not, a forced-ventilated motor, by means of duct work from a clean area, may be used; or in some cases totally enclosed motors with surface air coolers may justify the additional expense, particularly if the heat picked up in the motor can be returned to the system heat balance.

It is well to bear in mind that any large high-speed motor, particularly a 3,600-rpm, probably will have a long stator and rotor stacking. Where such motors are required, it is necessary to consider the additional cost of totally enclosed forced-ventilated motors, or totally enclosed motors with surface air coolers, as well as the fact that the ventilating passages are more numerous and more difficult to clean than is the case when lower-speed large motors are used, which are more open, and consequently have ventilating passages more easy to clean. However, the probable longer life of the totally enclosed motor may offset its higher costs.

Explosion-proof motors seldom are required, as hazardous atmospheric conditions are not present in most stations. The possible exception is a coal-dust condition which might require a totally enclosed "dust- and explosion-proof" class-II group-*F* motor.

Insulation

In general, for central-station auxiliary drives, class-*A* motors, rated 40 degrees centigrade rise have given satisfactory service. For low voltages, 440 volts or below, motors are given an extra insulation treatment for boiler-room applications. For voltages 2,300 volts and above, the high-voltage insulation is usually adequate.

Ambient temperatures should be checked to determine if the temperature will be above the standard of 40 degrees centigrade. In some cases, even in new well-ventilated central stations, the ambient may exceed 40 degrees centigrade; for example, in case the motors driving the fans are installed directly over boilers, the ambient might reach 125 degrees Fahrenheit or over 50 degrees centigrade. In case dripproof motors are to be recommended or used, class-*A* insulation would not be satisfactory for the normal rated motor. In this case, a larger motor with lower resulting temperature rise could be used with class-*A* insulation, as long as the maximum total temperature did not exceed 90 degrees centigrade; or a motor of normal rating corresponding to the load requirements, could be used with class-*B* insulation, which would allow a higher rise with the maximum total temperature not exceeding 110 degrees centigrade.

Before one concludes that a larger-size motor, or a motor with class-*B* insulation is required, the ambient temperature should be weighted over the entire operating year.

Of course, if for other reasons, it is necessary to use forced ventilation or motors with surface air coolers, such that the air entering the motor is held at a temperature not exceeding 40 degrees centigrade, class-*A* insulation would be satisfactory even for these conditions.

Bearings

The great majority of motors used for driving central-station auxiliaries are the sleeve-bearing type. When properly applied and serviced, sleeve bearings have an excellent operating record and ar
highly reliable. They are available i
motors ranging from the maximum to th
minimum horsepower requirements
Rolling-contact bearings, on the othe
hand, are available only in a limited rang
of sizes and speeds.

Figure 4 shows cross sections of larg
high-speed pedestal bearings, used o
high-speed motors driving boiler-fee
pumps. This particular drawing is for
forced-feed lubricated bearing, and i
will be noted that two oil rings are provided to lubricate the bearing initiall
when it is being started up, before the oi
pump begins to deliver oil (in case the oi
pump is directly connected to the moto
shaft), and also to give some lubricatio
in case of forced-feed-system failure
The bearing is of the spherical type, an
the double metal baffles of the labyrinth-type construction which prevent leakag
of oil vapor along the motor shaft, can b
seen. Sleeve bearings for lower-spee
motors of the end-shield type of construction are similar to these, except that the
are not of the spherical type and are no
forced-lubricated. Spiral oil grooving, i
the lower half of the bearing, is used instead of the rectangular oil grooves show
in Figure 4.

The oil deflectors machined in th
shaft as noted in the axial cross sectio
also serve to take a small amount o
thrust for intermittent conditions. I
will be noted that the babbitt or bearin
material is "turned up" on the ends t
furnish a rubbing surface for the smal
thrust collars. It is well to bear in min
that these thrust collars are for intermittent loading only, and have a ver
small thrust capacity, somewhere in th
range of 200 to 300 pounds, for the large
motors. This point is often overlooked

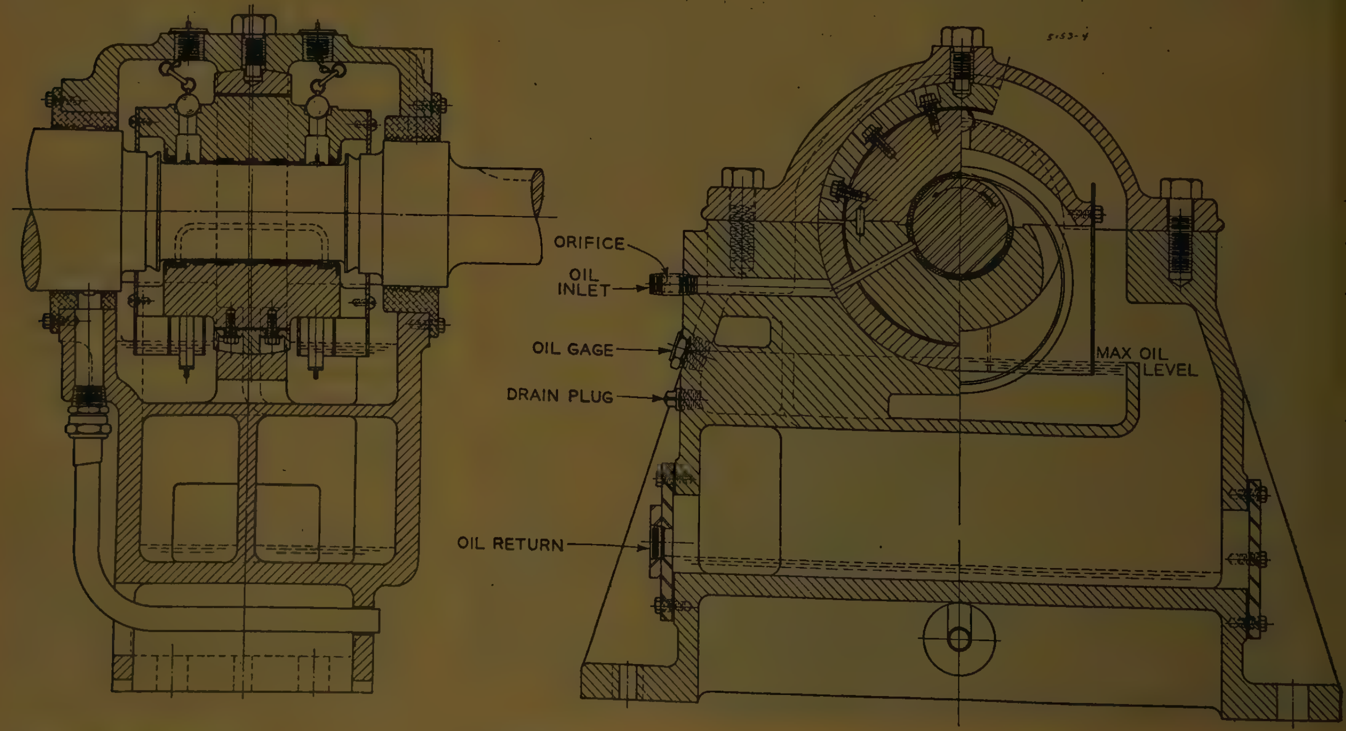


Figure 4. Sleeve bearing and pedestal for high-speed polyphase induction motor

Two vertical sections along and across shaft particularly in applying couplings between the motor and the driven auxiliaries. As is known, nearly all types of couplings, even the so-called "gear" type, can and do transmit thrust under certain conditions of loading. Quite frequently the thrust which can be transmitted by the coupling, or which actually may be generated in the coupling, because of the transmission of torque, will exceed the allowable thrust capacity of the thrust collars. Another misconception often associated with this same problem is the amount of axial magnetic pull developed by the motor under normal full-voltage operating conditions. Obviously, when the motor is moved off the magnetic center an axial force is generated which tends to bring the motor rotor back to the magnetic center. However, this force is zero when the rotor is on magnetic center, and it increases as the rotor is displaced from magnetic center to some maximum value. For practical purposes, the maximum value which will be produced is very small, usually not exceeding 200 or 300 pounds, even on large induction motors. This maximum value often will occur at a displacement which may be of little use in preventing the wiping of a thrust collar when the motor is subjected to axial thrust.

Motors with rolling-contact bearings are available at 250 horsepower at 3,600 rpm and 400 horsepower at 1,800 rpm. Certain applications will require the motor to carry a high axial thrust, which necessitates the use of a rolling-contact bearing. Such an application might be a vertical pump motor in smaller sizes.

In general the use of sleeve-bearing motors is recommended for central-station auxiliary drives where thrust is not involved.

Adjustable-Speed Drives

For adjustable-speed drives for auxiliaries, there are available multispeed motors, wound-rotor induction motors with secondary control, constant-speed motors with adjustable-speed hydraulic or magnetic couplings, brush-shifting a-c motors, and d-c motors, utilizing adjustable variable-voltage Ward Leonard control, or adjustable variable voltage by electronic means.

At the present time, the wound-rotor motor with secondary control is by far the most widely used of any of the types of drives previously mentioned. In particular, the wound-rotor motor has been applied to the boiler-feed pumps and the draft fans.

When boiler feed pumps are driven by wound-rotor motors, they are usually arranged for a nominal speed reduction of 20 to 30 per cent. This relatively small speed reduction is satisfactory to give the necessary control of feed water over a wide range of load conditions. When the boiler-feed-pump system involves one constant-speed and one adjustable-speed pump, the required adjustable speed range may be greater than indicated.

Table I. Range of Horsepower for Motors at Commonly Accepted Voltage Levels Used in Central-Station Auxiliary-Drive Systems

Volts	Motor Rating (Horsepower)
220	Minimum requirements to 500
440	1/2 to 1,000
2,300	100 at 60 cycles, and up
4,000	250 at 60 cycles, and up

For draft fans, when driven by wound-rotor motors, the speed reduction usually has not exceeded 50 per cent of rated speed. Where wider speed ranges are required, wound-rotor motors can be furnished for speed ranges of at least 5 to 1.

It is believed, based on experience and observation, that the wound-rotor motor with secondary control is as stable and as reliable as any other type of adjustable-speed drive, and much more reliable than some of the available types. There is only one piece of rotating equipment required, and it is of standard construction, with no particular mechanical difficulties to overcome. The controlling equipment is of the nonrotating type and highly reliable.

Secondary control of either the metallic-resistor secondary and multipoint switch or the liquid regulator with an infinite number of steps can be furnished. Recent developments have reduced the size and increased the speed of liquid rheostats.

For motors of all sizes up to 1,000 amperes at 1,000 volts 20-point cam switches are available for secondary control. For motors 500 horsepower and below, secondary-speed-control switches up to 40 or 50 points can be furnished.

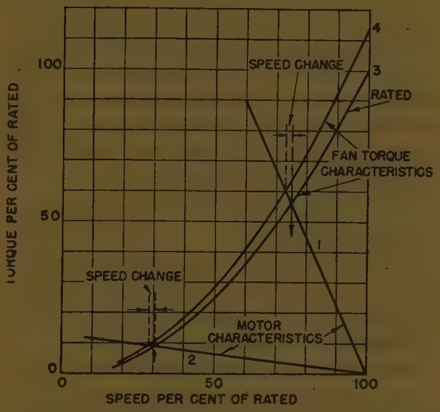


Figure 5. Speed regulation of wound-rotor induction motor driving a draft fan with change in fan load

1—Motor-torque characteristic, 75 per cent speed

2—Motor-torque characteristic, 30 per cent speed

3—Fan-torque characteristic, rated load

4—Fan-torque characteristic, 13.5 per cent overload

There has been some misconception in the past in regard to the stability of a wound-rotor motor, particularly at speeds below 50 per cent of rated speed, when it is operating with a fan or pump, whose torque curve decreases as the square of the speed. The wound-rotor motor is inherently stable, if by the definition of stability is meant that there is no tendency for hunting in the motor-torque characteristic, or there is no tendency for hunting when both the motor-torque characteristic and the torque characteristic of the load are considered. The wound-rotor induction motor has positive damping characteristics at low speed, as well as at high speed. It is true that the magnitude of this positive damping is lower at the low speeds, or, in other words, the drive inherently has more speed regulation at low speed than it does at high speed. This should not be confused with stability. Therefore, if the load torque does not change, there will be little or no cause for change of system speeds.

Table II was prepared to show the resultant system speed change, in per cent of running speed, which might be caused by certain changing conditions in the system of a draft fan. The conditions considered are:

1. A voltage change.
2. An arbitrary positive increment in fan-torque characteristic (the increment being equal at all speeds).
3. A change in the duct system resistance, such as change in the position of the dampers.
4. A change in the temperature of the flue gas.
5. Changes in secondary-resistor temperature.

For each of the conditions listed, two values of secondary ohms are shown, one value for approximately 75 per cent speed and the other for a 25 to 30 per cent speed, based on a fan or centrifugal-pump curve, with the torque varying as the square of the speed, and the load being equal to full load at rated speed. Also indicated is the change in fan- and motor-torque characteristics and the resultant change in system speed.

Figure 5 shows two fan-torque and two motor-torque characteristic curves. To analyze how Table II was derived, consider the intersection of curve 1 and curve 3. Curve 1 is the motor-torque characteristic curve with sufficient secondary resistance to result in 75 per cent of rated speed with the rated fan-torque characteristic. Similarly the intersection of curves 2 and 3 indicates that the motor operating with a value of secondary resistance results in a speed of 30 per cent with the rated fan-torque characteristic.

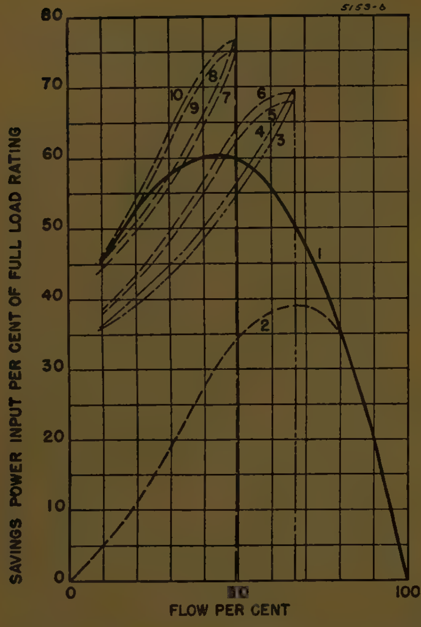


Figure 6. Input power savings of various types of driving motors and air-flow control over a constant-speed motor and throttling dampers

1—Wound-rotor motor
2—Constant-speed motor, inlet vanes
3—Two-speed motor, throttling dampers
4—Two motors, throttling dampers
5—Two-speed motor, inlet vanes
6—Two motors, inlet vanes

Curves 7, 8, 9, 10 are same as 3, 4, 5, 6, except that low speed is one half of rated speed instead of two thirds of rated speed

If the dampers are opened the fan-torque characteristic will be raised as shown by curve 4. If the same two values of secondary resistance are considered, the change in speed is evaluated from the intersection of curves 1 and 2 with curve 4, and this change in speed is expressed as a percentage of the speed resulting from the curves 1, 2, and 3, the values of the last column in the table will be found.

As will be noted, Figure 5 is for condition 3 for a 25-per-cent change in system resistance.

In all cases, the change in system speed is not considered a detriment. In the case of the 25-per-cent rise in stack temperature, the resultant speed increase will tend to offset some necessary manual or automatic speed change to compensate for the reduced weight flow, which will result at constant speed, but will increase gas temperature. For a fixed firing rate, the flow by weight probably should be held constant, even if changes do occur in the temperature of the stack gases entering the induced-draft fan owing to changes in position of superheater bypass dampers, and so on.

Table II. Resultant System Speed Change Which Might Be Caused by Certain Changin Conditions in System of a Draft Fan

Changed Condition	Secondary Ohms for	Torque Characteristic Change, Per Cent		System Speed Change, Per Cent o Running Speed
		Fan	Motor	
−5 per cent voltage	30 per cent speed	None	—10	—6
	75 per cent speed	None	—10	—1
Constant torque increase (drag)	30 per cent speed	+5	None	—2
	75 per cent speed	+5	None	—1.0
25 per cent system resistance	30 per cent speed	+13.5	None	—5.5
	75 per cent speed	+13.5	None	—2
25 per cent rise, stack temperature at 300 degrees Fahrenheit	30 per cent speed	—9	None	+3.6
	75 per cent speed	—9	None	+2.6
25 per cent rise, stack temperature at 500 degrees Fahrenheit	30 per cent speed	—13	None	+5.7
	75 per cent speed	—13	None	+3
200 degrees rise, secondary resistance	30 per cent speed	None	—1 to —2	—0.5 to —1.5
	75 per cent speed	None	—5	—0.5 to —1.5

Multispeed variable-torque motors are sometimes used for driving draft fans. In other cases two separate motors of suitable rating are used, one for maximum speed, the other for some reduced speed. The choice between a two-speed motor and two motors should be made on the basis of reliability, operating cost, first cost, and space requirements.

In the larger sizes, if one considers switchgear or control as well as the motors, first cost is about equal with the two motors showing a little lower first cost, especially if the two-speed motor is of the two-winding type, and the smaller of the two motors (considering the two-motor drive) is small enough to operate on low voltage while the larger must operate on the 2,300-volt or higher bus.

The separate two-motor drive might be considered to be more reliable, as the failure of one bearing or one winding would still allow the other motor to operate with damper control.

Figure 6 indicates approximate values of power savings using an adjustable varying-speed wound-rotor motor and no dampers, a two-speed motor, or two constant-speed motors with dampers over a single constant-speed motor and throttling dampers. It can be seen that the most economical combination of motor and control will depend on the adjustable speed range to be covered.

The two-motor drive will require a greater floor space.

The two-speed motor with a 2/1 speed ratio is or can be of the single winding type but the 1.5/1 two-speed motor must be of the two-winding type.

Brush-shifting adjustable-speed a-c motors have been applied successfully to constant-torque loads, such as positive-displacement fuel-oil pumps in oil-fired stations.

About the only place that an adjustable-voltage d-c drive is used in central stations at present is in the handling of the coal, such as in coal towers or skip hoists. The flexibility of the adjustable-voltage d-c control and the high accelerating torque of the d-c motors will provide more capacity per horsepower o motor in a coal tower than can be obtained with wound-rotor motor drive This type of system results in low coal handling-equipment maintenance, owin to the smooth accelerating and decelerating characteristics of the adjustable voltage d-c drive, which eliminates al sudden increments of torque and consequently is much easier on the mechanica parts of the coal-handling system.

An adjustable-voltage d-c system ca be designed in the case of coal-hoistin equipment, for future increase in coa tonnage, by increasing the hoisting speed

Also, some figures are available to indicate that the adjustable-voltage d- system takes less power from the distribution system than would an adjustable-speed a-c drive.

Recently some demand has been show by manufacturers of both pulverizers an operating companies for an adjustable voltage d-c drive for coal feeders, to oper ate from an adjustable-voltage system Such a system takes power from three phase or single-phase a-c power suppl and rectifies the alternating current t direct current, and also of controlling th voltage by means of grid control on th thyratron rectifiers.

Such coal feeders probably would b associated with pulverizers which hav little or no storage capacity in the pulverizers themselves, but which pulveriz the coal at a rate depending upon the rat of feeding coal into the pulverizers.

Similarly, the same type of adjustable voltage d-c drive has been quoted fo small stokers to operate in conjunctio with combustion systems.

Such applications have not been mad in central stations in sufficient numbe and for a sufficient length of time to giv any comments in regard to the succes and maintenance record of such drive although the record is excellent in othe industries.

Automatic Regulation of Turbine Generators to Relieve Power-Supply Systems of Steel-Mill Load Swings

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Synopsis: Certain steel-mill loads are fast, cyclic, and of considerable magnitude. Given a set of conditions involving such a load, a relatively small steel mill, speed-controlled generating capacity all connected to a large power-supply system, much of the swings in the steel-mill load will be taken by the large supply system. Of itself, this result may not assume special importance. When transmissions in the large power-supply system, well-loaded under steady-state conditions, are involved, and speed governing of the system is at the remote end of the transmissions, the cyclic loads tend to reappear in the transmissions. These circuits may, according to their finite transmitting abilities and the swing peaks imposed on them, be unable to carry the peaks without undesirable voltage depressions or cause undesirable frequency variations on the system. Generation of power from by-product fuels in a steel mill is generally economical but is dependent on the continuity and quality of the nonstorable fuel as well as on the steaming and generating capacity installed. Relief to a large power-supply system from the cyclic load swings normally carried by that supply system has been made possible by a new static load control and hydraulic servomotor to cause the steel-mill turbine generators to respond to the regulator. The combination has been applied successfully to two 20,000-kw turbine generators, causing one or both to take a base load, a percentage of the variable load, and to modify their total loads according to available steam pressure. The turbine-generator response can be as high as 5,000 kw per second on each unit. High-speed charts show the loads on the various circuits with and without the load control in operation.

The Basic Problem as Seen From the Standpoint of Power Systems Interconnected by Circuits of Finite Transmitting Capacities*

SUPPLYING POWER to large hot-strip continuous-rolling mills such as are now used so extensively in steel mills to produce wide thin-gauge metal strip has presented a new problem to the electric-power industry. The power drawn by a large continuous hot reduction mill is characteristically cyclic and, starting with the introduction of the slab, may build up to 25,000 kw in a period of ten seconds, stay nearly constant at peak value for a period of 40 to 60 seconds during the sojourn of the metal in all of the rolls, and then fall off to nearly zero in another ten-second period as the strip moves out of the rolls. This is followed by an idle period approximately equal to the rolling period while the succeeding slab is being oriented. The cycle then repeats itself, usually at the rate of 25 or more times per hour, depending on the size of the slabs.

A heavy cycling load of this kind, if not absorbed before it gets into the connected utility system, will produce: (1) wide frequency variations on an isolated system or (2) wide load swings on the tie lines of interconnected utility systems.

This problem is exemplified by the effect on the system of the Alabama Power Company of a 48-inch six-stand continuous hot-strip mill operated by the Tennessee Coal, Iron, and Railroad Company at their Fairfield plant near Birmingham, Ala. The steel company purchases power from the Alabama Power Company to supply its load requirements over and above the output of its own steam-generating plants which are fired principally with blast-furnace gas. As the availability of blast-furnace gas bears little relationship to the variable power-load requirements of the steel mill, the power taken from the Alabama Power Company system, consequently, is extremely variable. The cyclic nature of this load as recorded on the high-voltage side of the 70,000-kva 110/44/6.6-kv supply bank with the strip mill in operation is shown in Figure 1. It will be noted from this load chart that, when the hot-strip mill is shut down, the balance of the steel-mill load is comparatively free of rapid cyclic load swings, indicatin that the loads of the other steel-mil production processes are not only wel diversified, but also of smaller magnitud and less violent than the hot-strip-mil load cycle.

If the Alabama Power Company system were operated isolated from other utility systems, the effect of these load swings would be to produce wide frequency variations, probably of the order of plus or minus two tenths of a cycle, and the control problem would be primarily one of frequency control, as no tie lines would be involved. Isolated operation, however, is distinctly an abnormal condition, because the economics of interconnected operation between power systems, achieved through the minimization of generating reserves, the utilization of load diversity, and the interchange of economy energy between systems, long ago dictated the closure of tie lines as a normal operating condition between neighboring power companies in the Southeast and adjacent areas. As interconnections are extended between utility systems, thereby bringing more rotating equipment into parallel operation, the system flywheel effect is increased, and frequency variations from load changes become less pronounced, but wide load swings appear on the interconnecting tie lines, and the control problem then becomes predominately one of tie-line load control on the local systems. This is demonstrated very clearly in Figure 2 where frequency and tie-line loading are shown for both isolated and interconnected operation under identical load conditions.

As the Alabama Power Company's load-control problem is affected by the extent of its interconnections with other systems, a brief description of these interconnections will be in order. The Alabama Power Company together with the Georgia Power Company, South Carolina Power Company, Mississippi Power Company, and Gulf Power Company constitute the completely integrated Southern Group of systems of the Commonwealth and Southern Corporation, having a combined diversified peak load in excess of 1,250,000 kw, of which over 40 per cent is in Alabama. This Commonwealth and Southern composite system is normally interconnected through several ties with power systems in Florida on the South, and through three tie lines with the system of the Tennessee Valley Authority on the North. The Tennessee Valley Authority system, in turn, is normally tied in at Memphis, Tenn., with the interconnected group of power systems in the Southwest, and with the interconnected power systems in the northcentral area at Louisville, Ky.

This combined group of power systems, located in 24 states, normally is operated interconnected and has an estimated peak load of approximately 16,000,000 kw, not including the loads of large industries

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* Prepared by W. W. Eberhardt.

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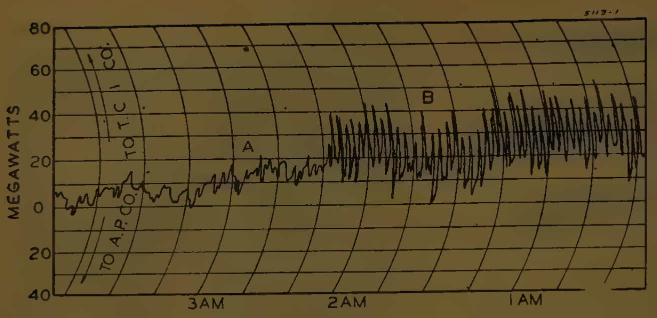

Figure 1 (above). Typical wattmeter chart of power supply to Tennessee Coal, Iron, and Railroad Company by Alabama Power Company

A—Hot-strip mill shut down
B—Hot-strip mill in operation

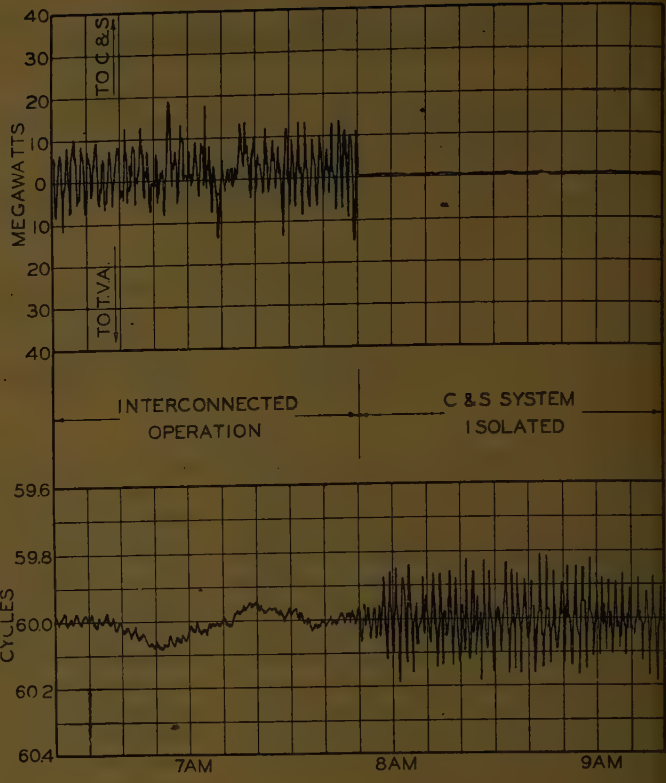

Figure 2 (right). Alabama Power Company system frequency chart and Commonwealth and Southern–Tennessee Valley Authority tie-line wattmeter chart for open and closed operation of tie line

carried on their own plants operating in parallel with the public utilities. While the torque-versus-frequency characteristics of this system as a whole have not been determined, a fairly close estimate can be made from tests and observations taken on the major portion of the system. In 1937 a series of tests was carried out on the portion of the interconnected system east of the Mississippi River extending from the Great Lakes to the Gulf of Mexico, during which speed was quickly varied 0.1 to 0.2 cycle by the regulating plant with all generator and tie-line regulating devices out of service, and the change in kilowatt output noted. For either increases or decreases, the change in kilowatt output was about one per cent per one-tenth-cycle variation in frequency. Different results may be obtained as the interconnection is extended and may vary over a period of years, if the ratio of motor load to noninductive load changes. The reason for this is that a noninductive lighting or heating load does not vary with frequency, provided constant voltage is maintained, whereas a motor load does. The load on the interconnected system has grown considerably during the past eight years and especially so during the last four years with the addition of war industries with heavy motor loads. It reasonably may be assumed, therefore, that the load variation per one-tenth cycle of frequency variation still represents a change of at least one per cent, if not more.

On the basis of the one-per-cent figure, a 25,000-kw load swing in the 16,000,000-kw system total load would vary the frequency 0.0156 cycle. This variation hardly would be noticeable on the interconnected-system frequency, as it would not ordinarily be outside of the normal frequency band width. Variations in loading of the close-in tie lines, due to such a load swing, however, are very pronounced and have created a difficult regulating problem when the swing is more rapid than the response time of the tie-line regulator.

When interconnections first were made about 15 years ago, joint regulation of frequency was attempted, but the results were unsatisfactory, because of tie-line tripouts due to overloads. This was due to the fact that load swings were shared in the various areas regardless of where the swings originated. The resultant load transfers over the tie lines were such that no definite value of tie-line loading, in the form of either load or capacity, could be scheduled and maintained. Based on experience and operating results over the years, it has become the accepted practice normally to assign the frequency regulation to a selected plant or system located near the interconnected load center and require all other systems normally to hold their tie-line loads on predetermined schedules towards that area. The frequency-regulating plant and the various tie-line-regulating plants complement each other in that close frequency regulation makes possible equally close adherence to tie-line schedules, and, conversely, if all tie lines are held on schedule, each system, by necessity, has to absorb its own load swings and thereby assist the frequency-regulating plant to the fullest extent in the control of frequency. In addition to assisting the frequency regulation, close tie-line regulation is also essential in obtaining maximum tie-line loading without risk of tripouts due to overload swings when large blocks of energy are being moved. This is a very important consideration, because tie lines, by necessity, are of finite transmitting capacity, depending on the economic considerations involved in the interconnection, and usually have a capacity rating less than ten per cent of the generating capacity of either of the participating systems.

Since the frequency regulation for the interconnected system ordinarily is assumed by plants in the northcentral area, the regulating problem of the Commonwealth and Southern group, therefore, becomes that of regulating the tie lines between it and the system immediately on the north, which is the system of the Tennessee Valley Authority.

The load south of the Commonwealth and Southern–TVA tie lines is approximately 1,600,000 kw including the Florida system, or approximately ten per cent of the whole interconnected-system load. On this basis 90 per cent of any instantaneous unbalance between load and generation in the Southeast is reflected north through the Commonwealth and Southern–TVA tie lines and ten per cent of any unbalance in northern and southwest areas is reflected south through the same ties. This apportionment of load swings applies only before governors and load controllers have had time to function. It is evident, therefore, that 90 per cent of rapid load swings in the Southeast, such as the type caused by continuous strip-mill operation, which are faster than the response time of tie-line regulators, will be reflected through the ties unless the tie-line load controller is nearly instantaneous in action.

Fast tie-line regulation is only possibl in the ideal case where only a single ti

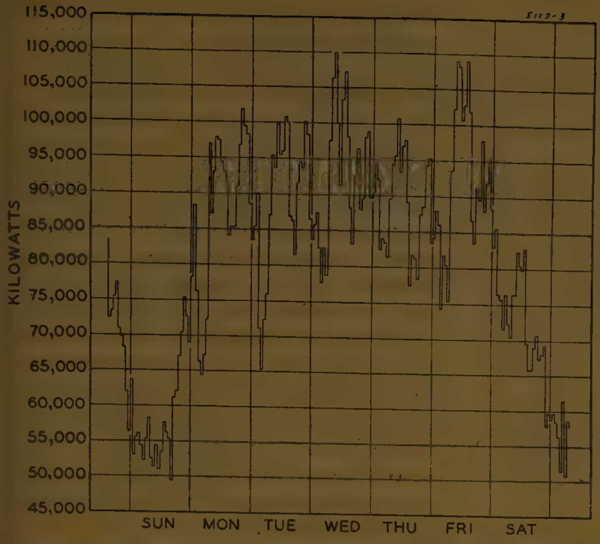

Figure 3. Typical hourly power demands of the Tennessee Company

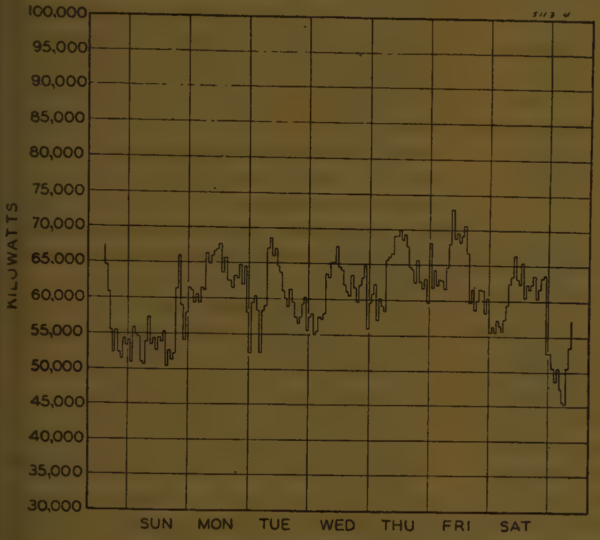

Figure 4. Typical hourly generation of electric power by the Tennessee Coal, Iron, and Railroad Company

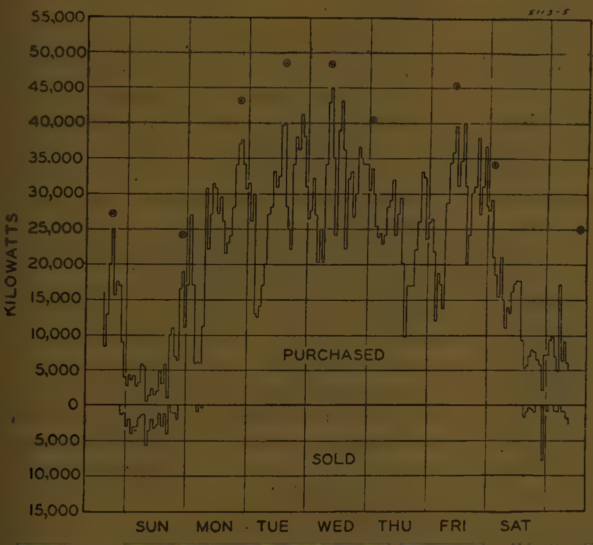

Figure 5. Typical unregulated hourly demands of the Tennessee Coal, Iron, and Railroad Company upon the Alabama Power Company (circles indicate maximum 15-minute daily demands)

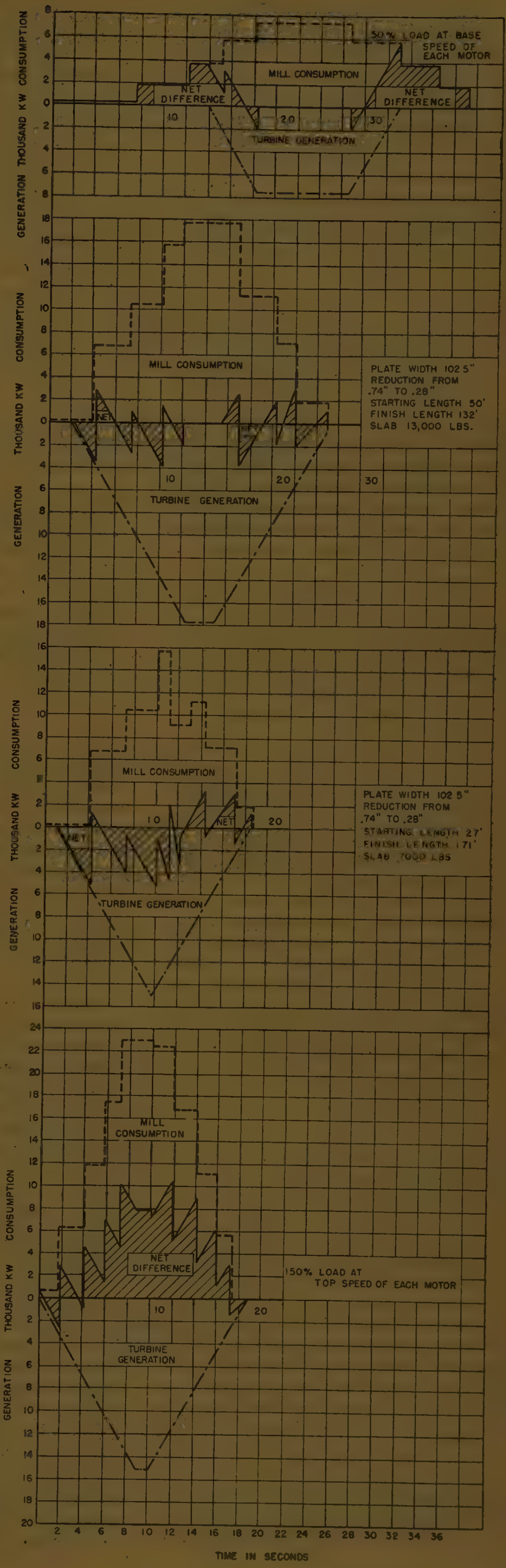

Figure 6 (right). Probable effect of an "anticipator" operating a conventional synchronizing motor of a turbine generator having a response of 2,000 kw per second (time lags in relaying neglected)

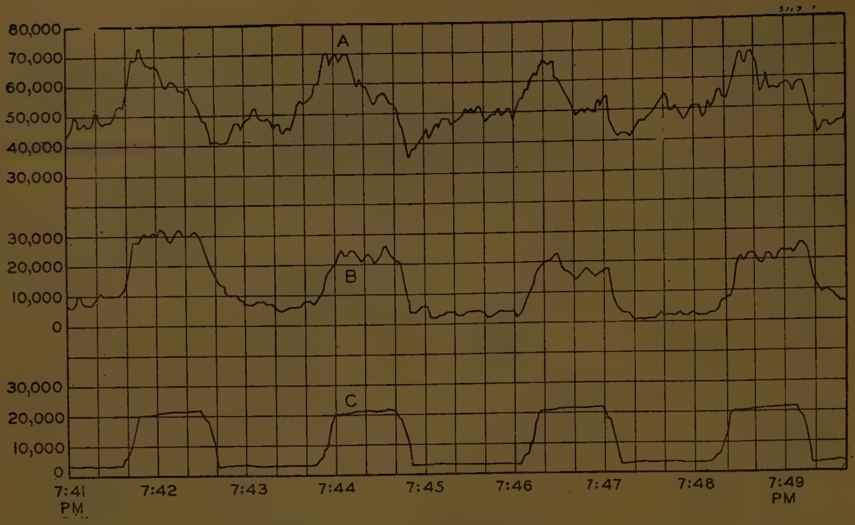

Figure 7. Typical hot-strip mill, Alabama Power Company–Tennessee Valley Authority, and Alabama Power Company–Tennessee Valley Authority tie-line wattmeter charts with Tennessee Coal, Iron, and Railroad Company load controller out of service

A—Kilowatt loading on Gorgas–Wilson Dam 154-kv tie line

B—Kilowatt supply to Tennessee Coal, Iron, and Railroad Company by Alabama Power Company at Fairfield 110/44/6.6-kv step-down substation

C—Kilowatt loading on hot-strip mill

line is involved and where the regulating plant is located at the controlling end of the tie line. In such a case the tie-line load indications can be made directly and instantly available for the operation of the tie-line regulator.

In the case of the Commonwealth and Southern–TVA interconnection, however, the tie-line control problem is far more complicated because of the multiple ties involved, and the great distance of the regulating plants from the tie-line metering points. The tie lines involved in the automatic-control scheme are:

1. Gorgas–Wilson Dam 154-kv tie, metered at Wilson Dam.

2. Gadsden–Guntersville 110-kv tie, metered at Guntersville.

3. Lindale–Chickamauga 110-kv tie, metered at Chickamauga.

Load indications on these tie lines, which are 75 to 90 miles apart, are telemetered an average distance of 150 miles by carrier current over the intervening transmission lines to the load-dispatching office at Birmingham. At the load-dispatching office, the individual tie-line loads are recorded and totalized on a totalizing recorder–controller. As the total tie-line load deviates from the scheduled setting of the load controller, correcting "Raise" or "Lower" impulses are transmitted another 80 miles to the regulating plant to change the plant load and restore the tie-line load to schedule.

The time consumed in completing these various operations is as follows:

Measure load on tie line at interchange point	1.2 seconds
Transmit load indication to load-dispatching office	2.4 seconds
Totalize load on three tie lines	1.2 seconds
Transmit load correcting impulses to regulating plant	2.4 seconds
Time for regulating plant to start responding to control impulse	2.0 seconds
	9.2 seconds

With this over-all response time of 9.2 seconds, it is obvious that the Commonwealth and Southern–TVA tie-line load regulator is completely ineffective so far as the control of continuous hot-strip-mill rapid load swings are concerned. As a matter of fact, the tie-line regulator must be set on "Low sensitivity" when the hot-strip mill is in operation to avoid overregulation or magnification of load swings.

The tie-line control equipment on the Commonwealth and Southern–TVA tie lines has been in service about eight months and represents about the fastest equipment available to the utility industry at the present time. After the war faster telemetering schemes undoubtedly will be available, but, if we assume that the telemeter time could be reduced 50 per cent, the total response time still would be on the order of five or six seconds which would not constitute a great deal of improvement so far as the control of rapid load swings is concerned.

The only effective solution to the problem, and the one that has been applied, is to absorb all or part of the hot-strip-mill load swings at their source: that is, by causing the steel-mill generating plant to increase and decrease its output in synchronism with the strip-mill load-swing cycle. This, in principle, satisfies the concept that disturbances should be contained on the system on which they originate, which concept has proved so successful in the operation of utility interconnections. The elements of the steel-mill load are described in the succeeding portion of this paper.

The Steel Mill as a Utility Load†

The Tennessee Coal, Iron, and Railroad Company's electric-power system is called upon to supply the energy for all stages of steel production from the mining of ore, coal, and limestone to the final processing of rails, shapes, plates, sheets, strip, tin plate, wire, and allied products.

The characteristics of the individual loads vary widely. The coal mines with their numerous large fans, pumps, motor generator sets, rectifiers, and belt conveyors have a fairly steady load of about 14,000 kw. Hoists and other intermittent loads represent a relatively small part of the total.

The ore mines have a widely fluctuating load owing to the fact that the ore is brought to the surface by means of slope hoists powered by slip-ring induction motors ranging in size from 1,800 to 3,500 horsepower. There are eight of these hoists which give some diversity, but it frequently happens that several of them start at approximately the same time, causing load swings of 10,000 to 14,000 kw. The peak loads on the ore-mine circuit, as a result of these swings, reach about 35,000 kw.

In the mills departments are many heavy power consumers which are, by nature, intermittent. Among the larger of these are the following:

One 4,000-horsepower a-c motor driving generators for 5,600-horsepower blooming-mill motor.
One 5,000-horsepower a-c motor driving generators for 7,000-horsepower blooming-mill motor.
One 6,000-horsepower a-c motor driving generators for 7,000-horsepower plate-mill motor.
One 4,900-horsepower a-c motor driving generators for 4,000-horsepower universal-plate mill.
Two 8,400-horsepower a-c motors driving generators for six stands and scale breaker of hot-strip mill.
Two 7,050-horsepower a-c motors driving generators for two five-stand cold-reduction mills.
One 5,000-horsepower a-c motor driving finishing stand of rail mill.
One 4,000-horsepower a-c motor driving converter blower.
One 4,000-horsepower a-c motor driving plate mill.
One 2,500-horsepower
One 3,000-horsepower } a-c motor driving stands of a structural mill.
One 2,000-horsepower
One 5,000-horsepower a-c motor driving billet mill.

The mill rolling schedules are worked out to have as few as possible of the heavy power consumers operating during the same shifts. However, there are not enough idle shifts for the several mills to prevent most of them operating together a good part of the time. The result is a rapidly varying demand on the power system. As the steel enters any one of the previously listed mills, a momentary peak equal to or greater than the motor rating may result.

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† Prepared by C. W. Harrison.

The greatest concentration of power is in the hot-strip mill. The peaks here are not only higher than for any other mill but are more consistent. The idling load on this mill is about 3,000 kw. It normally takes from 10 to 12 seconds for the steel to reach the last stand after entering the first stand. During this period, the load builds up to peaks as high as 25,000 kw for heavy slabs and 15,000 or 18,000 kw for lighter slabs. These peaks hang on for a number of seconds, varying with the length of slab, and fall off at about the same rate that they came on. When rolling a large order of the same size slabs, the load swings come with clocklike precision. The number varies from about 25 per hour for some of the long heavy slabs to over 100 per hour for some of the short ones.

The composite load from all departments of the Tennessee Coal, Iron, and Railroad Company is one that changes from day to day and from hour to hour. Figure 3 shows hourly demands for a week, picked at random. It will be seen that on Sunday this ranged from 49,000 to 75,000 kw. On weekdays, the range was from 64,000 to 110,000 kw. Each of these hourly figures represents the integrated total of a load with momentary swings having at times an amplitude in the neighborhood of 40,000 kw.

The Tennessee Coal, Iron, and Railroad Company's generators consist of three 20,000-kw units at the main power station of the Fairfield Steel Works and five units totaling 18,500 kw at the Ensley Steel Works power stations. As blast-furnace gas, which fluctuates both in volume and Btu content, is the primary fuel, and blast-furnace auxiliaries have first claim, the steam available for electric generation is not sufficient to maintain a steady output.

Figure 4 shows the total hourly generation of all the foregoing generators for the same week as was covered by Figure 3. This shows a variation from 50,000 to 72,000 kw which is not exceptional. There are times when the hourly generation exceeds 75,000 kilowatt-hours or drops below 40,000 kilowatt-hours.

Obviously another source of power is necessary to meet the demands of the Tennessee Coal, Iron, and Railroad Company system. If additional generators had been provided, additional boilers using fuel, other than blast-furnace gas, would have been necessary. The alternative was to purchase the additional power from the Alabama Power Company. To date, it has been found economical to purchase, rather than generate this power.

The procedure has been to operate two of the 20,000-kw machines, speed-governed, at or near their full capacity with the third 20,000-kw unit governed by pressure and allowed to generate as much as possible from whatever steam remained. Under such conditions the generation of this third machine swings up and down following variations in steam pressure rather than variations in system load. The small Ensley units are speed-governed and generate all that a limited supply of steam will permit. All of the Tennessee Coal, Iron, and Railroad generators operate in parallel with the Alabama Power Company system.

As might be expected, sudden power swings have been largely passed on to the power-company system. A small speed-governed power system inherently will not rush in and grab a large part of a sudden load from a large power system where the two are operating in parallel, unless special provisions are added to force it to do so. The small system, at best, normally will take little more than its pro-rata share of the new load based on the relative sizes of the two systems. If, as previously outlined, the small-system turbines already are running nearly wide open under speed regulation or held in check by pressure regulation when the upswing of load occurs, they will practically ignore the swing.

Figure 5 covering the same period as Figures 3 and 4 shows the interchange of power between the two systems. This chart, like Figures 3 and 4, shows the one-hour demands. Momentary peaks and valleys are, of course, much higher and lower than the values shown. The circles show the maximum 15-minute demand for each day. It is interesting to note that, during some hours over the weekend, the power flow was in both directions.

The contract between the power company and the Tennessee Coal, Iron, and Railroad Company has some special features to cover local conditions but basically consists of an energy charge for kilowatt-hours purchased and a demand charge. At present the demand charge is based on 50,000 kw and is a fixed amount so long as the 15-minute demand does not exceed 50,000 kw. In the event of an overrun, the new demand becomes the billing demand for that and the 11 succeeding months, provided another overrun does not occur during that 11-month period. A schedule is worked out for taking off the Tennessee Coal, Iron, and Railroad Company turbines for repairs during periods of the year when water power is most plentiful. At such times, a definite additional or seasonal demand is permitted over and above the 50,000-kw firm demand previously mentioned. A fixed charge is made for this seasonal demand.

Such in brief were the load conditions, the generators available, and the power-contract features which existed at the time the load control was installed.

The succeeding portion of the paper states the nature and sources of energy available in the Tennessee Coal, Iron, and Railroad Company plant as well as the effect the new load control has on these sources.

Fuel and Steam Facilities and Effect on Steam System of the Tennessee Coal, Iron, and Railroad Company Plant‡

The action of the peak-control regulator, operation of the steam turbines, and the resultant effect on electric-power generation are wholly contingent on the availability of steam with which to meet the cyclic peaks. As no additional steam-generating equipment or accessories were added when the regulator was installed, the method used and the effect on the existing steam-generating and consuming units are worthy of comment.

At Fairfield are three blast furnaces. The top gas of the three furnaces serves as the primary fuel for the steam-generating station. The availability of gas is contingent on the number of blast furnaces in operation. Not only is there a seasonal variation in gas volume and Btu heating content, owing to the change in the coke rate, but also a fluctuating and momentary change because of the furnace operating conditions. The furnaces are alternately checked each half-hour for approximately one minute. During the checking periods, practically no gas is generated from the furnace checked. Likewise, practically no gas is generated during the casting periods of the furnace which requires about ten minutes, four times each day. During periods of furnace delays, such as changes of tuyères and blow pipes, no gas is generated.

Approximately 14 per cent of the top gas is utilized at practically a constant rate for heating the hot-blast stoves. The balance, less leaks and waste, is available for steam generation. The blast-furnace gas supplied to the boilers is supplemented with a limited amount of pulverized coal and natural gas during prolonged high-load periods and intervals of blast-furnace gas shortage. Of the steam generated, approximately 34 per cent is used by the three turboblowers and miscellaneous users, leaving 66 per cent of the steam available for electric generation.

The turboblowers and miscellaneous users have first claim on the steam. The balance of steam available for electric current is subject to wide variations and fluctuations. The mill and mine electric loads also vary but bear no relations to the steam availability for electric generation.

Prior to the operation of the new load control and to utilize the widely varying

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‡ Prepared by E. J. Kohn.

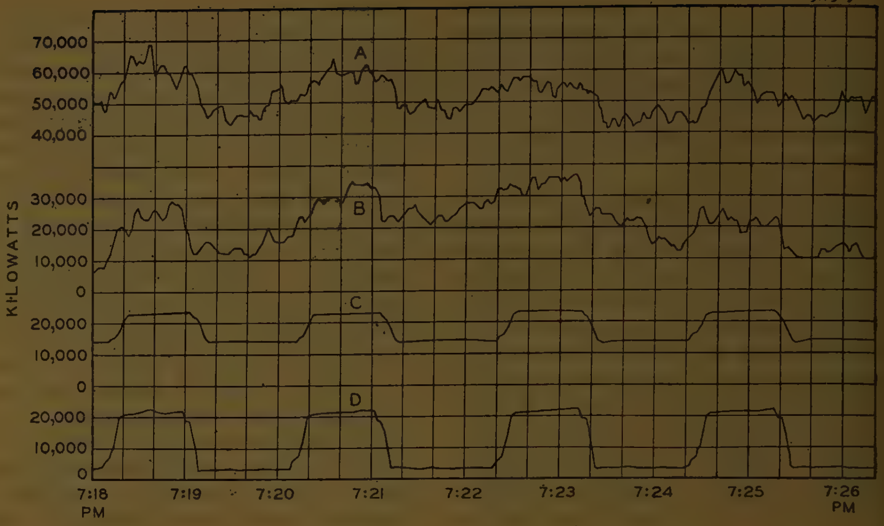

Figure 8. Typical hot-strip-mill regulating generator, Alabama Power Company–Tennessee Coal, Iron, and Railroad Company and Alabama Power Company–Tennessee Valley Authority tie-line wattmeter charts with Tennessee Coal, Iron, and Railroad Company load controller in service

A—Kilowatt loading on Gorgas–Wilson Dam 154-kv tie line

B—Kilowatt supply to Tennessee Coal, Iron, and Railroad Company by Alabama Power Company at Fairfield 110/44/6.6-kv step-down substation

C—Kilowatt loading on Tennessee Coal, Iron, and Railroad Company regulating generator

D—Kilowatt loading on hot-strip mill

and fluctuating volume of blast-furnace gas to full advantage, a steam-pressure regulator had been used which, in effect, maintained a constant steam pressure on the entire system by loading or unloading the turbogenerators in steps of 500 kw. When the new load control went in service, the old steam-control regulator was limited to the number 3 20,000-kw turbogenerator. To meet the peak demands for electric power, the steam, when required by the turbogenerators, must be available for instantaneous use. A gas holder, for the storage of blast-furnace gas, even if practicable, would not be of help in meeting the peak load, as the time element needed to generate steam in response to the peak load is too great for the existing boilers to carry all peak as well as a portion of the base load. An isolated boiler would be impractical as steam would only be utilized during the intervals of mill peak loads. During the off-peak loads, the steam would be lost at the relief valves of the boilers. Firing the boiler as required to meet the peak demand is impractical owing to the time element involved in generating the varying rate of steam and the time lag in the control equipment. An ample steam receiver, if practical, would be of advantage in meeting the peaks. The pipe lines serve, in part, as such a receiver. Supplementing the limited receiver capacity of the piping system, the boilers act as a heat flywheel and generate steam by flashing of the water stored in the boiler, due to the lowering of the steam pressure.

Normally the boiler steam pressure is maintained at 335 pounds per square inch by the steam-pressure regulator previously mentioned. With the new load control in service, the boiler steam pressure is pulled down to a new level, and a quantity of water in the boilers is flashed into steam. This added steam supplies the requirements for meeting the peak demand imposed on the generators. The magnitude and duration of the permissible peak demand determine the added steam requirements. The extent of generation by the Tennessee Coal, Iron and Railroad Company of the peak is determined largely by the permissible drop in steam pressure and the duration of no-peak load. The latter enables the boilers to recover full steam pressure in anticipation of rolling the next slab. The number, type, and size of the boilers, or in other words, the steam-releasing space and water storage in the boiler for flashing determine the permissible drop and ability to recover steam pressure. The type of control valve supplying the feed water to the boilers further influences the steam-releasing space in the boiler.

The steam-generating equipment consists of eight Babcock and Wilcox boilers of the four-drum type with integral two-drum economizer. Five of the boilers each have three 42-inch steam drums and one 48-inch mud drum with a heating surface of 14,607 square feet and a normal output of 134,000 pounds steam per hour with blast-furnace gas as fuel. Three of the boilers each have three steam drums 36 inches, 48 inches, and 42 inches in diameter and one 48-inch mud drum with a heating surface of 19,800 square feet and a normal output of 170,000 pounds per hour with blast-furnace gas as fuel. All eight boilers are equipped with superheaters. Normally only seven boilers are in service. With one of the larger boilers out of service the effect on steam generation and the ability to meet the peak demand is noticeable and accordingly reduced.

The numbers 1 and 2 generators since the regulator was put in service are each set to carry a base load of approximately 15,000 kw. The number 3 generator, with the constant-steam-pressure regulator in operation, is loaded or unloaded in steps of 500 kw in accordance with the steam available to maintain the normal operating steam pressure of 335 pounds per square inch. This pressure control is not so sensitive as the new load control and in a measure acts to permit some of the steam to go to the load-controlled turbine generators. For instance, when the load controller functions to supply the additional steam for the numbers 1 and 2 generators, the steam pressure drops and the number-3-generator output, in accordance with the constant-steam-pressure control is reduced in steps of 500 kw to meet the new low steam pressure.

Maintaining the load controller in continuous service resolves itself into the problem of determining the drop in steam pressure permissible during the peak load period to permit the boilers to recover fully their initial steam pressure during the off-peak period of the mill.

A study of the test curves made December 9, 1944, with 13,000-pound slabs through the hot-strip mill, shows time of rolling is one minute with a peak of 19,000 kw and the time between slabs to mill is also one minute.

Number 1 generator only was under the control of the load controller during the test. The generator load was increased from base load of 14,000 kw with steam rate of 163,000 pounds per hour to peak of 23,000 kw with steam rate of 260,000 pounds per hour or an increase of 9,000 kw requiring additional steam at the rate of 97,000 pounds per hour. For one-minute duration 16,160 pounds of steam were supplied from the line storage and the flashing of water in the boilers. The steam pressure dropped from the normal of 335 pounds per square inch to the predetermined low level of 315 pounds per square inch or 20 pounds per square inch drop. Only once during the test of over two hours duration, and then when the last slab was being rolled, did the steam pressure drop below the predetermined low limit. The pressure bias control acted automatically to transfer that portion of the peak which the boilers could not supply to the Alabama Power Company's system.

Two alternate operations present themselves: to operate as per test with an occasional peak thrown on the Alabama Power Company's system, or to operate at a lower peak generation. The latter recourse has been followed, because the fluctuating demands on the boilers are reduced with less resultant disturbance to the equipment. The control now is set to pick up 6,000 kw generation to meet the mill peaks. This peak, requiring steam at rate of about 64,000 pounds per hour, is divided between the two turbogenerators, numbers 1 and 2. No difficulty has been experienced even when rolling smaller slabs with a shorter cycle of operation.

From the viewpoint of reducing the peak electric demands, the control functions as intended. The favorable results obtained are due in part to the fact that excess electric-generating capacity is available. The electric generators are operated at less than full output, owing to the limited boiler capacity and the limited blast-furnace gas available as fuel.

The control equipment and the generators are adequate to extend further the favorable results obtained, but, as emphasized in the foregoing, the limiting factor is the steam available for the peak electric generation. Further limiting factors and of prime concern to the plant operators are the effects of the modified operations on the steam-generating and consuming units.

With the peak control inactive, one of the noticeable features of the plant is the generally uniform water and steam conditions.

Had the generating plant of the Tennessee Coal, Iron, and Railroad Company been designed to carry the mill load under isolated operation, adequate provisions would have been made both in the steam-generating and consuming units. Of primary consideration would have been steam generators with larger steam drums and provisions for continuous fuel supply. Not designed for such requirements, the generating-plant equipment therefore has to be operated within its capabilities. Of chief concern to the operators is the possibility of water carry-over from the boilers to the steam users which, more than the ability of the boilers to recover steam pressure, is the limiting factor in determining the peak electric generation possible. This condition is aggravated owing to the varying blast-furnace fuel supply. During the periods of diminished blast-furnace gas there is a cooling of the boiler firebox with resultant effect on the boiler conditions. When this condition occurs concurrently with the increased cycle demand of steam, the effect is a widely varying boiler water level with increased possibility of carry-over in the steam. Greater demands are made on turbine valves and control equipment, and additional stresses are imposed on the turbine wheels, condenser tubes, boilers and their control equipment under peak load regulation. It is to be expected that repair and maintenance costs will be increased over the corresponding costs incurred under speed governing of local turbines in parallel with a large power system.

Various alternative methods of imposing peak loads on speed-governed prime movers operating in parallel with large power systems are discussed in the succeeding portion of this paper.

Various Means of Effecting Load Control of Turbine Generators*

A study of the conditions previously stated led to the formulation of the following specifications which a load control should meet in order to relieve the external power-supply system from peak loads of the hot-strip mill:

1. Given a variable supply of steam for the turbine generators operating in parallel with a large power system, one or each of two turbine generators should be so controlled that:

(*a*). A constant base load (10 to 100 per cent of generator rating) can be carried.

(*b*). A percentage (0 to 100 per cent of generator rating) of the variable load in the strip-mill circuit can be added to the base load.

(*c*). The base load is to be divided in any desired ratio between the two prime movers or all put on one.

(*d*). The percentage of variable load assigned to the prime movers is to be divided between the two prime movers in any desired proportions or all put on one prime mover.

(*e*). The base and variable loads placed on the prime movers are to be reduced from their maximums at maximum predetermined steam pressure as the pressure is reduced below maximum until both such loads are reduced to zero at a predetermined minimum operating pressure.

2. The variable load may have maximum conditions of:

Total load	20,000 kw
Rate of change—average over period of change	5,000 kw per second
Duration of peak	100 seconds

Various approaches to a set of conditions like these have been suggested. Some of these are discussed to bring out their features.

Scheduling of loads tends to reduce a tendency toward a cumulative demand and is quite effective in reducing demands which otherwise would extend over appreciable time periods. It requires suitable instruments, meters and communication or remote-control systems, and an operator to supply the necessary intelligence from a given set of facts or a control system to accomplish the same result automatically. From the standpoint of correcting instantaneous changes of load, it is an ineffective method as the procedure is too slow.

Numerous types of *anticipators* have been suggested, usually to be applied to an existing prime mover and its control. Normally the rate of response from the correcting element is excessively slow compared to the rate of load change under consideration. The initiating signals suggested include photoelectric cells, mechanical triggers, limit switches, and characteristics of certain initial steps of load changes.

Incomplete consideration of such anticipators sometimes indicates a relatively inexpensive arrangement with the thought projected that it could be put together in a repair shop. The suggestion, generally made, is that the anticipator operate the synchronizing motor of the prime mover.

Synchronizing-motor controls of turbine-generators usually have a rate of response of full load in about 30 seconds. To offset load changes of the type under discussion, one or more prime movers of at least a total size equal to the load change should be used. Therefore, the anticipator is expected to compensate for, say, 20,000-kw load changes which pass from minimum to maximum or vice versa at rates of 1,500 to 7,000 kw per second. Contrasted with the actual rate of load change, the anticipator and its auxiliaries can only count on a rate of response from the prime mover corresponding to its rating divided by 30 seconds, which, if it were a 20,000-kw unit, would respond at 667 kw per second excluding delays due to relaying and acceleration.

In many instances the incremental rates of load change when a slab or billet enters a stand are much faster than the average rates suggested.

Some mills use a scale breaker as the first stand in a mill. The scale breaker requires a relatively small amount of power, but sufficient when a slab enters it to cause a change in the series field current of the motor. Such a change might be used as a signal to trigger the operation of the synchronizing motor on the turbine generator. Several charts, showing in Figure 6 what might be accomplished if no time lags in the operation of the relaying were considered, illustrate the effect of

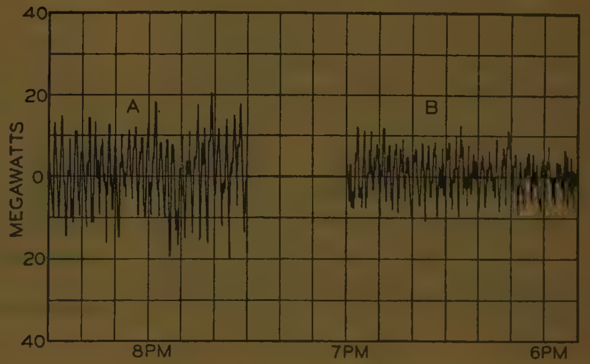

Figure 9. Gorgas–Wilson Dam 154-kv tie-line wattmeter chart showing the effect of the Tennessee Coal, Iron, and Railroad Company load controller on tie-line load swings

A—Load controller set on zero-load pickup

B—Load controller set on 50-per-cent-load pickup such an anticipator working on a turbine generator having a rate of response by synchronizing motor of approximately 2,000 kw per second. It will be observed from the several charts that:

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*Prepared by A. G. Darling.

1. The turbine generator never follows the load changes.

2. There are as many peaks created on the system when the regulator is in operation as the mill creates plus two more in some instances.

3. There is an opportunity for actual power reversal in the interconnecting line between the mill and the power supply system.

Lastly, it also will be observed that, for each schedule of the mill, a change must be made in the timing elements introduced in the response element to correspond with the time and load increments created by the mill load. The mill operator is the best source of information for presetting such an anticipator, but, as his job is to get out production, it is likely that the responsibility for prescheduling will be placed on the prime-mover operator who is at a disadvantage from lack of first-hand information.

Fixed-value modifications to local prime-mover loadings also have been suggested which modifications would follow a preset schedule, depending upon the type of product to be rolled. An initiating signal of some type is required, and a set of timing devices which can be modified readily are also necessary. This method involves the same disadvantages as the anticipator method and, in addition, is not sensitive to what actually occurs in the mill.

By the time all the relays, control cable, and safety devices are considered, the cost of the anticipator and the accomplishment of fixed-value modifications is likely to approach that of a good load controller.

The type of control which was furnished and is described in detail in another Institute paper[1] meets the specifications, responds to the power in the circuit or circuits to be measured from which the variable loads emanate, and reinterprets not only the rates of change but also the value of the loads as they occur to an hydraulic servomotor capable of modifying the load on the turbine at the rate of 25 per cent of turbine rating per second. The electric circuits and servomotor arrangements are such that, when the load control is in operation, the speed governor is made inactive in its "preemergency" speed setting, that is, slightly above the speed at which the power system normally operates. In the event of loss of connection of the controlled generator from the electric system, the speed of the load-controlled unit will rise until the speed governor takes effect, thus permitting the unit to continue operation at a safe speed with only sufficient steam passing through the turbine to carry running light losses and keep the turbine at a reasonable temperature. The load controller holds the desired base load on the turbine generator and compares the measured consumption of the strip mill with the excess output of the generator over base load. If any difference occurs in these values, the load controller operates the turbine valves through a servomotor to rebalance the equation. Steam-pressure variations work through pressure-control elements to apportion the total generator output to the pressure variations between preset limits. Electric circuits are used to interpret the conditions in the strip-mill and generator circuits to the pilot motor of the hydraulic servomotor. Steam-pressure modifications are accomplished mechanically since the rates of change are slow. The connection from the servomotor cylinder to the turbine steam valves is the same as used for speed governor and is a chain of mechanical linkages, rack, pinions and cams.

Means of transferring the prime movers from speed governing to automatic load control as well as full adjustment of the base and percentage of the variable load to be assigned to the prime movers and adjustment of the pressure regulator are incorporated in the load-controller panel, so placing facilities and responsibility on one power-station control-board operator. The details of design and construction of the regulator are given in the companion paper.[1]

The effect of the regulator on the tie lines of the external power system is discussed in the last portion of this paper.

Effect of Load Regulation of Local Prime Movers on the Power System†

In order to check the over-all effect of the load control, an operating test of the controller action was made while the hot-strip mill was rolling a series of 56 13,000-pound slabs in quick succession at the rate of about 25 slabs per hour. The load controller was set to pick up 50 per cent of the load swing which, in the case of these slabs, amounted to approximately 10,000 kw. During this test the controller was cut in and out of service at 30-minute intervals in order to evaluate the effectiveness of the regulator by making a direct comparison of the two operating conditions.

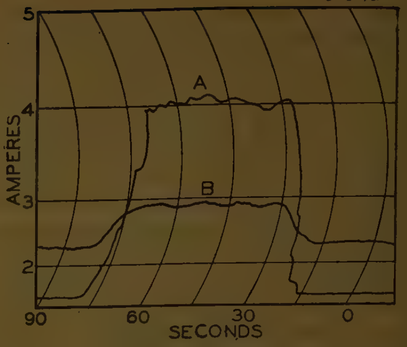

Figure 10. Timing chart of regulating generator and hot-strip-mill a-c secondary amperes

A—Hot-strip-mill current
B—Regulating generator current

Synchronized wattmeter charts, traveling three inches per minute, were obtained for the hot-strip-mill load, the steel-mill generator output, the Alabama Power Company's energy supply to the steel-mill load, and the load on the Gorgas–Wilson Dam 154-kv tie line. The Gadsden–Guntersville and Lindale–Chickamauga tie lines were open during the test to permit direct recording of the total tie-line load on a single chart. Representative load-test curves, replotted to a common kilowatt scale on rectangular co-ordinate paper, are shown in Figure 7 for the nonregulated condition, and in Figure 8 for the regulated condition. The effectiveness of the controller in suppressing the strip-mill load swings is clearly evident from a comparison of the two charts. The over-all effect as recorded on the tie-line wattmeter chart, traveling at the normal chart speed of three inches per hour, is shown in Figure 9, which indicates a considerable reduction in band width.

As the effectiveness of the controller in offsetting fast load swings is dependent on close synchronism between the response variations in generator output and the strip-mill load-swing cycle, a further check was made to determine the over-all response time of the generator by feeding the strip-mill and generator a-c amperes directly into two high-speed ammeters having their charts driven from a common drive shaft. The results of this test are shown in Figure 10 and indicate an over-all response time of approximately two seconds.

In conclusion, it can be said that the controller performs as it was intended to function, with quick response, good stability, and negligible overshoot. Judging from the performance of the load controller when set for a 50-per-cent load pickup, it may be concluded safely that if the steam-plant boiler conditions would permit operation with a 100-per-cent pickup setting, the hot-strip-mill load swings would be completely eliminated from the Commonwealth and Southern–TVA tie lines.

Reference

1. Automatic Load Control for Turbine Generators, F. E. Crever, R. L. Jackson. AIEE Transactions, volume 64, 1945, September section, pages 656–60.

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† Prepared by W. W. Eberhardt.

Dielectric Strength and Protection of Modern Dry-Type Air-Cooled Transformers

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TODAY there are in service over 1,000,000 kva of large dry-type air-cooled transformers, some in ratings as high as 4,000 kva and including the 15-kv voltage class. This wide acceptance in industry of the modern dry-type transformer and its continual growth, naturally has increased interest in its application and standardization. In a prior AIEE paper[1] the design characteristics, particularly with regard to thermal performance, have been discussed. The purpose of this paper is to review the development of the modern dry-type transformer, specifically with reference to its dielectric strength, and to present recommendations on impulse levels and methods of protection for applications which may be affected by lightning surges.

Application to Circuits Unexposed to Lightning

The development of the modern dry-type transformer was initiated in 1936, and at that time it was intended primarily for application to indoor and unexposed circuits where surge overvoltages are of internal origin. Until recently this has been the principal application for this type of apparatus. In this field of application surge overvoltages seldom will exceed three and one-half times the line-to-neutral crest voltage. Furthermore, the character of these surges is relatively slow compared to the abrupt effects resulting from impulse voltages. Figure 1, reproduced from reference 2, presents surge overvoltages that have been recorded in an extensive investigation on a 12-kv grounded-neutral system. A comprehensive survey of the subject is provided by the bibliography of reference 3. The data are typical of conditions encountered in these applications. On some ungrounded-neutral systems possibilities may exist of higher-surge overvoltages.

The low-frequency dielectric tests specified for dry-type transformers in American Standard C57.1, correspond to four or greater than four times the line-to-neutral operating voltage, and therefore in relation to the character of service the test has a practical significance. This and other requirements were taken fully into consideration in the design of dry-type transformers. In circuits unexposed to lightning no surge protection was deemed necessary, and none has been recommended. The experience with some 2,000 transformers in service amply bears out the adequacy of the design and the soundness of the practice recommended for these applications.

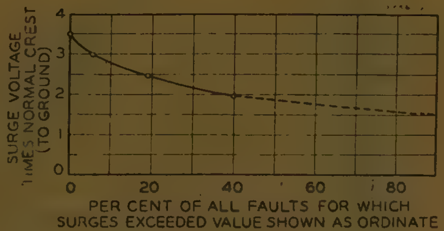

Figure 1. Surge voltages on 12-kv system[2]

Sphere-gap data 1932–38

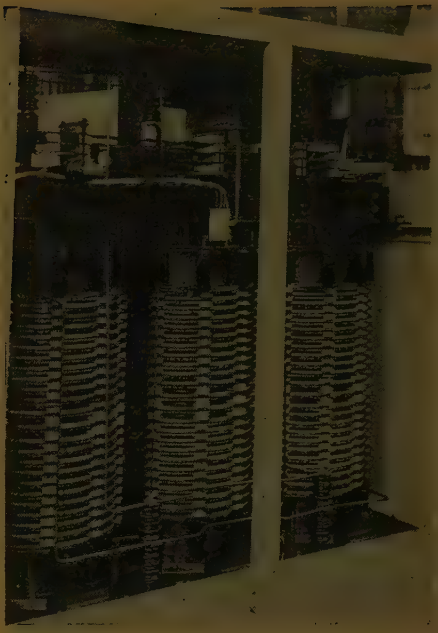

Figure 2. View of core and coil assembly of dry-type air-cooled transformer

Impulse Insulation Levels

Though the greater part of the dry-type transformers built since 1936 has been applied to unexposed circuits, as in industrial services, it was recognized that even circuits otherwise unexposed may be subject to surges transferred through the medium of transformers from circuits exposed to lightning.[4] For this reason attention was given from the beginning to a balanced insulation design. Even more compelling have been the specific applications where practical considerations and economy dictated connecting dry-type transformers directly to incoming overhead lines exposed to lightning. Also, it will be well to note that the possibility of developing the dry-type transformer for application generally to distribution systems—exposed and unexposed—definitely has been in the foreground.

Let us consider the insulation design and construction of modern dry-type air-cooled transformers. The core and coil assembly is the conventional core form of construction as shown in Figure 2. For operating voltages up to 5,000 a cylindrical winding commonly is used, and above 5,000 and up to and including 15,000 the winding consists of a column of disk-type coils. The winding conductors are insulated with glass. The high- and low-voltage windings are arranged symmetrically about the core with the low-voltage cylindrical winding on the inside. Both the high- and low-voltage coils are wound and assembled separately on insulating barriers. First, the individual windings and then the assembled core and coils are treated with a high-temperature thermosetting varnish to add strength, to bond the conductor and coil insulation in place, and to give the assembly a smooth tough glossy finish which makes it highly resistant to moisture and dirt. The high-voltage coil column is supported at the top and bottom by large porcelain spacers. Porcelain spacers separate the high-voltage coils and space the coils from the high-voltage insulating barriers. Porcelain spacers also separate the low-voltage coils from the core assembly. The high- and low-voltage windings are centered and supported, so that an unimpeded air duct separates them and forms the major insulation of the transformer. The transformer leads, taps and terminals, and other live parts are supported on porcelain insulators.

The problem of securing a balanced insulation design in dry-type transformers fundamentally does not differ from the corresponding problem in other transformer apparatus. Figure 3 gives a typical impulse-voltage distribution, and Figure 4 illustrates a detailed study of the stresses developed throughout the high-voltage winding of a dry-type transformer. It is from these studies and the strength of the insulation parts that the design is

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The authors acknowledge the assistance of their associates in securing the test data presented.

Table I. Typical Withstand Impulse Tests Applied to High-Voltage Terminals of Dry-Type Transformers

Transformer Rating	Impulse Wave*	Voltage, Kilovolts	Remarks
600 kva 13,800–480 volts Three phase, 60 cycles	1½x40-μs wave	50	With power-voltage excitation
500 kva 13,800–480 volts Three phase, 60 cycles	1½x40-μs wave	52	With power-voltage excitation
350 kva 13,800–480 volts Three phase, 60 cycle	1½x40-μs wave	51	With power-voltage excitation
	Steep-front wave, 0.30 μs	86	No power-voltage excitation
1,000 kva 12,000–480 volts Three phase, 60 cycles	1½x40-μs wave	54	No power-voltage excitation
	Chopped wave, two μs	62	No power-voltage excitation
200 kva 2,400–480 volts Three phase, 60 cycles	1½x40-μs wave	25	With and without power-voltage excitation

*Impulse wave of positive polarity applied in all tests.

established. Modern dry-type transformers are of balanced-insulation design; that is, the coil, turn, and the other minor insulation are balanced with the major insulation so that the transformer can withstand an impulse wave of the voltage specified.

The impulse strength of transformers designed with balanced insulation is the product of the 60-cycle strength and the impulse ratio of the insulation. The dry-type transformer depends for its dielectric strength on air which has an impulse ratio of 1.1 to 1.3, depending upon the construction. Based on these fundamental considerations and the verification of designs from actual impulse tests as shown in Table I, the impulse levels in Table II have been established, and these have been proposed for standardization.

The impulse levels in Table II correspond to the 1½x40-microsecond full wave. For purposes of standardization, in addition to the full wave, two chopped waves have been suggested to conform with present American Standards Association methods of testing. The chopped-wave voltage may well be 115 per cent of the full wave. Waves of positive polarity generally are more effective for testing dry-type apparatus and therefore are recommended.

Table I illustrates typical tests that have been applied to dry-type transformers, ranging from full-wave to chopped-wave to front-of-wave impulses. These are designated as withstand tests, for they represent test voltages at which the same relative margin is maintained with respect to ultimate strength as for the full wave at impulse level. Figure 5 illustrates corresponding oscillograms. These data demonstrate the rising volt-time characteristic of the insulation, the significance of which will become more apparent under the discussion of protection. A second point of practical interest to note is the ability of these modern dry-type transformers to withstand repeated impulses without any sign of deterioration, even when carried to the point of flashover. For instance, in a series of tests applied to 13,800-volt windings of the 350-kva transformer referred to in Table I, this transformer was subjected to a combined total of 100 impulses divided between full wave, chopped wave, and front of wave. Some of these tests were carried to the point of flashover of the insulation without causing any ill effect to the transformer. In the impulse tests to failure, flashover usually occurred across porcelain insulators supporting the leads or from lead terminals to ground, not affecting the more vital parts of the transformer.

Table II. Impulse Insulation Levels of Dry-Type Transformers

Insulation Class, Kilovolts	Low-Frequency, Dielectric Tests, Kilovolts Rms	Impulse Levels 1½x40-μs Wave, Kilovolts Crest
1.2	4	10
2.5	10	20
5.0	12	25
8.66	19	35
15.0	31	50

Protection Against Lightning Surges

When dry-type transformers are applied to circuits exposed to lightning surges, the impulse characteristics of the apparatus in relation to the protective devices available and to the degree of protection expected need to be considered. In such cases only lightning arresters having a low protective ratio are recommended, and for effective protection they should be connected at the terminals of the transformer as shown in Figure 6. These arresters are the types that have been developed for the protection of rotating machines and applied to motors and generators for many years. They are available in station and line types. In the application to dry-type transformers, only the line type is justified economically in most cases. This arrester is known as the type *RM*, indicating its application to rotating machinery. The characteristics of the arresters are given in Table III, which summarizes data from recent tests on commercial arresters.

Verification tests have been conducted on these lightning arresters in combination with transformers. In a series of tests the complete transformer assembly plus the arrester were excited at normal power voltage from a 2,000-kva power supply, much in the manner the installation would operate in service. The impulses applied were synchronized to the crest of the 60-cycle voltage as in the standard impulse tests. For instance, the oscillograms *A* and *B* in Figure 7 demonstrate the impulse voltages at the 13,800-

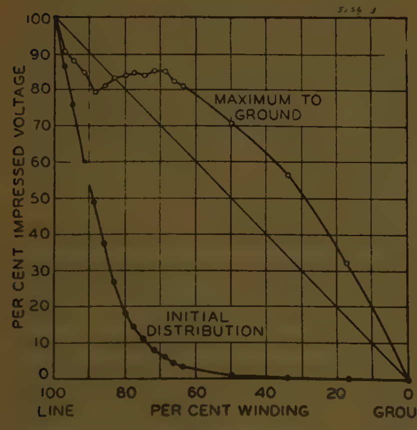


Figure 3. Voltage distribution in 13,750-volt winding, 1.6x37-microsecond wave applied

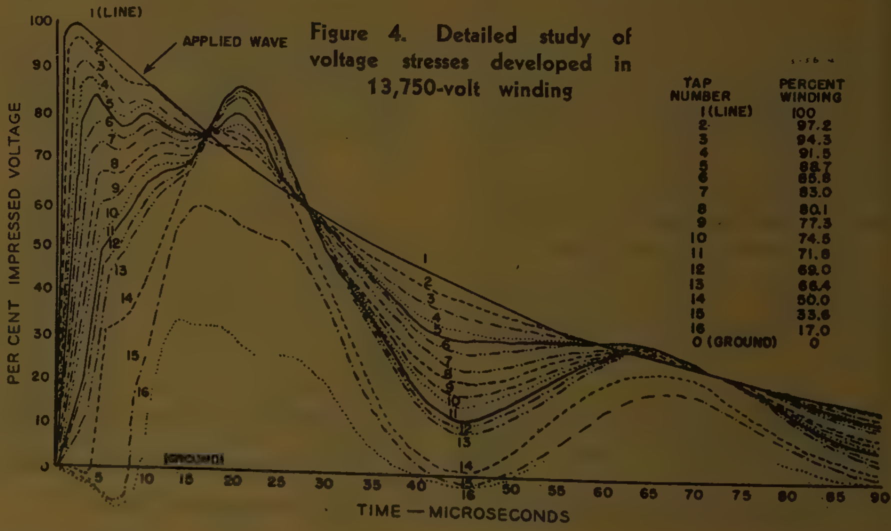


Figure 4. Detailed study of voltage stresses developed in 13,750-volt winding

Table III. Performance Characteristics of Type-*RM* Arresters*

Arrester Rating, Kilovolts Rms	Phase-to-Phase Circuit Voltage Rating		60-Cycle Gap Breakdown, Kilovolts Rms	Impulse Breakdown in Kilovolts Crest		Discharge-Voltage Kilovolt Crest on 10x20-μs Currents	
	Ungrounded Neutral System	Solidly Grounded Neutral System		Front of Wave**	$1^1/_2 \times 40$-μs Wave	1,500 Amperes	3,000 Amperes
3	2,400	4,160	6	13	9	9	10
6	4,800	7,200	12	24	18	18	20
9	7,200	11,500	18	32	27	27	30
12		13,800	24	40	36	36	40
15	13,800		30	48	45	45	50

*These arresters are special and should not be used interchangeably with standard distribution arresters.

**Test wave as recommended in AIEE Standard 28 for standard valve-type arresters.

volt terminal of a 350-kva transformer without protection and with a 15-kv *RM* arrester connected from terminal to case respectively. In another series of tests the voltage generated and applied was increased to several times the full-wave insulation level of the terminal. For instance, oscillogram *C* of Figure 7 is the generated voltage applied on repeated tests (ten) to one of the 13,800-volt terminals of the 350-kva transformer with a 15-kv *RM* arrester connected from the terminal to case. In effect this test corresponds to an incoming surge on a well-insulated overhead line having a surge impedance of 450 ohms, since the series resistance in the impulse generator was of this amount. As shown by oscillogram *D*, the voltage at the transformer is limited by the arrester. In this particular test lengths of leads included in the protective circuit produced an additional momentary voltage above the arrester characteristic, as shown by the initial instantaneous peak of 79 kv. In no instance during these tests was any ill effect whatsoever observed on the transformer insulation so that the conclusion definitely is drawn that even under these severe lightning surge conditions a good degree of protection can be provided to dry-type transformers with *RM* arresters, provided they are properly applied.

The rating of a lightning arrester for a particular installation is determined by the maximum dynamic voltage from line to ground. With solidly grounded-neutral systems the arrester rating can be less than line-to-line circuit voltage. Usually the next lower rating may be applied. For instance, on a 13.8-kv system with solidly grounded neutral, a 12-kv arrester may be applied with the advantage of an additional 20-per-cent increase in the margin of protection and at lower cost. Likewise, the availability of *RM* arresters at intermediate ratings as 4.5 kv and 7.5 kv should contribute to better protection in applications at the lower voltages.

Classes of Application and Protective Methods

In all problems of protection the question is to determine first, the extent apparatus may be endangered, and, second, whether the relative risk incurred, weighed against the importance of the installation and the cost of damage, warrants protective equipment. Where protection is required, the method should be effective and in keeping with economy. Let us consider the various possible applications and protective methods for modern dry-type transformers.

The extent dry-type transformers may be subject to lightning surges depends entirely upon the degree of exposure of the circuits to which they connect. For convenience, these are classified as follows:

1. Unexposed.
2. Quasiexposed.
3. Exposed.
4. Highly exposed.

The protective methods recommended for these various applications are outlined schematically in Figure 8.

The unexposed circuit is one on which lightning surges practically are nonexistent. That is, the circuit is either disassociated from lightning surges or only remotely coupled electrically to their source. Most industrial applications indoors, applications in mines and to underground cable systems, and many others come within this classification. As stated previously, system overvoltages on distribution and network circuits as commonly operated in this country seldom exceed three to four times line-to-ground normal crest voltage. On systems with the neutral isolated, conditions may develop resulting in somewhat higher-surge overvoltages. Considering that the strength of dry-type transformers to these surges is comparable to the impulse strength (Table II), system overvoltages five to six times, and higher for the lower classes, are permissible. Several years' experience from many transformer installations on various systems has indicated no instance of difficulty from system overvoltages. In general, no protection is required for these applications (Figure 8A).

The quasiexposed circuit would be one in which there is an intervening bank of oil-immersed transformers between the dry-type transformer and the exposed or highly exposed circuit, as shown in B of Figure 8. When a dry-type transformer is connected in the manner indicated, it is in general not necessary to use lightning-arrester protection at the transformer, provided the intervening transformers are protected properly on the exposed side.

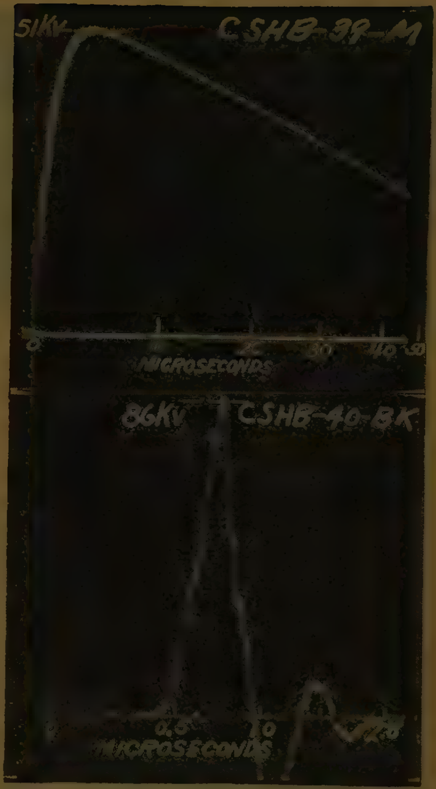


Figure 5. Typical full-wave and front-of-wave impulse voltages applied to dry-type air-cooled transformers, 15-kv insulation class

Oscillograms from tests on a 350-kva 13,800–480-volt three-phase 60-cycle transformer

Figure 6. Built-in lightning-arrester protection in dry-type air-cooled transformer

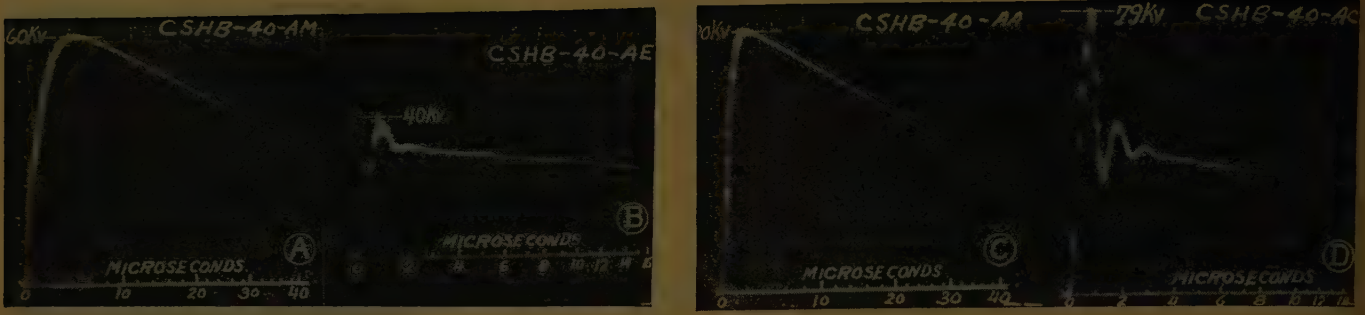


Figure 7. Impulse tests on transformer provided with lightning-arrester protection

350-kva 13,800–480-volt three-phase 60-cycle transformer and 15-kv *RM* arrester

It is assumed that the secondary circuit from the intervening transformers is unexposed; for instance, that it extends to the supply centers as a cable system or it enters directly into an industrial establishment. Should the dry-type transformer be connected closely adjacent to the secondary terminals of the intervening transformer and it alone become the entire load, lightning protection of the dry-type transformer would be necessary.[4]

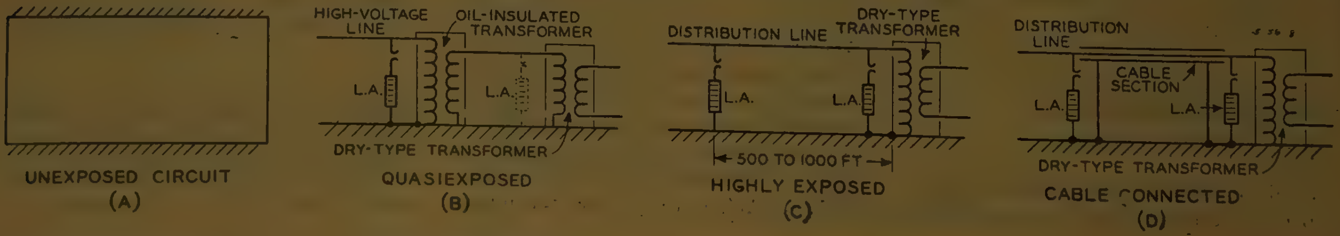
Exposed and highly exposed circuits are treated here as one. They differ in degree as to the frequency of occurrence and intensity of lightning surges to which each are subject. The exposed circuit would be one situated in urban districts or located such that there is some shielding from surrounding buildings or objects. It may have an impulse level of the order of 100 to 150 kv. The highly exposed circuit is one which is insulated with wood construction throughout, that is, wood poles, wood crossarms, wood braces, and so forth, and located in a territory which has a high lightning occurrence and intensity. Such a line would have an impulse level of 300 kv or greater and impulse voltages of this order actually may build up on it. In either case the incoming line connects directly to the dry-type transformers.

In these applications standard expulsion or valve-type distribution arresters are recommended on the incoming lines, in addition to the low-ratio arresters at the terminals of the transformer, preferably 500 to 1,000 feet from the transformer, as shown in C of Figure 8. The addition of the protective device on the line contributes to the over-all scheme of protection in that it takes the brunt of incoming high-current surges and permits the arrester at the transformer to function within its rating and provide a higher degree of protection to the transformer. Capacitors such as specified for rotating machines are not required. On urban circuits where "shielding" of the circuit is known to reduce the frequency and intensity of surges to a fractional part of the amount encountered in rural exposed circuits, the protective device on the line may be omitted. On highly exposed circuits some risk of direct strokes to the 500- or 1,000-foot section of line between the line arrester and the transformer may be present. This probability must be recognized, although it is fairly small. In general an overhead ground wire to shield the 500- to 1,000-foot end section of the line would not be justified. It is recognized that the special station-type arrester used for rotating-machine protection provides a somewhat greater margin of safety, because in the rare event of a stroke to the section of line adjacent to the transformer the station arrester gives added protection because of its greater current-discharge capacity and somewhat better protective level. For certain large and important installations its use may be justified, but for the general application the type *RM* arrester is the most practical.

Up to the present specific applications of dry-type transformers to exposed circuits have been almost entirely as shown schematically in D of Figure 8. The principal reason for this arrangement is that it lends itself quite conveniently to the entire installation. This commonly consists of the incoming distribution line, usually of wood construction, terminating outside the industrial establishment or adjacent to the site of power supply, the circuit continuing to the transformer proper as a cable section up to several hundred feet long. This arrangement has inherent merits also from a protection standpoint. As indicated, *RM* line-type arresters at the transformer combined with standard expulsion or valve-type distribution arresters applied to the cable pothead at the line end will provide a high degree of protection in this type of installation. At the line end the arrester ground terminals should connect directly to the cable sheath and to ground through a ground wire. The arresters at the transformer preferably should be built in, as shown in Figure 6, thus securing minimum length of arrester leads from the terminals to case and therefore the best over-all protection; this combination has other constructional and safety advantages. On distribution circuits which operate under conditions of low exposure, the arrester at the line end of the cable may be dispensed with.

The foregoing recommendations are based on fundamental engineering considerations. In addition they have been corroborated by test. For instance, tests were conducted on a test setup corresponding to the arrangement of Figure 8D, with a lead-sheath cable 250 feet in length. One end of the cable was connected to the high-voltage terminal (high-voltage insulation of 15-kv class) of a 1,000-kva air-cooled dry-type transformer, the other end to the impulse generator. In these tests a 15-kv *RM* line-type arrester was applied at the transformer, backed up by a line-type 15-kv arrester at the line-end of the cable, in the manner recommended for highly exposed circuits. The series resistance of the generator was approximately 450 ohms for generated voltages above 200 kv and considerably less for other voltages. The term "generated voltage" as used here simply means the voltage that the impulse generator would have developed with no connection made to the cable. It represents, so to speak, the impulse applied to the line terminal of the cable, modified and limited, of course, by the circuit and the arresters. The front of the generated voltage in these tests was close to one microsecond. The duration of the wave to half-crest value as

Figure 8. Classes of application and protective methods



Constant-Speed Drives for Aircraft Alternators

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IN LARGE MILITARY AIRCRAFT the demand upon the electric system has been increasing exponentially. At the start of the war, 50-ampere generators were considered ample; this rating soon jumped to 100, and now 200-ampere generators on our four-engine bombers are inadequate. As the limits of power output are reached, the necessity for satisfactory parallel operation and equality of division of the load becomes extremely important.

During ground operations, the main engines operate at low speed, and the battery is called upon to supply essential loads since at low speeds the generators are not supplying power. Before a flight, the crew frequently check the operation of turrets and other equipment and quickly discharge the low-capacity storage batteries of the airplane.

In this is seen the shaping of a demand for a more constant-speed type of drive for generators for airplanes. The problem of paralleling generators and obtaining equality of load distribution is greatly reduced if the generators can all be driven at a constant speed, and also power for ground operation or low engine speed would be provided automatically.

As the demands for more electric power and transmission distances have increased, it has become apparent that the present 30-volt d-c system is no longer adequate. In a current design of a large airplane, the conductor weight for a 30-volt system would be in excess of a ton and a half; whereas, if the voltage were increased to 250 volts, the weight would be reduced to approximately 250 pounds.

Contrary to a popular misconception, weight is just as important, if not more so, in large airplanes as in small. There is a tendency, in large airplanes, for designers to be more extravagant with weight and size; and, if this tendency is not curtailed and an effort made to use the most advanced metallurgy and efficient structural design, progress will be at a standstill, for advancement cannot be made by aerodynamic efficiency alone.

The demand for increased range also has emphasized the importance of weight in aircraft. It is not at all unusual to find present-day airplanes carrying once again their own weight in fuel and oil.

In other words, every pound that could be saved on the airplane represents two pounds' reduction in the gross weight of a long-range airplane.

Every ounce on an airplane must pay for itself. In military aircraft an increase in weight must provide either an important operational or tactical function, structural advantage, increased reliability, or decreased vulnerability. In commercial operation, extra pounds mean reduced pay-load and revenue. The cost of extra pounds has been evaluated quite accurately in dollars and cents by the commercial lines. Estimations range from $500 to $1,500 a pound. If dollars and cents talk, this should be a mighty strong talking point for the reduction in weight.

Constant-Frequency A-C System for Large Aircraft

Weight saving cannot be made, however, at the sacrifice of reliability. The "in-the-air" reliability of aircraft components must be very high. High-altitude flying conditions place additional burdens on equipment. The wide ranges of ambient temperatures encountered with military aircraft also add to the difficulties.

After careful study of the particular problems of some large-size long-range airplane designs, it was concluded that a high-voltage high-frequency three-phase a-c system would result in improved altitude performance and a weight saving. A voltage of around 200 volts permits a large reduction in conductor weight, and the reliability of a three-phase induction motor is unsurpassed by any other type of motor. Other factors such as circuit-breaker and switch design, and ease of voltage transformation, also influenced the choice of alternating current.

A variable-frequency variable-voltage system was ruled out because of the demands of many existing accessories which have been designed for constant-voltage supply. Because of the favorable speed ranges, light motor and transformer weights, and the possibility of direct application to many existing pieces of equipment, a frequency of 400 cycles was chosen.

This question then presented itself: How would the alternator be driven? A small auxiliary power plant was an obvious solution. However, upon more careful consideration, it is apparent that the installation would involve an excessive amount of weight. The auxiliary power plant must be pressurized for high-altitude operation, heated for the extreme cold of the upper atmosphere, and cooled for ground operation. The specific fuel consumption of the auxiliary power plant is greater than that of the main engines, and also prime mover capacity is, in effect, duplicated, resulting in a weight increase. A study on a large airplane indicated that on a long-range mission the use of an auxiliary power plant would add 1,500 pounds to the gross weight of the airplane. It was, therefore, apparent that the main engines should be used as a source of prime power for the electric system.

Requirements for a Constant-Speed Drive

With a decision to use the main engines as a source of driving power for the alter-

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recorded from the arrester oscillograms is in the order of 30 microseconds. The results of the tests may be summarized as follows.

The voltage generated and applied to the line-end of the cable was raised starting from below the *RM*-arrester operating voltage in steps to 100 kv, then to 186 kv and 300 kv, finally to 600-kv generated voltage. Many tests were applied. Protection was effective throughout the tests. To illustrate one of the tests: At 186-kv generated voltage the voltage developed at the transformer was limited to 51 kv at the crest of arrester spark-over and to 45 kv during arrester discharge. In this test the voltage at the line-end of the cable was held under 60 kv. The corresponding currents discharged at the line and transformer arresters were approximately 2,500 and 1,500 amperes.

In conclusion these are the general recommendations on methods of protection and protective schemes applicable to modern air-cooled dry-type transformers. In specific cases practical considerations may demand modifications or additions to fit the protection effectively and economically to the particular case.

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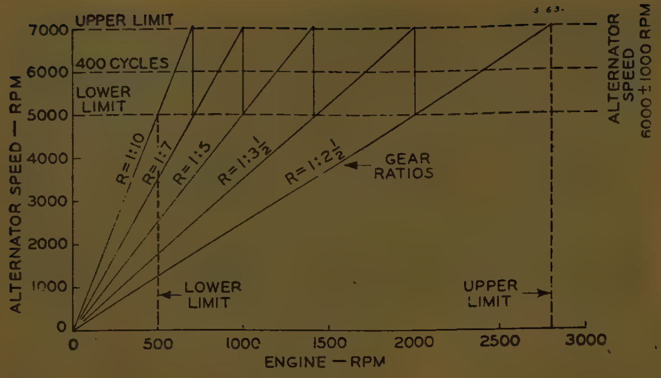


Figure 1. Alternator speed versus engine speed for a five-speed gearbox

Gear ratios chosen to provide an alternator speed 6,000 ± 1,000 rpm throughout operating range of the engine

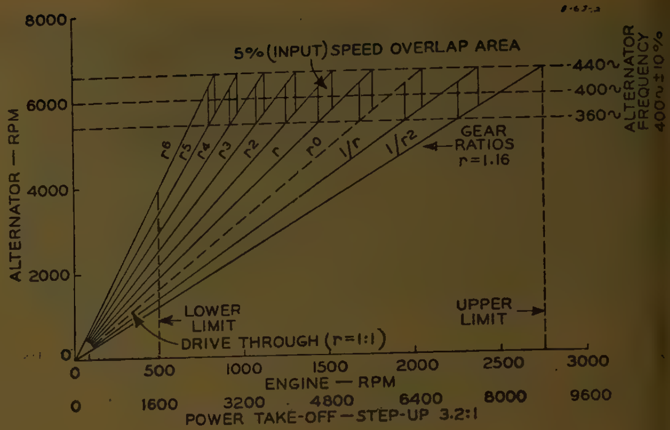


Figure 2. Alternator speed versus engine speed for a multiple staging of constant-ratio gear sets to provide a frequency regulation of ±10 per cent with a 5-per-cent speed overlap for clutching

nators, the necessity for some form of a variable-ratio coupling becomes apparent. An aircraft engine may vary from 500 rpm at idling to 3,000 rpm at military power. For a fixed-frequency system, the alternator requires a constant driving speed at all times. Since the power output of an induction motor is inversely proportional to the square of the frequency, it is desirable to maintain essentially constant frequency. If the system frequency is 400 cycles and an eight-pole alternator is used, the variable-ratio drive would be required to supply a constant 6,000-rpm output for input speeds ranging from 500 to 3,000 rpm. This is a ratio variation of 12:1 at low speed, to 2:1 at high speed.

The auxiliary power requirements of some of the new heavy bombers are upwards of 50 horsepower. Ordinarily an alternator must be capable of supplying a 100-per-cent overload for short-time periods. The variable-ratio drive, therefore, must be capable of supplying 50 horsepower continuously and 100 horsepower for short intervals.

This must be accomplished for a minimum of weight and mechanical complexity. If the device becomes too heavy and complex, it would lose its advantage over a direct-driven auxiliary power plant.

Finite-Ratio Versus Infinite-Ratio Drive

A selective fixed-ratio device could be used to maintain the alternator within a certain specific speed range. For example, a multiple-stage gearbox which would provide selective ratios of 10, 7, 5, $3^1/_2$, and $2^1/_2$ to 1 would maintain the alternator at speeds which would vary roughly 1,000 rpm above and below the required 6,000 rpm as the engine speed changed from 500 to 3,000 rpm (Figure 1). The speed variation would not be too objectionable so far as most of the motor applications were concerned, but a variation of plus or minus 1,000 rpm would result in a torque variation (assuming the voltage was held constant) of approximately 2:1 which would be objectionable.

Another disadvantage of a finite ratio is the difficulty which would be encountered in the parallel operation of alternators. Unless some auxiliary flexible coupling were used, it would not be possible to maintain parallel operation of the alternators.

This flexible coupling would have to provide for a speed differential of approximately 1,000 rpm if the afore-mentioned ratios were used in the fixed-ratio drive. A certain amount of droop would be desirable. These characteristics could be provided by a fluid coupling, but would add weight to the drive and reduce efficiency.

A planetary gear set can be switched from one ratio to another by means of electrically actuated clutches without too much difficulty. The clutches may be energized by suitable contacts on the governor. Overlap of the ratio-change contacts must be provided to eliminate chattering or hunting when the speed is in the vicinity of the ratio-change values.

The design and operation of an automatic ratio selector may be simplified considerably by using a constant ratio between successive gear sets. The ratios already indicated are not related by any common ratio. A separate gear set would have to be used for each ratio which has no common factor. A factorial relationship between gear sets permits various combinations of gear groups to be used to produce the desired over-all ratios.

Chrysler Corporation photo

Figure 3. An infinite-ratio mechanical-drive coupled to an experimental 37.5-kva aircraft alternator

The chart in Figure 2 shows the variation of alternator speed over the range in engine speed. The multispeed gearbox is connected to an engine-power take-off which has a step-up ratio of 3.2:1. The multistage gearing consists of two step-down ratios of $1/r$ and $1/r^2$, a drive-through or 1:1 ratio, and six step-up ratios of r, r^2, r^3, r^4, r^5, r^6. By the use of this common ratio, r, considerable simplification in the multistage gear can be accomplished by using various combinations of the factorially related gear sets to obtain the desired over-all ratios.

The arrangement shown in Figure 2 will provide an alternator speed range such that the frequency will vary only ± 10 per cent from the desired 400 cycles.

An overlap of five per cent of the input speed is provided to eliminate sporadic operation of the automatic ratio selector. This selector may be coupled to either the input or output shaft. The fixed percentage of input speed for overlap and sequential operation of the gear sets can probably best be provided for by a control governor attached to the input shaft. An overspeed protective governor would preferably be attached to the alternator shaft.

Infinite-Ratio Coupling

For parallel operation of alternators, the multistage gearbox would not be suitable unless some auxiliary flexible coupling, capable of adjusting the speed over a range of ten per cent, were used. This would not be entirely satisfactory, for there may be times when the main engines are operating in the same speed range. This coupling also would have to be capable of acting very rapidly, for, during the intervals of gear-set changes, large differential in output speeds would arise.

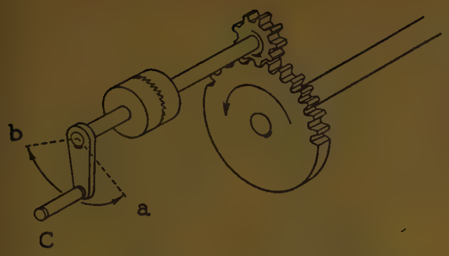


Figure 4. A spur gear driven by an oscillating small pinion gear

The pinion is connected by means of a freewheeling unit to a crank C which rocks from a to b

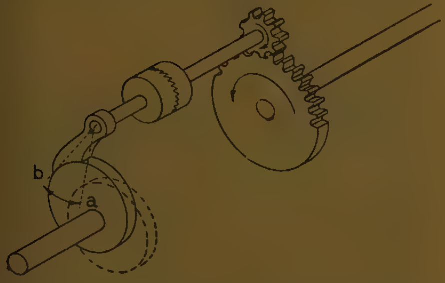


Figure 5. A cam and follower have been substituted for the crank

The same oscillating motion is provided

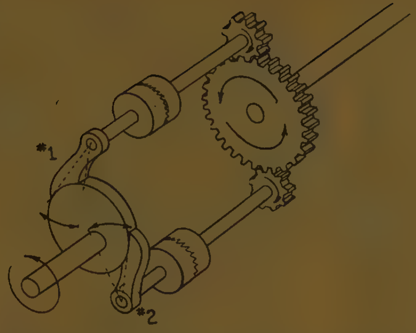


Figure 6. An additional pinion, freewheeling unit, and cam follower have been added to double the number of impulses from the rotating cam

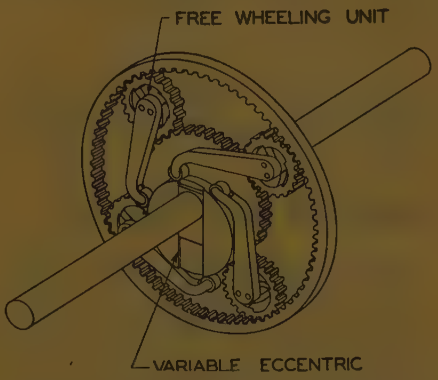


Figure 7. A four-pinion assembly with freewheeling units integral with the pinion gears and a variable eccentric cam on the input shaft

A more satisfactory drive would be one in which the ratio could be varied infinitely and rapidly. A fluid coupling of the hydraulic clutch would function, but its efficiency is so low over the required speed range that it is impractical to use.

The same criticism applies to any other type of drive which utilizes a slipping element for speed change, be it hydraulic, mechanical, or electrical. Slip, regardless of the type of element is manifest in energy loss and is wasteful of power.

In an airplane, efficiency is important from two standpoints:

(*a*). The amount of fuel which must be carried to furnish the auxiliary power.

(*b*). Cooling provisions which must be provided for the drive.

Added fuel increases the gross weight, and cooling provisions usually add to both weight and drag.

Two infinite-ratio coupling systems have been developed, which embody positive translations. One system is strictly mechanical, and the other is hydraulic. Because these systems do not depend upon slip, their efficiency is high. In general, it is easier to govern a slip device than a variable positive-translation device. This has been found to be true in the case of these devices. The governing has proved a problem, and it has been difficult to attain the desired rapidity of ratio change necessary to maintain constant-output speed throughout the range of speeds and accelerations of the engines.

Infinite-Ratio Mechanical Drive

A unique and efficient mechanical system has been developed by the Chrysler Corporation for providing continuously variable-ratio coupling (A drive attached to a 37.5-kva alternator is shown in Figure 3).

The device operates in the following manner:

In Figure 4 the output shaft is driven by means of the small pinion and spur gear. Oscillation of crank *C* from positions *a* to *b* transmits, through the freewheeling ratchet, intermittent rotation of the pinion gear in a clockwise direction. The pinion gear drives the output spur gear, revolving the output shaft in a counterclockwise direction.

In Figure 5 the crank has been replaced by a shoe riding on a cam. The cam is attached to the input shaft, and, as the input shaft revolves, the shoe will oscillate from *a* to *b*, driving the pinion and spur gear in a manner similar to the oscillating crank arrangement of Figure 4.

The output shaft will receive an impulse of acceleration each time the shoe is displaced from *a* to *b*. An additional impulse may be added by arranging another shoe and freewheeling pinion gear diametrically opposite shoe 1, as indicated in Figure 6.

Additional impulses likewise may be given the output spur gear by the additional shoe and pinion gear assemblies. The magnitude of the impulse is determined by the angular acceleration and displacement of the shoe, which is dependent

Chrysler Corporation photo

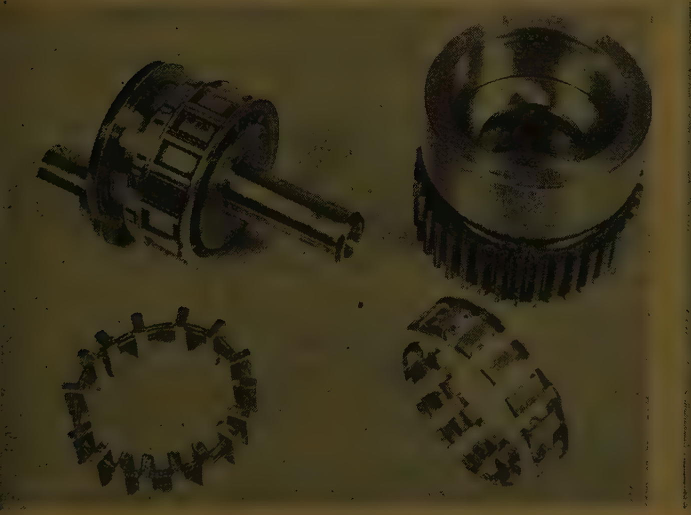


Figure 8. Components of a pinion gear and freewheeling assembly from the infinite-ratio mechanical drive

Chrysler Corporation photo

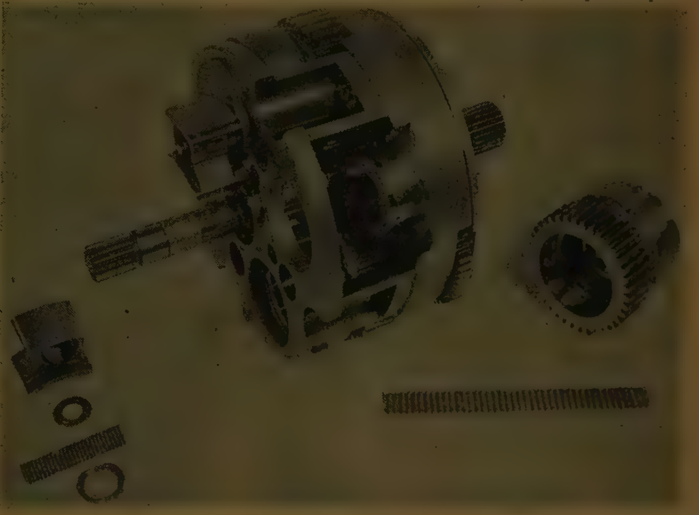


Figure 9. Assembly showing the pinion gears and one of the cam followers

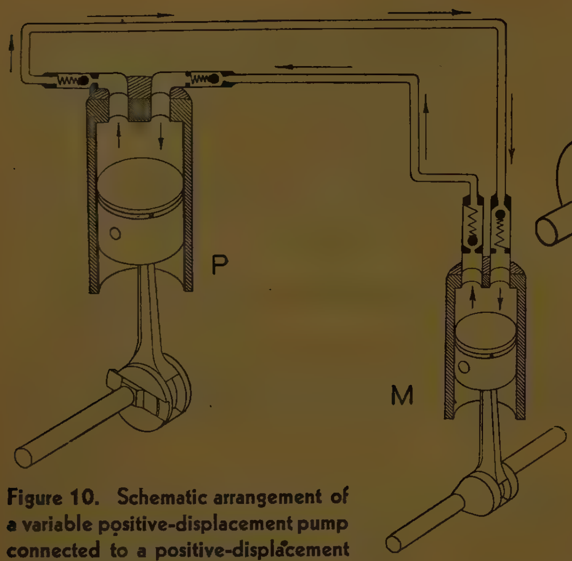


Figure 10. Schematic arrangement of a variable positive-displacement pump connected to a positive-displacement hydraulic motor

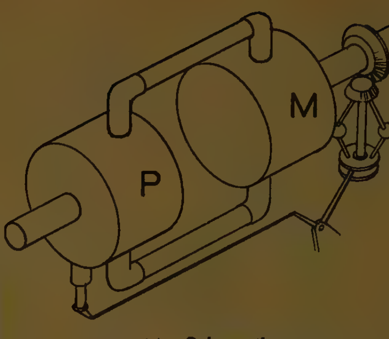


Figure 11. Schematic arrangement of a hydraulic-pump and motor combination with a governor controlling the pump displacement attached to the output shaft

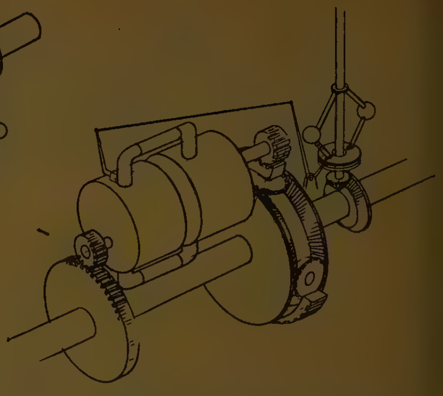


Figure 12. Schematic arrangement of a pump and motor combination utilizing a differential gear to limit the demands upon the hydraulic link

upon the speed and eccentricity of the cam.

In Figure 7 an arrangement is shown with four pinion units with concentric freewheeling units coupled by means of connecting rods to shoes which slide on the driving cam. In this arrangement the pinions are planets in a planetary gear system. As the cam revolves, these planet pinions impart driving impulses to the driven spur gear in the same manner as in Figure 6.

The ring gear is coupled by means of reduction gearing (not shown in the figure) to the drive shaft and drives the output shaft directly, since the planet pinions are locked effectively by the ratchet action of the freewheeling elements.

Since the ring gear is geared down from the drive shaft, the cam will revolve faster than the ring gear and will impart accelerating impulses to the planet pinions proportionate to the amount of eccentricity, which will add to the speed of the driven spur.

The acceleration impulses are proportional to the eccentricity of the cam. The additive speed differential may be varied by changing the amount of eccentricity. Therefore, the ratio of the drive can be changed from the fixed ratio of the ring gear to the fixed ratio plus the additional speed added by the planet pinions when the cam is in a position of maximum eccentricity.

The variation of cam eccentricity from zero to maximum may be perfectly continuous and, therefore, will provide an infinite number of ratios between the aforementioned limits.

This illustrates in principle the drive shown in Figures 3, 8, and 9; however, actual construction details of the drive differ considerably from the simplified arrangement of Figure 7.

One of the crank arms and freewheeling assembly is shown in Figure 8.

In Figure 9, one of the shoes is shown attached to the crank arm. A single shoe is in the lower left-hand corner, and the meshing pinions may be observed at the center.

Infinite-Ratio Hydraulic Drives

Fluid couplings, or the so-called hydraulic clutch, may be used with some means of controlling the amount of slip to provide an infinite number of ratios. Devices of this type, however, are not efficient over a very wide range in speed.

Drives have been developed which embody essentially the same basic principle as the previously described mechanical system and are quite efficient throughout a wide speed range. These drives are direct-coupled and depend upon a controllable positive displacement.

For example, Figure 10 shows a simplified arrangement of this type of drive. The unit consists of positive displacement motor *M*, coupled to a positive displacement pump *P*, in which the displacement may be controlled.

By varying the eccentricity of the drive cam, the volume of the fluid which is pumped each stroke is changed, and accordingly the ratio between the input shaft and output is changed. The ratio is infinitely variable over a large range of speeds.

Usually both the pump and motor are multicylinder. Figure 11 indicates schematically a governor placed on the output shaft which is linked to the displacement control in such a manner that the output speed of the unit is maintained constant for driving an alternator.

The use of a variable-displacement pump and positive-displacement motor for variable-ratio drives has been applied to turret drives in large battleships for a long time. These units, however, are very heavy and, because of their intermittent type of operation, are not designed for particularly high efficiency over a large range of ratios.

The weight of this type of drive may be reduced by two methods: by increasing the working hydraulic pressures, and by increasing the motor and pump speeds. An experimental drive has been built which maintains an output speed of 6,000 rpm with an input-speed variation of 1,000 to 3,000 rpm. The drive is capable of transmitting 60 horsepower and weighs approximately 50 pounds, or 0.83 pound per horsepower. The maximum working pressure is 4,500 pounds per square inch, and the motor pump speeds are on the order of 18,000 rpm. Even though the unit is extremely light and of high speed, the efficiency is surprisingly good, varying from about 70 per cent at the worst condition to 83 per cent at maximum.

Methods For Improving Efficiency

A method for improving the efficiency of a variable-ratio drive of this type is shown in Figure 12. The method is based upon the fact that the efficiency of a variable-displacement pump and constant positive-displacement pump is a maximum for one particular flow. This is a condition in which the reflected mechanical impedance of the motor load most nearly matches the effective internal impedance of the pump and the associated hydraulic circuit. This is analogous to the conditions for transferring maximum power in an electric circuit when the load impedance is made equal to the generator impedance.

The efficiency of the combination drops

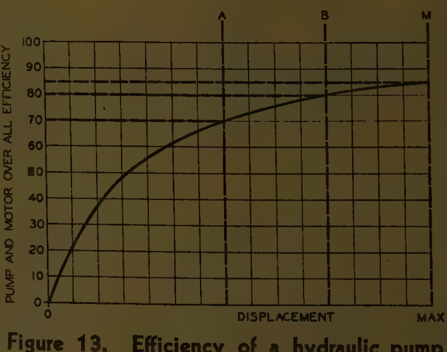


Figure 13. Efficiency of a hydraulic pump and motor combination versus displacement

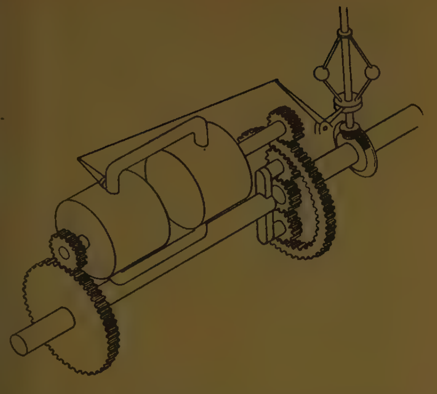


Figure 14. Schematic arrangement for improving the efficiency of a drive by the use of an epicyclic gear train to reduce the demands on the hydraulic units

slightly as the displacement is changed from this optimum value. When the pump and motor are of equal size, this point is usually the point of maximum displacement for the pump. See Figure 10.

In the arrangement of Figure 12, the pump and motor combination supplies only the differential speed which is necessary to bring the output shaft up to the desired constant speed. For example, if the drive-shaft speed varies from 2,000 to 6,000 rpm, and it is desired to maintain 6,000 rpm on the alternator, it will be necessary for the hydraulic unit to supply, through the differential gear, an additional 4,000 rpm to the alternator when the input is 2,000.

This would be the point of maximum demand on the hydraulic unit and would require the maximum displacement of the pump which occurs at maximum efficiency.

As the driver speed increases, the demands on the hydraulic units become less and less until an end point is reached when the driver is revolving at 6,000 rpm and there is zero demand from the differential gear. The efficiency falls off as zero is approached (see Figure 13), but, as the unit is supplying proportionately less power (the major portion of the power being supplied directly through the differential gear), the net over-all efficiency will remain high since it is a combination of the mechanical and hydraulic efficiency.

For purposes of rough comparison, assume a median efficiency of 70 per cent for a straight hydraulically coupled system, as indicated in Figure 13, and compare it with the foregoing. If one neglects the bearing and gear efficiency, it may be assumed roughly that the power demands will be divided between the hydraulic and mechanical sections proportional to the speed requirements.

With the driver varying from 2,000 to 6,000 rpm, the driver median speed is 4,000 rpm, and the speed demand on the hydraulic unit is 2,000 rpm. If one assumes the same median over-all hydraulic efficiency as above 70 per cent, the over-all efficiency of this arrangement is then 90 per cent.

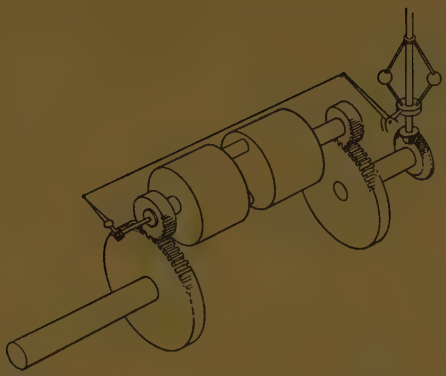


Figure 15. A hydraulic "differential" in which the pump and motor units revolve as a unit forming part of the gear set

With the gearing and bearing losses, which had previously been neglected, approximated as 10 per cent, the over-all efficiency of this arrangement would be 81 per cent, or an improvement of 11 per cent over the straight hydraulic drive.

The efficiency may be increased still further by using a pump and motor combination which can be reversed. The same type of arrangement, as illustrated in Figure 12, is used. In this arrangement the hydraulic unit is used to supply a differential speed above and below a median speed of the driver.

If the drive shaft had the same speed range as before (2,000 to 6,000 rpm), an epicyclic differential-gear set could be substituted for the conventional differential shown in Figure 14. The epicyclic gear set would incorporate a step-up to the output shaft of $1^{1}/_{2}$:1 such that the speed range of the output shaft would be 3,000 to 9,000 rpm if no differential-speed increment was supplied through the hydraulic unit.

At the median speed of the driver, 4,000 rpm, there would be no differential-speed demand as the output shaft would be revolving at the required 6,000 rpm. For input speed below this value, (2,000 to 4,000 rpm), the hydraulic unit would act as an overdrive and supply the necessary differential speed to bring the output up to 6,000 rpm (2,000 to 0 times the $1^{1}/_{2}$:1 ratio of the epicyclic gear). For the driver speed range of 4,000 to 6,000, the hydraulic unit would act as an underdrive, supplying the necessary negative differential to bring the speed down to the required 6,000 rpm.

The absolute speed range of the hydraulic unit, in this manner, is limited to 0 to 2,000 rpm, rather than the range of 0 to 4,000 rpm which was required in the previous case.

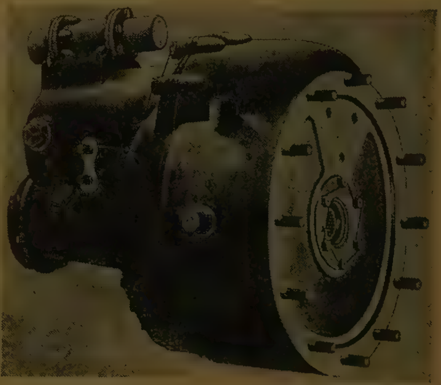
If one considers a hydraulic unit similar to the previous one, the efficiency at the median speed will be of the order as indicated by the dashed line *B* on Figure 13, or 80 per cent, which is a 10-per-cent improvement over the other arrangement. If the efficiency is calculated on the same basis as before, the over-all efficiency of the unit, with gearing and bearing losses neglected, would be approximately 96.6 per cent and, if a 10-per-cent loss in these two items were assumed, the total over-all efficiency would be 87 per cent, or a 6-per-cent improvement over the previous arrangement and a 17-per-cent improvement over the direct-coupled hydraulic arrangement shown in Figure 11.

The same results could be accomplished if the gear ratio of the power take-off were increased so as to provide a 3,000-to-9,000-rpm range of the driven shaft and a conventional differential gear were used instead of the epicyclic set.

Another arrangement which utilizes the same principle of overdrive and underdrive for increasing efficiency by limiting the speed range of the hydraulic units is shown in Figure 15. This arrangement does not use a differential gear, but the differential speed is supplied in a similar manner by the hydraulic unit.

For this method, the hydraulic pump and motor must be capable of rotating as a unit (zero displacement of hydraulic fluid) and also rotating in either direction relative to each other.

The pump is likewise a variable-displacement unit; and the motor, a con-



Sundstrand Machine Tool Company

Figure 16. A hydraulic infinite-ratio drive for a 40-kva aircraft alternator

Army standard spline and pad for 208/110-volt 400-cycle alternators

Sundstrand Machine Tool Company

Figure 17. Hydraulic infinite-ratio drive showing pad for connection of drive shaft to airplane main engine

Lightning Investigations on 33-Kv Wood-Pole Lines

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IN RECENT YEARS the importance of providing reliable service over lines in the transmission class of 22 to 44 kv has greatly increased. However, little fundamental data have been obtained on the severity of lightning duty on such lines or upon the detailed lightning performance of various types of line design and lightning protection. Most of such information has been gathered on the higher-voltage steel-tower lines. A comprehensive lightning investigation of this type has been undertaken jointly by the Public Service Company of Northern Illinois and the Westinghouse Electric Corporation on several types of 33-kv wood-pole circuits.

The study has consisted of four principal parts. A detailed climbing inspection program (correlated with operating results over a 2$^1/_2$-year period between 1939 and 1942) was carried out on a 28.8-mile line with the unprotected construction shown in Figure 1. This represents about the optimum amount of line insulation generally feasible for an unprotected line of this voltage class.[1] The inspections have provided valuable fundamental and correlating data on the nature of insulator flashover markings and on the distribution of lightning flashover and cases of sustained power follow between the different types of poles. At the beginning of the 1942 lightning season, two types of recording installations were put in service. One type is for the purpose of determining the character of line flashovers on unprotected lines. Sections of line were chosen where recorders could be placed on poles one-half to one mile apart whose insulation was comparable to a guyed pole, and with fully insulated tangent poles intervening. The line construction for these sections is also that of Figure 1. Photographic surge-current recorders and surge-crest-ammeter links were placed in the grounding circuit. The photographic surge-current recorder, which has been described in detail elsewhere,[2] employs the principle of photographing the luminosity produced by the flow of the surge current across a short spark gap inside of a light-tight case. The film image is produced through a special aperture and set of barriers so constructed that the light from the gap (Figure 5A) spreads in a nonuniform manner over the film perpendicular to its direction of motion. The barriers confine the image to a narrow wedge in the direction of film motion to enable high resolving power with time. The width of the image (in a direction perpendicular to the film motion) and its density provide a measure of the current magnitude. As current is increased, the film is saturated at increasing distances from the center axis. However, at a point just beyond this region, the film density can be measured to determine the approximate current magnitude. Current can be measured to an accuracy of about two to one but over the very great range of 0.1 to 150,000 amperes with a time-resolving power of 600 microseconds. Initially, there were 20 of these installations on three different line sections. However, the recorded prevalence of multiple flashovers extending over a mile or more made it advisable to select a single longer section of line. At present 14 poles on a ten-mile section of the line where the climbing inspection was made are equipped with recorders as shown in Figure 2. Two additional recorders have been placed at each pole to measure both the lightning and power-follow flashover currents from each side phase to a neutral point and from neutral to ground. Thus, it is possible to record for each of these paths the multiple character, duration, and approximate magnitude of the lightning-flashover currents together with the duration and approximate magnitude of the power-follow currents. A longer cross arm has

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stant-displacement unit. When the push–pull rod which controls the amount of displacement is in the middle position, there is no flow of hydraulic fluid and the pump–motor combination is hydraulically locked together and drives as a unit. The losses at zero differential speed are essentially mechanical—gear and bearings.

By varying the control rod on the pump, the flow is increased. The pump is capable of pumping in either direction, and, therefore, the rotation of the motor will be in one direction or the other, so that its speed will either add to, or subtract from, the speed of the hydraulic pump.

This type of drive has the same advantages of gain in efficiencies as the type previously described and, because of the effective overdrive and underdrive arrangement, is capable of operating over a large speed range.

An advantage of this design is that the speed of the pump and motor combination may be very easily stepped up so that the transmitted torque is low and the unit may be of very light weight.

A unit of this type has been built for less than one pound per horsepower and with an efficiency of 92 to 83 per cent over the speed range.

The sketches in Figures 16 and 17 are of a unit of this design built by the Sundstrand Machine Tool Company. Figure 16 shows the alternator-coupling flange and spline, and Figure 17 is the pad for connecting to the main engine.

Conclusion

The problem of providing a constant drive speed with a variable-speed prime mover is by no means new. There have been many solutions to this problem—electrical, mechanical, and hydraulic. Unfortunately, however, for an aircraft application they have all been too heavy or inefficient. In aircraft design, inefficiency is tantamount to weight for inefficient operation means more fuel is used, which means more weight is carried.

Many forms of constant-speed drives have been suggested and worked upon. It was not within the scope of this report to describe them all, many of which show considerable promise. The ones described in this report have all been at least in experimental production and have given us sufficient background information for us to draw a reasonable picture of the type of constant-speed drive which will soon be making its appearance in large aircraft.

The production units may be expected to have efficiencies of 85 to 95 per cent, a weight of less than one pound per transmitted kilowatt, will hold a preset output speed to within ±1 per cent total variation over an input-speed range of from 500 to 3,000 rpm, and will fit within a space of 50 cubic inches per kilowatt of rating.

The large airplanes of today with a-systems will have drives of this type. It is probable that with continued development the efficiency will be increased and the size and weight reduced to a point where it may be practical to use a constant-speed drive for d-c generators and other pieces of auxiliary equipment.

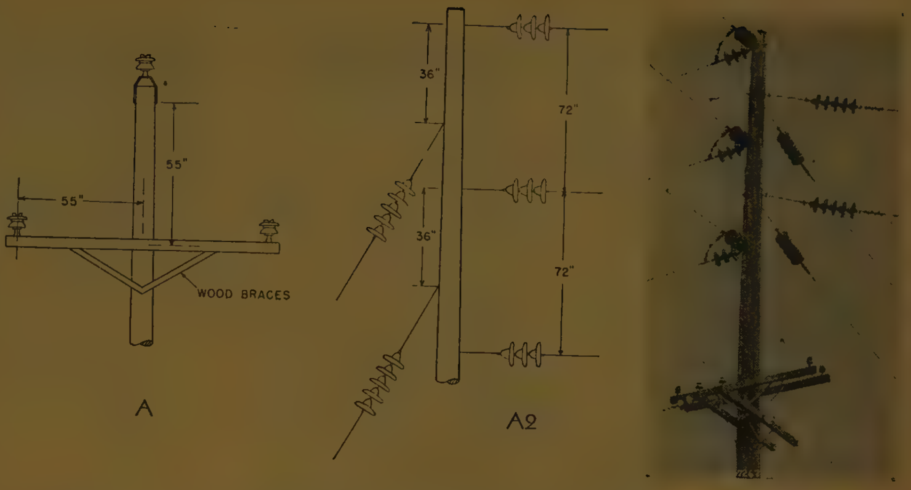
been installed lower down on the pole to provide the normal wood insulation path between the insulators and the recorder circuit.

For the second type of recorder installations, surge-crest-ammeter links have been placed in the ground leads of protected lines having the types-4 and 5 overhead-ground-wire construction of Figure 9, lines of type-3 construction (Figure 9) having deion protector tubes on the top conductor only and a line of type-6 construction (Figure 10) having protector tubes on each phase but on only one phase per pole and rotated between phases. The pole-ground-lead current records obtained in this way provide data on the frequency of direct strokes, the crest-stroke currents, and the division of current between ground leads as correlated with line performance. The magnetic links are placed so as to record crest currents ranging from a minimum of about 600 amperes to about 55,000 amperes. The presence of only a single straight conductor for each pole ground enables a more accurate recording of the pole currents than is possible on steel-tower lines where there is a multiplicity of paths down the tower and where only one tower-leg current is usually recorded. In addition magnetic links have been installed on a number of isolated protector-tube installations to correlate the difference in protector-tube duty under the two operating conditions.

In the fourth phase of the investigation a detailed record of the frequency and extent of lightning storms passing over the region occupied by the lines under investigation has been obtained. This was started at the beginning of the 1943 lightning season, and the manner in which it is accomplished is illustrated in Figure 3. Observers at the numbered substations submit a report on the apparent storm centers as illustrated by the samples at the bottom of the chart for every individual storm that they can observe. Each of their observations as to the location and direction of travel (at the time they are first able to observe it, when it passes closest to their observation station, and when they last observe it) and the time of the observation, are plotted, as shown, by an arrow. From these, the storm path, storm extent, and duration over each line can be determined. Such data provide a valuable correlation between the number of annual storm-days, number of storms, total annual storm duration, the frequency of strokes to the various lines, and the number of line tripouts.

Results of Climbing Inspection—Insulator Markings

In the course of the climbing inspection previously mentioned, 8.6 miles of line were inspected after about $1\frac{1}{2}$ years of operation of the line. The remaining 16.8 miles were inspected after about 2.5 years of operation. A total of 180 of the pin-type insulators that had flashover markings were removed from the pole for a more detailed inspection. Typical markings are illustrated by Figure 4. When flashover is first produced by lightning, the path of lowest dielectric strength is along the surface of the porcelain, so that breakdown occurs there first. Thus, for most lightning flashovers, heating of the porcelain glaze is more or less uniform along this flashover surface and results in the "lightning-only" markings illustrated by Figures 4A, B, C. Lightning markings are characterized by roughening of the insulator glaze (with a texture resembling fine sandpaper) over a path closely following the insulator contours. This frequently includes the underside of the top skirt. It may vary in width from a narrow line (Figure 4A) to a wide path extending several inches circumferentially around the insulator (Figure 4B). The length of the lightning markings varies greatly. In some cases there may be only a short narrow mark somewhere on the upper side of the top skirt. For others, the insulator surface may be marked almost continuously from tie wire to the lower edge of the inner skirt. In many cases a multiplicity of lightning paths is found on one insulator (Figures 4A and C). These can be formed by entirely separate lightning strokes or by different components of the same stroke.



Figures 1A (left) and B (right). Types of line construction on unprotected line used for detailed climbing inspection and photographic-surge-current-recorder study

A. Type 1, unguyed
When guyed, guy attached 30 inches below crossarm, and insulated with three $5\frac{3}{4}$-inch suspension units
A2. Type-2 corner pole

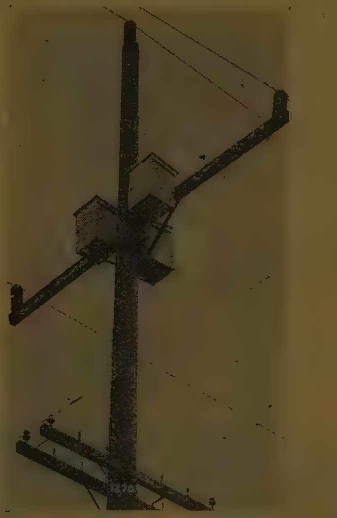


Figure 2. Installation of flashover recording equipment on line construction of Figure 1

Photographic surge-current recorders in both crossarm flashover paths and ground path. Crest-ammeter links in ground path

In case of power follow, the long-duration arc will tend to move away from the porcelain surface and maintain contact only at its terminals. This results in broad splashes of burned porcelain glaze at these points, often with considerable conductor and tie-wire pitting. The power-follow markings usually are localized in spots on the top and bottom of the insulator where the glaze has been burned through to the porcelain and frequently has a blistered appearance (Figure 4D). In less severe cases the glaze may be only partially burned through and is left with a color lighter than that of the normal surrounding glaze. Then the burned spot may have a crackled rather than blistered appearance. Sometimes the power-follow arc appears to strike from the conductor to the insulator pin, and the principal burn on the top of the insulator is at the edge of the top skirt under the conductor. In other cases, when the arc strikes from the tie wire, burns are found on the upper surface close to the tie wire. A power arc between line conductors usually will leave pits and beads or points of metal, often uniformly spaced out along the conductor from where the flashover started. These are the result of the arc traveling along the line, one melted spot occurring with each half cycle of current. The interpretation

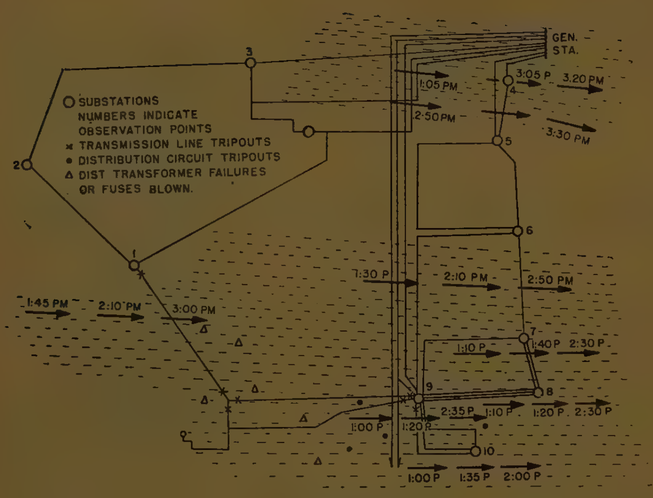


Figure 3. Sample storm chart prepared from reports submitted by substation attendants and used for determining storm frequencies and durations over lines being investigated

Sample Thunderstorm Reports

Observation Station	First Observation			When Over or Nearest Station			Last Observation		
	Time	Direction	Distance	Time	Direction	Distance	Time	Direction	Distance
1	1:45 p.m.	Southwest	7 mi	2:10 p.m.	South	2 mi	3:00 p.m.	East	6 mi
4	1:35 p.m.	Northwest	4 mi	3:05 p.m.	Over		3:20 p.m.	North	2 mi

No storm visible from stations 2 and 3

1—Lightning, thunder, no rain. Seemed to come from southwest, stay two miles south of station, then go east

4—Heavy rain at 3 p.m.

of evidence, as previously described, may not always be infallible, as some lightning strokes probably can have sufficient duration to produce effects resembling those attributed to power follow. Neither are so-called power-follow markings always evidence of sustained power follow causing circuit interruptions, as the 60-cycle arc may be extinguished before a circuit-breaker relay operation. These cases, however, are thought to be infrequent, as is borne out by correlation of similar inspection data with actual flashover-current records that subsequently will be discussed.

Distribution of Line Flashovers

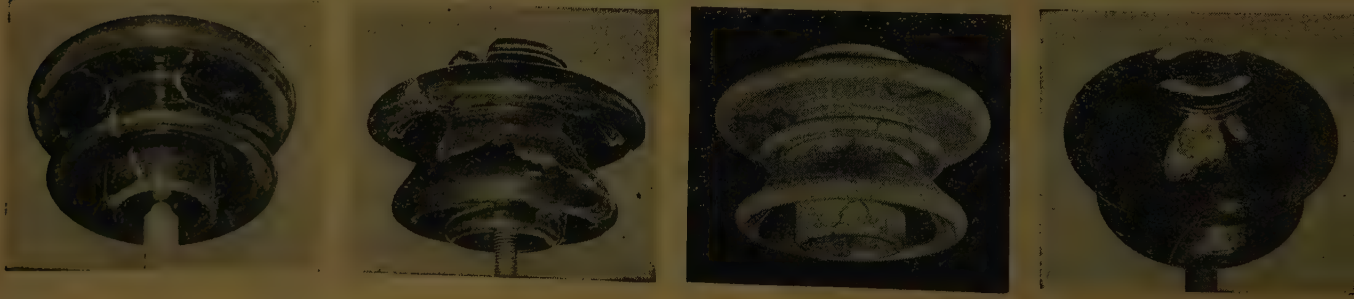
A length of 3.4 miles of the line inspected in detail parallels and is shielded by a 132-kv tower line, with overhead ground wire. The normal heights of the towers and poles above ground are 92.5 feet and 38 feet, respectively. The horizontal distance between tower-line ground wire and ridge-pin conductor on the pole line varies from 88 feet to 99 feet. No evidence of line flashover was found on the inspections. However, one stroke occurred to this section in 1943 as evidenced by a splintered pole. Thus, in $5^1/_2$ years the section has been struck once.

The remaining 25.4 miles of unshielded line on which the climbing inspections were made has 453 poles with an average span of 293 feet. A total of 411 poles are type 1 (Figure 1) unguyed with 108 having a distribution circuit below the high-voltage circuit, 12 poles are guyed type-2 vertical corner poles, and 30 are guyed type-1 poles. Thus about ten per cent of the poles are guyed. Table I presents a summary of the distribution of the two types of flashover as interpreted from the insulator markings and wood splintering inspection. This table shows that 36 per cent of the unguyed poles bore evidence of flashover, whereas about 80 per cent of poles with guys had been flashed over. Inspection of the distribution of flashed poles showed that unguyed poles tend to be protected by guyed poles several spans away. The trend in the case of power-follow arcs is more significant. Only 13 per cent of the tangent unguyed poles that flashed over had evidence of power follow, whereas the percentage for the guyed type-1 poles was 42 and for the vertical corner poles (with still lower insulation to ground) 90. As far as can be determined, the presence of distribution circuits or transformers (distribution arm five feet below transmission arm) has little bearing on the effects of lightning on the transmission line, 34 per cent of poles with distribution having flashover evidence. This particular analysis cannot determine accurately the actual number of individual cases of each type of flashover. However, it provides very significant evidence on the nature of the flashover of such lines which can be discussed more clearly when correlated with the additional data obtained from the photographic-recorder measurements.

Character of Flashovers on Unprotected Line

The section of line equipped with the photographic recorders is inspected after each storm, the photographic recorders and magnetic links being checked for flashover-current records together with a visual inspection for additional flashover evidence. Where necessary, a climbing inspection is made as frequently as possible to determine the location and type of insulator markings and other evidence not visible from the ground. This is for the purpose of detecting flashover on intervening poles not equipped with recorders and to provide correlating information with the flashover currents at the recorder poles. During the three-year period 49 cases of line flashover have occurred on the sections under investigation for which it could be determined whether or not a sustained power follow or a line tripout was produced. For 35 cases a detailed record was obtained of the number of poles flashed over, together with the type of lightning flashover and sustained power follow (if occurring) at each pole. It is recognized that the mechanism of arc restriking and dielectric recovery is quite

Figure 4. Typical example of flashover markings produced on 34.5-kv pin-type insulators by lightning flashover and power follow


A. Lightning only, seven paths

B. Lightning only

C. Lightning only after four recorded cases of flashover with crest currents of 12,500, 12,000, 35,000, and 22,000 amperes

D. Two power-follow markings and lightning track complex, being a function of a large number of factors that cannot be evaluated adequately from the data so far obtained. However, there are certain important facts that have been ascertained which are described here briefly.

One of these is the prevalent occurrence of simultaneous flashover produced over an extended section of line by a single stroke. It was found that a stroke occurring between recorder stations usually flashes over several of the unguyed poles in the vicinity of the stroke in addition to one or more of the recorder poles having the lower insulation to ground. This is illustrated by the records of Figures 5A and B. Figure 5A is typical of several obtained showing simultaneous flashover for each component of a multiple stroke. It is the usual case that, if an appreciable duration of the lightning current is indicated for any one stroke component, it frequently occurs only at one pole, the same pole having power follow when present. Figure 5B is another record showing simultaneous flashover produced over about 1.5 miles of line. For this case, although lightning flashover occurred between all three phases and ground at pole 280 power follow occurred only between the top and one side conductor. Evidence of similar multiple-flashover characteristics on 220-kv wood-pole lines has been presented elsewhere.[3]

Table I. Summary of Lightning and Power-Follow Inspection on 33-Kv Unprotected Wood-Pole Line—25.4-Mile Line—453 Poles—Average Span 285 Feet—Line Put in Service 1939, Last Inspection 1941

Type of Flashover Evidence	Number of Poles on Which Evidence Was Found: Unguyed Pole Type 1 No.	Per Cent	Insulated Guys Type 1 No.	Per Cent	Vertical Corner Pole Type 2 No.	Per Cent	Total Poles No.	Per Cent
Lightning only	127	31	14	47	1	8	142	31
Power follow also	19	5	10	33	9	75	38	8
No evidence	265	64	6	20	2	17	273	61
Totals	411		30		12		453	
Per cent of flashed poles of each type showing power follow		13		42		90		

	Positive Critical Dry Flashover Values in Kilovolts: Unguyed Type 1	Guyed Type 1	Vertical Corner Type 2
Top conductor to lower conductor	1,100	1,100	935
Top conductor to ground	(3,000–4,000)	1,000	950

Character of Voltages Produced by Direct Strokes to Unprotected Lines

The data presented by the foregoing two types of study provide definite substantiation of the nature of the voltage produced by direct strokes as visualized from our present knowledge of stroke mechanism.[4] This is illustrated by Figure 6. Strokes to open ground or to relatively low structures such as transmission lines are initiated by streamers propagating from cloud to ground (Figure 6A) which lower charge at a relatively slow rate onto the stroke channel being formed. During this period the electric field at the ground is being increased at an accelerating rate. During the initial stages of this process charge of opposite polarity to that on the stroke channel can flow along the conductor into the region below the stroke channel neutralizing this effect and holding the conductors at ground potential. However, at some instant before stroke contact, the rate of change of field produced by the stroke exceeds the rate at which charge can be drawn in on the conductors, and the conductor potentials start to rise (Figure 6B). It is indicated[4] that such induced voltages may start to be appreciable from 50 to 250 microseconds before actual stroke contact and reach magnitudes ranging from 100 to at least 1,000 kv. Just before the start of the "return-streamer" process (Figure 6A), short upward streamers may occur from the conductor, tending to modify the voltage to varying degrees. However, the general character of the voltage is one of an accelerated rate of rise which would become greatest after stroke contact (at the instant of maximum rate of current rise on the front of the discharge-current wave) if flashover did not occur first.

For such lines as being considered here, the voltage easily can reach the insulation strength to ground of a guyed pole in the initial period before actual stroke contact. Being of a slow rate of rise, it will cause flashover at the lowest insulation point, even though it be several miles distant. After stroke contact, the much higher rate of rise also may produce flashover at or near the point of contact. Further, the initial breakdown of wood insulation usually is just inside the surface, resulting in very high arc drops that frequently permit the simultaneous flashover of a number or adjacent unguyed poles (sometimes ten of more). Thus strokes to such lines should set up several simultaneous discharges as shown by the climbing inspection and photographic-recorder data. Parallel arc paths are very unstable, and one will tend to rob the others of the arc. This should be the path with lowest rate of dielectric recovery as characterized by the length and type of flashover medium and the magnitude and duration of current it has conducted. It would be expected that any sustained arcs would tend to concentrate at the guyed poles with flashover paths producing the lowest arc impedance and rate of dielectric recovery. As shown by Table I, 50 per cent of the poles showing power-follow evidence were guyed poles that constituted only ten per cent of the total poles. If the number of individual cases of power follow could have been counted accurately, the percentage probably would have been still higher. As previously mentioned, the unstable character of simultaneous arcs usually results in the concentration not only of the power follow but also the long-duration portion of each stroke component at

Table II. Lightning Exposure and Performance of Unprotected 33-Kv Wood-Pole Lines

Line No.	Predominate Construction	Length, Miles	Number of Storm-Days 1942	1943	1944	No. of Separate Storms 1942	1943	1944	Total Storm-Hours 1943	1944	Tripouts Per 100 Miles Per Year 1942	1943	1944
A	Types 1, 2	23.8	27	20	25	29	27	31	31.5	40.5	80	38	13
B	Types 1, 2	10.2	16	19	18	17	24	21	25	25	49	29	39
C	Types 1, 2, 7*	58.9	22	27	36	32	32	42	42.5	78	78	61	73
D	Type 7*	27.1	17	31	28	20	35	35	43	43	44	44	15
E	Types 1, 2	10.6	18	21	42	21	28	47	33	21	38	66	21

* Type-7 lines with flat configuration, ten-foot crossarms, steel braces, and no guy insulation.[1] Critical breakdown voltages at tangent poles = 590 kv line-to-line, 3,000–4,000 line-to-ground. At guyed crossarm poles 325 kv line-to-ground, at vertical corner poles 600 kv line-to-line and 350 kv line-to-ground. For 4,099 mile-years (1929–32) average tripouts per 100 miles per year = 38.

Lines of types 1 and 2 construction (see Figure 1 and Table I). For 571 mile-years (1938–44) average tripouts per 100 miles per year = 31.

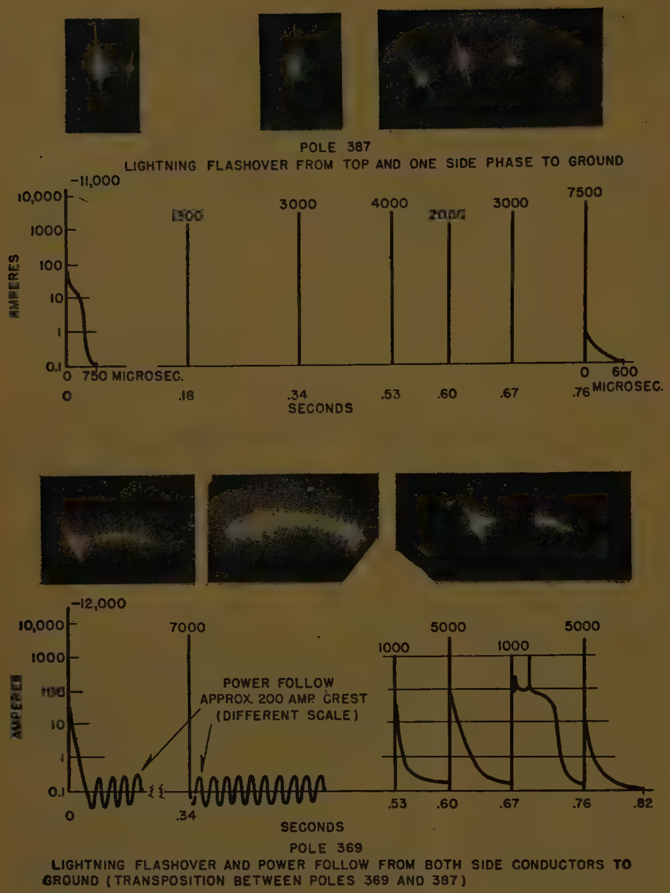


Figure 5A. Photographic surge-current record of multiple-lightning flashovers produced simultaneously at two recorder stations with power follow at one

Three tangent poles adjacent to pole 387 had lightning-flashover evidence. Photographic recorders were installed only in ground path. Distance between recorders is one mile. First component started power follow at pole 369 that apparently extinguished itself; second component started power follow extinguished by breakers. Additional components occurred after circuit was interrupted

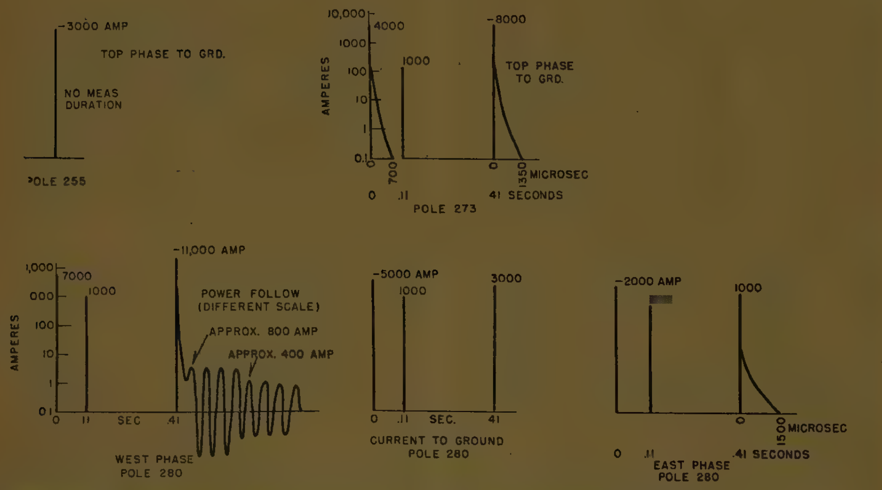


Figure 5B. Photographic surge-current record of lightning flashover produced at three recorder poles approximately one mile and one-half mile apart, by a three-component multiple stroke causing a phase-to-phase fault at pole 280

Recorders in ground path only at poles 255 and 273. Pole 280 showed minor pole and crossarm splintering indicating flashover from all three phases. All three insulators showed lightning flashover with power-follow marks on ridge and west phases

one location with only very short duration at the other flashed paths.

Character of Recorded Flashovers

For the total of 49 recorded cases of flashover 21 or 43 per cent caused a sustained power follow. For 35 of these the type of flashover at each pole could be identified. Of these the lightning flashovers at the recorder poles consisted of 18 single line to ground, eight double line to ground, and nine involving all three phases and ground. However, all but five involved lightning flashover between more than one phase at some point along the line. Fifteen or 40 per cent of the 35 caused a sustained power follow. All but three of these occurred at the guyed poles. Eight of the power follow cases were single line to ground, six double line to ground, and one line to line. Thus, although 30 out of 35 cases caused multiphase lightning flashovers, only seven or 20 per cent caused a sustained multiphase fault. In addition to the 15 out of 35 recorded cases of sustained power follow, one case of $3^1/_2$ cycles of power follow and one of one cycle of power follow were recorded (apparently self-extinguishing) together with a case in which sustained power follow was initiated after the third component discharge at one recording station while the same three-stroke components produced one, two, and one-half cycle, respectively, of power follow at an adjacent station.

It is of interest that the insulator inspection records of flashover at each recorder pole correlated quite well with the flashover-current records. In every case where only lightning-flashover currents were recorded, the insulator markings were interpreted as "lightning only." The same correlation was obtained for the cases of power follow except that no marked distinction could be made between the cases of sustained power follow, requiring operation of the eight-cycle breakers, and the power follow cases that lasted from 1 to $3^1/_2$ cycles. Also in one case power-follow evidence was not discernible at one of the two insulators shown by the recorders to have been involved in a sustained arc.

Storm Frequencies and Durations

Tables II and III list the data obtained on the number of storms, storm days, and total storm hours per year recorded over each of the unprotected and protected lines being investigated. These data were obtained from the storm charts prepared as illustrated in Figure 3. Correlated with the data are the number of tripouts per 100 miles of line per year and the number of strokes to the protected lines continuously equipped with magnetic links. It is of interest that although the ratio between storm-days, storms, and storm-hours varies considerably as does also the magnitude of each of these quantities from year to year and between different lines, the order of magnitude of all three is the same. No one of the above three indexes of storm activity appears to bear a closer relation to the frequency of direct strokes or line tripouts than the others. This indicates that the intensity of the electric storm or the density of direct strokes to ground is a variable of at least as great importance as the others.

Crest Stroke Currents

Before discussing in more detail the relation between the frequency of storms, direct strokes, and line tripouts, it would be best to examine the magnetic-link rec-

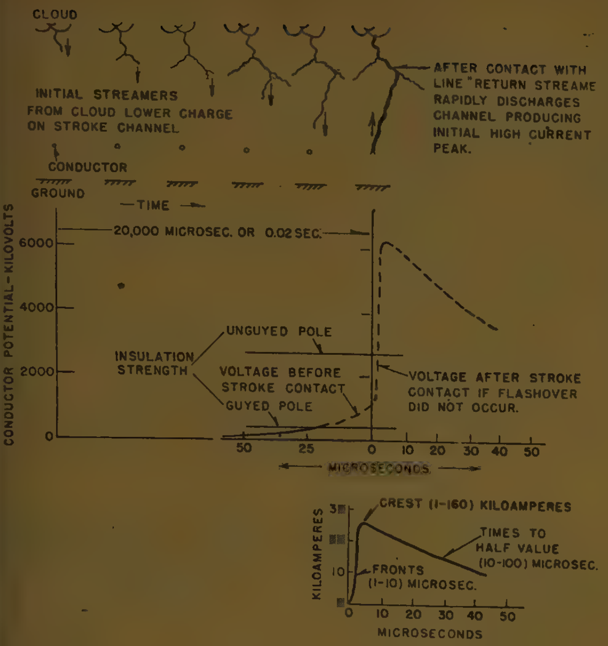

CLOUD
INITIAL STREAMERS FROM CLOUD LOWER CHARGE ON STROKE CHANNEL
AFTER CONTACT WITH LINE "RETURN STREAME RAPIDLY DISCHARGES CHANNEL PRODUCING INITIAL HIGH CURRENT PEAK.
CONDUCTOR
GROUND
—TIME →
CONDUCTOR POTENTIAL-KILOVOLTS
6000
4000
2000
0
20,000 MICROSEC. OR 0.02 SEC.
UNGUYED POLE
INSULATION STRENGTH
VOLTAGE BEFORE STROKE CONTACT
GUYED POLE
VOLTAGE AFTER STROKE CONTACT IF FLASHOVER DID NOT OCCUR.
50
25
0
10
20
30
40
50
← MICROSECONDS →
KILOAMPERES
10
CREST (1-160) KILOAMPERES
TIMES TO HALF VALUE (10-100) MICROSEC.
FRONTS (1-10) MICROSEC.
0
10
20
30
40
50
MICROSECONDS

Figure 6. Mechanism of the direct-stroke discharge by which relatively high voltage of slow rate of rise can be induced upon conductors of unprotected transmission lines before actual stroke contact

A (left). Voltage produced by direct stroke to conductor of unprotected 33-kv wood-pole line

B (below). Assumed stroke current and principal ranges of variation[2]

ords of pole-ground-lead currents upon which the stroke-frequency data are based. It is first necessary to consider the possible sources of surge currents that might produce a record. These are somewhat different for lines having an overhead ground wire than for lines equipped with protector tubes. As has been pointed out previously,[5] not only direct strokes but indirect strokes striking nearby can induce smaller magnitude currents in the ground leads. At least 90 per cent of all strokes to ground are negative which will induce positive currents in the ground leads. It was estimated[5] that for spans of 800–1,000 feet, induced currents as high as 5,000 amperes might be produced in a single tower of an overhead-ground-wire line. Similar estimates indicate possible induced currents as high as 1,500 amperes for spacings between ground leads of the order of 300 to 400 feet. For lines equipped with protector tubes higher currents can be produced, because such an indirect stroke will flash over a relatively few tubes and thus provide a limited number of paths to ground. Another important source for ground-lead-current records on lines equipped with tubes is the simultaneous flashover of a tube of lower breakdown voltage some distance away from the point of stroke contact due to the previously mentioned initial slow rate of voltage rise. Thus, an appreciable number of isolated pole-ground-lead-current records can be produced simultaneously with those close to the point of stroke contact. For the afore-mentioned reasons it is important to consider the crest-ammeter records carefully.

Such factors have been recognized to some extent in previous magnetic-link investigations on higher-voltage lines[6,7,8] and some sort of arbitrary method set up for throwing out certain records and not identifying them with direct strokes. However, in plotting the results of such previous studies it has been customary to plot percentage distribution curves showing the per cent of records exceeding a given magnitude. When this is done the character of the resulting curve depends to a great extent upon the accuracy of the judgment used and the number of records eliminated. A more fundamental way to approach this problem is to plot the total number of records (or the number per 100 miles of line per year) that exceeds each magnitude. The resulting curve will eliminate the afore-mentioned judgment factor except for the lower range in which other sources can produce records. This is important, as it is the higher-magnitude currents that are most likely to cause a flashover. Curves plotted in this way from this investigation are shown in Figure 7A together with a similar analysis of the Hansson and Waldorf data[6] in which all records thought by them to be direct strokes were used. Figure 7B presents a comparison of the more generally used percentage distribution curves from this and other investigations. The curve of Figure 7A for 33-kv lines with an overhead ground wire shows more high-current records than either the 33-kv protector-tube-line data or the steel-tower-line data. This same trend is shown for the individual maximum crest pole-ground-lead currents associated with each stroke. It was not found that any significantly greater number of grounds on the overhead-ground-wire lines tended to carry current than for the protector-tube lines. This indicates that more of the higher-current strokes actually were recorded on the ground-wire lines. This curve is based upon only 66 records, and for this reason a definite conclusion cannot be reached. However, it is of interest that an analysis of the data obtained by Hansson and Waldorf[6] from different lines indicates curves agreeing quite closely as long as about 100 records were available for each curve.

The factors which might affect such curves may be summarized briefly as follows: Because of the traveling-wave effect the sum of the crest tower currents should be greater than the actual crest stroke current. The difference will be greater the shorter the wave front and the longer the span. Further the sum of the crest tower currents is the sum of the respective maximum components as produced by several component discharges (in the case of multiple strokes) which

Figure 7A. Probability curves showing the number of recorded stroke currents per 100 miles of line per year exceeding a given crest magnitude

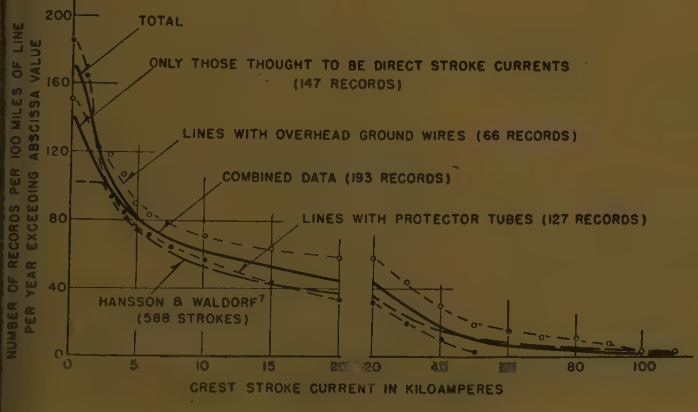
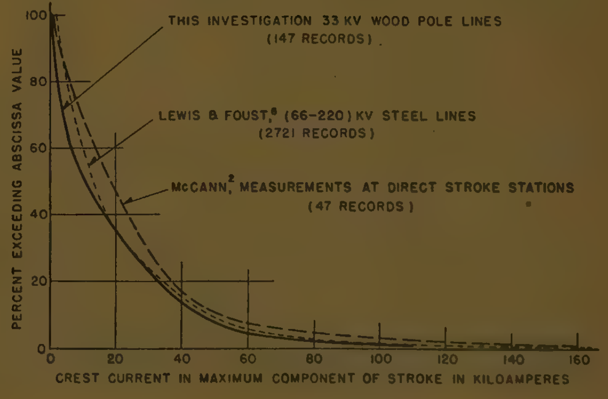


Figure 7B. Percentage distribution curves of direct-stroke crest currents



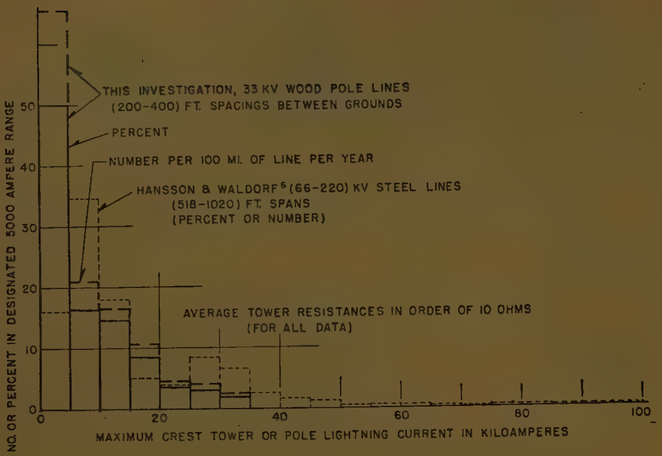


Figure 8A. Distribution curves of maximum crest tower or pole-ground-lead currents produced by direct strokes

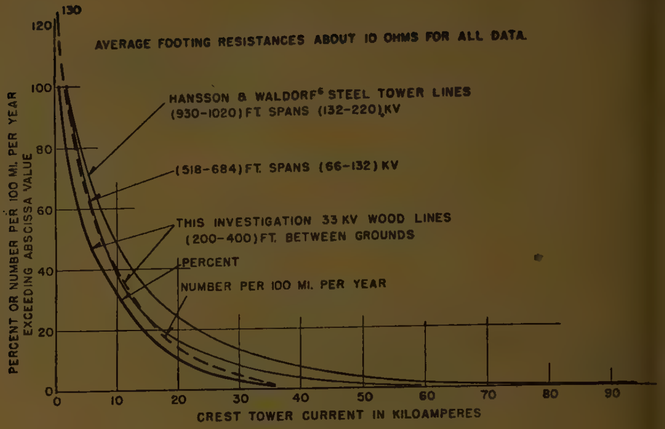


Figure 8B. Distribution curves of maximum crest tower or pole-ground-lead currents produced by direct strokes

might not bear the same proportionate relation to each other and cause the data to be high. Further, the effect of several strokes in some cases may be considered as one. On the other hand, magnetic-link measurements on steel-tower lines will indicate less than the total tower current, as no account usually is taken of the cross-brace currents.[5] There is one more very important effect that might account for the higher stroke currents on the 33-kv line with ground wires as compared to the 33-kv protector-tube-line data. The availability of the charge from ground that must combine with the stroke charge can be appreciably greater for a line with overhead ground wires. Then, there is a large number of paths provided by each ground lead, whereas for the tube line the only paths are those provided by the tubes that flash over. The possibility of more rapid combinations of the charges for the former case might result in a prevalence of higher crest currents. Although this effect cannot yet be definitely verified from the data so far obtained, it is indicated.

The higher number of total records (or low-magnitude records) obtained on lines with protector tubes is undoubtedly due to the two factors mentioned previously, induced surges, and simultaneous flashover. For instance, of 127 records to the tube lines, 57 were isolated surges involving only a single pole ground. Fifty per cent of these were positive. Of the 66 records from the ground-wire lines only eight (75 per cent positive) were of this nature. This latter would be expected, as they should be caused predominantly by indirect strokes. In order to form a basis for eliminating all but those records thought to be caused by direct strokes, all isolated tower-current records (involving only one pole) below 1,500 amperes for overhead-ground-wire lines and below 5,000 amperes for protector-tube lines were arbitrarily removed from the data. This produced the lower solid curve in Figure 7A.

Frequency of Direct Strokes

The curve obtained in this way gives an average of 130 direct strokes per 100 miles of line per year. Hansson and Waldorf[4] estimate a general average of 100 strokes per year from their studies on steel lines. However, as shown by the curve of Figure 7A their data represent the number exceeding about 2,500 amperes (the lower limit of their recorders). This point lies very close to the corresponding point on the combined curve of this investigation, which indicates that they also would have measured more actual direct strokes if conditions had permitted the recording of lower-magnitude direct strokes. On the other hand, when comparing the percentage distribution curve obtained by the foregoing method of removing certain records with the curve obtained in the Westinghouse direct-stroke investigation (Figure 7B) it is seen that a higher percentage of lower-magnitude records is still present in the data of this investigation. The direct-stroke study shows that about ten per cent of all direct strokes are below 2,000 amperes and 16 per cent are below 5,000 amperes, whereas the curve of this study shows corresponding percentages of 15 and 30. Adjustment of the data of this study to fit the direct-stroke data in this range would produce a curve in Figure 7A that would indicate about 95 direct strokes per 100 miles of line per year. It appears evident that, with a possible range of from 95 to 130, the frequency of strokes to the wood-pole lines (which are 35 to 50 feet in height and lie in isokeraunic regions of 25 to 45 storm-days) is about the same as for steel-tower lines 80 to 100 feet high. It should be pointed out that the lower-voltage lines of this investigation are in flat exposed terrain and not appreciably shielded by surrounding objects.

Table III. Lightning Exposure of Protected 33-Kv Wood-Pole Lines

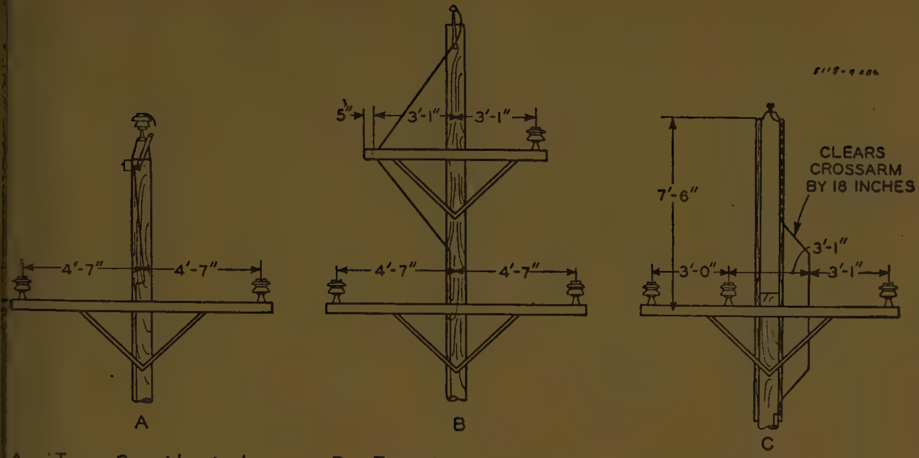
							Normal Tripouts From Lightning	
Line	Mile-Years	Storm-Days Per Year	Storms Per Year	Storm-Hours Per Year	Total Direct Strokes	Total Tripouts	Number	Number Per 100 Miles Per Year
Lines With Overhead Ground Wire (see Figure 9)								
F Type 4	21.9	25.3	30	43	21	0	0	2.3
G Type 4	9	23	28.5	33	19	1	1	
H Type 5	13.5	19.7	28.7	22	13	0	0	
					53	Average density—120 per 100 miles per year		
Lines With Single-Phase Protector Tubes (see Figure 9)								
I Type 3	13.8	19.7	28.7	22	24	2*	1	3.7
J Type 3	36.3	27	39	27	49	0	0	
K Type 3	3.4	23	25.3	33	11	1	1	
					84	Average density—136 per 100 miles per year		
Lines With Flat Construction, Three-Phase Tubes, Staggered Every Third Pole Per Phase (see Figure 10)								
L Type 6	8.5	23	25.3	33	10	2	2	43

Average number of strokes per 100 miles of line per year $= \frac{147}{112.7} = 130.$

*One occurred at end of line where it connects to an unprotected line. Pole ground resistances range from 3 to 20 ohms with an average for each line of approximately 10 ohms.

Stroke Polarity

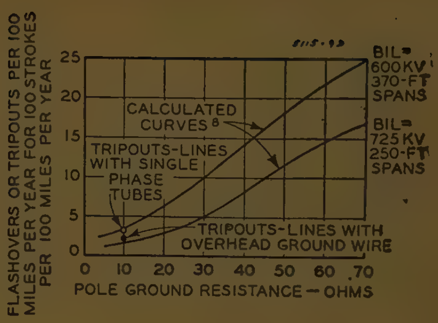
Of the 147 records considered as caused by direct strokes, 134 or 91 per cent were of negative polarity. The highest-magnitude recorded positive stroke had a



A. Type 3 with single-phase tubes
BIL 725 kv
370-foot average spacings between grounds
Average spans 185 feet

B. Type 4
BIL 725 kv
250-foot average spacings and spans

C. Type 5
BIL 600 kv
320-foot spacings and spans



D.

Figure 9. Comparison of actual and calculated performance of three types of 33-kv protected line construction employing an overhead ground wire or deion protector tubes on a top phase

crest current of 90,000 amperes and the highest negative stroke 113,000 amperes.

Pole-Ground-Lead Currents

Figure 8 presents the data on the maximum pole-ground-lead currents for each record identified as a direct stroke and compared with similar data of Hansson and Waldorf.[6,7] In these figures the data are plotted in two ways showing both the per cent and the number of tower currents per 100 miles of line per year corresponding to the given values. Since the average recorded direct strokes for the Hansson and Waldorf data is 100 per 100 miles per year, their curves for the two ordinate systems coincide. These curves show the benefit of short spacings between spans in reducing the maximum tower current. The highest-pole ground lead current recorded for the 33-kv lines of this investigation is 36,000 amperes as compared to a maximum of 95,000 amperes for the steel-tower lines with longer spans.[6] As shown by Figure 8B the maximum pole-ground-lead current produced by five per cent of the strokes to the wood-pole lines with the shorter spans is 26,000 amperes whereas the corresponding value for the 500–700-foot-span steel lines is 38,000 amperes and for the 900–1,000-foot-span lines 49,000 amperes.

Flashover Performance of Unprotected Lines

At the bottom of Table II is listed a general summary of the tripout performance of two types of unprotected line construction. The first type consists of flat construction using ten-foot crossarms and steel braces typical of installations prior to 1929.[1] The second type of construction is that of Figure 1 with the insulation levels listed in Table I. For both types of construction the average frequency of guyed poles is about once every half to one mile with spans of 150 to 300 feet. As shown in Table II, the flat construction experienced an average of 38 tripouts per 100 miles per year as compared to only 31 for the construction of types 1 and 2 (Figure 1). It is indicated that this improvement in performance is due partly to a reduction of lightning flashovers because of the higher insulation level but that a greater effect is the improvement of the dielectric-recovery characteristics of the line with greater wood insulation.

It has been recognized for some time that a path over wood has a greater "arc-quenching" action than through air or over insulators alone. Studies of the performance of higher-voltage steel-tower lines[5,9] show that about 85 per cent of the flashovers of lines 100 miles in length or less result in a tripout. Calculations from present direct-stroke data[2,9] (including the crest-stroke-current curve (Figure 7A) of this investigation) indicate that lines of the earlier flat construction should experience about 91 lightning flashovers per 100 miles per year, whereas the construction of Figure 1 should experience 88. From the foregoing tripout data the respective percentage ratios of tripouts to flashovers are thus 42 and 35. This latter figure compares with 40–43 per cent as determined from the photographic-surge-current-recorded studies previously described.

The concentration of cases of power follow at guyed poles suggests the possibility of materially reducing the outages on lines of Figure-1 construction by installing deion protector tubes either at or adjacent to all guyed poles. An investigation program of this type is planned. For best protection tubes should be installed on all three phases since, as shown by the special flashover study, most strokes to such lines result in multiphase lightning flashovers, even though the ridge-pin conductor shields the other two.

Performance of Continuously Protected Lines

Table III summarizes the lightning performance of the protected lines being studied in detail. Only one tripout and case of flashover evidence has occurred for an estimated 53 direct strokes to the lines with an overhead ground wire. The type-3 (Figure 9) construction with single-phase protector tubes has experienced three tripouts for 84 direct strokes. However, Line 1 (Table II) is connected to an unprotected line section, and one of the flashovers occurred at the end pole of the protected section which is also a transposition point. This, therefore, was not attributed to the protected construction. Both of the other two tripouts were associated with tube failures. Thus, the ground-wire lines averaged one tripout per 53 strokes as compared to one per 42 strokes for the single-phase tube construction.

Figure 9 gives the actual number of tripouts per 100 miles of line per year for the combined lines of the two aforementioned general types as compared to calculated performance[2,9] using the stroke-current wave-shape data of direct-stroke studies[2] and the crest-stroke-current frequency curve of Figure 7A. The character of the lightning voltages on an overhead ground wire or a conductor equipped with protector tubes is quite different from that appearing on an unprotected phase conductor.[9] The multiple grounds provided continually for the grounded wire and after a relatively low voltage has been reached by the tubes prevent any significant voltages occurring any appreciable time before stroke contact. Thus the voltages on such conductors are determined primarily by the stroke discharge current. Surprisingly close agreement is shown by the calculated and actual data of Figure 9, indicating that when the actual frequency of direct strokes is known our present knowledge of the characteristics of lightning-stroke currents[2,5,9] provides a reliable basis for determining line perform-

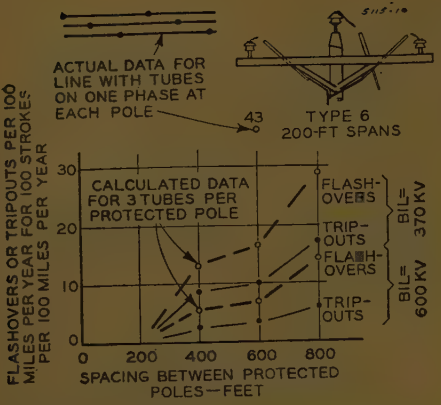


Figure 10. Calculated lightning performance of 33-kv line equipped with protector tubes on all three phases showing effect of spacing between protected poles and insulation at unprotected poles

Also comparison is made with operating performance of line with one tube per pole rotated between phases

ance. As shown in Figure 9, the somewhat higher outage rate for the tube construction is due to its lower insulation level and longer spans as compared to the ground-wire lines. By separating the pole ground lead from the pole,[9] it is quite practical to reduce the flashover rate for this type of construction by about one half. It should be pointed out that the calculated curves apply to flashovers whereas the operating records are tripouts. However, a detailed line-inspection program has been carried out on these lines which so far has detected no flashovers other than those resulting in the tripouts. These inspections are made periodically at all locations shown by the magnetic-link records to have received a direct stroke. That the application of protector tubes only on the ridge conductor of lines such as that of type 1 (Figure 1) provides both an effective and economical method of protection had been shown by previous experience.[10]

The second type of protector-tube application studied in detail is that shown in Figure 10 with a single tube at each pole but the tubes rotated between phases. As shown by the operating data for this construction (Table III) the performance has not been so good as for the other protected lines. For only ten direct strokes there have been two tripouts. One of these flashovers occurred at a switch pole near one end of the line section. The poorer performance is due primarily to the lower insulation level that results from the tube grounds at each pole. This is illustrated by Figure 10. The performance of such construction should be comparable to that with all three tubes installed at every third pole and the same phase-to-phase insulation level at the unprotected poles as provided by the staggered construction at each pole. In Figure 10 are curves showing the calculated[9] performance for three tubes at each protected pole showing the expected flashovers and tripouts as a function of frequency of protected poles and for two insulation levels at the unprotected poles. The upper set of curves applies to insulation levels typical of that provided by the staggered tube arrangement. The lower curves apply to the insulation level obtained with the given pole construction and no grounding circuit at the intervening poles. The curves applying to tripouts are estimated from the data of this investigation and others[9] on the ratios of outages to flashovers. The point representing the actual tripout rate for the staggered tube line shows 43 tripouts per 100 miles of line per year as compared to an estimated 17 flashovers and 12 tripouts, whereas, if the tubes actually had been placed three to a pole every third pole, the estimated flashover rate would have been only seven with four tripouts per 100 miles of line per year.

Duty on Deion Protector Tubes

The records of the lightning surge currents discharged by deion protector tubes are summarized in Figure 11. Two general types of application were studied. One consists of tubes applied continuously on the lines of Table III for which the bottom curve of Figure 11 applies. For the other type of application isolated groups of two to four poles adjacent to a vertical corner pole are equipped with one tube on the ridge-pin conductor. For both curves of Figure 11 all recorded surges are considered, although several tubes might be involved in one disturbance. The much greater tube duty for isolated applications is shown. Each protector tube at the infrequent installations discharged about once per year as compared to a frequency of 0.35 discharge per tube on the continuously protected lines. The tube duty should be considered from two viewpoints, one the duty imposed by high lightning currents and the other by the interruption of fault current. As shown by Figure 11 the tubes at isolated locations discharged about 12 times as many of the higher lightning currents. The actual duty from interrupting fault current depends upon the number of tubes involved for each disturbance. This is not known. In most cases it should be only one. Upon the assumption that one tube interrupts a fault current for every case of flashover, the average number of cases of fault-current interruptions is 0.7 per tube per year for the isolated installations and only 0.17 for the continuously applied tubes; a ratio of about four to one. These differences should be even more marked if cases of single isolated tube-equipped poles were considered. The foregoing data show the material benefit of closely spaced tubes in reducing the individual tube duty and maintenance. After 916 tube-years of experience with lines continuously protected by about 320 tubes, there have been three tube failures.

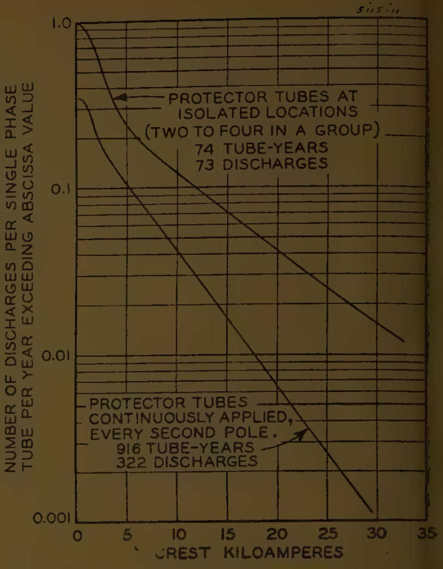


Figure 11. Surge-crest-ammeter records of the discharge duty on deion protector tubes installed on 33-kv wood-pole transmission lines

Conclusions

An analysis of the electric fields produced by the "initial streamers" of direct strokes to an unprotected transmission line indicates that surge voltages of appreciable magnitude can be induced on a conductor as much as 50 to 250 microseconds before actual stroke contact. These may reach potentials ranging from 100 to over 1,000 kv before the stroke actually discharges into the line. This is confirmed by a detailed climbing inspection of an unprotected 33-kv wood-pole line and by 49 recorded cases of flashover on sections of unprotected line equipped with photographic surge-current recorders. Having relatively slow rates of voltage rise, these "initial streamers" can flash over points of lower insulation several miles from the stroke, after which higher insulation close to the stroke also may be flashed over by the much higher rates of rise following actual stroke contact.

In the special flashover studies simultaneous multiple flashovers were recorded that extended over as much as two miles of line. In some cases they were caused by multiple strokes, and each component of the multiple discharge was recorded at several of the flashover points. In these cases one path usually (but not always) robs the others of the long-duration por tion of each stroke component so that the flashovers at the other points along the line are of very short duration with n evidence of power follow except at th one location.

About 40 per cent of the recorded case

of flashover caused a tripout. Although about 85 per cent produced a multiphase flashover at some point, only 20 per cent resulted in a sustained multiphase fault. The concentration of power follow at guyed poles shows that they determine to a great extent the tripout performance of such lines.

Examination of the flashover markings produced on 34.5-kv pin-type insulators and correlated with measurements of flashover currents shows that the character of the markings provides a method of detecting flashover from lightning alone and flashover accompanied by power follow.

Magnetic-link studies on continuously protected 33-kv wood-pole lines 35 to 50 feet high show that the direct-stroke frequency is of the order of 95 to 130 per hundred miles of line per year or the same order of frequency as higher-voltage steel-tower lines 80 to 100 feet high lying in regions of about the same lightning severity. Storm observations at substations provide a method of plotting storm maps for each lightning storm over the area covered by the lines being studied. Detailed records of storm frequencies and durations show a wide fluctuation in the number of storm-days, storms, and total annual storm durations both from year to year and between different lines only a few miles apart. As shown by the storm maps, this is caused by large local variations in the extent and paths of the storms. No one of the three indexes bears a close relation to the frequency of direct strokes to the line. This is probably due to the importance of an additional factor, the density of strokes emanating from each storm.

It was found that a more accurate and fundamental method of plotting crest stroke currents and tower currents is in terms of the number per unit length of line per year rather than the per cent that exceed a given value. Because of the uncertainty of the lower-magnitude surges the latter method can produce considerable error in the frequency of the more important higher-current records. There is an indicated prevalence of higher-magnitude strokes to lines equipped with overhead ground wire. A possible explanation for this is the better conducting path provided by the greater number of grounds in the case of a ground-wire line, so that the earth charges may combine more rapidly with the charge on the stroke channel. Comparison of the crest-pole ground lead currents recorded in this investigation (where average spacings between grounds range from 250 to 400 feet) with similar data from high-voltage lines (having spans ranging from 500 to 1,000 feet) shows the definite benefit of closely spaced grounds in reducing the maximum pole or tower currents.

Comparison of lightning-tripout records for unprotected 33-kv wood-pole lines of limited insulation and lines with about the optimum insulation for this voltage class (see Table II and Figure 1) shows an average rate of 38 tripouts per 100 miles of line per year for the former as compared to 31 for the latter.

Detailed data have been obtained on the lightning performance of 33-kv wood-pole lines continuously protected both by overhead ground wires and by protector tubes. Although only a relatively few mile-years of experience have been gained on each type of construction, the detailed nature of the records showing actual stroke frequencies together with line tripouts have established the degree of performance of the various types of construction. Lines equipped with an overhead ground wire and having insulation levels of 600 to 725 kv with spacings between grounds ranging from 250 to 320 feet (Figure 9) experience only about two tripouts per 100 miles of line per year when an average ground resistance of ten ohms is provided. The same degree of protection is shown to be provided by lines with one protector tube on the ridge-pole conductor of every other pole of lines with average spans of about 200 feet (Figure 10) if the same insulation level is provided. The performance record of a line with flat construction (Figure 10) having one tube per pole on alternate phases shows the importance of obtaining a reasonably high insulation level for unprotected phases at each pole. This line shows an outage rate of 43 tripouts per 100 miles of line per year, whereas the use of three tubes per pole on every third pole should result in only about four tripouts per 100 miles of line per year when the full insulation level of the unprotected poles is maintained.

Data have been obtained on the surge discharge duty of deion protector tubes installed continuously on protected lines and at isolated points on lines. For the former case the discharge rate for lightning surges is about 0.35 per individual tube per year, while the frequency of power-fault interruptions is only about 0.17 per individual tube per year. The corresponding rates of discharge are much higher for isolated tube installations. Likewise, the crest magnitudes of the surges that must be discharged at isolated installations are much greater.

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Control of Load, Frequency, and Time of Interconnected Systems

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Synopsis: The general problem is stated and typical solutions indicated. Characteristics of governors and of frequency- and load-control devices are briefly explained, and effects on generator and tie-line loads are summarized. Illustrating the principles enunciated, interconnected systems in southern Ontario and adjacent portions of Quebec totaling 1,650,000-kw generating capacity are described, also operating experience with automatic frequency control, and automatic load control of two ties, one of which is a 45,000-kw frequency-changer set interconnecting systems of relatively large capacity.

THIS IS the era of superpower. Two or three decades ago, engineers envisioned the day when the power resources of whole countries or even continents would be interconnected by vast networks of high-voltage transmission lines distributing electric power to all parts of the country. Today their dreams are approaching realization in large areas of the United States, in Britain,[6] and to a lesser degree in Canada and other countries.

On this continent these power networks or power pools as they are sometimes called, have grown up piecemeal by progressive interconnections within local areas, and finally by ties to more distant areas, which in turn may be connected to other areas beyond them. One of the earliest problems encountered in the operation of interconnected systems was the regulation of the transfer of power between one system and another, and closely related thereto, the regulation of frequency. A by-product of frequency control is time control, which demands precise *average* frequency, so that synchronous electric clocks may keep pace with the slow but inexorable advance of Father Time. These related problems are dealt with in this paper.

Purposes of Interconnection

At the outset, let us consider briefly the questions—why interconnection? and why control? Interconnections may be made for one or more of the following reasons:

1. Transfer of power from systems having a surplus to areas of shortage.
2. Diversity of loads on different systems.
3. Diversity of stream flow and storage facilities, affecting hydroelectric generation available in different areas.
4. Pooling of reserve generating facilities, with large savings in capital cost of stand-by generating capacity.
5. Greater reliability of service, and resources for emergencies.
6. Convenience in arranging outages of equipment for maintenance.

For example, water which would otherwise be spilled over the dam at some run-of-river plant might be used to permit storage of water at some other plant where storage facilities are available, which in effect transfers useable energy in the form of ponded water from one river to another which may be many miles distant; or alternatively, that water may be used to save expensive coal at some distant steam plant; or conversely, steam generation may be used to help out water-power areas in dry seasons. Diversity of loads and pooling of reserve capacity by interconnection make it possible to carry a greater total firm load on the combined systems than they could carry separately.

Control of some kind is necessary in order that the power transferred from one system to another over the interconnecting tie lines may conform to plans based on economic or contractual considerations. Tie-line load control is bound up with the control of frequency and time, since no one of the three may be changed without affecting the other two. Apart from their relation to load control, frequency and time demand close regulation on their own account; frequency, because many modern industrial processes with electric motor drives are dependent upon steady and accurate frequency; and time, because of the millions of electric clocks which time the movements of modern society to an accuracy measured in seconds. Some industries wish frequency to be controlled to a tolerance of plus or minus one quarter of one per cent on one-minute average values. This is not always attainable, but on the Southern Ontario system of the Hydro-Electric Power Commission of Ontario, that precision is attained for more than 98 per cent of the average day's run, using automatic frequency control. For the sake of electri clocks, the average frequency must b maintained even more precisely, since a error as small as 1/100 of one per cent wil in 24 hours create a time deviation o eight seconds on electric clocks. Time o the Commission's Southern Ontario system is usually controlled more precisel than this, but emergency conditions an heavy war-time demands occasionall cause temporary time deviations up t five seconds and sometimes more. Thes errors are automatically corrected over period of time after the emergency o overload condition has passed.

On interconnected systems, automati frequency control is now almost universa and automatic tie-line load control is becoming more and more extensively used Where tie-line capacity or stability limit

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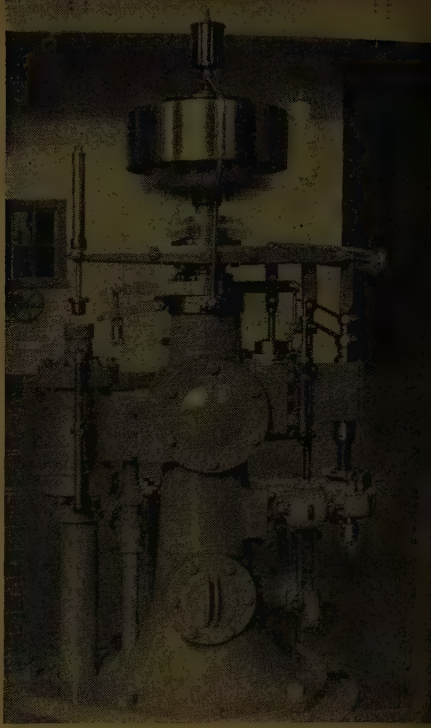


Figure 1. Governor of older type in use a
Queenston plant

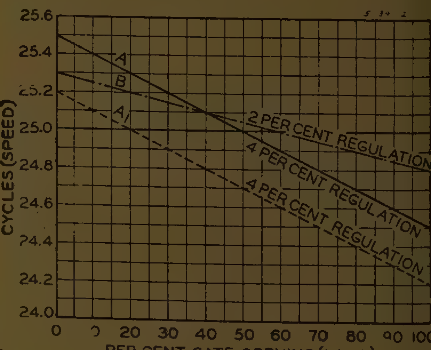


Figure 2. Characteristic curves of turbin governors, showing the relation between ga opening and speed

are critical factors, automatic tie-line load control is essential for the maximum utilization of line capacity.

A recent article[13] describes an interesting example of interconnected systems in the northwestern states, where 11 important power systems in five states have a total of $3^{1}/_{8}$ million kw generating capacity with 13,000 miles of major transmission lines ranging from 40 to 220 kv. This interconnection takes advantage of diversity of loads in two time zones and diversity of river flow in several watersheds on both sides of the Rocky Mountains, the resulting economies being equivalent to 100,000 kw of added generating capacity.

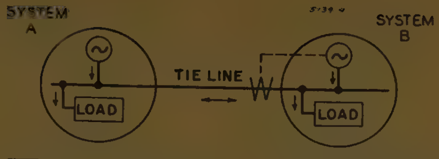
In this power pool we have examples of the two types of interconnection which are commonly met:

1. Systems connected like links of a chain, which may have side branches but no loops.
2. Multiple interconnections, forming loops.

One particularly complicated situation was solved by the unique method of phase-angle control,[3] but the usual problems were met by conventional methods of automatic frequency control and tie-line load control referred to later in the present paper.

Governor Characteristics

Regulation of frequency and tie-line load involves the regulation of the output of generators connected to the systems, and this requires regulation of the power input to their turbines. In a typical hydroelectric plant, the working fluid, water, is conducted from the forebay through the penstock to the turbine, and is discharged through the draft tube to the tail race. The discharge is controlled by the gates, consisting of a set of overlapping vanes arranged in a circle in the turbine casing at the point where the water enters the wheel. The gate stems pass through the upper wall of the turbine casing and all are connected to a gate ring, which is moved by servo-motor pistons under control of the governor.



1	2	3	4	5	6	7	8	9	10
LOAD CHANGE	FREQ.	T.L.L. (1)	F.F.C A	T.L.L. (2)	F.T.L. B	T.L.B B	SELEC. FREQ A	SELEC. FREQ B	T.L.L (3)
+A	LOW	←	R	0	L(R)	L−R	R	—	0
+B	LOW	→	R	→	R	R	—	R	0
−A	HIGH	→	L	0	R(L)	R−L	L	—	0
−B	HIGH	←	L	←	L	L	—	L	0

T.L.L.=TIE-LINE LOAD. F.T.L.=FLAT TIE-LINE LOAD CONTROL
F.F.C.=FLAT FREQUENCY CONTROL. T.L.B.=TIE-LINE BIAS CONTROL
S.F. (COL. 8, 9) = SELECTIVE FREQUENCY CONTROL.
R= RAISING; L= LOWERING (ADJUSTMENT TO GOVERNORS).

Figure 4. Schematic diagram of two systems interconnected by a single tie line

Tabulation shows effect of load change on frequency and tie-line load due to governor droop characteristics only (columns 1, 2, and 3), followed by the effect of flat frequency control only on system *A* (columns 4 and 5); same plus flat tie-line control on *B* (column 6), or tie-line bias control on *B* (column 7); or selective frequency control on *A* and *B* (columns 8 and 9)

Figure 1 shows one of the governors of the Queenston plant—not of the most modern make, but well adapted to an explanation of its operation, because its principal parts are large and exposed to view. The flyballs are on a vertical shaft, belted to the main generator shaft. The balls fly outward with increasing speed of the unit, and in so doing they raise a collar to which the center of the floating lever is coupled by a sliding shoe. The left end of the floating lever is connected to the pilot valve, which controls the turbine gates through the medium of the servo motor. The right end of the floating lever is positioned by a rod coming back from the gates. This mechanical construction determines a fixed relation between the steady-state speed and gate opening, which is called the speed-droop characteristic of the governor.

One other feature which should be noticed is the handwheel at the lower right and near it a small motor, called the synchronizing motor, which is geared to the handwheel. The synchronizing motor is used by the operator when starting the unit to adjust the generator to the proper speed for synchronizing to the bus, and after synchronizing, the same motor is used for adjusting load on the unit, or for regulating frequency of the system.

Automatic load or frequency regulators act through the medium of governor synchronizing motors. A change in the governor speed setting by means of this motor has the same effect as lengthening or shortening the gate return rod, causing a change in the speed associated with a given gate opening.

The relation between speed and gate opening is shown graphically in Figure 2 by the speed-droop curves of a governor. The base line is actually gate opening, starting from the no-load gate opening as zero, but it can also represent output or load on the unit, assuming that output is directly proportional to gate opening. This is not usually the case, but it will simplify our explanations to assume that it is true, and that our base represents per cent of full gate or per cent of full load.

As applied to a single isolated generator, curve *A* means that at no load, the frequency will be 25.5 cycles per second (hereinafter usually abbreviated to "cycles"). As load increases, the governor holds a lower frequency, which at

Figure 3. Frequency, and load of two Queenston generators, one (number 10) controlled by a frequency regulator and the other (number 3) on free governor control

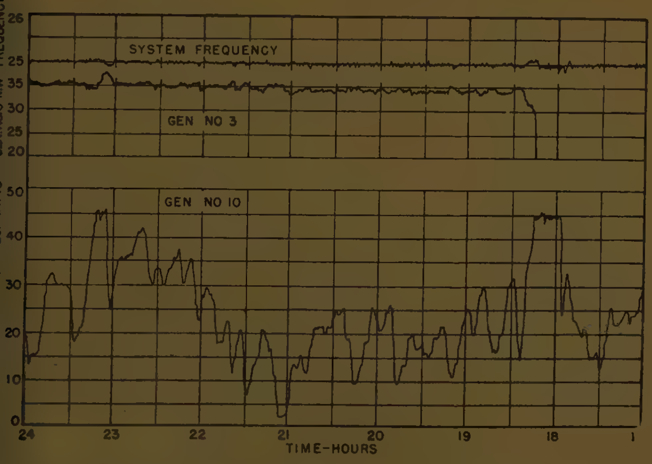
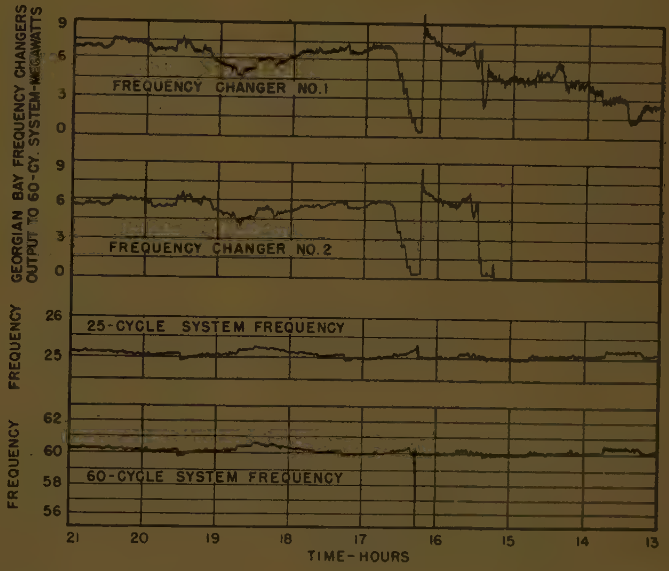


Figure 5. Fluctuating load on two frequency-changer sets in parallel constituting the tie between the Georgian Bay 60-cycle system and Niagara 25-cycle system

Frequency was manually controlled on the Niagara system and the frequency-changer load was unregulated



half load will be 25 cycles. Further increase in load causes further reduction in speed, till at full load the frequency is 24.5 cycles—a total droop from no load to full load of one cycle on 25, or four-per-cent regulation.

When generators are operating in parallel, as on interconnected systems, frequency is the same for all generators, no matter whether they are side by side in the same generating station, or miles apart in different stations. Hence this speed-droop characteristic provides a basis for load sharing among generators operating in parallel, for example, two generators having speed-droop curves like *A* and *B* in Figure 2. All may not have the same per-cent regulation or the same full-load capacity, and these factors will affect the amount of any system load change which will be taken by each generator; but the main point is that generators in parallel tend to share all changes in system load due to their governor speed-droop characteristics, provided that no changes are made to the governor speed settings.

Effects of Frequency Control

In the early days of electric power, when frequency was not so important as it is now, the governor speed-droop characteristic was satisfactory, both for its main purpose as a basis of stable load sharing, and also for a crude sort of frequency control. Now, when frequency must be precisely controlled, the governors must be adjusted from time to time to compensate for changes in system load. Instead of making simultaneous equal adjustments to governors of all generators on the system, it is more convenient to adjust one or a few large generators, which are called the frequency-regulating generators of the system.

The effect of making an adjustment to the speed setting of a governor by means of the synchronizing motor shown in Figure 1 is to raise or lower the speed-droop curve parallel to itself. For example, curve *A* may be lowered to A_1, which would reduce the output of generator *A* and lower the frequency of the system. If frequency of the system is being regulated by adjustments to generator *A*, with no adjustments to generator *B*, then generator *A* will be compelled to take care of all load changes of the system, because when frequency is restored to 25 cycles, generator *B* will always be at the 60-per-cent load point. In other words, frequency control has the effect of counteracting the speed droop of generators to which it is applied—not eliminating the drooping characteristic, but shifting it to a new level after each load change.

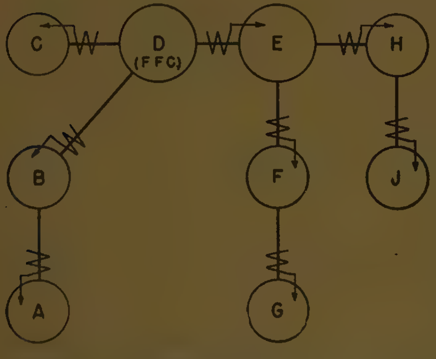


Figure 6. Schematic diagram of nine systems interconnected like links of a chain which may have side branches but no loops

Flat frequency control on system *D*. Flat tie-line or tie-line bias control on others

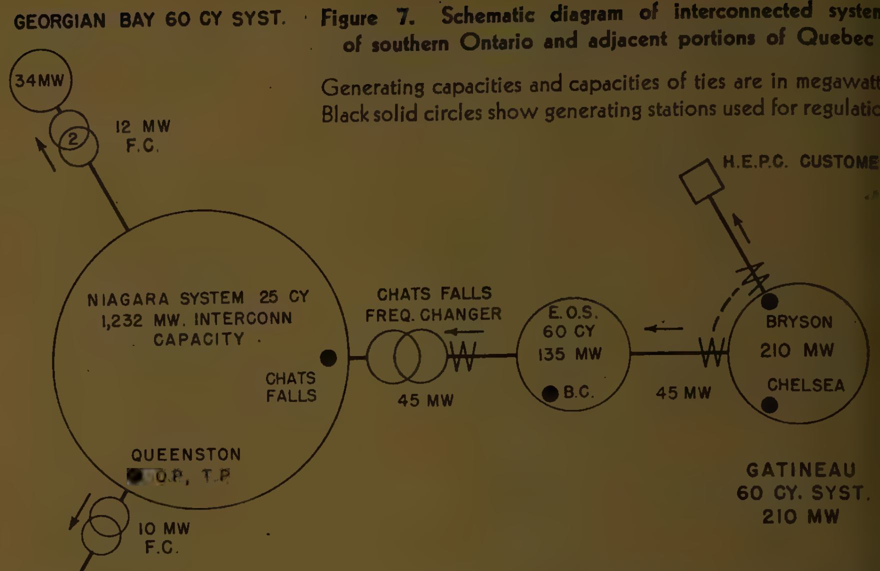


Figure 7. Schematic diagram of interconnected system of southern Ontario and adjacent portions of Quebec

Generating capacities and capacities of ties are in megawatt Black solid circles show generating stations used for regulati

When load change occurs suddenly, there are three stages in the restoration of normal frequency. In the first stage, inertia of the generator and of other rotating machinery connected to the system is the governing factor, and so the frequency falls for the first few seconds following a sudden increase of load, drawing upon the total kinetic energy of the system, while water in the penstocks is accelerating. In the second stage, the incremental load is shared by all generators according to their capacities and speed-droop characteristics. Finally, the whole of the load change is taken over by the frequency-regulating generator,* and other generators return approximately to their original loading when frequency has been restored to normal. On sudden load changes of considerable magnitude, all three stages can be distinguished on graphic meter charts, but when load changes are gradual, the first two stages are hardly noticeable, and the result is that the frequency-regulating generator* tends to vary its output with all changes i system load, while other generators maintain approximately constant output.

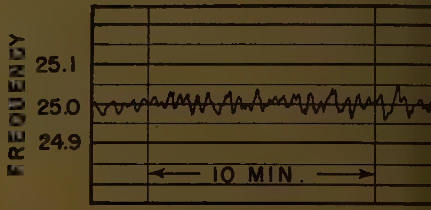


Figure 8. Uncontrollable fluctuations o Niagara system frequency (1936) showin irregular swings with a period averaging abo 25 seconds

This is illustrated in Figure 3.

Time runs from right to left. Th lower curve shows the fluctuating load o generator 10 at Queenston when it wa being used as a frequency-regulatin generator for the Niagara system, wit automatic frequency control. A littl before 1800, a sharp increase in syste load depressed the frequency temporaril and caused the output to increase to fu load, in response to raising impulses fro the frequency regulator. The frequenc was slightly low for 20 minutes until 181 when generator 3 was loaded to about 8 per cent of its maximum capacity b manual control. This raised the fr quency, and generator 10, under contr of the frequency regulator, reduced i output by a nearly equal amount. Fro that point on, generator 10 takes the ma jor swings of system load, while generat 3 holds a constant average load wit temporary departures only when frequenc deviates noticeably from normal. Th at 2300, when generator 10 went u sharply to full load and frequency becam low, generator 3 increased its output d to its governor speed-droop characteristi until the system load began to drop a fe minutes later, permitting the frequenc to rise. This caused generator 3 to reve to normal output, while generator follows the variations of system lo

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* The word "generator" in the singular should be understood to include the case of two or more generators sharing the regulation.

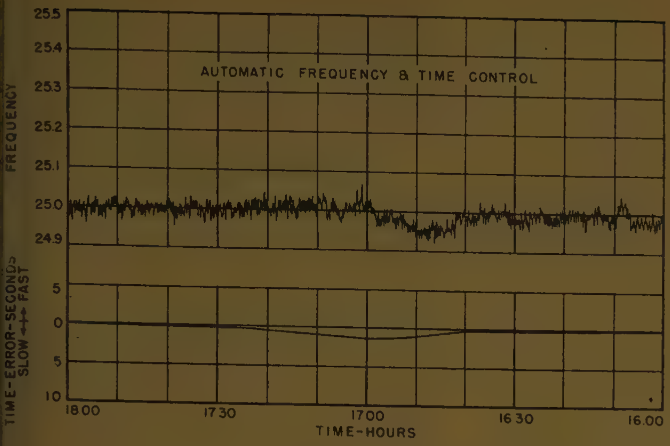

Figure 9. Automatically regulated frequency and time error of Niagara system (interconnected as in Figure 7)

Low frequency between 1640 and 1700 is due to regulating generators being at full load. Time lost owing to low frequency is later regained by automatic bias holding frequency slightly high from 1700 to 1725

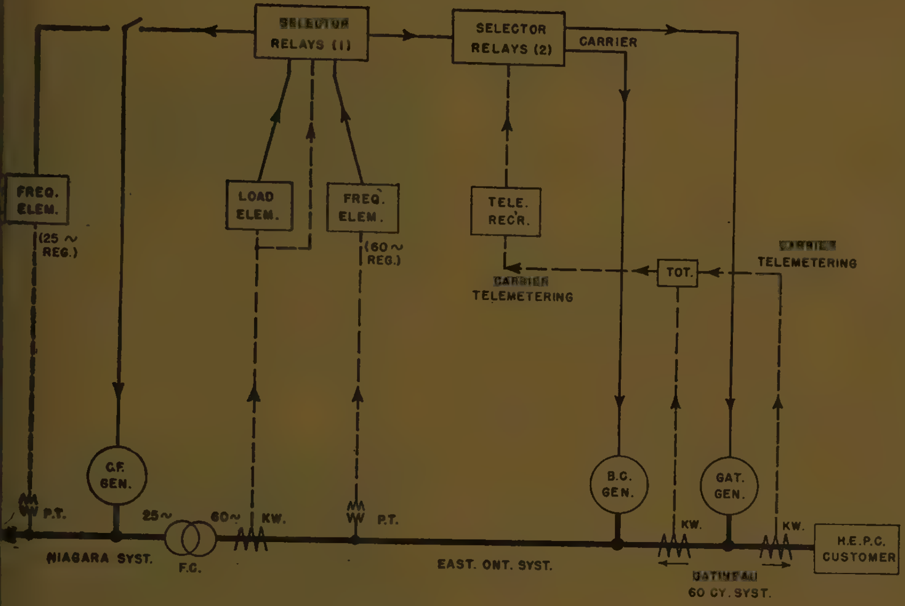

Figure 10. Schematic diagram of frequency- and load-control system centered at Chats Falls

Power circuits are in heavy lines, governor control circuits in light full lines, and metering circuits in broken lines

under control of the frequency regulator. The cause of the load increase on generator 10 at 2300 was reduction of generation at a base load plant elsewhere on the system. This has the same effect as increase of system load.

Figure 4 represents two power systems with an interconnecting tie line, numerous generators and numerous loads being reduced to a single equivalent generator and equivalent load for each system. Circuit breakers, transformers, and other details are omitted. This differs from the case of two generators in parallel feeding a common load only in the separation of the generators and loads into two divisions. With no external control of the governors, the two generators will still share all incremental load of the combined systems, according to their speed-droop characteristics and generating capacities. Power flow in the tie will depend upon these two factors and also upon the origin of the load change—whether the change occurred in area *A* or in area *B*. For example, if load increases in system *A*, frequency will become low, and tie-line load will deviate in the direction *B* to *A*.

In a consideration of regulation of frequency of these systems, it is usual to make one system responsible for the regulation of frequency and time. This will usually be the larger system, or the one having greater generating reserves, or ability to handle load swings more efficiently. Let us choose system *A* as the frequency-regulating system. In accordance with the principles already explained, all changes in total load of both systems will appear as changes in the load of generator *A*, while the load of generator *B* will remain substantially constant. Therefore, if load change occurs in system *A*, it will not permanently affect the tie-line load, but all load changes occurring in *B* will appear as changes in the tie-line loading, because the changes are being absorbed by the frequency-regulating generator *A*.

Under the conditions just considered, unregulated tie-line loads tend to vary with all fluctuations in load of systems remote from the frequency-regulating system. This is illustrated in Figure 5, which shows load fluctuations on an unregulated tie consisting of two frequency changers in parallel, interconnecting two systems of the Hydro-Electric Power Commission in southern Ontario, the Niagara 25-cycle system, and the Georgian Bay 60-cycle system. The power flow is all in one direction, toward the 60-cycle system, which was deficient in stream flow and was receiving assistance from the Niagara system. One machine alone was on at first, and then two in parallel. At 1615, when the load on the frequency-changers was high, a sudden loss of load on the Niagara system caused the frequency to rise suddenly, and forced extra power through the frequency-changers to accelerate the 60-cycle system. This tripped off the already heavily loaded machines and the systems separated, Niagara frequency rising due to the loss of load, and Georgian Bay frequency dropping due to the loss of supply from Niagara. The drop of 60-cycle frequency was great because most of the generators on the Georgian Bay system were blocked and could not increase their output until the operators raised the blocks. The systems were paralleled again a few minutes later. The frequency here was manually controlled, which accounts for sustained deviations up to 0.3 cycle on 25 cycles.

Tie-Line Load Control

In many cases it is necessary to regulate the interchange of power through a tie line, holding the tie-line load substantially constant at a predetermined value. Referring again to Figure 4, when system *A* is regulating frequency, tie-line load may be regulated by control of generation in system *B*. A regulator sensitive only to tie-line load exerts what is called "flat tie-line load control." This tends to make each system "mind its own business," the generators in system *A* taking care of load fluctuations in that system under control of the frequency regulator, and generators in system *B* taking care of load fluctuations in *B* under control of a tie-line load regulator.

Another means of combining regulation of frequency with regulation of tie-line load is called selective frequency control,[3] in which a regulator or regulators sensitive only to frequency act selectively upon generators in *A* and *B* under supervision of watt relays, responsive to tie-line load. For example, if frequency is low, a raising impulse will be initiated by the frequency regulators. If the tie-line load at that instant is too great in the direction *B* to *A*, the tie-line watt relay will permit the raising impulse to pass to the governors of system *A*, since this will tend to correct the tie-line loading as well as to correct the frequency, but raising impulses will be blocked from going to the governors of *B*, since this would make the tie-line deviation greater.

When systems are connected like links of a chain, which may have side branches, but no loops, it is usual to assign frequency control to one large central system such as *D* in Figure 6, and have other systems regulate load in the tie leading toward the frequency-regulating system. This is simply an extension of the principle of load control on a tie line between two systems as already explained. If any system such as *F* is unable for any reason to regulate load in the tie between *F* and *E*, the tie *F–E* will become an unregulated tie, and system *E* will automatically assume the regulation for the two systems *E* and *F*, holding load on tie *E–D* up to the limit of the available generating capacity in system *E*. An overload on system *E* would make it impossible further to control the load in tie *E–D*, and would throw additional burden on the frequency-regulating system *D*.

Figure 11. View in Chats Falls control room showing frequency-control supervisory panel switches on operator's desk

Round dial at left indicates total 60-cycle delivery from Gatineau Power Company, with load-setting controls in upper part of indicator case

On large interconnections it is usually desirable to provide for automatic assistance to the frequency-regulating system on the occurrence of unexpected overloads or emergency conditions. For this purpose it is customary to use tie-line load regulators in which the load setting is automatically biased by deviations in frequency, and perhaps also by time error.[7] This is known as "tie-line load control biased by frequency," abbreviated to "tie-line bias control." This system of mutual aid enables interconnections of vast magnitude to operate successfully, meeting demands which the systems individually could not meet without much greater generating reserves.

When interconnections form loops or networks, load on each individual tie cannot readily be controlled, but each system may totalize all its tie-line loads and regulate the "net interchange" or total power supplied to or received from the pool, leaving the division of load among the ties to the natural laws of power flow in networks.[12] As in the case of single tie lines, net-interchange control may be applied either on the flat-load-control principle, or with the tie-line-bias modification.

The characteristic effects of automatic frequency- and load-control devices on systems interconnected in chain fashion exemplified in Figure 6, may be summarized in the following five statements, covering the general case where some ties are regulated and others may be unregulated, with some systems regulating and others nonregulating.

1. Frequency of the whole interconnection tends to be maintained at normal value by action of the frequency regulator on regulating generator(s) in the frequency-regulating system.*

2. The load of each regulated tie tends to be maintained at the preset value by action of a load regulator on regulating generator(s) in a regulating system more remote from the frequency-regulating system.

3. Output of the regulating generator(s) in each regulating system* tends to vary with all load changes within a group consisting of the local system and all contiguous nonregulating systems as far as the nearest regulated ties.

4. Load of each unregulated tie tends to vary with all load changes of contiguous nonregulating systems of the group (as defined in 3) on the side of the tie remote from the regulating system of the group.

5. The output of generators other than regulating generators tends to remain approximately constant.

Interconnections in Ontario and Quebec

In Canada, the largest interconnection of power systems is in the province of Quebec, where three large systems operate with interconnected generating capacity of about 2,000,000 kw (1942). Frequency is regulated automatically by either of two of these systems, but automatic tie-line control is not used because of the long distances between load metering points and the generating stations, making installations difficult and expensive.[11]

Next in magnitude is the Southern Ontario system of the Hydro-Electric Power Commission of Ontario which is the product of the amalgamation of the
former Niagara, Eastern Ontario, Geo
gian Bay, and Dominion Power system
now called divisions, but still physicall
"systems" as they will be termed herei
The total load is slightly over 1,500,00
kw. The largest system is the Niagara,
25-cycle system, the Eastern Ontario an
Georgian Bay being 60-cycle, and th
Dominion Power system with load in th
Hamilton area being $66^2/_3$-cycle.

The Niagara system receives power no
only from Niagara district, but also fro
sources far outside the load area: Chat
Falls plant on the Ottawa River, Pauga
plant on the Gatineau River, Masso
plant of the McLaren-Quebec Power Com
pany on the Lievre River, and Beau
harnois on the St. Lawrence.[5,9] Each
the other systems is connected to th
Niagara system through one or two fr
quency-changer sets. The single fr
quency-changer set for the $66^2/_3$-cyc
system is located at Niagara Falls; for th
Georgian Bay system, two sets at Ha
over, at the end of a 110-kv 25-cycle li
from Kitchener; and for the Easte
Ontario system, one set at Chats Fal
plant. Generation on the Eastern O
tario system is supplemented by pow
from High Falls plant on the Liev
River, and from the Gatineau Pow
Company's 60-cycle system, both in th
Province of Quebec. Tie lines interco
necting the Eastern Ontario system a
Gatineau 60-cycle system cross the O
tawa River at a point just above the ci
of Ottawa.

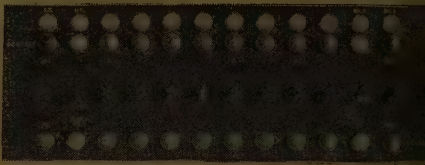

Figure 12. Frequency-control panel o
operator's desk at Chats Falls, with maste
control switches for 25- and 60-cycle regula
tors, and transfer control switches for eigh
generators in Chats Falls plant

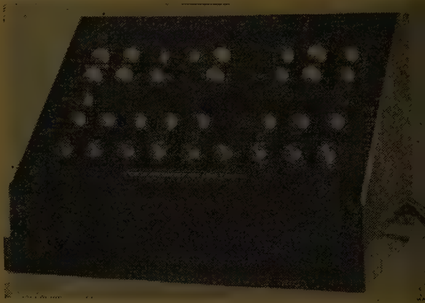

Figure 13. Load-control supervisory cabin
on operator's desk at Chats Falls, with switche
and lamp indications for all load-contro
functions

These four systems of the Commissi

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* "Regulating system" is a general term which includes "frequency-regulating system" as a special case.

and the Gatineau 60-cycle system operate normally interconnected, with total connected generating capacity of about 1,650,000 kw.

Figure 7 shows schematically the five interconnected systems with the generating capacity in each system and the capacity of the ties indicated in megawatts. The normal direction of power flow is shown by the arrows. Flat frequency control with automatic time-error correction is maintained on the Niagara system by regulation at one of two points, Queenston plant on the Niagara River, or Chats Falls plant on the Ottawa River. The Georgian Bay and Dominion Power system frequency-changer ties are floating without automatic regulation. The other two ties—the frequency changer at Chats Falls and the Gatineau tie line—are equipped for automatic regulation, the first because of limited capacity of the frequency-changer set, and the second because of contractual obligations. The Gatineau Power Company delivers power to the Commission at two points, the principal supply being over the tie lines to the Eastern Ontario system, and a minor portion fed directly to an isolated customer of the Commission and metered at a point about 45 miles from the tie-line terminal. The isolated load is telemetered and totalized with the tie-line load. The totalized quantity is again telemetered 30 miles to Chats Falls where the load-regulating equipment is located. Control therefore may be applied to the totalized quantity rather than to the tie line only. Telephone-line carrier is used for telemetering and control.

Frequency-Regulator Characteristics

The type of frequency-regulating equipment in use at Queenston and Chats Falls was designed at the School of Engineering Research of the University of Toronto, under the supervision of H. W. Price, and all regulators were built in the Commission's shops. The initial installation at Queenston has been in service since 1934, and two regulators—one 25-cycle and one 60-cycle—have been in service at Chats Falls since 1942.

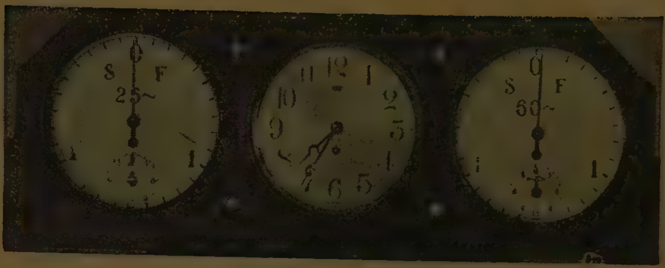

Figure 14. Time-error dials (25- and 60-cycle) and station clock in control room at Chats Falls

In a preliminary study of Niagara system frequency and governor characteristics, it was observed as shown in Figure 8 that frequency of a large system is continually fluctuating with an irregular period averaging about 25 seconds, even when the system load is relatively steady. These swings are too rapid to be controlled, and too small to be of vital consequence when they are of short duration. Therefore, the frequency regulator was designed to disregard these minor fluctuaions, but to be very sensitive to deviaions in average frequency. The regulaor maintains a close comparison between otal integrated cycles of system frequency and the nominal cycles correspondng to rated frequency of the system. Whenever the difference changes by one half cycle—not one half cycle per second, but one half cycle in phase position, or 180 electrical degrees—on the 25-cycle system, or 1.2 cycles on the 60-cycle system, a frequency-corrective impulse is sent out to the governors. This is repeated as often as necessary to correct the frequency. In the average day, about 2,000 impulses are produced by the frequency regulator, or one in 45 seconds on the average, though in time of need this can be as often as one every two seconds. This characteristic of the regulator minimizes the number of impulses per day, and avoids undue wear on relays, governors, and gate mechanisms.

Automatic time correction is provided to bias the frequency by about 1/20 of one per cent when the system time error reaches 0.4 second fast or slow. Within the limits of 0.4 second error, a vernier bias, imperceptible on the frequency charts, continues to urge the time error gently toward zero. The reference standard for time error is the station master clock, which may develop errors of its own with regard to radio time signals, but is usually not more than five seconds out.

Figure 9 shows the uncontrollable frequency fluctuation at a slower chart speed. It also shows a frequency dip resulting from overload on the regulating unit, and the resulting time error; then after relief from overload, the automatic frequency bias, just perceptible as a change in the average level of the frequency line, and finally restoration of normal average frequency when the time error is less than 0.4 second.

Scheme of Frequency and Load Control

Figure 10 is a schematic diagram showing the essential elements of the scheme of combined frequency and load control from the 60-cycle control equipment centered at Chats Falls. Corrective impulses may be initiated either by the frequency-sensitive element, supplied from 60-cycle potential, or from the load-sensitive element, responsive to load measured on the 60-cycle side of the frequency-changer set. These impulses, from both sources, are sorted by the first set of selector relays, raising impulses being sent to 25-cycle generators in Chats Falls plant and lowering impulses toward 60-cycle generators, or vice-versa, depending upon the direction of the existing deviation of frequency-changer load from the scheduled value as set on a watt relay. Impulses which are directed toward 60-cycle generators must now pass a second set of selector relays, which determine on the basis of the total Gatineau delivery, totalized and telemetered back to Chats Falls, whether raising impulses should be permitted to go to Gatineau plants and lowering impulses to Barrett Chute, or vice-versa.

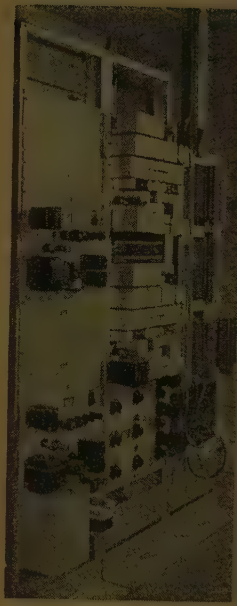

Figure 15. Telephone-line carrier transmitting and receiving equipment for telemetering and remote control at Chats Falls

Figure 16. Telemetering and load-control panel in control room at Bryson plant of Gatineau Power Company

The principle of this control system is a combination of selective frequency control and flat tie-line load control. The frequency-sensitive element, initiating impulses on the basis of frequency deviation only, and the selector relays, distributing these impulses on the basis of deviations in the tie-line loads, embody the principles of selective frequency control. This control alone is prompt in regulation of frequency, and also has a tie-line load-regulating effect, but is rather slow in shifting load after a change in setting of the watt relay, because impulses are initiated only by random swings of frequency. Selective frequency control is therefore supplemented by load-regulating impulses initiated by the load-sensitive element, regardless of frequency. All the impulses from the load element go to 60-cycle generators, either Barrett Chute or Gatineau. Impulses from the load element are sent at either of two rates, once every $6^2/_3$ seconds for deviations exceeding 5,000 kw or once every 20 seconds for smaller deviations. The combination of a 20-second time interval with a suitable dead band on the watt relay gives the regulating generators time to respond to each impulse, and gives stable operation. The watt relay is designed to be insensitive to the rapid swings of synchronizing power which are typical of tie-line loads, but to be sensitive to sustained deviations in either direction.

Figure 11 is a view of part of Chats Falls control room showing the large dial on which the Gatineau 60-cycle delivery is indicated. Adjustable contacts in this instrument are used to set any desired delivery. In front of the farther operator is the control panel for supervision of the 25- and 60-cycle frequency regulators, and for the eight generating units in this plant. At his left and hidden from view is a control cabinet for supervision of the load regulator.

Chats Falls frequency regulator and generator control panel is shown in Figure 12. Master control keys for the two frequency regulators are at the extreme ends of this panel, 25-cycle at the right and 60-cycle at the left. The 60-cycle regulator is always in use, and the 25-cycle regulator is also required when the systems are separated. With the systems separated, Chats Falls generators may be transferred to the 25-cycle regulator as suggested in Figure 10. Next to each master key is the frequency bias key for automatic time correction. This key is "on" except when Queenston is regulating frequency and time. The other eight keys are for placing the eight Chats Falls units on automatic control, from the 60-cycle regulator if the keys are up, or from the 25-cycle regulator if the keys are down. Lamps indicate the position of the keys, and other lamps (upper row) indicate when the units are approaching full gate, which is their limit of control. Limit switches on the governor prevent the automatic control from raising the units above 90 per cent gate or lowering below 50 per cent, in order to keep them in a fairly efficient operating range.

Figure 13 shows Chats Falls load control supervisory cabinet. This cabinet, located on the operator's desk, is for control and supervision of the 60-cycle frequency and load regulation centered at Chats Falls. Indicating lamps show where each impulse goes—to Chats Falls generators, to Barrett Chute, or to Gatineau generators—and whether it is a raising or a lowering impulse. Lamps also indicate whether the tie-line loads are above or below the scheduled setting, and a special red lamp and intermittent buzzer signal draw the operator's attention when the frequency-changer load deviates more than 5,000 kw from schedule. The switches may be used to set up the control for abnormal operating conditions. For instance, when the frequency changer is out of service for maintenance, thus separating the 25- and 60-cycle systems, Chats Falls units are taken off the 60-cycle control, and the 60-cycle regulator is then used for regulation of 60-cycle generators only. This change in control setup also takes place automatically when the frequency changer trips off for any reason, and the change is indicated by a red lamp signal. Other lamps are for supervision of the three carrier sets, two transmitters and one receiver, which may be seen in Figure 15.

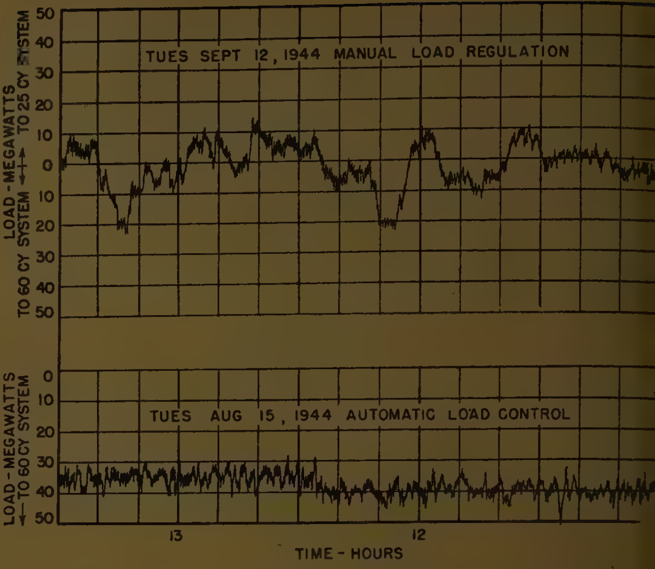

Figure 17. Comparison of load on Chats Falls frequency-changer set with and without automatic control

The clock case shown in Figure 14 is mounted above the switchboard in front of the operator's desk in Chats Falls control room. The two outer dials are time error indicators, 25- and 60-cycle respectively, and the center one is a time clock operated electrically by impulses at half-minute intervals from the station master clock which is checked daily by radio time signals. The large time-error dial covers a range of $1^1/_2$ seconds fast or slow, and a smaller inset dial goes to 12 seconds fast or slow. These indicators operate by position-type telemeter system, from the time-error element of the frequency regulators. When the systems are interconnected both dials read alike, and normally not more than one half second fast or slow. Time-error recorders give a graphic record on a chart with range of 25 seconds fast or slow, which is rarely overrun even in times of serious trouble. The ordinary range of time deviations during present conditions is from five seconds fast to five seconds slow, which is due to shortage of spinning reserve because of war loads. The reference standard, as already stated, is the station master clock.

Figure 15 shows telemetering and load-control carrier terminal equipment at Chats Falls. This consists of two carrier transmitters on the nearest panel, one transmitting to Barrett Chute and one t Gatineau plants, and one receiver on the lower part of the second rack, receivin telemetering signals representing Gatineau total 60-cycle delivery. This equipment is located at the terminals of the open-wire telephone circuits on which the carrier is superimposed on audio-fre quency voice communication.

The telemetering transmitter and loa control panel at Bryson is shown in Figur 16. A carrier receiver, connected to th telephone line, brings in load-regulatin impulses to the relays on this panel, an these in turn operate on the governors At the same time, the telemetering transmitter, measuring the local delivery to th Commission, is sending out telemeterin signals on the same telephone line at different frequency. Indicating lamp and buzzer give all necessary indication and warnings. The instrument at the to of the panel is a frequency relay which i intended to block the incoming contro signals from Chats Falls in case of a separation between the Eastern Ontario an Gatineau systems. If such a separatio should occur the Gatineau frequenc would rise, closing a high-frequency contact of the frequency relay and locking ou the automatic control until reset by th operator. Except for the telemeter transmitter, this equipment is typical of th installations at each of the three remot generating stations.

Operating Performance

Figure 17 shows the comparison be tween automatic and manual control frequency-changer load at Chats Fall The noon hour was picked as the tim when load changes occur at the greate rates on both 25- and 60-cycle system and when load swings are likely to b severe. Manual control was considere permissible only because the desired tran fer was about zero, and swings would n likely overload the set, if watched. A operator was assigned to watch the loa

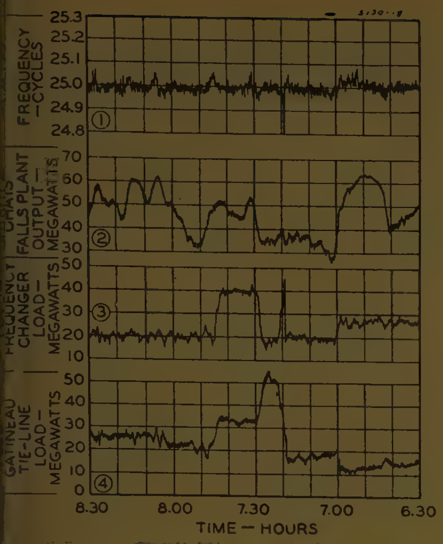

Figure 18. Charts showing results of automatic frequency and load control during trouble conditions on Eastern Ontario system

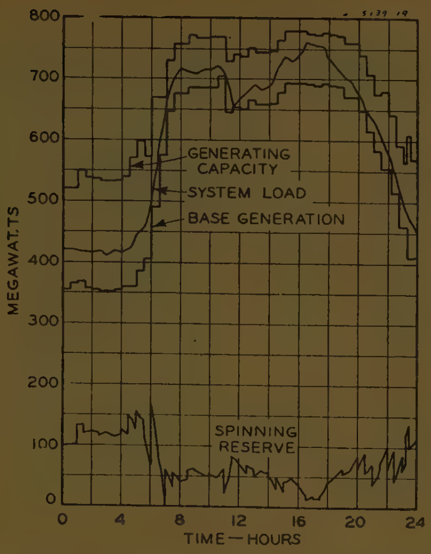

Figure 19. Daily load curve for a portion of the Niagara system, showing total connected generating capacity, base load generation, and spinning reserve, based on half-hourly readings

Thursday November 6, 1941

throughout this period and to telephone for correction of 60-cycle generation when required. With automatic control, the set carries as much as 40,000 kw average load at any time it is required with little or no attention on the part of the operator and rarely a telephone call for load correction. The chart shows a change in the load setting from 40,000 kw to 35,000 kw at 1225, an operation which is quickly and easily performed by a setting on the watt relay. The load responds promptly to this.

The capacity of this frequency-changer set is 45,000 kw which is about 13 per cent of the connected generating capacity of the 60-cycle systems. During two years of interconnected operation it has tripped off and separated the systems 13 times, ten of these separations being due to severe disturbances on one or both systems, and the other three due to overload occasioned by lack of experience in interconnected operation during the first year. The set has withstood surprisingly large disturbances without tripping, but on these occasions the initial load on the set happened to be small, or was in such a direction that the disturbance reduced or reversed the direction of the power flow.

Figure 18 illustrates the behavior of the control system and the interconnected generators during trouble. Generators at High Falls supplying 35,000 kw to the Eastern Ontario system suddenly tripped off. The meter charts shown are:

1. Twenty-five-cycle system frequency.
2. Output of Chats Falls plant, which was being used as the regulating station for the Niagara system.
3. Chats Falls frequency-changer load, which was being regulated by the automatic load regulator, acting on Gatineau 60-cycle generators.
4. Gatineau tie-line load, which was not being regulated at the time because Barrett Chute plant was shut down and was therefore not available for regulation.

Under these conditions it is to be expected that the Gatineau tie-line load will fluctuate with all changes in load or generation in the Eastern Ontario system, since the only 60-cycle generators available for regulation were in the Gatineau system.

At 7:19 p.m., when the 60-cycle generators at High Falls tripped off, depriving the Eastern Ontario system of their 35.000 kw generation, the frequency suddenly dropped to 24.8 cycles on the 25-cycle system, and by a corresponding amount on the 60-cycle systems, but quickly recovered to 24.95 cycles approximately, and finally to 25 cycles in about three minutes. It is apparent that the sudden change in load was carried initially by the kinetic energy of the system while frequency was dropping during the first few seconds, then was shared by both 25- and 60-cycle generators because of their speed-droop characteristics, and finally was absorbed by the regulating generator of the 60-cycle systems. The swing of 25,000 kw in the frequency-changer load is the share of the load change which was contributed temporarily by the 25-cycle system, a small part (5,000 kw) coming from Chats Falls generators, and the balance, or 20,000 kw, from other generators, at Queenston and elsewhere. The other 10,000 kw came from the Gatineau generators, as shown by the initial rise in the lower curve. The automatic regulator immediately began to send raising impulses to Gatineau generators, in an effort to raise the low frequency and also to correct the deviation in the frequency-changer load. This was accomplished in three minutes, when the frequency-changer chart shows its load was back to schedule at 7:22, and the whole of the 35,000 kw lost generation was made up by the Gatineau generators. Notice that Chats Falls output returned to its original value. This shows how sudden load change is shared initially by all generators having any spinning reserve, but is absorbed ultimately by regulating generators in the system or group where the disturbance originated.

Although the automatic load control had successfully performed its function in relation to the frequency-changer load, the effect was to increase the Gatineau 60-cycle delivery above the contract amount. Since Chats Falls generators had spinning reserve capacity available, the load setting on the frequency changer was manually changed at 7:27 from 20,000 kw to 40,000 kw toward the Eastern Ontario system. This increased Chats Falls output by 20,000 kw and reduced Gatineau output by a similar amount by 7:30 p.m. Finally, at 7:45, High Falls generators came back on load, and all conditions were restored to normal.

Automatic Controls and the Human Element

Although much can be said for automatic controls, it must be recognized that they have definite limitations. Automatic regulators do a job which human operators either cannot successfully perform, or can perform only by constant attention to an exacting and never-ending task. Nevertheless it should not be inferred that a system equipped with a few control devices at strategic points practically runs itself. Regulators can function only within the limits of capacity of the generators under their control, and this may be in the order of five to ten per cent of the total generating capacity of the system or group of systems for which these generators are regulating. The remaining generation, on base load, must be adjusted up or down from hour to hour to conform to the general trend of the system daily load curve as estimated by the power supervisor, so as to keep the regulating generators within their regulating range. This is illustrated by Figure 19 showing a 24-hour curve of connected capacity, load, and base load generation for a portion of the Niagara system. Load assignments are made partly by prearranged schedule, but the supervisor must be prepared at any moment to meet unforeseen conditions with such power resources as are available. For this purpose, automatic controls must be supplemented by adequate communication facilities, and by telemetering of critical quantities such as forebay levels, loads on ties, and loads on generating stations, especially those that are under control of automatic regulators.

Instability in D-C Aircraft Systems

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Synopsis: A laboratory and theoretical study was made to determine the causes and methods of combating an unstable oscillatory voltage condition which has been encountered in aircraft d-c electric systems, using types *P*-1, *P*-2, and *R*-1 generators with carbon-pile voltage regulators.

The generator characteristics were found to be such that oscillatory instability would not occur without the presence of a voltage regulator which was out of adjustment and which had inadequate damping.

A method is given for determining the amount of damping required for stable operation in terms of the generator field time constant and certain regulator characteristics.

INSTABILITY has been encountered in aircraft electric systems employing 28.5-volt generators, 24-volt batteries, and carbon-pile voltage regulators. In systems in which the regulators were out of adjustment, sudden application or removal of load has initiated sustained oscillations in load current and system voltage. The frequency of these oscillations has been between 10 and 60 cycles per second with amplitudes as large as the maximum value of load current.

In an analysis by the Army Air Forces it was determined that this type of system instability was associated with wear of the regulator stack and could be corrected by readjustment of the regulators. The analysis also disclosed that instability occurred at a smaller value of stack wear when certain generators were used, particularly at high speeds. A damping transformer was developed to prevent unstable operation when the stack became worn. This transformer was not entirely satisfactory in that it did not prevent unstable operation for the maximum possible stack wear with all types of generators.

At the request of the Army Air Forces a further analysis was made to determine

1. The causes and nature of the unstable condition.

2. A means of evaluating the generator and regulator characteristics to determine whether they tend to be unstable.

3. A means of preventing instability.

Generator Stability Analysis

The first approach was to determine whether a generator without a regulator could sustain an oscillation initiated by a switching transient.

EQUIVALENT CIRCUIT FOR GENERATOR AND LOAD

From the mathematical analysis given in Appendix II, it is possible to determine the relation of the electrical quantities and characteristics of the generator and load circuits which would be necessary to produce oscillatory instability without the effect of a voltage regulator. These quantities and characteristics were measured on a particular generator and used in the evaluation of the operational equations. The operational equation for the equivalent circuit, given in Figure 1, which relates generator load current with time is

$(p^2+\alpha p+\beta)i_1=C$ (equation 11 of Appendix II)*

This equation is identical in form with the classic differential equation of the electric circuit involving resistance, inductance, and capacitance,

$$\frac{d^2i_1}{dt^2}+\alpha\frac{di_1}{dt}+\beta i_1=C$$

As the equations in the two cases are identical, there must, of course, be found in the solution of the former the same oscillations and transients as are contained in the well-known solution to the latter.

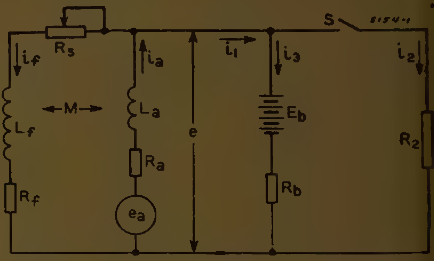

Figure 1. Equivalent circuit of generator with battery and resistance load

In Figure 1, although there is no condenser capacity involved and therefor no electrostatic energy, there are nevertheless two different magnetic circuits i which energy can be stored. During a oscillation all of the energy stored in the generator field cannot be transferred t the armature circuit. Most of it is dissipated as i^2r. One of the objects of thi investigation is to determine whether th characteristics are such as to transfe enough energy between the field an armature circuits that oscillations may b produced. Such oscillations, althoug small and highly damped, might be amplified by the voltage regulator so as to produce subsequent oscillations.

GENERATOR CHARACTERISTICS

The generator characteristics necessar to evaluate the above operational equation were measured for *P*-1, *P*-2, and *R*- generators, with the test equipmen shown in Figure 2. Values of r_f, L_f, L

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* For explanation of symbols see Appendix I.

In conclusion, it is interesting to note that interconnection has developed a high degree of co-operation without which its fullest benefits would be unattainable. Operating staffs and managements of interconnected systems have learned that what is best for the whole is usually best for each. Best results are attained when all systems are prepared to contribute according to their ability, to meet the need of any—a principle which works well in all human relationships.

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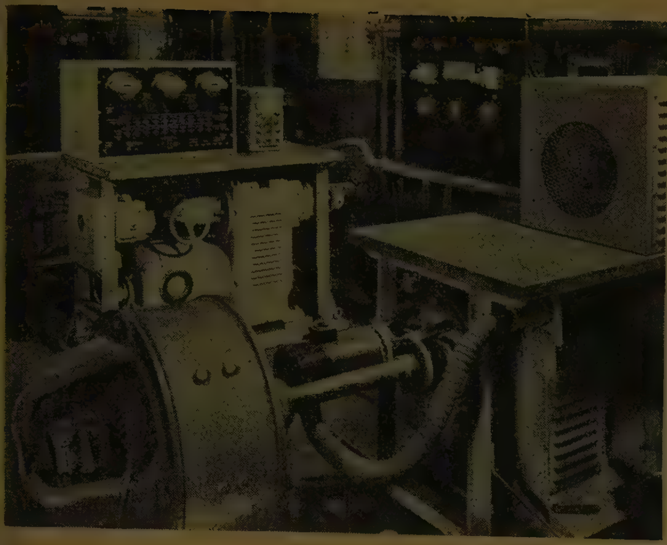

Figure 2. General view of test equipment showing in left foreground special d-c drive motor used to obtain minimum speed regulation on sudden application of load to the aircraft generator

and field time constant for all types of generators are given in Table I.

Oscillatory Instability

In the solution of equation 11 it is found that the condition for no oscillation in the circuit containing generator, battery, and load is that existing when $\beta < \alpha^2/4$. With the characteristics of the *P*-1 generator of manufacturer *A* from Table I and the following:

Generator	Load
$M = -0.16$ millihenry	$E_b = 27.6$ volts
$R_a = 0.03$ ohm	$R_b = 0.028$ ohm
$r_f = 14.76$ ohms	$R_2 = 0.57$ ohm
$Kk_a = 0.031$ volt per ampere	
$Kk_f = 12.0$ volts per ampere	
$e_0 = 5.18$ volts	

it may be shown that the circuit is nonoscillatory. There is no value of Kk_a which can make the circuit oscillatory with the other parameters remaining the same as given above. However, with the initial value of Kk_a and a value of $M = -0.8$ millihenry, it can be shown by calculation that a damped oscillation of about six cycles per second theoretically could be produced. This value of M is five times the measured value for this generator. The brushes as set to give maximum output were about 1/4 bar behind neutral. It is possible that a similar generator with a larger brush shift might have a greater M, but it is not likely that it will approach 0.8 millihenry. Mutual inductance in a generator is a very difficult quantity to measure with accuracy.

Inasmuch as the *P*-1 generator of manufacturer *A* has shown the greatest tendency for unstable operation and its measured parameters were so far short of the values necessary to produce instability without a regulator, these equations were not evaluated for any other generator.

It is interesting to note that in all cases where oscillations may be produced it is necessary to have cumulative compounding (this does not imply overcompounding) with armature current; that is, a positive value for Kk_a. Practically, this will be obtained only with generators having series, interpole, or compensating windings and with brushes shifted against rotation, the cumulative compounding direction. Generators without interpole or compensating windings normally have their brushes shifted in the direction of rotation, because they do not commutate satisfactorily with brushes shifted against rotation. In the laboratory rising volt-ampere characteristics were obtained at maximum rated speed for *P*-1 generators of manufacturer *A*, which have interpole and compensating windings. In all cases it was necessary to have

1. Brushes set for maximum output.
2. Well-polished and well-seated brushes.
3. A black carbon film deposited on the commutator.

For generators without interpole or compensating windings volt-ampere output was increased by application of a film, thus indicating a cumulative effect. *P*-2 and *R*-1 generators exhibited the same compounding characteristics as the *P*-1 generator of manufacturer *A*, except that the film was more difficult to maintain at the maximum speed of 10,000 rpm.

A theoretical calculation and laboratory test were made to determine load current as a function of time for the generator of manufacturer *A* running at 4,500 rpm in parallel with a *G*-1 battery while switching a 50-ampere load. The circuit arrangement is shown in Figure 1. The generator and load characteristics are given in the foregoing table.

The calculated transient value of load current after switching load R_2 in parallel with the battery (battery previously floating on line) is

$$i_1 = -63.06\epsilon^{-8.39t} + 2.96\epsilon^{-179.1t} + 64 \text{ amperes}$$

At $t = 0.66$ second the load R_2 is switched off, and the calculated expression for load current becomes

$$i_1' = 62.9\epsilon^{-8.48t} - 2.8\epsilon^{-188t} + 3.9 \text{ amperes}$$

These expressions are plotted as a function of time in Figure 3. A comparison with data from an actual test oscillogram for the same generator and the same circuit conditions also is shown in Figure 3. Discrepancy between the test and calculated values is caused by variations in generator and load circuit parameters under operating conditions and by unavoidable inaccuracies in measurement of parameters. In the expression for load current it should be noted that there are no oscillatory terms.

It can be seen from the test curve that, although no oscillations were present, load current reversed, and the system became unstable. There was actually a reversal of polarity of the residual field in the generator each time this unstable operation occurred. This reversal of polarity probably is caused by the resultant armature, interpole, and compen-

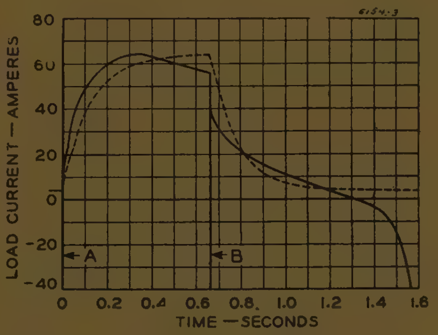

Figure 3. Comparison of calculated and test curves of load current as a function of time for switching a resistance load in parallel with a battery (*P*-1 generator, manufacturer *A*)

---- Calculated
——— Test oscillogram
A—Switch *S* closed
B—Switch *S* opened

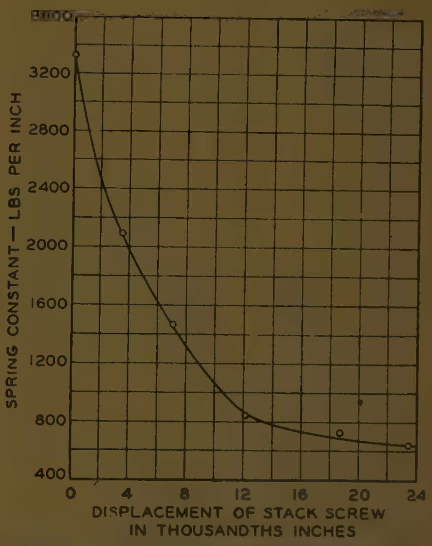

Figure 4. Spring constant versus stack displacement (regulator manufacturer *C*)

sating-winding flux overcoming the main-field-winding flux, as in a series motor with brush shift against rotation.

Although this latter phenomenon constitutes a form of instability, its action is sufficiently slow that a voltage regulator easily can prevent the voltage collapse. As all of the instability troubles recorded in service were oscillatory in nature, the analysis of this phenomenon was not pursued further.

Regulator Stability Analysis

Once it was found that the generator of manufacturer *A* without a regulator would not sustain an oscillation, the next approach was to determine what characteristics of carbon-pile regulators would produce instability in combination with generator characteristics.

A voltage regulator in any system behaves in a manner similar to an amplifier. Small deviations in the regulated voltage are made to produce relatively large changes in the generated voltage of the final machine to offset the internal voltage drop caused by variations in load. Such devices as vibrating contacts, motor-operated rheostats, and carbon-pile rheostats have been used to produce the large changes in excitation for small errors in the regulated voltage, and, although in some cases their action is intermittent, the net result produced has been amplification.

The chief difficulty in stabilizing all regulating systems is the presence of time delays. These may be analyzed in terms of the time constants of the various inductive circuits, such as the fields of rotating machines and also the time constants of the regulator itself. The latter may be electrical, mechanical, or both. Such a system may be idealized by showing the amplifier or regulator without delay but with an extra exciter to represent its delay, so that all the constants can be represented by simple inductive and resistive, or inductive, resistive, and capacitive electric circuits.

Two basic types of stabilizing means may be used for regulating systems, namely, electrical damping and mechanical damping. At the time this analysis was made no mechanical damper had been devised for aircraft carbon-pile regulators because of the difficulty of obtaining sufficient damping with the extremely small armature travel.

In this analysis a damping transformer was assumed to be the most satisfactory method of electrical damping. The problem then resolved into finding the correct size damping transformer to provide stable operation with all combinations of generators and regulators which would be used.

In Appendix III a mathematical analysis of the problem is given which includes all the characteristics of the system that have a major effect upon stability. Some of the constants for this system are difficult to measure, and a few assumptions are necessary, but from this analysis one can determine the approximate size damping transformer necessary for stable operation. Once the approximate amount of damping necessary is known, the optimum size damping transformer can be determined quickly from laboratory tests.

It had been shown in service and laboratory tests that wear or maladjustment of the carbon stack was the initiating cause of oscillatory instability. Decrease of the spring constant with increasing stack displacement, which is shown for a particular regulator in Figure 4, appears as an increase in the value of C_m in equation 30, Appendix III. This equation also shows that the value t_d $(=M/R_p)$ is the determining factor in the design of the damping transformer. Additional factors in the design of a damping transformer are discussed in another AIEE paper.[1]

Results of Calculations and Regulator Tests

Curves of stack displacement versus the value of M/R_p necessary to maintain stable operation for several generator and regulator combinations are shown in Figures 5, 6, and 7.

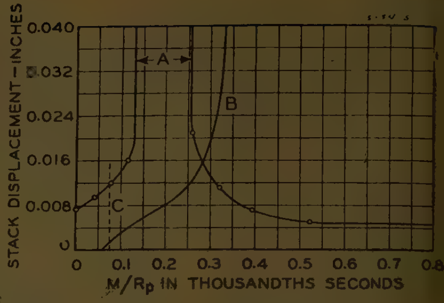

Figure 5. Distance stack screw can be moved out before instability occurs versus M/R_p of damping transformer (regulator manufacturer *C*, *P*-1 generator manufacturer *C*)

A—Test
B—Calculated (mechanical damping=0)
C—Present transformer

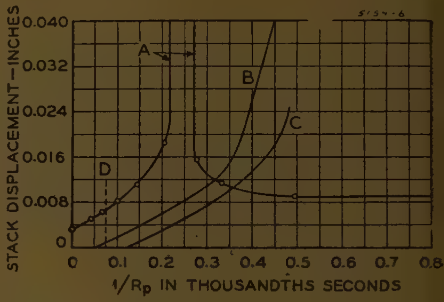

Figure 6. Distance stack screw can be moved out before instability occurs versus M/R_p of damping transformer (regulator manufacturer *C*, *P*-1 generator manufacturer *A*)

A—Test
B—Calculated (mechanical damping=0.1 pound-inch per second)
C—Calculated (mechanical damping=0)
D—Present transformer

In Figure 5 curves are shown for a regulator of manufacturer *C* in combination with a *P*-1 generator of manufacturer *C*. This regulator was the earlier model containing no antihunt resistor. The test curve is plotted from points where stability was just maintained with the regulator at operating temperature. The calculated curve shows the value for critical* damping using the following constants in equation 30, Appendix III:

$R_c=30$ ohms
$L_c=0.0875$ henry
$R_m=0$ and 0.1 pound-inch per second
$1/C_m=$value obtained from Figure 4
$AR_c=200$

The curves shown in Figure 6 are for the same regulator, manufacturer *C*, with a *P*-1 generator of manufacturer *A*.

The curves shown in Figure 7 are test curves taken with a regulator of manufacturer *B* in combination with a *P*-1 generator of manufacturer *A*. This regulator contained an antihunt resistor, and curves were taken with and without this resistor to illustrate its effect.

There can be noted a discrepancy of about two to one in the calculated and

Table I. Generator Characteristics

Generator Type	Manufacturer	Revolutions Per Minute	Field Resistance in Ohms		L_f,* Henrys	L_a,* Millihenrys	Field Time Constant, L_f/r_f, Seconds
			R_f(gen.)	r_f(tot.)			
P-1	*A*	4,500	2.3	14.0	0.28	0.14	0.020
P-1	*A*	4,500	2.4	15.3	0.20	0.14	0.013
P-1	*B*	4,500	2.8	20.0	0.36 (calc. 0.28)	0.22	0.018
P-1	*B*	4,500	2.8	18.6	0.33	0.22	0.018
P-1	*C*	4,500	2.9	16.5	0.51	0.19	0.031
P-2	*D*	8,000	2.7	14.0	0.22	0.10	0.016
R-1	*E*	10,000	2.6	20.0	0.27	0.08	0.013
R-1	*F*	10,000	2.8	20.0	0.43	0.07	0.021
M-2	*G*	2,500	3.6	20.0	0.53		0.026

* Inductance values obtained from oscillographic measurements.

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* For definition see Appendix III.

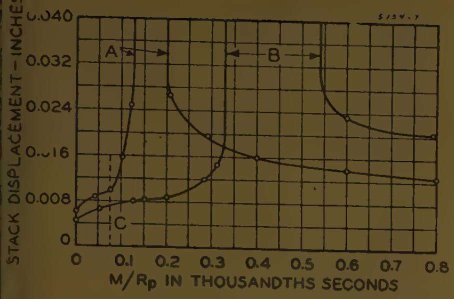

Figure 7. Distance stack screw can be moved out before instability occurs versus M/R_p of damping transformer (regulator manufacturer *B*, *P*-1 generator manufacturer *A*)

A—500 damping resistor
B—No damping resistor
C—Present transformer

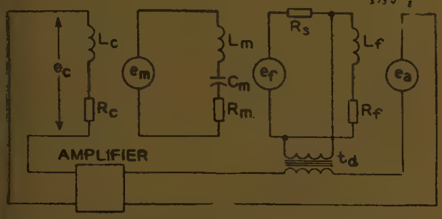

Figure 8. Representation of system with three time delays and one damping transformer

test values for the amount of damping required for a given stack displacement. This can be attributed to several factors. The measurement of the spring constant of the regulator was a tedious procedure, requiring observations of armature displacement in the order of 1/10,000 inch where a small error would be greatly magnified. Also the values of M/R_p for the test curve might be in small error because of interpolation from the inductance curves. The value of mechanical damping of 0.1 pound-inch per second was obtained from equation 30, Appendix III, using the spring constant for the stack position from the observed critical damping of the regulator with no damping transformer or antihunt resistor. The mechanical damping may be greater than this, which would make the calculated and test curves in closer agreement. There is no direct method of measuring the mechanical damping of this type regulator. It is also likely that the assumed amplification of 200 is higher than the actual value.

Test curves show that for a given transformer there is an upper limit to the value of M/R (obtained by decreasing R_p) which will provide stable operation. Although the feedback voltage (proportional to M/R_p) can be further increased by reducing R_p, the self-inductance in the primary winding (neglected in Appendix III) becomes appreciable and causes the feedback correction to lag the error voltage, so that the resultant damping is reduced. A lower limit for the value of R_p to give maximum feedback for effective damping with a given transformer, generator, and regulator combination could be obtained by solving a fifth-order equation. The practical way to obtain this limiting value is from test curves.

Conclusions

Factors Affecting Stability

Mutual Coupling in Generator. It is theoretically possible for an oscillatory condition to occur following a switching transient in a system composed of a generator (without voltage regulator), a battery, and a resistance load. The condition necessary to produce oscillation is large mutual coupling between the armature and shunt field windings. This coupling must be in the direction to produce cumulative compounding.

Cumulative Compounding in Generator. Generators having compensating and interpole windings and having the brushes shifted against rotation (in order to give maximum output) most nearly approach this condition. However, the measured constants of present *P*-1, *P*-2, and *R*-1 generators fall so far short of the values necessary to produce oscillatory instability (without a regulator) that the compounding tendency was concluded to be a secondary factor in the creation of an oscillatory condition.

Generators having rising volt-ampere characteristics may become unstable when operating with other generators or a battery. This is a nonoscillatory type of instability. This phenomenon was reproduced in the laboratory with a *P*-1 generator having compensating windings and a film on the commutator such as to give maximum compounding. However, this transient was so slow that the addition of a voltage regulator prevented occurrence of the phenomenon. This characteristic may tend to make the task of the voltage regulator more difficult but is again a secondary factor in the creation of system oscillations.

Time Constant of Generator Shunt Field. The aircraft generators investigated have high amplification and very small field time constants, particularly at maximum rated speed. Both of these factors are contrary to the usual case with large industrial machines and make the regulating problem more difficult. The fact that a compensated generator is less stable than an uncompensated machine can be linked directly with their field time constants. A compensated machine needs less main-pole flux to maintain a given voltage and for the same field current requirements will have a weaker shunt field winding with less inductance. This difference in inductance between machines is greater than the difference in total field resistance of the machines for no-load and light-load operating conditions, so that the compensated machine will have a smaller field time constant. The test and calculated curves in Figures 4 and 5 bear out this relationship. Both curves were made with the same regulator, but in Figure 5 the *P*-1 generator having the largest field time constant was used, whereas in Figure 6 the generator having the smallest field time constant was used. The use of interpole or series windings will have a similar effect.

Regulator Mechanical Construction and Assembly. The original assembly and adjustment of the regulator has a bearing on the amount of wear necessary to cause system instability. No two springs are identical, but the differences in individual springs should be maintained as small as possible by checking before assembly. This spring discrepancy and a noncritical final adjustment of the regulator can affect the amount of allowable stack wear by a thousandth of an inch or more. This is illustrated by comparison of the test curve in Figure 6 and the test curve without the antihunt resistor in Figure 7 at $M/R_p=0$.

Regulator Antihunt Resistor

The effect of the antihunt resistor is shown in Figure 7. This resistance allows an additional wear of about 0.0015 inch without any damping transformer and about 0.002 inch with the present Westinghouse damping transformer. Although this antihunt resistor has a damping effect, its full benefit will not always be obtained, because part of the additional wear may be lost in the final adjustment and the regulator still satisfy all regulation and stability requirements.

From the above it can be seen that, if any electrical damping is to be used on this type regulator, the total damping should be in the form of a transformer which can be inserted after final stack and core adjustments have been made.

Optimum Damping Transformer

It can be seen from the curves of stack wear versus amount of feedback necessary for stability that one damping transformer will not compensate for infinite stack wear for all combinations of regulators and generators. Allowance for infinite stack wear is not necessary to maintain stable operation for all operating conditions.

The amount of wear a regulator can allow and still operate satisfactorily as a regulator is limited by the range of its variable resistance. After the stack has worn about 0.012 inch, the regulator cannot be adjusted for 28.5 volts or less, and the operator will see that the regulator should be returned for readjustment.

With this fact in mind, if a damping transformer is designed to allow about 0.012-inch stack wear before instability occurs, the only possibility of unstable operation is the failure of an operator to replace a regulator which obviously should be replaced. A regulator set for 28.5 volts on a 28-volt system still may share load because of its equalizing coil, but the operator can see that it needs replacing from its no-load voltage.

To obtain the correct value of damping transformer, curves similar to the ones shown in Figures 5, 6, and 7 are necessary for each combination of generator and regulator. From these curves it can be seen that an M/R_p of about 0.0003 second would allow a maximum stack wear of 0.013 inch or greater with all combinations shown in these figures. If the regulators of manufacturer *B* without antihunt resistor, Figure 7, were omitted from this analysis (none of this type is in use) an M/R_p of 0.00022 second would allow a maximum stack wear of 0.0245 inch or greater with the other combinations.

A well-designed transformer with the foregoing amounts of damping could be made weighing five ounces or less. Present carbon-pile regulators are 2.5 pounds.

RATE OF RESPONSE

Damping transformers reduce overshoot but do not materially reduce the speed of response.

PARALLEL OPERATION

In laboratory tests covered in a previous report to the Army Air Forces it was shown that, if one regulator is poorly adjusted and does not have a damping transformer, this regulator will affect the whole system and cause instability. Damping transformers on the other regulators in the system have little effect upon the poorly adjusted regulator. Therefore, for stable system operation, each individual regulator must have a damping transformer, and, if each regulator is stable in itself, the system will be stable.

Appendix I. Symbols

Generator:

R_f=resistance of shunt field

R_s=control or stack resistance in series with shunt field

r_f=total resistance of shunt field circuit (R_f+R_s)

R_a=resistance of armature, interpole, and compensating windings

L_f=self-inductance of shunt field

L_a=self-inductance of armature, interpole, and compensating windings

M=mutual inductance between shunt field and armature circuit windings, including mutual inductance of any demagnetizing or magnetizing armature, interpole, or compensating turns

$T_f=(=L_f/r_f)$ time constant of shunt field circuit

e_a=generated armature voltage

Kk_f=volts generated in the armature per ampere in the shunt field. Determined from the no-load saturation curve at maximum rated generator revolutions per minute, Kk_f is the slope of a line tangent to the saturation curve at the operating voltage

e_o=fictitious residual voltage determined by intersection of straight line assumed for Kk_f with voltage axis at zero field current; e_o includes brush drop

Kk_a=volts generated in the armature per ampere in the armature. Kk_a includes any cumulative or differential effect of brush shift or interpole and compensating windings

Load:

R_2=resistance of load

R_b=internal resistance of battery

E_b=open-circuit battery voltage

Regulator:

R_c=resistance of regulator coil

L_c=inductance of regulator coil

$T_c=(=L_c/R_c)$ time constant of regulator coil

R_m=mechanical damping of regulator in dyne centimeters per second

$1/C_m=S$, spring constant of regulator in dynes per centimeter. This includes the combination of both spring and carbon stack and is assumed to be a constant for small deviations from a given position

L_m=mass of moving portion of regulator in grams. (Value taken equal to weight of armature plus one third of carbon stack)

$t_d=M/R_p$ feed-back ratio of transformer which has the dimensions of a time constant (=transformer mutual inductance/primary resistance)

A=amplification factor of regulator. (The total amplification of the system can be expressed as AR_c which is in the order of 200 for this system)

Appendix II. Operational Equations for Generator and Load Circuits

Figure 4 shows the arrangement of the system analyzed to determine the effect of generator characteristics on the oscillatory instability condition which had been observed in service. From preliminary tests and field reports it was observed that instability occurred in a system composed of a single generator and resistance load just as readily as in more complicated systems. For this reason it was not considered necessary to analyze the more general (and more complicated) system with inductive or overhauling loads and more than one generator. Lead resistance between the generator and battery was ignored for the sake of simplicity.

The operational equations for the voltage in the different circuits are as follows:

Load circuits

$$e=E_b+i_3R_b \qquad (1)$$

$$e=i_2R_2 \qquad (2)$$

Generator shunt field circuit

$$e=(R_s+R_f+L_fp)i_f+Mpi_a \qquad (3)$$

Generator armature circuit

$$e=e_a-(R_a+L_ap)i_a-Mpi_f \qquad (4)$$

Other relations

$$i_a=i_f+i_1 \qquad (5)$$

$$i_1=i_2+i_3 \qquad (6)$$

$$e_a=e_o+K(k_fi_f+k_ai_a) \qquad (7)$$

$$r_f=R_s+R_f \qquad (8)$$

If the generator is assumed to be operating at low flux density (on the straight-line portion of the saturation curve), the equation for the generated voltage is

$$e_a=e_o+K\phi \qquad (9)$$

where

ϕ=flux per pole (megalines)

K=total generated armature volts per megaline of flux per pole

The flux, ϕ, is a function of both the shunt field current and the armature current.

$$\phi=k_fi_f+k_ai_a \text{ (megalines)} \qquad (10)$$

k_f=megalines per pole per shunt field ampere

k_a=megalines per pole per ampere in the armature circuit

If the foregoing simultaneous equations are solved for the relationship between i_1 and t, the following well-known operational equation is obtained:

$$(p^2+\alpha p+\beta)i_1=C \text{ (amperes per second}^2\text{)} \qquad (11)$$

where

$$\alpha=\frac{(R_2+R_b)[(L_f+M)(Kk_a-R_a)-r_fL_a-(Kk_f+Kk_a-R_a)M]-R_bR_2(2M+L_a+L_f)}{(R_2+R_b)(M^2-L_fL_a)} \qquad (12)$$

$$\beta=\frac{(R_2+R_b)(Kk_a-R_a)r_f+R_bR_2(Kk_f+Kk_a-r_f-R_a)}{(R_2+R_b)(M^2-L_fL_a)} \qquad (13)$$

and

$$C=\frac{-e\hat{o}(R_2+R_b)r_f-E_bR_2(Kk_f+Kk_a-r_f-R_a)}{(R_2+R_b)(M^2-L_fL_a)} \qquad (14)$$

Equation 11 is a second-order linear operational equation whose solution is

$$i_1=\epsilon^{-\alpha/2\,t}[c_1\epsilon^{\gamma t}+c_2\epsilon^{-\gamma t}]+i_o \qquad (15)$$

where

$$\gamma=\sqrt{\alpha^2/4-\beta} \qquad (16)$$

and

$$i_o=C/\beta$$

The integration constants c_1 and c_2 can be determined as follows from the boundary conditions of switch *S* having been opened previously and then closed at $t=0$:

$$c_1+c_2+i_{ss}=i_o \qquad (17)$$

where

$$i_o=\frac{C}{\beta},\ R_2=\infty \text{ initial condition} \qquad (18)$$

Capacitors for High-Frequency Induction-Heating Circuits

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Synopsis: Increased military production has accelerated the application of high-frequency heating where accurate control of temperature for short intervals is fundamentally important. The heating of metals by induction has been widely applied. The present paper describes the development of a new type of dielectric liquid called Lectronol. Capacitors containing this liquid are particularly well adapted for use in the tank circuit of electronic heaters used in induction heating. The capacitor is water-cooled and is housed in a hermetically sealed completely filled nonmagnetic container so constructed as to provide sufficient flexibility to take care of the liquid expansion due to thermal changes. The capacitor is noteworthy because of the absence of cellulose sheet insulation, satisfactory operation being entirely dependent on the superior insulating properties of the Lectronol. Capacitors containing this liquid are characterized by low dielectric loss and high dielectric strength over the frequency range utilized in power oscillators. The capacity per unit volume is approximately twice that obtained with mineral oil. The electrical characteristics of the capacitor are stable under severe conditions of use.

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The Need for a New Capacitor

HIGH-FREQUENCY ELECTRIC power has been applied to practical industrial uses for more than 20 years. The use of high-frequency generating equipment has increased steadily during that time. Not only has such equipment been familiar to many in the communication, radio, medical, and scientific-investigation fields, but in recent years it has come to be recognized as an economic industrial machine. Particularly since the start of industrial production for military use, there has been a tremendous increase in the application of powerful high-frequency generators for concentrated heating applications. The use of high-frequency currents and voltage industrially is largely for heating where speed in attaining a localized temperature for a very short interval is fundamentally important. The heating of metals by induction and heating of nonconductive materials by dielectric absorption has been widely described in recent technical literature.[1–6].

The generation of high-frequency current, whether by rotating generators, by spark gaps, or by electronic-tube oscillators, involves tuned circuits of inductance and capacitance. Since large amounts of heat energy are most readily transferred from the power source to the part heated by the use of high voltage, high current, or both combined, we must deal with coils and capacitors as well as other components of the oscillating circuit which will stand the high-voltage stresses and carry large currents. Such a circuit is commonly referred to as a "tank" circuit because of its energy-storage properties.

Capacitors for tuning such circuits are a fundamental part of the high-frequency generating set, in fact, representing up to one third of the cost of a complete equipment in some representative designs in use. In the past, capacitors used in tank circuits operating at frequencies above 15,000 cycles have been mostly mica-di-

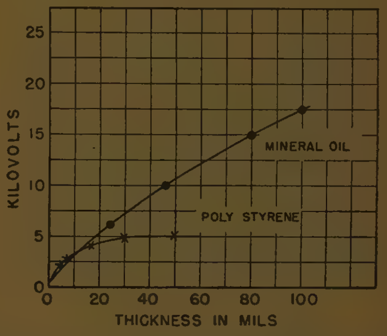

Figure 1. The dielectric-strength–thickness relation for mineral insulating oil and polystyrene films

Test temperature—25 degrees centigrade
Test frequency—500 kilocycles

$$i_{ss}=\frac{C}{\beta},\ R_2=R_2 \text{ final condition} \qquad (19)$$

$$c_1=\frac{(i_o-i_{ss})(\gamma+\alpha/2)}{2\gamma} \qquad (20)$$

$$c_2=\frac{(i_o-i_{ss})(\gamma-\alpha/2)}{2\gamma} \qquad (21)$$

Appendix III. Application of Damping Transformer

The present system is idealized in Figure 8.[2] The regulator is represented by two time constants, one electrical and one mechanical which is shown by an electrical equivalent. The circuit equations for voltage changes can be written as follows:

$$e_c=(R_c+L_cp)i_c=(1+T_c)i_cR_c \qquad (22)$$

$$R_ci_c=e_m=(R_m+1/C_mp+L_mp)i_m \qquad (23)$$

This circuit represents the mechanical part of the regulator, where the resistance is dependent upon the spring position for any one condition or, in electrical terms, the voltage output of the regulator is equal to the voltage across the condenser times the amplification of the regulator. In this analysis the amplification of the regulator is represented by a separate amplifier.

$$i_m/C_mp=e_f=(r_f+L_fp)i_f=(1+T_fp)i_fr_f \qquad (24)$$

$$i_fr_f=e_a \qquad (25)$$

$$e_c=-A[e_a+t_dp(e_f-i_fR_s)] \qquad (26)$$

(The negative sign is used because corrections must be opposite to deviations.)

From equations 22, 23, and 24

$$e_c=(1+T_cp)(1/C_m+R_mp+L_mp^2)(1+T_fp)C_me_a \qquad (27)$$

From equations 24 and 25

$$e_f=(1+T_fp)e_a \qquad (28)$$

Substituting 25 and 28 into 26 gives

$$e_c=A[1+t_dp(1-R_s/r_f+T_fp)]e_a \qquad (29)$$

Combining equations 27 and 29 yields

$$e_a=\frac{-Ae_a/C_m[1+t_dp(R_f/r_f+T_fp)]}{(1+T_cp)(1+T_fp)(1/C_m+R_mp+L_mp^2)} \qquad (30)$$

Upon using the constants for any particular regulator and generator combination in equation 30, a value of t_d can be obtained to give critical damping. The value necessary for critical damping can be obtained by the use of Routh's criterion without the labor of solving a fourth-order equation.

For a fourth-order equation, such as

$$ap^4+bp^3+cp^2+dp+e=0$$

the necessary condition for critical damping is that

$$bcd\geq b^2e+d^2a$$

where critical damping is defined that the system will be stable and nonoscillatory in the steady state.

References

1. DESIGN OF REACTANCES AND TRANSFORMERS WHICH CARRY DIRECT CURRENT, C. R. Hanna. AIEE TRANSACTIONS, volume 46, 1927, pages 155–60.

2. RECENT DEVELOPMENTS IN GENERATOR VOLTAGE REGULATION, C. R. Hanna, K. A. Oplinger, C. E. Valentine. AIEE TRANSACTIONS, volume 58, 1939, pages 838–44.

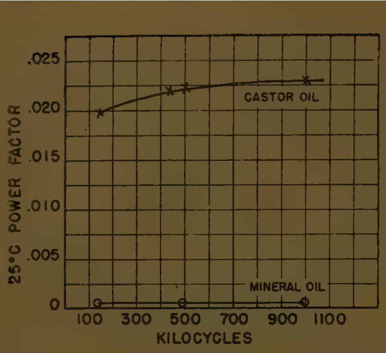

Figure 2. The comparative power-factor characteristics of mineral insulating oil and castor oil

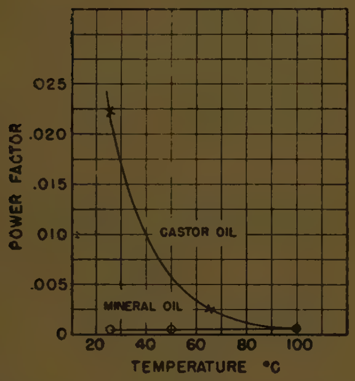

Figure 3. The effect of temperature on the power factor of commercial insulating liquids when tested at 500 kilocycles

electric types. However, the tremendous requirements for mica capacitors by the military services for communication and radar applications, along with difficulty in importing the higher grades of mica, was expected to restrict its use to those applications for which another dielectric could not be readily used. Fortunately other dielectric materials have been found possessing both better operating characteristics and economic advantages for use in induction-heating tank circuits.

What Are the Requirements of a Suitable Capacitor?

Basically, the characteristics required of capacitors for use in industrial high-frequency heaters are not widely different from those required for the familiar use in broadcast transmitters. However, the following requirements should be noted:

1. The current and voltage rating must be high to allow handling more energy. Thus it becomes desirable and economical to build higher kilovolt-ampere (kilovolts × amperes) ratings into single units to avoid a multiplicity of series and parallel connections of smaller ratings.

2. Dissipation of capacitor losses with the continuous operation necessary to meet high-production requirements in industrial plants requires adequate consideration. Although capacitor losses may be very small as compared to the total losses in a complete heater equipment, the effective dissipation of these losses with low-temperature rise is essential. Since cooling water is necessary for quenching of the heated parts, for load coils, spark gaps, vacuum tubes, or other circuit components, water-cooled capacitors are also practical. Forced-air cooling is a practical solution for the lower range of kilowatt ratings where water-cooling is undesirable.

3. Variation in operating frequency is not so critical as for radio transmitters. Therefore, the allowable variation of capacitance with temperature is not so critical in capacitors for induction heaters. This makes it possible to use a number of different dielectric materials.

4. Industrial usage requires much longer periods of operation without shutdown for maintenance or repair than has been accepted as satisfactory for earlier high-frequency radio and laboratory equipment. Reliability is, therefore, an important consideration.

5. The physical size, mounting features, means for both electric and water connections to other circuit components, and accessibility must be suitable for industrial use.

Since electronic heaters use circuits developed first for radio transmitters, the natural course was to use the same circuit components. However, as new development will permit, circuit components more adapted for industrial heating are to be desired. For example, most manufacturers recognize that economics dictates a policy of building equipment for industrial use having operating features and factors of safety which allow operation for very long periods without maintenance or inspection by technical personnel. Such has not often been true with radio equipment or laboratory equipment where constant technical supervision and frequent replacement of parts have been the practice.

The Capacitor Dielectric Material

A rather exhaustive search of the entire field of dielectric materials suitable for high-frequency operation has been made.

Liquid dielectrics have been traditionally accepted as adaptable for high-voltage use only when used in conjunction with a solid insulation. In capacitors and cables, the solid used has invariably been paper sheets or tapes. These alone have not been satisfactory for high-voltage use because of their low-dielectric-strength value. When carefully dried and impregnated with a liquid dielectric, however, the replacement of the air normally present in the fibrous structure of the cellulose sheet by the liquid dielectric has produced a combination which has proved to be highly efficient as a dielectric for high-voltage use. In the low-frequency-capacitor application, a noninflammable synthetic liquid is particularly well adapted because of its high dielectric constant.

With the advent of the power oscillator however, the traditional interdependency

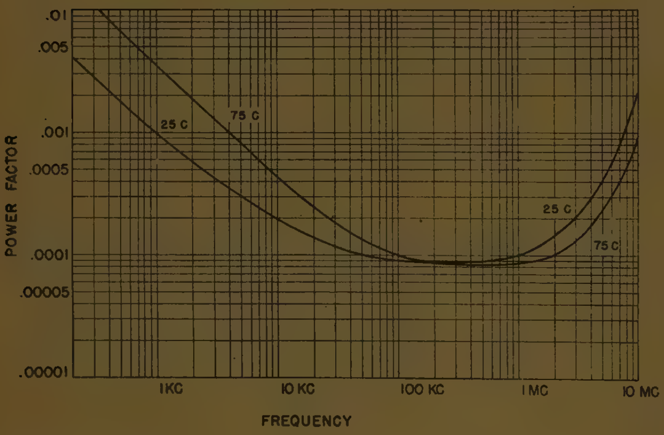

Figure 4. Power-factor versus frequency characteristics for Lectronol measured on standard concentric test cell with low-voltage oscillator

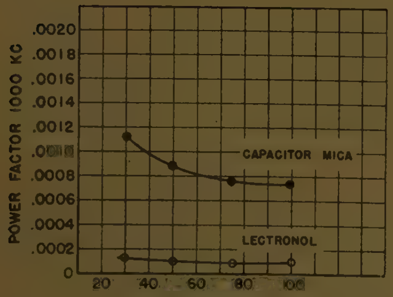

Figure 5. Comparing the temperature effect on the power factor of capacitor mica and Lectronol tested at 1,000 kilocycles

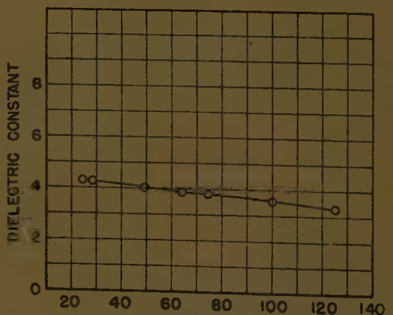

Figure 6. The dielectric-constant–temperature relation for Lectronol in the range of 100–1,000 kilocycles

of the liquid and solid insulation for high-voltage use has been subjected to critical review. The presence of the cellulose sheet insulation in capacitors operating at high voltage in the range of frequencies above 100 kilocycles has not been found to be entirely practicable. The result has been the development of a capacitor which is noteworthy because of the entire absence of cellulose sheet insulation between the electrode plates. The satisfactory operation of the capacitor is entirely dependent on the insulating liquid used. Such capacitors have been designated as "liquid-filled high-frequency parallel-plate capacitors" or more simple "class-*HFP* capacitors."

The Effect of Frequency on the Dielectric Strength

Montsinger[7] has suggested that the dielectric strength of solid insulation varies with frequency and the duration of voltage application in accordance with the relation:

$$E=1.225+\frac{0.525}{\sqrt[4]{T}}\times f^{-0.137}$$

where

E = dielectric strength (volts)
T = duration of voltage application (seconds)
f = frequency of test voltage (cycles per second)

Although the effect of frequency on the breakdown of solid insulation is dependent to some extent on the thickness of insulation tested, the relationship suggested by Montsinger for frequencies up to 90 kilocycles has been found to be approximately true at frequencies in the range of 100 to 1,000 kilocycles. Thus, Montsinger's relation would predict that the dielectric strength of solid insulation at 500 kilocycles would equal 30 per cent of the corresponding 60-cycle value. By actual test using dielectric pads made up of styrene films, a material of good dielectric properties at both low and high frequencies, the dielectric strength at 500 kilocycles averages 21 per cent of the 60-cycle breakdown value.

In contrast with this severe loss in dielectric strength which is exhibited by solid insulation, liquid dielectrics show marked stability. When tested at 60-cycle voltages the dielectric strength of insulating liquids in good condition averages about 30 kv, using one-inch-diameter square-edge brass electrodes, in accordance with the procedures prescribed by the American Society for Testing Materials.[8] The dielectric strength tested under the same conditions at a frequency of 500 kilocycles average about 60 per cent of the 60-cycle value. The greater dielectric-stability characteristic of insulating liquids when tested at the higher frequencies renders this type of dielectric well suited for application in high-frequency high-voltage capacitors. This is illustrated in Figure 1, which compares the dielectric strength of polystyrene and mineral oil tested at 500 kilocycles 25 degrees centigrade. It is only for the lower dielectric thickness that the solid dielectrics are possessed of dielectric strengths comparable to the liquids. It is in this manner that they have heretofore been widely used. For high-voltage use at high frequency, the liquid dielectrics have marked superiority.

Table I

Voltage	Microfarad	Microhenrys	Megacycles Resonant Frequency
2,000	0.034	0.075	3.5
2,000	0.025	0.085	3.0
6,000	0.010	0.12	4.5
6,600	0.009	0.10	5.3
9,000	0.0056	0.13	5.8
9.000	0.0029	0.12	9.0

Commercial Dielectric Liquids

Mineral transformer oil and castor oil heretofore have been considered to possess characteristics suitable for use under high-frequency voltage. In the range from 100 to 1,000 kilocycles, mineral transformer oil has low electrical loss. As illustrated in Figure 2, the power factor is less than 0.0005 at 30 degrees centigrade. Castor oil, however, has excessively high losses at room temperature. A capacitor using castor oil as a dielectric therefore would be expected to operate at temperatures well above room temperature. As illustrated in Figure 3, an increase in temperature results in a continuously decreasing electrical loss. At 100 degrees centigrade, the electrical losses in castor oil are reduced to values equally as low as those characteristic of mineral oil.

In high-frequency-capacitor use, neither of these insulating liquids is satisfactory. Mineral transformer oil, although possessing excellent high-frequency loss values, has a dielectric constant of such low value (2.18) that capacitors using this liquid possess large physical volume per microfarad. Castor oil possesses a suitably high dielectric constant (4.2 at 30 degrees centigrade). Its high electrical losses, however, coupled with its recognized chemical instability at elevated temperature, greatly restrict its application in high-frequency electric capacitors.

Electrical and Chemical Requirements

A liquid dielectric suitable for use in high-frequency plate capacitors must meet exacting electrical and chemical requirements. It must possess low electrical losses over the whole range of possible operating temperatures and frequencies. It must possess high dielectric strength. It must be chemically stable under high-voltage stress at the electrical frequencies to which it will be subjected. If subject to oxidation, the products formed must be of a type which do not affect its electrical losses, dielectric strength, or other properties which would prevent its continued successful operation under high-frequency electrical stresses. Lectronol is such a liquid. This synthetic oil is a thin, colorless, mobile, synthetic, organic liquid possessing the following characteristics:

Flash point—180 degrees centigrade
Reaction—Neutral
Pour point——8 degrees centigrade
Dielectric constant (25 degrees centigrade)—4.4
Dielectric strength (25 degrees centigrade)—40 kv (average for 0.1-inch gap)

POWER FACTOR OR LOSSES

Over the frequency range utilized in power oscillators the synthetic oil described is characterized by low dielectric loss, as shown in Figure 4. As illustrated in Figure 5, the losses are even lower than those obtained with mica over the tem-

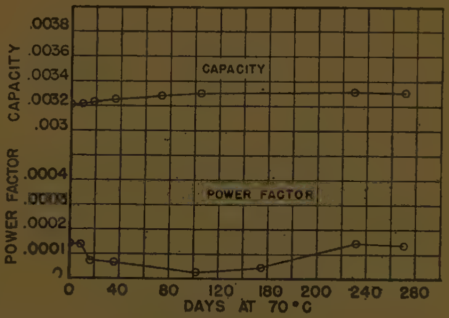

Figure 7. Illustrating the electrical stability of high-frequency parallel-plate capacitors filled with Lectronol heated at 70 degrees centigrade in contact with air

Test frequency—500 kilocycles
Temperature of electrical measurement—25 degrees centigrade

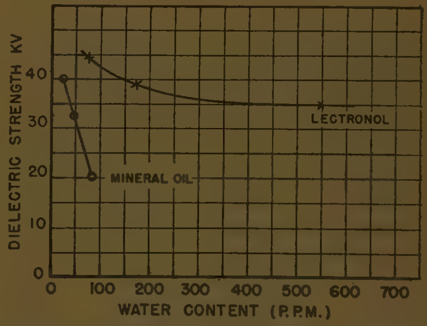

Figure 8. The dielectric strength of Lectronol is maintained at a high value even in the presence of moisture of sufficient amount to produce a marked drop in the dielectric strength of mineral oil

Test temperature—25 degrees centigrade
Test frequency—60 cycles
Test gap—100 mils

perature range from 30 to 100 degrees centigrade.

Most liquids characterized by low dielectric loss at high frequency are also characterized by a low dielectric constant. Lectronol is an exception. Although having a loss even lower than that of mica, its dielectric constant is approximately twice that of mineral oil. In Figure 6 is illustrated the dielectric constant as affected by temperature over the range from 25 to 125 degrees centigrade. The dielectric constant falls in a linear relation to the increase in temperature.

A liquid to be satisfactory for capacitor use must not only possess a suitably low dielectric loss but also be able to maintain this favorable characteristic under the most adverse operating conditions. Parallel-plate capacitors having Lectronol as the sole high-frequency dielectric show substantially no electrical deterioration even when heated for 200 days at 70 degrees centigrade (Figure 7).

Repeated studies with Lectronol substantiate its remarkable suitability for use in high-frequency capacitors. Those factors which normally would be expected to result in increased power factor have been found to have but little effect. Oxidation produces neither sludge nor dielectric deterioration. Neither power factor nor dielectric strength are affected measurably by arcing a hundred or more times when the current in the arc is of low value and not many cycles duration. Accidental arcing between capacitor plates through the liquid does not cause measurable deterioration, if the arc currents are limited to low values of a few amperes which avoids melting of the plates.

Dielectric Strength

Lectronol is characterized by a high and stable dielectric strength. Most insulating liquids, such as mineral oil, show a rapid drop in dielectric strength with increasing water content. Such liquids have but little capacity for holding water in solution. This insulating liquid, on the other hand, has substantially greater solvency for moisture. Its saturation value is approximately ten times that of mineral oil. Substantial increases in its water content have only minor effect on its dielectric strength. This is illustrated in Figure 8, the data of which have been obtained in accordance with the American Society for Testing Materials, testing method *D*117, using one-inch-diameter square-edge brass electrodes spaced 0.100 inch apart. The marked ability of the liquid to resist the deteriorating effects of contaminants, such as moisture, is reflected in its excellent dielectric strength on high-frequency voltage under conditions of practical use.

In Figure 9 are illustrated the comparative dielectric-strength values of dry liquid-filled class-*HFP* capacitors, plotted as a function of the gap distance under 550-kilocycle voltage. Even under the condition of low water content, most favorably adapted to produce the highest dielectric-strength values for mineral oil, it shows no superiority to the synthetic insulating liquid under high-frequency voltage. The excellent dielectric strength of the latter is not materially affected by temperature increase from 25 to 100 degrees centigrade (Figure 10).

Capacitor Construction

After the selection of the liquid with suitable characteristics, the plate assembly, insulators, supports, and terminal bushings are next important. The two sets of parallel plates must be spaced accurately and permanently, for which heavy-gauge aluminum stock is most desirable. The two plate assemblies must be rigidly supported and yet allow for thermal expansion, which is accomplished with Mycalex plates. This insulating material has very good high-frequency voltage strength as well as low loss under high-frequency stress and also high mechanical strength. More than ample voltage-creepage surfaces are obtained readily with the large sheet used.

Very effective water cooling of the assembly is obtained by connecting the cooling coil to the grounded set of plates, thus giving direct heat flow from the plates through the connectors into the tubing and into the water. Also the water tubing serves as a current conductor, making possible small-size high-current connections going directly to the load transformer or tank coil. Current-carrying connections between series or parallel connected units may be combined with cooling water tubing connections.

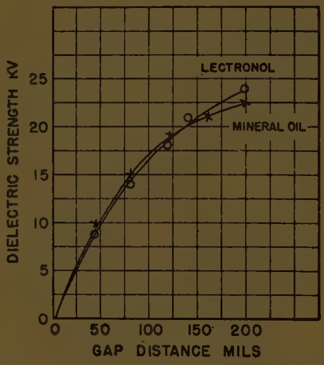

Figure 9. A comparison of the dielectric-strength characteristics of high-frequency parallel-plate capacitors filled with dry mineral oil and Lectronol measured at 25 degrees centigrade under 500-kilocycle test voltages

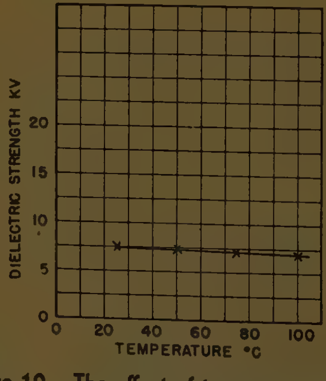

Figure 10. The effect of temperature on the dielectric strength of high-frequency parallel-plate capacitors filled with Lectronol

Test frequency—500 kilocycles
Gap distance—36 mils

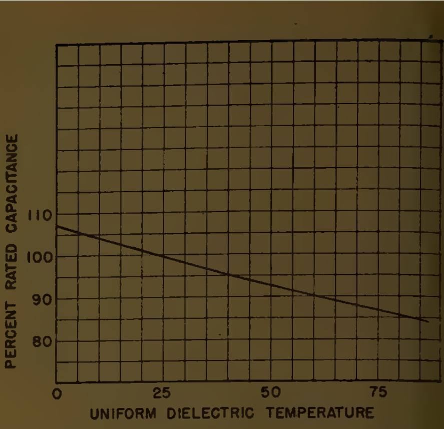

Figure 11. Variation of capacitance with temperature in degrees centigrade measured at 60 cycles

The container is nonmagnetic of double-lock seam construction, providing sufficient flexibility in side walls to take care of liquid expansion due to thermal changes. The case is completely filled with liquid and hermetically sealed. Removable footed-type mounting brackets are provided for supporting the capacitor in any desired position, protecting the ends from damage and yet not interfering with flexibility of the capacitor sides.

Terminal bushings require particular design consideration to obtain sufficiently low-voltage stress distribution to avoid dielectric heating in the insulating material and yet have high current-carrying capacity. When a hollow shell type of ceramic bushing is used and the interior is filled with the low-loss, high-strength liquid, the dielectric loss in the bushing assembly is held to a minimum, and liquid cooling about the current-carrying parts is effective in dissipating losses. Two or more such bushings, connected in paralle to split the current and having water-cooling tube connections at the top, allow high-current operation of a singl capacitor. For voltage ratings of si kilovolts and above, corona shields adde at the base of the bushing are an advantage, making it possible to use smaller bushing and less creepage surfac and at the same time providing a definit flashover voltage value at a point awa from the ceramic surface.

Electrical Characteristics

It is comparatively easy to maintain close tolerance in capacitance of ±

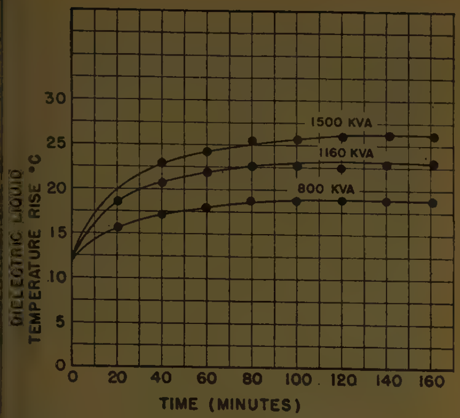

Figure 12. Typical dielectric-liquid temperature rise in degrees centigrade above outlet water temperature variation with time in operation for high-frequency parallel-plate capacitor rated 0.0056 microfarad, 500 kilocycles, and one-half-gallon-per-minute water flow

per cent in production of such capacitors, as capacitance variation between units is almost entirely a result of variation in physical dimensions.

Change in capacitance with temperature is dependent on both change in dielectric constant of liquid temperature and change in physical spacing of plates with linear metallic expansion. Capacitance variation with temperature is between one fourth and one third of 1 per cent per degree centigrade as shown by Figure 11. This change in capacity is recognized as being too great for use where the frequency of the tank circuit must be held within narrow limits, such as for radiobroadcasting. However, a slight change in frequency of three to four per cent which is typical of changes from no-load to full-load operating temperature is not objectionable for induction heating. The use of cooling water aids appreciably in holding a constant operating temperature in capacitors, as the full-load continuous temperature rise is held to a reasonably low value.

The internal inductance of such capacitors is low with only a small variation due to change in physical dimensions such as height of terminal bushings above the case. Since internal inductance varies only between 0.075 and 0.13 microhenrys, the resonant frequency varies with the microfarad rating as shown by Table I.

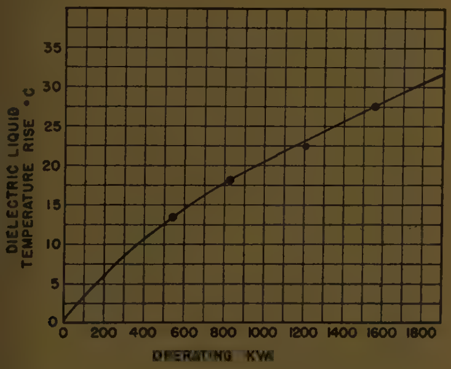

Figure 13. Typical dielectric-liquid temperature rise in degrees centigrade above outlet water-temperature variation with operating kilovolt-amperes for capacitor rated 0.0056 microfarad, nine kilovolts, 540 kilocycles with one-half-gallon-per-minute water flow

Though individual capacitors with minimum length of bus connections are suitable for operation at several megacycles, particular consideration must be given to keeping the length of the tank circuit and loop area of the circuit connections at a minimum, if the higher-frequency range of the capacitors is to be utilized.

An outstanding characteristic is the low loss of these capacitors at high frequency. Losses are a combination of I^2R or metallic-conductor loss and the dielectric loss in the liquid. Metallic losses are low, because design features include parallel-current paths, good electric conductors, and well-made joints between conductors. In fact in the practical application losses in the tank coil and the two capacitor terminals are a much larger factor than the internal losses of the capacitor. For example, a capacitor rated 0.0056 microfarad operating at 540 kilocycles, 9,000 volts rms, and 171 amperes, on continuous operation will show 3.3 degrees rise in cooling water with a flow rate of one-half gallon per minute. Using the constant of 31.6 watts per pound of water per minute per degree centigrade, we calculate a total loss of 448 watts.

Thus, the calculated power factor with full-load operation at 1,539 kva is 0.00029. With reference to Figure 5, the liquid dielectric power factor is 0.0001, leaving a power factor of 0.00019 assignable to I^2R or metallic losses.

Some designers prefer the use of Q factor for comparison of circuit component losses. As the Q factor of a capacitor is equal to the reciprocal of the per cent power factor times 100, we get a factor of approximately 3,450 for this rating and operating load.

The use of water cooling tends to stabilize operating temperatures with constant temperature rise reached in about $1^1/_2$ hours continuous operation.

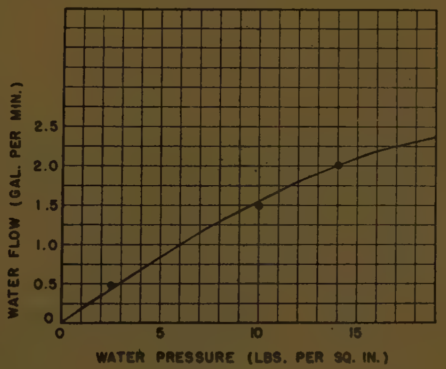

Figure 14. Water-flow variation with pressure applied to cooling coil of high-frequency parallel-plate capacitors

Typical temperature-rise curves are shown in Figure 12. The outlet water temperature controls the operating temperature of the capacitor. In order that the temperature of the dielectric liquid does not exceed 68 degree centigrade, the outlet water temperature is limited to 35 degrees centigrade at 1,500 kva and above. With a limit on operating temperature of liquid, it is reasonable that outlet temperature may be higher when kilovolt-ampere loading is lower, as shown by Figure 13. For economy in use of cooling water, several capacitors may be connected in series for water flow, depending on the pressure available. Figure 14 shows the pressure drop per capacitor for a range of different water-flow rates. The temperature rise in the cooling water is low and dependent on the operating kilovolt-amperes as shown by Figure 15.

Basis of Rating

The voltage rating is fixed by two factors:

1. The maximum peak transient voltage which the capacitor is expected to withstand.

2. The continuous rms operating voltage which contributes to the temperature rise or heating load on a capacitor.

Consistent with dielectric strength of the liquid and plate spacing, all capacitors are designed for operation with transient voltage peaks not to exceed 200 per cent of the rms high-frequency rating.

The usual practice of listing test-voltage ratings on mica capacitors in place of maximum continuous rms rating has not proved to be satisfactory for industrial application. Having only the ultimate short-time voltage strength listed as a rating makes it necessary for each designer of equipment in which mica capacitors are used to decide for himself the factor of safety to be allowed on operating voltage as against test voltage. In too many cases the pressure of space requirements and economics, along with inex-

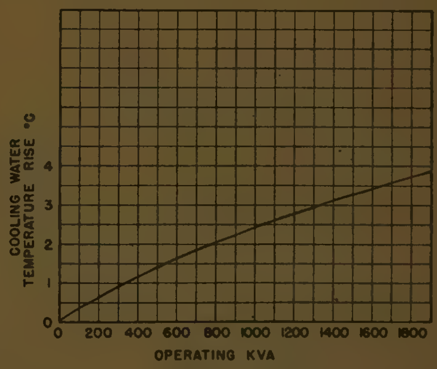

Figure 15. Variation in cooling-water temperature rise in degrees centigrade with operating kilovolt-amperes for high-frequency parallel-plate capacitor rated 0.0056 microfarad, nine kilovolts, 540 kilocycles

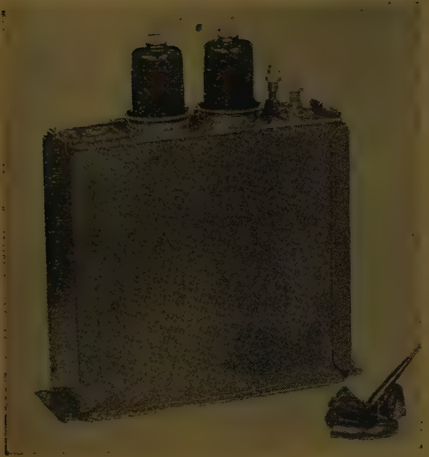

Figure 16. General Electric high-frequency parallel-plate capacitor unit for electronic-heater resonant-circuit applications

High-frequency, parallel-plate, water-cooled, rated 0.0056 microfarad, 9,000 volts, 171 amperes, at 540 kilocycles

perience on life statistics if at all available, all lead to the result that mica capacitors are used with insufficient factor of safety for continuous operation.

It is essential that adequate consideration be given to the effective dissipation of losses for continuous operation of all capacitors and that operation is in accordance with predetermined ratings under which the capacitor is designed to operate. For a safe economical industrial installation, a capacitor designer must establish the operating rating.

For class-*HFP* capacitors both voltage and current ratings are based on a frequency of 540 kilocycles which has been selected as being in the most popular range for electronic induction-heater operation. As the capacitor is rated for a maximum voltage at 540 kilocycles, the current rating at lower frequencies must be reduced in direct proportion to the frequency change, while the voltage rating remains unchanged. For frequencies above 540 kilocycles, the current rating can be increased to limits allowed by operating temperatures of the dielectric liquid and terminal conductors. Water cooling of terminal conductors has been found practical because of much higher current rating allowed. For example, a typical pair of terminals as shown by Figure 16 will carry 95 amperes at 540 kilocycles with 45 degrees rise when connected with conductors having natural air cooling but will carry 171 amperes when connected to water-cooled tubing with approximately the same rise.

Conclusions

1. A much improved and more economical high-voltage high-current tank circuit capacitor has been developed for induction-heating and dielectric-heating oscillators.

2. A new high-dielectric-strength liquid with high dielectric constant and very low loss over a wide frequency range has been applied in a practical fixed-plate capacitor design. Having the container, including terminal bushings, completely filled with liquid and hermetically sealed insures that the capacitor will keep its original electrical characteristics.

3. Installation of capacitors with larger sizes required for electronic heaters has been simplified. Maintenance cost has been reduced because of larger factor of safety from overheating on continuous operation in high ambient temperatures.

4. Large amounts of mica previously used for capacitors in electronic heating oscillators have been saved for military radio and radar applications requiring capacitors with minimum change in capacitance with temperature and requiring physical dimensions interchangeable with standard mica capacitor sizes previously available.

Figure 17. Rear view of electronic heater, 15 kw, type *F*-15, model *4F15E1*, without rear door

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Guides for Operation of Transformers, Regulators, and Reactors

AIEE TRANSFORMER SUBCOMMITTEE

Guide for Loading Oil-Immersed Distribution and Power Transformers

SCOPE

THIS guide covers general recommendations for loading all types of oil-immersed transformers with class-*A* insulation except water-cooled transformers built before 1929, self-cooled transformers rated at 200 kva or less, and transformers having a rated full-load current less than ten amperes. They include transformers immersed in synthetic or noninflammable liquids.

The manufacturer should be asked for recommendations relating to water-cooled transformers built before 1929 and for more specific recommendations for large and important transformers if required.

CAUTION

It must be recognized that loads above rating should not be applied without a thorough study of the various limitations involved. Among these limitations are oil expansion, pressure in sealed-type units, bushings, leads, soldered connections, tap changers, and the thermal capability of associated equipment such as cables, reactors, circuit breakers, disconnecting switches and current transformers. These may constitute the practical limit in load-carrying ability.

Before loading apparatus above rating to the full extent covered in these guides, it is recommended that load capabilities be checked with the manufacturer.

GENERAL

Recommendations in this guide are based on life expectancy of transformer insulation as affected by operating temperature and time. Transformer life expectancy at various operating temperatures is not accurately known, but the information given regarding loss of life of insulation is considered to be conservative and the best that can be produced from present knowledge of the subject. The effects of temperature on insulation life are being investigated continuously by the transformer subcommittee, and new findings may affect future revision of the guides. The word "conservative" is used in the sense that the expected loss of insulation life for a single recommended overload will not be greater than the amount stated.

The rated kilovolt-ampere output of a transformer is that load which it can deliver continuously at rated secondary voltage without exceeding a given temperature rise measured under prescribed test conditions. The temperature rise on which the rating is based takes into consideration the experience of the industry regarding

1. Insulation life as affected by operating temperature.
2. The ambient temperatures assumed to exist throughout the life of the transformer.

The actual output which a transformer can deliver at any time in service without undue deterioration of the insulation may be more or less than the rated output, depending upon the ambient temperature and other attendant operating conditions.

Practically all of the data in reference to the aging of insulation at different temperatures has been obtained in laboratory tests in which the decrease in mechanical strength has been measured. The relation between the life expectancy of insulation as indicated by laboratory tests and the actual life of a transformer is largely theoretical, so that loading based on such information must be tempered by sound judgment based on experience.

As the evaluation of the cumulative effects of temperature and time in causing deterioration of transformer insulation is not thoroughly established, it is not possible to predict with any great degree of accuracy the length of life of a transformer even under constant or closely controlled conditions, much less under widely varying service conditions.

The change in rate of deterioration with change in temperature has received much study, but the actual values for different operating temperatures are not completely agreed on. An average value which has been used for a number of years is that the rate of deterioration doubles with each eight degrees centigrade that the temperature of the insulation increases. This value is not accurate for all temperatures nor for all points in the life of a transformer. It is given here to show the importance of controlling ambient temperatures in indoor installations by means of ventilation, thus controlling the insulation temperature and life.

Owing to the many variables mentioned and particularly to the many varying conditions of load and ambient to which a transformer can be subjected in service, it is not possible to give definite rules for the loading of transformers. It is only possible to give suggested loadings under specified conditions and look to the user to make the best use of this information for his particular problem.

AMBIENT TEMPERATURES AND LOADING FOR NORMAL LIFE EXPECTANCY

The basic loading condition of a transformer for normal life expectancy is continuous loading at rated kilovolt-amperes and rated delivered voltage with the temperature of the cooling air at no time exceeding 40 degrees centigrade and with the average temperature of the cooling air during any 24-hour period equal to 30 degrees centigrade. For water-cooled transformers comparable temperatures are 30 and 25 degrees centigrade, the five-degree-centigrade lower average temperature of the cooling medium being to allow for possible less efficient cooling due to coils that are not clean. (It is assumed that operation under the foregoing conditions is equivalent to operation at a continuous ambient temperature of 30 degrees centigrade for cooling air and 25 degrees centigrade for cooling water.)

The hottest-spot copper temperature is the sum of the temperature of the cooling medium, the average temperature rise

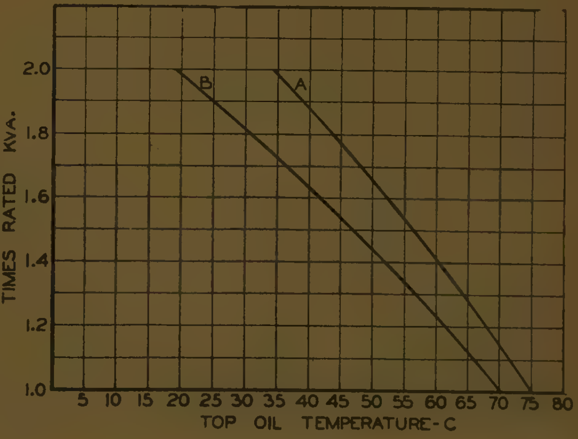

Figure 1. Permissible loading for normal life expectancy based on top-oil temperature

A—Water-cooled, self-cooled, and forced-air-cooled transformers rated 133 per cent or less of self-cooled rating

B—Forced-oil-cooled transformers or forced-air-cooled transformers rated over 133 per cent of self-cooled rating

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Paper 45-130, recommended by the AIEE committee on electric machinery for publication in AIEE TRANSACTIONS. Manuscript submitted February 26, 1945; made available for printing May 23, 1945.

This paper is a complete revision of AMERICAN RECOMMENDED PRACTICE—GUIDES FOR OPERATION OF TRANSFORMERS AND REGULATORS, American Standard C57.3—1942. It will be presented to the American Standards Association sectional committee on transformers C57 for inclusion in proposed revision of transformer Standards.

of the copper and the hottest-spot allowance. For oil-immersed transformers operating continuously under the foregoing conditions with normal life expectancy this temperature has been assumed to be 95 degrees centigrade.

Other usual service conditions, as given in American Standard C57.1, section 2.000, must be complied with.

Years of experience have indicated that a transformer rated in accordance with the Standard and operated under the foregoing conditions will have a reasonably long life.

AMBIENT TEMPERATURES

Ambient temperature is an important factor in determining the load capability of a transformer. Whenever the actual ambient temperature can be determined from readings taken at the time of the load being considered, such temperatures should be used.

It is often necessary to predict the load that a transformer can carry safely at some future time when the actual ambient temperature is unknown. A method of approximating the ambient temperature for the month in which the expected load is anticipated is as follows (Temperatures referred to are contained in reports prepared by the Weather Bureau of the United States Department of Commerce and are available for various sections of the country):

1. For loads with normal life expectancy:

Use the average temperature over a period of years for the month involved. Some believe that this temperature should be increased by as much as five degrees centigrade because

(*a*). The average temperature for a given month may be above the average temperature of the same month for a period of years.

(*b*). Increased aging at higher than average temperature is not offset by decreased aging at lower than average.

2. For short-time loads with moderate sacrifice of life expectancy:

Use the average of the daily maximum temperatures for the month involved averaged with similar values for the same month over a period of years.

During any one day there is an even chance that the maximum temperature will exceed this value. Some feel that five degrees centigrade should be added to the average of the daily maximum temperatures. If this is done, the assumed temperature will not be exceeded on more than a few days of the month, and, where it is exceeded, the loss of life will not be more than twice that for the temperature assumed.

LOADING UNDER SPECIFIED CONDITIONS WITH NORMAL LIFE EXPECTANCY

Loading Determined by Measured Temperature. Transformers may be loaded above rating for any period of time provided the hottest-spot temperature is not greater than 95 degrees centigrade. Hottest-spot temperature indicators are available by which this temperature may be measured.

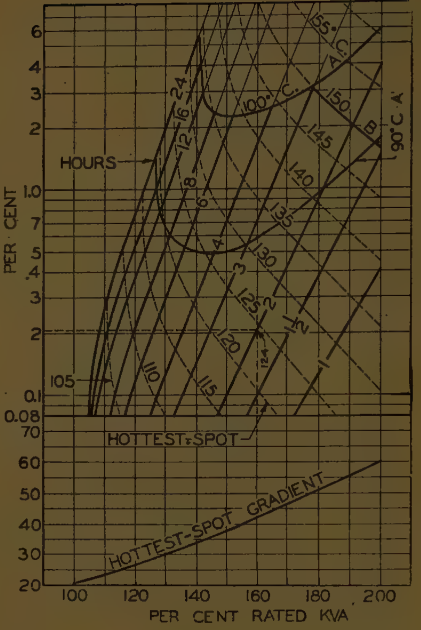

Figure 2. Short-time loading with moderate sacrifice of life expectancy, self-cooled and water-cooled transformers

Following 50 per cent or less of rated kilovolt-amperes or top-oil temperature rise of 25 degrees centigrade

Ambient temperature 30 degrees centigrade for air, 25 degrees centigrade for water

Subtract hottest-spot gradient from hottest-spot temperature to obtain top-oil temperature

A—Assumed temperature limitation for top oil (protected)

A'—Assumed temperature limitation for top oil (unprotected)

B—Assumed temperature limitation for hottest-spot copper

Time, Hours	Times Rated Kilovolt-Amperes to Use not More Than Following Life (Per Cent)			
	0.1	0.25	0.50	1.0
1/2	2.00	2.00	2.00	2.00
1	1.76	1.91	2.00	2.00
2	1.50	1.62	1.72	1.82
4	1.27	1.38	1.46	1.53
8	1.13	1.21	1.30	1.37
24	1.05	1.10	1.15	1.23

For explanation of degree of accuracy, see Appendix II.

On account of large variations in the difference between the hottest-spot copper and the top-oil temperatures of various designs under full load, oil temperature alone should not be used as a guide in loading transformers.

However, by taking the temperature rise of the hottest-spot copper over top oil (hottest-spot copper gradient) into account, oil temperatures may be used as an approximate guide in loading. When so operated for any period of time, the measured top-oil temperature should not exceed the values given in Figure 1 for the load being carried. These loads can be applied to any transformer covered by these guides as they are conservative.

Oil temperatures may be used as a more accurate guide, by using the hottest-spot copper gradient of any particular transformer for the load it is carrying at the time. If the hottest-spot copper gradient at full load is known, the gradient at other loads may be determined easily by using Figure 10 in Appendix II. The limiting oil temperature for normal life expectancy for any load, regardless of the time carried, can be determined by subtracting the hottest-spot copper gradient for that load from 95 degrees centigrade.

Loading on Basis of Test Temperature Rise. For each degree centigrade in excess of two degrees that the test temperature rise is below the standard temperature rise specified in the Standard, the transformer

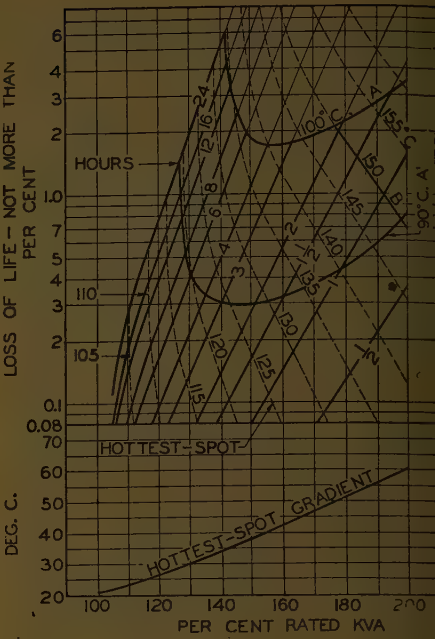

Figure 3. Short-time loading with moderate sacrifice of life expectancy, self-cooled and water-cooled transformers

Following 100 per cent of rated kilovolt-amperes or top-oil temperature rise of 45 degrees centigrade

Ambient temperature 30 degrees centigrade for air, 25 degrees centigrade for water

Subtract hottest-spot gradient from hottest-spot temperature to obtain top-oil temperature

A—Assumed temperature limitation for top oil (protected)

A'—Assumed temperature limitation for top oil (unprotected)

B—Assumed temperature limitation for hottest-spot copper

Time, Hours	Times Rated Kilovolt-Amperes to Use not More Than Following Life (Per Cent)			
	0.1	0.25	0.50	1.0
1/2	1.75	1.92	2.00	2.00
1	1.54	1.69	1.81	1.92
2	1.35	1.48	1.58	1.68
4	1.20	1.32	1.40	1.48
8	1.11	1.20	1.28	1.35
24	1.05	1.09	1.15	1.23

For explanation of degree of accuracy, see Appendix II.

load may be increased above rated kilovolt-amperes by the percentages shown in second column of Table I. Making use of this factor gives the kilovolt-amperes that the transformer can deliver with 55 degrees centigrade temperature rise. The leeway of two degrees is to provide for a negative tolerance in the measurement of temperature rise.

Some transformers are designed to have the difference between hottest-spot and average copper temperatures greater than the nominal allowance of ten degrees centigrade. This will result in a temperature rise for average copper of less than 55 degrees centigrade, but the hottest-spot copper temperature rise may be at the limiting value of 65 degrees centigrade.

Such transformers should not be loaded above rating as outlined under this heading. The manufacturer should be consulted to give information as to design hottest-spot allowances.

Loading on Basis of Ambient Temperature. For each degree centigrade that the average temperature of the cooling medium is above or below 30 degrees centigrade for air or 25 degrees centigrade for water, a transformer may be loaded for any period of time below or above its kilovolt-ampere rating as specified in Table I. Average temperature should be for periods of time not exceeding 24 hours with maximum temperatures not more than ten degrees centigrade greater than average temperatures for air and five degrees centigrade for water. On the basis used in this guide for calculating loss of life, life expectancy will be approximately the same as if it had been operated at rated kilovolt-amperes and standard ambient temperatures over that period.

The use of transformers in cooling air above 50 degrees centigrade, or below 0 degrees centigrade, or with cooling water above 35 degrees centigrade is not covered by Table I and should be taken up with the manufacturer.

Loading on Basis of Load Factor. When the load factor for a period of time not exceeding 24 hours is below 100 per cent, the maximum loading of a transformer during that period may be increased above rated kilovolt-amperes by the percentages shown in Table II for each per cent that the load factor is below 100 per cent. On the basis used in this guide for calculating loss of life, life expectancy will be approximately the same as if it had been operated at rated kilovolt-amperes during that period.

Loading on Basis of Short-Time Loads Above Rating. When short-time loads above rating occur not more than once in any 24-hour period, the maximum loading of a transformer during that period may be increased conservatively above rated kilovolt-amperes, as given in Table III. On the basis used in this guide for calculating loss of life, life expectancy will be approximately the same as if it had been operated at rated kilovolt-amperes during that period.

Effect of Various Factors Existing at One Time. When two or more of the following factors affecting loading for normal life expectancy exist at one time, the effects are cumulative and the increase in loads due to each may be added to secure the maximum suggested load (each increase must be based on rated kilovolt-amperes):

1. Loading on basis of test temperature rise.

2. Loading on basis of ambient temperature.

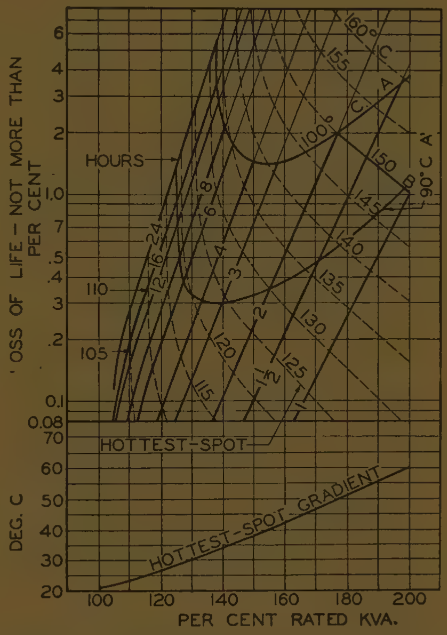

Figure 4. Short-time loading with moderate sacrifice of life expectancy, forced-air-cooled transformers rated 133 per cent or less of self-cooled rating

Following 50 per cent or less of rated kilovolt-amperes or top-oil temperature rise of 25 degrees centigrade

Ambient temperature 30 degrees centigrade for air

Subtract hottest-spot gradient from hottest-spot temperature to obtain top-oil temperature

A—Assumed temperature limitation for top oil (protected)

A'—Assumed temperature limitation for top oil (unprotected)

B—Assumed temperature limitation for hottest-spot copper

Time, Hours	Times Rated Kilovolt-Amperes to Use not More Than Following Life (Per Cent)			
	0.1	0.25	0.50	1.0
½	1.97	2.00	2.00	2.00
1	1.66	1.79	1.90	2.00
2	1.39	1.51	1.59	1.68
4	1.21	1.31	1.38	1.45
8	1.11	1.19	1.26	1.33
24	1.05	1.09	1.15	1.22

For explanation of degree of accuracy, see Appendix II.

3. Either loading on basis of load factor or loading on basis of short-time overloads. Do not use both.

SHORT-TIME LOADING WITH MODERATE SACRIFICE OF LIFE EXPECTANCY

To be able to give an idea of the relative loss of life due to loads above rating under various conditions, we have assumed a conservative aging rate of insulation based on tests and operating experience. The information is not intended to furnish a basis for calculating the normal life expectancy of transformer insulation.

Transformers may be loaded above rated kilovolt-amperes following various

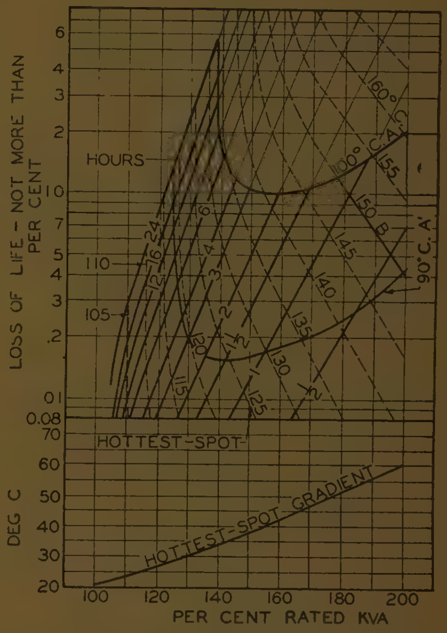

Figure 5. Short-time loading with moderate sacrifice of life expectancy, forced-air-cooled transformers rated 133 per cent or less of self-cooled rating

Following 100 per cent of rated kilovolt-amperes or top-oil temperature rise of 45 degrees centigrade

Ambient temperature 30 degrees centigrade for air

Subtract hottest-spot gradient from hottest-spot temperature to obtain top-oil temperature

A—Assumed temperature limitation for top oil (protected)

A'—Assumed temperature limitation for top oil (unprotected)

B—Assumed temperature limitation for hottest-spot copper

Time, Hours	Times Rated Kilovolt-Amperes to Use not More Than Following Life (Per Cent)			
	0.1	0.25	0.50	1.0
½	1.67	1.82	1.94	2.00
1	1.47	1.60	1.71	1.81
2	1.29	1.41	1.50	1.58
4	1.18	1.28	1.35	1.43
8	1.10	1.18	1.26	1.33
24	1.05	1.09	1.15	1.21

For explanation of degree of accuracy, see Appendix II.

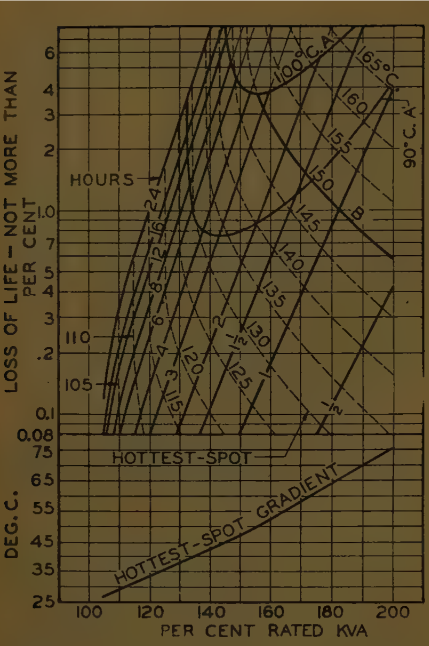

Figure 6. Short-time loading with moderate sacrifice of life expectancy, forced-oil-cooled transformers or forced-air-cooled transformers rated over 133 per cent of self-cooled rating

Following 50 per cent or less of rated kilovolt-amperes or top-oil temperature rise of 22 degrees centigrade

Ambient temperature 30 degrees centigrade for air

Subtract hottest-spot gradient from hottest-spot temperature to obtain top-oil temperature

A—Assumed temperature limitation for top oil (protected)

A'—Assumed temperature limitation for top oil (unprotected)

B—Assumed temperature limitation for hottest-spot copper

Time, Hours	Times Rated Kilovolt-Amperes to Use not More Than Following Life (Per Cent)			
	0.1	0.25	0.50	1.0
1/2	1.78	1.92	2.00	2.00
1	1.53	1.64	1.73	1.82
2	1.32	1.42	1.49	1.57
4	1.18	1.26	1.33	1.40
8	1.10	1.17	1.24	1.31
24	1.05	1.08	1.14	1.20

For explanation of degree of accuracy, see Appendix II.

loads by various amounts for various times, and the approximate relative loss of life may be estimated.

The load capability of transformers varies widely and is affected by the following characteristics:

1. Top-oil rise over ambient temperature.
2. Hottest-spot rise over top-oil temperature (hottest-spot copper gradient).
3. Ratio of load losses to no-load loss.
4. Time constant.*
5. Ambient temperature.

It is not possible to give a single set of loads which will apply to all transformers for a given set of operating conditions.

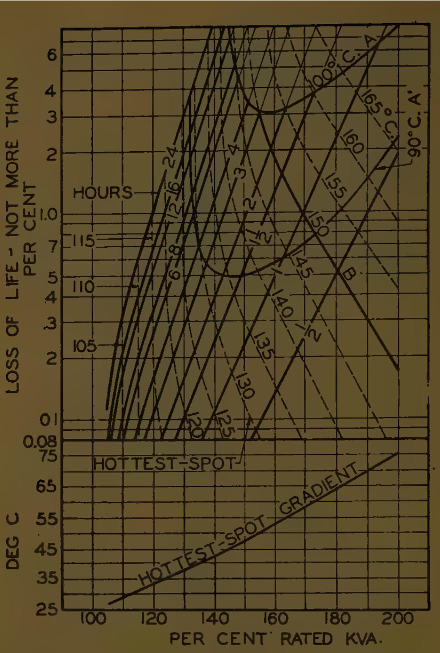

Figure 7. Short-time loading with moderate sacrifice of life expectancy, forced-oil-cooled transformers or forced-air-cooled transformers rated over 133 per cent of self-cooled rating

Following 100 per cent of rated kilovolt-amperes or top-oil temperature rise of 40 degrees centigrade

Ambient temperature 30 degrees centigrade for air

Subtract hottest-spot gradient from hottest-spot temperature to obtain top-oil temperature

A—Assumed temperature limitation for top oil (protected)

A'—Assumed temperature limitation for top oil (unprotected)

B—Assumed temperature limitation for hottest-spot copper

Time, Hours	Times Rated Kilovolt-Amperes to Use not More Than Following Life (Per Cent)			
	0.1	0.25	0.50	1.0
1/2	1.56	1.70	1.80	1.90
1	1.39	1.50	1.59	1.69
2	1.26	1.36	1.43	1.51
4	1.16	1.25	1.32	1.39
8	1.10	1.18	1.24	1.31
24	1.05	1.09	1.14	1.20

For explanation of degree of accuracy, see Appendix II.

Therefore, such values are given for a transformer having specified characteristics, and these characteristics have been selected so that allowable loads for practically all other combinations of characteristics will be at least as great as those given.

As the operating conditions under which transformers may be loaded are so extremely variable, particularly with regard to frequency of load, data are given so that the approximate relative loss of insulation life for various loads for various times may be determined. The user can then determine the allowable load for his conditions by taking into account the probable number of such loads during the life of the transformer and the approximate percentage of life which he is willing to sacrifice.

Appendix I gives a method by which the user can produce similar data for transformers having other characteristics, and the loads for a given hottest-spot temperature may be somewhat greater than those given hereafter.

In the curves and tables that follow, the following limitations have been used:

Maximum oil temperature (unprotected)* .... 90 degrees centigrade‡
Maximum oil temperature (protected)*.... .... 100 degrees centigrade‡
Maximum hottest-spot copper temperature .... 150 degrees centigrade
Maximum short-time loading (one-half hour or more)..200 per cent**

Short-Time Loads Above Rating for Transformer With Specified Characteristics

Assumed Transformer Characteristics. (See Table IV.) By the use of the data in the Table V the user may determine hottest-spot copper temperature for various loads and times following 50-per-cent load or less and 100-per-cent load and from this determine the approximate loss of life for such loading. Or he may determine loading for any selected loss of life. The user can select loadings and losses of life most suitable to his conditions. An average loss of life of one per cent per year incurred during emergency operations is considered reasonable.

Top-oil temperature, when known, is a better criterion of conditions at the start of a given load than the previous conditions of load, because the oil may not have reached its ultimate temperature for the previous load.

On each of the afore-mentioned figures ultimate top-oil temperature rise for the preceding load is given. This value may be used to determine the curve to use and as a guide in interpolation.

Hottest-spot copper temperatures versus loss of life assumed in making up figures are given in Figure 11.

Additional Short-Time Loading Available. In addition to the short-time loads outlined, the transformer may be loaded above or below kilovolt-ampere rating in accordance with directions given under "Loading for Normal Life Expectancy." (Each increase must be based on rated kilovolt-amperes.)

1. Loading on basis of test temperature rise.

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*See Appendix II for explanation.

†Oil is considered to be protected, or unprotected, depending on whether the transformer is or is not equipped with conservator or inert gas above the oil.

‡At high oil temperatures precautionary measures, such as draining oil, may be necessary to prevent overflow or excessive pressures.

**Current-carrying parts, other than windings, may impose a lower limit.

2. Loading on basis of ambient temperature.

THERMAL RELAYS

Thermal relays designed for application to individual transformers are available. They make use of hottest-spot temperature in their operation and automatically take into account ambient temperature and previous conditions of loading, since the actuating element is influenced by the hottest oil and the load current. Higher loads are permitted for short times than are permitted for long times. These relays can be designed, at the option of the user, to permit loads which will produce practically no sacrifice of life expectancy or some predetermined moderate sacrifice of such expectancy. Contacts may be wired to cause sounding an alarm, starting of fans, or disconnecting of the transformer. Application of such relays should be discussed with the manufacturer.

SUPPLEMENTAL COOLING OF EXISTING SELF-COOLED TRANSFORMERS

The load that can be carried on existing self-cooled transformers can be increased by as much as 66 per cent by adding auxiliary cooling equipment such as fans, external forced-oil coolers, or water-spray equipment. The amount of additional loading varies widely, depending on:

(a). Design characteristics of the transformer.

(b). Type of cooling equipment.

(c). Permissible increase in voltage regulation.

(d). Limitations of associated equipment.

No general guides can be given for such supplemental cooling, and each transformer should be considered individually.

Table I. Loading on Basis of Ambient Temperature

Type of Cooling	Per Cent of Rated Kva: Decrease Load for Higher Temp	Per Cent of Rated Kva: Increase Load for Lower Temp
Self-cooled	1.5	1.0
Water-cooled	1.5	1.0
Forced-air-cooled	1.0	0.75
Forced-oil-cooled	1.0	0.75

Guide for Loading Pole-Type Constant-Current Transformers

INCREASED PERIODS OF OPERATION OF POLE-TYPE CONSTANT-CURRENT TRANSFORMERS FOR AMBIENT TEMPERATURES LOWER THAN NORMAL

Since pole-type constant-current transformers operate outdoors at night when the ambient temperature is comparatively low, it is often desirable to take advantage of the longer period of operation within the limiting temperature rise which is made possible by an ambient temperature lower than normal. The time of operation of a pole-type constant-current transformer may be increased as the average ambient temperature decreases, as shown in Table VI.

Guide for Loading Step-Voltage and Induction-Voltage Regulators

A voltage regulator has a short-time load capability greater than the nameplate rating, the magnitude depending on the initial load and ambient temperature, as in the case of a transformer. However, there are some additional factors that must be considered in this type of equipment. The fact that the induction machine has a winding in slots on a rotating member mounted in bearings necessitates a limit to the load to prevent excessive hottest-spot temperatures and undue vibration. Also, a regulator must be capable of operating or holding against the torque produced by load. In the case of a step-voltage regulator, the burning of contacts is the limiting feature.

A working group of the AIEE transformer subcommittee is now reviewing the loading of regulators to determine if the loads given here can be increased safely.

Table II. Loading on Basis of Load Factor

Type of Cooling	Increase in Per Cent of Rated Kva	Maximum Per Cent Increase*
Self-cooled	0.5	25
Water-cooled	0.5	25
Forced-air-cooled	0.4	20
Forced-oil-cooled	0.4	20

* Corresponds to 50 per cent load factor.

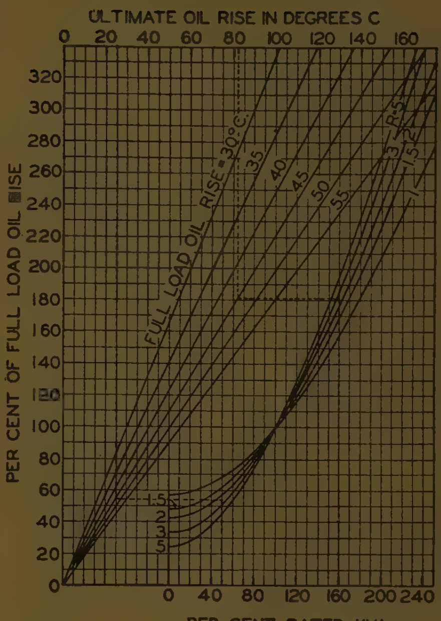


Figure 8. Ultimate top-oil temperature rise

SHORT-TIME LOADS ABOVE RATING FOR OIL-IMMERSED SELF-COOLED REGULATORS

Short-time loads above rated kilovolt-amperes may be applied to oil-immersed self-cooled regulators with daily average cooling air at 30 degrees centigrade in accordance with the limits given in Table VII.

Loads with normal life expectancy may be carried once every 24 hours, whereas those with moderate sacrifice of life expectancy should be carried infrequently.

With cooling-air temperatures other than 30 degrees centigrade the short-time loads may be increased by one per cent of

Table III. Loading on Basis of Short-Time Loads Above Rating

Time, Hours	Times Rated Kilovolt-Amperes: Self-Cooled and Water-Cooled Transformers			Forced-Air-Cooled Transformers Rated 133 Per Cent or Less of Self-Cooled Rating			Forced-Air-Cooled Transformers Rated More than 133 Per Cent of Self-Cooled Rating and All Forced-Oil-Cooled		
*Initial Load (per cent)	90	70	50	90	70	50	90	70	50
1/2	1.59	1.77	1.89	1.45	1.58	1.68	1.36	1.47	1.50
1	1.40	1.54	1.60	1.31	1.38	1.50	1.24	1.31	1.34
2	1.24	1.33	1.37	1.19	1.23	1.26	1.14	1.18	1.21
4	1.12	1.17	1.19	1.11	1.13	1.15	1.09	1.10	1.10
8	1.06	1.08	1.08	1.06	1.07	1.07	1.05	1.06	1.06

*Percentages fix the load which is assumed to exist before the short-time load is applied. Use either average load for two hours previous to load above rating or average load for 24 hours (less overload period), whichever is greater.

Ambient temperature assumed for this table is 30 degrees centigrade for air and 25 degrees centigrade for water.

As the loads may be applied once every 24 hours, and as there is some evidence that the rate of insulation deterioration at about 100 degrees centigrade doubles with less than 8 degrees centigrade increase in insulation temperature the values have been based on 4 degrees centigrade rather than on 8 degrees centigrade.

rated kilovolt-amperes for each degree centigrade that the average temperature is below 30 degrees centigrade, except that no further increases are recommended beyond the 25-per-cent limit cited in the table. Average temperature should be for periods of time not exceeding 24 hours with maximum temperatures not more than ten degrees centigrade greater than average temperatures.

Short-time loads should be reduced two per cent of the continuous self-cooled rating below the recommended load for each degree centigrade that the daily average temperature of the cooling air exceeds 30 degrees centigrade.

Table IV. Assumed Transformer Characteristics

	Self-Cooled Water-Cooled	Forced-Air-Cooled*	Forced-Air-Cooled† or Forced-Oil-Cooled
Hottest-spot rise (degrees centigrade)	65	65	65
Top-oil rise (degrees centigrade)	45	45	40
Time constant at full load (hours)	3.0	2.0	1.5
Ratio of copper to iron losses	2.5 to 1	3.5 to 1	5 to 1
Ambient temperature (degrees centigrade)	30	30	30

*Forced-air-cooled rating is 133 per cent or less of self-cooled rating.

†Forced-air-cooled rating is over 133 per cent of self-cooled rating.

Table V. Reference to Figures

	Self-Cooled Water-Cooled	Forced-Air-Cooled*	Forced Air-Cooled† or Forced-Oil-Cooled
Following 50-per-cent load or less	Figure 2	Figure 4	Figure 6
Following 100-per-cent load	Figure 3	Figure 5	Figure 7

Values between 50 and 100-per-cent load may be interpolated with sufficient accuracy.

*Forced-air-cooled rating is 133 per cent or less of self-cooled rating.

†Forced-air-cooled rating is over 133 per cent of self-cooled rating.

Guide for Loading Current-Limiting Reactors

See Interim Report on Overloading Current-Limiting Reactors, by transformer subcommittee of the AIEE committee on electric machinery.[1]

Application at Altitudes Greater Than 3,300 Feet (1,000 Meters)

OPERATION AT LESS THAN STANDARD AMBIENT TEMPERATURES

It is recognized as good practice to use apparatus of standard temperature rise at rated kilovolt-amperes at altitudes greater than 3,300 feet (1,000 meters), provided the temperature of the cooling air does not exceed the values for the respective altitudes given in Table VIII. Standard temperature limits will not be exceeded.

Table VI. Loading Pole-Type Constant-Current Transformers

Average Ambient Temperature, Degrees Centigrade	Period of Operation, Hours
30	8
25	10
20	12
15	14
10	16

Table VII. Loading Regulators

Starting at an Oil Temperature Corresponding to Continuous Operation at	Normal Life Expectancy, Per Cent		Moderate Sacrifice of Life Expectancy, Per Cent	
	1 Hr	2 Hr	1 Hr	2 Hr
Full load	0	0	35	30
$^3/_4$ load	20	14	50*	45
$^1/_2$ load	25*	17	50*	50*

*This load is the maximum that can be allowed, owing to hottest-spot temperature, vibration, torque, or contact-burning considerations on this class of equipment.

Table VIII. Application at Altitudes Greater Than 3,300 Feet (1,000 Meters)

Method of Cooling Apparatus	Maximum Allowable Average Temperature of Cooling Air for Carrying Rated Kilovolt-Amperes, Degrees Centigrade			
	3,300 Ft (1,000 M)	6,600 Ft (2,000 M)	9,900 Ft (3,000 M)	13,200 Ft (4,000 M)
Oil-immersed self-cooled	30*	28	25	23
Oil-immersed forced-air-cooled	30*	26	23	20
Oil-immersed forced-oil-cooled with oil to air cooler	30*	26	23	20
Dry-type self-cooled				
Class-*A* insulation	30	27	24	21
Class-*B* insulation	30	26	22	18
Dry-type forced-air-cooled				
Class-*A* insulation	30	24	19	14
Class-*B* insulation	30	22	14	6

*See section 2.000 of American Standard *C57.1* for explanation of average ambient temperature.

Obviously when advantage is taken of lower ambient temperatures at the higher altitudes to offset the increased temperature rise caused by decreased air density, the output cannot be increased to the full extent covered under "Loading on Basis of Ambient Temperature." For such loading the temperature base should be in accordance with Table VIII.

OPERATION AT STANDARD AMBIENT TEMPERATURES

It is recognized as good practice to use apparatus of standard temperature rise at altitudes greater than 3,300 feet (1,000 meters), provided the load to be carried is reduced below rating by the percentages given in Table IX for each 330 feet (100 meters) that the altitude is above 3,300 feet (1,000 meters). Standard temperature rise will not be exceeded.

INSULATION

The dielectric strength of equipment which depends upon air for its insulation decreases as the altitude increases. The dielectric strength at 3,300 feet or less for a given insulation class should be multiplied by the proper altitude correction factor to obtain the dielectric strength at the required altitude (see Table X).

Appendix I. Calculations of Temperature and Loss of Life for Short-Time Loading of Transformers

Short-time loads above rating that use up different amounts of life expectancy for transformers having definite characteristics have been given in Figures 2 through 7. Generally, transformers included in these guides which have other characteristics will have greater load capabilities. This appendix gives information by which similar curves may be derived for such transformers.

In explaining the use of Figures 8 and 9 through 11, a self-cooled transformer having the same characteristics as those selected in the main portion of the guides, and having a load of 160 per cent for two hours following 50 per-cent load, is used as an example. The figures also apply to forced-air-cooled and forced-oil-cooled transformers.

Characteristics at Rated Kilovolt-Amperes Required to Be Known	Example
1. Hottest-spot rise	65 C
2. Top-oil rise	45 C
3. Time constant at full load	3.0 hr
4. Ratio of copper to iron losses	2.5 to 1
5. Ambient temperature	30 C

Top-Oil Rise

1. Determine ultimate top-oil rise for initial load (50 per cent). On Figure 8 start at per cent for initial load (50) and go up to curve for ratio of losses ($R=2.5$). Go to left to line for full-load oil rise (45 degrees

centigrade). Go vertically and read ultimate oil rise (25 degrees centigrade).

2. Determine ultimate top-oil rise for short-time load in question. Repeat procedure in paragraph 1, starting at per-cent load in question (160) and read ultimate oil rise (82 degrees centigrade).

3. On Figure 9 set a straightedge at the initial oil temperature rise on the left scale (25 degrees centigrade) and at the ultimate oil temperature rise on the right scale (82 degrees centigrade). At the intersection of the line so determined and the vertical line through the intersection of lines for the desired time (two hours) and time constant (three hours), determine the oil temperature rise (52 degrees centigrade) at the end of the assumed time (two hours).

Hottest-Spot Copper Rise Over Top-Oil Temperature (Copper Gradient)

On Figure 10 start at per-cent load (160) on horizontal scale and go up to the curve for full-load hottest-spot rise (20 degrees centigrade). Go to left and read copper gradient (42 degrees centigrade) for the per-cent load in question (160 .

Hottest-Spot Copper Temperature

Obtain hottest-spot copper temperature by adding ambient temperature (30 degrees centigrade), top-oil rise (52 degrees centigrade), and hottest-spot rise (42 degrees centigrade), giving 124 degrees centigrade.

Per Cent Loss of Life

From Figure 11 obtain the loss of life (0.2 per cent) for the hottest-spot copper temperature (125 degrees centigrade) and the duration of the load (two hours). In calculating these curves it has been assumed that hottest-spot temperature is constant at its final value during the overload period. When this assumption is made, the elevated temperature during the cooling period need not be considered.

Table XI is suggested as a convenient method of tabulating values obtained by the foregoing procedure and covers values for the example assumed.

Appendix II. Formulas for Calculating Transient Heating of Oil-Immersed Transformers

List of Symbols

θ_a=ambient temperature (degrees centigrade)

θ_g=hottest-spot rise over top-oil temperature (degrees centigrade)

$\theta_{g(fl)}$=hottest-spot rise over top-oil temperature at full load

θ_{hs}=hottest-spot temperature (degrees centigrade)

θ_o=top-oil rise over ambient temperature (degrees centigrade)

θ_{fl}=full-load top-oil rise (degrees centigrade)

θ_i=initial oil rise for $t=0$ (degrees centigrade)

θ_u=ultimate oil rise for load P (degrees centigrade)

Table IX. Application at Altitudes Greater Than 3,300 Feet (1,000 Meters)

Types of Cooling	Correction Factor, Per Cent
Oil-immersed self-cooled apparatus	0.4
Oil-immersed water-cooled apparatus	0.0
Oil-immersed forced-air-cooled apparatus	0.5
Oil-immersed forced-oil-cooled apparatus with oil to air cooler	0.5
Oil-immersed forced-oil-cooled apparatus with oil to water cooler	0.0
Dry-type self-cooled apparatus	0.3
Dry-type forced-air-cooled apparatus	0.5

P=ratio of load to full load

P_t=ratio of equivalent initial load to full load (load which results in an ultimate rise of θ_i)

R=ratio of load loss to no-load loss

e=2.71828

t=time load is applied in hours

B=thermal time constant of transformer for any load or any ratio of initial to ultimate oil temperature (hours)

B_r=time constant for full load beginning with initial temperature rise of 0 degrees (hours)

W_{fl}=total loss at full load (watts)

C=thermal capacity of transformer (watt-hours per pound per degree centigrade)

Hottest-Spot Temperature

The hottest-spot temperature

$$\theta_{hs}=\theta_a+\theta_o+\theta_g \tag{1}$$

Transient heating and cooling equation for top-oil rise over ambient temperature

$$\theta_o=(\theta_u-\theta_i)(1-e^{-\frac{t}{B}})+\theta_i \tag{2}$$

Ultimate top-oil rise for load P

$$\theta_u=\theta_{fl}[(P^2R+1)/(R+1)]^{0.8} \tag{3}$$

Hottest-spot rise over top oil

$$\theta_g=\theta_{g(fl)}P^{1.6} \tag{4}$$

$$\theta_{g(fl)}=\frac{\text{average copper rise over top oil}^*+\text{10 degrees centigrade}}{\text{Time constant at rated kilovolt-amperes}} \tag{5}$$

Table X. Application at Altitudes Greater Than 3,300 Feet (1,000 Meters)

Altitude	Altitude Correction Factor A for Insulation Class
3,300 feet (1,000 meters)	1.00
4,000 feet (1,200 meters)	0.98
5,000 feet (1,500 meters)	0.95
6,000 feet (1,800 meters)	0.92
7,000 feet (2,100 meters)	0.89
8,000 feet (2,400 meters)	0.86
9,000 feet (2,700 meters)	0.83
10,000 feet (3,000 meters)	0.80
12,000 feet (3,600 meters)	0.75
14,000 feet (4,200 meters)	0.70
15,000 feet (4,500 meters)	0.67

Altitude of 15,000 feet is considered a maximum for standard apparatus.

$$B_r=\frac{C\theta_{fl}}{W_{fl}} \tag{6}$$

C=0.06 (weight of core and coils)+0.04 (weight of tank)+1.33 (gallons of oil)

Theoretical Corrections for Preceding Formulas

Theoretically. several corrections should be made in the foregoing formulas when one is calculating transient oil rises, such as corrections for change in:

1. Time constant for loads above rating.
2. Ultimate copper loss at end of load period.
3. Oil viscosity.

Practical Application of Formulas for Calculation of Oil Rises

In making general calculations based on assumptions of transformer characteristics and maximum hot-spot temperatures which generally have a large factor of safety, results close enough for all practical purposes are obtained if all of these corrections are omitted and the simpler formulas are used.

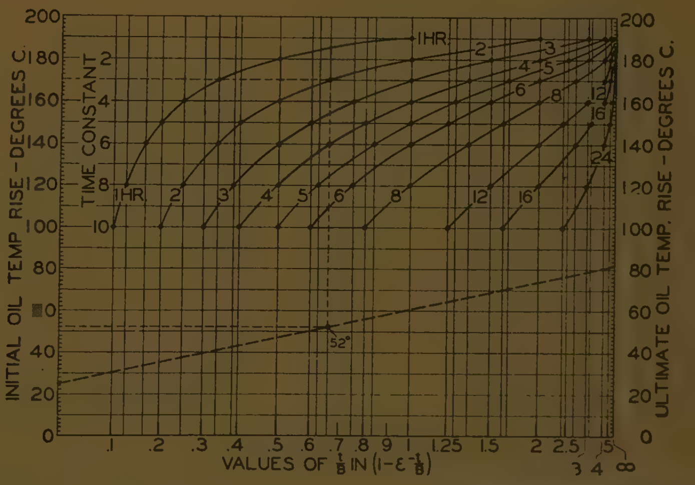
The overload values given in Figures 2 to 7 were calculated by the formulas without corrections.

1. Time Constant. The time constant is the length of time which would be required for the temperature of the oil to change from the initial value to the ultimate value if the initial rate of change were continued until the ultimate temperature was reached.

*At rated kilovolt-amperes from manufacturer's test.

The time constant is usually the length of time required for a specified percentage of

Figure 9. Top-oil temperature rise at end of specified load period



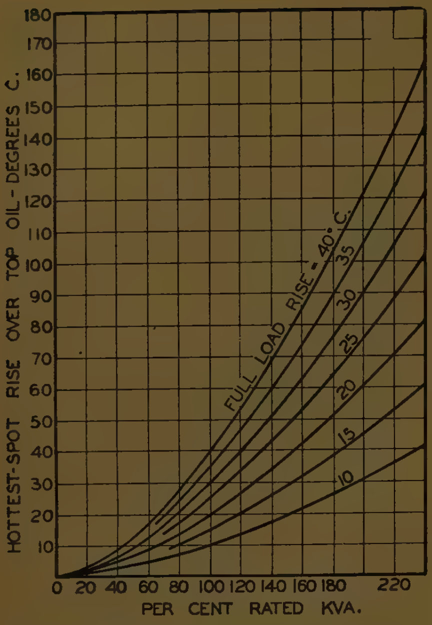


Figure 10. Hottest-spot copper rise over top-oil temperature

the change in temperature from initial value to ultimate value to take place.

If n* equals unity, 63 per cent of the temperature change occurs in a length of time equal to the time constant, regardless of the relationship of initial temperature and ultimate temperature. If n is not unity, the percentage varies and is a function of both initial temperature rise and ultimate temperature rise. In particular, if n equals 0.8, the percentage is 67 if the initial temperature rise is zero.

If the initial temperature rise is greater than zero, the percentage is lower than 67 and decreases as the initial temperature rise increases for a given ultimate temperature rise. If the initial temperature is approximately equal to the final temperature, whether just above or just below it, the percentage is approximately 63. If the initial temperature is greater than the ultimate temperature, the percentage is less than 63.

Since evaluation of the exact percentage for cases where n is not unity and the initial temperature rise is not zero becomes very laborious, it is frequently advisable to use the value of 63 per cent as an approximation. In the more frequently encountered cases where n is approximately 0.8, the error resulting from this procedure is not large compared to the expected error in transient thermal calculations.

If $n=1.0$, equation 6 is correct for any load and any starting temperature. If n is less than 1.0, equation 6 holds only for full-rating starting cold. If $n=0.8$, the time constant for any load and for any starting temperature for either a heating cycle or cooling cycle is given by equations 7 and 8.

$$B=B_r\left[\frac{\frac{\theta_u}{\theta_{fl}}-\frac{\theta_i}{\theta_{fl}}}{\left(\frac{\theta_u}{\theta_{fl}}\right)^{1.25}-\left(\frac{\theta_i}{\theta_{fl}}\right)^{1.25}}\right] \qquad (7)$$

If starting cold ($\theta_i=0$) equation 7 reduces to

$$B=B_r\left(\frac{\theta_{fl}}{\theta_u}\right)^{0.25} \qquad (8)$$

2. *Copper Loss.* As the resistance of the copper for ultimate conditions is greater when the temperature is greater for loads above rating than for rated load conditions, a resistance correction factor should be added to equations 3 and 4.

3. *Viscosity of Oil.* The ultimate temperature rise of oil for a constant loss decreases slightly as the temperature of the oil increases, owing to a decrease in the viscosity of the oil. For hot-spot rise over oil the viscosity correction tends to offset the effect of increased resistance.

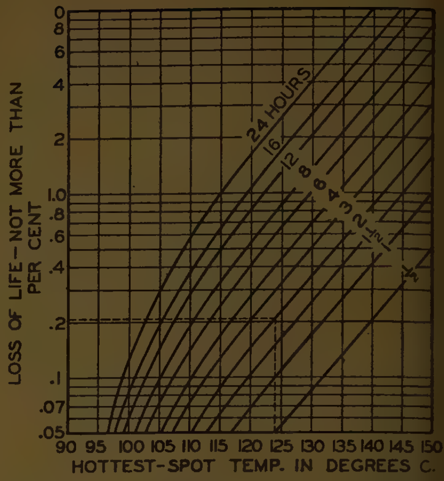


Figure 11. Hottest-spot copper temperature versus loss of life

Time, Hours	Temperatures in Degrees Centigrade to Use not More Than Following Life (Per Cent) 0.1	0.25	0.50	1.0
½	132	142	150	
1	124	134	142	150
2	117	126	134	142
4	111	119	126	134
8	105	112	119	126
24	99	104	109	115

For explanation of degree of accuracy, see Appendix II.

Explanation of Degree of Accuracy of Figures 2 to 7 and Figure 11

The percentage loss-of-life values which will not be exceeded, shown on these figures, of necessity have been made quite conservative, as they depend on the rate of insulation

Table XI. Summary of Values Used in Plotting Figure 2

Per Cent Rated Kva	Ultimate Oil Rise		Hours Duration 0.5	1.0	1.5	2.0	3.0	4.0	6.0	8.0	12.0	16.0	24.0
Top Oil Rise—Degrees Centigrade (θ_o), From Figure 9													
105	48		28	31	34	36	39	42	45	47	47	48	48
110	51		28	32	35	37	41	44	47	49	50	51	51
120	56		29	33	37	40	44	48	52	54	55	56	56
140	68		31	36	42	46	52	57	62	65	68	69	69
160	82		34	41	47	52	61	67	74	78	81	82	82
180	97		36	45	53	60	71	78	87	92	96	97	97
200	113		37	48	59	66	80	89	101	107			
Hottest-Spot Copper Temperature—Degrees Centigrade													
	θ_g (From Figure 10)	$\theta_g+\theta_a$					$\theta_{hs}=\theta_o+\theta_g+\theta_a$						
105	22	52	80	83	86	88	91	94	97	99	99	100	100
110	24	54	82	86	89	91	95	98	101	103	104	105	105
120	27	57	86	90	94	97	101	105	109	111	112	113	113
140	34	64	95	100	106	110	116	121	126	129	132	133	133
160	42	72	106	113	119	124	133	139	146	150	153	154	154
180	51	81	117	126	134	141	152	159					
200	61	91	128	139	150	157							
Loss of Life—Not More Than—Per Cent, From Figure 11													
105											0.05	0.07	0.11
110										0.06	0.13	0.18	0.27
120									0.12	0.21	0.38	0.53	0.80
140							0.14	0.30	0.76	1.43	2.49	3.60	5.40
160					0.10	0.22	0.74	1.65	4.70	8.40			
180				0.13	0.40	0.96	3.65	9.50					
200			0.07	0.40	1.46	3.90							

The above values contain numbers to the second de cimal place to give figures used in plotting curves. No such high degree of accuracy in actual life loss is implied.

See Appendix II for explanation of symbols.

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*Exponential power of temperature rise versus loss.

The Self-Inductance of a Toroidal Coil Without Iron

H. B. DWIGHT
FELLOW AIEE

IF A NONMAGNETIC BAR is bent into the shape of a ring and a single layer of wire is wound around the bar, the self-inductance of the "toroidal coil" formed in this way is given by well-known short expressions.

If the cross section of the nonmagnetic bar, that is, the cross section of the toroidal coil, is rectangular, the self-inductance is

$$L=2hN^2 \operatorname{logn} \frac{r_2}{r_1} \text{ abhenrys} \qquad (1)$$

where

h = axial length of the coil
r_1 = radius to inner face of the coil, centimeters
r_2 = maximum radius of the coil, centimeters
N = number of turns in the coil
logn denotes natural logarithm

If the cross section of the toroidal coil is round, that is, if each turn is a circle, the self-inductance is

$$L=4\pi N^2\left(R-\sqrt{R^2-a^2}\right) \text{ abhenrys} \qquad (2)$$

where

R = mean radius of the complete ring
a = radius of a single turn, centimeters[1]

It was suggested to the writer by R. F. Field of the engineering department of the General Radio Company that a computation for inductance of a toroidal coil with a rectangular section and a comparatively thick winding would be useful. In this paper is derived a formula for such a coil.

If, when the current in the wire is taken to be one abampere, the number of lines in each portion of magnetic flux is multiplied by the number of turns which it surrounds or links, the sum of such products or linkages is equal to the self-inductance of the coil in abhenrys.

Let the thickness of the winding on the inner side of the coil be t_1 (see Figure 1). Then

$$r_3=r_1-t_1$$

$$\text{Length of element } dx=h+(h_1-h)\frac{(r_1-x)}{t_1}$$
$$=h+(h_1-h)\frac{r_1}{t_1}-$$
$$(h_1-h)\frac{x}{t_1}$$

Let it be assumed that the wire follows along dx and has a sharp corner when it goes to the right, instead of a rounded corner.

Let the density of turns per square centimeter be n, a constant depending on the size of wire and the amount of insulation.

$$n=\frac{N}{\pi(r_1^2-r_3^2)} \qquad (3)$$

Also

$$t_2=\frac{N}{2\pi r_1 n} \qquad (4)$$

since the wires cross the axial line t_2 practically at right angles. Similarly,

$$t_3=\frac{N}{2\pi r_2 n} \qquad (5)$$

The dimension h_1, Figure 1, is found by drawing a straight line through the ends of t_2 and t_3.

$$h_1=h+2\left\{t_3+\frac{(w+t_1)(t_2-t_3)}{w}\right\} \qquad (6)$$

Turns linking element dx are

$$T_{(x)}=\pi n(x^2-r_3^2)$$

$$\text{Flux density at } dx=\frac{4\pi T_{(x)}}{2\pi x}=\frac{2T_{(x)}}{x}$$

Linkages for flux in element dx are

$$\text{Flux density}\times T_{(x)}\times\left\{h+(h_1-h)\frac{r_1}{t_1}-\right.$$
$$\left.(h_1-h)\frac{x}{t_1}\right\}dx$$
$$=\frac{2\pi^2n^2}{x}(x^4-2x^2r_3^2+r_3^4)(h+pr_1-px)dx$$
$$=\frac{2\pi^2n^2}{x}\left\{-px^5+(h+pr_1)x^4+2pr_3^2x^3-\right.$$
$$\left.2(h+pr_1)r_3^2x^2-pr_3^4x+(h+pr_1)r_3^4\right\}dx$$

where

$$p=\frac{h_1-h}{t_1} \qquad (7)$$

Integrate from $x=r_3$ to r_1.

Linkages due to flux in the inner trapezoid of base h_1 (Figure 1) are

$$2\pi^2n^2\left[-\frac{px^5}{5}+(h+pr_1)\frac{x^4}{4}+2pr_3^2\frac{x^3}{3}-\right.$$
$$\left.(h+pr_1)r_3^2x^2-pr_3^4x+(h+pr_1)r_3^4 \operatorname{logn} x\right]_{x=r}^{r_1} \qquad (8)$$

The triangle ABC, Figure 1, is filled with wires that are nearly horizontal. The linkages of the magnetic flux in ABC are about one per cent of the total for the coil in the example given. To simplify the computation of this part, the triangle ABC can be replaced by the triangle ADC. These triangles have the same

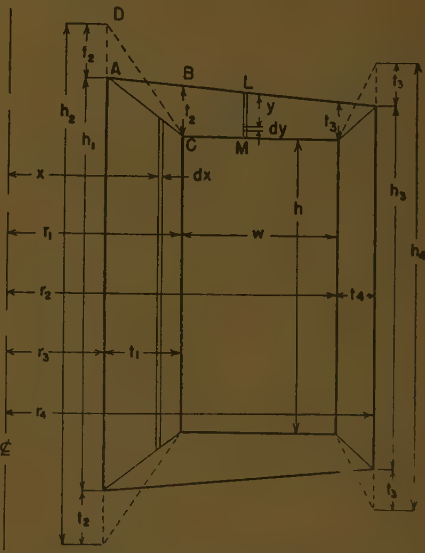

Figure 1. Section of toroidal coil

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deterioration occuring at various temperatures.

Transformer life expectancy with continuous operation at 95 degrees centigrade is not known, but past experience indicates that the life is satisfactory. As a basis for these tables, operation at 115 degrees centigrade for 24 hours has been assumed to give not over one-per-cent loss of insulation life, as determined by tensile strength.

Information given is considered to be the best that can be produced from the present knowledge of the subject. In spite of its approximate nature, it is believed that it will be of much value to the industry as a guide.

The subject of insulation life is receiving the continuous attention of the transformer subcommittee. New findings may make future revision and modification of these guides necessary.

The transformer subcommittee does not recommend that any particular transformer life expectancy be assumed. It specifically points out that more than approximate accuracy in loss of life values is not implied.

Reference

1. INTERIM REPORT ON OVERLOADING CURRENT-LIMITING REACTORS, AIEE transformer subcommittee. AIEE TRANSACTIONS, volume 62, 1943, October section, pages 611–12.

area, as they have the same height t_1, and equal bases $AD = BC = t_2$. In both triangles the flux density is 0 along the side AD or AB and increases toward the vertex C and so the triangle ADC, treated as an extension of the trapezoid of base h_1, gives a close approximation to the linkages of triangle ABC.

Therefore, instead of computing equations 7 and 8, take

$$h_2 = h_1 + 2t_2 \tag{9}$$

$$q = \frac{h_2 - h}{t_1} \tag{10}$$

and linkages due to flux in the enlarged trapezoid of base h_2 are

$$2\pi^2 n^2 \Big[-\frac{qx^5}{5} + (h+qr_1)\frac{x^4}{4} + \frac{2}{3} q r_3^2 x^3 - (h+qr_1) r_3^2 x^2 - q r_3^4 x + (h+qr_1) r_3^4 \operatorname{logn} x \Big]_{x=r_3}^{r_1}$$

$$= 2\pi^2 n^2 \Big[(h+qr_1)\Big(r_3^4 \operatorname{logn} \frac{r_1}{r_3} - r_3^3 t_1 + \frac{1}{2} r_3^2 t_1^2 + r_3 t_1^3 + \frac{1}{4} t_1^4 \Big) - q t_1^3 \Big(\frac{4}{3} r_3^2 + r_3 t_1 + \frac{1}{5} t_1^2 \Big) \Big] \tag{11}$$

since $r_1 = r_3 + t_1$.

Consider a vertical elementary strip LM of width dx, at a distance x from the axis.

$$LM = t_3 + (t_2 - t_3)\frac{(r_2 - x)}{w} = \frac{wt_3 + (t_2 - t_3)(r_2 - x)}{w}$$

The turns which link the flux in $dxdy$ are

$$T = \frac{y}{LM} N = \frac{wyN}{wt_3 + (t_2 - t_3)(r_2 - x)}$$

$$\text{Flux density at } dxdy = \frac{2T}{x}$$

Linkages of the flux in $dxdy$ are

$$\frac{2T^2}{x}\, dxdy = \frac{2}{x} \frac{w^2 y^2 N^2}{\{wt_3 + (t_2 - t_3)(r_2 - x)\}^2}\, dxdy$$

Integrate from $y = 0$ to

$$\frac{wt_3 + (t_2 - t_3)(r_2 - x)}{w}$$

Linkages for the element of width dx are

$$\frac{2}{3x} \frac{\{wt_3 + (t_2 - t_3)(r_2 - x)\} N^2}{w}\, dx$$

Integrate from $x = r_1$ to r_2 and multiply by 2.

Linkages for the trapezoids at the ends of the coil

$$= \frac{4N^2}{3w} \Big[\{wt_3 + r_2(t_2 - t_3)\} \times \operatorname{logn} x - x(t_2 - t_3) \Big]_{x=r_1}^{r_2}$$

$$= \frac{4N^2}{3} \Big[\Big\{ t_3 + \frac{r_2(t_2 - t_3)}{r_2 - r_1} \Big\} \operatorname{logn} \frac{r_2}{r_1} - (t_2 - t_3) \Big] \tag{12}$$

For the outer trapezoid of the coil section, Figure 1,

$$n = \frac{N}{\pi(r_4^2 - r_2^2)} \tag{13}$$

from which r_4 and t_4 may be found.

Note that measured values of t_2, t_3, and t_4 may be used instead of values computed on the basis of constant density of turns.

The length of a vertical element at radius x in the outer trapezoid is

$$h + \frac{x - r_2}{t_4}(h_3 - h) = h - (h_3 - h)\frac{r_2}{t_4}\ (h_3 - h)\frac{x}{t_4}$$

where

$$h_3 = h + 2\Big\{ t_3 - \frac{t_4}{w}(t_2 - t_3) \Big\} \tag{14}$$

Turns linking the element at radius x are

$$T_{(x)} = \pi n (r_4^2 - x^2)$$

$$\text{Flux density at } dx = \frac{2T_{(x)}}{x}$$

Linkages for flux in element of width dx are

$$\text{Flux density} \times T_{(x)} \times \Big\{ h - (h_3 - h)\frac{r_2}{t_4} + (h_3 - h)\frac{x}{t_4} \Big\} dx$$

$$= \frac{2\pi^2 n^2}{x} (x^4 - 2x^2 r_4^2 + r_4^4)(h - p_1 r_2 + p_1 x)\, dx$$

$$= \frac{2\pi^2 n^2}{x} \{ p_1 x^5 + (h - p_1 r_2) x^4 - 2 p_1 r_4^2 x^3 - 2(h - p_1 r_2) r_4^2 x^2 + p_1 r_4^4 x + (h - p_1 r_2) r_4^4 \}\, dx$$

where

$$p_1 = \frac{h_3 - h}{t_4}$$

Integrate from $x = r_2$ to r_4. Linkages for flux in trapezoid of base h_3 are

$$2\pi^2 n^2 \Big[\frac{p_1 x^5}{5} + (h - p_1 r_2)\frac{x^4}{4} - \frac{2}{3} p_1 r_4^2 x^3 - (h - p_1 r_2) r_4^2 x^2 + p_1 r_4^4 x + (h - p_1 r_2) r_4^4 \operatorname{logn} x \Big]_{x=r_2}^{r_4}$$

In order to include the effect of the two outer triangles, change h_3 to

$$h_4 = h_3 + 2t_3 \tag{15}$$

Then

$$q_1 = \frac{h_4 - h}{t_4} \tag{16}$$

Linkages for flux in the enlarged trapezoid of base h_4 are

$$2\pi^2 n^2 \Big[(h - q_1 r_2)\Big(\frac{4}{3} r_4 t_4^3 + \frac{1}{5}\frac{t_4^5}{r_4} + \frac{1}{6}\frac{t_4^6}{r_4^2} + \ldots \Big) + q_1 t_4^3 \Big(\frac{4}{3} r_4^2 - r_4 t_4 + \frac{1}{5} t_4^2 \Big) \Big] \tag{17}$$

since $r_2 = r_4 - t_4$, and t_4 is small compared with r_4.

The self-inductance of the coil in abhenrys is given by the sum of formulas 1, 11, 12, and 17.

Example. The following coil was constructed and measured by R. F. Field. If centimeter dimensions are used,

$r_1 = 2.54$, $r_2 = 7.62$, $r_3 = 0.698$
$r_4 = 8.23$, $t_2 = 1.173$, $t_3 = 0.635$
$h = 12.7$, $N = 1{,}680$

According to these dimensions, the density of the turns in the inner and outer parts is different.

$w = r_2 - r_1 = 5.08$
$t_1 = r_1 - r_3 = 1.842$
$h_1 = 15.44$, $h_2 = 17.78$, $q = 2.76$, $h + qr_1 = 19.71$
Linkages for the rectangle $wh = 78.8 \times 10^6$
Linkages for enlarged inner trapezoid of Figure $1 = 17.0 \times 10^6$
Linkages for the two end trapezoids $= 3.9 \times 10^6$
Linkages for enlarged outer trapezoid $= 2.0 \times 10^6$
Inductance of coil $= (78.8 + 17.1 + 3.9 + 2.0)10^6$ abhenrys $= 0.1018$ henry

The measured value agreed with this within less than one per cent.

Reference

1. Theory of Alternating Currents, Alexande Russell. Edition of 1914, page 74.

TRANSACTIONS SECTION

Preprint of Corresponding Pages From the Current Annual AIEE Transactions Volume

Resonant-Circuit Constant-Current Regulators

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Synopsis: Airport-approach and runway-marker light circuits are quite similar to the usual street-lighting circuits, except that the important additional feature of brightness control is required. Standard moving-coil regulators were found to be unsatisfactory for this purpose, and so the development of resonant-circuit constant-current regulators with built-in brightness control was fostered by the Army Air Forces. Four years of service have proved regulators of this type to be entirely satisfactory, and they have been adopted as standard by the Army, Navy, and Civil Aeronautics Administration.

Resonant-circuit regulators have no mov- ng parts and operate at high efficiencies nd power factors, even at light loads. De- ailed performance data, together with a omparison of resonant-circuit and moving- oil regulators, are presented in this paper.

ALTHOUGH the theoretical aspects of resonant-circuit constant-current egulators were published by Steinmetz s early as 1900, the lack of satisfactory apacitors prevented their use until omparatively recently.[1] Moving-coil egulators were universally adopted for treet-lighting circuits and were found so atisfactory that only a very few experi- ental resonant regulators were built rior to 1940, even though satisfactory apacitors were then available.

In 1940, the Army Air Forces were con- onted with the problem of providing rightness control for airport-approach nd runway lighting circuits, for the llowing reasons:

The high-candle-power lamp assemblies ing installed in an attempt to permit land- gs under adverse weather conditions were metimes blinding under clear weather con- tions.

Black-out operations necessitated a eat reduction in candle power in addition providing hoods for the lamp assemblies.

The brightnesses found to be necessary, together with the average currents which most nearly produce those brightnesses on all the lamp assemblies used, are as enumerated in Table I.

Runway and approach lighting circuits in use in 1940 were nearly identical with conventional street-lighting circuits, except, of course, that a different type of lamp assembly was used. The circuit consisted of a moving-coil regulator, together with a series circuit from two to six miles in length. A series–multiple or series–series transformer frequently was provided for each lamp assembly in order to isolate the assembly from the high-voltage circuit, and to prevent service interruptions in the event that a lamp assembly was damaged by an airplane or other vehicle.

Moving-Coil Regulators

The first method of brightness control considered was the installation of a tapped autotransformer in the output circuit of a moving-coil regulator. Although the desired current values were obtained in this manner, the scheme was found to be unsatisfactory for the following reasons:

1. When connected to a rated lamp load, the regulator operates at only 20.5 per cent of its rated kilowatt output, when the one-per-cent brightness tap is selected.

2. Conventional moving-coil regulators overheat when operated at less than 50 per cent load. This permits only intermittent use of the ten-, three-, and one-per-cent brightness taps.

3. On the one-per-cent brightness tap, primary power factor and efficiency are excessively low, being only 22 per cent and 66 per cent, respectively, for the three-kilowatt combination tested.

4. Voltage encountered on open circuit is increased by approximately the same ratio as the current was decreased.

For comparison with resonant-regulator performance, curves showing results of tests conducted upon a conventional three-kilowatt 220-volt-input 6.6-ampere-output moving-coil regulator connected to a variable lamp load are included in Figure 1. Within the region of short circuit to rated load, it can be seen that the input current is approximately constant, and the primary power factor is nearly proportional to the load.

Resonant-Regulator Development

The first experimental resonant-circuit regulator was received by the Army Air Forces and installed in the early part of 1941. Test results and service experience proved satisfactory, and so it was decided to use resonant regulators for all future installations. The first regulator to be produced in quantity was an oil-immersed self-cooled five-kilovolt-ampere 230/115-volt 6.6-ampere unit which included five brightness-selector relays, four runway-selector relays, a power-supply relay, and an open-circuit protector relay. The brightness- and runway-selector relays were interlocked with the power relay in such a manner as to insure that the regulator would be de-energized when they were operated. This regulator was adopted as standard by the Army Air Forces (type *A*-1, specification 32360), and later standardized by the Army, Navy, and Civil Aeronautics Administration (specification *AN-R-3a*). A monocyclic square, and later a resonant *T* circuit, were used to obtain the constant-potential–constant-current transformation in this regulator.

In 1943 a 7.5-kw 2,300-volt regulator was developed in order to meet increased power demands, and to fit into the complete airport lighting system then being standardized. This regulator (specification *AN-R*-17) is the present Army,

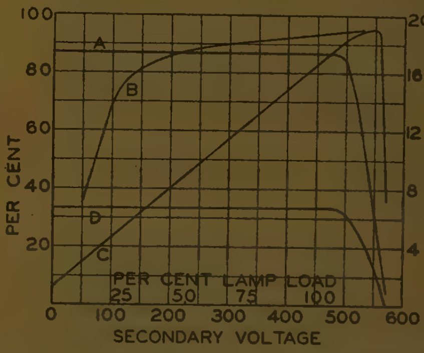

Figure 1. Moving-coil regulator, three-kilowatt, 220-volt primary winding, 6.6-ampere secondary winding

A—Primary current
B—Efficiency
C—Primary power factor
D—Secondary current

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per 45-165, recommended by the AIEE com- ttees on air transportation and electric machinery publication in AIEE TRANSACTIONS. Manu- ipt submitted August 1, 1945; made available printing August 17, 1945.

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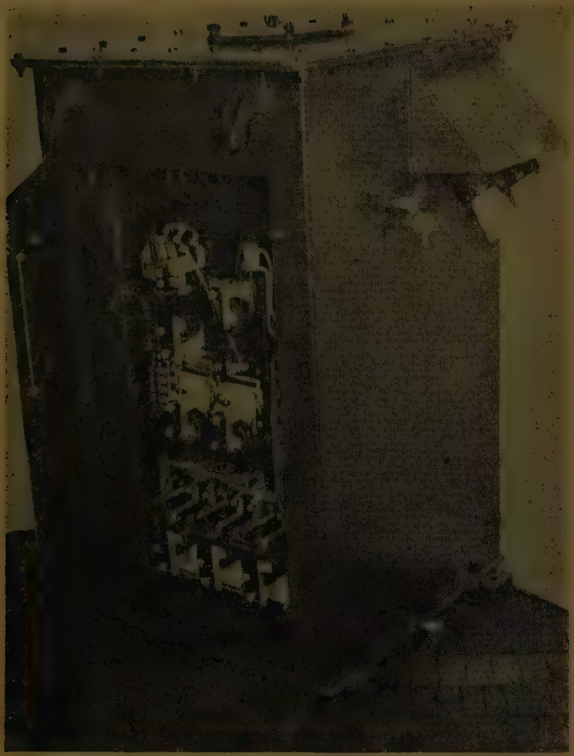


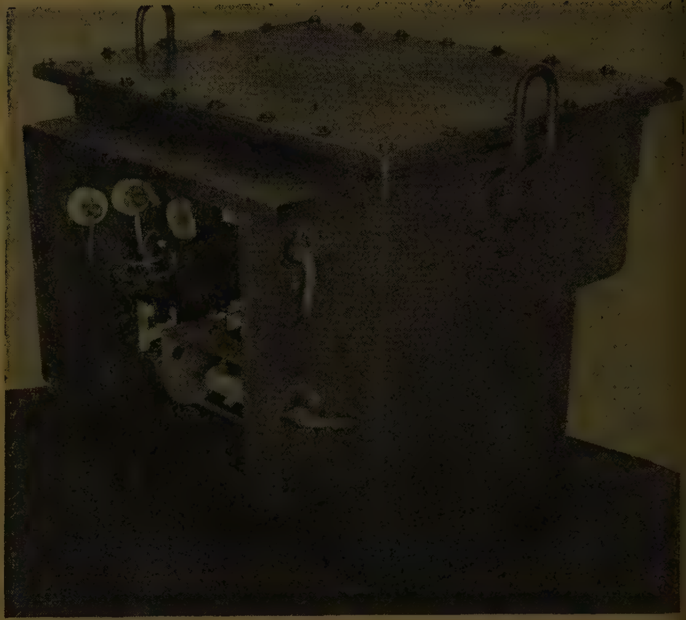


Figure 2 (left). Resonant-circuit regulator, 7.5 kw, 2,300 volts, 6.6/5.5/4.8/4.2/3.8 amperes, remote brightness control

Figure 3. Resonant-circuit regulator, type-*B*-1, six kilowatts, 230 volts, 6.6/5.5/4.8/4.2/3.8 amperes, manual brightness control

Navy, and Civil Aeronautics Administration standard, and several hundred have been procured from two separate sources. It is illustrated in Figure 2. The 7.5-kw regulator is oil-immersed, self-cooled, weighs approximately 1,000 pounds, and includes five brightness-selector relays and one open-circuit protective relay. It is designed for use with a separate remote-controlled oil switch, and a control panel (specification *AN-P*-59) which is installed in the airport control tower.

Shortly afterwards, a requirement arose for a 230-volt manually controlled regulator for overseas installations of high-intensity runway and approach lights. This resulted in the development of the six-kilowatt regulator known as the type-*B*-1 (Army Air Forces specification 32470) which is illustrated in Figure 3. This type of regulator is similar to the 7.5-kw unit, except for the lower primary voltage, and the use of a manually operated brightness tap changer in lieu of the brightness-selector relays.

Description—Typical Resonant Regulator

The type-*B*-1 regulator illustrated in Figure 3 has been selected for detailed discussion, because it is a typical resonant-circuit regulator with performance characteristics similar to the majority of units tested. This regulator consisted of the major items shown on the wiring diagram of Figure 4, inclosed in or mounted on the front of a sheet-steel tank containing 28 gallons of oil. The complete assembly weighs 965 pounds and is approximately 24 by 29 by 31 inches high.

The essential portion of the regulator is the resonant circuit, sometimes called "monocyclic square," which consists of two five-microfarad, 1,400-volt mineral-oil capacitors (items *F*, Figure 4), and two 530-ohm 60-cycle reactors (items *E*, Figure 4). The constant-potential–constant-current transformation is performed by this circuit. It has been demonstrated mathematically that the following relation exists, assuming that there are no losses, and that all four reactances are equal in magnitude:[1]

$$I_s = \frac{jE_p}{X}$$

where

I_s = output current
E_p = input voltage
X = reactance of one side of square
j = operator denoting 90 degrees counterclockwise rotation

Inspection of this relation reveals that output current is independent of the load, but that it is directly proportional to and 90 degrees out of phase with the input voltage. Experience with actual regulators indicates that the relation may be used with negligible error over the normal operating range. Losses in well-designed reactors and capacitors have little effect on the output current, but the reactances should be held within one per cent of the desired value (or compensated for by means of factory adjustment taps on the input or output transformer).

Table I

Brightness in Per Cent	Current in Amperes
100	6.6
30	5.5
10	4.8
3	4.2
1	3.8

The functions of the two coil 200-210-220-230/1,900-1,580-1,380-1,210-1,090-volt 6.5-kw-input transformer (item *C*) are to provide the correct voltage for the monocyclic square and to isolate the high-voltage from the low-voltage-input circuit. The tap changer (item *D*) is used to compensate for sustained variations in voltage impressed upon the monocyclic square; the five values of lamp brightness are easily obtained by means of the brightness tap changer (item *B*) which may be operated without deenergizing the regulator.

The manually operated 35-ampere circuit breaker (item *A*) provides on–off control, and open-circuit protection by means of the shunt trip which is energized through the normally closed contacts of the series relay (item *H*) when open circuit occurs. The resistor (item *J*) provides a slight time delay, thereby preventing undesirable tripping during switching.

The purpose of the two-coil 3.5/6.6-ampere, six-kilowatt-output transformer (item *G*) is to limit the open-circuit voltage. Theoretically the open-circuit output voltage of the monocyclic square would be infinite; in an actual regulator (without output transformer), it would rise dangerously high, and might cause immediate failure of the capacitors. When the secondary winding of the regulator under discussion is opened, the monocyclic square maintains the current in the primary winding of the output transformer, the core becomes saturated, and the output voltage is limited to approximately 175 per cent of the full-load voltage.

Performance Data—Typical Resonant Regulator

The regulator illustrated in Figure 3 was subjected to extensive laboratory tests, and the results obtained are shown in Figures 5 to 9, inclusive.

The data shown in Figures 5, 6, and 7 were taken by operating the regulator on a 230-volt 60-cycle power supply with a variable lamp load, and making direct measurements of input and output current, voltage, and power. The current-regulation curves for all five brightness taps, shown in Figure 5, indicate a maximum drop in current of 3.5 per cent from short circuit to full load. Figure 6, when compared with Figure 1, illustrates the basic difference in performance between resonant and conventional moving-coil regulators. The moving-coil unit has a constant input current and a power factor which is nearly proportional to the load, while the resonant circuit unit has an input current which is nearly proportional to the load, and a power factor of unity from 25-per-cent load to full load. Figure 7 shows that the resonant regulator has excellent power factors and efficiencies even at the lowest brightness tap (3.8 amperes) whereas the moving-coil regulator would have a power factor of less than 25 per cent lagging.

The data shown in Figure 8 were taken with two regulators placed in a large cold room, connected to a 230-volt 60-cycle power supply and to a six-kilowatt lamp load. Meters were placed outside the cold room, with provision made for switching from one regulator to another. Test runs were made at several temperatures, with each temperature held constant during the test and for at least 24 hours before the test run was started. Readings were taken immediately after energizing the regulator, and the time required for the output current to reach normal room-temperature values was recorded. The two regulators, although made by different manufacturers, were quite similar in performance at room temperature. The curves of Figure 8 show very plainly that chlorinated-diphenyl capacitors are not satisfactory, unless the regulator is to be installed in a heated room.

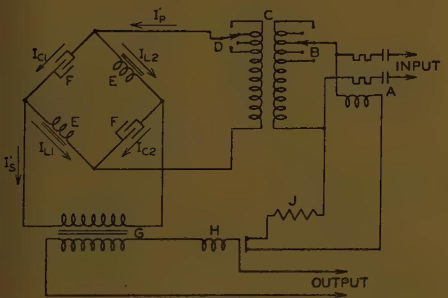


Figure 4. Wiring diagram, type-*B*-1 resonant-circuit regulator

A—Manual circuit breaker with shunt trip
B—Brightness tap changer
C—Input transformer
D—Primary-voltage tap changer
E—Reactor
F—Capacitor
G—Output transformer
H—Series relay
J—Resistor

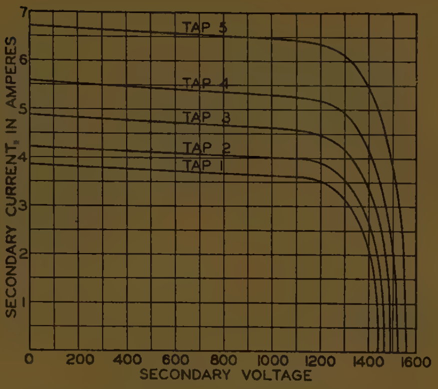


Figure 5. Current-regulation, type-*B*-1 resonant-circuit regulator

The vector diagrams shown in Figure 9 were plotted from data taken with the regulator connected to a 60-cycle power supply held constant at 230 volts and a lamp load which was varied from short circuit to open circuit. The two diagrams are oriented so that the correct relation between corresponding currents and voltages exists. The values of currents and voltages existing in the monocyclic square under rated load conditions are shown by the solid lines, while the dotted lines show the loci of the corners of the diagrams, so that the vector diagrams for any load may be visualized. The change in voltage applied to the square due to voltage drop in the input transformer as the load is increased from short circuit to rated load is too small to be shown, but the increase in voltage due to the large leading current flowing in the input transformer at overloads is illustrated.

Resonant-Regulator Characteristics

The current regulation of the resonant-circuit regulator depends primarily upon the accuracy of tuning of the resonant circuit, and upon the regulation of the input and output transformers. Since the output current of existing regulators varies in proportion to variations in a supply voltage, it has been considered uneconomical to specify better than four-per-cent regulation (±2 per cent of the specified current), although it could be obtained without difficulty. The importance of current regulation is minimized by the fact that, after installation, the normal load current may be corrected by means of taps on the regulator input transformer, and/or adjustment of the supply voltage.

Efficiency of a resonant regulator is determined by the losses in reactors, capacitors, and transformers, and is maintained at a comparatively high value, even at light loads. Power consumed by built-in relays ordinarily is included in the regulator losses, unless supplied from separate control circuits.

Except at extremely light loads, a properly designed resonant regulator has a primary power factor which is approximately equal in magnitude but opposite in sign to the secondary power factor. For example, the input power factor of a regulator feeding a 90-per-cent-lagging series load would be approximately 90 per cent leading. The actual input power factor is determined by the following factors:

1. Manufacturing tolerance in monocyclic square.
2. Exciting current of input transformer.
3. Exciting current of output transformer.
4. Series coil of protector relay.
5. Inductive reactance of series load.

Factors 3, 4, and 5 cause leading power factors, factor 2 has a lagging effect, and the first factor may have either a lagging or leading effect.

The effect of frequency variations upon resonant-circuit regulators is much less than one would expect at first thought. The regulator illustrated in Figure 3 has been used satisfactorily on 50-cycle power supplies without modification. The output current at full load is approximately three per cent less on 50 cycles than it is on 60 cycles.

Oscillographic studies indicate that the output current from a resonant regulator operating near full load has no switching surges. This is in direct contrast to the effective value of the output current from a moving-coil regulator, which surges to a high value and then oscillates about the steady-state value, immediately after the unit is energized. Observations made on a monocyclic square regulator operating at rated lamp load also indicate that the input and output current waves approximate sine waves. Although time limitations have prevented a complete study of harmonics under all load conditions, no trouble due to harmonics has been reported during four years operating experience with loads consisting of lamps, and lamps connected to well-designed isolating transformers.

Summary

The satisfactory service obtained from over 1,500 resonant regulators procured

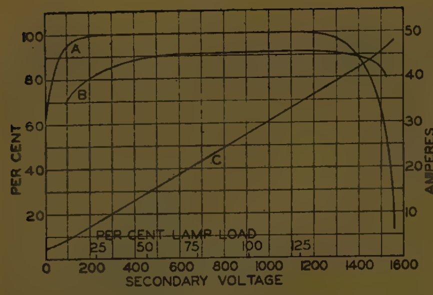


Figure 6. Performance data, brightness tap 5, type-*B*-1 resonant-circuit regulator

A—Primary power factor
B—Efficiency
C—Primary current

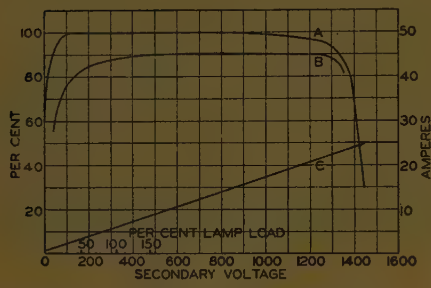


Figure 7. Performance data, brightness tap 1, type-*B*-1 resonant-circuit regulator

A—Primary power factor
B—Efficiency
C—Primary current

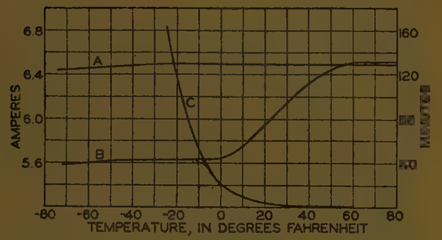
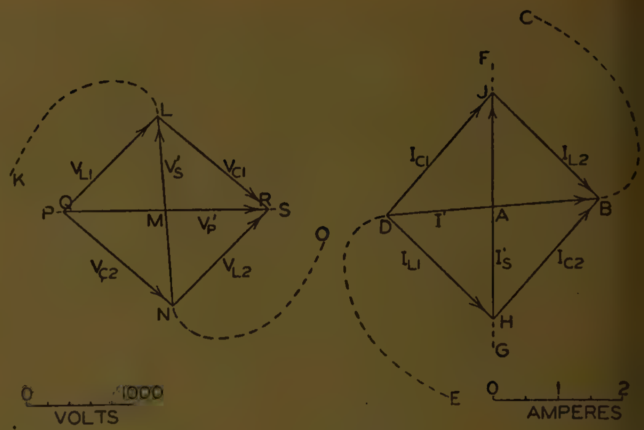


Figure 8. Low-temperature performance, resonant-circuit regulators

A—Initial output current, resonant regulator with mineral-oil capacitors
B—Initial output current, resonant regulator with chlorinated-diphenyl capacitors
C—Time required for output current of regulator with chlorinated-diphenyl capacitors to reach rated value

Figure 9. Vector diagram, type-*B*-1 resonant-circuit regulator



Voltages:
As load is increased from short circuit to rated load:
Point *L* moves from *M* to *L*
Point *N* moves from *M* to *N*
As load is increased from rated load to open circuit:
Point *L* moves from *L* to *K*
Point *N* moves from *N* to *O*
Point *R* moves from *R* to *S*
Point *Q* moves from *Q* to *P*

Currents:
As load is increased from short circuit to rated load:
Point *B* moves from *A* to *B*
Point *D* moves from *A* to *D*
As load is increased from rate load to open circuit:
Point *B* moves from *B* to *C*
Point *D* moves from *D* to *E*
Point *J* moves from *J* to *F*
Point *H* moves from *H* to *G*

since 1940 is evidence that this type of regulator has moved out of the experimental category. Resonant regulators are the most satisfactory means of providing power for airport runway and approach lighting circuits, which must be operated at reduced intensities on clear nights, and they have been standardized by the Army, Navy, and Civil Aeronautics Administration. Moving-coil regulators still are used for applications where operation at reduced intensities is not required, such as street lighting and airport boundary lighting.

Existing resonant-circuit regulators have the following definite advantages and disadvantages:

A. Advantages:

1. No moving parts (except relays).

2. Unity power factor with resistance loads, leading power factor with inductive loads.

3. High power factor and efficiency maintained with loads as low as 20 per cent. This permits satisfactory operation of lighting circuits at one-per-cent brightness.

4. Brightness control easily obtained by varying input voltage to resonant circuit.

5. No switching surges of sufficient duration to affect lamp life.

B. Disadvantages:

1. Output current varies with input voltage.

2. Not familiar to most operating and maintenance personnel.

In contrast to the foregoing, moving-coil regulators have the following characteristics:

A. Advantages:

1. Output current constant regardless of primary voltage.

2. Conventional equipment, familiar to most electrical personnel.

B. Disadvantages:

1. Moving parts which require accurate adjustment.

2. Power factors of 75 to 85 per cent lagging at full load, unless supplied with capacitors for power-factor correction.

3. May not be used at less than 50-per cent load, owing to overheating.

4. Unsatisfactory for brightness control owing to overheating, and extremely low power factors and efficiencies at low brightnesses.

5. Current surges immediately after energizing shorten lamp life, especially for airport use where frequent switching may be required.

Sufficient data for an accurate cos comparison of the two types of regulator are not available, owing to the fact that al resonant regulators procured to date included an input transformer and brightness tap-changing equipment. Elimination of these items, however, should mak the cost of a resonant regulator equal to o less than the cost of a moving-coil regulator.

In conclusion, the author predicts tha resonant regulators will continue to b used for all airport runway and approac lighting circuits, and that they wi gradually come into use for other applications, such as street lighting. The successful conclusion of the following two developments, now being actively encouraged by the Army Air Forces, wi do much to further the postwar use o resonant regulators:

1. A lightweight air-cooled design.

2. A method of compensating for primary voltage variations.

Reference

1. Theory and Calculation of Alternating Current Phenomena (fourth edition), Charle Proteus Steinmetz. McGraw Publishing Company, 1908. Pages 78–129.

Parallel Operation of Main-Engine-Driven 400-Cycle Aircraft Generators

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Synopsis: This paper reports progress in the development of variable-ratio drives and governors for parallel operation of aircraft alternators driven from the main engines. Preliminary tests have been made demonstrating successful parallel operation of such drives under various conditions of acceleration and loading. The general system of operation and results of some of the first tests are described in this paper.

POTENTIAL military requirements may create a demand for very large aircraft. Four-hundred-cycle three-phase 208Y–120-volt a-c power appears to be quite suitable for most of the electrically operated devices. The electrical manufacturers in this country have been asked by the Army Air Forces to develop an electric system to generate, distribute, and utilize this power to perform various required functions on such aircraft.

For reasons of space, weight, economy, and reliability, it was decided to drive these generators from the main engines. Several other means of driving such generators were considered, but because of the very limited time available for the development this method was selected, as it appeared to hold most promise for a quick solution. Among the other means considered were:

1. Auxiliary engines or gas turbines.
2. Exhaust-gas turbines.
3. Waste-heat boilers and steam turbines.
4. Hydraulic torque converters.
5. Mechanical transmissions of various types.
6. Rotating electric machines in various combinations.
7. Electronic means.

Some of these now are being perfected, and much of the experience gained with the present system will be applicable to these other systems.

Parallel operation of the generators on a multiengine airplane is desired for military service, because it provides the following benefits:[1]

1. Greater motor-starting capacity.
2. Reduction of time and magnitude of voltage disturbance from fault, motor starting, or loss of generation.
3. More effective use of spare capacity, so that smaller or fewer generators can be used without excessive load-transfer switching.
4. Greater ability to withstand fault without disruption of service and greater ability to burn faults clear.
5. Continued operation of load equipment following engine or generator failure; does not require any automatic-transfer equipment nor any immediate attention from the operator.

Statement of the Problem

The aircraft engines are relatively large compared with the generators, as they are used for main-propulsion power. Consequently, their rating may be from 50 to 100 times the rating of the generators which are used for auxiliary electric power on the airplane. In addition the main-engine speed may vary over a wide range (three to one or more), depending upon the conditions of flight. Furthermore, all engines are not always operated at equal speeds at all times. These factors presented several new problems to the electrical industry.

Parallel operation of a-c generators is common in industrial or central-station practice. However, usually most of the following conditions prevail for constant-frequency a-c systems with generators operated in parallel:

1. The prime mover is comparable in rating with the generator.
2. The prime mover usually is operated at constant speed.
3. The prime mover is governed to operate in accordance with the requirements of the generator.
4. All prime movers on the system are operated at a speed exactly proportional to the system frequency.
5. Transient speed changes of the prime movers are relatively small.

In contrast with this, in the proposed aircraft system:

1. The prime mover is large, frequently being 50 to 100 times the rating of the generator.
2. The prime movers are operated over a wide speed range to meet changing conditions of flight.
3. The prime mover is governed to operate in accordance with the propeller and propulsion requirements of the airplane.
4. Different engines on the same airplane may be operated at different speeds at the same time.
5. Transient speed changes of the prime movers may be very rapid.

The problem is, with this combination of conditions, to provide a means of driving the alternators from the main engines, maintaining frequency and voltage at all times, and keeping all the machines operating in synchronism and sharing the load. Of course, in addition to these fundamental requirements the system must be suitable for use in aircraft, which conditions bring in a host of additional requirements which have been listed elsewhere.[1]

The alternators now in use have eight poles and operate at 6,000 rpm to deliver 400 cycles. This means that one mechanical degree is equivalent to four electrical degrees. Thus to maintain synchronism it is necessary that the mechanical displacement among generators does not exceed about 20 mechanical degrees, and to divide load properly they should be within one or two mechanical degrees of each other. This close relationship should be maintained, even though the individual engine speeds vary widely and are subject to high rates of acceleration. To the uninitiated, it looks like an impossible job, except with some type of slip device such as a fluid coupling. A slip device is out of the

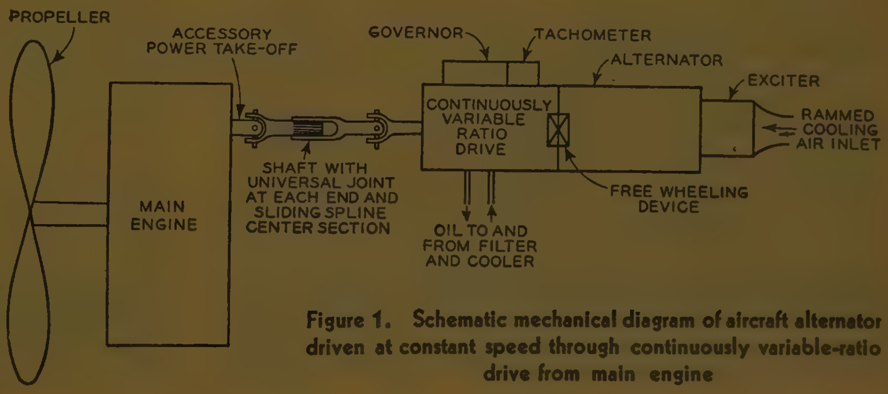
PROPELLER
ACCESSORY POWER TAKE-OFF
GOVERNOR
TACHOMETER
ALTERNATOR
EXCITER
CONTINUOUSLY VARIABLE RATIO DRIVE
RAMMED COOLING AIR INLET
SHAFT WITH UNIVERSAL JOINT AT EACH END AND SLIDING SPLINE CENTER SECTION
FREE WHEELING DEVICE
MAIN ENGINE
OIL TO AND FROM FILTER AND COOLER

Figure 1. Schematic mechanical diagram of aircraft alternator driven at constant speed through continuously variable-ratio drive from main engine

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The author wishes to express his appreciation to colleagues in his own organization for their cooperation in carrying on these experiments and to the engineers of the Sundstrund Machine and Tool Company for providing the variable-ratio drive units.

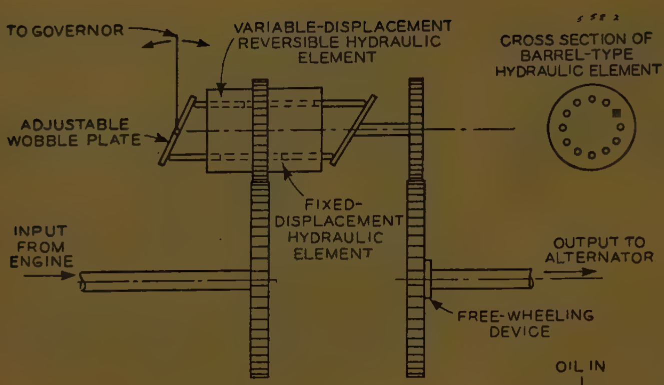
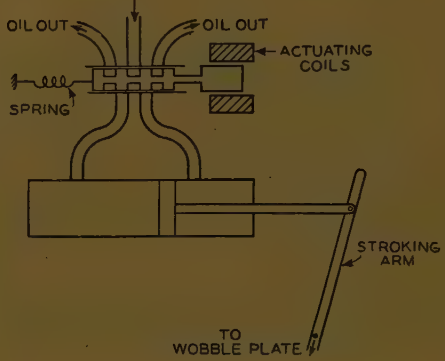

TO GOVERNOR
VARIABLE-DISPLACEMENT REVERSIBLE HYDRAULIC ELEMENT
ADJUSTABLE WOBBLE PLATE
FIXED-DISPLACEMENT HYDRAULIC ELEMENT
INPUT FROM ENGINE
OUTPUT TO ALTERNATOR
FREE-WHEELING DEVICE
CROSS SECTION OF BARREL-TYPE HYDRAULIC ELEMENT

Figure 2. Schematic mechanical diagram of one form of continuously variable-ratio-drive unit

Figure 3 (below). Schematic mechanical diagram of governor and servo mechanism



question, however, because of the wide speed range involved, which makes it too inefficient to be applicable. The use of a positive-displacement type of drive to improve efficiency requires the development of a high-speed governor.

For constant frequency to be obtained from conventional generators the latter must be operated at constant speed. This means that, if the prime mover has a variable speed, some type of variable ratio transmission must be interposed between the prime mover and the generator and governed to provide the desired result.

Procedure

An analytical approach[2] provided specifications for governor design and indicated that the characteristics required for parallel operation were well within practical design values and that parallel operation could be achieved.

It was necessary to overcome many obstacles, and some are yet to be mastered. Nevertheless, the fundamental principles of accomplishing this job have been established and demonstrated.

Because the solution of this problem required the development of a governing system with a very high speed of response, it was thought that many engineers would be interested in a description of the general method of accomplishing this result. The purpose of this paper is to give a brief description of the governor and what is believed to be the first 400-cycle a-c system of this type which has been successfully demonstrated.

Description of the System

Figure 1 shows a schematic mechanical diagram of the system arrangement of one main-engine-driven generator unit which is part of a proposed system for use on large aircraft. Note that this plan contemplates mounting of the continuously variable-ratio drive and generator separate from the engine to avoid excessive overhung weight on the engine. The unit is driven from an accessory drive shaft on the engine through a shaft designed to permit the engine to move in its resilient mounting without injury to the driven unit. A free-wheeling device is located between the variable-ratio drive and alternator, which permits driving the alternator but never permits the alternator to try to drive the engine. The governor adjusts the ratio of the variable-ratio drive to maintain frequency and load division, as explained later, in spite of variations in engine speed.

The setup used in test differs from Figure 1 in that the driving power was taken from the main output shaft of the engine through a suitable gear box, as the accessory drive shafts on these engines were not large enough to provide the required power.

Figure 2 is a schematic mechanical diagram of one form of continuously variable-ratio drive unit. This unit is a differential type of hydraulic drive using oil as the working medium. A simple valve arrangement (not shown) is located between the pump and motor end of the unit, providing a properly valved pat for the flow of the working fluid betwee the pump and motor.

The input end of the unit has a variabl stroke. This stroke is varied by the angl of the wobble plate, which in turn is positioned by the governor. The output en of the unit has a fixed stroke. Thus th angle of the adjustable wobble plate o the input end of the unit establishes th relative speed of the fixed stroke elemen with respect to the variable displacemen element. When the stroke of the latter i zero, both elements are locked hydraulically and rotate as a unit. This i the straight-through drive ratio. At thi ratio hydraulic losses in the unit are at minimum. The variable-ratio drive uni should be designed to have maximu efficiency at engine speeds correspondin to the cruising speed of the airplane.

Figure 3 is a schematic mechanical diagram of the governor and servo mechanism which operates to position the wobbl plate and thereby adjusts the drive rati to provide the desired output speed.

Oil at a moderate pressure is supplie to an electromagnetically operated pilo valve, which is balanced against a spring A valve movement of a few thousandth of an inch permits oil to enter one side o the piston and leave from the other side thus causing the piston to move. Th piston is linked mechanically to th wobble plate of the variable-ratio drive As a result of the very small valve movement required, and by proper design o the ports, a very high speed of response i achieved. The pilot valve position i governed by the electrical signal introduced into its actuating coil. The governor uses a constant-current balanced valve system. The pilot valve always returns to its neutral position after th speed correction is made. The transient position of the valve is determined b the change in signal above or below th balance-position signal.

Figure 4 is a schematic electrical diagram of the governor control circuit showing the principle of operation. Fo

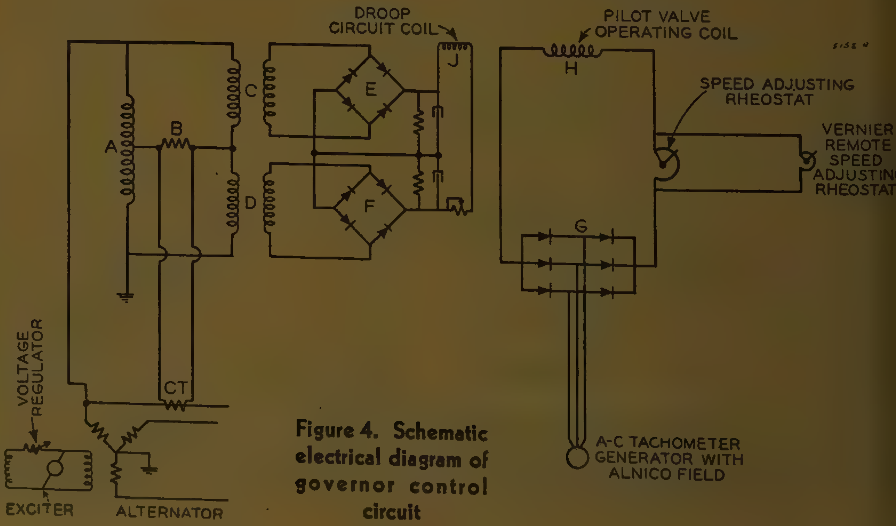
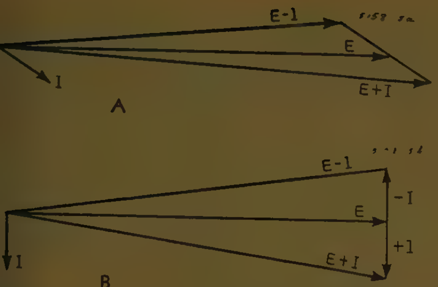

DROOP CIRCUIT COIL
J
C
E
A
B
D
F
PILOT VALVE OPERATING COIL
H
SPEED ADJUSTING RHEOSTAT
VERNIER REMOTE SPEED ADJUSTIN RHEOSTAT
G
VOLTAGE REGULATOR
CT
EXCITER
ALTERNATOR
A-C TACHOMETER GENERATOR WITH ALNICO FIELD

Figure 4. Schematic electrical diagram of governor control circuit



'igure 5. Vector relations in load-frequency droop circuit

implicity certain other circuits including tabilizing and emergency overspeed cir-uits are not shown.

The operation is as follows: A small a-c achometer generator using a permanent-nagnet field is driven from the output of he variable-ratio drive. The output of his tachometer is rectified by rectifier *G* nd applies a signal, which is directly pro-ortional to speed on the operating coil *H*.

Another signal, which is proportional o load, is placed on droop-circuit coil *J*, vhich is wound on the same core as *H*. This provides a five-per-cent frequency lroop at full load. This load-frequency lroop causes the generators to share the ilowatt load when operating in parallel.

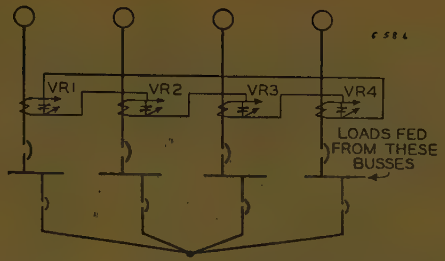


Figure 6. Typical star bus arrangement showing current-transformer connections for differential cross-current compensation for division of reactive load among generators

The load–frequency-droop signal is derived from a circuit which is sensitive to the inphase component of current. This provides a signal approximately proportional to power load on the alternator and is accomplished in the following manner. Terminal voltage of the alternator is fed into a midtapped autotransformer *A*. Another signal, which is proportional to current, is obtained from a current transformer and fed into the resistor at *B*. This latter signal is added and subtracted vectorially from the voltage signal, the results of the addition and substraction appearing in transformers *C* and *D* respectively. The outputs of these transformers are rectified by full-wave dry-type rectifiers *E* and *F*. The resulting two d-c signals are subtracted, and their difference placed on the droop-circuit coil *J*.

Figure 5 shows the vector relations involved in the load–frequency-droop circuit.

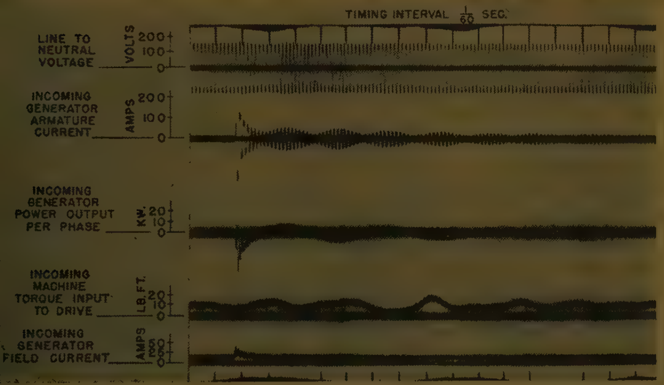
Figure 5a shows the condition with a lagging power-factor load. Note that there is a substantial difference between the voltage vectors $E+I$ and $E-I$, which provides a signal to the droop coil.

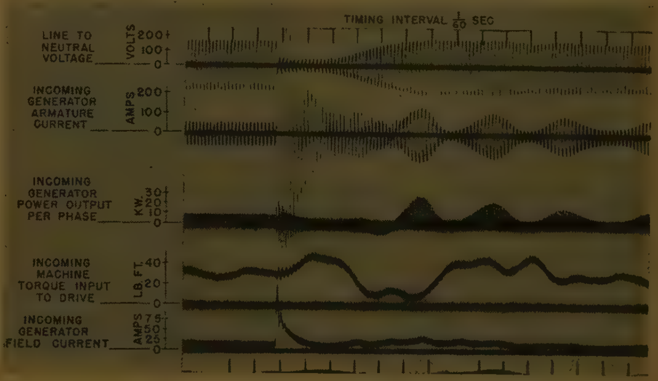
Figure 5b shows the condition with a zero power-factor load. Here $E+I$ and $E-I$ are equal, hence their difference is zero, and no signal is introduced into the droop coil.

The droop circuit is responsive to real power and not to reactive power.

The division of reactive kilovolt-amperes among several generators connected in parallel can be achieved by means of differential cross-current compensation in the voltage-regulator circuits. This provides for reactive division without voltage droop. Schematically this connection is shown in Figure 6. A normally closed interlock is provided on each generator circuit breaker to short-circuit its reactive division current transformer when the circuit breaker is open and the generator is taken out of service.

This differential reactive-division circuit is ideal for parallel operation, providing the units are to operate in parallel at





'igure 7. Synchronizing disturbance with ncoming machine at full voltage with no load on system

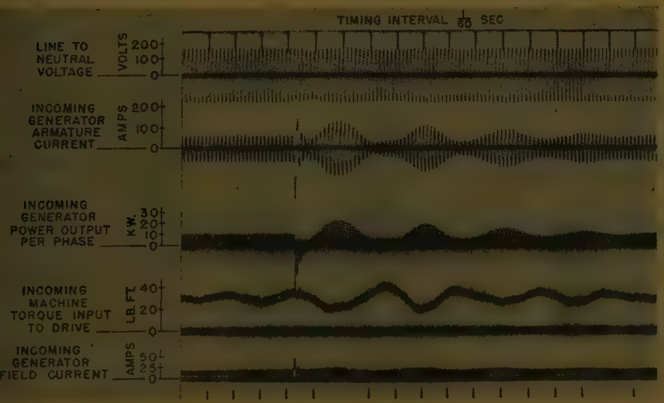
'igure 8. Synchronizing disturbance with ncoming machine at full voltage and both ncoming and running machines carrying load

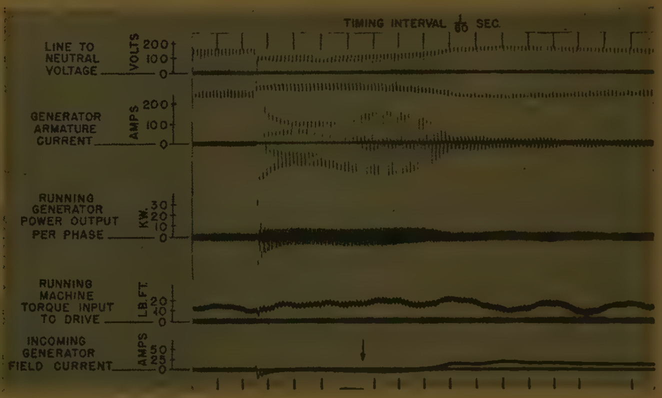
Figure 9. Synchronizing disturbance with incoming machine at full voltage and both incoming and running machines carrying load

Incoming machine purposely synchronized 180 electrical degrees out of phase with running machine

Figure 10 (below). Synchronizing disturbance with incoming machine running unexcited

Arrow indicates time of closure of exciter field switch





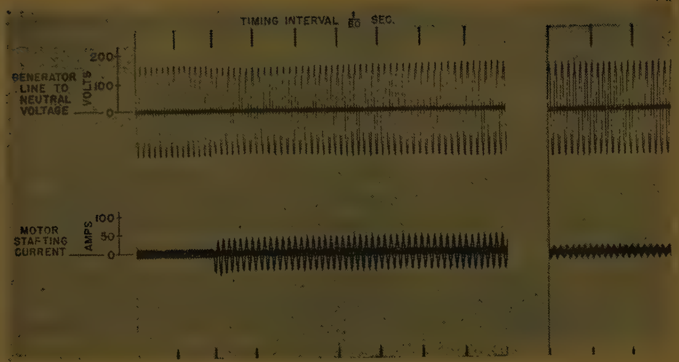


Figure 11. Voltage disturbance and recovery caused by simultaneous starting of two 750-watt Amplidyne motor-generator sets driven by squirrel-cage induction motors

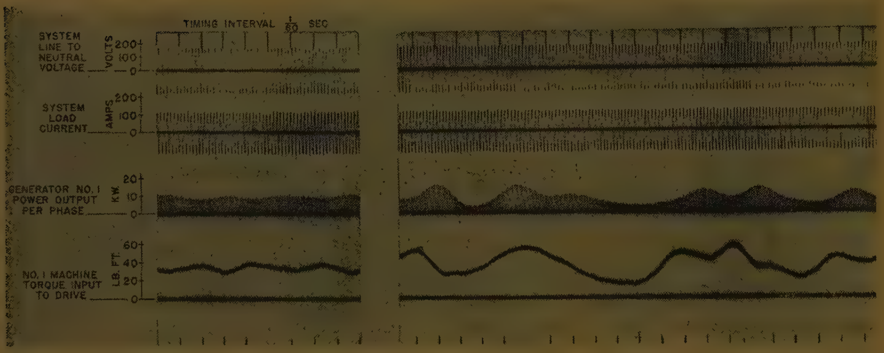


Figure 12. First section of oscillogram shows normal parallel operation with smooth engine, and second section shows effect of making engine rough by short-circuiting spark plugs on two cylinders in consecutive firing order

all times. If, however, nonparallel operation is desired, or operation with the system split into various combinations other than all machines operating in parallel, a rather complicated switching of the reactive-division control circuits is involved. To avoid this, individual cross-current compensation can be used. For this method there is no interconnection between reactive-division circuits for different generators. Instead, each individual voltage regulator is biased by reactive current output of its corresponding generator to give a regulated-voltage characteristic which droops slightly with increasing reactive load. To avoid excessive droop of voltage with increasing load, compensation can be added to give a rising voltage characteristic with resistive load and the combination designed to hold substantially flat voltage at the average load power factor

Operating Experience

Tests have been made with a laboratory setup using two 450-horsepower air-cooled radial aircraft type engines, each driving a 30-kw 40-kva 400-cycle alternator through a step-up gear and variable-ratio drive. Reciprocating gasoline engines were selected for tests to simulate appropriately acceleration rates and torque pulsation effects which will be obtained in actual installations. Torque measurements were made with an electric-strain-gauge-type torque meter located in the input shaft to one variable-ratio drive unit.

The presence of the free-wheeling between the alternator and drive was found to make synchronization very easy even with purely random synchronization. The machines have been synchronized at random at full voltage with and without load at widely different speeds. They also have been synchronized with the incoming-machine exciter field circuit open, closing the field simultaneously with the generator circuit breaker and permitting the field to build up while synchronizing.

Figure 7 is an oscillogram showing a typical synchronizing disturbance with full voltage on the incoming machine and no load on the system. The generator circuit breaker was closed at random, although in this case it happened to close nearly in synchronism. Note that there is very little disturbance to the system voltage. With automatic speed matching and synchronizing equipment, equal or better performance could be achieved consistently.

Figure 8 is an oscillogram showing a synchronizing disturbance similar to Figure 7, except that both machines had an initial load on them prior to synchronization.

Figure 9 is an oscillogram showing a synchronizing disturbance on the two generator system with each generator carrying load at normal voltage, and with the synchronizing switch purposely closed when the generators are 180 electrical degrees out of phase. At the instant of closure this corresponds to a three-phase short circuit on both machines with correspondingly high armature currents. The machines pull into step, and system voltage has recovered in 0.1 second.

The lower trace on Figure 9, showing alternator field current on the incoming machine, is of special interest in this case. Note the high transient field current which is induced at the instant of closure of the synchronizing switch. This current flows through the exciter armature and depresses the exciter voltage. If severe enough, such a surge may cause a reversal of the exciter voltage, which in turn may reverse the stabilizing signal to the voltage regulator, causing it to oscillate. To avoid this, the exciter should be designed so that it will not reverse polarity or lose its residual magnetism on such surges. These surges will be more severe on a system with more generators and with running loads.

The power trace indicates that the generator is free-wheeling for a short interval as the machine is pulling into step. The drive input torque does not go negative, however, as the generator is free-wheeling and the drive losses are supplied

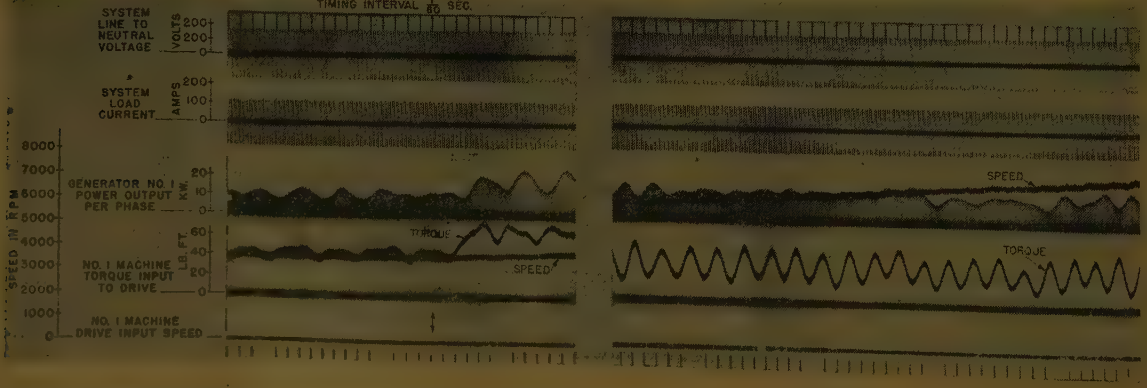


Figure 13. Oscillogram showing transient division of loa between generator by governor actio during acceleratio of one engine whe operating in paralle

Accelerating engin is very rough durin accelerating perio by the input shaft. The power and torque curves pulsate for a few cycles as the generators oscillate about their equilibrium position, but no destructively large torques appear during the synchronization in this case.

Figure 10 is an oscillogram showing the disturbance resulting from bringing the incoming machine on the bus running at approximately full speed but unexcited. In this case the exciter field switch was closed slightly later than the alternator armature switch at the time indicated by the arrow. There was no load on the system in this case. When the incoming generator switch is closed, the incoming machine operates as an induction motor running light, taking magnetizing current from the line, but practically no power. (See power trace.) When the exciter field switch is closed, the exciter of the incoming machine builds up, pulling the machine into step and furnishing its own excitation so that the circulating current drops to practically zero. The bus-voltage disturbance would be substantially less with more generators on the system.

Figure 11 is an oscillogram showing voltage disturbance and recovery from a suddenly applied load. In this case two generators were running in parallel at no load, and two squirrel-cage induction motors, each driving a 750-watt Amplidyne generator were started simultaneously. There is a momentary voltage dip of about four per cent and the voltage is restored to normal in about 0.05 second. The induction-motor Amplidyne sets accelerated to full speed in about 0.7 second, and, as the Amplidynes were unloaded, the induction-motor current dropped to exciting current only. The oscillogram was shortened for reproduction purposes, and the portion between the two parts shown represents a time interval of 0.6 second.

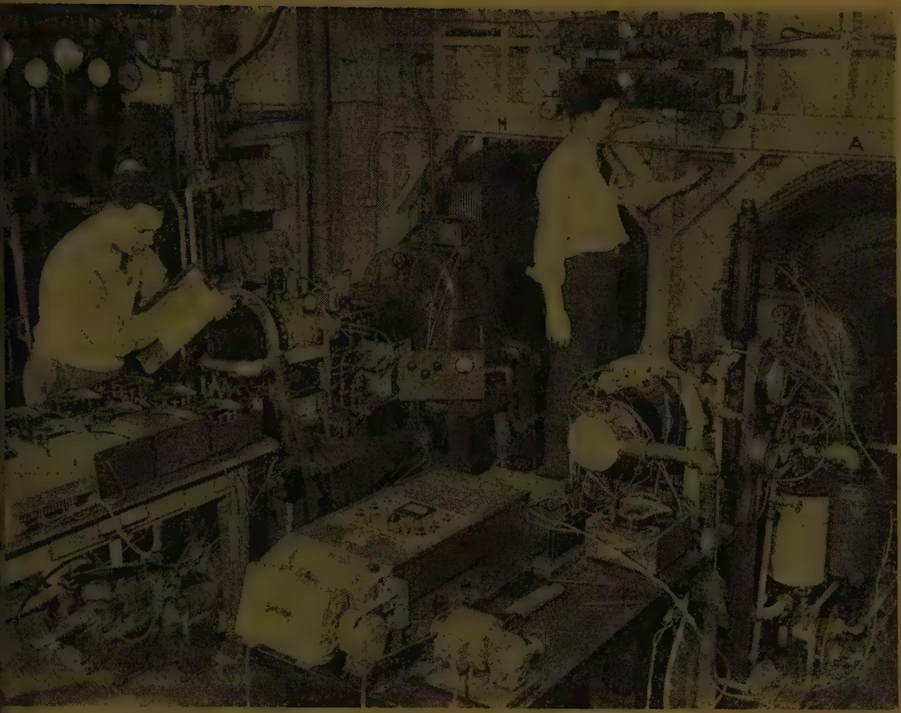


Figure 14 (above). General view of test setup

Driving engines are located behind wall

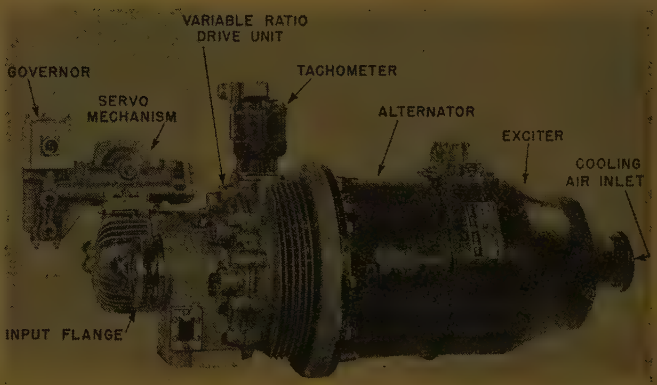


Figure 15. Side view of combined variable-ratio drive and alternator used in tests

This oscillogram gives some idea of the nature of voltage disturbances resulting from switching loads an the system. With more generators on the system, the disturbance would be less for a given load increment.

Figure 12 is an oscillogram showing two machines operating in parallel. The first part of the oscillogram shows normal operation with a normal engine. The second part of the oscillogram shows the effect of purposely making one engine rough by short-circuiting spark plugs on two cylinders in consecutive firing order. Note that the power division between generators pulsates but that, even in this severe case, the generator does not free-wheel. The system load current and voltage are influenced only slightly. In a four-engine set the influence on the system would, of course, be less.

Figure 13 is an oscillogram showing transient division of load during an acceleration of one engine, when the machines are operating in parallel. The accelerated engine was very rough during the accelerating interval in this case. Acceleration begins at the time indicated by the double-headed arrow. The oscillogram was shortened for reproduction purposes, and the portion between the two parts shown represents a time interval of 1.4 seconds. The rate of acceleration is about 1,700 rpm per second at the input to the accelerated drive. Note that the machines remain in step and that, though the accelerated machine picks up load initially, the governor reduces the load even before the acceleration is completed.

The oscillograms presented here are typical of more than 100 oscillograms which have been taken under a great variety of conditions in exploring the effect of various factors and combinations of conditions.

Figure 14 is a general view of the test setup. The engines are behind the wall and are coupled directly to the gear boxes which in turn are coupled to the variable-ratio drives which drive the alternators.

Figure 15 is a side view of an early design of one of the variable-ratio drive units and alternators used for these tests. This particular unit is rated 50 horsepower continuous and has an input speed range of 2,700 to 7,000 rpm. The over-all length of the combined unit is 29 inches, and the weights are as follows: alternator and exciter, 78 pounds; variable-ratio drive, 55 pounds; governor and servo mechanism, nine pounds; tachometer, 2.9 pounds.

Final designs have not been completed, and efficiency and life tests have not been run. However, these tests are planned for the near future. It is expected that these new variable-ratio drive units will weigh about 70 pounds and have a maximum efficiency of about 85 per cent. The new units will cover a wider speed range and be rated to correspond with the continuous and overload ratings of the generator.

This test program should contribute much data which will help to evaluate the a-c system for aircraft.

Electric Power in a Steel Plant

R. W. GRAHAM
MEMBER AIEE

IF ALL THE APPLICATIONS of electricity were listed, it would be found that this servant of man is used more extensively and in a greater variety of applications in the manufacture of steel than in any other industry. From the tiny photoelectric cell to the giant slow-speed reversing rolling-mill motor delivering torques of 2,000,000 pounds at one-foot radius, the application of electric power with its ease of control in the manipulation of steel is amazing as well as fascinating to anyone.

The use of electric power to replace hand labor and other less efficient methods of power supply has been most pronounced in the following applications in a steel plant:

1. Material handling, as coal and ore unloading.
2. Plant water supply for cooling and other purposes.
3. Overhead electric traveling cranes.
4. Transportation facilities.
5. Electric arc furnaces.
6. Rolling-mill drives.

Electric-power requirements in the steel industry are more demanding than those in many other industries. The two outstanding requirements to be considered are:

1. Continuity of service.
2. Methods and equipment to eliminate interruptions.

It is well known, that, when large tonnages of steel are in the process of pouring or rolling, an interruption of minutes and even seconds may in some instances make it necessary to scrap the entire mass of steel which is being processed. Because of the rapid drop in the working temperature of the steel there is always the question, after the restoration of a short power interruption, whether to attempt to roll the steel then in the mills or to take the loss in tonnage, as the bar of steel is then too long to be reheated in the soaking pits.

For adequate protection against power interruptions there are generally two or more sources of current supply. The Bethlehem Steel Company plant in Lackawanna has two outdoor substations with a combined capacity of 70,000 kva receiving 60,000-volt 25-cycle power over two transmission lines from the Niagara system, which is located approximately eight miles from the plant. Supplementing this is an auxiliary steam-generating station of 20,000-kw capacity. These stations feed into a tie network on opposite corners of a rectangular loop system. The steam station generates energy at 6,600 volts while the power purchased from the public utility system is stepped down from 60,000 to 6,600 volts, which gives a complete 6,600-volt distributing system for the main power supply to the larger consuming units and to the departmental substations.

The normal operation of the Niagara 60,000-volt system is for both incoming lines to be in service with oil circuit breakers at both ends of the line and a tie circuit breaker between them. These lines are tee-taped to the 20,000-kva transformer station which feeds the strip and bar mills through selective switching, permitting either line to feed the mills.

Because of the large number of synchronous motors on the Ward-Leonard-control motor generator sets and the synchronous motors on large mill drives, a leading power factor easily can be obtained on the system.

The Bethlehem Company has 2,250 horsepower synchronous motors on moto generator sets starting directly across th line (6,600 volts) with an inrush of 9,50 kva and with a voltage drop no mor noticeable than starting through reactor of the 8,500-horsepower synchronou motors, which drive 6,000-kw moto generator sets. It also has a 1,000 horsepower squirrel-cage induction moto on a mill-scale-breaker drive, starting directly across the line at 6,60 volts. These machines have specia bracing to withstand the heavy inrush o across-the-line starting. Elimination o reduced voltage starters simplifies th switchgear required for these units.

The amount of energy back of this system together with the synchronous machines operating in the plant, confined i such a congested area, allow short circuit of a high magnitude. These short-circui currents require oil circuit breakers o 500,000-kva rupturing capacity to clea faults when they occur.

The outdoor substation transformer originally were water-cooled, but, o account of the excessive cost of filtere water, they have been changed to air cooling and provided with blowers. Thi eliminates the danger of failure due t freezing or leakage in the water system which might contaminate the transforme oil. The larger transformer installation comprise three single-phase units in delta In case of trouble in any unit, operatio can be continued on open delta at reduce capacity until such time as the transformer giving trouble has been repaired.

In addition to the split-up in the mai Niagara power supply to the larger consuming units, which probably represent 75 per cent of the load, the steam-generated power supply has been split into tw branches, the regular load bus, *B*, and th emergency bus load, *A*. In case o Niagara power failure or trouble withi the plant, the load bus *B* is dropped immediately by relaying, and bus *A* is kep alive from the steam-station source

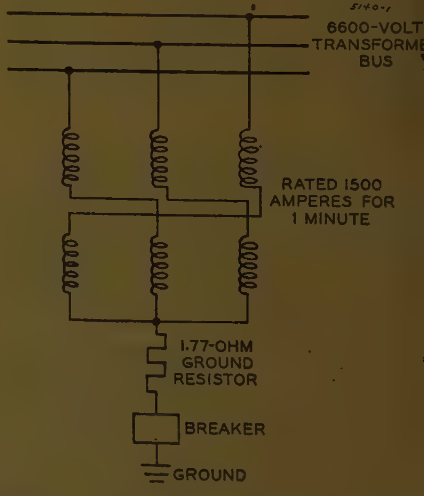


Figure 1. Three-phase zigzag groundin transformer

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Conclusions

The operation of 400-cycle a-c alternators for the supply of auxiliary power in large aircraft, driven from the main engines through variable ratio drives, is feasible, and parallel operation of these alternators is practical. No elaborate synchronizing equipment is required. Thus, the use of higher-voltage alternating current now can be considered as another form of electric power supply for large aircraft, and we can begin to evaluate its properties in a realistic manner.

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2. GOVERNOR REQUIREMENTS FOR AIRCRAFT ALTERNATOR DRIVES, W. K. Boice, L. G. Levoy, Jr. AIEE TRANSACTIONS, volume 64, 1945, July section, pages 534–40.

This emergency bus *A* furnishes power to steam-plant auxiliaries such as boiler induced-draft fans, forced-draft fans, and powdered-fuel drives.

The steam station normally is supplied with surplus gas from the blast furnaces, fed to the boilers as fuel. This is supplemented with an auxiliary powdered-coal and fuel-oil supply. The emergency bus *A* also takes care of boiler-room auxiliary drives, exciters, and plant water supply.

The water supply in a steel plant is particularly important. It must be maintained constantly for cooling purposes, in the blast-furnace hearths, in the charging doors and valves of the open-hearth furnaces, in the gas scrubbers, and in many other applications where high temperatures are encountered.

The emergency bus *A* also supplies power to a few motor generator sets which furnish d-c power to open hearths and coke ovens. This d-c power must be a dependable supply, as the open-hearth cranes may be handling hot molten metal, and the coke oven machinery may be pushing hot coke, which cannot be interrupted without damage to the large pusher rams.

An industrial plant with its large integrated power system is quite similar to a public utility, except that the distances between consumers are very short.

In general the transmission system consists of several three-phase 500,000-circular-mil or 1,000,000-circular-mil copper cables supported on steel towers with wide spacing to prevent the conductors from whipping together under unfavorable wind and sleet conditions. Wide spacing is permissible with the heavy conductors, as both the supporting steel spans and the distances between the various mills are short, and with 25-cycle current line reactances are not objectionable.

Due to the proximity of the various mills and substations, it is rather difficult to set up any relaying scheme with the time elements necessary to give much selection in clearing the various lines in the system. Recent relay developments have been applied from time to time, but it is hard to keep up with advancements made in relay protection.

Relays

The original relay installation in our plant for line protection made use of *CZ* impedance-distance relays. These did not prove satisfactory, due to the relatively short lines and high currents involved, and they have been replaced with various other types, the purpose of which is to sectionalize the plant by clearing a section of the power system rather than an individual line. This is a simpler scheme and does not involve the coordination of as large a number of relays, and yet outages due to line troubles are kept relatively low.

In order to provide still better protection for the equipment, grounding transformers have been installed, as shown in Figure 1, to establish a definite ground neutral and provide adequate ground current for ground relaying. The relaying system thus is divided into two parts:

1. **Phase-to-phase fault relaying.**
2. **Phase-to-ground fault relaying.**

With the normal power system setup, the system is sectionalized into zones by means of relays, as shown in Figure 2. In case of a fault in zone 1 it is separated from zone 2 by means of overcurrent phase relays, type *IAC*, and directional overcurrent ground relays, type *CR*, at *D*89 and *D*90. Within zone 1 further separation on lines 89 and 90 is obtained by means of directional overcurrent relays, type *IBC*, and directional ground relays, type *ICC*, at *BB*89 and *A*90. Lines 98 and 99 between the hot-mill and the cold-mill busses are protected by type *IJD* pilot-wire relays, which function on either a phase or ground fault between the ends of the line while being inoperative on a through fault beyond the ends of the lines.

Pilot-wire, type *HCB*, relays are installed on lines 101 and 102 to the bar mill. The *HCB* relay takes the current in each end of the line and passes it through a positive- and zero-sequence network in each relay. The voltages across these networks are compared and are equal and opposite for any through fault beyond the line terminals. For either a phase or a ground fault between line terminals, these voltages are not equal and opposite but add up to operate

ϕ—Phase-to-phase-fault protection
GR—Ground-fault protection
HCB—Westinghouse type *HCB* pilot-wire relay
IAC—General Electric Company induction overcurrent relay
CO—Westinghouse induction overcurrent relay
IBC—General Electric Company directional overcurrent relay
CR—Westinghouse directional overcurrent relay
IJD—General Electric Company pilot-wire relay
ICC—General Electric Company current polarized ground directional relay
CWC—Westinghouse current polarized ground directional relay

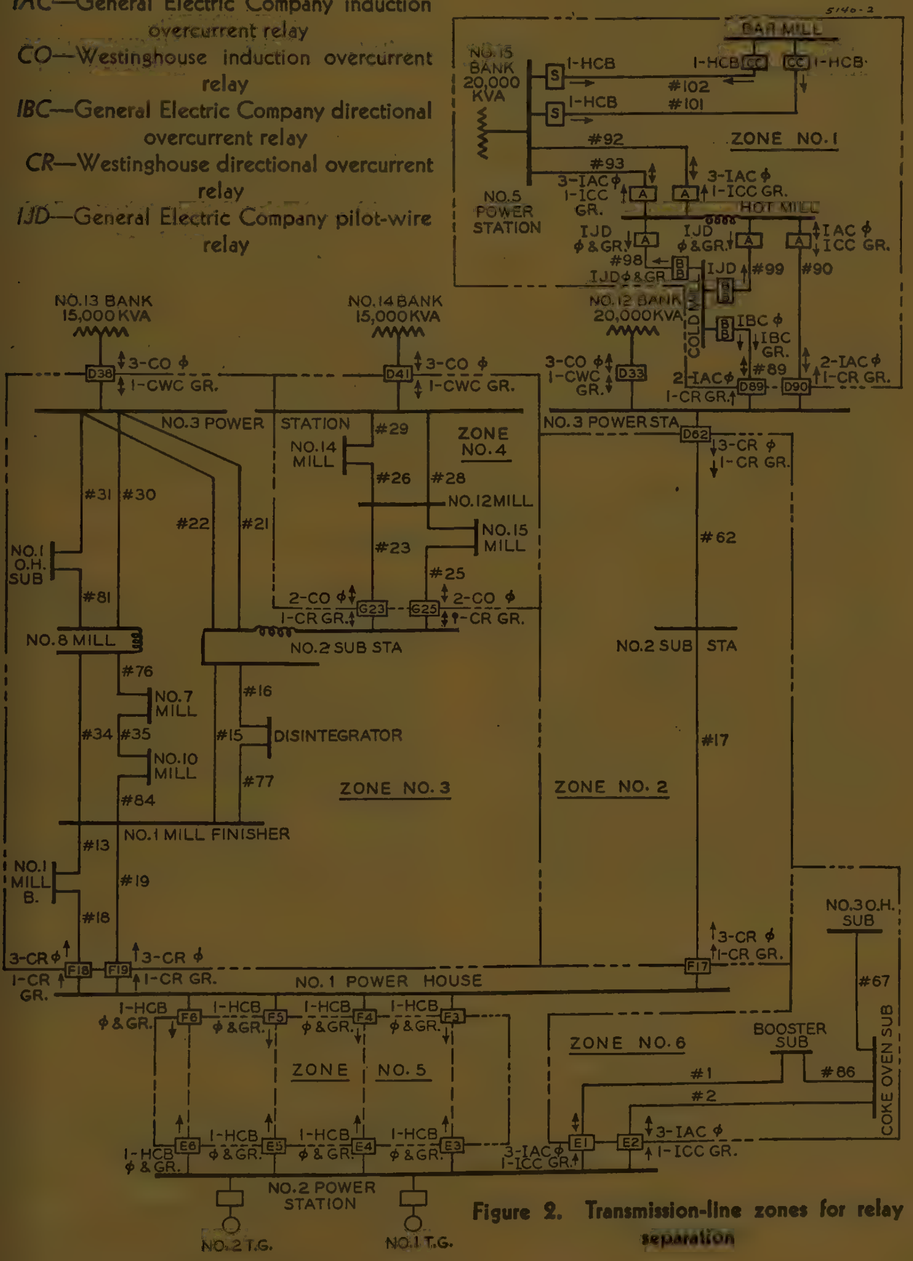

Figure 2. Transmission-line zones for relay separation

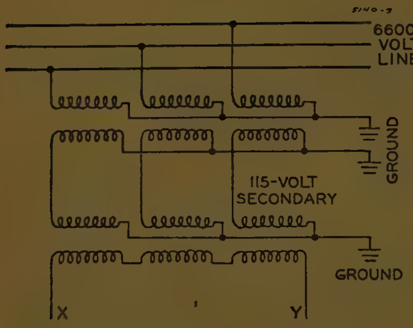

Figure 3. Potential connections for *CR* directional ground relays

relays at both ends of the line, thus clearing the faulted section.

In the original installation the 6,600-volt strip-mill substation consisted of one solid line of 24 metal-clad oil circuit breakers. After a failure of an oil switch which ruptured its tank and started a fire which destroyed several units, the bus was split into three sections, with a reactor placed between two sections and a third section relocated closer to the apparatus which it served.

All the 6,600-volt machines operating in this section of the plant are of recent design, with neutrals brought out with a double set of current transformers arranged for differential-relay protection as well as for protection from overload, phase failure, or undervoltage.

Surge protection equipment consisting of capacitors and Thyrite arresters is located in pockets in machine foundations as close as possible to the machine leads.

When the station was rebuilt, a reactor of three-per-cent reactance on 10,000-kva base was installed to reduce the asymmetrical amperes on a short circuit from 81,000 to 63,600 amperes under the worst conditions. Cross-differential protection was installed on both sections of the two hot-mill busses, and differential protection was installed on the cold-mill bus at the time of this change-over.

*A*92 and *A*93 circuit breakers are operated by straight overcurrent relays, type *IAC* on phase faults, and by ground directional relays, type *ICC*, for ground faults. This is a current-polarized relay obtaining its directional characteristic from a second current winding which is energized by the ground current flowing in the neutral of grounding transformer 30 at power station 5. The same type of relay is used for ground protection at *A*90. These ground relays operate to clear faults away from the bus.

Zones 2 and 3 separate from the rest of the system by means of directional overcurrent relays, acting as phase relays and ground relays and operating when the fault is within the zone. A typical case would be at *F*17 where there are three type-*CR* phase relays and a low-energy *CR* ground relay for protection. These relays consist of a directional element operated from phase current and voltage and an overcurrent element. The relays operate only:

1. When the direction of the current is proper.
2. When the amount of current exceeds the setting.

The ground relay is similar, but the potential for the directional element is supplied from a set of star–delta-connected potential transformers with one corner of the delta secondary open (Figure 3) and the potential coil of the relay connected across the open corner. Normally there is no voltage across these terminals, but in case of a ground fault at any place of the system a zero-sequence or ground-fault voltage will occur across the terminals of the open corner of the delta. Similar relays are installed at *D*62, *F*18, and *F*19.

At the *D*38 end of zone 3 straight overcurrent relays with sufficient time and type *CWC* current-polarized ground relays are depended upon, with the current coils of the *CWC* connected in series to act similarly to plain overcurrent ground relays.

Zone 4 normally is operated as a radial feeder out of *D*41, and *G*23 and *G*25 open. The only relays involved are overcurrent phase and ground relays, similar to those at *D*38, *G*23, and *G*25 circuit breakers are equipped with overcurrent relays set relatively low to operate on any abnormal current, and they completely sectionalize the fault, should *G*23 and *G*25 be closed. *G*23 and *G*25 have a ground overcurrent relay to operate on ground faults in either direction.

Zone 5 covers four underground lead-cable circuits between power stations 1 and 2. Each of these circuits is equipped with *HCB* pilot-wire relays, similar to the relays on lines 101 and 102 to the bar mill previously described. These relays have operated on two occasions to clear cable faults, performing satisfactorily without disturbing the rest of the system.

Zone 6 is treated as a radial feeder out of power station 2 with *IAC* phase relays. At present there are no ground relays in this section, but it is planned to install *ICC* current-polarized directional ground relays, with the direction of operation away from power station 2.

All transformer banks and 6,600-volt motors are protected with either differential relays or phase and ground relays, the ground relays being set to operate almost instantaneously in order to clear any faulty machine from the line at once.

A general over-all picture of our ground relaying scheme is shown in Figure 2. Not all lines but only the circuits controlled by the ground relays on lines are shown on this sketch. Ground relaying is particularly desirable, as the greater portion of all faults usually develop first as ground faults which may turn into phase-to-phase faults. Ground relays are also very desirable in the protection of motors and transformers, because grounding of windings to the frame is a common type of fault. By means of ground relays the machine with such a fault can be removed quickly from the line before any serious phase-to-phase fault can develop with the attendant damage due to short-circuit currents. This is an important point, as the short-circuit currents on our system are approximately 50,000 amperes while ground currents never can exceed 5,000 amperes. Because ground relays on machines are set to operate very quickly, the damage to the machine due to a ground fault in the machine is thus kept at a minimum.

Summary

The electric load in a steel plant may be compared to a public utility feeding many consumers in a congested area.

Continuity of service is guaranteed by obtaining power from more than one power supply.

Interruptions may be held to a minimum by proper phase and ground-relay protection.

Zones may be set up to isolate troubles and confine interruptions to the particular zones in which they occur.

Maintenance of Good Brush Performance

W. C. KALB
FELLOW AIEE

BRUSHES used on electric equipment have three distinct functions to per-orm. First, they must carry the load current into and away from the rotating element of the machine. Second, they must function as bearings on the surface of the commutator or slip ring, preferably without any applied lubrication, and at surface speeds sometimes in excess of 100 miles per hour. Third, on commutating machines, the brushes must control within safe limits the short-circuit current result-ng from uncompensated voltages in the armature coils undergoing commutation and must resist the destructive action of the very high local current densities which are frequently characteristic of the commutation cycle.

This is quite an assignment. In addition to the need for good electrical conductivity, heavy demands are placed on the lubricating, thermal, and refractory properties of the brush material. The combination of these characteristics best adapted to one application obviously may not be best for another in which a different relationship of the three major brush functions is encountered. Before undertaking a discussion of the maintenance of good brush performance it is therefore advisable to consider the criteria by which acceptable performance of these essential brush functions is judged.

Criteria of Good Brush Performance

1. Sparking, the most obvious indication of unsatisfactory brush performance, may result from a variety of causes. Among the numerous causes may be mentioned poor commutator surface; faulty commutation cycle, such as completion of current reversal too early or too late; faulty machine adjustment; various mechanical or electrical faults in the machine; bad load conditions; vibration; and incorrect brush grade. Freedom from sparking of injurious intensity may well be considered the first criterion of good brush performance.

2. Operation with minimum commutator wear is another measure of satisfactory brush performance. Commutator wear accompanied by evidence of burning on the commutator surface is usually traceable to imperfect contact between brushes and commutator. This, in turn, may be due to low spring pressure, tight brush holders, excessive vibration, high or feather-edge mica, or brush chattering. A bright surface on the worn commutator indicates actual abrasion, either from foreign material in the brush face, or from the use of an abrasive brush grade.

3. The nature of the surface film on the commutator is being given increasing recognition as a gauge of brush performance. Satisfactory operation requires that a smooth surface film of uniform character be maintained. This subject will be discussed in greater detail later.

4. Brush performance is also judged on the basis of associated losses or, conversely, machine efficiency. Energy losses from short-circuit currents during commutation, from the resistance of a glazed commutator surface to the passage of load current, from the unbalancing effects of selective action, and from brush friction may at times be enough to affect seriously the efficiency or the satisfactory performance of a motor or generator. However, with proper brush selection and operation, these losses can be kept to a small value.

5. Quietness of operation is dependent, primarily, on the maintenance of uninterrupted contact between brushes and commutator. On some applications an audible tone from the brushes in operation may be sufficient reason for condemning the grade. Even where freedom from noise is not an essential feature of performance, the firm contact on which quiet operation depends has an important influence on satisfactory performance in other respects.

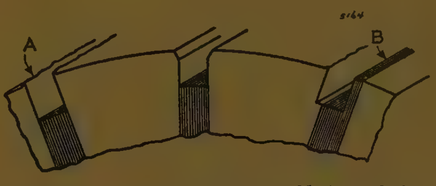

Figure 1. Undercut mica with U-shaped slots

A—Rough edges may be raised by the undercutting tool. These should be removed by beveling

B—Slotting off center leaves mica fins

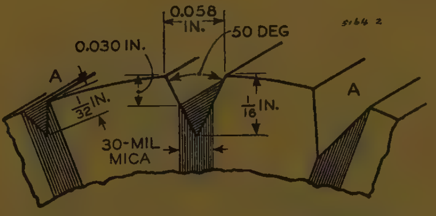

Figure 2. Undercut mica with V-shaped slots

A—Mica fins are left in V slots that are too shallow or off center

6. Brush life is often given undue weight in judging brush performance. While long brush life is desirable, its attainment at the sacrifice of good performance in some other respects may prove far from economical. When satisfactory brush performance in respect to the criteria previously cited is achieved, reasonable brush life may well be expected to follow as a natural result.

Good Commutator Surface Essential to Good Brush Performance

Economical maintenance and satisfactory operation of electric equipment is dependent to a large degree upon the care used in preparing the surface of the commutator or slip rings and the attention given thereafter to keeping the surface in good condition. Systematic inspection and immediate correction of incipient faults greatly reduce liability to shutdown, as well as the cost of repairs incurred when faults such as bar burning, high mica, and flat spots are allowed to develop to a serious stage.

SURFACING

The best way to produce a true cylindrical surface on a commutator or slip ring is by turning or grinding. Before undertaking this operation on a commutator it should be carefully examined to make sure that all segments are firmly clamped by the V rings and, if necessary, it should be retightened. On large machines the grinding or turning operation is usually performed with the armature in the machine but on small machines it is usually transferred to a lathe. In either case, rigid mounting of the turning tool or grinding equipment is essential to good results. Windings should be shielded from copper clips and dust during this operation. A diamond-shaped tool and fine-pitch feed are usually preferred for turning, finishing with a very light cut to minimize "threading." Some manufacturers use an actual diamond tool on small high-speed commutators where a high degree of precision is essential. Mounting the armature and its bearings on V blocks, rather than on lathe centers, is also recommended practice for lathe finishing. Grinding fixtures are available carrying a revolving abrasive wheel and there are also other types using a stationary stone held in a manner similar to a lathe tool.

The commutator stone is definitely superior to sandpaper for hand surfacing of commutator and rings. Such stones, of suitable grit and bond and with handles attached, are readily available. The stone should be formed or worn to the same curvature as the surface being finished and should have a span in the direction of rotation greater than the broadest flat spot to be removed. Presenting a rigid surface, the hand stone, properly applied,

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will remove high mica, flat spots, burned bar edges and other surface defects without disturbing the cylindrical contour. By the use of a hand stone when such faults first appear, the need for turning or grinding can be postponed or entirely avoided. A hand stone should also be used after turning to remove all traces of the thread-like ridges left by the lathe tool. The stone should be pressed firmly against the commutator or ring and moved very slowly from side to side to avoid diagonal scratches.

Sandpapering is an acceptable method of removing deposit from a commutator surface, correcting roughness, or reducing high mica, provided the accurate contour of the surface has not been seriously disturbed. However, sandpaper cannot be depended on to remove flat spots. Because the paper is flexible it tends to broaden, rather than remove, the flat spot. The rigid face of the commutator stone, on the other hand, will cut down the high areas and remove the spot by restoration of a true cylindrical surface. Best results with sandpaper are obtained by attaching it to a wooden block formed to the radius of the commutator or ring. Paper of relatively fine grain is usually preferable.

UNDERCUTTING

The practice of undercutting the mica of commutators has now attained such general approval that there is no need to argue its merits. There are relatively few applications on which flush mica commutators have any advantage over undercut. The tools used for undercutting range from the hacksaw blade and triangular file for hand operation to a great variety of motor-driven tools, most of which use revolving disk-shaped cutters. Shape of slot is largely a matter of individual preference. The square bottom or U-shaped slot shown in Figure 1 has the advantage of being effective until the commutator is worn down the full depth of the undercut. Accumulated dust, however, is not thrown out from the U-shaped slot as readily as it is from the V-shaped slot, Figure 2. Depth of U-shaped slots is generally made about 1/32 inch to 3/64 inch. Cutters for V-shaped slots are made with different angles between the cutting edges. Forty-degree cutters are generally preferred for thin mica, 50-degree for medium mica, and 60-degree for thick mica. The depth of cut should be sufficient to leave approximately 1/32 inch of mica-free copper at each side of the slot. Before undercutting a commutator, it should be turned, ground, or stoned depending on its previous condition, so that all surface faults, such as grooves, ridges, and flat spots, are eliminated. After undercutting, the commutator surface should be lightly polished with a fine-grain commutator stone or sandpaper to remove the small burrs raised at the edges of the slots by the undercutting tool. It is also good practice to burnish the commutator surface with hardwood blocks applied under heavy pressure. Hard maple is an excellent wood for this purpose and should be applied with the end grain against the commutator. This produces a well polished surface and, since considerable heat is developed, tends to form an oxide film on the segments which has a favorable influence on brush performance.

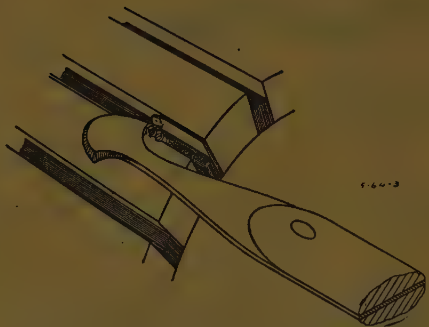

Figure 3. Removing mica fin and chamfering edge of commutator bar with home-made tool

Dimensions of slots exaggerated for clarity

MICA FINS

A frequent source of trouble on undercut commutators is variously described as "feather-edge mica," "side mica," or "mica fins." These terms refer to the thin edges of mica often left along the sides of the slots through failure to center the cutter accurately on the mica, deviation from a straight line parallel to the commutator segments, or insufficient depth of V-shaped slots. Commutator wear may also reach mica fins not in evidence immediately after undercutting. Troubles traceable to feather-edge mica are encountered with such frequency on commutating equipment, and the cause is so often overlooked in the search for less tangible faults, that the avoidance of this condition justifies special care. It results in difficulties of the same type as those encountered on nonundercut commutators when the mica becomes "high." In fact, it may be even more troublesome than ordinary high mica due to the tendency for small particles of mica to break loose and become embedded in the brush faces. Sparking, bar burning, picking up of copper by the brush faces, and noisy operation are among the operating difficulties often traced to this fault. After undercutting the commutator, or whenever faulty operation indicates the possible presence of feather-edge mica, every slot on the commutator should be inspected carefully. If mica fins are detected they should be removed with a knife blade or other suitable tool. An effective tool for this purpose can be made by grinding a hook-shaped cutting edge at the end of a hacksaw blade and attaching a suitable handle. This type of tool can be drawn through the slot, as illustrated in Figure 3, cutting at the root of the side mica on one side and slightly chamfering the opposite edge of the slot at the same time. Of course, frequent resharpening will be necessary to insure complete removal of the mica. After completing this operation the slots and armature windings should be blown out thoroughly to remove all loose particles of mica and copper.

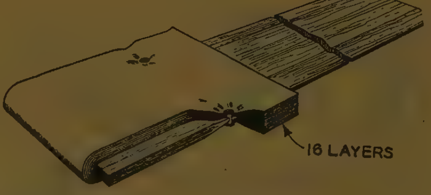

Figure 4. Canvas wiper of the type described in text

CANVAS WIPER

Another home-made device which has proved effective in maintaining good brush performance is the canvas wiper, Figure 4. Several layers of six- or eight-ounce hard-woven canvas or duck are folded over the end of a strip of strong pliable wood of suitable dimensions and secured by rivets. Countersinking the strip at the points where the rivets are inserted reduces the danger of the rivet heads making contact with the commutator. The canvas is held against the commutator under heavy pressure and rubbed slowly back and forth. This removes oil, grease and smudge from the commutator surface without hazard of the hand coming in contact with live parts of the machine. Applied with sufficient frequency, it removes the oxidation at bar edges caused by sparking and may forestall the development of serious bar burning. However, it does not destroy the desirable oxide or electrographitic surface film on which good brush performance is largely dependent. Maintenance departments which have adopted the use of the canvas wiper as regular practice have found commutator maintenance greatly reduced. Frequency of application depends on the tendency for surface deposit to accumulate. With moderate accumulation, daily application is recommended while more severe conditions may require application at shorter intervals. This has proved a very effective method of establishing and maintaining a good commutator surface with a minimum of major resurfacing operations.

Installation and Setting of Brushes

Users of electric equipment seldom have any choice in regard to the type of brush holders supplied on motors and generators but are not thereby relieved of the responsibility of keeping brush holders in good condition, free from any accumu-

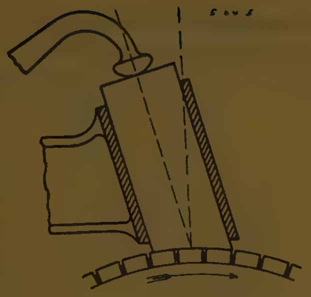

Figure 5. Brush held in trailing position

lation which might interfere with normal movement of the brushes, accurately located and firmly secured, and correctly adjusted in respect to spring pressure.

BRUSH ANGLE

The angle at which the brush is held against the commutator is a subject which might be discussed at great length. For the present, however, it is probably sufficient to make the following general recommendations. The radial position is usually preferred for reversing motors although there are numerous examples of successful operation with the brushes inclined at a substantial angle from the radial. For trailing operation, that is, with the commutator revolving toward the wider of the two angles at the brush face, as in Figure 5, an inclination of 15 degrees from the radial is representative of general practice. A certain amount of wedging action may be encountered when brushes having a relatively high coefficient of friction are operated at larger trailing angles. With leading operation, where the commutator revolves toward the acute angle at the brush face, as in Figure 6, an inclination of from 30 to $37\frac{1}{2}$ degrees is preferred. This insures a reactive force against the leading side of the brush holder which gives stability to the brush position. The critical angle for leading operation is in the neighborhood of 20 degrees. At this and smaller angles, disturbances to brush friction are liable to induce chattering of the brushes. In practice it is customary to bevel the top or free end of brushes operated at a leading angle, as indicated by dotted line in Figure 6, thus giving increased stability to the brush position.

STAGGERING

Most brushes now used have very mild abrasive properties, or none at all, so that the need for staggering the position of the brushes around the commutator to prevent the brushes wearing deep grooves no longer exists. However, the practice is continued for the sake of uniformity in commutator-surface appearance. A rule which should be observed when setting the brush holders in staggered relation is to make this arrangement in positive and negative pairs, as shown in Figure 7. A brush of one polarity should be followed, in the same path, by one of the opposite polarity. The next pair, in a circumferential direction, should then be shifted in an axial direction from the first pair. The character of the surface film developed under the positive brushes differs from that under the negative brushes, but when the brushes are staggered in pairs as described, no part of the commutator is contacted by brushes of one polarity more than by those of the other.

SYMMETRY

Lack of symmetry, either electrical or mechanical, results in inequalities of voltage between different pairs of brush-holder studs and may be the source of heavy circulating currents in the armature. Overheating of armature windings and more severe sparking on some brush studs than on others are operating faults often traceable to lack of electrical or mechanical symmetry. When such indications are noted, check the spacing of the brushes from toe to toe, the alignment of brushes on each stud with the commutator segments, the spacing of field poles, the length of air gaps, and the security of cross connections. Any inequalities found should be corrected.

SANDING FACES

New brushes, when first installed, should be carefully sanded to the contour of the commutator. A coarse grade of sandpaper is suitable for removing most of the carbon, but the final surface should be formed with paper of fine grain or with a brush-seating stone. The latter is a fine-grain free-wearing stone of special composition sold by various manufacturers of commutator-maintenance equipment. Sandpaper should be drawn only toward that side of the brush holder against which the brush rests when in operation and the brush should be lifted to return the sandpaper strip for succeeding strokes. This avoids forming a double face on the brush. The time required for facing in the brushes will be reduced by using considerably heavier pressure than that used in operation. Keep the sandpaper in contact with the commutator during the cutting stroke to prevent rounding the edges of the face.

SPRING PRESSURE

Careful attention should be given to the adjustment of spring pressure on the brushes. Many instances of faulty brush operation have been traced to improper or unequal spring tension. Unequal spring tension is a frequent source of selective action, the brushes with heavy tension tending to take a proportionately larger share of the load current than those with less. The resulting higher temperature at the faces of the brushes under heavy pressure lowers their contact drop and causes them to take a still heavier share of the load. It can readily be seen that this effect is cumulative and may result in excessive overloading of individual brushes. Pressure on all brushes on a machine should, therefore, be kept as nearly equal as the adjustment provided on the brush holders will permit. For any grade of brush and type of service there is a certain range of pressure within which best performance is obtained. At higher pressures friction losses and brush wear are increased, while at lower pressures the brushes are not kept in intimate contact with the commutator. Sparking, injury to the commutator surface, and rapid brush wear are likely to result from operating brushes at too low a pressure. One pound below the optimum range of pressure will usually result in more rapid brush wear than one pound above the optimum range. In other words, the disintegration of the brush face from electrical causes at low pressure is greater than the increased mechanical abrasion produced by pressure correspondingly higher than the normal range. That is why more brush complaints originate from low pressure than from high pressure. The relationship of brush pressure to rate of wear for a typical electrographitic grade is shown in Figure 8. Brush pressures are recommended by the manufacturer in terms of pounds per square inch. To determine this value, divide the pressure in pounds parallel to the length of the brush by its cross sectional area in square inches. As a rule no correction is made for the increased area of face resulting from a face bevel. Manufacturers' specific recommendations should be followed, but the following figures, in pounds per square inch, are indicative of representative practice.

Industrial service:	
Electrographitic grades	2–3
Carbon and carbon graphite grades	$1\frac{3}{4}$–$2\frac{1}{2}$
Soft graphite grades	$1\frac{1}{4}$–2
Metal–graphite grades (slip rings)	$2\frac{1}{2}$–$3\frac{1}{2}$
Fractional-horsepower motors	4–5
Traction service	5–7

NEUTRAL POSITION

The fact is generally understood that brush position has a pronounced influence on commutation. On noninterpole machines the neutral position, that is, the

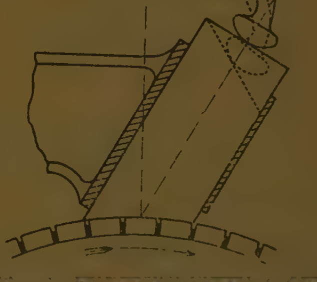

Figure 6. Brush held in leading position

position giving best commutation, shifts with the load. A permanent setting of brush position must, therefore, be a compromise between no-load and heavy-load conditions and can only be determined by a consideration of load characteristics. Interpole machines, on the other hand, have a fixed neutral point, theoretically unaffected by change in load. There is accordingly one best brush position for all load conditions. Time will not be taken for a discussion of the several methods in use for locating the neutral point on interpole machines. It should be mentioned however, that there is little latitude in neutral position on most machines of interpole design. If the brushes are shifted appreciably from the correct position a compounding effect from the interpole field flux is introduced.

Table I. Indications of Unsatisfactory Performance and Their Source

Indications	Immediate Causes	Primary Faults (See Table II)
Appearing at brushes		
Sparking	Commutator surface condition	1, 2, 3, 43, 44, 45, 46, 49, 58, 59
	Overcommutation	7, 12, 31, 33
	Undercommutation	7, 12, 30, 32
	Too rapid reversal of current	7, 12, 30, 32
	Faulty machine adjustment	8, 9, 11
	Mechanical fault in machine	14, 15, 16, 17, 18, 19, 20, 21, 22
	Electrical fault in machine	25, 27, 28, 29
	Bad load condition	38, 39, 40, 41, 42
	Poorly equalized parallel operation	7, 13, 23, 34
	Vibration	50, 51
	Chattering of brushes	See "Chattering or noisy brushes"
	Wrong brush grade	54, 56, 58
Etched or burned bands on brush face	Over commutation	7, 12, 31, 33
	Undercommutation	7, 12, 30, 32
	Too rapid reversal of current	7, 12, 30, 32
Pitting of brush face	Glowing	See "Glowing at brush face"
	Embedded copper	See "Copper in brush face"
Rapid brush wear	Commutator surface condition	See specific surface fault in evidence
	Severe sparking	See "Sparking"
	Imperfect contact with commutator	11, 14, 15, 16, 50, 51
	Wrong brush grade	53, 57
Glowing at brush face	Embedded copper	See "Copper in brush face"
	Faulty machine adjustment	7, 12
	Severe load condition	38, 39, 41, 42
	Bad service condition	46, 47
	Wrong brush grade	56, 60, 61
Copper in brush face	Commutator-surface condition	2, 3
	Bad service condition	43, 46, 47, 48, 49
	Wrong brush grade	58, 60
Flashover at brushes	Machine condition	14, 35
	Bad load condition	38, 39, 41, 52
	Lack of attention	5, 11
Chattering or noisy brushes	Commutator surface condition	See specific surface fault in evidence
	Looseness in machine	15, 16, 17
	Faulty machine adjustment	10, 11
	High friction	43, 45, 49, 51, 57, 58
	Wrong brush grade	54, 57, 58
Brush chipping or breakage	Commutator surface condition	See specific surface fault in evidence
	Looseness in machine	15, 16, 17
	Vibration	51
	Chattering	See "Chattering or noisy brushes"
	Sluggish brush movement	14

Nature of Brush Contact and Commutator Surface Film

It is now widely recognized that brush performance is influenced to a pronounced degree by the interface conditions between the brush and the commutator on which it bears. A great deal of study has been and is still being given to this subject. The commutator surface and the brush face, regardless of the high polish acquired, do not present to each other surfaces of exactly identical curvature or profile. The play of the brush in its holder and the yielding of supporting members under stress may give the brush face a slightly longer radius of curvature than the commutator. The effective area of contact for a particular brush at a given instant may represent, therefore, a relatively short span of the brush face in the direction of rotation. Furthermore, within this area, actual contact between brush and commutator is limited to a few points, or very small areas, carrying the load current at an extremely high current density. Rotation of the commutator, mechanical abrasion, and the disintegrating effects of the high localized current densities cause constant shifting of these areas of contact so that contact at any given point of either brush face or commutator is probably of extremely short duration, except for firmly embedded points which protrude to a substantial degree from the surrounding areas. The paths through which current flows between brush and commutator are of three types:

1. The points of solid contact just described.

2. Adjacent areas in which free particles of carbon, graphite, or copper provide a conducting path.

3. The minute gap, resulting from the difference in commutator and brush-face curvature, across which some current may pass in the form of an arc.

These areas of current flow are constantly and rapidly shifting within the boundaries of the brush face and in a very short space of time, possibly comparable to one full revolution of the commutator, current flow will have occurred through every portion of the brush face, giving it an appearance indicating complete and uniform contact.

Surface Film

Increased recognition is also being given to the influence of the commutator surface film on brush performance. Emphasis has been placed on the importance of this surface film by experiences encountered on military aircraft at altitudes of 25,000 feet and higher. Ordinary grades of brushes which perform in an entirely satisfactory manner at normal altitudes, wearing at a rate of one inch in several thousand hours, may wear out completely in the course of one flight at high altitude. This failure has been shown to be due to the failure of the commutator surface film under the atmospheric conditions existing thousands of feet above the earth's surface. This brings us to a consideration of the character of this surface film. It appears to be of complex structure, including in its make-up a thin layer of copper oxide on the surface of the commutator bars and a thin deposition of graphite over this oxide. On this solid layer of protective film an adsorbed layer of oxygen and water vapor is formed under normal atmospheric conditions. This adsorbed water vapor and oxygen film, although probably of submicroscopic thickness, provides boundary lubrication between the brush face and the graphite-copper-oxide film beneath. Wear of the latter film is thereby reduced and oxygen released by electrolysis of the water vapor tends to maintain the copper-oxide film. The graphite film, of course, is deposited by the brushes. At high altitude there is much less oxygen in the atmosphere and practically no water vapor, so that the boundary film disappears and the brushes come into such close contact with the commutator that atomic cohesive forces become effective. The protective graphitic and copper-oxide film is then quickly stripped off and rapid brush and commutator wear follow. A similar effect has been observed on the slip rings of rotary converters and on some commutating equipment during winter periods of extremely low humidity. Dusting and rapid brush wear has been stopped in such instances by humidifying the atmosphere of the substation or by adding some lubricating agent to the brush to take the place of the missing film of adsorbed water vapor. Such means of correction not being applicable to aircraft equipment

Table I. Indications of Unsatisfactory Performance and Their Source (Continued)

Indications	Immediate Causes	Primary Faults (See Table II)
Appearing at commutator surface		
Rough or uneven surface		1, 2, 3, 4, 17
Dull or dirty surface		5, 44, 59
Eccentric surface		1, 19, 22, 51
High commutator bar		17
Low commutator bar	Sparking	2, 25
Streaking or threading of surface	Sparking	43, 44, 45, 46, 49, 58
	Copper or foreign material in brush face	2, 3, 46, 47, 48, 60
	Glowing	See "Glowing at brush face"
Bar etching or burning	Sparking	2, 3, 7, 12, 30, 31, 32, 33
	Flashover	5, 11, 14, 35, 38, 39, 41, 52
Bar marking at pole-pitch spacing	Sparking	25, 37
Bar marking at slot-pitch spacing	Sparking	7, 12, 30, 56, 59
Flat spot	Sparking	19, 23, 25, 41, 42, 52
	Flashover	5, 11, 14, 35, 38, 39, 41, 52
	Lack of attention	1, 5, 11
Discoloration of surface	High temperature	See "Heating—at commutator"
	Atmospheric condition	44, 46
	Wrong brush grade	59
Raw-copper surface	Embedded copper	See "Copper in brush face"
	Bad service condition	43, 45, 47, 49
	Wrong brush grade	58, 60
Rapid commutator wear with blackened surface	Burning	2, 3, 11, 14
	Severe sparking	See "Sparking"
Rapid commutator wear with bright surface	Foreign material in brush face	43, 45, 47, 49
	Wrong brush grade	60
Appearing as heating		
Heating in windings	Severe load condition	38, 41, 42, 52
	Unbalanced magnetic field	18, 19, 20, 21, 27, 28, 29
	Unbalanced armature currents	8, 19, 22, 25, 27, 28, 29, 37
	Poorly equalized parallel operation	7, 13, 23, 34
	Lack of ventilation	6
Heating at commutator	Severe load condition	38, 41, 42
	Severe sparking	7, 8, 9, 12, 20, 33, 45, 56
	High friction	10, 11, 36, 43, 45, 49, 57, 58
	Poor commutator surface	See specific surface fault in evidence
	Depreciation	24
	High contact resistance	55
Heating at brushes	Severe load condition	38, 41, 42
	Faulty machine adjustment	7, 10, 11, 12, 26
	Severe sparking	See "Sparking"
	Raw streaks on commutator surface	See "Streaking or threading of surface"
	Embedded copper	See "Copper in brush face"
	Wrong brush grade	56, 57, 58, 60, 61

Table II. Primary Sources of Unsatisfactory Brush Performance

Preparation and care of machine

1. Poor preparation of commutator surface.
2. High mica.
3. Feather-edge mica.
4. Bar edges not chamfered after undercutting.
5. Need for periodic cleaning.
6. Clogged ventilating ducts.

Machine adjustment

7. Brushes in wrong position.
8. Unequal brush spacing.
9. Poor alignment of brush holders.
10. Incorrect brush angle.
11. Incorrect spring tension.
12. Interpoles improperly adjusted.
13. Series field improperly adjusted.

Mechanical fault in machine

14. Brushes tight in holders.
15. Brushes too loose in holders.
16. Brush holders loose at mounting.
17. Commutator loose.
18. Loose pole pieces or pole-face shoes.
19. Loose or worn bearings.
20. Unequal air gaps.
21. Unequal pole spacing.
22. Dynamic unbalance.
23. Variable angular velocity.
24. Commutator too small.

Electrical fault in machine

25. Open or high-resistance connection at commutator.
26. Poor connection at shunt terminal.
27. Short circuit in field or armature winding.
28. Ground in field or armature winding.
29. Reversed polarity on main pole or interpole.

Machine design

30. Commutating zone too narrow.
31. Commutating zone too wide.
32. Brushes too thin.
33. Brushes too thick
34. Magnetic saturation of interpoles.
35. High bar-to-bar voltage.
36. High ratio of brush contact to commutator surface area.
37. Insufficient cross connection of armature coils.

Load or service condition

38. Overload.
39. Rapid change of load.
40. Reversing operation of noninterpole machine.
41. Plugging.
42. Dynamic braking.
43. Low average current density in brushes.
44. Contaminated atmosphere.
45. "Contact poisons."
46. Oil on commutator or oil mist in air.
47. Abrasive dust in air.
48. Humidity too high.
49. Humidity too low.

Disturbing external condition

50. Loose or unstable foundation.
51. External source of vibration.
52. External short circuit or very heavy load surge.

Wrong brush grade

53. "Commutation factor" too high.
54. "Commutation factor" too low.
55. Contact drop of brushes too high.
56. Contact drop of brushes too low.
57. Coefficient of friction too high.
58. Lack of film forming properties in brush.
59. Lack of polishing action in brush.
60. Brushes too abrasive.
61. Lack of carrying capacity.

pecial grades of brushes have been developed containing an ingredient which roduces and maintains a protective film n the commutator or ring surface under he adverse atmospheric conditions of exreme altitudes. It is too early to say to hat extent these or similar grades will ind application under normal altitude onditions, but investigations are being onducted along this line on various types f power, traction, and industrial equipnent.

The thin layer of copper oxide normally leveloped on the commutator surface is ery beneficial to brush performance, iding commutation and reducing brush riction. Certain forms of atmospheric ontamination, particularly those of a educing character, tend to destroy the xide film and interfere with good operaion. These contaminants injurious to the ommutator surface have been termed contact poisons." Some of the most acve materials of this type are carbon etrachloride, alcohol, acetone, turpentine, chlorine, smoke, acid, and other corrosive fumes.

ELECTROGRAPHITIC SURFACE FILM

Electrographitic brushes possess, to greater or less degree, the property of depositing an electrographitic surface film on the commutator which is a highly effective aid to good brush performance. It not only has its own inherent value in reducing friction and maintaining intimacy of contact between brushes and commutator, but it also provides protection to the underlying layer of oxide on the commutator surface which apparently is a factor in the maintenance of normal contact drop and commutation. To some extent this deposition occurs through the graphite of the brush being rubbed into the minute crevices of the commutator surface. There is evidence, however, that a substantial portion of this deposition results from electrolytic action and is influenced both by the characteristics of the brush and by the current density in the brush faces. A series of brush grades commercially available provides a good illustration of this point. The regular grades of this series have normal film-forming properties. They will develop and maintain a good electrographitic surface film under average load conditions on heavy-duty power equipment.

However, on applications where the average load is under 40 amperes per square inch, film may not be deposited as rapidly as it is being worn away. Spots and streaks of raw copper sometimes appear on the commutator surface, often followed by the deposition of copper particles in the brush faces and serious threading or grooving of the commutator. The rate of film deposition is doubtless a function of the combined load and short-circuit currents at the brush face since it has been observed that a machine on which the brushes are slightly off neutral may maintain a good electrographitic surface film at a lower load-current density than a similar machine in perfect adjustment. In general, 40 amperes per square inch may be considered the minimum safe current density for the regular grades of this series. The foregoing figure assumes a clean atmosphere, free from oil vapor or contaminating gases which tend to produce a film or deposit on the commutator surface. In the presence of such contamination these grades may be able to maintain the surface film at an average current density as low as 30 amperes per square inch.

A corresponding series of grades with increased film forming properties has been developed for applications where the average load current density is consistently under 40 amperes per square inch. These brushes will maintain a good surface film at current densities as low as 30 amperes in a clean atmosphere or 20 amperes where oil vapor or other atmospheric contamination tends to accelerate film formation.

In the presence of some types of contaminating atmosphere or excessive oil vapor, a surface film of such high resistance may develop that brush performance is impaired rather than improved. For such conditions there is a third group of grades which have a mild polishing action. These grades tend to remove excess film deposition and have also proved advantageous in overcoming a moderate tendency toward edge burning or marking of the commutator bars from over oxidation, such as that produced by very high current peaks in the commutation cycle. The minimum safe current density for grades of this class is usually about 50 amperes per square inch under clean atmospheric conditions or 40 amperes in the presence of a contaminated atmosphere.

Intimacy of Contact

The trend of design toward higher angular velocities and higher surface speeds for commutators and slip rings has given prominence to the importance of intimate contact being maintained between the brush and the moving surface. Interruption of contact resulting from very slight irregularities of the commutator or ring surface may be sufficient to cause sparking and injury to the surface.

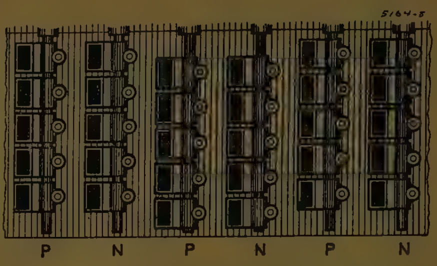

Figure 7. Correct method of staggering brushes

P—Positive N—Negative

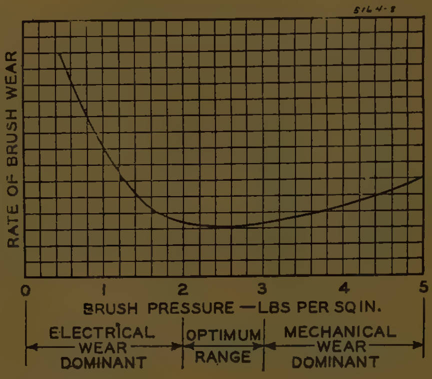

Figure 8. Relationship between brush pressure and rate of brush wear for typical electrographitic grade

Such a condition tends to become cumulative and may interfere seriously with machine performance. The nature of the surface film on a commutator, atmospheric conditions, frictional properties of the brushes, brush pressure, and angle of inclination all have an influence on the intimacy of contact at the brush face. Another factor, possibly more important than any of the foregoing, is related to the elastic property of the brush. This property is subject to accurate quantitative determination and, because of its direct influence on commutating performance, has been termed the "commutation factor." The commutation factor is an index of the capacity of the brush to absorb the shocks resulting from slight irregularities on the revolving surface. Experience with applications under widely varied conditions of service has demonstrated the importance of this property in respect to brush operation. By selecting a brush of suitable commutation factor, intimate contact between brushes and commutator can be maintained, with good commutation and quiet operation, in the face of disturbing conditions that could not readily be overcome in any other manner.

Correction of Unsatisfactory Performance

Maintenance of good brush performance is facilitated by seeking the primary source of trouble as soon as the first indication of unsatisfactory operation is observed. Immediate correction of the primary fault will avoid interruption of ser
ice and the necessity for more expensi
repairs at a later date. There are nume
ous indications of unsatisfactory perforn
ance and a great many primary faul
from which they may result. Most
these primary sources of poor perforn
ance can be grouped under the followi
headings:

1. Poor preparation and care of the m
chine.
2. Faulty machine adjustment.
3. Mechanical fault in machine.
4. Electrical fault in machine.
5. Machine design.
6. Load or service conditions.
7. Disturbing external conditions.
8. Wrong brush grade.

The problem of determining the p
mary source of trouble in a specific ca
can be simplified to some extent by a sy
tematic analysis of symptoms. A plan f
making such an analysis, together wi
lists of the more common indications
unsatisfactory performance and the p
mary faults from which they may resu
is given in the appendix to this paper.

Summary

It is difficult to say what is the mo
important characteristic of a carbon brus
The property which makes a brush gra
the ideal selection for one applicati
may be of secondary importance on a
other. For heavy duty service it seer
preferable that those characteristics whi
aid or permit the maintenance of a u
form surface film on the commutat
should be given first consideration. Pa
ticular attention should be given to t
ability of the brush to maintain close co
tact with the commutator or ring surfa
In addition to care in the selection
brush grade, equal attention should
given to the initial preparation of t
commutator or ring surface, avoidance
high or feather-edge mica, accurate m
chine adjustment, careful brush install
tion, maintenance of correct and unifor
spring pressure, and periodic use of t
canvas wiper. Such routine care co
little but aids greatly in maintaining go
brush performance at minimum expen
Finally, at the first indication of unsat
factory performance, an effort should
made to determine the primary fault a
to correct it in time to prevent seri
disturbance of operation.

Appendix

The accompanying tables have be
prepared as a guide to systematic analy
of unsatisfactory brush performance w
a view to determining the primary fa

Phase-Comparison Carrier-Current Relaying

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DIFFERENTIAL PROTECTION is recognized as the ideal means of quickly isolating faulty electric equipment. The carrier-current relaying system described herein is a differential system comparing the phase angles, via a carrier-current channel, of the currents entering and leaving a transmission-line section, and is known as phase-comparison carrier-current relaying.

Functional Summary of Relay System

The equipment operates when fault currents are above predetermined limits, and simultaneously energizes the trip coils of the circuit breakers at both ends of a faulty line section when the fault-current magnitudes are large enough at both terminals. On all unbalanced faults, that is, other than three-phase faults, the sensitivity is such that operation is obtained with fault currents of less than full-load magnitude. For three-phase faults, the current magnitude for operation must be greater than full load. Since the system operates in response to current alone, no potential supply is necessary.

On all unbalanced faults, the carrier-current blocking signal is sent automatically, that is, independently of the operation of the contacts of any mechanical relay.

The relay operating time is, on the average, less than 2.5 cycles, and sensitivity is obtained with unusually low burden.

Only one carrier-current frequency is required and the channel is available, except during faults, for other uses such as telemetering and supervisory control. The equipment can be used on a two-frequency duplex channel if other services make such an arrangement desirable.

Carrier-Current Channel Requirement

The ideal relaying system should require only one carrier frequency in order to conserve channel space. This eliminates any system requiring simultaneous transmission from the two terminals. Comparison of more than one quantity requires either tone modulation for each quantity or a separate carrier frequency for each quantity, either of which exceeds the single-channel goal. Consequently, a single quantity is derived from the phase currents and this single quantity is used in such a way that transmission in both directions over one carrier-current channel is possible.

The means of accomplishing two-way transmission over a single carrier-current channel and the method of obtaining a single quantity for comparison are discussed hereinafter.

Fundamental Operating Principle

Fundamentally, the operating principle of this equipment is exactly the same as the first carrier-current relaying system,[1] but in the present application, the use of modern methods has eliminated the limitations of certain components of the earlier equipment. Since the fundamental principle has been explained before,[1,2] it should suffice here to review the explanation only briefly.

At each line terminal, the local single quantity derived from the polyphase currents has two functions, namely:

1. During one half cycle it causes a blocking signal to be transmitted, via the carrier-current channel, to the remote line terminal.
2. During the alternate half cycle, it causes tripping if no blocking signal is received from the remote terminal.

Since blocking requires that a signal must be received from the remote terminal in the half cycle assigned to tripping at the local terminal, the two terminals must alternate in transmitting blocking impulses during successive half cycles. The current transformers are connected so that this action occurs on external faults.

During internal faults, there is a relative current reversal of approximately 180 degrees and the two line terminals transmit blocking signals during one half cycle, leaving both terminals free to trip during the other half cycle. Since lack of a signal from the remote terminal permits tripping, tripping at one terminal (but not at both terminals) is obtained for an internal fault fed from that end only.

Derivation of Single Quantity for Comparison

In order to determine the best method of combining the three individual phase currents into a single quantity for comparison, it is helpful to examine the currents present during each type of fault in terms of symmetrical components as follows:

Single-phase-to-ground fault	I_1..I_2..I_0
Two-phase-to-ground fault	I_1..I_2..I_0
Phase-to-phase fault	I_1..I_2
Three-phase fault	I_1
Normal load current (no fault)	I_1

POSITIVE-PHASE-SEQUENCE CURRENT

Since positive-phase-sequence current (I_1) is present during all types of faults, it immediately appears to be the logical choice for the single quantity for comparison. However, except during three-phase faults, the use of I_1 has serious objections. These are the following:

1. While I_1 is present on all types of faults, it is also present during normal load conditions. Thus, a fault can be recognized only if the magnitude of I_1 is greater than full-load current. For different types of faults at the same location, the magnitude of I_1 varies, being a maximum on three-phase faults and a minimum on single-phase-to-ground faults. Hence, if the three-phase short-circuit current is just sufficient for pickup, operation would not be obtained on any unbalanced fault. As an example, on a phase-to-phase fault, I_1 is approximately one half of its value during a three-phase fault at the same location (Appendix I).

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In Table I are listed some of the more common indications of unsatisfactory performance together with the immediate cause and primary faults from which these conditions most frequently arise. The primary faults are indicated in Table I by numbers which refer to corresponding items in Table II.

There may be several primary faults from which the same indication of unsatisfactory performance can result. This is one reason it is often very difficult to determine the exact cause of poor brush operation. On the other hand, there is usually more than one indication of a faulty condition. If the primary faults usually responsible for each of these indications are noted, and those selected which are common to all observed indications, the number of probable sources of trouble will be considerably reduced and isolation of the existing fault thereby simplified.

It is not practicable to include in these tables all indications of unsatisfactory brush performance nor all primary sources of trouble. These lists are representative of the difficulties more commonly encountered and, supplemented by other items from the experience of the observer, offer a systematic plan for diagnosing faulty brush performance and determining what corrective measures should be applied.

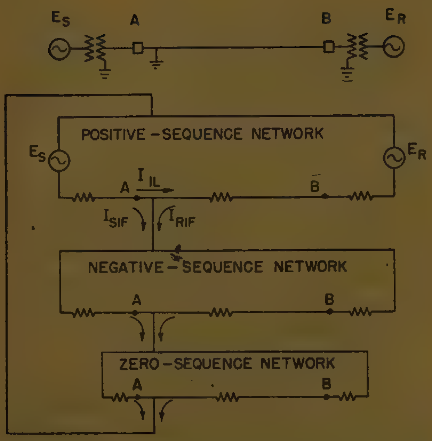

Figure 1. Symmetrical-component currents during a single-phase-to-ground fault

This means that three-phase fault current must be twice the pickup value in order to obtain sufficient I_1 for operation on a phase-to-phase fault.

2. The I_1 component is influenced by load conditions for all types of internal faults except three-phase faults. This is illustrated in Figure 1 for an internal single-phase-to-ground fault and in Appendix I, equations 7 and 8, for a phase-to-phase fault. In the positive-phase-sequence network, there are two components of each line-terminal current, a fault component which enters the faulted line at each terminal, and a through-current component. The through-current component may adversely affect the relaying system in two ways:

1. Decrease the I_1 sensitivity.
2. Cause incorrect blocking.

To summarize, the use of I_1, necessarily operating only above the maximum load value, does not provide the desired sensitivity for faults involving only one or two phases.

NEGATIVE-PHASE-SEQUENCE CURRENT

Since negative-phase-sequence current (I_2) is present during all except three-phase faults, its use should be considered; two of its advantages being the following:

1. Under balanced load conditions there is no I_2 component, and on most transmission lines the load normally will be practically balanced.

2. The I_2 component under fault conditions is substantially unaffected by load conditions (Figure 1 and Appendix I, equations 9 and 10).

These advantages show that the use of I_2 would permit attainment of the desired sensitivity on phase-to-phase faults, and perhaps on ground faults as well, without interference from load current.

ZERO-PHASE-SEQUENCE CURRENT

The presence of zero-phase-sequence current is, of course, the best evidence of a ground fault. For ground faults, I_0 has the same advantages as I_2, that is, the desired sensitivity can be obtained without interference from load current.

Phase-Sequence Currents Used in This Equipment

Since the use of negative-phase-sequence current appears desirable for all types of faults except the three-phase fault, this equipment uses a negative-phase-sequence network with provision for developing a network output on three-phase faults. This provision consists of three phase-overcurrent relays, operation of all three shifting the network so as to introduce a positive-phase-sequence component into the network output. Thus, the equipment functions on three-phase faults at a current magnitude above full load, but for unbalanced faults, phase currents of less than full-load magnitude will cause operation.

Any relaying system for the general application must provide high sensitivity during ground-fault conditions. Because of possible unbalanced load, it is not considered practical to attempt to attain the necessary high sensitivity by means of the use of negative-phase-sequence current alone. For that reason, it is possible to use zero-phase-sequence current should the available minimum negative-phase-sequence current be too low to assure relay operation. Since the phase-to-phase fault still requires the use of negative-phase-sequence current, a combination of negative-phase-sequence and zero-phase-sequence currents is then used for unbalanced faults, the network output being of the form ($I_2 + KI_0$).

When two different phase-sequence components are added without taking account of their relative magnitudes, it may be possible to obtain an internal ground-fault condition wherein one of the phase-sequence quantities has control at one terminal with the other phase-sequence quantity having control at the other terminal. Such a condition might easily result in a "blind spot" (failure t trip at any terminal).

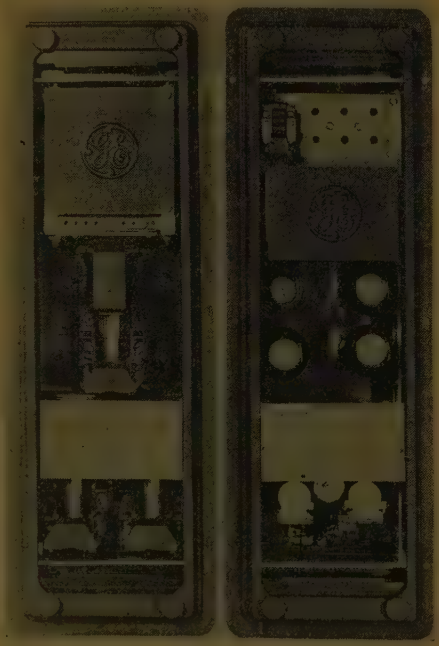

Figure 2. Network unit (left) and tripping unit (right) in draw-out cases for semiflush mounting

Therefore, when zero-phase-sequenc current is used, it is essential that the zero-phase-sequence current outweigh (by adjustment of K) the negative-phase-sequence current at both terminals under all internal-ground-fault conditions. The determination of the value of K should be as simple as possible, and here again there is a further advantage in the use of negative-phase-sequence current. Since the flow of negative-phase-sequence current is practically independent of load current, the determination that the system is fre of blind spots is relatively simple. This discussed more fully in Appendix VII.

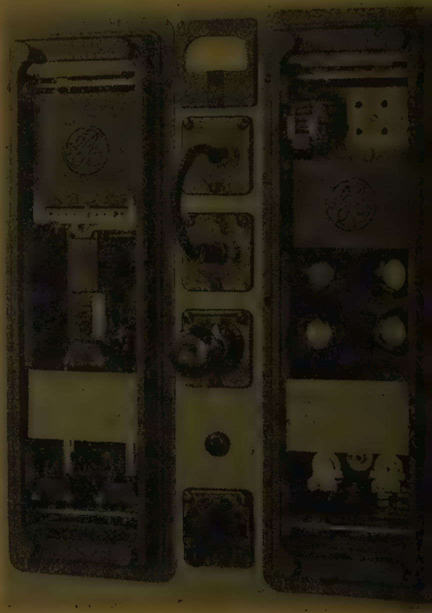

Figure 3. Network and tripping units mounte with a-c and d-c test equipment on a pan section

Description of Relaying Equipment

The physical embodiment of the oper ating principles developed and outline in the foregoing is shown in Figure 2 whic illustrates all the relay elements mounte in drawout cases for convenience in ad justment and maintenance. Figure shows these cases assembled with tes equipment on a panel. The relations the various elements are shown in th block diagram, Figure 4, and Figure 5 is simplified elementary diagram of the ci cuit.

All of the tubes are type 25*B*6*G* lik those used in the associated carrier-cu rent transmitter–receiver. All of th auxiliary relays except the seal-in rela are of the telephone type, operating their normal contact pressures, thus insu ing reliability proved by long experience.

Phase-Sequence Network

With reference to Figures 2 and 5, the network is housed in the left-hand case, and consists of transformers $T1$ and $T2$, resistors $R1$, $R2$, and $R3$, and capacitor $C1$. $T1$ and $T2$ are fed from phases a and c, respectively, and $R3$ and $C1$ provide a 60-degree phase shift of the output of $T1$ so that, with balanced load, the voltage across $C1$ is in phase with the output of $T2$, which is then adjusted to equal magnitude by $R2$ so that the vector difference is zero. Zero-phase-sequence current is neutralized by ground-current windings, one of which has taps for adjustment of K. Equation 18, Appendix II, shows that the output of such a network is proportional to negative-sequence and zero-sequence currents only, and independent of positive-sequence current. On a phase-to-phase fault, such a network gives the same magnitude of output voltage regardless of which pair of phases is involved.

In order to provide for a three-phase fault, overcurrent relays in the three phases (O, Figure 5) are so connected that if all three pick up, they change the setting of $R1$ so as to unbalance the network and thus introduce a positive-sequence component into the output (equation 18a, Appendix II).

Amplifier

The amplifier allows the use of a low-burden network by permitting it to operate into a high impedance, and provides energy to drive the carrier-current transmitter as explained later. Also, the amplifier output controls the fault detector and provides the comparer with a local voltage for comparison with the incoming carrier-current signal. These elements are described in the following sections.

The amplifier consists of two tubes operating in push-pull through an output transformer which provides a match with the load resistance consisting principally of the transmitter screen grids.

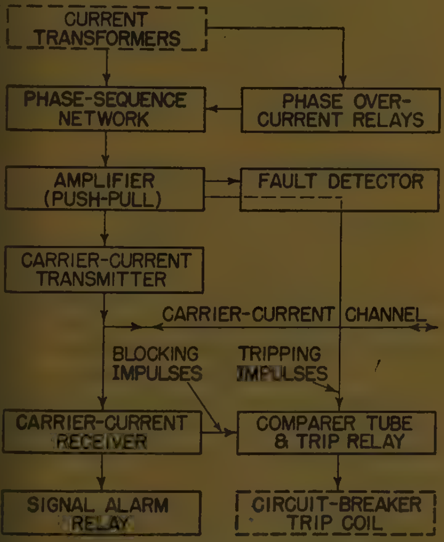

Figure 4. Block diagram showing relations between elements of the protective system

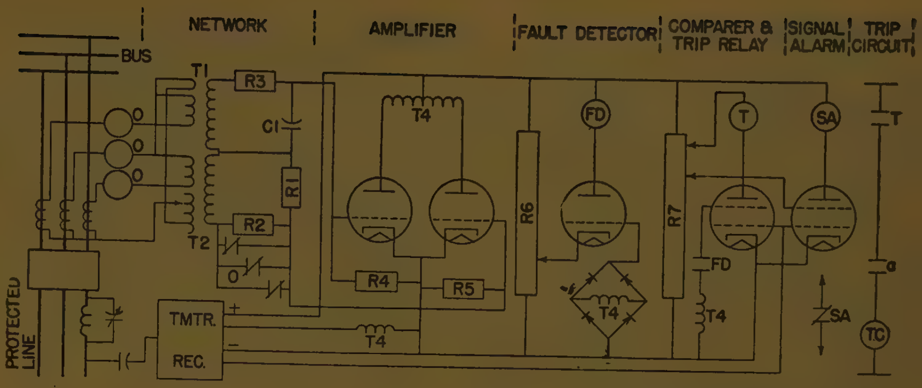

Figure 5. Schematic elementary diagram of the protective system

C—Capacitor
FD—Fault detector
O—Phase overcurrent relay
R—Resistor
REC.—Carrier-current receiver
SA—Signal-alarm relay
T—Trip relay
TC—Circuit-breaker trip coil
T1, *T2*, *T4*—Transformers
TMTR.—Carrier-current transmitter

Fault Detector

The fault detector is controlled by the amplifier, rather than by the network output directly, in order to take advantage of its voltage amplification.

This element is necessary to insure (on an external fault) that the network output is sufficient to drive the transmitter and produce an adequate blocking signal at the remote terminal, before permitting any attempt to trip. If the same carrier channel is used for communication, the fault detector also serves to transfer control of the channel temporarily from communication to relaying for the duration of a fault.

The fault detector, Figure 5, consists of a voltage-divider resistor, a full-wave rectifier, a tube, and an auxiliary relay for its plate circuit. The rectifier is supplied from a separate winding on the amplifier output transformer. The use of a fault-detector tube permits compensation, by proper biasing, for the reduction of amplifier output voltage with d-c supply voltage, thus providing a fault-detector pickup which is substantially constant over an adequate range of d-c supply voltage.

Comparer and Trip Relay

All of the elements described in the foregoing, together with the carrier-current transmitter and receiver, are subsidiary to the comparer. They gather and deliver to the comparer the information necessary to determine the fault location. The comparer makes the decision in accordance with the previous discussion under "Fundamental Operating Principle."

The comparer consists of a tube, a voltage-divider resistor to adjust its plate voltage, and the trip relay in its plate circuit. The tube is mounted on the relay auxiliary unit in the carrier cabinet to minimize the pickup in the lead from the carrier receiver to the comparer control grid. The other elements are mounted in the tripping relay case, Figure 2.

A winding of the amplifier output transformer $T4$, Figure 5, energizes the screen grid of the comparer tube, positive relative to the cathode, during one half cycle, and negative during the other half cycle. During the positive half cycle, the comparer tube will conduct and cause the trip relay to operate if no incoming carrier-current signal is received during that half cycle. On the other hand, reception of a carrier-current signal during that half cycle causes a negative voltage (rectified carrier signal) to be applied to the control grid, thereby blocking tube conduction, and consequently, operation of the trip relay. Figure 6 shows oscillograms of the carrier-frequency voltage during internal and external faults, and the d-c blocking impulses on the comparer control grid during an external fault.

Signal Alarm

The signal alarm provides a means for calling a person to the carrier telephone (simplex) and for indicating the failure of the d-c supply or opening of the heater of any tube (since all five heaters are in series).

This element consists of a tube, a voltage-divider resistor to adjust its screen-grid voltage, and an auxiliary relay in the plate circuit. The tube is mounted in the carrier cabinet with the comparer tube to minimize pickup, the other elements being mounted in the tripping relay case, Figure 2.

The screen-grid voltage is adjusted so that the signal-alarm tube normally carries enough current to pick up the signal-alarm relay. When carrier current is received, the same negative voltage that served as blocking voltage in the comparer is applied to the control grid, reducing the signal-alarm plate current, thereby causing the auxiliary relay to drop out and close the external alarm circuit.

SEAL-IN UNIT

The seal-in unit shunts out the contacts of the trip relay and provides a hand-reset target indication. Provision is made for tripping two circuit breakers.

CARRIER-CURRENT TRANSMITTER-RECEIVER

The carrier-current equipment employed for phase-comparison relaying is essentially the same as that used for directional-comparison relaying. The transmitter is controlled by the voltage on the screen grids of the tubes. During standby conditions, this voltage is zero and no carrier is transmitted.

When the 60-cycle excitation voltage from the network amplifier is applied to the screen-grid circuit, the transmitter sends a signal on the positive half cycle and is definitely blocked during the negative half cycle of the excitation wave. The transmitter cannot lock in and transmit continuously.

The carrier-current receiver rectifies and filters the incoming carrier wave and thus provides a pulsating d-c blocking voltage which is applied to the control grid of the comparer tube. A twin-diode tube is used in the receiver; one of the diodes is used for the relaying function while the other is used to drive an audio amplifier for telephone or other services employing audio modulation. These diodes operate on opposite half waves of the carrier-frequency cycle to obtain a balanced load on the receiver circuit and to reduce interaction between the two circuits.

The transmitter consists of a single-tube master oscillator driving a four-tube power amplifier. The operating frequency range is 50 to 150 kilocycles, and the temperature stability is better than 0.005 per cent per degree centigrade. Variable permeability tuning is used in all tuned circuits. Taps on the power-amplifier tuned circuit serve to match the transmitter plate impedance to the load over a range of 50 to 500 ohms.

A variable inductance in the transmitter output circuit serves to tune the coupling-capacitor circuit and eliminates the need for an external line-tuning unit except where unusually long lead-in cables are required.

A carrier-frequency milliammeter and a d-c milliammeter which can be switched into various circuits in the transmitter or receiver provide the necessary instruments for checking the operation.

While many carrier relay sets have been installed outdoors, the present trend is for indoor installation. The connection between the carrier set and the coupling capacitor is made with low-loss concentric cable which may be run in ordinary conduits or ducts, or buried in the ground.

OTHER FUNCTIONS

Like other carrier relaying installations where the carrier is controlled by a fault detector, the carrier channel is available for other functions such as telephone, remote control, telemeter readings, and supervisory control. The operation of the fault detector cuts off the auxiliary services and gives the relaying function full use of the carrier channel during a fault.

CONSTRUCTION

The carrier-current equipment is built on chassis units having standardized unit dimensions. Many standard units, such as audio oscillators, modulators, and receivers, are available to make up any desired combination. This type of construction is illustrated in Figure 7 which shows a transmitter-receiver and auxiliary unit for phase-comparison relaying, together with a modulator unit for communication.

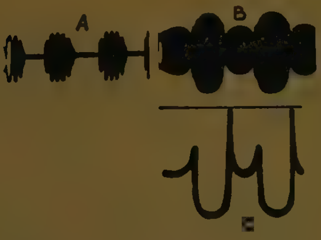

Figure 6. Oscillograms of voltages

A—At terminals of carrier-current set during internal fault. Ripples are due to heterodyne between simultaneous signals of local and remote transmitters. (Length of trace—three cycles at 60 cycles per second)

B—At terminals of carrier-current set, during external fault. Large amplitude is local signal; small amplitude, remote signal

C—At control grid of comparer tube, during external fault, 2.5×pickup current. This pulsating d-c blocking signal results from rectification of the carrier-frequency voltage of B. Zero axis added at top

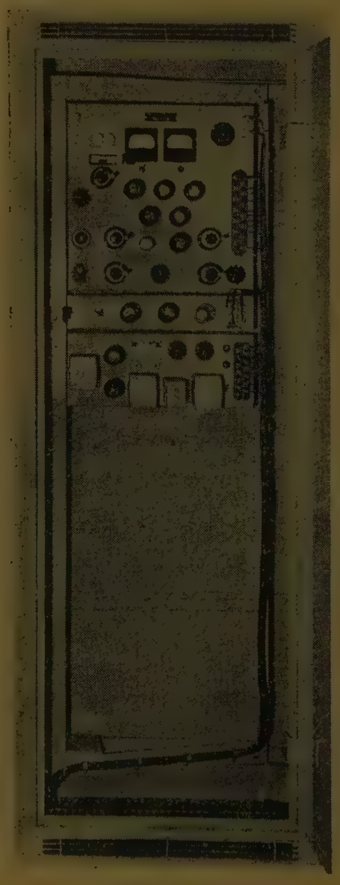

Figure 7. Carrier-current equipment for phase-comparison relaying

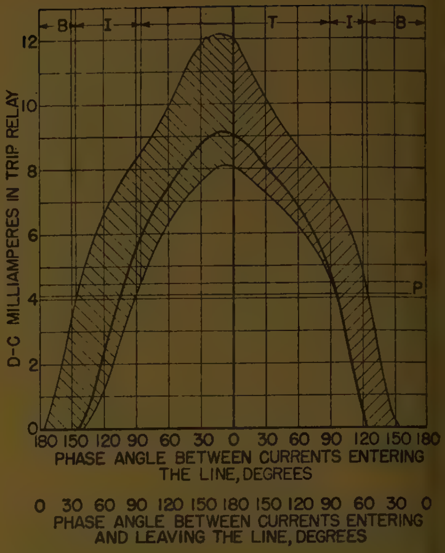

Figure 8. Phase-angle characteristic of one of a pair of production units

Heavy line is for:
1.5×pickup current
ten-decibel line loss
129 volts direct current at each station
Shaded band covers the range from
1.5×pickup current in each terminal
five-decibel line loss
110 volts direct current
to
20×pickup current in each terminal
15-decibel line loss
140 volts direct current

B—Blocking zone
I—Indeterminate zone, where tripping depends on the afore-mentioned variables
P—Range of pickup of tripping relay
T—Tripping zone
(On external faults, the through current is at the same phase angle so that only a narrow band is required for blocking)

Combined Characteristics

PHASE-ANGLE CHARACTERISTIC

The principal characteristic, and the one that determines whether tripping will occur, is the relation between comparer plate current (which is also trip-relay coil current) and the phase angle between corresponding currents at the two ends of the protected section. This is called the phase-angle characteristic. This plate current will also be affected to some extent by fault-current magnitude, fault-current distribution if the fault is internal, carrier-frequency line losses, d-c source voltage at the local terminal; and d-c source voltage at the remote terminal.

This phase-angle characteristic indicates the margin available for blocking on an external fault, and for tripping on an internal fault. Since no single value of fault current or line losses can be considered as normal, the choice of standard conditions is arbitrary; these have been chosen as 1.5 times fault-detector pickup, ten-decibel line loss, and 129 volts, direct current. The solid line of Figure 8 is typical of the average comparer plate (trip relay) current of one unit as a function of phase displacement between the two terminals. This curve is for normal conditions of control voltage, line loss, and fault-current magnitude. The shaded area shows the cumulative effect, on that characteristic, of the most adverse combination of all variables.

Figure 9 shows separately the effects of each variable on the phase angle at which tripping occurs, with all the other variables at standard value.

Fault-Detector Pickup

Figure 10 shows the dependence of fault-detector pickup and drop-out on d-c source voltage.

Operating Time

Figure 11 shows the dependence of operating time on current magnitude. These are typical values, not necessarily maximum for any unit.

Burden

The maximum burden on any one current transformer for any fault not involving ground is 0.2 ohm. The maximum burden on any current transformer for a single-phase-to-ground fault is 1.1 ohms.

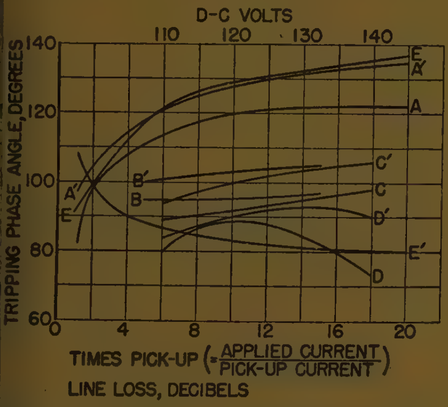

Figure 9. Variations in tripping phase angle due to changing one variable at a time

A to *E* are with one terminal leading, and *A′* to *E′* are with the other terminal leading

A,A′—Equal current in both terminals, 1.1 to 20×pickup current

B, B′—Line loss, 5 to 15 decibels

C, C′—D-c control voltage at local station 110 to 140 volts direct current

D, D′—D-c control voltage at local station dropped to 100 volts for two seconds after being at 110 to 140 volts for two minutes

E, E′—Current at local station 1.5×pickup, current at remote station 1.1 to 20×pickup

Testing Facilities

D-C Test Equipment

The tripping unit is provided with six jacks for measuring the five plate currents and the heater current, on a panel-mounted milliammeter provided with a cord and plug, as shown in Figure 3.

A-C Test Equipment

A five-position switch (shown in Figure 3) and an artificial-load resistor are provided to cut the relays out of service and make an over-all check of both pairs of relays and the carrier apparatus. If the switches are turned in like directions, the test currents applied to the relays simulate an internal fault, and the tripping impulse which should result lights a lamp but is blocked from the trip coil by a switch contact. If the switches are turned in opposite directions, the test currents simulate an external fault, and failure to again light the indicating lamp proves successful blocking. The test current is purposely chosen about ten per cent above the pickup value, and failure to trip or to block at this current indicates that maintenance is required.

Application

Since the equipment described here is based upon current magnitude, it is essential that currents in excess of the relay pickup (with a margin) be available. If simultaneous operation is required, pickup current must be present at each terminal upon the inception of the fault. On the other hand, sequential tripping can be obtained with initial current below pickup at one terminal, if the redistribution of current, upon the operation of the first circuit breaker, is such as to provide pickup current for operation of the second relay.

The amount of margin of calculated fault current above pickup is a matter of personal opinion. However, it is believed that a margin of 50 per cent above pickup is acceptable. This margin can reasonably be reduced at one terminal if a margin of 50 per cent above pickup is available after the other terminal circuit breaker has opened.

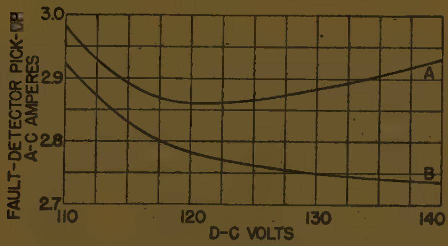

Figure 10. Fault-detector pickup

A—At 110 to 140 volts direct current

B—After operating at 100 volts for two seconds preceded by operation at 110 to 140 volts

It is conservative to assume that the currents which actuate the relay will be those corresponding to transient generator reactance. Since most short-circuit studies are based upon subtransient reactance, currents so based should be decreased by an estimated factor.

The application of any relaying system usually passes through two distinct phases. First, it is necessary to determine, with minimum effort, that a particular system is applicable in a given situation. Later, the application is studied in greater detail to determine the actual settings, operating margins, and other items.

Regarding the relaying system described herein, the first phase, the determination of its applicability, is discussed in the following section. The second phase is outlined in Appendix VII.

Three-Phase Faults

In order for the equipment to trip for an internal three-phase fault, the secondary current must be above the three-phase pickup of the relay, eight amperes.

Ground Faults (Using Negative-Phase-Sequence Current Only)

For ground-fault operation, sufficient negative-phase-sequence current will be available if the total current in the fault is approximately the same for three-phase and single-phase-to-ground faults. More precisely, negative-phase-sequence current alone may be used if the ratio of the *total* three-phase fault current to the *total* single-phase-to-ground fault current $(I_{3\phi}'/I_G')$ is between 0.8 and 1.5 for an internal fault at either terminal (Appendix IV).

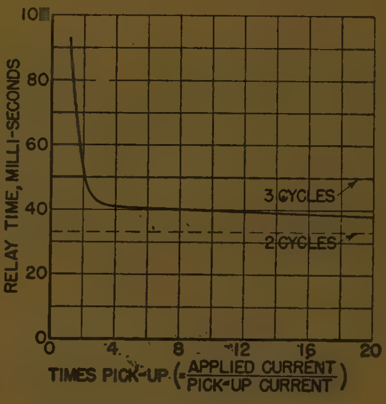

Figure 11. Operating time

Ground Faults (Using Zero-Phase-Sequence Current)

If the ratio $(I_{3\phi}'/I_G')$ is outside of the limits 0.8–1.5, it may be necessary to add ground-current excitation. For zero-phase-sequence excitation to predominate, one of the following conditions must be met:

If $(I_{3\phi}'/I_G')$ is less than 1, then C_1/C_0 must be less than 8.5 (equation 45, Appendix V). If $(I_{3\phi}'/I_G')$ is greater than 1, then

$$\frac{C_1}{C_0}\left[3\frac{(I_{3\phi}')}{(I_G')}-2\right]$$

must be less than 8.5 (equation 46, Appendix V), where

$C_1 = I_{3\phi}/I_{3\phi}'$
$C_0 = I_G/I_G'$
$I_{3\phi}$ = portion of $I_{3\phi}'$ at terminal in question
I_G = portion of I_G' at terminal in question

In order for the equipment to trip for internal ground faults when ground-current excitation is used, the secondary ground current must be above the following:

Single-end feed—0.5 ampere*
Double-end feed—1.65 amperes (maximum)*

Appendix I. Effect of Load Current

To determine the effect of load current on positive-phase-sequence and negative-phase-sequence currents during a phase-to-phase fault and to determine the relative magnitudes of those currents during three-phase and phase-to-phase faults, refer to Figure 12 and assume that the positive-phase-sequence and negative-phase-sequence impedances are equal.**

Before fault (before closing switch S)

$$I_{S1} = -I_{R1} = \frac{E_S - E_R}{Z_{S1}+Z_{R1}} \qquad (1)$$

$$I_{S2} = I_{R2} = 0 \qquad (2)$$

E = voltage across switch $S = E_S - I_{S1}Z_{S1}$

$$= E_S - \frac{E_S Z_{S1} - E_R Z_{S1}}{Z_{S1}+Z_{R1}} = \frac{E_S Z_{R1} + E_R Z_{S1}}{Z_{S1}+Z_{R1}} \qquad (3)$$

With switch S closed and $E_S = E_R = 0$ (Thévenin's theorem)

$$I_1 = -I_2 = I_{S1} + I_{R1} = -(I_{S2} + I_{R2})$$

$$= \frac{E}{2Z_1} = \frac{\dfrac{E_S Z_{R1}+E_R Z_{S1}}{Z_{S1}+Z_{R1}}}{\dfrac{2Z_{S1}Z_{R1}}{Z_{S1}+Z_{R1}}} = \frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{S1}Z_{R1}}$$

$$= \frac{E_S}{2Z_{S1}} + \frac{E_R}{2Z_{R1}} \qquad (4)$$

$$I_{S1} = -I_{S2} = \left(\frac{E_S}{2Z_{S1}} + \frac{E_R}{2Z_{R1}}\right)\frac{Z_{R1}}{Z_{S1}+Z_{R1}}$$

$$= \frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{S1}(Z_{S1}+Z_{R1})} \qquad (5)$$

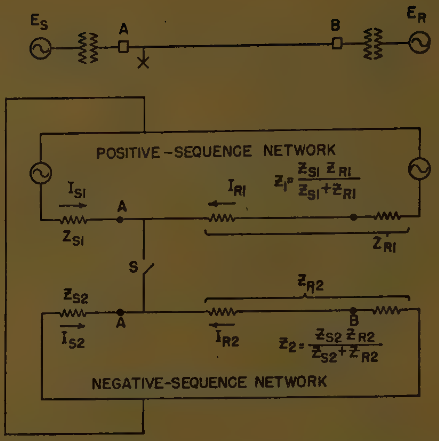
ES
A
B
ER
POSITIVE-SEQUENCE NETWORK
IS1
IR1
$Z_1 = \frac{Z_{S1}Z_{R1}}{Z_{S1}+Z_{R1}}$
A
ZS1
ZR1
S
ZS2
ZR2
A
B
IS2
IR2
$Z_2 = \frac{Z_{S2}Z_{R2}}{Z_{S2}+Z_{R2}}$
NEGATIVE-SEQUENCE NETWORK

Figure 12. Symmetrical-component currents during a phase-to-phase fault (for Appendix I)

$$I_{R1} = -I_{R2} = \left(\frac{E_S}{2Z_{S1}} + \frac{E_R}{2Z_{R1}}\right)\frac{Z_{S1}}{Z_{S1}+Z_{R1}}$$

$$= \frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{R1}(Z_{S1}+Z_{R1})} \qquad (6)$$

Total Current

From equations 1 and 5

$$I_{S1} = \frac{E_S - E_R}{Z_{S1}+Z_{R1}} + \frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{S1}(Z_{S1}+Z_{R1})} \qquad (7)$$

From equations 1 and 6

$$I_{R1} = -\frac{E_S - E_R}{Z_{S1}+Z_{R1}} + \frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{R1}(Z_{S1}+Z_{R1})} \qquad (8)$$

From equations 2 and 5

$$I_{S2} = -\frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{S1}(Z_{S1}+Z_{R1})} \qquad (9)$$

From equations 2 and 6

$$I_{R2} = -\frac{E_S Z_{R1}+E_R Z_{S1}}{2Z_{R1}(Z_{S1}+Z_{R1})} \qquad (10)$$

Comparison of the magnitude of I_{S1}, equation 7, with the three-phase short-circuit current (E_S/Z_{S1}) is obscured by the two different voltages E_S and E_R.

If $E_S = E_R$, equation 7 becomes:

$$I_{S1} = \frac{E_S}{2Z_{S1}} \qquad (7a)$$

This same result may be obtained more easily by considering only the left-hand portion of Figure 12, from which equation 7a is obtained directly.

From equation 7a, it is seen that the positive-phase-sequence current during the phase-to-phase fault is one half its value during a three-phase fault at the same location. In a similar manner, it can be shown that the 2/1 ratio holds in regard to negative-phase-sequence current. The 2/1 ratio is modified slightly if the positive- and negative-phase-sequence impedances are unequal. Also, with regard to the positive-phase-sequence component, the 2/1 ratio can be modified in either direction by the load component, the first term of equation 7.

Comparison of equations 7 and 8 shows that the fault components have the same sign, indicating an internal fault, whereas the load components have different signs, indicating a through-current condition.

Appendix II. Network Output

Referring to Figure 13, windings N have one-third the number of turns of windings A and C, and tapped winding KN has K times the number of turns of winding N. For an unbalanced fault (that is, a fault not operating all three overcurrent relays, O),

$$E_c = K_c[I_c + \frac{(K-1)}{3}I_G] = K_c[I_c + (K-1)I_0] \qquad (11)$$

$$E_a = K_a[I_a - (I_G/3)] = K_a(I_a - I_0) \qquad (12)$$

where K_c and K_a are constants of proportionality.

By adjustment of $R3$, E_C lags E_a by 60 degrees.

$$E_C\alpha - aE_a \qquad (13)$$

where a is the vector operator which rotates a vector 120 degrees in the positive direction.

$$I_a = (I_1 + I_2 + I_0) \qquad (14)$$

$$I_c = (aI_1 + a^2I_2 + I_0) \qquad (15)$$

where I_1, I_2, and I_0 are positive-, negative-, and zero-phase-sequence components of I_a.

From equations 12, 13, and 14

$$E_C = K_a'(-aI_1 - aI_2) \qquad (16)$$

From equations 11 and 15

$$E_c = K_c(aI_1 + a^2I_2 + KI_0) \qquad (17)$$

By design, and adjustment of $R2$, $K_c = K_a'$, and the network output voltage

$$E = E_C + E_c = K_c[(a^2 - a)I_2 + KI_0]$$
$$= K_c[-j\sqrt{3}I_2 + KI_0] \qquad (18)$$

With a balanced three-phase fault $I_2 = I_0 = 0$, and the operation of overcurrent relays O changes to another tap on $R2$, which is adjusted to reduce E_c by a factor k, or

$$E_c = kK_c(aI_1 + a^2I_2 + KI_0) = kK_c aI_1 \qquad (17a)$$

$$E_C = -K_a'aI_1 \qquad (16a)$$

Again,

$$K_c = K_a'$$

Then

$$E = E_C + E_c = K_c aI_1(k-1) \qquad (18a)$$

Comparison of equations 18 and 18a

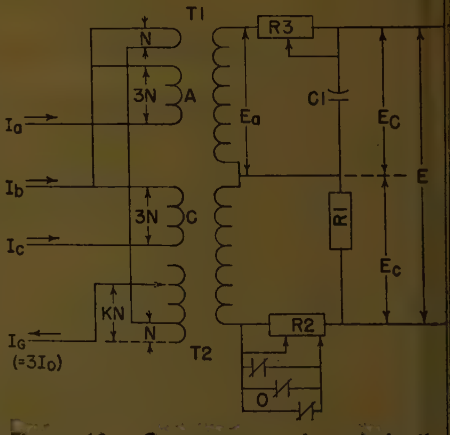

Figure 13. Connections and symbols (for Appendix II)

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* On two-phase-to-ground faults, ground current may be calculated by means of equation 36, Appendix III.

** Negative-phase-sequence machine reactance is nearly equal to transient and subtransient reactance and hence is nearly equal to positive-phase-sequence reactance during the first few cycles after a fault.

shows that the network output voltage is less as a function of I_1 during three-phase faults than it is as a function of I_2 during unbalanced faults. Consequently, the network is more sensitive to unbalanced faults than it is to three-phase faults.

Appendix III. Negative- and Zero-Phase-Sequence Currents

Negative-phase-sequence and zero-phase-sequence currents and their ratio during single- and two-phase-to-ground faults in terms of three-phase and single-phase-to-ground short-circuit currents now will be determined.

General Relations

Refer to Figure 14 for the negative-phase-sequence network, and if $Z_1=Z_2$,

$$\frac{I_2}{I_2'}=\frac{I_{3\phi}}{I_{3\phi}'} \qquad (19)$$

Where $I_{3\phi}'$ = total three-phase short-circuit current; $I_{3\phi}=C_1I_{3\phi}'$ = three-phase short-circuit current from same direction as I_2.

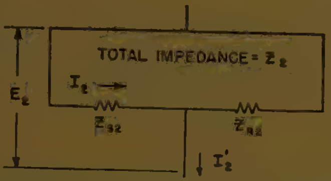

Figure 14. Negative-phase-sequence network (for Appendix III)

Referring to Figure 15 for the zero-phase-sequence network,

$$\frac{I_0}{I_0'}=\frac{I_G}{I_G'} \qquad (20)$$

where

I_G' = total fault current on single-phase-to-ground fault

$I_G=C_0I_G'$ = single-phase-to-ground fault current from same direction as I_0

Equations 19 and 20 apply to any type of fault during which those values are present. From equation 19

$$I_2=I_2'\frac{I_{3\phi}}{I_{3\phi}'} \qquad (21)$$

From equation 20

$$I_0=I_0'\frac{I_G}{I_G'} \qquad (22)$$

From equations 21 and 22

$$\frac{I_2}{I_0}=\frac{I_2'}{I_0'}\left(\frac{I_{3\phi}I_G'}{I_{3\phi}'I_G}\right) \qquad (23)$$

SINGLE-PHASE-TO-GROUND FAULT

$$I_0'=I_2'$$

From equation 21

$$I_2=I_2'\frac{I_{3\phi}}{I_{3\phi}'}=I_0'\frac{I_{3\phi}}{I_{3\phi}'}=\frac{I_{3\phi}}{I_{3\phi}'}\,\frac{I_G'}{3}=\frac{C_1I_G'}{3} \qquad (24)$$

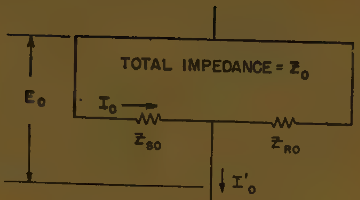

Figure 15. Zero-phase-sequence network (for Appendix III)

From equation 23

$$\frac{I_2}{I_0}=\frac{I_2'}{I_0'}\left(\frac{I_{3\phi}I_G'}{I_{3\phi}'I_G}\right)=\frac{I_{3\phi}I_G'}{I_{3\phi}'I_G}=\frac{C_1}{C_0} \qquad (25)$$

TWO-PHASE-TO-GROUND FAULT

$$\frac{I_2'}{I_0'}=\frac{Z_0}{Z_1}=\frac{Z_0}{Z_1}+2-2=\frac{Z_0+2Z_1}{Z_1}-2 \qquad (26)$$

But $Z_0+2Z_1=E/I_0'$ for single-phase-to-ground fault.

$$Z_0+2Z_1=\frac{3E'}{I_G{}^*} \qquad (27)$$

From equations 26 and 27

$$\frac{I_2'}{I_0'}=\frac{3E}{Z_1I_G'}-2=3\left(\frac{I_{3\phi}'}{I_G'}\right)-2=R \qquad (28)$$

From equations 23 and 28

$$\frac{I_2}{I_0}=\frac{I_2'}{I_0'}\left(\frac{I_{3\phi}I_G'}{I_{3\phi}'I_G}\right)=R\left(\frac{I_{3\phi}I_G'}{I_{3\phi}'I_G}\right)=R\frac{C_1}{C_0} \qquad (29)$$

$$I_1=\frac{E(Z_1+Z_0)}{Z_1(Z_1+2Z_0)}$$

$$E_2=E-I_1Z_1=E-\frac{E(Z_1+Z_0)}{Z_1+2Z_0}$$

$$E_2=\frac{EZ_0}{Z_1+2Z_0}=\frac{E}{(Z_1/Z_0)+2} \qquad (30)$$

From equations 26 and 28

$$\frac{Z_1}{Z_0}=\frac{1}{R} \qquad (31)$$

From equations 30 and 31

$$E_2=\frac{E}{(1/R)+2} \qquad (32)$$

$$I_2'=-\frac{E_2}{Z_1}=-\frac{E}{Z_1}\left(\frac{R}{1+2R}\right)=-I_{3\phi}'\left(\frac{R}{1+2R}\right) \qquad (33)$$

From equations 19 and 33

$$I_2=-I_{3\phi}\left(\frac{R}{1+2R}\right)=-C_1I_{3\phi}'\left(\frac{R}{1+2R}\right) \qquad (34)$$

From equations 29 and 34

$$I_0=\frac{I_2}{R}\,\frac{C_0}{C_1}=-C_0I_{3\phi}'\left(\frac{1}{1+2R}\right) \qquad (35)$$

$$\text{Ground current}=3I_0-3C_0I_{3\phi}'\left(\frac{1}{1+2R}\right)$$
$$=-\frac{3I_{3\phi}'I_G}{I_G'}\left(\frac{1}{1+2R}\right) \qquad (36)$$

---

*I_G' previously defined as applying to a single-phase-to-ground fault *only*.

Appendix IV. Operation During Ground Faults

Conditions during which sufficient negative-phase-sequence current is available for relay operation during ground faults will now be determined.

SINGLE-PHASE-TO-GROUND FAULT

From equation 24, Appendix III,

$$I_2=\frac{C_1I_G'}{3}=\frac{I_{3\phi}}{3}\,\frac{1}{(I_{3\phi}'/I_G')} \qquad (37)$$

When $I_{3\phi}$ is just equal to the three-phase relay pickup (eight amperes), the magnitude of I_2 must be equal to or greater than the negative-phase-sequence pickup ($\sqrt{3}$ amperes) or

$$I_2=\frac{8}{3}\,\frac{1}{(I_{3\phi}'/I_G')}\geqq\sqrt{3}$$

$$\frac{I_{3\phi}'}{I_G'}\leqq\frac{8}{3\sqrt{3}}=1.5\text{ (approximately)} \qquad (38)$$

TWO-PHASE-TO-GROUND FAULT

From equation 34, Appendix III,

$$I_2=I_{3\phi}\left(\frac{R}{1+2R}\right)=I_{3\phi}\left[\frac{3(I_{3\phi}'/I_G')-2}{6(I_{3\phi}'/I_G')-3}\right] \qquad (39)$$

Again, I_2 in equation 39 must be equal to or greater than $\sqrt{3}$ when $I_{3\phi}=8$ amperes.

$$8\left[\frac{3(I_{3\phi}'/I_G')-2}{6(I_{3\phi}'/I_G')-3}\right]\geqq\sqrt{3}$$

$$24(I_{3\phi}'/I_G')-16\geqq6\sqrt{3}(I_{3\phi}'/I_G')-3\sqrt{3}$$

$$13.6\ (I_{3\phi}'/I_G')\geqq10.8$$

$$\frac{I_{3\phi}'}{I_G'}\geqq0.8\text{ (approximately)} \qquad (40)$$

Appendix V. Blind Spots

Conditions necessary to avoid blind spots during ground faults (when ground-current excitation is used) will now be determined. The network output from equation 18, Appendix II, is

$$E=K_c(-j\sqrt{3}I_2+KI_0) \qquad (18)$$

If it is assumed that I_2 can be at any angle relative to I_0, KI_0 will overcome $\sqrt{3}I_2$ when

$$KI_0>\sqrt{3}I_2$$

or when

$$K>\sqrt{3}\left(\frac{I_2}{I_0}\right) \qquad (42)$$

If (as an additional safeguard against failure to trip due to excessive phase displacement between the network outputs at the two terminals) it is specified that $(-j\sqrt{3}I_2+KI_0)$ shall not be in excess of 45 degrees from I_0, at either terminal, KI_0 must be larger than $\sqrt{3}(I_2/I_0)$ by an additional multiplier, $\frac{1}{\sin 45°}=\sqrt{2}$

$$KI_0>\sqrt{2}\sqrt{3}I_2;\quad K>2.45\left(\frac{I_2}{I_0}\right) \qquad (43)$$

With the maximum available value of K as 21,

$$21 > 2.45\left(\frac{I_2}{I_0}\right)$$

or

$$\frac{I_2}{I_0} < 8.5 \text{ (approximately)} \qquad (44)$$

But from equation 25, Appendix III, $I_2/I_0 = C_1/C_0$ or

$$\frac{C_1}{C_0} < 8.5 \text{ for a single-phase-to-ground fault} \qquad (45)$$

From equation 29, Appendix III, $I_2/I_0 = (RC_1/C_0)$ or

$$\frac{C_1}{C_0}\left[3\left(\frac{I_{3\phi}'}{I_G'}\right)-2\right] < 8.5 \text{ for two-phase-to-ground fault} \qquad (46)$$

Thus, if $I_{3\phi}'/I_G'$ is less than 1, equation 45 gives the limiting condition, whereas if $I_{3\phi}'/I_G'$ is greater than 1, equation 46 gives the limiting condition.

Appendix VI. Relay Pickup

Relay pickup in terms of ground current (when ground-current excitation is used) is determined as follows. From equation 18, Appendix II, the network output is

$$E = K_c(-j\sqrt{3}I_2 + KI_0) \qquad (18)$$

For any condition for which I_2 and I_0 are known at pickup, E/K_c can be found. Knowing E/K_c, pickup in terms of I_G can be determined knowing only I_2/I_0.

The single-phase-to-ground single-end-feed pickup values given in Appendix VII are for a fault on the phase giving the highest pickup, that is, the phase for which I_2 and I_0 are most nearly in opposition. For a fault on phase b, I_2 (phase a being the reference phase) lags I_0 by 120 degrees. The quantity $-jI_2$ lags I_0 by 210 degrees ($I_2 = -ja^2I_0$), or 30 degrees from complete opposition. In magnitude, $I_2 = I_0$. By choosing any combination of K and ground current at pickup (for example, $K = 21$, $I_G = 0.5$), and by substitution in equation 18b,

$$E = K_c(-ja^2\sqrt{3}I_0 + KI_0)$$

$$-\frac{K_cI_G}{}[(K-\sqrt{3}\cos 30)^2 + (\sqrt{3}\sin 30)^2]$$

$$= \frac{K_c 0.5}{3}[(21-1.5)^2 + (\sqrt{3}/2)^2]$$

$$E/K_c = 3.25$$

With double-end-feed, and with the assumption that I_2 and I_0 can be in complete opposition,

$$E/K_c = -\sqrt{3}I_2 + KI_0 = \frac{I_G}{3}\left(K - \sqrt{3}\frac{I_2}{I_0}\right)$$

$$I_G = \frac{E}{K_c}\frac{3}{\left(K-\sqrt{3}\frac{I_2}{I_0}\right)}$$

and at pickup

$$I_{GPU} = 3.25\frac{3}{\left(K-\sqrt{3}\frac{I_2}{I_0}\right)} = \frac{10}{K-\sqrt{3}\frac{I_2}{I_0}} \text{ approximately} \qquad (41)$$

Appendix VII. Detailed Application of Relay System

The application of the relay system can be reduced to the following steps:

1. Determination of margin above pickup during three-phase faults with minimum generating conditions.
2. Determination of margin of the negative-phase-sequence current, above pickup, during single-phase-to-ground and two-phase-to-ground faults.
3. The determination of K (the proportion of added excitation from I_0) to avoid blind spots if sufficient negative-phase-sequence current is not available in 2.
4. If I_0 is used, the determination of relay pickup in terms of ground current, to check the margin of fault current above pickup.

Three-Phase Faults

Under minimum generating conditions, determine the current in the relay at A (Figure 16) for a fault at 2. Repeat for the relay at B for a fault at 1. The three-phase pickup of the relay is eight amperes. If in either instance one of the currents is insufficient to operate the relay, assume sequential operation and determine the current at the questionable terminal with the opposite circuit breaker open. Hereafter, this note is not repeated but is assumed.

Phase-to-Phase Faults

It is not necessary to consider this type of fault, unless the incidence of three-phase faults is assumed negligible, since the relay system is more sensitive to phase-to-phase faults than to three-phase faults. The relay pickup for this condition is three amperes ($\sqrt{3}$ amperes negative-phase-sequence).

Ground Faults—Determination of I_2

In the determination of the available negative-phase-sequence current, it is necessary to know only the three-phase and single-phase-to-ground short-circuit currents which must be known in any overcurrent relay application.

For a single-phase-to-ground fault (equation 24, Appendix III):

$$I_2 = \frac{I_{3\phi}}{I_{3\phi}'}\frac{I_G'}{3} \qquad (24)$$

where

I_G' = total ground amperes in single-phase-to-ground fault

I_G (used in equation 25 in this appendix) = portion of I_G' from terminal in question

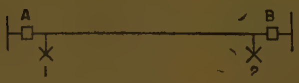

Figure 16. Fault locations (for Appendix VII)

$I_{3\phi}'$ = total phase amperes in three-phase fault

$I_{3\phi}$ = portion of $I_{3\phi}'$ from terminal in question

With a single-phase-to-ground fault at 2 (Figure 16), determine the negative-phase-sequence current I_2 at terminal A from equation 24. Repeat for terminal B for a fault at 1.

For a two-phase-to-ground fault, repeat the determination of I_2 with this type of fault using equation 34 (from Appendix III).

$$I_2 = I_{3\phi}\left(\frac{R}{1+2R}\right) \qquad (34)$$

where

$$R = \frac{3I_{3\phi}' - 2I_G'}{I_G'}$$

If there is sufficient negative-phase-sequence current available during all of these conditions, no further calculation is necessary (that is, it is not necessary to consider the introduction of I_0 into the network). If I_2 is to be checked after the opening of the first breaker, the same formulas can be used with $I_{3\phi} = I_{3\phi}'$, each of these being the short-circuit current calculated under this single-end-feed condition.

If I_2, as so determined, is not considered to be sufficient for relay operation, I_0 must be added to the network.

Ground Faults—Determination of K

With a single-phase-to-ground fault at 2 (Figure 16), determine the ratio I_2/I_0 at terminal A from equation 25 as given here (from Appendix III):

$$\frac{I_2}{I_0} = \frac{I_G'I_{3\phi}}{I_GI_{3\phi}'} \qquad (25)$$

Repeat for terminal B with a fault at 1.

Repeat for two-phase-to-ground faults using equation 29 from Appendix III.

$$\frac{I_2}{I_0} = \left(\frac{I_G'I_{3\phi}}{I_GI_{3\phi}'}\right)R \qquad (29)$$

By use of the highest of the four values ob I_2/I_0 found, choose an available value of K (6, 8, 11, 15, or 21) so that

$$K > 2.45\frac{(I_2)}{(I_0)} \text{ (from Appendix V)} \qquad (43)$$

It is noted from equation 43 that if it is necessary to introduce I_0 into the network, ground current must be available from both terminals. The relay system is not applicable if an internal fault will, under any circumstances, draw I_0 from one direction only and I_2 from the other direction, unless negative-phase-sequence currents alone are sufficient for at least sequential relay operation.

Ground Faults—Determination of Pickup in Terms of Ground Current

If it is assumed that the negative-phase-sequence current can be at any phase angle relative to the zero-phase-sequence current, the resultant pickup in terms of ground cur

The Electrical Resistivity of Resin-Treated Wood and Laminated Hydrolyzed-Wood and Paper-Base Plastics

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WOOD in the dry state is an excellent dielectric, having a volume resistivity in the range of 3×10^{17} to 3×10^{18} ohm-centimeters.[2] The resistivity decreases rapidly, however, with an increase in moisture content, reaching values of 10^5 to 10^6 ohm-centimeters at the fiber-saturation point (30-per-cent moisture content on the basis of the dry weight of the wood).[5] This tremendous change in the electrical resistance of wood with changes in moisture content, which so overshadows the variations due to varying species and specific gravity and ash content of the wood,[4,6] furnishes an excellent means of measuring the moisture content of wood.[6,12]

The treatment of wood with aqueous phenol-formaldehyde resin-forming solutions according to the methods developed at the United States Forest Products Laboratory[7–11] has been shown to reduce greatly the moisture adsorption and swelling even under equilibrium conditions. Because of the reduced moisture content of resin-treated wood (*impreg*[10]) and the corresponding resin-treated compressed wood (*compreg*[11]) in equilibrium with any relative humidity, the electrical resistivity should be correspondingly increased. The data given in this report show that this is the case and that wood, hydrolyzed-wood sheet, and paper properly treated with resins are suitable for various dielectric uses even under relatively high-humidity conditions.

The electrical resistivity of normal wood and *impreg* and *compreg* of ten different species was determined on single sheets of veneer after kiln drying and in equilibrium with four different relative humidities. The changes in resistivity resulting from variations in resin content, degree of compression, and the presence of glue lines in laminated material were determined. The extent of surface resistivity was investigated for the resin-treated wood. The volume resistivities of a limited number of hydrolyzed-wood sheet specimens containing both aniline-furfural and phenol-formaldehyde resins, and laminated Mitscherlich sulphite paper treated with both water- and alcohol-soluble phenolic resins were also determined.

Electrical Measurements

D-c resistivity measurements were made with a sensitive galvanometer used as an ammeter in an ammeter–voltmeter circuit as shown in the circuit diagram of Figure 1. The galvanometer had a sensitivity with the scale at four meters of 7.60×10^{-10} amperes per centimeter scale deflection. High potentials ranging from 20 to 300 volts were obtained with a d-c motor generator set. Low potentials, from 15 to 0.01 volts, were obtained from dry cells with the use of potential dividers. The apparatus was thus capable of measuring any resistance from 30 ohms to 4×10^{18} ohms.

All the switches were coated with ceresin wax, which has a high-surface resistivity, to minimize current leakage. Switches S_3, S_4, and S_7 were shielded by wrapping bare copper wires about the bases in three directions and connecting these to the circuit at α, thus leading any stray current around the galvanometer. To avoid confusion, these leads are not shown in Figure 1. In spite of these precautions, stray currents were found to be excessive in damp weather. Hence the apparatus was set up in a constant relative-humidity room held at 30-per-cent relative humidity and 80 degrees Fahrenheit. With these precautions the volume leakage of the system was reduced to the point that the conductivity was only about 5×10^{-13} ohms, which was insufficient to affect the results seriously.

The electrodes consisted of two rectangular brass plates, each with a surface area of 35.0 square centimeters and a brass guard rim one centimeter wide that fitted around the perimeter of one of the electrode plates with a 0.5-centimeter gap between the guard rim and the electrode. This, together with the other shield wires, was connected at α in Figure 1, so that any current flowing along the surface of the sample was led around the galvanometer.

The electrode plates were held against the sample by a screw clamp with hard rubber jaws. The clamp had a resistance of 2×10^{13} ohms, which was about ten times that of the highest resistance measured and from 100 to 1,000 times the usual resistance measured.

It was found that the resistance of a specimen varied inversely with the pressure applied to the electrodes at lower pressures, but above a pressure of 23 pounds per square inch the resistance change became negligible. The clamp was tightened to exceed this pressure for all the measurements.

The surface resistivity was measured with six bar electrodes ten centimeters long and one centimeter wide. Three parallel electrodes on each surface were paired opposite each other on the two faces of a specimen and bolted through holes in the specimens slightly larger than the bolts. The two outer bars on both surfaces were electrically coupled so that a flow of current occurred from the cen-

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rent ($3I_0$) may be increased if I_2 opposes I_0. The resultant pickup may be as high as 3.25 (Appendix VI) times the single-end-feed ground-fault pick-up values ($3I_0=2.2$, 1.55, 1.05, 0.75, or 0.5 amperes corresponding respectively to the values of K just given).

The ground-fault currents ($3I_0$) are the following:

Single-phase-to-ground $=I_G$ (one of the known currents)

$$\text{Two-phase-to-ground} = \frac{3I_{3\phi}'I_G}{I_G'}\left(\frac{1}{1+2R}\right) \quad \text{(from Appendix III)} \qquad (36)$$

If the available ground currents calculated are less than 3.25 times the single-end-feed pickup, the actual pickup in terms of ground current (I_{GPU}) can be calculated by means of equation 41, Appendix VI.

$$I_{GPU}=\frac{10}{K-\sqrt{3}\left(\dfrac{I_2}{I_0}\right)} \qquad (41)$$

If the available ground current is not sufficient to operate the relay (pickup calculated from equation 41), the value of K may be increased. K should not be increased unnecessarily, since a higher value of K means higher burden on the current transformers.

References

1. A CARRIER-CURRENT PILOT SYSTEM OF TRANSMISSION LINE PROTECTION, A. S. Fitzgerald. AIEE TRANSACTIONS, volume 47, 1928, pages 22–9.

2. A NEW CARRIER RELAYING SYSTEM, T. R. Halman, S. L. Goldsborough, H. W. Lensner, A. F. Drompp. AIEE TRANSACTIONS, volume 63, 1944. August section, pages 568–72.

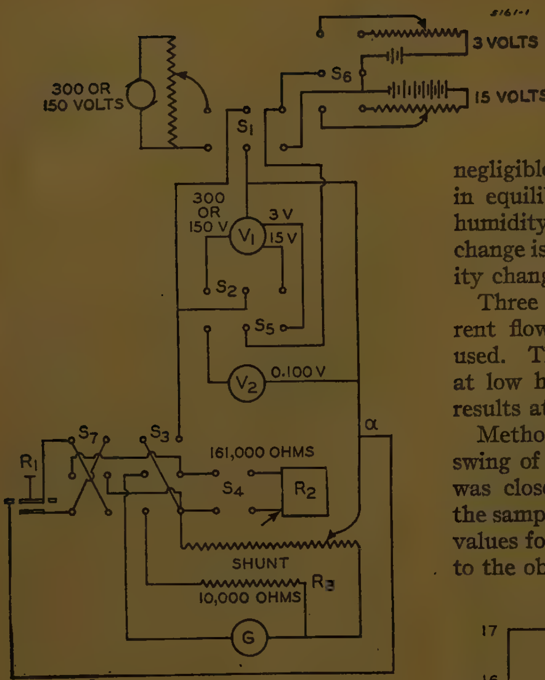

Figure 1. Wiring diagram of resistivity equipment

tral to the two outer bars on both faces across a 0.10-centimeter gap. The surface resistivity r_s was calculated from the equation $r_s = 4R_s\frac{l}{d}$ where R_s is the measured surface resistance, l is the bar length, and d is the distance between the bars.

The wood specimens were cut from $^1/_{16}$-inch rotary-cut veneer (0.16 centimeter). The *compreg* specimens were one third to one half as thick, as a result of compression. The hydrolyzed-wood sheet and laminated-paper plastic specimens varied from 0.06 to 0.15 centimeter in thickness.

The specimens were kiln dried or conditioned in humidifiers containing saturated salt solutions. At 30 degrees centigrade the barium chloride, sodium chloride, manganese chloride, and magnesium chloride used gave 90.8-, 75.8-, 53.9-, and 32.7-per-cent relative humidity, respectively.

The specimens were moved from the lower to the higher relative-humidity conditions. The resistivity–moisture content curves to follow are hence all adsorption curves.

Resistivity measurements made after different times of humidification indicated that moisture equilibrium was obtained in all cases in 20 days or less. The measurements recorded in the following figures were all made after 30 days' humidification to insure definitely the attainment of equilibrium conditions.

All the volume-resistivity measurements were made in the 30-per-cent relative-humidity room as rapidly as possible after the specimens were removed from the humidifiers, which required about three minutes. During this time the moisture-content change for all the resin-treated specimens was practically negligible. Even in the untreated wood in equilibrium with 90-per-cent relative humidity, for which the moisture-content change is most rapid, the volume-resistivity change was only five to ten per cent.

Three methods of measuring the current flowing through the samples were used. They gave widely varying results at low humidities, but closely checking results at high humidities (Figure 3).

Method 1 observed the maximum swing of the galvanometer as the switch was closed and current surged through the sample. This method gave the lowest values for the resistivity. It was subject to the objection that the electrode plates acted as a capacitor and so contributed to the magnitude of the swing.

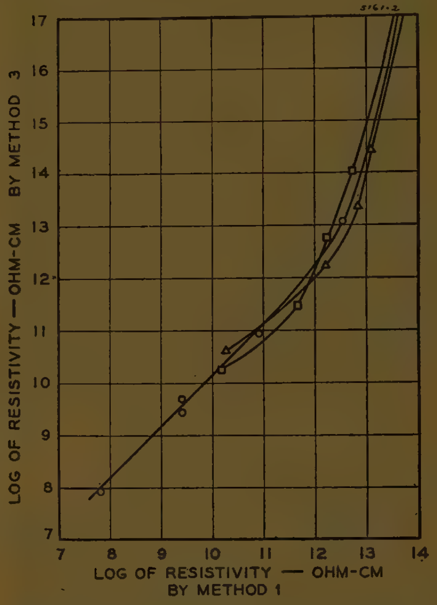

Figure 2. Logarithm of average volume resistivity of ten species of wood (basswood, yellow birch, cottonwood, Douglas fir, white fir, sweetgum, Western hemlock, yellow poplar, redwood, and Sitka spruce), and of *impreg* and *compreg* made therefrom determined by method 3 versus the corresponding values determined by method 1

○ Untreated wood
△ *Impreg*
□ *Compreg*

Method 2 observed the first steady deflection of the galvanometer after the switch was closed. The polarity of the electrodes was changed by means of the reversing switch S_7 (Figure 1), and the average of the readings for the flow of current in the two directions was used. The value of the resistivity obtained by this second method probably approximated most nearly the true value of the resistivity. The chief difficulty was that some samples polarized so rapidly that no steady deflection could be observed.

Method 3 was that used by the American Society for Testing Materials,[1] namely, that of reading the galvanometer deflection after the current had been flowing through the sample for one minute. This, however, gave a value of the resistivity which had been considerably increased by polarization.

By plotting corresponding values of the resistivity obtained by methods 1 and 3 against each other and extrapolating the curve, it was possible to determine method-3 resistivities from method-1 resistivities that were too high to measure directly but which were still within the range of the apparatus for method 1 (Figure 2). This procedure was used to obtain method-3 resistivities for the kiln-dried samples before they were exposed to the 32.7-per-cent relative humidity. The kiln-dried samples contained from 0.5 to 2-per-cent moisture and had method-3 resistivities of the order of 10^{16} ohm-centimeters.

Volume Resistivity of Wood

Volume-resistivity measurements were made on matched specimens of the ten

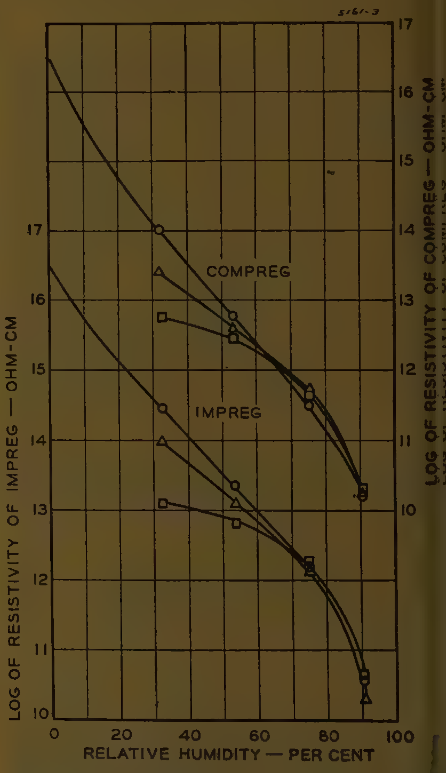

Figure 3. Logarithm of the average volume resistivity of *impreg* and *compreg* made from ten species of wood (basswood, yellow birch, cottonwood, Douglas fir, white fir, sweetgum, Western hemlock, yellow poplar, redwood, and Sitka spruce), determined by three methods versus the equilibrium relative humidity

□ Method 1
△ Method 2
○ Method 3

species of wood, listed in Figure 2, in the untreated form and made up into *impreg* and *compreg* by previously described methods[8–11] using Bakelite Corporation resinoid *BR*5995. Both the *impreg* and the *compreg* were cured in a hydraulic press for 45 minutes at 300 degrees Fahrenheit, *impreg* under a negligible pressure and *compreg* under 1,250 pounds per square inch. The resin contents of the different specimens varied from 31 to 57 per cent on the basis of the weight of the dry untreated wood. It will be shown later (Figure 7) that within this range the volume resistivity changes but slightly with variations in resin content. Because of this and the fact that the variation in volume resistivity with species was practically within the range of experimental error, average values for all the species are plotted in Figures 2 through 5.

Figure 3 gives a comparison between the average volume resistivities of *impreg* and *compreg* determined by the three different methods plotted against the equilibrium relative humidity. Variations between the values obtained by the three methods are appreciable only for low relative humidities at which the resistivities are relatively high. The resistivities given in the following figures were all determined by method 3, that recommended by the American Society for Testing Materials.[1]

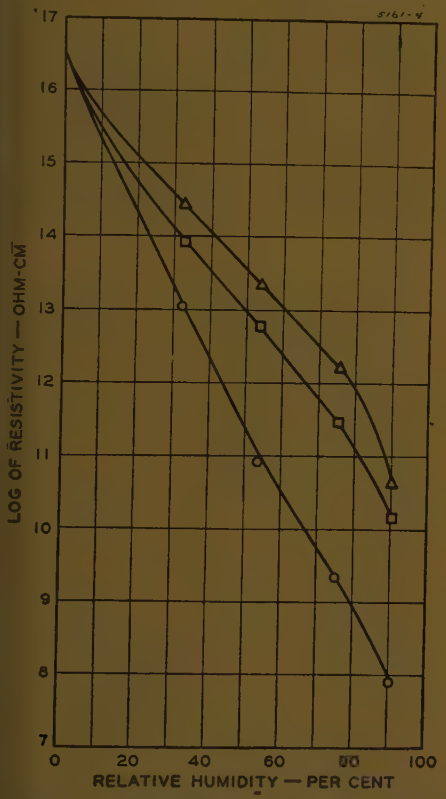

Figure 4. Logarithm of the average volume resistivity (method 3) of ten species of wood (basswood, yellow birch, cottonwood, Douglas fir, white fir, sweetgum, Western hemlock, yellow poplar, redwood, and Sitka spruce), and of *impreg* and *compreg* made therefrom versus the equilibrium relative humidity

○ Untreated wood
△ *Impreg*
□ *Compreg*

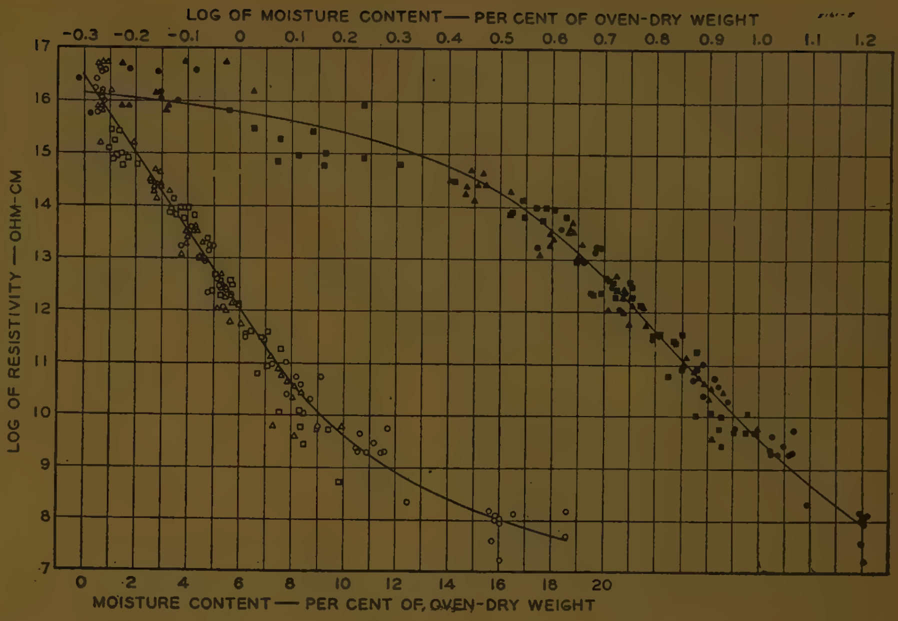

Figure 5. Logarithm of the average volume resistivity (method 3) of ten species of wood (basswood, yellow birch, cottonwood, Douglas fir, white fir, sweetgum, Western hemlock, yellow poplar, redwood, and Sitka spruce), and of *impreg* and *compreg* made therefrom versus the moisture content

Per cent moisture content:
○ Untreated wood
△ *Impreg*
□ *Compreg*
Log per cent moisture content:
● Untreated wood
▲ *Impreg*
■ *Compreg*

Figure 4 gives the logarithm of the average resistivity for untreated wood, *impreg*, and *compreg*, made from ten different species, plotted against the equilibrium relative humidity. The resistivity of *impreg* is slightly higher than that of *compreg* and considerably higher than that of normal wood under the same relative-humidity conditions. The three curves, when extrapolated to zero relative humidity, give practically identical resistivities. The reason for this is shown in Figure 5, in which the same resistivity data are plotted against the moisture content. On this basis, the values for the three materials fall on a single curve, indicating that the resistivity depends primarily upon the moisture content. Differences between the curves of Figure 4 merely show differences in hygroscopicity (or apparent hygroscopicity, as will be shown later) of the three materials.

Figure 6 gives the volume resistivity of birch *impreg* and *compreg*. The values are very similar to the average values for the ten species given in Figures 4 and 5.

Figure 7 shows the effect of the resin content of both *impreg* and *compreg*, made from two different species, upon the volume resistivity at two different relative humidities. The resistivity increases rapidly with an increase in resin content up to about 30-per-cent resin content on the basis of the dry weight of the wood, and then remains relatively constant, but again decreases with very high resin content. The original increase is due to the decrease in hygroscopicity with increasing resin content.[7] On the other hand, electrolyte introduced with the resin will decrease the resistivity as the proportion of resin to wood increases. This decrease in resistivity is evidently small, compared to the increase in resistivity due to decreased hygroscopicity up to about 30-per-cent resin content. Above this resin content the decrease in hygroscopicity is small.[7] Between this resin content and one of 70 per cent the two effects evidently neutralize each other, whereas above 70-per-cent resin content the effect of the electrolyte predominates.

The rapid decrease in the hygroscopicity in both the *impreg* and the *compreg* with increasing resin content up to about 30 per cent, followed by a smaller decrease in hygroscopicity with a further increase in resin content, is clearly shown in Figure 8 for the specimens of Figure 7.

Figures 4 through 8 all show that *impreg* has a higher resistivity than *compreg* for the same relative humidity, although it is the same for the same moisture content. This difference between the two materials makes it appear that *compreg* is more hygroscopic than *impreg*. This explanation does not seem so probable as the possible entrapping of water within the resin of the *compreg* which was not lost in the curing or in the kiln drying of these samples prior to humidification, but which is lost during the final oven drying to determine the dry weight for moisture-content calculations. It thus would be advisable to oven-dry *compreg*, to be used as an insulator, just after hot pressing. If this is done, the *compreg* curves should coincide with the *impreg* curves.

Birch specimens were made at 0, 300, 600, 900, and 1,200 pounds per square inch pressure. All the compressed panels gave similar volume-resistivity curves to those of the birch *compreg* of Figure 6. The tendency of the material to trap water was evidently about the same for all compressed specimens.

Seven-ply *impreg* and *compreg* panels were made from redwood and Western hemlock, using added glue between the plies to determine if lamination or the glue had any effect upon the volume resistivity. The panels contained about 40 per cent of treating resin and 9 per cent of bonding resin on the basis of the weight of the dry untreated wood. No significant differences in the volume resistivity between these specimens and the single-ply specimens were observed.

Surface Resistivity of Wood

Figure 9 shows the relationship between the surface resistivity and the relative humidity to which oven-dry samples of *impreg* and *compreg* were exposed for 15 minutes. The relative humidity of the chamber in which the measurements were made was controlled by blowing air over different concentrations of sulphuric acid. Relative humidities corresponding to different concentrations were taken from tables given by Wilson.[13] Fifteen minutes' exposure was adequate to allow the surface film to come to approximate equilibrium without causing the volume resistivity to decrease to the point where it would affect the results appreciably. During this period two to three check readings were made, using the first steady galvanometer deflection. The specimens were freshly dried before exposure to the next higher relative humidity. No difference between *impreg* and *compreg* was observed; hence a single curve was plotted for both. Below a relative humidity of 40 per cent the surface resistivity is so small that it becomes of the same order as the volume resistivity, so that an appreciable part of the current flow is through the specimens. The curve shows that volume resistivity measurements made in a 30-per-cent relative-humidity room are inappreciably affected by the surface conductivity. Listray[3] similarly found that the surface resistivity of several different dielectrics became constant below a relative humidity of about 40 per cent.

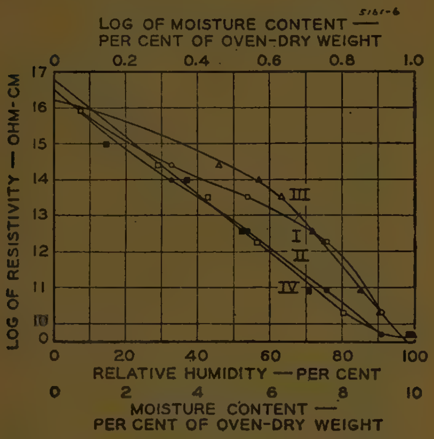

Figure 6. Logarithm of the volume resistivity (method 3) of yellow birch *impreg* and *compreg* versus the equilibrium relative humidity and the moisture content

Resistivity of birch *impreg* and *compreg*

I ○ *Impreg* } relative humidity
II ● *Compreg* }
III { △ *Impreg* } log of moisture content
{ ▲ *Compreg* }
IV { □ *Impreg* } moisture content
{ ■ *Compreg* }

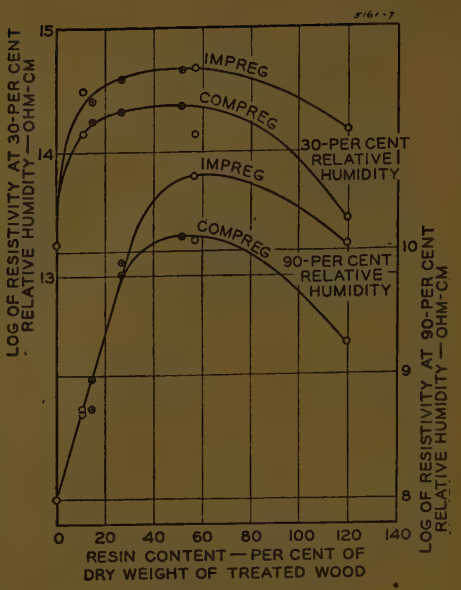

Figure 7. Logarithm of the volume resistivity (method 3) of white fir and yellow birch *impreg* and *compreg* at two different relative humidities versus the resin content

○ White fir
⊕ Yellow birch

Volume Resistivity of Paper and Hydrolyzed-Wood Laminates

The volume resistivities of resin-treated paper and hydrolyzed-wood sheets are given in Figures 10 and 11, together with the average values for normal wood and *compreg*. When plotted against the relative humidity (Figure 10) all the values fall between those for normal wood and for *compreg*. When plotted against the moisture content (Figure 11), most of the values fall on a single curve below the composite curve for wood, *impreg*, and *compreg*. It appears that the position of a point on one of the curves of Figure 11 is determined by the hygroscopicity and the relative humidity at which the material i conditioned, while the position of th curve is determined by the amount o other electrically conductive materia present in the specimens besides moisture such as resin catalyst or residual electrolyte in the wood, hydrolyzed wood, o paper. Evidently all the papers and th hydrolyzed wood retained more processin

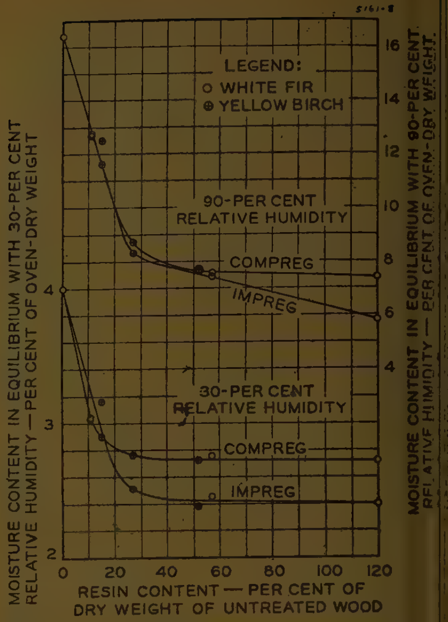

Figure 8. Equilibrium moisture content o white fir and yellow birch *impreg* and *compre* at two different relative humidities versus th resin content

○ White fir
⊕ Yellow birch

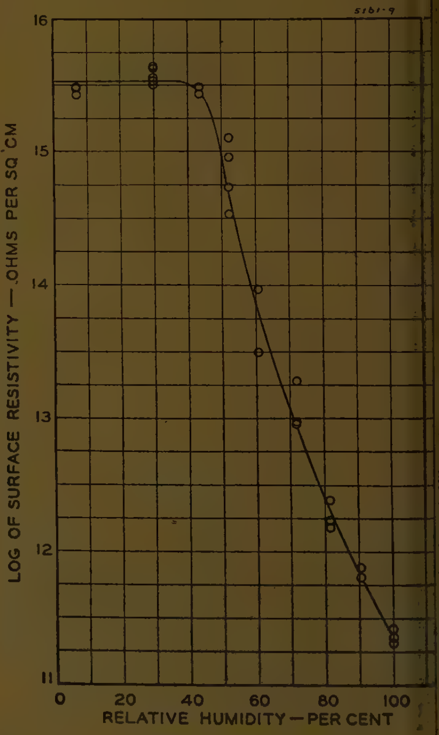

Figure 9. Logarithm of the surface resistivit of several species of *compreg* versus the atmos pheric relative humidity

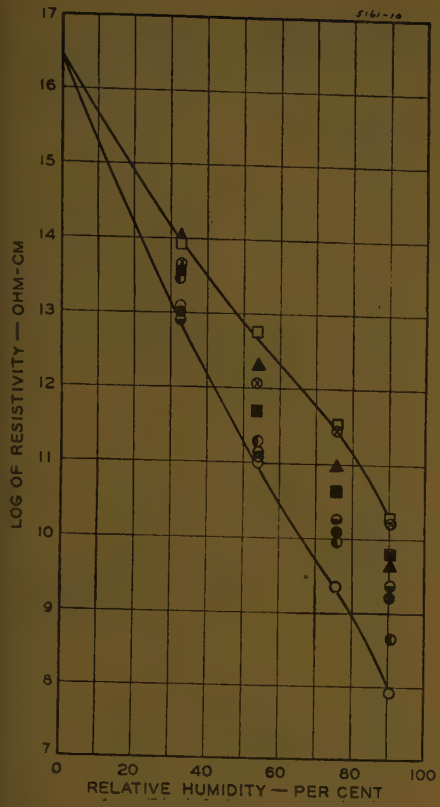

Figure 10. Logarithm of the volume resistivity of *compreg* (average for ten species, Figure 4), resin-treated paper, and hydrolyzed-wood sheet versus the equilibrium relative humidity

○	Normal wood	
□	*Compreg*	
▲ 1		Resin-treated paper
◐ 2		
■ 3		
⊖ 4		
● 5		Hydrolyzed-wood sheet
⊗ 6		

1—Soft (low-strength) Mitscherlich paper treated with Bakelite spirit-soluble resin *BR*13850, resin content 44 per cent (of total weight), water absorption (*AAF* specification 15065) 1.5 per cent

2—High-strength modified Mitscherlich paper treated with Bakelite spirit-soluble resin *BR*13850, resin content 34 per cent (of total weight), water absorption 7.5 per cent

3—High-strength Mitscherlich paper treated with Bakelite water-soluble resin *BR*5995 on the paper machine with no diffusion period, followed by treatment with Bakelite spirit-soluble resin *BR*13850; resin content, water-soluble, 20 per cent; spirit-soluble 13 per cent (of total weight), water absorption 1.3 per cent

4—High-strength Mitscherlich paper (same as 3) treated only with Bakelite water-soluble resin with no diffusion period; resin content 35 per cent (of total weight)

5—Hydrolyzed-wood sheet containing 15 per cent (of total weight) of aniline–furfural

6—Hydrolyzed-wood sheet containing 15 per cent (of total weight) of Bakelite water-soluble resin *BR*5995

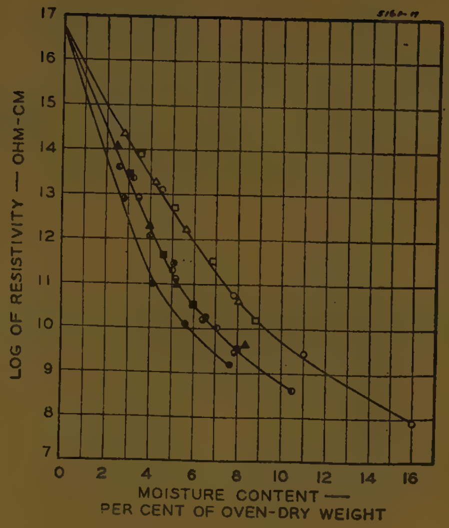

Figure 11. Logarithm of volume resistivity of normal wood, *impreg* and *compreg* (average of ten species, Figure 4), resin-treated paper and hydrolyzed-wood sheet, versus the moisture content. For key to specimens see Figure 10

○	Normal wood	
△	*Impreg*	
□	*Compreg*	
▲ 1		Resin-treated paper
◐ 2		
□ 3		
⊖ 4		
● 5		Hydrolyzed-wood sheet
⊗ 6		

electrolyte than the normal electrolyte content of the wood, or the resins used contained more electrolyte. It is of interest that all the curves extrapolate to the same volume resistivity at zero relative humidity and zero moisture content. This might be expected, as the conductivity of any contained electrolyte is largely dependent upon moisture being present. Hydrolyzed-wood sheet 5 with an aniline–furfural plasticizer is the only treated material tested which does not contain a phenolic resin. Its definitely poorer electrical insulating properties may be due either to the difference in resins used or the presence of more electrolyte.

Resistivity According to ASTM Method

A few measurements of electrical resistance were made by the American Society for Testing Materials method.[1] This test determines the resistance between adjacent binding posts that are bolted through panel specimens. Three binding posts were bolted through 2$^1/_2$-by-4-inch specimens, of thicknesses shown in Table I, one at the center and two at 1$^1/_4$ inches (center to center) from the central binding post in the length direction of the specimen and equally distant from each of the edges. Brass washers 9/16 inch in outside diameter were used on both surfaces of the bolted-in binding posts, giving a distance between adjacent washers on the surfaces of the specimens

Table I. Electrical Resistance According to the ASTM Method

Material	Resin*	Resin† Content, Per Cent	Thickness, Inch	Cycle	Electrical Resistance, Megohms
Birch *impreg*	*BR*5995	28.7	0.060	1 2	5,605 6,490
Cottonwood *compreg*	*BR*15100	21.8	0.252	1	146
Cottonwood *compreg*	*BR*15100	21.8	0.074	1 2 3	317 439 412
Hydrolyzed-wood sheet	*BR*5995	15.0	0.043	1	13,300
Hydrolyzed-wood sheet	*BR*5995	15.0	0.065	1 2	1,890 1,020
Hydrolyzed-wood sheet	Aniline–furfural	15.0	0.059	1	550
Soft (low-strength) Mitscherlich paper	*BR*13850	44.0	0.034	1	2,035
High-strength Mitscherlich paper	*BV*16303	36.9	0.145	1 2	5 5
High-strength Mitscherlich paper (waxed surfaces)‡	*BV*16303	36.9	0.142	1 2	3,560 7,120
High-strength Mitscherlich paper	*BV*16526	36.0	0.256	1 2 3	100 110 95
High-strength Mitscherlich paper	*BV*16526	36.5	0.132	1 2	82 100
High-strength Mitscherlich paper (waxed surfaces)‡	*BV*16526	36.5	0.130	1 2	279,000 39,800

* All resins but aniline–furfural are Bakelite Corporation resins.

† Resin content is on the basis of total weight of product. The resin content of the *impreg* and the *compreg*, on the basis of the dry weight of the untreated wood, on which basis they are normally expressed, are 48.3 and 28.0 per cent, respectively.

‡ The waxed specimens were coated with a thin film of wax by dipping the heated panels in cooler molten wax. The films were so thin that they were barely perceptible to the eye.

of 11/16 inch. The specimens were dried for seven hours at 160 degrees Fahrenheit according to the ASTM specifications and then exposed to a relative humidity of 90.5 ± 0.5 per cent for 96 hours at 80 degrees Fahrenheit (specified 87 degrees Fahrenheit ± 2 degrees Fahrenheit). Resistance measurements were made using the previously described apparatus. The results are shown in Table I. The insulation resistances are the averages of values obtained between adjacent pairs of binding posts. Most of the specimens were heated a second time for seven hours at 160 degrees Fahrenheit and again humidified for 96 hours to give second-cycle measurements. In some cases third-cycle tests were made. The agreement between the values for the different cycles was quite good for all materials tested except when waxed specimens were used. The *impreg* and the hydrolyzed-wood sheet plasticized with phenolic resin gave the highest electrical resistance values of the unwaxed specimens. The soft readily penetrated paper (number 1, Figure 10) gave the highest values of the treated but not waxed papers. The high-strength paper plastic made with *BV*16303 resin gave the lowest values while the high-strength paper plastic made with *BV*16526 gave intermediate values. The low values obtained with the *BV*16303 resin are probably due to the electrolyte content of the resin. The *compreg* values are not so high as they might have been if the material had contained more resin, as is evident in Figure 7. The waxed specimens gave high but not so consistent results. Differences are probably due to slight imperfections of the surface films. Although considerable wax drained off the specimens while heating prior to the second-cycle measurements, the electrical resistances were still high on the second measurement.

A large manufacturer of electrical material who uses the ASTM test for specifying panel material requires, for exacting purposes, a minimum average electrical resistance of 100 megohms with no single value falling below 60 megohms. All the materials tested except one of the resin-treated papers meet these requirements.

Conclusions

The volume resistivity of all the laminated plastics tested is equal to or better than that of normal wood. The materials with the lower water absorptions give higher resistivities. Normal wood, *impreg*, and *compreg* give identical volume - resistivity – moisture - content curves but different volume-resistivity–relative-humidity curves, indicating that their difference in insulating properties is due to differences in hygroscopicity. *Impreg* and *compreg* have volume resistivities about ten times that of normal wood at 30-per-cent relative humidity and about 100 to 1,000 times that of normal wood at 90-per-cent relative humidity, the *impreg* values tending to be somewhat greater than the *compreg* values. Resin contents in the range of 35 to 70 per cent have a negligible effect in changing the volume resistivity of *impreg* and *compreg*.

The surface resistivity at relative humidities below about 40 per cent is sufficiently small to be ineffective in controlling the insulating properties. The insulating properties of dry material exposed to higher relative humidities are largely determined by surface conductivity effects.

The volume resistivities of the laminated-paper plastics and hydrolyzed-wood plastics tested are somewhat lower than the values for *impreg* and *compreg*. This is, in part, due to greater hygroscopicity, but it also appears to be due to a somewhat greater electrolyte content.

Electrical-resistance measurements by the ASTM method indicate that practically all the resin-treated materials herein reported are suitable for insulating purposes. The fact that the resistance values obtained by this method can be markedly increased with high-resistance surface films of wax, which do not affect the hygroscopicity of the panels, indicates that this method measures surface resistance primarily. The volume-resistivity curves furnish a better means of judging the insulating value of the materials under prolonged exposure to high relative humidities.

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Per-Unit Impedances of Synchronous Machines—II

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MEMBER AIEE

THE preceding part[1] of this paper derived the generalized per-unit equations of the synchronous machine. The machine impedances were divided into two groups: those measurable from the stator terminals (x_d, x_d', r, and so forth), called the definitive impedances, and those internal impedances which cannot be measured from the stator terminals (X_{afd}, X_{ffd}, R_{ffd}, and so forth), called the nondefinitive impedances. The per-unit values of the definitive impedances are independent of the base rotor currents, but part I showed that the per-unit values of the nondefinitive impedances are functions of the base rotor currents, and accordingly are subject to misuse and misinterpretation unless the rotor-current bases are explicitly defined. These rotor-current bases and their corresponding base-current ratios are the fundamental quantities in any per-unit system, and this concluding part II is devoted to a fuller exposition of the base-current ratios of the more important per-unit systems.

To a considerable extent, the subject of base-current ratios is of primary interest only to the design engineers directly responsible for the calculation of the per-unit impedances. They must continuously guard against any inadvertent change of base during the analysis of new rotor circuits, since this would result in an impedance not in accord with the other rotor circuits. However, the application engineer also is directly interested in base-current ratios, because:

1. Such knowledge is necessary in order that he may be assured that all the per-unit impedances are in accord.

2. In many applications it is of advantage to introduce new bases or to use impedances calculated on some other base.

One specific base-current ratio is defined and evaluated in this paper, and its adoption or use as a preferred base in the calculation of per-unit impedances is suggested. Such standardization is essential in order that published values of per-unit impedances may be properly interpreted and used. Many advantages will accrue to the general use of a preferred base rotor current. The chief of these is that, if all impedances are calculated on the preferred base, they then become immediately available for use in general analysis or general equivalent circuits without fear of misapplication, and without the trouble and possible errors involved in determining the base current on which they were calculated and in converting this to a preferred base. As another example of the advantages of a preferred base rotor current, it should be noted that without recourse to a preferred base it is not possible when studying two per-unit impedance formulas to separate an improvement in the quality of the underlying analysis from a mere change in the rotor-current base.

It is obvious that the adoption of some preferred rotor-current base will have no effect upon vector diagrams as seen from the machine terminals. The base field current for such diagrams will continue to be that corresponding to normal open-circuit air-gap-line voltage, although the preferred base rotor currents will be used in calculating the impedance values.

This paper also brings together and evaluates the several base-current ratios which are found in contemporary literature; this evaluation is necessary in order to harmonize the various published per-unit impedance formulas. The method of converting all per-unit impedances to preferred-base impedances is evident from the expressions given in "Results", part I.

Suggested Preferred Bases

It is unnecessary to discuss the stator bases since by universal practice these are taken as the name-plate values (e_{ao}, i_{ao}, L_{ao}, x_{ao}). No corresponding universal practice exists in the selection of the rotor base quantities, and the adoption of some base-current ratio as a preferred standard for use in the calculation of machine impedances would be of advantage.

The base field current which in the author's experience offers the most advantages is that field current which will induce in each stator phase a voltage equal to $x_{ad}i_{ao}$. This will hereinafter be termed the "x_{ad}" base. Because of the advantages of this base itemized in the following, it is suggested that this x_{ad} base be adopted as a preferred standard:

(*a*). It makes X_{afd} numerically equal to x_{ad}, and at once makes unnecessary any further differentiation between these quantities.

(*b*). It makes the stator leakage reactance $(x_d - X_{afd})$ identical with the more familiar "leakage" reactance $(x_d - x_{ad})$. With a similar definition of the base current in the fictitious quadrature-axis field circuit, the stator leakage reactance is the same for both axes.

(*c*). It is in more common use at present than any other base (although implicitly only), and many of the present impedance formulas are based on it.

The base-current ratio for this field current is given by equation 1c. This expression is derived in the following section under definition 1.

$$\frac{I_{fdo}}{(3/2)i_{ao}} = \frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}} \tag{1c}$$

The foregoing definition of base current is also suggested for the additional rotor circuits of machines with one additional rotor circuit in each axis. The advantages are the same as those given for the base field current. The base-current ratio for the additional rotor circuits is then as given by equation 1d. This expression is also derived in the following section under definition 1.

$$\frac{I_{xdo}}{(3/2)i_{ao}} = \frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\frac{N}{K_pK_dP_1}\frac{1}{N_{xd}} \tag{1d}$$

Synchronous machines with more than one additional rotor circuit in each axis require an involved analysis in which it is often advantageous to use specialized base quantities. For that reason it does not appear advisable to adopt preferred bases for multiple rotor circuits.

Specific Base-Current Ratios

The more important base-current ratios with practical advantages are given by the following four definitions. These definitions are summarized in Table I.

Definition 1: Let base field current I_{fdo} be that current which will induce in each stator phase a voltage equal to $x_{ad}i_{ao}$; let base rotor-circuit current I_{xdo} be similarly defined. These are the base currents presented in the preceding section. These bases are hereinafter termed the x_{ad} bases.

With three-phase currents of magnitude i_{ao} in the stator, and with the rotor cir-

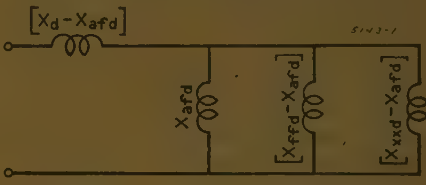

Figure 1. *d*-axis equivalent circuit

$X_{afd} = X_{axd} = X_{fxd}$

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cuits open, the maximum of the fundamental-component flux density is as given by equation 1a.

$$\beta_{ga}(\max)=0.4\pi\frac{4}{\pi}\frac{1.5i_{ao}N}{K_pK_dP_1}\frac{2.54}{g}A_{d1} \tag{1a}$$

The corresponding phase voltage is $x_{ad}i_{ao}$. By definition 1, I_{fdo} is the field current which will give a fundamental-component density equal to equation 1a. The maximum of the fundamental component due to I_{fdo} is as given by equation 1b.

$$\beta_{gf}(\max)=0.4\pi\, N_{fd}I_{fdo}\frac{2.54}{g}F_{d1} \tag{1b}$$

The base-current ratios of definition 1 are thus as given in equations 1c, d.

$$\frac{I_{fdo}}{(3/2)i_{ao}}=\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}} \tag{1c}$$

$$\frac{I_{xdo}}{(3/2)i_{ao}}=\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\frac{N}{K_pK_dP_1}\frac{1}{N_{xd}} \tag{1d}$$

It is evident from definition 1 that these x_{ad} bases make X_{afd} and X_{axd} numerically equal to x_{ad}.

Definition 2: Let base field current I_{fdo} be that current which will give a magnetomotive force per pole equal to the flat-topped armature reaction at normal stator current; let base current I_{xdo} be similarly defined. These bases are hereinafter termed the magnetomotive-force bases.

The base-current ratios can be written directly as in equations 2a, b.

$$\frac{I_{fdo}}{(3/2)i_{ao}}=\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}} \tag{2a}$$

$$\frac{I_{xdo}}{(3/2)i_{ao}}=\frac{N}{K_pK_dP_1}\frac{1}{N_{xd}} \tag{2b}$$

Note that the base currents of these magnetomotive-force bases are easily evaluated since equations 2a, b do not contain pole-shape coefficients such as A_{d1}, F_{d1}, D_{d1x}, etc.

Definition 3: Let base field current I_{fdo} be the field current required to produce normal, open-circuit, air-gap-line stator voltage; let base current I_{xdo} be similarly defined. These bases are hereinafter termed the unit-voltage bases.

$$\psi_{ao}=10^8L_{afd}I_{fdo}=10^8L_{ao}i_{ao} \tag{3a}$$

Therefore

$$\frac{I_{fdo}}{(3/2)i_{ao}}=\frac{L_{ao}}{(3/2)L_{afd}} \tag{3b}$$

$$\frac{I_{xdo}}{(3/2)i_{ao}}=\frac{L_{ao}}{(3/2)L_{axd}} \tag{3c}$$

Dividing equation 1a by equation 1b with I_{fdo} defined as in definition 3 will give x_{ad}, and equations 3d, e are then obtained from equations 3b, c.

$$\frac{I_{fdo}}{(3/2)i_{ao}}=\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}}\frac{1}{x_{ad}} \tag{3d}$$

$$\frac{I_{xdo}}{(3/2)i_{ao}}=\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\frac{N}{K_pK_dP_1}\frac{1}{N_{xd}}\frac{1}{x_{ad}} \tag{3e}$$

The major advantage of the unit-voltage bases is that the stator–rotor mutual coupling coefficients are unity for the field circuit and the base additional rotor circuit.

Definition 4: Let the base rotor currents be defined so that the mutual impedances (X_{afd}, X_{axd}, X_{fxd}) for a machine with one additional rotor circuit are all equal. These bases are hereinafter termed the equal-mutuals bases.

$$X_{afd}=X_{axd}=X_{fxd} \tag{4a}$$

Equating 2a, 2b, and 3c of part I gives the base-current ratios in terms of the mutual inductances corresponding to equation 4a. These are evaluated in terms of design dimensions in equations 4b, c, d, and the base-current ratios of equations 4e, f thereby obtained.

$$L_{afd}=3.19\times10^{-8}\frac{4Rl}{P_1g}\frac{N}{K_pK_d}N_{fd}F_{d1} \tag{4b}$$

$$L_{axd}=3.19\times10^{-8}\frac{4Rl}{P_1g}\frac{N}{K_pK_d}N_{xd}D_{d1x} \tag{4c}$$

$$L_{fxd}=3.19\times10^{-8}\frac{4Rl}{g}N_{fd}N_{xd}D_{dox}y_{xd} \tag{4d}$$

$$\frac{I_{fdo}}{(3/2)i_{ao}}=\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}}\frac{D_{d1x}}{D_{dox}y_{xd}} \tag{4e}$$

$$\frac{I_{xdo}}{(3/2)i_{ao}}=\frac{N}{K_pK_dP_1}\frac{1}{N_{xd}}\frac{F_{d1}}{D_{dox}y_{xd}} \tag{4f}$$

The equal-mutuals bases are of particular importance with machines having only one additional rotor circuit in each axis, as they result in the simple equivalent circuit of x_{la} in series with the parallel combination of X_{afd}, X_{lfd}, and X_{lxd}. It should be noted, however, that it is not now possible to replace X_{afd} by the more common x_{ad}, as these quantities are no longer equal. This fact seems to have been overlooked in the application of this simplified circuit.

Rotor Bases in Current Use

From the preceding analysis, it is evident that impedance formulas are subject to misuse unless the base-current ratios are known. A compendium of the base-current ratios of the present technical literature is needed in order to harmonize the various published results. This section offers the first step toward such a general compendium.

It is not necessary to study those publications which considered only the definitive impedances, since these are independent of the rotor bases. Examples of the papers which presented formulas for the rotor impedances as well as the stator quantities are Doherty and Shirley,[2] Linville,[3] and Kilgore.[4] The papers of Doherty and Nickle,[3] Park and Robertson,[4] Park,[7] and Waring and Crary[8] are illustrative of the papers on the operational machine equations. It is obvious that these two groups fail to include many important publications, but those listed are adequate to illustrate the need for a more complete survey.

Table I. Base Rotor Currents Used in the Calculation of Per-Unit Machine Impedances

Definition Characteristic	Base Field Current and Base Additional Rotor-Circuit Current Will
1. . x_{ad}	Induce in each stator phase a peak voltage equal to $x_{ad}i_{ao}$
2. . Magnetomotive force	Give in their respective windings a magnetomotive force equal to the flat-topped armature reaction at normal stator current
3. . Unit voltage	Induce in each phase normal open-circuit air-gap-line stator voltage
4. . Equal mutuals	Result in equal mutual impedances ($X_{afd}=X_{axd}=X_{fxd}$) for a machine with one additional rotor circuit

The base field current used by Doherty and Shirley may be derived from their expression for the increase in field magnetomotive force* due to the short-circuit current. It is the field current which will produce a magnetomotive force equal to $1/K_\phi$ times the flat-topped armature reaction, and differs only by the factor K_ϕ from the base field current of the magnetomotive-force base (definition 2). The corresponding base-current ratio is given by equation 5.

$$\frac{I_{fdo}}{(3/2)i_{ao}}=\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}}\frac{1}{K_\phi} \tag{5}$$

Doherty and Shirley did not discuss any additional rotor circuits.

The base rotor currents used by Linville are explicitly defined as those which will produce in their respective windings a magnetomotive force equal to the flat-topped armature reaction. These are the base currents of the magnetomotive-force bases (definition 2), and the corresponding base-current ratios are evaluated in equations 2a, b.

Kilgore uses "effective" rotor leakage reactances defined as the quantities which when added to (x_d-x_{ad}) will give x_d' and x_d''. The base rotor currents are then such as to make the base stator–rotor mutual inductances numerically equal to x_{ad}. These are the base currents of the x_{ad} base (definition 1), and the corresponding base-current ratios are evaluated in equations 1c, d.

Doherty and Nickle specified as base field current that current which would produce normal open-circuit air-gap-line stator voltage. For a per-unit system with reciprocal mutual impedances, this is the base field current of the unit-voltage base (definition 3), and the corresponding base-current ratio is evaluated in equation 3a.

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* Equation 2, page 1259, reference 2.

A relatively large amount of machine analysis is based on the simplified circuit of Figure 1 although x_{ad} is commonly used in place of X_{afd}. The base currents which result in this circuit are the equal-mutuals bases of equations 4e, f (definition 4). Note that x_{ad} should not be used in place of X_{afd} as these two are equal only for the conditions of definition 1, the x_{ad} bases.

The base field current used by Park and Robertson was that which would give normal open-circuit air-gap-line stator voltage, but nonreciprocal mutual-impedance relations were used which resulted in the following expression for x_d'.

$$x_d' = x_d - \frac{X_{afd}}{X_{ffd}}{}^{*} \tag{6a}$$

If equation 6a is compared with equation 6b which is the more general expression for x_d',

$$x_d' = x_d - \frac{X^2_{afd}}{X_{ffd}}{}^{**} \tag{6b}$$

it is evident that satisfactory values of x_d' can be obtained only if X_{afd} is calculated on a base which recognizes the nonreciprocal mutual impedances.

The base current of Park is that of definition 3, but nonreciprocal mutual impedances are used which for all practical purposes restrict the application to machines with no additional rotor circuits.

The equations of Waring and Crary are general and dimensionally correct, and are not restricted to any specific base current. The base quantities are set by the design engineer in the calculation of the impedances. If it is desired to use the restricted operational equations of references 5 and 6 on machines with multiple rotor circuits, the necessary nonreciprocal mutual impedances can be best obtained by modifying the corresponding reciprocal mutual impedances of Waring and Crary as recently published by the author.[9]

To illustrate quantitatively the effect which the foregoing multiplicity of bases has upon the machine impedances, consider a machine with the design constants given by equation 7.

$$\left.\begin{array}{lll} K_\phi = 0.98 & \dfrac{4}{\pi}\dfrac{A_{d1}}{F_{d1}} = 1.10 & x_{ad} = 1.20 \\ D_{d1x} = 0.84 & D_{dox}y_{xd} = 0.73 & \end{array}\right\} \tag{7}$$

Assume that x_d, X_{fd}, X_{ffd} are calculated by formulas using the base-current ratios of the x_{ad} bases (definition 1). If these same quantities are calculated for the same machine but by formulas based on the other ratios given in the foregoing review, the numerical values so obtained will be as shown in Table II. The values in this table are in per cent of the values calculated by formulas based on the x_{ad} bases.

The magnitude of the possible error due to the misapplication of a per-unit impedance which has been calculated on a rotor-current base not explicitly defined is evident from Table II. The nonreciprocal relations of references 6 and 7 are not included in this table as their application is relatively rare. It is self-evident that this example is applicable only to the machine with the constants given by equation 7. The variations for other machines may be much greater or much less than those illustrated, depending on the design data.

Table II. Calculated Numerical Values for x_d, X_{afd}, and X_{ffd}

	x_d	X_{afd}	X_{ffd}
By the x_{ad} bases (definition 1)	100	100	100
By the magnetomotive force bases (definition 2)	100	91	83
By the unit-voltage bases (definition 3)	100	83	69
By the equal-mutuals bases (definition 4)	100	105	110
By equation 5	100	93	86

Nomenclature

The nomenclature consists of the quantities given in the following plus the nomenclature of part I. All dimensions are in inches. As in part I, all quantities are in per unit except those in bold-face type which are in the physical ampere-inch system (ohms, henrys, and so forth). Quantities such as N, P_1, K_p, etc., for which it is obviously unnecessary to distinguish the ampere-inch values by a special script are in standard type only.

ar = flat-topped armature reaction at base current $= \dfrac{1.5Ni_{ao}}{K_pK_dP_1}$ ampere turns per pole

A_{d1}† = factor by which the maximum flux density is multiplied to obtain the maximum of the fundamental component of flux density; machine excited by a sine-wave armature magnetomotive force in direct axis

$\frac{2}{\pi}D_{dox}$ = factor by which the maximum flux density is multiplied to obtain the average density within the damper circuit of span $2y_{xd}$; machine excited by the damper circuit of span $2y_{xd}$ in the direct axis

D_{d1x} = same as A_{d1}, but with machine excited by the magnetomotive force of the xth additional rotor circuit in the direct-axis

F_{d1}‡ = same as A_{d1}, but with machine excited by the magnetomotive force of the direct-axis field winding

g = effective air gap

K_p K_d = pitch and distribution factors, respectively (greater than 1.0)

K_ϕ# = ratio of total flux per pole to the total fundamental flux per pole, at no load; machine excited by the magnetomotive force of the direct-axis field winding

l = machine stacked length, inches

N = stator series turns per phase

N_{fd} = turns per pole of field-winding circuit

N_{xd} = turns per pole of xth additional rotor circuit

P_1 = number of poles

R = stator bore radius, inches

x_d', x_d'' = transient and subtransient reactances, respectively

x_{la}, X_{lfd}, X_{lxd} = leakage reactances; armature, field, and xth additional rotor circuits; for use with equal-mutuals bases only

$2y_{xd}$ = span of xth additional rotor circuit, direct axis, in per unit of half pole pitch

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* Equation 20, reference 6.

** Equation 20, reference 8.

† See Figure 13, reference 10.

‡ This is the A_1 factor given by Figure 8, reference 10.

# See Figure 10, reference 10.

Wound- and Dummy-Rotor Methods of Quality Control and Trouble Shooting of Induction-Motor Windings and Cores

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ONE OF THE FUNDAMENTAL principles in the solution of problems is the division of the large problem into several smaller ones. This principle is used throughout electric-motor engineering. One of the most common examples is the use of design constants or parameters. For instance, the complete characteristics of an induction motor are fixed and performance is determined if six constants are known.

r_1 = primary resistance
r_2 = secondary resistance
X = short-circuit reactance
X_o = open-circuit reactance
F_e = iron loss
$F+W$ = friction plus windage

If quality-control methods are used in the factory to prevent the values of these constants from straying beyond desired limits for a given design, then final assembly and test will not be plagued with rejections of defective motors.

It is the function of this paper to discuss means of controlling the variation of F_e iron loss and X_o open-circuit reactance. The open-circuit reactance is also a measure of the ampere turns necessary to set up the magnetic flux and a measure of the final magnetizing current and no-load amperes.

Because most induction motors have permanently short-circuited secondary windings in the form of squirrel-cage rotors, it has become customary to measure X_o and F_e by running the motors at synchronous speed. Under this condition, there is no current in the rotor of a polyphase motor, and a condition equivalent to open-circuited secondary winding exists. Actually, except for very small motors, satisfactory results are obtained at no-load speeds slightly under synchronism.

While the measurement of X_o and F_e on completed motors is necessary and desirable in quality control and trouble shooting, its obvious objection is that defective parts have not been eliminated at the source, but have continued through all the expensive and time-consuming manufacturing operations. Another very serious disadvantage is that small variations in the iron loss and ampere turns of the magnetic circuit are obscured and confused by ordinary variations in friction and air gap.

The stator is the most frequent source of troublesome variation, and one method of overcoming the difficulties mentioned is to test all stators in a fixture with rotor on bearings and duplicating normal operating conditions. The main objections to this method are its practical difficulties of construction, cost, and the necessity of applying it to a completely machined and ground stator.

Since the purpose is to duplicate the condition of open-circuited secondary winding, why not abandon the substitute method of running at synchronous speed and go directly to the open-circuited rotor? If only a single motor were involved, of course, there would be the expense of making a special rotor without squirrel cage. When hundreds or thousands of motors are in quantity production, however, the cost of a special "dummy" rotor for testing is negligible.

Dummy Rotors

Dummy test rotors are constructed exactly like the regular rotors, except without squirrel cage, and ground to the finished diameter of the regular rotor to obtain the regular air gap when testing finished stators. If it is necessary to provide means for holding the lamination in place, such means must not be conducting if it includes the magnetic circuit. The slots usually are left in to keep flux distribution as near normal as possible. Regular shafts are not necessary, because they do not rotate, but usually some sort of unfinished shaft is used to provide a convenient handle.

If polyphase power is applied to a polyphase stator and dummy rotor, the magnetic conditions differ somewhat from the open-circuit conditions in the regular motor. The first difference is that the high-frequency pulsation and surface losses caused by motor teeth and slot openings passing each other are not present. Partly offseting these, the rotor iron loss is at line frequency instead of slip frequency. The second difference is that the helpful effect of rotor currents in improving the sinusoidal flux distribution are not present. The nearer the stator winding is to having a sinusoidal distribution itself, the less will be this difference.

However, for quality control and trouble shooting, relative values and difference between good and bad stators are much more important than the actual value.

One of the difficulties in both no-load and dummy rotor tests with polyphase stators is the low power factor. This gives the difference of two large values when reading input by the two-wattmeter method. Since relative values are desired, and not actual, dummy-rotor test investigations very often are made with only single phase on the stator. This excites one phase only and gives about half the polyphase iron loss.

In general for comparative purposes, it is necessary to deduct only the primary copper loss before comparing readings; however a more accurate measure of the percentage variation is obtained when the constant-rotor iron loss also is deducted. The main difficulty in this is a means to measure it. Since it is usually small, a calculation from iron-loss curves may be

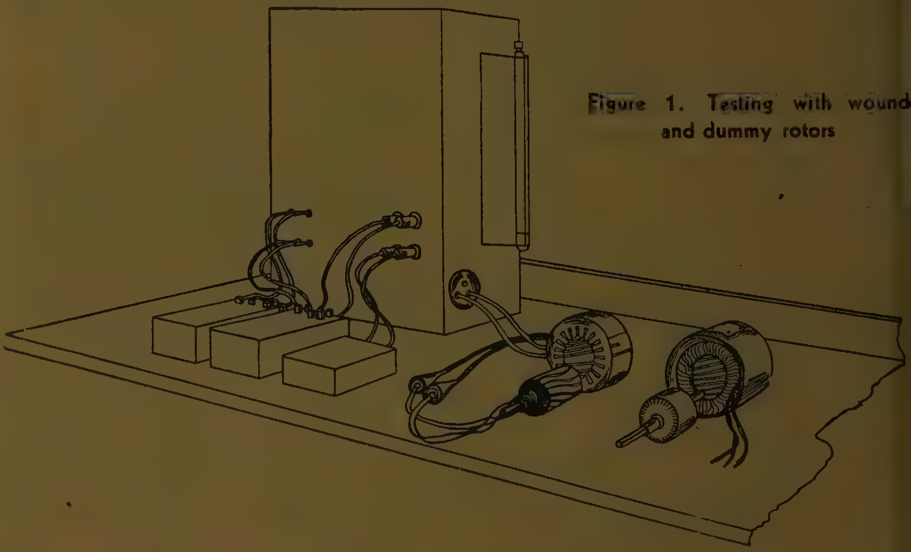

Figure 1. Testing with wound and dummy rotors

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sufficiently accurate. If a test value is desired, it may be necessary to put a temporary winding on the rotor and test with an oversize dummy stator as described later.

For quality control of polyphase wound-rotor motors or single-phase repulsion motors, readings may be taken on every new design with the brushes lifted, thus providing a dummy-rotor reading for comparison in case of future trouble. In case of regular squirrel-cage motors, the special dummy rotor must be provided. It is not necessary that a separate rotor be provided for every core stack. One rotor longer than any expected size is usually sufficient. In fact, it is better to have the rotor longer than the stator to eliminate the necessity for special care on axial location in testing.

The dummy-rotor method provides one of the most accurate tests of a wound stator that there is. Its accuracy may be increased even further for iron-loss or short-circuit investigation by using higher frequencies such as 500 or 1,000 cycles. Of course, any comparison to actual value disappears, and the tests become relative only.

Very often, the high-frequency test will provide a high-potential test between turns as a by-product.

Table I. Regular Test of Stator Cores

Volts = 91

	Stator Number					
	1		2		3	
	Max	Min	Max	Min	Max	Min
Amperes	2.13	1.82	2.08	1.82	2.13	1.92
Watts	24.0	22.0	25.5	24.5	23.0	22.0
Resistance after test	1.410		1.410		1.410	
I^2R	6.4	4.7	6.1	4.7	6.4	5.2
Total iron loss	17.6	17.3	19.4	19.8	16.6	16.8
Total iron loss from item 1	5.5	5.5	5.5	5.5	5.5	5.5
Stator iron loss	12.1	11.8	13.9	14.3	11.1	11.3

Wound Rotors

We have mentioned the advantage of the dummy rotor over the running test in eliminating confusion due to variation in friction. However, the dummy-rotor test does require a winding in the stator with the possibility that expensive winding and machining operations may be completed before it is found that the stator cores have excessive iron loss. Under certain conditions it is difficult to distinguish between excessive iron loss and stator short circuits of one turn only. Also small variations in the number of stator turns may be confused with excess iron loss.

To overcome these disadvantages, the "wound-rotor" method may be used. This method consists of providing a winding on the test rotor and exciting from the rotor instead of the stator. Tests may then be made on bare cores without any windings, housing, or any machining whatsoever, thus eliminating any question of defective windings when iron loss is being investigated or controlled. Also progressive tests may be made after each winding and machining operations to see if any harm has been done to the magnetic structure and increased the iron loss.

The same differences occur between the condition of actual operation and the condition of wound-rotor test as occur with the dummy rotor. That is, the flux in the rotor is at line frequency instead of slip frequency, and the flux distribution may be less sinusoidal than in the actual machine with squirrel-cage rotor. If the wound rotor is designed for this purpose, it can have sinusoidally distributed windings which will keep this difference fairly small. The main difference in the flux distribution of the wound rotor is that the excitation comes from the rotor. This means that the gradual loss of leakage flux from the useful flux progresses from rotor to stator instead of from stator to rotor. To be specific, in the regular case the air-gap flux is less than the stator flux by the leakage but in the wound-rotor test, the air-gap flux must be kept greater than the stator flux by the leakage in order to give the same stator flux; actually the matter can be easily taken care of by adjusting the turns of the rotor, and except for extremely high densities the difference in flux distribution does no harm.

Wound rotors are usually made single phase to eliminate the error of two large wattmeter readings which occurs on the two-wattmeter method with three phase.

It is usually necessary to deduct the rotor copper loss when comparing iron-loss readings, particularly if some of the stator cores are ground to finished diameter and some are as punched. In order to obtain a better representation of percentage differences, the rotor iron loss also may be deducted. The value may be obtained by calculation or by test with a special oversize dummy stator, bored to size, but having no slots and a diameter much bigger than the regular stator. The densities in this will be very small, and even a rough calculation is sufficient to determine the iron loss. To be more specific, a test is made of the wound rotor in the dummy stator. The rotor copper loss and calculated stator iron loss are subtracted from the input. The remainder is used as the rotor iron loss for all further testing with the wound rotor.

For some work it is desirable to keep all conditions of the wound-rotor tests as near like actual conditions as possible. In these cases, the same number of poles are used as in the motor windings. However, many times the readings are magnified and more useful when two-pole rotors are used.

As in the case of the dummy rotors, high frequency may be used to make the tests more sensitive.

It is often necessary to turn the wound rotor in the stator being tested and take maximum and minimum readings. The effect of the grain of rolling the steel, and the effect of rivet and bolt holes are often quite pronounced.

Very often the need of a wound rotor for test analysis or trouble shooting occurs without warning, and there is no time to design and build a special rotor. Sometimes in these cases, a d-c or repulsion-motor armature may be obtained and leads soldered to opposite commutator bars for connections. Even if the air gap is larger than desirable or unusual test voltage and frequency necessary, comparative readings usually can be obtained which are usable in separating good and bad stator cores.

If wound rotors are used continuously, the windings and core structure are apt to become damaged. Protective coatings of varnish or other material should be used. Sometimes it is desirable to have a master standard kept in the laboratory for calibrating new or rewound rotors.

Wound rotors may be used as turn counters for stator windings by reading the induced voltage in the maximum position. If the test is a routine one comparing production stators with a master or sample stator, a direct reading is sufficient. If turns of an unknown winding are desired, it is necessary to apply voltage and read induced voltage on both rotor and stator. In this case

$$K = \frac{\text{effective turns of stator}}{\text{effective turns of rotor}}$$

$$= \sqrt{\frac{E_s \text{ read}}{E_r \text{ applied}} \times \frac{E_s \text{ applied}}{E_r \text{ read}}}$$

Very often stator short circuits can be distinguished from excessive iron loss by turning the wound rotor. Usually stator short circuits are of a single-phase nature such as one turn or one coil short-circuited. In this case, the maximum reading shows up the short circuit very prominently, and the minimum reading shows no effect at all.

A typical example of the use of wound and dummy rotors occurred some years ago. The first indication of trouble was an excessive number of rejects on final motor test. After a number of the

Power Systems for Auxiliary Drives in Steam-Electric Stations

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A STEAM-ELECTRIC power station in general consists not only of such major items as boiler equipment, steam-turbine-generator units and condensers, but also of many auxiliaries. These auxiliaries now have come to be almost exclusively a-c-motor-driven, although steam-driven auxiliaries are used occasionally in some topping installations where their use improves the heat cycle and in other cases for stand-by service. Since the reliability of a station can be no better than the reliability of its auxiliaries, the power-supply system for these auxiliaries, as well as the associated motors and control, become important items in the design of a steam-electric-power station.

These considerations in conjunction with the recent rapid rise in the quality and acceptance of factory-assembled switchgear, control, and transformer equipments make it now opportune to consider the design of the power system for these auxiliaries, taking full advantage of factory-assembled equipments. It is hoped that this discussion will not only direct proper attention to this over-all problem, but also indicate how a new level of reliability, economy, simplicity both in installation and operation, and safety to personnel can be obtained readily on power systems for auxiliaries by proper design approach based on such factory-assembled equipments.

As a result of discussions with several operating groups and after studying various types of auxiliary-power systems, together with the existing methods of obtaining auxiliary power, the following general conclusions have been reached. These conclusions should serve as a guide in planning power systems for auxiliaries.

Conclusions

1. A separate auxiliary-power system should be considered for each boiler.
2. Power supply from the main-generator leads is being increasingly used.
3. The preferred voltages are 460 and 2,400 volts.
4. It is economical to use a single-voltage 460-volt system, where the required transformer capacity per boiler does not exceed 1,000 kva.
5. A dual-voltage system comprising 2,400 and 460 volts should be considered where the required transformer capacity per boiler is 1,500 kva or greater.
6. In a dual-voltage system motors up to at least 250 horsepower can be economically applied at 460 volts.

Power Requirement of the Auxiliaries

The over-all auxiliary-power requirements for any given new steam-turbine-generator capacity can never be determined precisely until the plant is placed in operation. However, it can be reasonably estimated if proper attention is given to such details as the number and type of boilers, the pressure and maximum flow of steam, the nature of the feed-water piping, and the type and treatment of fuel. This last item is particularly significant, as it is evident that pulverized-coal-fired boilers will require more auxiliary power than boilers using other types of fuel, because of the pulverizers and exhauster fans.

Figure 1 gives the approximate total auxiliary power in per cent of the maximum output of a turbine-generator set. In addition, the probable individual power requirements for the major auxiliaries are indicated. The auxiliary-power requirement should be based upon the actual maximum output of the turbine-generator set. In a specific case this output may be any value between the nominal rated and the maximum permissible from the set, depending upon the capacity of the boiler plant. Thus, since the boiler capacity is selected normally with a margin to cover adverse conditions of fuel, draft, and boiler maintenance, it is evident that the boiler-plant capacity really determines the auxiliary-power requirements, through its effect on the actual maximum output of the set.

The total auxiliary power, of course, is the difference between the gross and net output of the turbine-generator set. The total auxiliary transformer capacity can be approximated by assuming a power factor of 0.8 to 0.85 for the power requirements obtained from Figure 1. In the final design, however, transformer capacities should be determined conservatively by totalizing the coincidental loads with due regard for power factor. In addition, provision must be made for general station power facilities such as lighting, machine shop, and coal-handling equipment. These general station power requirements are very often associated with the auxiliary-power supply for the first one or two units in a station.

The size of the individual motors for the major auxiliaries is determined largely by the rating of the individual boilers and

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The authors express their appreciation to the representatives of the operating companies who contributed during the discussions on this subject, and acknowledge the helpful suggestions of A. H. Fish D. M. Jones, and J. C. Page, in the preparation o this paper, and the assistance of F. S. Li in makin the economic studies summarized herein.

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good and bad motors were tested by the dummy-rotor method, it became certain that the trouble was in the stator and was due either to short circuits or excessive iron loss. Resistances and torques indicated that the number of turns were probably correct.

After the good and bad stators were tested by the wound-rotor method, it was found that the bad stators were bad in both maximum and minimum positions. This created considerable suspicion of short circuits in the stator windings. This point was settled when the stators were stripped. The good stators remained good, and the bad stators remained bad, with the readings the same as before the windings were removed.

This was considerable relief to the winding department which had previously been rewinding so called "defective" stators.

The next step was to institute a 100-per-cent wound-rotor test on all stator cores before winding. This immediately removed the rejects.

Upon further investigation of the good and bad cores as they came from the core building department, the trouble was found to be due to a lack of insulating iron-oxide scale which was the only insulation between individual laminations. Once this condition was corrected, and proper insulation provided between laminations, the rejects of defective cores prior to winding dropped to practically zero and the 100-per-cent core test was taken off.

Appendix. Typical Test and Analysis

1. Wound rotor tested with dummy stator to determine rotor iron loss:

Volts = 91
Amperes = 0.64
Watts = 8.30
Resistance after test = 1.406
I^2R = 0.58
Total iron loss = 7.72
Calculated stator iron loss = 2.22
Rotor iron loss = 5.50

2. The regular test of stator cores is shown in Table I.

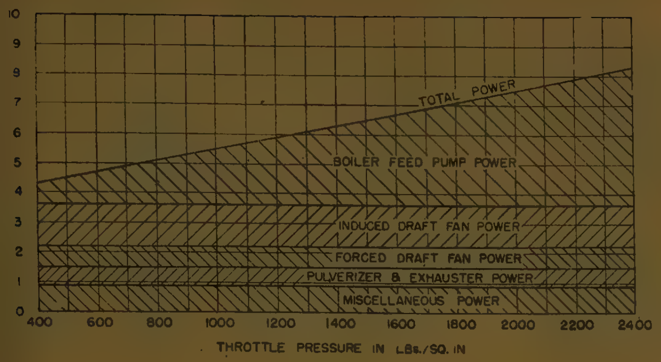


Figure 1 (left). Approximate total auxiliary power in per cent of maximum generator output in kilowatts and approximate division of power between major auxiliaries

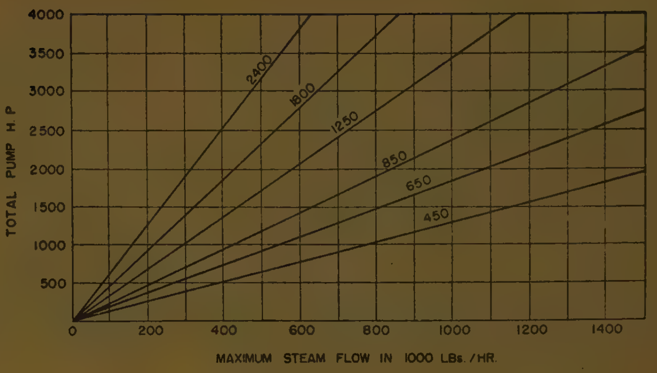


Figure 2 (right). Boiler-feed-pump power for various pressures and maximum steam flows

oes not necessarily bear any direct rela-on to the size of the turbine-generator t or the total auxiliary power associated ith that set. As an aid in estimating e probable boiler arrangement for any ontemplated installation, present boiler ractice may be broadly summarized as llows:

Steam-turbine-generator units up to pproximately 30,000 kw maximum rat-g generally have a single boiler; units etween 30,000 kw and 60,000 kw maxi-um rating may have either one or two oilers, and practice is about equally ivided in this region; units above 60,000 w maximum rating generally have two oilers, although single boilers are now vailable and occasionally used up to a apacity of 1,000,000 pounds of steam per our.

Boilers are usually rated in pounds of team per hour, and a reasonable average team rate for modern turbine-generator ets is nine pounds of steam per kilowatt-our. Thus, it is evident that a 1,000,000-ound boiler would supply steam for a set utput of at least 110,000 kw.

Figure 2 shows the estimated boiler eed-pump power for various pressures nd flows, including liberal allowance for ead loss in piping. In the smaller plants t is the usual practice to have a minimum f two pumps, each large enough to de-iver maximum flow. This results in at east 100-per-cent spare pump capacity. n the larger plants two pumps often are equired to provide maximum flow, and a hird serves to provide 50-per-cent spare apacity. In some of the larger plants, oiler feed-water supply is provided by a ombination of tank and booster pumps n series. In such arrangements, the booster pump is usually variable speed and of sufficient capacity to give the required outlet pressure range for the particular plant conditions and at the same time offset the rising pressure characteristic of the constant-speed pump at the lower flows. The capacity of such a combination may vary from 50 to 100 per cent of the total boiler requirements.

The actual motor ratings, in most cases, will be slightly greater than the values derived from Figure 2.

Figures 3 and 4 show approximate motor ratings for induced-draft and forced-draft fans, respectively. These curves are based on average values obtained by plotting actual motor ratings for several boiler ratings. The discontinuity between 200,000 pounds and 300,000 pounds flow results from a change in boiler construction in this region. It will be noted that induced-draft fans require more power than forced-draft fans. Thus, in practice, as the size of a boiler increases, two induced-draft fans often will be used with one forced-draft fan. Finally, in the largest boilers, both induced- and forced-draft fans may be split into two groups.

Power System Application Guides

A power-supply system for auxiliary drives in a steam-electric station is unique in many respects, for example:

1. There are a few large motors.
2. There are many small motors.
3. The cable runs are short.
4. The load is relatively constant
5. Voltage regulation is very seldom a problem.

In planning a system the following principles and practices which have demonstrated their soundness in service should be recognized:

(*a*). An adequate source of power must be available for starting up the plant.

(*b*). The entire system should be considered essential, since the essential motors are both large and small and are not confined to any one location in the plant.

(*c*). In cases where two voltages are used, it simplifies operation and switching arrangements to bring all the power to the higher-voltage bus.

(*d*). The reliability of metal-clad, insulated five-kilovolt busses and metal-enclosed 600-volt busses justifies a single-bus system.

(*e*). The reliability of power transformers justifies the use of a conventional radial system.

(*f*). High short-circuit currents should be avoided.

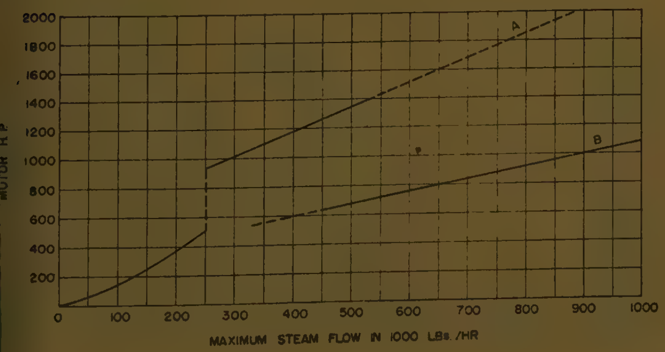


Figure 3 (left). Induced-draft-fan power

A—One fan per boiler or total power
B—Two fans per boiler—power per fan

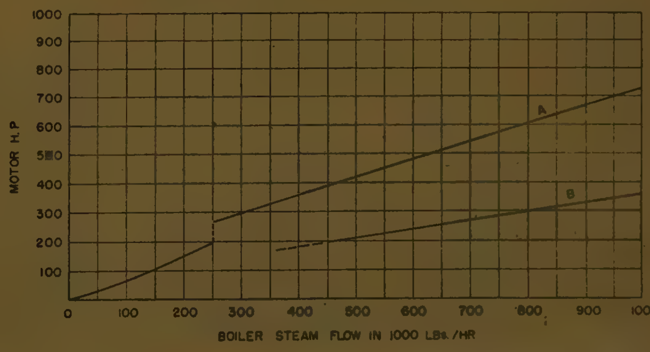


Figure 4 (right). Forced-draft-fan power

A—One fan per boiler or total power
B—Two fans per boiler—power per fan

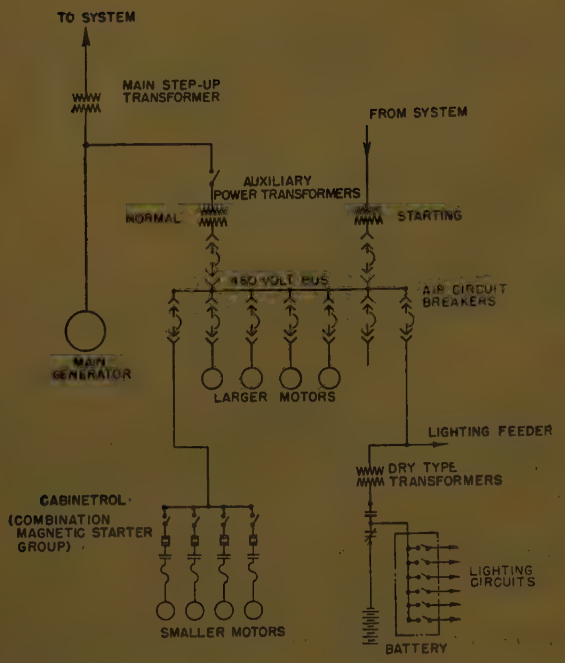
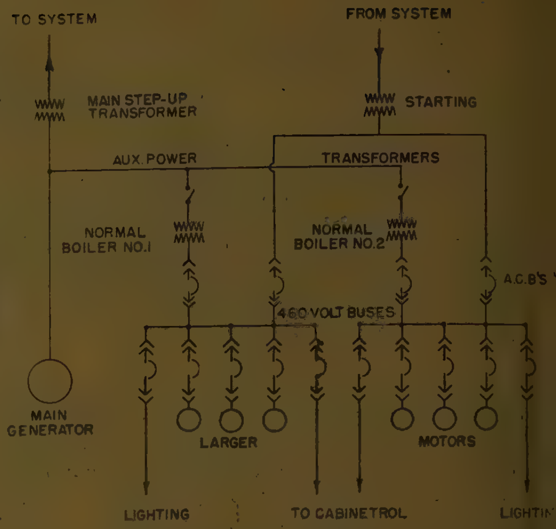


Figure 5 (left). Single-voltage system supplied from generator leads with one or two boilers per unit

Figure 6 (right). Single-voltage system supplied from generator leads with two boilers per unit



The Auxiliary Power Source

Auxiliary power may be obtained from any of the following sources:

(*a*). Main-generator leads.
(*b*). Generator bus.
(*c*). House turbine-generator set.
(*d*). Direct-driven shaft generator.

The choice among the afore-mentioned sources of power is frequently based upon opinion which may in turn be founded upon some annoying experience. When a shaft generator or house turbine-generator set is used, the performance of the auxiliary-power system is not affected by disturbances on the main system. So far as efficiency is concerned, the house turbine-generator set is at a disadvantage because of the inherent lower efficiency of the smaller turbines. In cases where either type of auxiliary generator is used, there are the disadvantages of the resultant added cost and complexity.

In addition there are such detailed difficulties as starting large motors from a relatively small generator, and the possible troubles with paralleling main and auxiliary generators and transferring from the starting power source to the separate generator. Auxiliary generators used in these applications are usually of the high-speed turbine-generator type, and, when equipped with an automatic voltage regulator, a motor with a rating up to one-third that of the generator generally can be started acceptably. A more severe voltage condition may occur aft
the transfer of a large group of moto
from a starting source to an auxiliar
generator, resulting from the increased r
active motor currents at the reduce
motor speeds.

Auxiliary Power From Generator Leads

There seems to be, however, an increa
ing trend toward the unit system a
with it an increased tendency to obta
power directly from the leads of the ma
generator. So far as cost and simplici
are concerned, this method has defini
advantages. This provides auxiliar
power at the efficiency of the main set a
permits the unit to go back into servi
immediately after having been tripped
as a result of a system disturbance. Pra
tice indicates that the short length
cable plus the auxiliary transformer co
nected to the leads of the main generat
very seldom is responsible for an outa
of the unit. Of course, a starting sour
of power is required, but one such sour

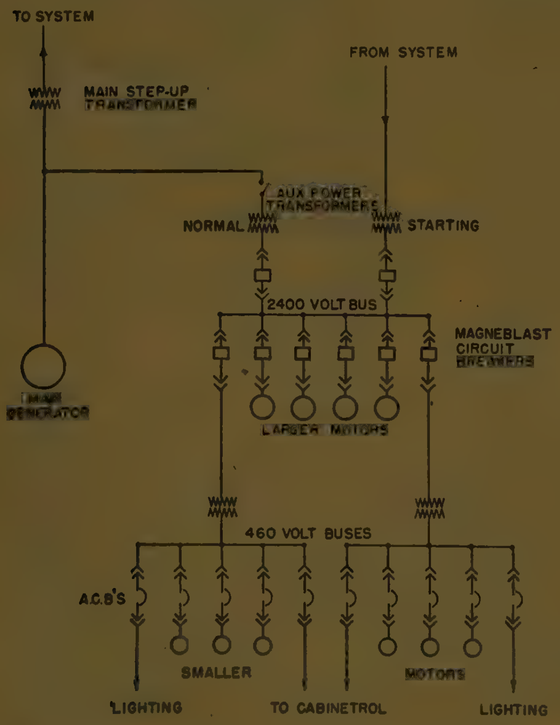


Figure 7 (left). Dual-voltage system supplied from generator leads with one or two boilers per unit

Figure 8 (right). Dual-voltage system supplied from generator leads with two boilers per unit

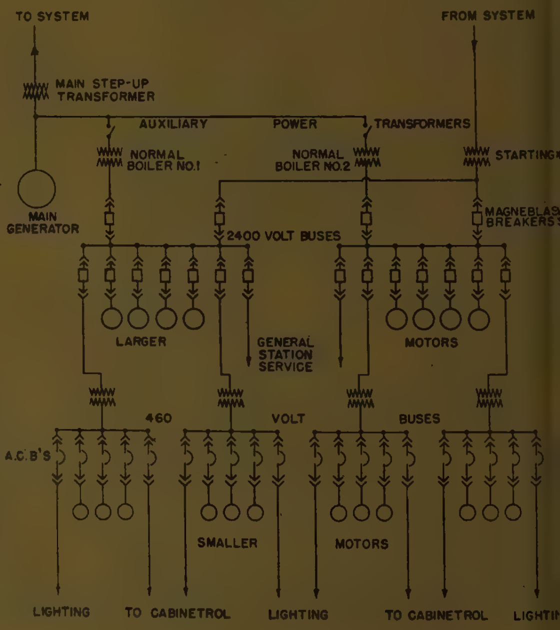


Table I

Transformer Rating		Feeder Breaker Interrupting Rating	Transformer-Secondary Breaker Rating	
Kva	Secondary Voltage		Current	Interrupting
300	480	15,000 amperes	800 amperes	50,000 amperes
500	480	25,000 amperes	800 amperes	50,000 amperes
750	480	25,000 amperes	1,600 amperes	50,000 amperes
1,000	480	50,000 amperes	2,000 amperes	50,000 amperes
1,000*	480	25,000 amperes	1,600 amperes	75,000 amperes
1,500	480	50,000 amperes	4,000 amperes	50,000 amperes
1,500	2,400	50,000 kva	600 amperes	100,000 amperes
,000	2,400	50,000 kva	600 amperes	50,000 kva
,500	2,400	50,000 kva	600 amperes	50,000 kva
,000	2,400	100,000 kva	1,200 amperes	50,000 kva
,750	2,400	100,000 kva	1,200 amperes	100,000 kva
,000	2,400	100,000 kva	1,200 amperes	100,000 kva

* It will be noted that one transformer rating, 1,000 kva at 480 volts, has an impedance higher than the enerally accepted standard. This permits the use of feeder breakers having a lower-than-normal interupting rating for this transformer size and results in a more economical installation in spite of the special ransformer. It will be found that attempting to use special impedance values for smaller transformer ratngs at 480 volts will result in only a minor saving in equipment cost and a small reduction in fault currents vhich are already considered acceptable. In the case of larger transformers at 480 volts, the high impedance equired to permit the use of lower breaker ratings results in both impractical transformer design and exessive voltage regulation.

vill suffice for several unit busses. When he supply is through a transformer, the oltage dip incident to motor starting is eldom a problem.

The Auxiliary-Power Voltage

In laying out an auxiliary-power-supply ystem, there are two voltage classes of quipment to be considered, namely, 600 olt and 5,000 volt. The cost of cable nd control equipment in any voltage lass is dependent largely upon its current ating. Thus, unless there are other offetting factors, equipment and apparatus hould be used at as high a voltage rating n its class as is practical.

In the 600-volt class, nominal system oltages of 115, 230, 460, and 575 volts ave been used. In this discussion 460 olts is considered throughout as the referred level in the 600-volt class for he following reasons:

The 460-volt system is considered to be e conservatively high voltage in this class.

Motors and control at this voltage are idely used and therefore readily available.

At 440 volts practical motor ratings are rge enough to overlap the lower practical otor ratings for both 2,300 volts and 4,000 olts.

At 440 volts practical motor ratings inude those down to one-half horsepower and us overlap the higher practical ratings of gle-phase 120-volt motors.

Since motor prices are the same for all vels in the 600-volt class, there is no disvantage in cost for the 440-volt motors.

In the 5,000-volt class, nominal system ltages of 2,400 and 4,160 volts are in e. However, the increase in cost of 00-volt motors over 2,300-volt motors kes it desirable to use 2,400-volt sysns, except in special cases, which will discussed later.

Thus, in this discussion 440, 2,300, and 00 volts are recognized as the standard tor voltages, with corresponding nominal bus voltages of 460, 2,400 and 4,160 volts, respectively. To insure 460 volts at the bus a transformer secondary voltage of 480 is normally used. At the two higher voltages transformer secondary voltages are usually the same as the bus voltages, in view of the short cable runs encountered.

600-Volt Equipments

There are various types of control for low-voltage a-c motors, all of which include means for starting and stopping the motors, and also for providing any combination of motor overload and short-circuit protection. These types may be grouped as follows:

(*a*). Combination magnetic starters, using thermal air circuit breakers for short-circuit protection. These are available in sizes up to and including size-V starters.

(*b*). Combination magnetic starters, using fusible disconnects for short-circuit protection. These are available in sizes up to and including size-III starters.

(*c*). Drawout electrically operated air circuit breakers, which are available in all ratings of the breakers.

The specific ratings of the foregoing equipments are shown later in Tables V and VI, and it should be noted that all ratings of these equipments may be combined in the same respective assembly.

Drawout air circuit breakers generally are used for transformer secondary circuits, bus ties, feeder circuits, larger motors, and for most essential motors, regardless of size. The air breakers have the advantage of being a latched-in type of device and therefore do not drop out on voltage dips. In addition, the drawout feature provides disconnecting means which facilitates maintenance and improves service continuity. Their use, however, should be confined to applications where the required number of operations is not excessive.

Combination magnetic starters are used customarily for the smaller motors and also on applications where repetitive starting conditions are encountered. They can be furnished in factory-assembled, metal-enclosed groups, known as Cabinetrols. In general, it also will be found that drawout air circuit breakers will be less expensive than size-IV or larger combination starters, whereas combination starters will be less expensive up to and including size III.

Electrically operated drawout air circuit breakers for this type of service are furnished in metal-enclosed groups and referred to as drawout-air-circuit-breaker equipments.

5,000-Volt Equipments

There are two arrangements available for controlling high-voltage a-c motors, both of which provide combination short-circuit and overload protection:

(*a*). Metal-enclosed high-voltage contactors with current-limiting fuses.

(*b*). Vertical-lift metal-clad switchgear, with Magneblast circuit breakers.

The complete ratings of these equipments are given later in tables.

The fused starters are available for motors up to 700 horsepower at 2,300 volts. Fuses will, in general, clear a short-circuit more rapidly than the breaker. The contactors have long mechanical life and are best applied where a large number of operations will be encountered. From

Table II

2,400–480 Volts		13,200–480 Volts		13,200–2,400 Volts	
Kva	Per Cent Impedance	Kva	Per Cent Impedance	Kva	Per Cent Impedance
200	5.0	200	5.0	750	5.5
300	5.0	300	5.0	1,000	5.5
500	5.0	500	5.0	1,500	5.5
750	5.5	750	5.5	2,000	5.5
1,000*	5.5	1,000*	5.5	2,500	5.5
				3,000	5.5
1,500	5.5	1,500	5.5	3,750	5.5
				5,000	5.5
				6,000	5.5
				7,500	5.5

* In some cases it will be advantageous to specify 8.0 per cent impedance for these ratings. See Table I for their application.

a cost standpoint 50,000-kva metal-clad switchgear and fused starters are about equal, while for the higher interrupting ratings the first cost of fused starters is less.

On the other hand, when fuses are applied, there is always the possibility of single-phase motor operation due to the blowing of one fuse. Also, most large motors for station auxiliary drives do not require frequent starting and stopping. Further, most installations of this size will have one or more motors larger than 700 horsepower. It has thus been general practice to use circuit breakers for controlling the larger motors, and in the transformer, bus tie, and feeder circuits.

Transformers

Three-phase transformers are used almost exclusively in both the single- and dual-voltage auxiliary-power systems. Oil-filled or Pyranol-filled transformers for outdoor use and Pyranol-filled or dry-type transformers for indoor use are available. Transformers having ratings and characteristics as indicated in Table II may be obtained and are regularly provided with manually adjustable taps.

Unit Substations

The metal-clad switchgear equipments, using vertical-lift Magneblast breakers, and their transformers are available in unit equipments as shown in Figure 10 and are known as master unit substations.

The metal-enclosed drawout-air-circuit-breaker equipments and their transformers are generally furnished in unit equipments as shown in Figure 11 and are known as load-center unit substations. Cabinetrol equipments and transformers also can be furnished in unit equipments known as Cabinetrol unit substations.

Power-System Arrangements

Figures 5 to 9 suggest several auxiliary-power-system arrangements, and there are obviously many more which could be shown. Most of these arrangements show the auxiliary power coming from the main-generator leads primarily because of its increased use.

Figure 5 applies to a plant with one or two boilers, where all the auxiliary power may be properly supplied at 460 volts. Individual feeder positions control the larger motors, and others supply one or more lighting circuits, as well as one or more groups of magnetic starters which control the smaller individual motors. Transformer secondary breakers are used so that transfer from the starting to running supply may be readily made. Interrupter switches, where shown, permit isolating the transformers for maintenance purposes.

Table III

Interrupting Rating in Kva	Ampere Rating
50,000	600 1,200
100,000	600 1,200 2,000
150,000	600 1,200 2,000

Figure 6 applies particularly for a small unit having two boilers in which the auxiliary power is split into a supply for each boiler. This arrangement reduces the short-circuit duty on the low-voltage busses and prevents outages on one boiler system from involving the other boiler's system. One transformer is shown as a starting supply for either bus, but a separate starting transformer for each bus will give complete reserve capacity.

Figure 7 applies to plants using either one or two boilers, where the amount of auxiliary power is sufficient to justify the use of two voltages. The high-voltage transformer-interrupter switches and secondary breakers at 2,400 volts serve functions similar to those indicated on Figure 5. The larger motors are supplied at 2,400 volts, and one or more 2,400-volt feeder positions supply transformers which step down a portion of the total power to 460 volts. Bus ties are not shown between the 460-volt busses, and thus transformer secondary breakers are not required at 460 volts. When reserve transformer capacity to the 460-volt busses is considered desirable, transformer-secondary and bus-tie breakers may be added. In order to keep within the interrupting rating of the 600-volt air circuit breakers it is sometimes advantageous to supply two or more transformers from one 2,400-volt feeder. If bus-tie breakers are used in this arrangement, they should be used between 460-volt busses which are fed from different 2,400-volt feeders.

Table IV

Voltage	Interrupting Rating—Kva	Horsepower Rating
2,400	150,000	50–700
4,160	250,000	50–1,250

Figure 8 applies particularly for the larger units having two boilers. Dividing the power into a supply for each boiler reduces short-circuit duties and prevents outages on one boiler system from involving the other boiler's system. One transformer is shown as a starting supply for either 2,400-volt bus, but one for each bus would give 100 per cent reserve capacity.

Figure 9 shows how a power supply for auxiliaries can be taken directly from the low-voltage station bus when such a bus is available.

Table V

Interrupting Amperes	Continuous Ampere Rating	Maximum Motor Horsepower at 440 Volts	Maximum* Setting of Instantaneou Trip—Amperes
15,000	15–225	150	3,375
25,000	15–225	150	3,375
25,000	250–400	275	6,000
25,000	450–600	400	9,000
50,000	100–225	150	3,375
50,000	250–600	400	9,000
50,000	800–1,200	800	18,000
50,000	1,600		24,000
75,000	2,000–2,500		30,000
75,000	3,000		36,000
100,000	4,000		48,000

* The maximum setting of the nonadjustable in
stantaneous short-circuit trip on the air circu
breakers is important in determining the rating
the transformer-secondary breaker, so that it wi
not trip for feeder faults.

For the larger stations, it may be de
sirable to establish a separate centrall
located bus from which the general statio
service power is distributed. The s
called starting transformer can then
used to bring power from the system t
this bus which in turn can also be a
ranged to provide starting power for t
individual unit busses.

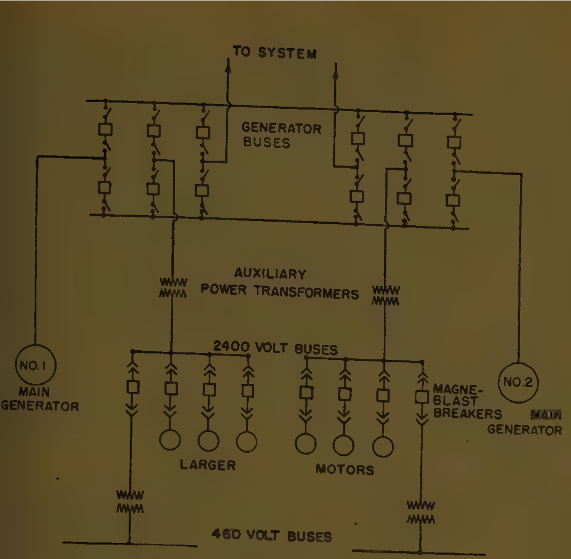
Single-Voltage Systems

Studies show that, so far as investme
is concerned, on the basis of the typic
number of feeder positions usually r
quired by the smaller turbine-generat
units, the straight 460-volt system and t
2,400–460 dual-voltage system are pra
tically equal when the normal pow
supply demands a 1,500-kva transforme
Below this rating there is a definite co
advantage in favor of the straight 46
volt system. In fact, if the auxilia
power supply can be held within the capa
ity of a 1,000-kva transformer, the sho
circuit duty on such a 460-volt auxilia
system will be limited accordingly, and t
440-volt motors and control for the larg
auxiliaries associated with this amount
total auxiliary power, will not result in
impractical solution.

Dual-Voltage Systems

As previously indicated, studies sh
that a dual-voltage system should be co
sidered when the auxiliary-power supp
requires a 1,500-kva or larger transforme
In most of these applications, 2,400 vol
will be more economical than 4,160 vol
Actually transformer capacities up to a
including 5,000 kva at 2,400 volts may
used with economical and practical swit
gear having an interrupting rating
100,000 kva.

A 5,000-kva transformer will supp
power to a boiler having a flow of appro
mately 600,000 pounds per hour. Boil
larger than this are most always provid
with split auxiliaries such as two induce



OTHERWISE SIMILAR TO FIGURE 7 OR 8

Figure 9. Dual-voltage system supplied from generator bus

draft fans, two forced-draft fans, and two or more boiler-feed pumps. So far as auxiliaries are concerned, such a boiler has the advantages of a two boiler installa-ion, and the 2,400-volt auxiliary power nay be divided as shown in Figure 8. Thus, by using divided auxiliaries and two auxiliary-power transformers rated either 3,750 or 5,000 kva, the power for most of the largest boilers may economi-cally be supplied at 2,400 volts.

For the larger installations, where indi-vidual transformer ratings greater than ,000 kva may be required, the use of ,160 volts should be considered. Sys-ems using 4,160 volts have the advantage f lower-current ratings and lower short-ircuit currents but may not necessarily how an advantage in cost owing to the igher cost of 4,160-volt motors. What-ver the relative economics of the motors nd control are, it should be recognized hat raising the voltage always reduces he short-circuit amperes, and thus at ,160 volts the kilovolt-ampere capacity of he supply transformer can be larger than t 2,300 volts.

In the case of 2,400–460 dual-voltage ystems, studies indicate that, by evalua-ion of such cost items as difference in ontrol equipment, cable, motors, and dditional transformer capacity including valuation of losses, it is economical for ne short runs normally encountered in a team station to include motors up to 250 orsepower in the 460-volt class. Simi-rly, in the case of 4,160–460 dual-oltage systems, it is generally economical include motors up to 350 horsepower in ne 460-volt class.

It will be seen from Figure 1 that in the ormal auxiliary-power system the few otors associated with the larger auxil-ries account for two thirds to three uarters of the total auxiliary power. hus, when a complete list of the motors d their ratings is available, there should little difficulty in selecting not only the otors which are to be supplied at 460 lts but also the total transformer capacity required to serve these motors.

In plants where medium-size boilers are used, this one third of the total kilovolt-ampere is often stepped down from 2,400 volts to 460 volts through one transformer on a radial basis. Where larger boilers are employed, two or more transformers may be used on a radial basis to step down power from the 2,400-volt bus. This has the advantage of permitting the use of control equipment of lower interrupting rating and of allowing the control to be located nearer individual groups of motors.

When a strictly radial system is not considered adequate, the first departure will consist usually of adding a tie breaker between 460-volt busses. In order to take full advantage of this feature, excess transformer capacity must be provided. The magnitude of this excess capacity admittedly is a matter of judgment, however, and 50-per-cent excess capacity should be sufficient in most cases. Provision should be made to prevent closing the bus-tie breaker between any pair of busses unless one of the two source breakers is open, or at least will be opened promptly after the bus-tie breaker is closed.

In a plant with two boilers per generating unit, with a separate auxiliary-power supply for each boiler, it will simplify operation to adhere strictly to the unit-system concept in the auxiliary-power supply and thus avoid such complications as providing bus ties between 460-volt busses which normally supply auxiliaries for different boilers.

Relaying Selectivity

For essential types of service such as power-station auxiliary drives, selective operation of feeder breakers and transformer-secondary breakers is desirable during fault conditions on the feeders. When a transformer-secondary, bus-tie, or group feeder breaker is to be selective with the feeder breakers it must not trip instantaneously. In this way the feeder breaker is given an opportunity to trip and clear the fault, and therefore it must have an interrupting capacity at least equal to the available fault. Low-voltage air circuit breakers are all equipped with instantaneous trips, and thus any of these breakers in backup positions must be of sufficient rating so that their instantaneous trips will not cause breaker operation on feeder faults.

Power circuit breakers on motor circuits usually are equipped with instantaneous over-current relays for fault protection; thus, the breakers in backup positions may be timed to be selective with respect to motor breakers and other feeder breakers.

Table I summarizes the present breaker ratings which can be used with the transformer ratings listed in Table II and can be expected to give selective operation of 600-volt air circuit breakers during short-circuit conditions. The maximum setting of the nonadjustable instantaneous short-circuit trip on low-voltage air breakers is given in Table V. For convenience the ratings of 5,000-volt breakers for larger transformers also are included in Table I. The higher system short-circuit duties, encountered in this type of service, have been used in selecting these ratings.

Cabinetrol equipments as normally applied are not selective with the group feeder breakers over a wide range of ratings. This will not be serious if all essential motors, regardless of how small they may be, are supplied from their individual 600-volt air circuit breakers. In some cases, small essential motors will require contactors because of the frequency of operation of their control. If such auxiliaries are in duplicate, they may be supplied from two different Cabinetrol groups and in this way still meet the service reliability requirements.

Selectivity of Cabinetrol as normally applied may be expected with certain com-

Table VI

NEMA Size	Interrupting Element	Interrupting Rating—Amperes	Maximum Horsepower at 440 Volts
1	Fuse*	50,000	7½
2	Fuse*	50,000	25
3	Fuse*	50,000	50
1	Thermal air circuit breakers†	15,000	7½
2		15,000	25
3		15,000	50
4		15,000	100
5	Air circuit breaker	25,000	200

*The fusible-combination magnetic starters, when equipped with nonrenewable nonindicating National Electric Code fuses, are adequate for installing in circuits where the available symmetrical short-circuit current at the incoming-line terminals of the starter does not exceed 50,000 amperes and the short-circuit is at or beyond the load terminals of the starter.

† Where these combination magnetic starters with thermal air circuit breakers are supplied through a 25,000-ampere 460-volt air circuit breaker, the starters may be applied on a bus having an available short-circuit duty of 25,000 amperes.

bination ratings. Fused Cabinetrol equipment at 460 volts up to and including size II will, in general, be selective with a 25,000-ampere group feeder breaker. For larger-fused Cabinetrol units and for all Cabinetrol units using thermal air breakers, a 25,000-ampere group feeder breaker can be expected to trip during a short-circuit.

However, when selectivity of Cabinetrol in a wider range of ratings is desirable, it may be obtained either by choosing a group feeder breaker with a sufficiently high instantaneous setting or by using a series feeder reactor. Either of these approaches increases the spread between the available short-circuit current and the maximum instantaneous trip setting for the feeder breaker.

Protective Features

Instantaneous short-circuit protection is provided on all types of control equipment. Combination magnetic starters are equipped with either fuses or thermal air circuit breakers, and low-voltage air circuit breakers are equipped with instantaneous trips. Instantaneous overcurrent relays are also used on the Magneblast air breakers.

Inverse time-delay short-circuit protection should be normally provided as backup protection on transformer-secondary, bus-tie, and transformer-feeder circuit breakers. When these breakers are of the 600-volt type, this feature preferably should be obtained by using separate relays with current transformers. On Magneblast breakers separate relays must be used.

Thermal devices are available in all the types of equipments discussed, and their function is to provide inexpensive overcurrent protection for motors against stalled rotor and single-phase operation or prolonged overloads. They may be arranged either to trip the motor circuit or to sound an alarm, and the opinions of operators are not uniform on this subject.

When the thermal device is arranged for tripping, it opens the coil circuit on magnetic starters; it acts directly on the trip element of low-voltage air circuit breakers and on the larger breakers is used in conjunction with current transformers to energize the breaker shunt trip coil. In applications where a special thermal characteristic must be met, a separate thermal relay, energized from current transformers can be used to provide the trip function on any of the equipments.

When alarm features only are desired, the thermal devices in all the equipments discussed can be arranged to sound an alarm instead of opening the motor circuit. Since an alarm is of no use except where there is time to profit by it, this feature is of advantage only during overloads and not during stalled rotor conditions and probably may be tolerable only on the smaller inexpensive motors.

A combination feature which gives alarm on overload and which trips on stalled rotor conditions is available. This consists of a thermal relay, energized from current transformers, and an overcurrent relay, which transfers the thermal relay from the alarm to the trip circuit upon the occurrence of excess current.

Undervoltage time-delay protective features are sometimes desirable. A voltage dip which drops out contactors and does not return to the required pickup value in a few seconds may damage the a-c operating coils of contactors equipped with undervoltage release, and protection against such a voltage dip may be arranged to trip any of the feeder circuits having devices of this type.

Differential protection of the main generator should be extended to include the normal auxiliary-power transformer. Generally, the ratios are not sufficient to allow the main differential relaying to detect faults in the auxiliary transformer; therefore an additional relay system should be provided for the auxiliary transformer to trip the generator breaker, field breaker, auxiliary-transformer-secondary breaker, and, in some cases, the turbine stop valve. In addition, automatic throwover may be provided to energize the auxiliary bus from the starting supply whenever differential relays trip off the main source. This allows the boiler to be brought down under full control of its auxiliaries.

Figure 10. Typical master unit substation

It is common practice to operate 460-volt and 2,400-volt auxiliary-power systems ungrounded but with ground indication facilities.

Lighting

Station lighting can be economically taken from the 460-volt busses through one or more three-phase feeder positions. Single-phase dry-type transformers, of proper rating and number to balance the feeder load, may be located about th station to supply the individual lightin circuits. By interspersing these circuit the loss of one feeder for a short time wi not be serious.

It is customary to provide an emergenc source of lights from a battery. This ma be done by arranging the battery fo automatic throwover to a fraction of th circuits which are again interspersed o by having an entirely separate emergenc lighting circuit distributed over the area.

General Station Service and Coal Handling

Certain items, which are not directl associated with any one unit in the st tion, require power. These include coa handling equipment, yard floodlightin service water, machine shop, and crane and the amount of power required wi vary widely for different plants. Th power may be supplied from the auxiliar bus for the first unit in the plant, in whic case provision should be made for a alternate source when additional uni are added, or for the larger stations from separate centrally located bus. It will advantageous to supply this power fro the higher auxiliary voltage since lo cable runs are often involved.

Calculation of Short-Circuit Currents

In preparing the tabulated applicati guides, conventional methods of calcula ing short-circuit kilovoltamperes and cu rents were used.

The interrupting rating of Magnebla air circuit breakers is customarily given kilovoltamperes at a certain voltage. T relay and interrupting times for the breakers are such that induction-mot contributions and offset currents may neglected. Thus, the short-circuit ki voltampere is determined directly fr transformer reactance and system rea ance beyond. In these applications t ratio of transformer reactance to syste reactance is such that the system may well be considered infinite. The mome tary duty in these cases is obtained multiplying the short-circuit kilovo amperes by 1.4 and also adding the ind tion-motor contribution.

The interrupting rating of low-volta air circuit breakers is the rms curre averaged for the three phases and det mined at the first half cycle, which is time at which contacts part. This av age value is determined by applying multiplying factor of 1.25 to the sy metrical contribution through the tra former and then adding the contribut of induction- and synchronous-mo loads. The motor contribution is custo arily taken into account by adding times the connected motor load, which these applications, will be approximat

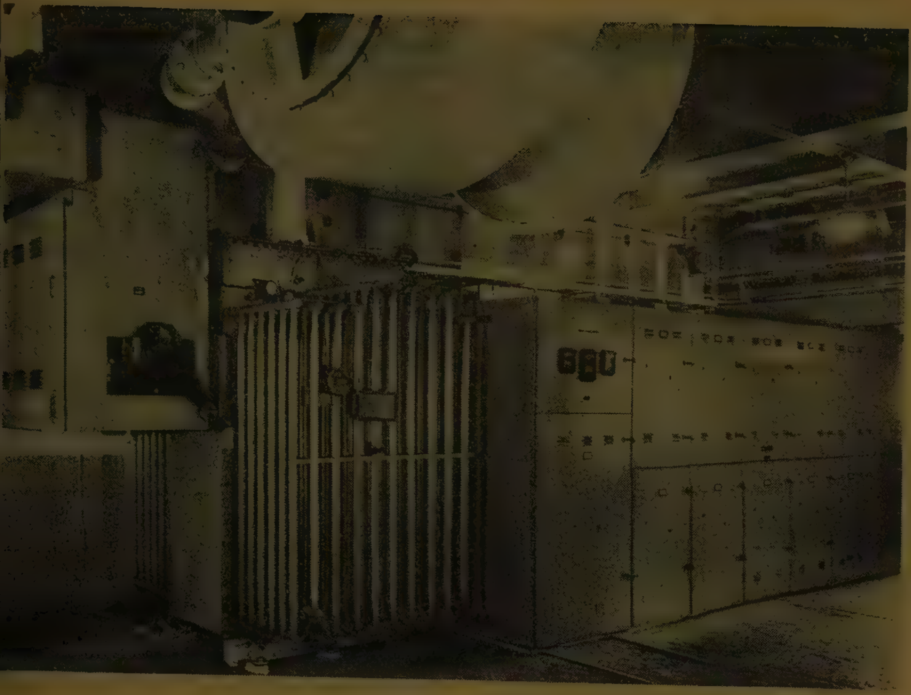


Figure 11. Typical load-center unit substation

five times the transformer rating. The momentary duty for this class of breaker is considered to be equal to the interrupting duty.

The interrupting rating of 600-volt fusible-combination magnetic starters is given in amperes based on the available symmetrical short-circuit amperes. This value may be obtained in a similar manner to that for low-voltage air circuit breakers, except omitting the 1.25 factor.

Location of Equipment

In steam-electric stations the major part of the power is used at or near the boiler. There is usually space available n the bay between the boiler plant and he turbine room where the auxiliary-power-system equipment can be advanageously located. This location should esult in short power and control cable runs.

In the smaller plant where the power is supplied entirely at 460 volts, the complete switchgear and associated stepdown transformer may well be at one centralized location. The Cabinetrol may be at a remote location near a group of small motors which it serves.

In the larger plant where a dual-voltage system is used, the 2,400-volt switchgear logically should be at a central location in the plant. The 460-volt load-center units and Cabinetrol panels may be at remote locations near the load they serve. If the normal starting transformer is of moderate rating, consideration should be given to placing it at the centralized location, where it can be combined readily with the 2,400-volt switchgear into a unit substation. When the normal transformer is in the larger ratings, it will be located outdoors in most cases. Similarly, the starting transformer will be located outdoors, since it generally will be energized from the high-voltage bus, and in this way long high-voltage cable runs may be avoided.

Appendix

Transformer Ratings

Table II lists the standard transformer ratings which will be used normally for this type of service.

5,000-Volt Switchgear Equipments

Metal-clad switchgear, with electrically operated removable oilless circuit breakers, is available in the ratings shown in Table III.

Combination High-Voltage Fused Contactors

Metal-enclosed contactors with high-interrupting-current limiting fuses and with air or oil-filled contactors are available in the ratings shown in Table IV.

600-Volt Switchgear Equipments

Metal-enclosed low-voltage switchgear, with electrically operated drawout air circuit breakers have the ratings shown in Table V.

Combination 600-Volt Magnetic Starters

Combination magnetic starters with either fusible disconnects or thermal air circuit breakers are available for use as low-voltage motor starters. These starters are available in the ratings shown in Table VI.

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Synopsis: This is a study of high-voltage underground-cable systems, consisting of the cable line and the terminal equipment at each of its ends. The study undertaken indicates the economic zone of application of cables and attempts to find an economic relationship between the voltage and the block of power transmitted over distances up to 15 miles. Thirty transmission lines have been designed using both the solid and the oil-filled cables. The lines can be considered as either transmission links with a unidirectional power flow or as tie lines between large metropolitan steam-power stations. The prices of underground cable and terminal equipment were obtained from manufacturers, and refer to the 12-month period immediately preceding World War II. The costs of various system parts, as installed on foundations, were obtained from the study of actual construction projects in different parts of the country and reduced to the 12-month period. The system capital cost and its annual cost are analyzed separately. The quantities of major equipment items in each system are given, and costs of major equipment groups, or of major items, are represented on a percentage basis. Such a representation permits the estimating of the cost of an underground line, or of the entire transmission project, when copper prices and the cost of equipment differ from those taken in the study.

THE RECORDS of engineering thought and development as reflected in the technical literature show that some 20 years ago there was displayed in the United States a great interest in the newly developed oil-filled cable. At that time 138-kv oil-filled cable was being installed in the United States and abroad. Since then extensive installations of this cable have been made, and the art has taken several steps forward—not only by developing an oil-filled cable for 230 kv but by evolving the gas-pressure cable and the Oilostatic system. The most notable installation of the 230-kv cable took place in France around the year 1936 when the intra-Paris cable 11.6 miles long was installed.[1–3] Cables of the newer type have also been placed in a few initial installations and may be considered as having amply justified their development.

The economic phase of underground transmission systems, however, did not receive during these years the attention given to the technical end of this problem. The only outstanding record in the literature is the study made by Roper[4] in 1931. In view of this, therefore, it seems that an economic study of this problem may be in order.

This study covers underground power-transmission and tie lines using impregnated-paper-insulated cables of the so-called solid and oil-filled types. Cables of other designs, such as the Oilostatic system in which the cables are installed in a pipe and maintained under a 200-lb.-per-square-inch oil pressure, or the high-pressure gas cables installed in a pipe or drawn into ducts, have not been considered, as at the time of preparation c this paper cost data on both systems wer not available. It has been stated, however, that their over-all costs compar very favorably with those of an oil-fille system, showing costs in some case smaller and in other cases slightly highe than those of an oil-filled system. It appears, therefore, that when a study is t be made of the cost of a projected underground power-transmission system consideration also must be given to thes systems.

No attempt has been made to compar the economics of the underground and th overhead lines of equal length and equa

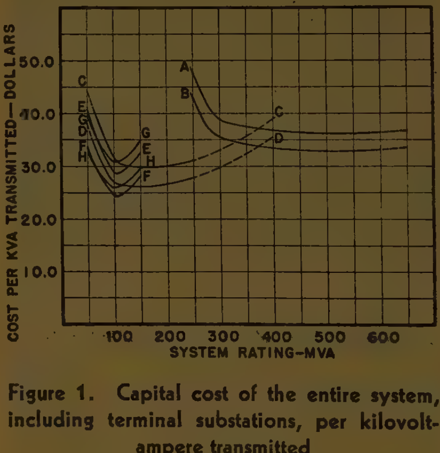

Figure 1. Capital cost of the entire system, including terminal substations, per kilovolt-ampere transmitted

A...15..230...Oil-filled cable
B...10..230...Oil-filled cable
C...15..138...Oil-filled cable
D...10..138...Oil-filled cable
E...15.. 69...Oil-filled cable
F...10.. 69...Oil-filled cable
G...15.. 69....Solid cable
H...10.. 69....Solid cable

The first figure represents transmission distance in miles; the second, kilovolts of transmission voltage

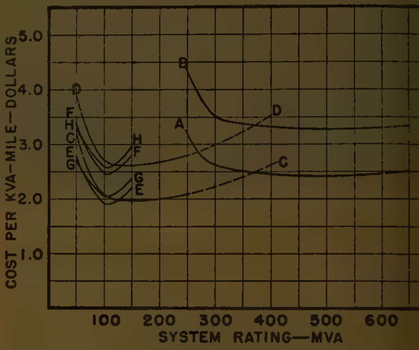

Figure 2. Capital cost of the entire syste including terminal substations, per kilovo ampere-mile

For explanation of identification letters, s Figure 1

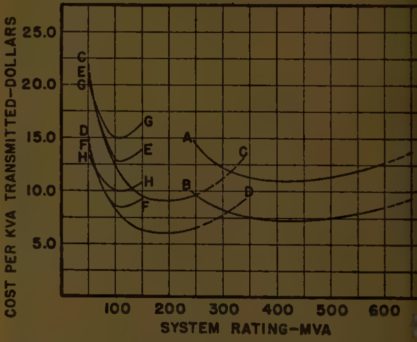

Figure 3. Capital cost of the transmission li per kilovolt-ampere transmitted

For explanation of identification letters, i Figure 1

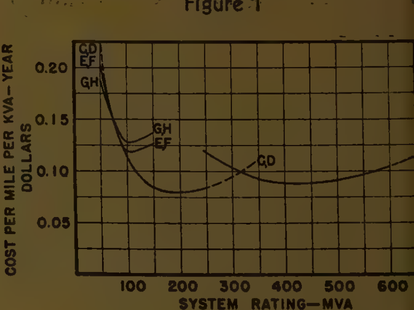

Figure 4. Capital cost of the transmission l per kilovolt-ampere-mile

For explanation of identification letters, Figure 1

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To the engineers of the General Electric Company, the Westinghouse Electric Corporation, the General Cable Corporation, the Okonite Company, the Simplex Wire and Cable Company, and to several electrical-public-utility and consulting engineers the author expresses appreciation for valuable data and advice.

block of power transmitted. The systems differ from those studied by Roper in that in all cases they have terminal substations at both ends; they are studied and analyzed, however, both as complete systems with terminal equipment included, and as underground cable lines alone. The indirect expenses, which seem to have been purposely left out of Roper's study, are included in this paper. The general structure of the present study is similar to that used by the writer in his study of the costs of overhead power-transmission systems.[5] Therefore only an outline of the approach to the problem will be given here, and for a detailed method of analysis the reader is referred to similar parts of the other paper.

The underground systems investigated in this paper have no definite geographic location. The prices taken are those prevailing in the years preceding World War II. The prices of equipment are obtained from the manufacturers, and the cost of land, structures, substation secondary equipment, labor, and installation on foundation were obtained from various public-utility companies and from the literature. All these data were reduced to the common one-year period, with appropriate labor indexes being used. Irrespective of the variations in costs of particular items, the relationship between the costs of individual systems will remain basically unchanged. The study of the transmission of various blocks of power at two different voltages permits one to obtain the relationship of the voltage, the transmission distance, and the most economical block of power transmitted. It shows the general trend of high-voltage underground-system costs and defines the economic zone of application of the various cables studied.

The system capital and annual costs are calculated and analyzed separately.

System Description

The study covers 30 systems rated at 50,000 to 600,000 kva and transmitting power over one to four circuits of underground lines 10 and 15 miles long, at voltages of 69, 138 and 230 kv, at 60-cycle frequency. The systems are lines in metropolitan areas transmitting from steam-power stations, or lines tying in two great metropolitan load centers and permitting power to be transmitted in both directions. The transformer kilovolt-ampere capacity is assumed to be slightly greater at the sending end of the system.

The 69-kv cables are of both solid and oil-filled type, and all the 138-kv and 230-kv cables are oil filled, all cables being single-conductor cables. The underground lines are connected to high-voltage busses at both ends, and at no intermediate point are they connected to overhead transmission lines. Since the lines pass through metropolitan areas, there is no right-of-way problem, the assumption being made that permission to dig the trenches will be obtained from the municipalities.

The system and cable data are given in Table I.

The systems as taken in this study include the underground line, completed with all accessories, and the sending and the receiving substations. All systems have high-voltage busses and high-voltage circuit breakers at both ends of the system. All single-circuit systems have a single nonsplit high-voltage bus. The 149,700-kva-rated systems, with two circuits, have a single bus, split in two. For systems rated from 247,500 to 600,000 kva, double busses are provided; these busses are split into as many sections as there are circuits in the system. The load assumed on the system for the purpose of calculating the circuit-breaker interrupting capacity is at both system ends. The total load is the function of the block of power transmitted,[5] 60 per cent of the total being placed at the system receiving end, and the remaining 40 per cent at its sending end.

The underground lines have standard relay protection. Lines with solid cable

Table I. List of Systems Designed—Type and Size of Cables

	System Data			Cable Data					
System Identification Number	System Rating, Kva	Transmission Distance, Miles	Transmission Voltage, Kv	Type of Cable	Cross Section, Thousand Circular Mils	Number of Circuits in System	Cable Insulation Thickness, Inch	Thickness of Lead Sheath, Inch	Outside Diameter of Cable Over Lead Sheath, Inches
U-33*a*	54,200	10	69	Solid	600	1	0.656	0.125	2.452
U-331*a*	52,800	10	69	Oil-filled	350	1	0.315	0.125	1.790
U-34*a*	65,200	10	138	Oil-filled	168	1	0.560	0.141	2.165
U-35*a*	54,200	15	69	Solid	600	1	0.656	0.125	2.452
U-351*a*	52,800	15	69	Oil-filled	350	1	0.315	0.125	1.790
U-36*a*	65,200	15	138	Oil-filled	168	1	0.560	0.141	2.165
U-361*a*	75,300	10	69	Solid	1,000	1	0.656	0.125	2.872
U-362*a*	75,300	15	69	Solid	1,000	1	0.656	0.125	2.872
U-363*a*	76,900	10	69	Oil-filled	700	1	0.315	0.125	2.014
U-364*a*	76,900	15	69	Oil-filled	700	1	0.315	0.125	2.014
U-37*a*	103,600	10	69	Solid	1,750	1	0.656	0.125	3.482
U-371*a*	106,000	10	69	Oil-filled	1,250	1	0.315	0.125	2.300
U-38*a*	103,600	10	138	Oil-filled	400	1	0.560	0.141	2.342
U-39*a*	103,600	15	69	Solid	1,750	1	0.656	0.125	3.482
U-391*a*	106,000	15	69	Oil-filled	1,250	1	0.315	0.125	2.300
U-40*a*	103,600	15	138	Oil-filled	400	1	0.560	0.141	2.342
U-401*a*	149,700	10	69	Solid	1,250	2	0.656	0.125	3.052
U-402*a*	149,700	15	69	Solid	1,250	2	0.656	0.125	3.052
U-403*a*	149,700	10	69	Oil-filled	800	2	0.315	0.125	2.070
U-404*a*	149,700	15	69	Oil-filled	800	2	0.315	0.125	2.070
U-405*a*	160,000	10	138	Oil-filled	900	1	0.560	0.141	2.646
U-406*a*	160,000	15	138	Oil-filled	900	1	0.560	0.141	2.646
U-41*a*	255,000	10	138	Oil-filled	750	2	0.560	0.141	2.562
U-42*a*	247,500	10	230	Oil-filled	400	2	0.925	0.156	3.234
U-43*a*	255,000	15	138	Oil-filled	750	2	0.560	0.141	2.562
U-44*a*	247,500	15	230	Oil-filled	400	2	0.925	0.156	3.234
U-441*a*	378,000	10	230	Oil-filled	1,000	2	0.925	0.156	3.556
U-442*a*	378,000	15	230	Oil-filled	1,000	2	0.925	0.156	3.556
U-45*a*	600,000	10	230	Oil-filled	600	4*	0.925	0.156	3.352
U-46*a*	600,000	15	230	Oil-filled	600	4*	0.925	0.156	3.352

The system identification number is kept throughout the entire study.

The 69-kv and 138-kv oil-filled cables have an 0.500-inch hollow core, and the 230-kv cables have an 0.690-inch core.

*Systems *U*-45*a* and *U*-46*a* have two duct banks, two circuits per duct bank.

Table II. System Major Equipment Items

System Identification Number	Sending-End Transformers—Total System Kva	Receiving-End Transformers—Total System Kva	High-Voltage Oil Circuit Breakers—Number of Breakers in System and Their Interrupting Capacity*	High-Voltage Air-Break Disconnectors—Total Number in System and Their Ampere Rating†
U-33*a*	67,900	65,800	6–500	12–600
U-331*a*	66,733	64,050	6–500	12–600
U-34*a*	81,200	78,400	6–750	12–600
U-35*a*	68,367	65,800	6–500	12–600
U-351*a*	67,550	64,050	6–500	12–600
U-36*a*	81,783	78,400	6–750	12–600
U-361*a*	94,033	91,233	6–1,000	12–600
U-362*a*	94,617	91,233	6–1,000	12–600
U-363*a*	96,600	93,217	6–1,000	12–600
U-364*a*	97,417	93,217	6–1,000	12–600
U-37*a*	129,150	125,533	6–1,000	12–600
U-371*a*	132,533	128,333	6–1,000	12–600
U-38*a*	127,983	124,250	6–1,500	12–600
U-39*a*	129,733	125,533	6–1,000	12–600
U-391*a*	133,350	128,333	6–1,000	12–600
U-40*a*	128,683	124,250	6–1,500	12–600
U-401*a*	200,000	194,500	14–1,500	28–600
U-402*a*	200,875	194,500	14–1,500	28–600
U-403*a*	201,125	194,500	14–1,500	28–600
U-404*a*	202,500	194,500	14–1,500	28–600
U-405*a*	211,125	205,625	10–1,500	20–600
U-406*a*	211,750	205,625	10–1,500	20–600
U-41*a*	336,500	327,750	20–2,500; 4–1,500	48–600
U-42*a*	321,375	314,500	24–2,500	48–1,200
U-43*a*	337,625	327,750	20–2,500; 4–1,500	48–600
U-44*a*	322,375	314,500	24–2,500	48–1,200
U-441*a*	392,700	384,500	20–3,500; 4–2,500	48–1,200
U-442*a*	393,400	384,500	20–3,500; 4–2,500	48–1,200
U-45*a*	624,000	611,000	24–5,000; 12–3,500	72–1,200
U-46*a*	625,500	611,000	24–5,000; 12–3,500	72–1,200

Sending-end and receiving-end transformer kilovolt-amperes include spare transformer capacity as follows: Systems *U*-33*a* to *U*-40*a*, inclusive, have one single-phase spare unit; systems *U*-401*a* to *U*-44*a*, inclusive, have a spare three-phase bank. Systems *U*-441*a* to *U*-46*a*, inclusive, have no spare kilovolt-ampere capacity, their transformers permitting a 25-per-cent overload for two hours (Overload accounted for in system cost).

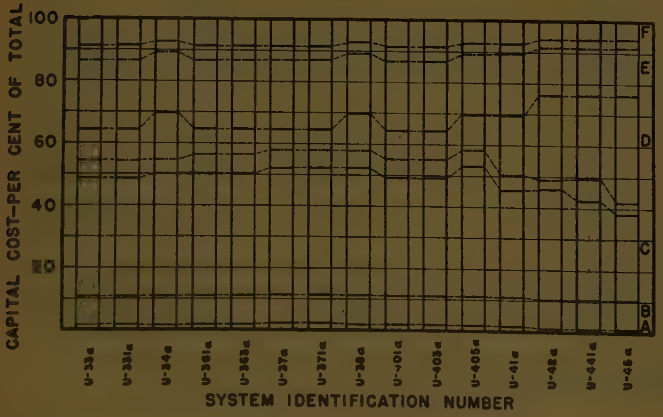
*First figure indicates number of three-phase oil circuit breakers; second figure indicates breaker interrupting capacity in megavolt-amperes.

†First figure indicates number of three-phase air-break disconnectors; second figure indicates disconnector current rating.

Figure 5. Division of capital costs of sending and receiving substations into their major component parts, in transmission systems ten miles long

A—Land
B—Structures
C—Power transformers
D—High-voltage switching and protective equipment
E—Low-voltage switching, protective and control equipment
F—Auxiliary substation equipment

Dash-and-dot lines represent those parts of the receiving-end substation which differ substantially from the sending-end percentages (*C* and *E*)



are provided with a relay cable (pilot wire), and the oil-filled-cable lines have an oil-signal cable for each power-cable circuit. Two wires of the relay and the oil-signal cables are used for communication purposes.

The system major equipment items are given in Table II.

General Principles of System Design

Since the systems studied are parts of existing metropolitan systems, the position is taken that no other transmission systems in the vicinity depend on them for continuity of operation, but rather the reverse is the case. Therefore the underground lines are designed to have as many circuits as are necessary for the transmission of the specified block of power, no provision being made for spare circuits or spare single cables.

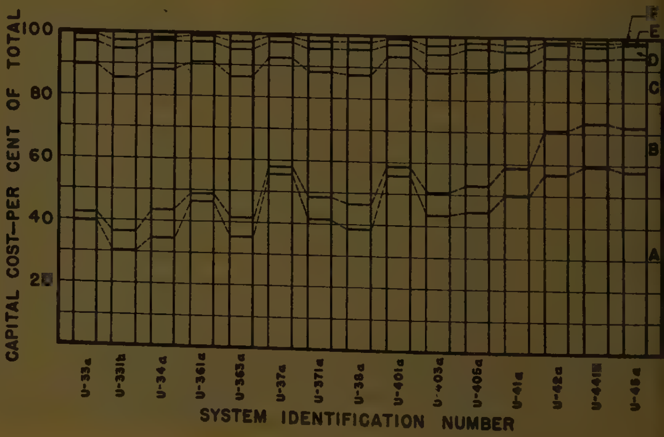
Voltages at which transmission is being effected are those which appear suitable for the particular block of power transmitted. No attempt is being made to secure the economical loading of cables for a certain cyclic load. However, the accepted annual load factor, loss factor relationship,[6,7] is used throughout the study. The selection of the cable copper cross section is made from temperature considerations, calculations being based on the cable summer rating. No credit is given for the charging current of cables.

A study of underground transmission systems shows that stability is no problem in these systems, neither steady-state nor transient stability. Therefore round-rotor machines of standard design are applicable, no stability-improving devices are required, and standard breakers are used of eight-cycle opening time. For this reason, as well as for voltage regulation purposes, no synchronous condensers are placed at the receiving end of the system. Voltage drop in an underground line being very small, voltage is assumed to be controlled by adjustment of the reactive power of the receiving-end steam-station generators, supplementing the plus–minus-ten-per-cent load-ratio control of the receiving transformers.

System rating is expressed in kilovolt-amperes and is equal to the rating of all cable circuits. The rating of terminal equipment installed is obtained by adding to the system rating all the losses in all parts of the power-transmission system, at appropriate power factors.

Figure 6. Division of capital cost of the transmission line into its component parts, in transmission systems ten miles long

A—Cables
B—Cable accessories
C—Duct banks
D—Manholes
E—Bonding
F—Equipment for communication and relaying



Underground-Line Design Data

In choosing the size of the underground cable required for the transmission of a certain block of power, the maximum allowable temperature of cable insulation is taken as the determining factor. This temperature is taken at 60 degrees centigrade for the solid cables, and at 70 degrees centigrade for the oil-filled cables, with the base temperature of the earth at 20 degrees centigrade. Losses and voltage drops are taken as obtained for these current–temperature conditions.

The selection of the 69-kv solid cables is made on the basis of the Insulated Power Cable Engineers' Association tables,[8] for cables laid in ducts, for 100-per-cent daily load factor. The kilovolt-ampere rating of these cables is taken for current-carrying capacities as given in these tables. The selection of the 69-kv, 138-kv, and 230-kv oil-filled cables is made from data obtainable in literature[9, 10] and from tables supplied by the cable manufacturers. In every case tables were used for cables laid in ducts, and where the cable winter rating is given the ratio of summer to winter rating was taken at 0.85. There being no standards for the current-carrying capacity of the oil-filled cables generally accepted by the industry, in the same sense as are accepted the IPCEA tables for solid cables, the current rating of these cables is taken as obtained by calculation for each case; and the kilovolt-ampere rating of the oil-filled cables is based on the permissible current calculated.

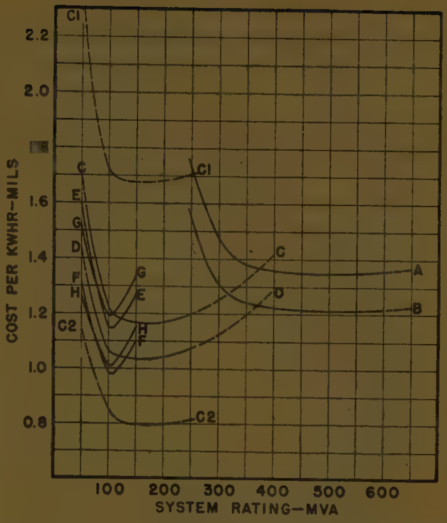


Figure 7. Cost of the entire system, including terminal substations, per kilowatt-hour

For explanation of identification letters, see Figure 1. All costs at 50 per cent annual load factor except as follows:

C1...15..138 at 35 per cent
C2...15..138 at 75 per cent
annual load factor

For the calculation of permissible current-carrying capacities, the insulatio thickness, thickness of lead sheath, an the dielectric power factor of cables wer taken as recommended by the Associatio of Edison Illuminating Companies[11] an as given by the manufacturers. The calculation of the cable outside diameter, o its geometric factor, of the thermal resistance to I^2R loss and to dielectric los between conductor and base, the calculation of current rating and of the cable three-phase dielectric loss were made a recommended by Simmons.[12, 13]

All cables have special bonding.[1 Sheath voltage per length of cable between manholes was calculated, and wherever this voltage at rated current loading exceeds 11 volts the series impedance bonding is used. Bonding transformers are installed in the manholes for the joints. The percentage residual sheath losses allowed after special bonding is 5 in 69-kv cables, ten in 138-kv and 20 in 230-kv cables.

In determining the number of normal joints and stop joints, the practice recommended by cable manufacturers and given in the literature[15] was followed. The length of a section for oil-filled cables is taken at one mile; lines are assumed to be of a substantially level profile. The distance between manholes for solid cables is 754 feet, and for oil-filled cables 660 feet. The pilot-wire and oil-signal cables are placed in separate ducts, so

Table III. Ducts, Manholes, and Cable Accessories

System Identification Number	Joints, Number in System		Ducts		Manholes, Number in System			Reservoirs—Type, Capacity, and Number in System*		
	Normal Joints	Stop Joints	Conduit Size, Inches	Number of Ducts in a Bank	For Normal Joints	For Stop-Joints	For Reservoirs	CC—Feeder or Gravity Type	AC—Gas-in-Cell Type	DC—Oil-in-Cell Type
U-33*a*	207		3.5	4	69					
U-331*a*	210	33	3.5	4	70	11	9	6–9	6–9	48–9
U-34*a*	210	33	3.5	4	70	11	9	6–15	6–14	48–15
U-35*a*	312		3.5	4	104					
U-351*a*	315	48	3.5	4	105	16	14	6–9	6–9	78–9
U-36*a*	315	48	3.5	4	105	16	14	6–15	6–14	78–15
U-361*a*	207		3.5	4	69					
U 362*a*	312		3.5	4	104					
U-363*a*	210	33	3.5	4	70	11	9	6–12	6–12	48–12
U-364*a*	315	48	3.5	4	105	16	14	6–12	6–12	78–12
U-37*a*	207		4.5	4	69					
U-371*a*	210	33	3.5	4	70	11	9	6–15	6–14	48–15
U-38*a*	210	33	3.5	4	70	11	9	6–18	6–14	48–15
U-39*a*	312		4.5	4	104					
U-391*a*	315	48	3.5	4	105	16	14	6–15	6–14	78–15
U-40*a*	315	48	3.5	4	105	16	14	6–18	6–14	78–15
U-401*a*	414		4.5	7	69					
U-402*a*	624		4.5	7	104					
U-403*a*	420	66	3.5	7	70	11	9	12–12	12–12	96–12
U-404*a*	630	96	3.5	7	105	16	14	12–12	12–12	156–12
U-405*a*	210	33	3.5	4	70	11	9	6–19	12–9	48–18
U-406*a*	315	48	3.5	4	105	16	14	6–19	12–9	78–18
U-41*a*	420	66	3.5	7	70	11	9	12–19	24–9	96–18
U-42*a*	420	66	4.5	7	70	11	9	24–18	36–12	192–15
U-43*a*	630	96	3.5	7	105	16	14	12–19	24–9	156–18
U-44*a*	630	96	4.5	7	105	16	14	24–18	36–12	312–15
U-441*a*	420	66	4.5	7	70	11	9	24–19	36–12	192–18
U-442*a*	630	96	4.5	7	105	16	14	24–19	36–12	312–18
U-45*a*	840	132	4.5	7	140	22	18	48–18	72–12	384–18
U-46*a*	1,260	192	4.5	7	210	32	28	48–18	72–12	624–18

Number of normal joints in 69-kv solid cables is seven per mile. In all the oil-filled cables seven normal joints and one stop joint per mile are used.

All manholes are of straight type, made of concrete. All manholes and duct banks paved with eight-inch concrete.

*First figure indicates number of reservoirs in system; second, the reservoir unit capacity in gallons.

that in each system a duct bank has one more duct than there are single-conductor cables. Separate manholes are provided for normal joints, for stop joints, and for the reservoirs. Three types of reservoirs are used—the *CC*, *AC*, and *DC* types.

The detailed data on ducts, manholes, and cable accessories are given in Table III.

Method of Estimating System Capital Cost

For the purpose of estimating the capital cost, the system is divided into the following three groups:

1. The sending-end substation.
2. The underground line.
3. The receiving-end substation.

The subdivisions of the groups are shown in Table IV, which gives a typical calculation of system capital cost.

The capital cost of each group is arrived at in the following manner:[5] The cost of all equipment (material) is obtained from the manufacturers. In cases where the cost of freight and delivery to site of erection is not included, certain percentages are added to account for it. On top of this, a percentage is added to cover costs of labor and installation on foundation; this percentage is determined from actual projects and varies with the type of equipment and voltage. In this fashion the costs are determined of structures, of high- and low-voltage switching, protective and control equipment, and of the auxiliary substation equipment. To the cost of equipment as installed on foundation, two more items are added: the indirect expenses and the cost of money during construction. To cover the indirect expenses, 22.5 per cent is added. The cost of money is taken at six per cent for one-half the length of the construction period, which is taken at six months for all the systems under study.

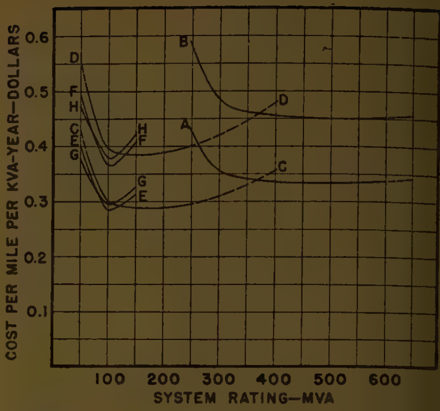


Figure 8. Cost of the entire system, including terminal substations, per mile per kilovolt-ampere-year at 50 per cent annual load factor

For explanation of identification letters, see Figure 1. Costs per mile per kilovolt-ampere-year at 35 and 75 per cent annual load factors differ very little from the cost at 50 per cent annual load factor

All prices are based on a cost of copper of $10^1/_8$ cents per pound of ingot copper, and on a cost of lead of 4.6 cents per pound, fob, New York. Cost of fabricated steel of shapes involved in substation structures is taken at $102 per ton, fob, Pittsburgh.

The kilovolt-ampere rating of power transformers at both ends of the system having been increased to cover appropriate losses in system parts, the losses thus become capitalized.

Discussion of System Capital Cost

Capital cost of the systems under study is calculated per kilovolt-ampere transmitted and per kilovolt-ampere-mile for the system as a whole, including terminal substations, and for the underground line alone. The total capital cost figures are given in Table V. The capital cost of the entire system is represented in Figures and 2, and that of the transmission line in Figures 3 and 4. The costs of major subdivisions of the costs of the substations and of the transmission line, in per cent of their total costs, are represented in Figures 5 and 6.

The results obtained show that there is a definite relationship between the block of power transmitted (system rating) and the transmission voltage. As the block of power transmitted increases, the cost of the entire system and of the transmission line per kilovolt-ampere and per kilovolt ampere-mile at first decreases; then, hav ing reached a certain minimum, it in creases again. The most economica block of power, therefore, is not the larg est that may be transmitted with a certai type of cable at a certain voltage.

At a rating under 65,000 kva the 69-k solid cable is unquestionably the mos economical. As the system rating in creases, the 69-kv oil-filled cable come

Table IV. Typical Calculation of Total System Capital Cost—System *U-41a*

In Dollars

		Cost of Item as Delivered (Material)	Cost of Item as Installed (on Foundation Where Needed) Including Labor Cost	Group Cost
1.	Sending substation			
1.1	Land	33,700	33,700	
1.2	Structures		206,500	
1.3	Power transformers	629,000	754,800	
1.4	High-voltage switching equipment			
	1.41 oil circuit breakers	271,600	325,900	
	1.42 air-break disconnectors	61,200	73,450	
	1.43 busses and supports		115,400	
1.5	Low-voltage switching, protective, and control equipment		500,000	
1.6	Auxiliary substation equipment		154,000	
			2,163,750	
	Indirect expenses		486,850	
	Cost of money during construction		39,800	2,690,400
2.	Line			
2.1	Underground cables*	669,000	669,000	
2.2	Cable accessories*			
	2.21 terminals	4,550	4,550	
	2.22 joints	64,100	64,100	
	2.23 reservoirs	48,150	48,150	
	2.24 oil	900	900	
2.3	Duct banks*		421,300	
2.4	Manholes*		68,400	
2.5	Bonding		32,400	
2.6	Equipment for communication and relaying	24,700	29,600	
			1,338,400	
	Indirect expenses		301,150	
	Cost of money during construction		24,550	1,664,100
3.	Receiving substation			
3.1	Land	33,700	33,700	
3.2	Structures		206,500	
3.3	Power transformers	727,000	872,400	
3.4	High-voltage switching equipment			
	3.41 oil circuit breakers	271,600	325,900	
	3.42 air-break disconnectors	61,200	73,450	
	3.43 busses and supports		115,400	
3.5	Low-voltage switching, protective, and control equipment		432,500	
3.6	Auxiliary substation equipment		154,000	
			2,213,850	
	Indirect expenses		498,100	
	Cost of money during construction		40,650	2,752,600
	Total system capital cost			7,107,100

System *U-41a* is rated at 255,000 kva. Power is transmitted over a distance of 10 miles, at a voltage of 138 kv, over two circuits of a 750,000-circular-mil oil-filled cable.

Construction period taken at six months. Cost of money during construction taken at six per cent for one-half construction period.

*Cost of duct banks includes the cost of laying cable and affixing terminals. Cost of manholes includes affixing joints and placing reservoirs.

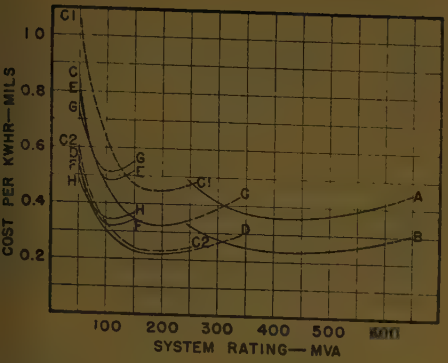


Figure 9. Cost of the transmission line per kilowatt-hour

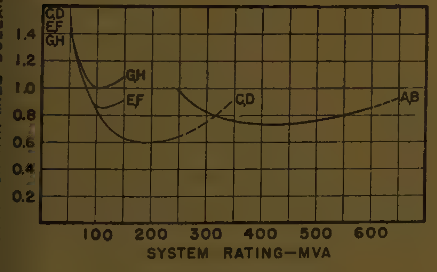
For explanation of identification letters, see Figure 1. All costs at 50 per cent annual load factor except as follows:

C1...15..138 at 35 per cent
C2...15..138 at 75 per cent
annual load factor

definitely into the picture. The over-all cost of the entire system, and of the underground line alone, for this cable is the lowest when somewhat over 100,000 kva is transmitted. With system ratings above 65,000 kva and up to almost 100,000 kva there is very little difference in the cost of the underground line alone whether a 69-kv oil-filled or a 138-kv cable is used; with still higher system ratings, a 138-kv oil-filled cable is very much more economical. When, however, the system as a whole is considered, the lower cost of the 69-kv terminal equipment extends the application of the 69-kv oil-filled cable up to ratings as high as 125,000 kva. For the 138-kv oil-filled cable the over-all cost of the entire system is the lowest when about 150,000 kva is transmitted; for the cable line alone the lowest cost is obtained with a transmission of about 200,000 kva. The different economical power limits in the two cases, when the costs of the entire system and of the transmission line alone are considered, is due to the high cost of the terminal substations compared with the cost of the underground line in systems of this voltage. The 138-kv oil-filled cable may be economically applied for transmission up to somewhat over 300,000 kva when the underground line alone is considered; and it may be used up to 375,000 kva when the entire system is evaluated, including the terminal substations. When still larger blocks of power are transmitted, then either a 230-kv oil-filled cable may be used, or the block of power should be split into a greater number of 138-kv oil-filled-cable circuits. Within the economical range of power quantities investigated, the cost of transmission with the 230-kv cable changes rather little. It is quite likely, however, that as the block of power exceeds 600,000 kva, the transmission costs will rise again, and either a different type of cable will have to be selected or an increased number of circuits will have to be used. The differences in the cost of transmission, as affected by the magnitude of the block of power, on the one hand, and the type and voltage of cable, on the other hand, are so great that a thorough study of any project is certainly warranted.

The division of the system capital cost into its three component groups, given in Table V, shows that in the great majority of cases the underground line is the largest cost item in the total cost, the percentage cost depending on the relationship between the total cost of the line and the cost of the terminal equipment required for the transmission of the block of power involved. The analysis of the costs of the substations, given in Figure 5, shows that the power transformers constitute the largest cost item of both the sending and the receiving substations in all the systems except the two rated at 600,000 kva, where the circuit-breaker interrupting capacity is so high that the cost of the high-voltage switching equipment exceeds that of the power transformers. The percentage division of the capital cost of the transmission line into its main component parts, given in Figure 6, indicates that in the systems rated at 106,000 kva and more the cost of the underground cables forms the largest item of the cost of the line. Only in systems of smaller rating the cost of the duct banks represents the largest of all items; this is due to the high proportion of labor costs in the duct banks. The cost of cable accessories in the 230-kv oil-filled cable lines is very much higher than that in the 69- and 138-kv cable systems.

The percentage division of the capital cost into its component parts makes it possible to evaluate the effect of changes in the cost of system parts or in the cost of basic materials, when it is desired to consider costs different from those employed in the paper. By multiplying the percentage change in the cost of a particular item, or, for example, in the price of copper, by the corresponding percentage given for that item in the figures, the de-



igure 10. Cost of the transmission line per ile per kilovolt-ampere-year at 50 per cent annual load factor

or explanation of identification letters, see igure 1. Costs per mile per kilovolt-ampere-ear at 35 and 75 per cent annual load factors iffer very little from the cost at 50 per cent annual load factor

Table V. Capital and Annual System Cost

In Thousands of Dollars

System Identification Number	Capital Cost				Total Annual System Cost at 50-Per-Cent Annual Load Factor
	Entire System	Sending Substation	Receiving Substation	Transmission Line	
U-33a	1,759	496	511	752	251
U-331a	1,725	489	504	732	255
U-34a	2,221	717	735	769	320
U-35a	2,138	498	513	1,127	301
U-351a	2,093	493	505	1,095	309
U-36a	2,613	721	736	1,156	364
U-361a	2,140	634	658	848	311
U-362a	2,567	638	659	1,270	367
U-363a	2,113	646	670	797	315
U-364a	2,516	650	673	1,193	373
U-37a	2,679	810	835	1,034	390
U-371a	2,584	827	852	905	388
U-38a	2,744	915	952	877	402
U-39a	3,195	812	836	1,547	457
U-391a	3,037	828	853	1,356	453
U-40a	3,185	919	954	1,312	463
U-401a	4,472	1,399	1,451	1,622	640
U-402a	5,307	1,404	1,453	2,450	748
U-403a	4,232	1,399	1,451	1,382	626
U-404a	4,928	1,406	1,454	2,068	708
U-405a	4,330	1,632	1,700	998	628
U-406a	4,827	1,634	1,701	1,492	697
U-41a	7,107	2,690	2,753	1,664	1,026
U-42a	10,805	4,164	4,197	2,444	1,457
U-43a	7,938	2,694	2,754	2,490	1,139
U-44a	12,052	4,189	4,207	3,656	1,613
U-441a	12,663	4,854	5,000	2,809	1,750
U-442a	14,096	4,878	5,013	4,205	1,928
U-45a	19,810	7,272	7,412	5,126	2,722
U-46a	22,379	7,287	7,418	7,674	3,046

The total annual system cost at 35- and 75-per-cent annual load factors closely approximates the cost at the 50-per-cent factor.

Table VI. Annual Fixed Charges and Annual Expenses for Maintenance and Operation on System Parts

In Per Cent of Their Capital Cost

	Annual Fixed Charges					Annual Expenses for Maintenance and Operation
Item	Taxes	Insurance	Depreciation and Obsolescence	Interest on Money Invested	Total	
Sending substation	2.0	0.5	1.823	6.0	10.323	2.0
Transmission line	2.0		1.823	6.0	9.823	1.3
Receiving substation	2.0	0.5	1.823	6.0	10.323	2.0

Charges for depreciation and obsolescence calculated by the sinking-fund method, at six per cent per annum money cost, for a 25-year life span of all system parts.

sired changes may be evaluated[5] and applied, on a percentage basis, to the calculated costs per kilovolt-ampere or kilovolt-ampere-mile. Changes in the system design, in the accepted permissible current rating for particular cables, or in the amount of equipment used, changes in the percentage allotted to cover the indirect expenses, or in the cost of money during construction, may be similarly introduced.

Method of Estimating System Annual Cost

The system annual cost is taken as the sum of the annual fixed charges and the annual operating expenses and is calculated for annual load factors of 35, 50, and 75 per cent. Both the annual fixed charges and the annual expenses for maintenance and operation are calculated as percentages of the capital cost separately for each of the three major groups into which the capital cost structure is divided. These are given in Table VI. The cost-of-energy loss, forming a separate part of the annual operating expenses, is taken equal to the sum of the cost of generation and the cost of transmission.

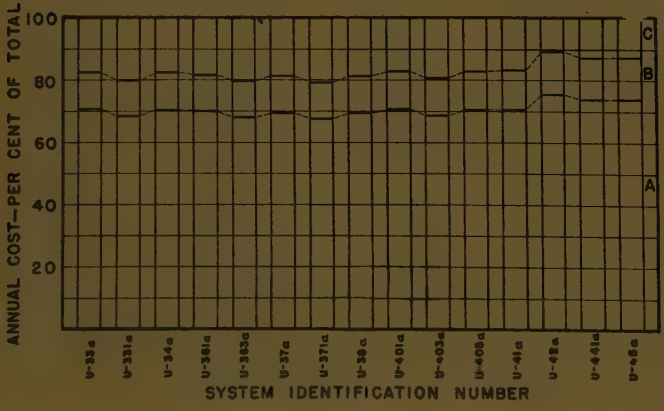


Figure 11. Division of annual cost of the entire system, including terminal substations, at 50 per cent annual load factor, in transmission systems ten miles long

A—Fixed charges
B—Expenses for maintenance and operation
C—Cost of energy loss
Division of annual cost at 35 and 75 per cent annual load factors differs very little from that at 50 per cent annual load factor

The capital cost of the steam-power station is taken at $100 per kilowatt for all systems designed. The annual fixed charges of the power plant are taken at the same rate as for the substations of the system, that is 10.323 per cent. The cost of maintenance and operation of the power plant consists of three parts: fixed operating costs, taken at $1.25 per kilowatt per year; variable maintenance costs, which at the annual load factors of 35, 50, and 75 per cent are taken correspondingly at $0.875, $1.25, and $1.85 per kilowatt; and operating supplies (labor) taken at 0.25 mills per kilowatt-hour. The cost of fuel at the annual load factors of 35, 50, and 75 per cent is taken correspondingly at 1.897, 1.795, and 1.70 mills per kilowatt-hour.

The losses in various system parts are given in Table VII. For simplification purposes, losses in the underground-cable line have been taken as fully dependent on load, although, strictly speaking, this is not entirely correct, especially in the 230-kv cables. It will be observed from the table that losses in the cable line represent the largest single item of the system losses. A careful design of the cable line is therefore exceedingly important.

The cost of energy loss per kilowatt-hour for the entire system is given in Table VIII. It is calculated for an 85-per-cent power-factor load.

Discussion of System Annual Cost

The total annual system cost at 50-per-cent annual load factor is given in Table V. The cost of the entire system per kilowatt-hour (at 85-per-cent power-factor load) and per mile per kilovol ampere-year is represented in Figures and 8. Corresponding costs for t underground transmission line are giv in Figures 9 and 10.

The shape of the annual-cost curves similar to that of the capital-cost curve because the annual fixed charges form t largest part of the total annual cost. view of this, identical conclusions may drawn from the annual-cost curves from the capital-cost curves.

The division of the annual cost into three major component parts at the 5 per-cent annual load factor is given Figure 11 for the entire system and Figure 12 for the underground line alo The fixed charges form by far the large percentage of the total annual cost. A changes which may take place in t capital cost may be introduced into t annual cost through the percentag which the fixed charges form of the to annual cost. Similarly, the effect changes which it may be desired to ma in the rate of taxes, in the insurance, in t rate of depreciation and obsolescence, in the interest on money invested may estimated easily.[5] When the main nance and operation costs, the cost of t energy loss, or the cost of generation other than those taken in the study, th effect on the annual cost of the ent system, or on the cost of the undergrou line alone, may be evaluated in h fashion.

Effect of Design Conditions a Economic Factors on System C

Within the scope of this paper it is i possible to estimate the effect which modification of the various design con tions adopted and of the economic fact assumed in the study would have on t

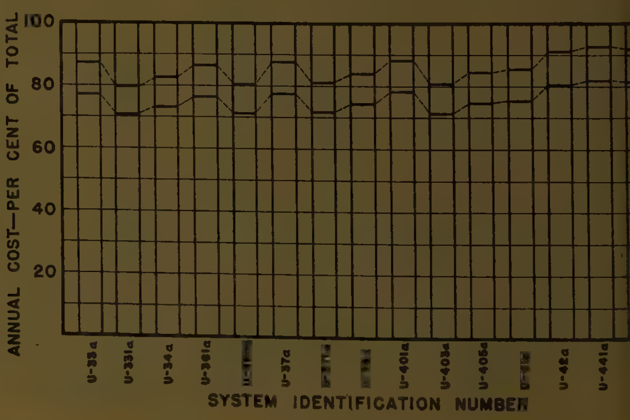


Figure 12. Division of annual cost of transmission line at 50 per cent annual l factor, in transmission lines ten miles lon

A—Fixed charges
B—Expenses for maintenance and opera
C—Cost of energy loss
Division of annual cost at 35 and 75 per c annual load factors differs very little from at 50 per cent annual load factor

cost of the system or on the cost of the transmission line.

Among the design conditions affecting the cost is the condition of service reliability. This could have been modified either by installing an extra single-conductor cable, or by requiring that systems carry full load with a loss of one circuit. The length of the oil-filled cable section and the number of joints and manholes assumed per mile could be changed in ther direction. The power factor of the ad could be taken other than the as-med 85-per-cent lagging power factor, ith its consequent effect on kilowatt-our cost. Self- instead of water-cooled ower transformers could be used, and eir spare capacity could be modified. inally, the system of high-voltage busses d their sectionalization, the number of gh-voltage circuit breakers and their terrupting capacity, calculated for a rtain assumed load, could be made mewhat different from those taken in e study.

Table VII. System Losses in Kilowatts

System dentification Number	Losses Independent of Load			Losses Dependent on Load			
	Sending-End Transformers	Receiving-End Transformers	Total Losses	Line	Sending-End Transformers	Receiving-End Transformers	Total Losses*
U-33a	194	177	371	723	291	264	1,378
U-331a	196	176	372	1,206	281	263	1,850
U-34a	230	215	445	1,040	344	322	1,836
U-35a	195	177	372	1,081	293	264	1,738
U-351a	193	176	369	1,810	289	263	2,462
U-36a	231	215	446	1,560	347	322	2,359
U-361a	269	251	520	871	402	376	1,799
U-362a	270	251	521	1,306	405	376	2,237
U-363a	275	255	530	1,292	413	384	2,239
U-364a	278	255	533	1,938	417	384	2,889
U-37a	358	345	703	955	536	517	2,208
U-371a	367	353	720	1,433	549	529	2,711
U-38a	361	341	702	1,115	541	512	2,368
U-39a	359	345	704	1,433	537	517	2,687
U-391a	368	353	721	2,150	552	529	3,431
U-40a	363	341	704	1,673	546	512	2,931
U-401a	533	498	1,031	1,384	799	747	3,230
U-402a	535	498	1,033	2,076	803	747	3,926
U-403a	536	498	1,034	2,176	804	747	4,027
U-404a	539	498	1,037	3,265	809	747	4,121
U-405a	546	527	1,073	1,215	818	791	3,144
U-406a	548	527	1,075	1,822	822	791	3,755
U-41a	845	840	1,685	1,884	1,373	1,260	5,017
U-42a	732	705	1,437	1,464	1,100	1,059	4,123
U-43a	888	840	1,728	2,827	1,336	1,260	5,923
U-44a	734	705	1,439	2,211	1,103	1,059	4,873
U-441a	1,270	1,076	2,346	1,476	1,900	1,614	5,740
U-442a	1,274	1,076	2,350	2,214	1,906	1,614	6,484
U-45a	1,780	1,710	3,490	2,954	2,665	2,565	9,384
U-46a	1,785	1,710	3,495	4,431	2,675	2,565	10,871

ine losses are the sum of copper losses and dielectric losses in cables.

These include losses in the current-limiting reactors.

The method of cost estimating has by elf an important bearing on the final st. For example, had the winter rating en used for the cables instead of the mmer rating, or had credit been al-wed for the charging current of the derground cables, the total costs could ve been appreciably lowered. The rious economic factors, such as the ice of copper and lead; cost of the terminal equipment, of the underground cable, of labor, and installation on foundation; financial methods of cost estimates; annual fixed charges; and the cost of maintenance and operation, may also vary from those taken in the study and may affect the system cost.

In estimating the magnitude of error in the final cost figures of this study, it may be said that the error in the design is very likely less than ten per cent. Of the same order is the error in the cost figures obtained from various manufacturers. The error in estimating costs of labor and installation, and other miscellaneous costs is around 15 per cent.

Summary and Conclusions

Results of design and cost estimates of cable-transmission systems and tie lines may be summarized as follows:

1. The type and voltage of the underground cable selected for a certain transmission line or tie line depend on the magnitude of the block of power to be transmitted. It is shown in the paper that, as the block of power increases beyond certain values, it becomes more economical to use a cable of higher voltage—or of a different type—rather than to employ a cable of the type and voltage which are more suitable for a smaller block of power. In particular this is evident when the 69-kv solid and oil-filled cables are being compared with the 138-kv oil-filled cable.

2. The most economical block of power is neither the smallest nor the largest that may be transmitted at a particular voltage. When properly selected, however, it will lower considerably the cost per kilovolt-ampere-mile, as well as the annual cost.

3. At prices for the 230-kv oil-filled cable and its accessories as taken in this study, this cable can compete with the 138-kv oil-filled cable only as a tie line between two 230-kv systems with available 230-kv terminal equipment. If an underground cable line requires installation of new terminal equipment, it will be very much less expensive to employ a 138-kv oil-filled cable. Otherwise a 230-kv cable tie line alone, without terminal equipment, is not much more expensive than a 138-kv cable line when a block of power of optimum value is transmitted, and at some system ratings it is the more economical.

4. The cost of the 230-kv oil-filled cable accessories as taken in this study appears to represent a very high percentage of the cost of the cable tie line. It is the cost of these accessories that limits the application of the 230-kv cable, and the lowering of it will extend considerably the region of the economical application of this cable. On the other hand, it might perhaps be possible to use sections of greater length and to place the normal joints farther apart than would follow from present-day practice.

5. Since cables in this country, as a general rule, use greater insulation for a certain voltage and operate at higher temperatures than is the practice in some European countries, the thought is expressed that it perhaps would lead to useful economic results, should a broad study be made of the relationships of the cable insulation for a certain voltage, the maximum operating temperature, the cable current-carrying capacity, and the cost of the cable. The relationship between the insulation and the current rating at the operating temperature must not be more favorable than that which would result in a satisfactory cable of a minimum cost. Such a study probably would involve an analysis of the various parts of the cable structure, such as the diameter of the hollow core and the thickness of the lead sheath. Even if such studies are being made by some companies, indications are that more extensive studies carried on by the industry as a whole, as a co-operative effort between the producers and the consumers, would bring gratifying results both to the cable manufacturers and the public utilities.

6. Figures are given showing the division on a percentage basis of the system capital and annual costs into their major component parts. Tables show the

Table VIII. Cost of Energy Loss Per Kilowatt-Hour

In Mills

System Identification Number	Annual Load Factor		
	35 Per Cent	50 Per Cent	75 Per Cent
U-33*a*	7.67	6.00	4.98
U-331*a*	7.67	6.01	4.98
U-34*a*	7.76	6.07	4.73
U-35*a*	7.97	6.20	4.82
U-351*a*	7.98	6.23	4.82
U-36*a*	7.95	6.20	4.82
U-361*a*	7.48	5.88	4.60
U-362*a*	7.73	6.05	4.72
U-363*a*	7.45	5.85	4.59
U-364*a*	7.67	6.01	4.98
U-37*a*	7.38	5.80	4.55
U-371*a*	7.32	5.76	4.52
U-38*a*	7.42	5.82	4.57
U-39*a*	7.59	5.94	4.65
U-391*a*	7.49	5.88	4.61
U-40*a*	7.60	5.95	4.64
U-401*a*	7.57	5.93	4.64
U-402*a*	7.80	6.09	4.75
U-403*a*	7.49	5.88	4.60
U-404*a*	7.69	6.02	4.69
U-405*a*	7.45	5.84	4.59
U-406*a*	7.58	5.94	4.64
U-41*a*	7.48	5.88	4.60
U-42*a*	8.23	6.39	4.94
U-43*a*	7.63	5.98	4.67
U-44*a*	8.45	6.53	5.04
U-441*a*	7.75	6.06	4.72
U-442*a*	7.92	6.17	4.80
U-45*a*	7.72	6.04	4.71
U-46*a*	7.90	6.17	4.80

Cost of steam generation is taken at 6.20 mills per kilowatt-hour at 35-per-cent annual load factor, 4.97 mills at 50-per-cent annual load factor, and 3.99 mills at 75-per-cent annual load factor.

Cost of transmission is calculated from the sum of the annual fixed charges and annual expenses for maintenance and operation, both taken for the entire system, terminal substations including.

amount of various equipment used in the system. Through their use, changes in the system cost may be evaluated when costs other than those used in this study are taken for the cable and its accessories, for the system equipment, for labor and installation, or for the duct banks; when prices of basic materials change; or when some of the financial factors affecting the capital or annual cost structure are modified. Similarly, the data may be used for estimating the cost when the design of some system parts, or the amount of terminal equipment, is modified; when it is desired to modify the current-carrying capacity of the cable line; when the duct banks or the number of cable accessories is changed, while at the same time the costs and economic assumptions made in the study are retained. The tables and figures may be used either for complete systems, including terminal equipment, or for the underground cable line alone, whether it be a straight transmission line, a tie line, or a cable termination of an overhead transmission line approaching the power station. The cost curves given may be interpolated within the range of values investigated.

7. This paper is an indication of the general trend of costs of underground power-transmission and tie lines. For an exact determination of the capital or annual costs of a projected system, or of an underground cable line, the costs must be calculated in detail. In some cases, with certain combinations of the various factors involved, costs lower or higher than those given in this study may be obtained. The relative costs and the effect of various factors, however, will remain as presented.

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The Direct- and Quadrature-Axis Equivalent Circuits of the Synchronous Machine

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THE EQUIVALENT-CIRCUIT method of analysis is a tool of unquestioned competence in the solution of machine and system problems involving a number of simultaneous equations. Its efficacy in obtaining practical results has been demonstrated in many diverse applications. In particular, the complete equivalent circuit of the synchronous machine—complete in the sense that the field-winding circuit and the multiple damper-winding circuits are individually included—is used whenever a detailed knowledge is needed of the operation of all the rotor circuits. Specific examples of its use are in the design of the damper-winding circuits, in problems involving single-phase and asynchronous operation, in the determination of damping and synchronizing torque, and in the determination of the transient and subtransient impedances; these few by no means exhaust the list.

When we consider the many practical applications of the complete equivalent circuit of the synchronous machine, it is somewhat surprising to note that it has been analyzed only somewhat superficially in the technical literature and has been the subject of very few technical papers, the most important of these being the papers of Linville[1] and Liwschitz.[2] Linville presented an equivalent circuit which was complete within the limits of certain well-defined approximations and also derived formulas for all the machine impedances. Liwschitz recognized that there were many applications in which the complexity of Linville's equivalent circuit would not be warranted and accordingly presented simplified equivalent circuits which were easier to use. The simplified circuits of Liwschitz give satisfactory results in many important problems, but in those applications which require a knowledge of the details of damper-winding operation the complete equivalent circuits are indispensable.

Because these complete equivalent circuits of the synchronous machine are becoming of ever-increasing importance, especially in these times with systems and machines being subjected to higher and higher specific loadings, it is the purpose of this paper to derive more complete and more exact equivalent circuits than heretofore have been available. These circuits are developed primarily for use on a-c network analyzers, since the modern analysis of problems of the type discussed in this paper tends more and more toward the use of such mechanical aids.

This paper also presents formulas for all the impedances needed by the equivalent circuits. An assemblage of the impedance formulas such as is here given is necessary when presenting an equivalent circuit in order to be certain that all the impedances are in accord and calculated on the same base. It will be found that the per-unit impedance formulas presented in this paper differ from those of Linville in that, in addition to improved permeance coefficients, the rotor current base is the x_{ad} base[7] which is more familiar to designers than the magnetomotive-force base of Linville. These impedance formulas are presented in this paper in a direct systematized form which considerably simplifies the determination of the per-unit values, and which is not subject to the misinterpretation which sometimes causes errors in the determination of the per-unit impedances of the multiple rotor circuits.

The permeance coefficients which are an integral part of the reactance formulas are determined in this paper by means of a gap-reluctance expression whose accuracy is proved by comparison with similar coefficients evaluated from flux plots. These permeance coefficients are evaluated numerically for a typical pole configuration and presented in tabular form.

The specific results and a discussion of their superiority over presently available data are given in the following section.

Results and Discussion

The complete direct-axis equivalent circuit of the synchronous machine is given by Figure 1. The quadrature-axis equivalent circuit is obtained from Figure 1 merely by substituting q for d. This circuit is an improvement over previous equivalent circuits in the following particulars:

(*a*). The impedances are given in their most general form and are all present, although some can be eliminated by a suitable choice of base-current ratio in the calculation of the per-unit impedances. For instance, this paper uses a base-current ratio which makes $X_{gnnd}=X_{fnd}$, and, accordingly, if the end-ring impedances can be neglected, the coupling transformers of Figure 1 are unnecessary.

(*b*). The end-ring impedance is correctly represented, and is separated from the field-winding circuit by means of 1/1 coupling transformers.

(*c*). The component impedances are given directly in terms of resistance and capacitive reactance, so that the circuit can be set up on an a-c network analyzer without introducing the resistance errors of inductive reactance. The circuit impedance $\hat{x}_d(j\omega)$ is obtained by direct measurement of the terminal voltage and current.

The equivalent circuit of Figure 1 is developed directly from the operational equations of the synchronous machine by noting the physical relations which exist between the various impedances. This development is given in the section, "Development of Equivalent Circuits."

The per-unit impedances for use in the equivalent circuit of Figure 1 are developed in the sections, "Impedances of Direct-Axis Circuits" and "Impedances of Quadrature-Axis Circuits." Because of the large number of formulas so obtained, it is not practical to collect and present them in this section. The origin of most of the difficulties in the determination of these multitudinous impedances is in the stator–rotor turn ratio of the short-pitched damper-winding circuits. In order to maintain a consistent stator–rotor turn ratio and thereby obtain an harmonious system of per-unit impedances, the latter are evaluated herein by first determining the physical ampere-inch-second impedances, and converting these to per unit by the direct conversion factors previously published by the author.[3] In the author's judgment, this method is superior to any other method presently available, since the physical concept of the impedance is maintained up to the last step at which point the per-unit impedances are obtained merely by introducing the conversion factor. In addition, the stator–

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rotor turn ratio is used systematically without possibility of error.

The rotor-current base selected for the per-unit impedances of this paper is the so-called x_{ad} base which is the base which makes X_{afd} numerically equal to x_{ad}. This base has been selected, because it is the base in most common use among designers, and most design formulas are based on it. If desired, however, the per-unit impedances can be converted to any other base by the consistent expressions previously published.[3]

The permeance coefficients necessary for the reactance formulas of the multiple damper-winding circuits are evaluated in Appendixes I and II by numerical integration of definite integrals based on a reluctance expression developed from a study of the work of Doherty and Nickle.[4] These coefficients are evaluated for a typical pole configuration, and the resulting numerical values are given in Table I. Design experience has indicated that these permeance coefficients are more accurate than any previously published, especially in the quadrature axis. A direct indication of their accuracy is given by comparison with the work of Wieseman[5] who evaluated similar quantities for the field-winding circuit from actual flux plots. The correspondence with the work of Wieseman is given in Table II.

The definite-integral expressions for the pole-shape coefficients as derived in this paper are not intended to compete with any future evaluation by flux plotting but instead are offered as an acceptable and satisfactory solution until more accurate values can be obtained from flux plots and design experience. The author believes, however, and the correspondence with the work of Wieseman substantiates this belief, that these integrals are sufficiently accurate that future investigations can be directed at complementary correction factors for strategic points rather than at complete new integrals or tabular values.

Numbering of Damper-Bar Circuits

Particular attention is called to the numbering of the damper-bar circuits. This numbering system is shown on Figure 2. The numbering of the physical bars proceeds outward from the polar axis. If a single bar lies directly on the polar axis, it should be hypothetically divided at its center line with the two halves thus becoming bar 1 on each side of the polar axis.

The numbering of the direct-axis circuits is identical with the numbering of the physical bars, and proceeds outward from the direct or polar axis. In contradistinction to the direct-axis circuits, the numbering of the quadrature-axis circuits proceeds outward from the quadrature or interpolar axis. The advantage of this method is that all the logic and equivalent circuits developed for one axis are immediately applicable to the other axis with only a change of subscript from d to q or vice versa. The disadvantage is that the term n, upon superficial examination, seems somewhat ambiguous since the same bar has one number in the direct axis and another in the quadrature axis. This superficial ambiguity is eliminated by noting that the subscript n followed by d specifies the nth circuit numbered from the direct axis, the subscript n followed by q specifies the nth circuit numbered from the quadrature axis, and the subscript n without either d or q following specifies the nth physical bar numbered from the polar axis.

Analysis

DEVELOPMENT OF EQUIVALENT CIRCUITS

The generalized per-unit equations 1a, b, ... and 2a, b, ... were derived in a recent publication by the author.[3]

$$\psi_d = X_{afd}I_{fd} + X_{a1d}I_{1d} + X_{a2d}I_{2d} + \ldots - x_d i_d \quad \text{(1a)}$$

$$\Psi_{fd} = X_{ffd}I_{fd} + X_{f1d}I_{1d} + X_{f2d}I_{2d} + \ldots - X_{fad} i_d \quad \text{(1b)}$$

$$\Psi_{1d} = X_{1fd}I_{fd} + X_{11d}I_{1d} + X_{12d}I_{2d} + \ldots - X_{1ad} i_d \quad \text{(1c)}$$

$$\Psi_{2d} = X_{2fd}I_{fd} + X_{21d}I_{1d} + X_{22d}I_{2d} + \ldots - X_{2ad} i_d \quad \text{(1d)}$$

$$E_{fd} = p\Psi_{fd} + R_{ffd}I_{fd} + R_{f1d}I_{1d} + R_{f2d}I_{2d} + \ldots \quad \text{(2a)}$$

$$E_{1d} = p\Psi_{1d} + R_{1fd}I_{fd} + R_{11d}I_{1d} + R_{12d}I_{2d} + \ldots \quad \text{(2b)}$$

$$E_{2d} = p\Psi_{2d} + R_{2fd}I_{fd} + R_{21d}I_{1d} + R_{22d}I_{2d} + \ldots \quad \text{(2c)}$$

The corresponding equations in the quadrature axis can be written from equations 1a, b, ... and 2a, b, ... by substituting q for d.

If all the rotor-circuit voltages are zero, the foregoing equations reduced to equations 3a, b, ...

$$\psi_d = X_{afd}I_{fd} + X_{a1d}I_{1d} + X_{a2d}I_{2d} + \ldots - x_d i_d \quad \text{(3a)}$$

$$0 = \left(X_{ffd} + \frac{R_{ffd}}{p}\right)I_{fd} + \left(X_{f1d} + \frac{R_{f1d}}{p}\right)I_{1d} + \left(X_{f2d} + \frac{R_{f2d}}{p}\right)I_{2d} + \ldots - X_{fad} i_d \quad \text{(3b)}$$

$$0 = \left(X_{1fd} + \frac{R_{1fd}}{p}\right)I_{fd} + \left(X_{11d} + \frac{R_{11d}}{p}\right)I_{1d} + \left(X_{12d} + \frac{R_{12d}}{p}\right)I_{2d} + \ldots - X_{1ad} i_d \quad \text{(3c)}$$

$$0 = \left(X_{2fd} + \frac{R_{2fd}}{p}\right)I_{fd} + \left(X_{21d} + \frac{R_{21d}}{p}\right)I_{1d} + \left(X_{22d} + \frac{R_{22d}}{p}\right)I_{2d} + \ldots - X_{2ad} i_d \quad \text{(3d)}$$

It is evident from equations 3a, b, ... that Ψ_d and all the rotor currents are

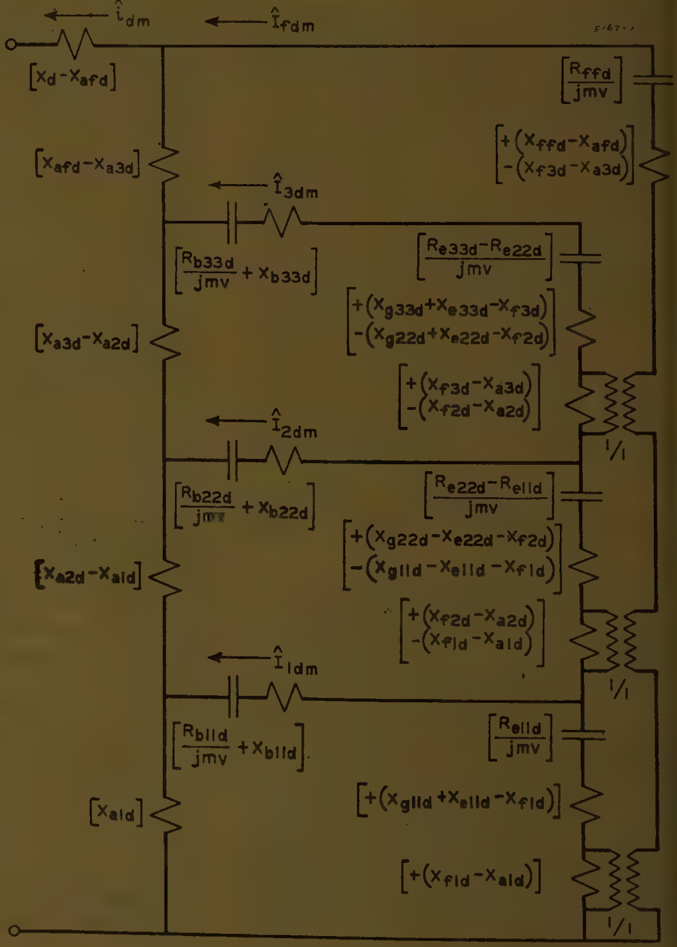

Figure 1. Direct-axis complete equivalent circuit for five or six bars per pole, $\hat{x}_d(jmv)$

$\frac{R}{jmv}$ = capacitive reactance

x = resistance

The quadrature-axis complete equivalent circuit is obtained from this circuit by substituting q for d

Table I. Pole-Shape Coefficients Evaluated for a Typical Pole Configuration

$\rho=1.50 \quad \alpha=0.70 \quad p_p^{-1}g=0.03$

y_{nd}, y_{nq}	$D_{don}y_{nd}$	$D_{gon}y_{nq}$	D_{d1n}	D_{q1n}
0	0	0	0	0
0.1	0.157	0.0316	0.198	0.0394
0.2	0.310	0.0690	0.388	0.0858
0.3	0.457	0.123	0.562	0.148
0.4	0.598	0.210	0.713	0.243
0.5	0.726	0.329	0.839	0.348
0.6	0.845	0.460	0.938	0.466
0.7	0.933	0.598	0.996	0.558
0.8	0.986	0.747	1.022	0.630
0.9	1.024	0.900	1.034	0.676
1.0	1.056	1.056	1.037	0.692

linear operational functions of i_d. If i_d is assumed to have the vectorial form, equation 4, then ψ_d and the rotor currents must have the vector forms, equations 5a, b, c. The *mv* notation of equations 4 and 5a, b, c is used, because it is adaptable to different machine operating conditions. The *m* specifies the order of the harmonic when any are present, and the *v* is a generalized rotor-velocity term. For asynchronous operation at constant slip *s* and with only fundamental currents flowing, *v* is replaced by *s*, and *m* equals unity only and may be dropped. For asynchronous single-phase operation, harmonics are present, and *m* is needed to define the particular harmonic being studied, and *v* becomes equal to the rotor velocity.

$$i_d=\hat{i}_{dm}\epsilon^{+jmvt} \tag{4}$$

$$\psi_d=\hat{\psi}_{dm}\epsilon^{+jmvt};\ I_{fd}=\hat{I}_{fdm}\epsilon^{+jmvt};\ I_{nd}=\hat{I}_{ndm}\epsilon^{+jmvt} \tag{5a, b, c}$$

Substituting equations 4 and 5a, b, c into equations 3a, b, ..., using only the steady-state solution of the operational equations, and canceling the exponentials gives equations 6a, b, ...

$$\hat{\psi}_{dm}=X_{afd}\hat{I}_{fdm}+X_{a1d}\hat{I}_{1dm}+X_{a2d}\hat{I}_{2dm}+\ldots-x_d\hat{i}_{dm} \tag{6a}$$

$$0=\left(X_{ffd}+\frac{R_{ffd}}{jmv}\right)\hat{I}_{fdm}+\left(X_{f1d}+\frac{R_{f1d}}{jmv}\right)\hat{I}_{1dm}+\left(X_{f2d}+\frac{R_{f2d}}{jmv}\right)\hat{I}_{2dm}+\ldots-X_{fad}\hat{i}_{dm} \tag{6b}$$

$$0=\left(X_{1fd}+\frac{R_{1fd}}{jmv}\right)\hat{I}_{fdm}+\left(X_{11d}+\frac{R_{11d}}{jmv}\right)\hat{I}_{1dm}+\left(X_{12d}+\frac{R_{12d}}{jmv}\right)\hat{I}_{2dm}+\ldots-X_{1ad}\hat{i}_{dm} \tag{6c}$$

$$0=\left(X_{2fd}+\frac{R_{2fd}}{jmv}\right)\hat{I}_{fdm}+\left(X_{21d}+\frac{R_{21d}}{jmv}\right)\hat{I}_{1dm}+\left(X_{22d}+\frac{R_{22d}}{jmv}\right)\hat{I}_{2dm}+\ldots-X_{2ad}\hat{i}_{dm} \tag{6d}$$

Equations 6a, b, ... can be applied to the subject problem by recognizing the physical relations which exist between the self- and mutual impedances of the various rotor circuits. These relations are introduced in the following paragraphs. They are the foundation for the direct- and quadrature-axis equivalent circuits.

Consider the *n*th additional rotor circuit in the direct axis. The reactance X_{nnd} is the sum of the reactance due to the air-gap flux within the bars which form the *n*th additional rotor circuit, the reactance due to the leakage flux in the bar slots, and the reactance due to the end-ring flux.

$$X_{nnd}=X_{gnnd}+X_{bnnd}+X_{ennd} \tag{7a}$$

The mutual reactance X_{nkd} between the *n*th additional rotor circuit and any outer additional rotor circuit *k* is the sum of the reactance due to the air-gap flux and the reactance due to the end-ring flux; the bar-slot flux is pure leakage.

$$X_{nkd}=X_{gnnd}+X_{ennd}\ (k>n) \tag{7b}$$

The mutual reactance between the *n*th additional rotor circuit and any inner additional rotor circuit is obtainable from the inner-circuit reactances. This important relation is a direct result of the reciprocal per-unit mutual impedances to which attention was directed in previous publications.[3,6]

The mutual reactance between the *n*th additional rotor circuit and the field-winding circuit depends upon only the air-gap flux of the *n*th additional rotor circuit, since the bar-slot flux and the end-ring flux are not mutual with the field winding. These mutual reactances are reciprocal because of the reasons stated in the preceding paragraph.

Concerning the resistance components, the resistance of the *n*th additional rotor circuit is the sum of the bar and end-ring resistances of that circuit.

$$R_{nnd}=R_{bnnd}+R_{ennd} \tag{7c}$$

The mutual resistance between the *n*th additional rotor circuit and any outer additional rotor circuit *k* is the end-ring resistance of the *n*th circuit, since the bar resistance is not mutual with the *k*th circuit.

$$R_{nkd}=R_{ennd}\ (k>n) \tag{7d}$$

The mutual resistance between the *n*th circuit and any inner circuit is obtained from the inner-circuit resistances as these mutual impedances are reciprocal.

There are no mutual resistances between the additional rotor circuits and the field-winding circuits since these circuits are coupled only magnetically.

The impedances of the quadrature-axis circuits can be written directly from the preceding expressions by substituting *q* for *d*.

When the physical relations of the preceding paragraphs are introduced into equations 6a, b, ... the equations obtained can be electrically duplicated by the a-c circuit of Figure 1 with $\hat{\psi}_{dm}$ the alternating voltage across the terminals. Figure 1 is therefore the equivalent circuit of the direct-axis equations of a synchronous machine.

In regard to equations 3a, b, ..., the relation between ψ_d and i_d may be written operationally as in equation 8a.

$$\psi_d=-x_d(p)i_d \tag{8a}$$

Substituting equations 4 and 5a, b, c and taking only the steady-state solution reduces equation 8a to equation 8b.

$$\hat{x}_d(jmv)=-\frac{\hat{\psi}_{dm}}{\hat{i}_{dm}} \tag{8b}$$

Reversing the direction of $\hat{i}_{dm}$ in Figure 1 permits $\hat{x}_d(jmv)$ to be defined as the a-c impedance of the direct-axis equivalent circuit since $\hat{\psi}_{dm}$ is the voltage across the circuit and $-\hat{i}_{dm}$ is the current into it. $\hat{x}_q(jmv)$ can be obtained in an analogous manner from the quadrature-axis equivalent circuit.

The current moduli $\hat{U}_{ndm}$ are defined as the rotor-circuit currents obtained when the terminal current $\hat{i}_{dm}$ is equal to $1.00\underline{/0}$. $\hat{U}_{nqm}$ is analogously defined.

$$\hat{U}_{ndm}=\hat{I}_{ndm}\text{ for }\hat{i}_{dm}=1.00\underline{/0} \tag{9a}$$

The rotor-circuit currents $\hat{I}_{ndm}$ and $\hat{I}_{nqm}$ are obtained from the product of the actual terminal current $\hat{i}_{dm}$ and the corresponding modulus as shown in equation 9b, c. The terminal currents $\hat{i}_{dm}$ and $\hat{i}_{qm}$ are determined by the machine operating conditions.

$$\hat{I}_{ndm}=\hat{U}_{ndm}\hat{i}_{dm};\ \hat{I}_{nqm}=\hat{U}_{nqm}\hat{i}_{qm} \tag{9b, c}$$

The actual currents in the physical damper bars (as distinguished from the sub-*d* and sub-*q* currents in the hypothetical direct- and quadrature-axis circuits which are introduced only to simplify the mathematics) are given by equations 9d, e. It is evident from these expressions that, in general, there is unequal loading of the bars in the leading and trailing pole halves. The degree of this inequality depends upon the pole saliency and lack of symmetry between the direct- and quadrature-axis circuits. Equal loading of the trailing and leading pole halves exists only when $\hat{I}_{nd}$ and $\hat{I}_{nq}$ are in time quadrature.

Bars on trailing pole halves:

$$\hat{I}_{nm}=+\hat{I}_{ndm}+\hat{I}_{nqm} \tag{9d}$$

Bars on leading pole halves:

$$\hat{I}_{nm}=-\hat{I}_{ndm}+\hat{I}_{nqm} \tag{9e}$$

Due consideration must be given to the numbering system previously described by which the direct-axis circuits are numbered from the direct axis, and the quadrature-axis circuits are numbered from the quadrature axis. For instance, the third bar from the direct axis in Figure 2 is the second bar from the quadrature axis, and the total current in this bar (in the trailing half) is as given by equation 9f.

$$\hat{I}_{3m}=\hat{I}_{3dm}+\hat{I}_{2qm} \tag{9f}$$

IMPEDANCES OF DIRECT-AXIS CIRCUITS

Experience in the use of these equivalent circuits has shown that the safest and shortest method of calculating the per-unit impedances is first to determine the ampere-inch-second values and convert these to per-unit values by the conversion factors previously published.* This method is followed in this paper.

Unit field current as used hereinafter is that field current which will induce in each stator phase a voltage of $x_{ad}i_{ao}$. Let this current be I_{fdo}. The corresponding base-current ratio is given by equation 10a. This base current has the advantage that it makes X_{afd} numerically equal to x_{ad} and at once eliminates the necessity for differentiating between these two quantities. In a previous paper[7] the author has suggested the universal adoption of this current as a preferred base.

$$\frac{I_{fdo}}{(3/2i)_{ao}}=\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\frac{N}{K_pK_dP_1}\frac{1}{N_{fd}} \qquad (10a)$$

The unit current for the additional rotor circuits as used hereinafter will be that current which when flowing in the additional rotor circuit of 100-per-cent pitch will induce in each stator phase a voltage $x_{ad}i_{ao}$. Let this current be I_{xdo}. The corresponding base-current ratio is given by equation 10b. A 100-per-cent-pitch circuit is not usually present in modern synchronous machines, but it is convenient to use it as a base circuit since it has maximum effectiveness.

$$\frac{I_{xdo}}{(3/2i)_{ao}}=\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\frac{N}{K_pK_dP_1} \qquad (10b)$$

Base stator inductance L_{ao} and fundamental flux per pole at rated voltage will be needed for the evaluation of the per-unit impedances. These quantities are expressed in the following in terms of machine dimensions.

$$\beta'_{\max}=3.19\frac{F_g}{g} \quad \beta'_{\text{fund}}=3.19\frac{F_g}{g}F_{d1} \qquad (11a, b)$$

$$\phi_f=12.76\frac{Rl}{P_1g}F_gF_{d1} \qquad (11c)$$

$$\frac{1}{L_{ao}}=10^8\frac{i_{ao}}{\psi_{ao}}=\frac{10^8}{1.5P_1}\left(\frac{K_pK_dP_1}{N}\right)^2\frac{A}{\phi_f} \qquad (11d)$$

$$\frac{1}{L_{ao}}=\frac{10^8}{19.14}\frac{A}{F_gF_{d1}}\frac{g}{Rl}\left(\frac{K_pK_dP_1}{N}\right)^2 \qquad (11e)$$

$$\begin{aligned}19.14&=1.5\times12.76=1.5\times4\times3.19\\&=1.5\times4\times0.4\pi\times2.54\end{aligned} \qquad (11f)$$

Stator Synchronous Reactance, x_d. The stator synchronous reactance x_d can be obtained most easily by arbitrarily separating it into the components x_{ad} and (x_d-x_{ad}). The former is the reactance of armature reactance, and the latter, somewhat unfortunately, has been termed the "leakage" reactance.

Assume the stator windings sinusoidally distributed with three-phase currents i_{ao} flowing therein. At the instant when the resulting sine wave of armature magnetomotive force is in the direct axis, the fundamental flux per pole will be as given by equation 12a. The corresponding per-unit generated voltage is given by equation 12b; this is x_{ad} by definition.

Fundamental flux per pole =

$$\frac{3.19}{g}\frac{4}{\pi}\frac{1.5Ni_{ao}}{K_pK_dP_1}A_{d1}\frac{2}{\pi}\frac{2\pi Rl}{P_1} \qquad (12a)$$

$$x_{ad}=\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\frac{A}{F_g}=k_{ad}\frac{A}{F_g} \qquad (12b)$$

The quantity $(x_d\text{-}x_{ad})$ has been evaluated several times in the technical literature. The most accurate published expressions are probably equation 37 of Alger[8] and equation 4a of Kilgore,[9] and the reader is referred to these references. In both cases, this quantity has been termed the armature "leakage" reactance.

Field-Winding Reactance, X_{ffd}. With a current of one ampere flowing in the field winding, the flux per pole due to air-gap flux is given by equation 14a. The corresponding inductance in henrys for the entire field-winding circuit of P_1 poles is given by equation 14b.

$$\text{Flux per pole}=3.19\frac{N_{fd}}{g}\frac{2}{\pi}F_{do}\frac{2\pi Rl}{P_1} \qquad (14a)$$

$$L_{ffd1}=12.76\times10^{-8}\frac{Rl}{g}N_{fd}^2F_{do} \qquad (14b)$$

Let Ψ_b and Ψ_t be the effective permeance† of the pole-body and pole-tip "leakage" paths per axial inch of machine length per pole, respectively. The inductance in henrys of the field-winding circuit of P_1 poles due to the pole-tip and pole-body flux is given by equation 14c.

$$L_{ffd2}=3.19\times10^{-8}N_{fd}^2lP_1(\Psi_b+\Psi_t) \qquad (14c)$$

The total inductance in henrys of the field-winding circuit is given by equations 14d, e. Introducing equation 10a gives the per-unit value of the field-winding reactance, equation 14f.

$$L_{ffd}=3.19\times10^{-8}N_{fd}^2P_1l\left[\frac{4RF_{do}}{P_1g}+(\Psi_b+\Psi_t)\right] \qquad (14d)$$

$$L_{ffd}=10^{-8}N_{fd}^2P_1l\left[\frac{K_\phi\phi_f}{F_gl}+3.19(\Psi_b+\Psi_t)\right] \qquad (14e)$$

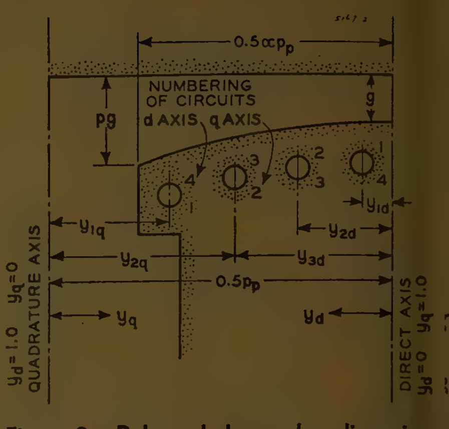

Figure 2. Pole and damper-bar dimensions and numbering of damper-winding circuits

y_d and y_q are measured in per unit of half the pole pitch (0.5p_p). y_{nd} and y_{nq} are the values of y_d and y_q, respectively, to circuit n, measured from and numbered from the corresponding (*d* or *q*) axis

$$X_{ffd}=k_{ad}K_\phi x_{ad}+3.19k_{ad}^2\frac{Al}{\phi_f}(\Psi_b+\Psi_t) \qquad (14f)$$

Mutual Reactance Between Stator and Field Winding, X_{afd}. The fundamental flux per pole per ampere field-winding current is given by equation 15a. If sinusoidal distribution of the armature winding is assumed, the corresponding mutual inductance in henrys is given by equation 15b. Introducing equation 10a into equation 15b gives X_{afd} as shown in equation 15c.

Fundamental flux per pole

$$=3.19\frac{N_{fd}}{g}\frac{2}{\pi}F_{d1}\frac{2\pi Rl}{P_1} \qquad (15a)$$

$$L_{afd}=3.19\times10^{-8}\frac{N_{fd}}{g}F_{d1}\frac{4R_l}{P_1}\frac{N}{K_pK_d} \qquad (15b)$$

$$X_{afd}=\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\frac{A}{F_g}=k_{ad}\frac{A}{F_g}=x_{ad} \qquad (15c)$$

Equation 15c illustrates a major advantage of the selected base-current ratio: X_{afd} and x_{ad} are numerically equal.

nth Additional-Rotor-Circuit Reactance, X_{nnd}. With one ampere in the *n*th additional rotor circuit, the average flux density in the air gap within the bars which bound the *n*th circuit is given by equation 16a. The corresponding inductance in henrys for the entire circuit of P_1 poles is given by equation 16b.

$$B_{\text{avg}}=\frac{3.19}{g}\frac{2}{\pi}D_{don} \qquad (16a)$$

$$L_{gnnd}=12.76\times10^{-8}\frac{Rl}{g}D_{don}y_{nd} \qquad (16b)$$

The inductance due to the bar-slot leakage flux is given by equation 16c. This expression neglects the small amount of leakage flux around the bars where they emerge from the pole body. Equa-

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[*] See "Results", reference 3.

†The permeance factor $(\Psi_b+\Psi_t)$ is difficult to evaluate and depends considerably upon the characteristics of the machine type being studied. Its complete derivation would be too long to be included in this paper. Satisfactory expressions have been given by Kilgore and Linville, and the permeance factor previously given is related to the corresponding quantities of Kilgore and Linville as follows:

$$3.19(\Psi_b+\Psi_t)=\lambda_{Fs}+\lambda_{Fe}=L_b+L_t$$

wherein λ_{Fs} and λ_{Fe} are evaluated in equations 15a and 16a of Kilgore, and L_b and L_t are evaluated in equation 14a of Linville.

tion 16c assumes that the slots are in open round slots as shown in the nomenclature; for rectangular slots, the factor 0.625 within the bracket should be replaced by $(0.333\, d_{sr}/w_{sr})$; for closed slots, the bracketed expression should be replaced by test or estimated values of the slot permeance.

$$\mathbf{L}_{bnnd}=6.38\times10^{-8}P_1 l\left(\frac{d_r}{w_r}+0.625\right) \qquad (16c)$$

The inductance due to the end-ring leakage flux can be approximated by considering the end rings as two wires of a single-phase transmission line with a distance between centers of D_e and an (effective) cross-sectional radius of r_e. This inductance is given by equation 16d.

$$\mathbf{L}_{ennd}=0.508\times10^{-8}P_1 l_{end}\left(9.2\log_{10}\frac{D_e}{r_e}+1\right) \qquad (16d)$$

The per-unit values of equations 16b, c, d, based upon the base-current ratio 10b, are given by equations 16e, f, g respectively.

$$X_{gnnd}=\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{F_g}\frac{D_{don}}{F_{d1}}y_{nd} \qquad (16e)$$

$$X_{bnnd}=0.5\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{F_g}\frac{P_1 g}{F_{d1}R}\left(\frac{d_r}{w_r}+0.625\right) \qquad (16f)$$

$$X_{ennd}=0.04\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{F_g}\frac{P_1 g}{F_{d1}R}\frac{l_{end}}{l}\times\left(9.2\log_{10}\frac{D_e}{r_e}+1\right) \qquad (16g)$$

The per-unit reactance of the nth additional rotor circuit is given by equation 16h.

$$X_{nnd}=X_{gnnd}+X_{bnnd}+X_{ennd} \qquad (16h)$$

Mutual Reactance Between Stator and nth Additional Rotor Circuit, X_{nad}. With one ampere flowing in the nth additional rotor circuit, the maximum value of the fundamental component of flux density is given by equation 17a, and the mutual inductance in henrys between the stator and the nth additional rotor circuit is given by equation 17b.

$$\beta_{fund}=\frac{3.19}{g}D_{d1n} \qquad (17a)$$

$$\mathbf{L}_{nad}=12.76\times10^{-8}\frac{Rl}{P_1 g}\frac{N}{K_p K_d}D_{d1n} \qquad (17b)$$

If the base-current ratio 10b is used, the per-unit mutual reactance corresponding to equation 17b is given by equation 17c.

$$X_{nad}=\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)\frac{A}{F_g}\frac{D_{d1n}}{F_{d1}}=\frac{D_{d1n}}{D_{d1x}}x_{ad} \qquad (17c)$$

Note that with the base-current ratio 10b, the mutual reactance X_{nad} becomes numerically equal to x_{ad} for the xth additional rotor circuit.

Mutual Reactance Between Additional Rotor Circuits, X_{nkd} $(k>n)$. The mutual inductance $\mathbf{L}_{nkd}$ between the nth additional rotor circuit and any outer rotor circuit k is equal to the sum of $\mathbf{L}_{gnnd}$ and $\mathbf{L}_{ennd}$ since the fluxes which define the latter inductances are mutual with the outer circuit. $\mathbf{L}_{nkd}$ is therefore as given by equation 18a. The per-unit mutual reactance X_{nkd} of equation 18b is based on the base-current ratio 10b. X_{gnnd} and X_{ennd} are given by equations 16e and 16f, respectively. The mutual reactance between the nth additional rotor circuit and any inner circuit can be obtained from the impedances of the inner circuit, since all mutual impedances in this article are reciprocal.

$$\mathbf{L}_{nkd}=\mathbf{L}_{gnnd}+\mathbf{L}_{ennd} \qquad (18a)$$

$$X_{nkd}=X_{gnnd}+X_{ennd}\quad(k>n) \qquad (18b)$$

Mutual Reactance Between Field-Winding Circuit and nth Additional Rotor Circuit, X_{nfd}. The mutual inductance $\mathbf{L}_{nfd}$ between the field-winding circuit and the nth additional rotor circuit is $N_{fd}\mathbf{L}_{gnnd}$, since the flux which defines the latter inductance is mutual with the field-winding circuit. $\mathbf{L}_{nfd}$ is given by equation 19a, and the corresponding per-unit value, based on the base-current ratios equations 10a and 10b is given by equation 19b.

$$\mathbf{L}_{nfd}=N_{fd}\mathbf{L}_{gnnd}=12.76\times10^{-8}\frac{Rl}{g}D_{don}y_{nd}N_{fd} \qquad (19a)$$

$$X_{nfd}=\left(\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\right)\frac{D_{don}}{D_{d1x}}y_{nd}x_{ad} \qquad (19b)$$

Stator Resistance, r_a. The stator resistance is a definitive impedance, and its per-unit value is given by equation 20a.

$$r_a=\frac{\mathbf{r}_a}{\mathbf{x}_{ao}} \qquad (20a)$$

Field-Winding Circuit Resistance, $\mathbf{R}_{ffd}$. $\mathbf{R}_{ffd}$ is easily obtained from the design or name-plate data. The corresponding per-unit value, based on the base-current ratio 10a, is given by equation 21a.

$$R_{ffd}=\frac{10^8}{\omega P_1}\left(\frac{4}{\pi}\frac{A_{d1}}{F_{d1}}\right)^2\frac{1}{N_{fd}^2}\frac{A}{\phi_f}\mathbf{R}_{ffd}$$

(no external resistance) (21a)

Table II. Comparison of Pole-Shape Coefficients Obtained in This Paper by Numerical Integration With Corresponding Values Obtained by Wieseman

$\rho=1.50\quad \alpha=0.70\quad p_p{}^{-1}g=0.03$

	A_{d1}	A_{q1}	F_{d1}	K_ϕ
1	0.874	0.472	1.037	1.020
2	0.876	0.481	1.062	1.010

1 = values obtained in this paper by numerical integration.

2 = values obtained by Wieseman[6] from flux plots.

Resistance of nth Additional Rotor Circuit, $\mathbf{R}_{nnd}$. The resistance of the two bars which form the nth additional rotor circuit in each pole is given by equation 22a. The resistance of the corresponding end-ring section is given by equation 22b. The resistance of the entire nth additional rotor circuit of P_1 poles is given by equation 22c, and the corresponding per-unit value is given byequation 22d.

$$\mathbf{R}_{bnnd}=1.67\times10^{-6}\frac{l_{bn}}{a_{bn}}P_1 \qquad (22a)$$

$$\mathbf{R}_{ennd}=3.33\times10^{-6}\frac{l_{end}}{a_{end}}P_1 \qquad (22b)$$

$$\mathbf{R}_{nnd}=1.67\times10^{-6}P_1\left(\frac{l_{bn}}{a_{bn}}+2\frac{l_{end}}{a_{end}}\right) \qquad (22c)$$

$$R_{nnd}=\frac{10^8}{\omega P_1}\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{\phi_f}\mathbf{R}_{nnd} \qquad (22d)^*$$

Mutual Resistance Between Additional Rotor Circuits, R_{nkd} $(k>n)$. The mutual resistance between the nth additional rotor circuit and any outer rotor circuit k is the end-ring resistance of the nth additional rotor circuit. $\mathbf{R}_{nkd}$ is given by equation 23a, and the corresponding per-unit value by equation 23b. The mutual resistance between the nth additional rotor circuit and any inner rotor circuit is obtained from the latter, since in this article all the mutual impedances are reciprocal.

$$\mathbf{R}_{nkd}=\mathbf{R}_{ennd}=3.33\times10^{-6}\frac{l_{end}}{a_{end}}P_1 \qquad (23a)$$

$$R_{nkd}=\frac{10^8}{\omega P_1}\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{\phi_f}\mathbf{R}_{nkd} \qquad (23b)^*$$

IMPEDANCES OF QUADRATURE-AXIS CIRCUITS

The currents in the additional rotor circuits in the quadrature axis are expressed in per unit of I_{xdo} which is the base current of the direct-axis additional rotor circuits. This base is adopted so that the per-unit currents in the direct- and quadrature-axis additional rotor circuits may be added directly.

The concept of a field winding in the quadrature axis appears academic, but it is convenient to assume such a winding (but of infinite resistance), with a base current defined analogously to I_{fdo}. By this concept the stator flux which is leakage with respect to all the rotor circuits is (x_a-x_{ad}) for both the direct and quadrature axis. In addition, the assumption of the existence of a quadrature-axis field winding is convenient, since it maintains symmetry between the direct- and quadrature-axis equations.

The equations of the quadrature-axis circuits, including the quadrature-axis field winding, are identical with the direct-axis equations with the d replaced by q. It would appear therefore that the formulas for the quadrature-axis impedances could be obtained from the corresponding direct-axis formula by merely substituting q for d. This is actually true for the ampere-inch-second values (ohms and henrys), but is not true for the per-unit values because I_{zdo} is the base current for the additional rotor circuits in both axes. The per-unit quadrature-axis impedances are obtained by substituting q for d in the corresponding direct-axis ampere-inch-second impedance formula, and converting the result into a per-unit value based on the base-current ratio 10b.

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*Equations 22d and 23b are evaluated for copper at 75 degrees centigrade. For any other material at/or any other temperature, these equations should be multiplied by the material resistivity in per unit of the resistivity of copper at 75 degrees centigrade.

Stator Synchronous Reactance, x_q. x_q is evaluated by separating it into the two components x_{aq} and (x_q-x_{aq}) analogously to the method used in the evaluation of x_d. x_{aq} is given by equation 24a. (x_q-x_{aq}) is independent of the rotor position and is equal to (x_d-x_{ad}).

$$x_{aq}=\frac{4}{\pi}\frac{A_{q1}}{F_{d1}}\frac{A}{F_g}=\frac{A_{q1}}{A_{d1}}x_{ad} \tag{24a}$$

$$x_q-x_{aq}=x_d-x_{ad} \tag{24b}$$

nth Additional-Rotor-Circuit Reactance, X_{nnq}. The components of L_{nnq} can be obtained from the corresponding components of L_{nnd} by substituting q for d. If the base-current ratio 10b is used, the corresponding per-unit values are given by equations 25a, b, c. The total reactance X_{nnq} is given by equation 25d.

$$X_{gnnq}=\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{F_g}\frac{D_{qon}}{F_{d1}}y_{nq} \tag{25a}$$

$$X_{bnnq}=0.5\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{F_g}\frac{P_1g}{F_{d1}R}\left(\frac{d_r}{w_r}+0.625\right) \tag{25b}$$

$$X_{ennq}=0.04\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{F_g}\frac{P_1g}{F_{d1}R}\frac{l_{enq}}{l}\times\left(9.2\log_{10}\frac{D_e}{r_e}+1\right) \tag{25c}$$

$$X_{nnq}=X_{gnnq}+X_{bnnq}+X_{ennq} \tag{25d}$$

Mutual Reactance Between Stator and nth Additional Rotor Circuit, X_{naq}. L_{naq} can be obtained from equation 17b by substituting q for d. The corresponding per-unit value, based on the base-current ratio 10b, is given by equation 26a.

$$X_{naq}=\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)\frac{A}{F_g}\frac{D_{qn}}{F_{d1}}=\frac{D_{q1n}}{D_{d1x}}x_{ad} \tag{26a}$$

Mutual Reactance Between Additional Rotor Circuits, X_{nkq} $(k>n)$. In a manner similar to that used in the derivation of X_{nkd} it can be shown that X_{nkq} is given by equation 27a.

$$X_{nkq}=X_{gnnq}+X_{ennq}\quad(k>n) \tag{27a}$$

Resistance of nth Additional Rotor Circuit, R_{nnq}. R_{nnq} as given by equation 28a is obtained from equation 22c by substituting q for d. The corresponding per-unit value, based on the base-current ratio 10b, is given by equation 28b.

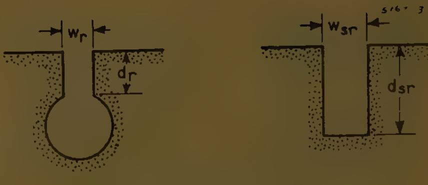

Figure 3. Rotor slots

$$R_{nnq}=1.67\times10^{-6}P_1\left(\frac{l_{bn}}{a_{bn}}+2\frac{l_{enq}}{a_{enq}}\right) \tag{28a}$$

$$R_{nnq}=\frac{10^8}{\omega P_1}\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{\phi_f}R_{nnq} \tag{28b*}$$

Mutual Resistance Between Additional Rotor Circuits, R_{nkq} $(k>n)$. R_{nkq} as given by equation 29a is obtained from equation 23a by substituting q for d. The corresponding per-unit value, based on the base-current ratio 10b, is given by equation 29b.

$$R_{nkq}=R_{ennq}=3.33\times10^{-6}\frac{l_{enq}}{a_{enq}}P_1 \tag{29a}$$

$$R_{nkq}=\frac{10^8}{\omega P_1}\left(\frac{4}{\pi}\frac{A_{d1}}{D_{d1x}}\right)^2\frac{A}{\phi_f}R_{nkq}\quad(k>n) \tag{29b*}$$

Appendix I. Evaluation of Pole-Shape Coefficients

The pole-shape coefficients D_{don}, D_{qon}, D_{d1n}, and D_{q1n} must be evaluated by flux plots if extreme accuracy is desired since the actual magnetic gap obviously cannot be represented exactly by any known mathematical expression. However, the flux-plotting method is too time-consuming to be used in the majority of actual problems. A more practical method is to obtain, by any means, whatsoever, a satisfactory mathematical expression for the gap permeance, and by means of numerical integration then evaluate the pole-shape coefficients. Such a method is used in this appendix.

The gap-reluctance expression by which satisfactory values of the pole-shape coefficients are obtained in this paper is given by equations 30a, b, ...h; these expressions give the gap length in per unit of the minimum gap g.

$$g_y=g_d \text{ in the region } 0<y_d<\alpha_\eta \tag{30a}$$

$$g_y=g_q \text{ in the region } \alpha_\eta<y_d<1 \tag{30b}$$

$$g_d=1+(\rho-1)\left(\frac{y_d}{\alpha}\right)^2 \tag{30c}$$

$$g_q=g_\eta+\beta g_\eta\sin^2\left(\frac{y_d-\alpha_\eta}{1-\alpha_\eta}\right)\frac{\pi}{2} \tag{30d}$$

$$g_\eta=1+(\rho-1)\left(\frac{\alpha_\eta}{\alpha}\right)^2 \tag{30e}$$

$$\beta=-1+\sqrt{1+\frac{1}{4}\left(\frac{1-\alpha}{p_p^{-1}gg_\eta}\right)^2} \tag{30f}$$

$$\alpha_\eta=\alpha-3.5\rho p_p^{-1}g \tag{30g}$$

$$y_d+y_q=1.0 \tag{30h}$$

D_{don}, D_{qon}, D_{d1n}, and D_{q1n} are defined in the following subsections, and definite-integral expressions for each are derived in terms of the reluctance expression 30a, b, ...h. Numerical values obtained from these definite-integral expressions are given in Table I for a typical pole configuration.

Evaluation of D_{don}

$(2/\pi)D_{don}$ is defined as the factor by which the maximum gap density must be multiplied to obtain the average density within the damper circuit of span $2y_{nd}$ with the machine excited by the damper circuit of span $2y_{nd}$ in the direct axis. Expressing the corresponding flux per pole in the two ways shown by equation 31a leads directly to the evaluation of $D_{don}y_{nd}$ as given by equation 31b.

$$\phi=2\int_0^{y_{nd}}\frac{3.19}{gg_y}\,dy_d=2\,\frac{3.19}{g}\,\frac{2}{\pi}\,D_{don}y_{nd} \tag{31a}$$

$$D_{don}y_{nd}=\frac{\pi}{2}\int_0^{y_{nd}}g_y^{-1}dy_d \tag{31b}$$

Evaluation of D_{qon}

$(2/\pi)D_{qon}$ is analogous to $(2/\pi)D_{don}$, but in the quadrature axis. Its value as given by equation 32a is obtained by the same derivation as was used for D_{don}. The integral value given by equation 32b is obtained from equation 32a by changing the origin of integration by means of equation 30h.

$$D_{qon}y_{nq}=\frac{\pi}{2}\int_0^{y_{nq}}g_y^{-1}dy_q=\frac{\pi}{2}\int_{1-y_{nq}}^{1}g_y^{-1}dy_d \tag{32a, b}$$

Evaluation of D_{d1n}

D_{d1n} is defined as the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by the nth additional rotor circuit in the direct axis. The flux density per ampere turn at any point along the gap can be written as in equation 33a; the maximum value of the fundamental component is then given by equation 33b. By the definition of D_{d1n}, the maximum value of the fundamental component can also be written as in equation 33c, and D_{d1n} can be evaluated as in equation 33d by equating expressions 33b and 33c.

$$\beta_y=\frac{3.19}{gg_y} \tag{33a}$$

$$\beta_{y\,\text{fund}}=2\int_0^{y_{nd}}\frac{3.19}{gg_y}\cos\frac{\pi}{2}y_ddy_d \tag{33b}$$

$$\beta_{y\,\text{fund}}=\frac{3.19}{g}D_{d1n} \tag{33c}$$

$$D_{d1n}=2\int_0^{y_{nd}}g_y^{-1}\cos\frac{\pi}{2}y_ddy_d \tag{33d}$$

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*Equations 28b and 29b are evaluated for copper at 75 degrees centigrade. For any other material at/or any other temperature, these equations should be multiplied by the material resistivity in per unit of the resistivity of copper at 75 degrees centigrade.

Evaluation of D_{q1n}

D_{q1n} is analogous to D_{d1n}, but in the quadrature axis. Its value as given by equation 34a is obtained by the same derivation as was used for D_{d1n}. The integral value given by equation 34b is obtained from equation 34a by changing the origin of integration by means of equation 30h.

$$D_{q1n}=2\int_{0}^{y_{nq}} g_y{}^{-1}\cos\frac{\pi}{2}y_q dy_q \tag{34a}$$

$$D_{q1n}=2\int_{1-y_{nq}}^{1} g_y{}^{-1}\sin\frac{\pi}{2}y_d dy_d \tag{34b}$$

Appendix II. Accuracy of the Derived Pole-Shape Coefficients

It is obviously not possible at present to obtain complete checks on the accuracy of the foregoing integral expressions, because the necessary experimental information is lacking. If such information were available, the reluctance expression 30a, b, ...h would be unnecessary, since the pole-shape coefficients could be obtained directly from the experimental data. It is possible, however, to use the work of Wieseman as follows to check the accuracy of the limiting points $y_{nd}=1.00$, $y_{nq}=1.00$, and thereby obtain a qualitative estimate of the over-all accuracy.

Wieseman presented curves, based on actual flux plots, for the quantities A_{d1}, A_{q1}, F_{d1}, and K_ϕ. A check on the reluctance expression 30a, b, ...h, can be obtained by evaluating these same expressions by numerical integration based on equation 30a, b, ...h, and comparing these derived values with the corresponding quantities calculated by Wieseman. A_{d1}, A_{q1}, F_{d1}, and K_ϕ are defined in the following subsections, and definite-integral expressions for each are derived in terms of the gap-reluctance expression 30a, b, ...h. Numerical values as obtained from these definite integrals are given in Table II for a typical pole configuration and are there compared with the corresponding values obtained by Wieseman from flux plots. The agreement with Wieseman's results is sufficiently close as to warrant fully the use of the integral expressions for the pole-shape coefficients D_{don}, D_{qon}, D_{d1n}, and D_{q1n} until absolutely accurate values can be obtained from actual flux plots.

Evaluation of A_{d1}

A_{d1} is defined as the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by a sine-wave armature magnetomotive force in the direct axis. Assuming a sine-wave magnetomotive force of one ampere turn peak value in the direct axis, the flux density at any point in the gap is given by equation 35a, and the maximum value of the corresponding fundamental component of flux density is given by equation 35b. Based on the definition of A_{d1}, the maximum value of the fundamental component can also be written as in equation 35c, and A_{d1} can be evaluated as in equation 35d by equating expressions 35b and 35c.

$$\beta_y=\frac{3.19}{gg_y}\cos\frac{\pi}{2}y_d \tag{35a}$$

$$\beta_{y\,\text{fund}}=2\int_{0}^{1}\frac{3.19}{gg_y}\cos^2\frac{\pi}{2}y_d dy_d \tag{35b}$$

$$\beta_{y\,\text{fund}}=\frac{3.19}{g}A_{d1} \tag{35c}$$

$$A_{d1}=2\int_{0}^{1} g_y{}^{-1}\cos^2\frac{\pi}{2}y_d dy_d \tag{35d}$$

Evaluation of A_{q1}

A_{q1} is analogous to A_{d1} but in the quadrature axis. Its value as given in equation 36a is obtained by the same derivation as was used for A_{d1}. An alternative expression for A_{q1} in terms of y_d measured from the polar axis is given by equation 36b; this was obtained by substituting equation 30h into equation 36a.

$$A_{q1}=2\int_{0}^{1} g_y{}^{-1}\cos^2\frac{\pi}{2}y_q dy_q \tag{36a}$$

$$A_{q1}=2\int_{0}^{1} g_y{}^{-1}\sin^2\frac{\pi}{2}y_d dy_d \tag{36b}$$

Evaluation of F_{d1}*

F_{d1} is defined as the factor by which the maximum flux density must be multiplied to obtain the maximum of the fundamental component of flux density with the machine excited by the direct-axis field winding.

The field winding links only the flux which enters the pole, and its effective span or pitch is accordingly somewhat less than 100 per cent. However, the effective span is greater than the physical-pole arc, because the field winding links the flux which enters the side of the pole tip. Hence, the effective span of the field winding approaches but remains slightly less than the full 100-per-cent value. A study of Table I, however, shows that D_{d1n} is relatively constant (for given values of ρ, α, and $p_p{}^{-1}g$) for values of y_{nd} in the neighborhood of 1.00. Relatively little error is introduced therefore by assuming that F_{d1} is given by D_{d1n} for $y_{nd}=1.00$. Wieseman's values of F_{d1} thus may be compared directly with D_{d1n} for $y_{nd}=1.0$ as obtained in this paper by numerical integration.

Evaluation of K_ϕ

K_ϕ is defined as the factor by which the total fundamental flux per pole must be multiplied to obtain the total flux per pole, with the machine excited by the direct-axis field winding. By means of the previously justified assumption that the field-winding circuit is equivalent to a damper-winding circuit of 100-per-cent pitch (with negligible error), K_ϕ can be evaluated in terms of D_{don} and D_{d1n}, for $y_{nd}=1.00$, as shown in the following.

By the definition of D_{don} and D_{d1n}, the total flux per pole and the total fundamental flux per pole are given by equations 37a, b, respectively, with y_{nd} taken as unity. The ratio of equation 37a to equation 37b then gives K_ϕ as in equation 37c, by definition.

$$\text{Total flux per pole}=\frac{3.19}{g}\frac{2}{\pi}D_{don} \tag{37a}$$

$$\text{Fundamental flux per pole}=\frac{3.19}{g}\frac{2}{\pi}D_{d1n} \tag{37b}$$

$$K_\phi=\left.\frac{D_{don}}{D_{d1n}}\right|_{y_{nd}=1.0} \tag{37c}$$

Nomenclature

The quantities given below are per-unit values unless in bold-face type in which case they are in the physical ampere-inch-second system (amperes, ohms, henrys, and so forth). Quantities such as N, P_1, R, l, for which it is obviously unnecessary to distinguish the ampere-inch-second values by a special script are in standard type only. Vector values are indicated by a circumflex (^) over the symbol.

All dimensions are in inches.

The subscript notation is as follows.

a, b, c = armature phases
f = field-winding circuit
n, k, x = damper-winding circuits. n is the general term; k refers to a damper-winding circuit external to $n (k > n)$; and x refers to the 100-per-cent-pitch (most external) circuit which is used as a base
d, q, 0 = direct, quadrature, and "zero" axes
o = base quantities
m = order of harmonic

Only the direct-axis quantities are defined in the following, as the quadrature-axis quantities are obtained from these by merely substituting q for d.

a_{bn} = cross-sectional area of bar n
a_{end} = average cross-sectional area of end ring for bar n
A = flat-topped armature reaction at base stator current
$A=\dfrac{1.5\mathbf{i}_{ao}N}{K_pK_dP_1}$
A_{d1} = defined in Appendix II
D_{don}, D_{d1n} = defined in Appendix I
D_e = axial distance between end rings minus axial length of pole, or twice the distance between one end ring and side of pole
$\mathbf{e}_{ao}$ = peak value of rated stator phase voltage
E_{fd}, E_{nd} = rotor-circuit applied voltages
$F_{do}=D_{don}$ for $y_{nd}=1.0$
F_{d1} = defined in $B3$
F_g = field-winding ampere turns for rated air-gap-line stator voltage
g = minimum gap, effective
g_y = gap length at any point y_d, in per unit of g
$\mathbf{i}_{ao}$ = peak value of rated stator line current
$\mathbf{I}_{fdo}$, $\mathbf{I}_{xdo}$ = base-current; field winding, and xth additional rotor circuit, respectively
i_d, I_{fd}, I_{nd} = direct-axis currents; stator, field winding; and nth additional rotor circuit, respectively
I_n = total current in nth physical bar
$k_{ad}=\dfrac{4}{\pi}\dfrac{A_{d1}}{F_{d1}}$
K_p, K_d = pitch and distribution factors, respectively (greater than 1.0)

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*Wieseman called this quantity A_1.

K_ϕ = defined in Appendix II

l = machine stacked length

l_{bn} = length of bar n

l_{end} = circumferential length of end ring measured from direct axis to center of bar n

L = ampere-inch-second inductance corresponding to per-unit reactance x

L_{ao} = base stator inductance

$$L_{ao}=\frac{10^{-8}\psi_{ao}}{i_{ao}}=\frac{x_{ao}}{\omega}$$

N = stator series turns per phase

N_{fd} = turns per pole of field winding

p = differential operator $=\dfrac{d}{d(\omega t)}$

p_p = pole pitch $=\dfrac{2\pi R}{P_1}$

P_1 = number of poles

r_a = stator resistance per phase

r_e = effective radius of end-ring cross section; $\pi r_e^2 = a_{end}$

R_{bnnd}, R_{ennd} = resistances of nth additional rotor circuit; bar and end ring, respectively

R_{ffd}, R_{nnd} = resistance of field-winding and nth additional rotor circuits, respectively

R_{nfd}, R_{nkd} = mutual resistances, nth additional rotor circuit to field winding and kth circuit, respectively

s = slip in per unit

$\hat{U}_{ndm}$ = current modulus defined in equation 9a

v = general rotor-velocity term of equation 4

x_{ao} = base stator ohms

$$x_{ao}=\frac{e_{ao}}{i_{ao}}=\omega L_{ao}$$

x_{ad} = reactance of armature reaction

x_d, X_{ffd}, X_{nnd} = direct-axis reactances; stator, field winding, and nth additional rotor circuit, respectively

X_{afd}, X_{and}, X_{nfd}, X_{nkd} = mutual reactances; stator field, stator–nth rotor circuit, nth rotor circuit to field winding, and nth rotor circuit to kth rotor circuit

X_{bnnd}, X_{ennd}, X_{gnnd} = reactance components of nth rotor circuit; due to bar-slot flux, end-ring flux, and air-gap flux, respectively

$x_d(p)$ = operational stator impedance as viewed from stator terminals

y_d, y_q = peripheral distances on pole face measured in per unit of half the pole pitch; see Figure 2

α = ratio of pole arc to pole pitch

β'_{max} β'_{fund} = maximum flux density and maximum value of the fundamental component of flux density at normal voltage no load

ρ = ratio of maximum gap to minimum gap; see Figure 2

ϕ_f = fundamental flux per pole at normal voltage no load

ψ_{ao} = peak value of rated stator phase linkages; $e_{ao}=10^{-8}\omega\psi_{ao}$

ψ_d, Ψ_{fd}, Ψ_{nd} = direct-axis linkages; stator, field-winding, and nth additional rotor circuits, respectively

Ψ_b, Ψ_l = effective permeance of the pole-body and pole-tip leakage paths per axial inch of machine length per pole

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Modern Practice in Power-Plant Auxiliary Equipment and Systems

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THIS DISCUSSION of central-station auxiliary equipment and systems cannot cover the entire field of practice, but must touch only selected high spots. The particular points selected were chosen to include those items where the user may exercise judgment in choosing alternate arrangements of supply or items of apparatus, or where his specifications may influence apparatus design by emphasis on particular requirements.

The discussion is timely, because power-plant auxiliary equipment has undergone important development in the past several years, and because the interruption in normal construction of new facilities, necessitated by war activities, provides an opportune moment for analysis of what is available and how to apply it.

Supply systems for auxiliaries have received extensive study with the result that loop and network systems now find application along with the more conventional radial distribution system. The change in equipment is keynoted by the swing to air circuit breakers and the currently marked trend toward the use of air-cooled transformers. Both motors used for central-station auxiliary service and the associated motor starters have been given close attention with the general result that more economical reliable drives will be available to meet rigid requirements. One fact is ever apparent: the central-station operator demands maximum reliability. Nowhere else on the entire power system is the requirement of reliability so great compared to the investment in equipment involved.

There follows, thus, a discussion of particular design features of motors, desirable arrangements of switchgear and control, and data on distribution systems that may be helpful when selection of the method of supply is under consideration.

Motor Design

As in the past, the squirrel-cage induction motor for across-the-line starting is the best selection for an auxiliary drive wherever constant-speed operation is desired. The simplicity, reliability, and low cost all enter to make it the number-one choice. Its control is simple as well. The higher power factor and efficiency of synchronous drives is offset by the increased complication of synchronizing control and a source of excitation. Also, since the cost of both real and reactive power is a minimum at the generator bus, higher efficiency and power factor is not a large incentive.

Rotor Design

Draft fans and some pulverizers have high inertia, and motors for these loads are best designed to limit the temperatures and expansion of the rotors to safe values. The WR^2 of each drive of this nature should be supplied to the motor designer for an analysis of the particular application. Certain types of pulverizers are subject to jamming, and it is known from experience that operators may try several times to start them. Motors for these pulverizers require an extra margin in rotor thermal capacity. An ample air gap is especially desirable for motors on

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hese two types of loads because of rotor xpansion and should not be discouraged by unnecessary emphasis on high power actor. Similarly, adequate thermal capacity with ample starting torque may equire higher starting current than might be obtained with a less reliable design, and unnecessary restrictions in this respect should not be imposed. Motors or high-inertia loads also should be designed to withstand the high transient orque which occurs when the motor is nergized.

Another consideration which affects reiability but does not appear on the perormance sheet is magnetic stability. This requires a satisfactory relation between the stiffness of the shaft and the nbalanced magnetic pull which occurs when the rotor is displaced radially with espect to the stator. Such displacement nay be either deflection of the shaft and ther structural parts caused by vibration r an initial displacement caused by mahining tolerances. Here again an ample ir gap is required for a reliable motor.

NSULATION OF WINDINGS

The insulation of motor windings for entral-station auxiliary service should be ighly resistant to moisture, coal dust, nd fly ash. Although the atmospheric onditions are no more severe than in nany industrial plants, the extreme imortance of reliability of the auxiliaries equires the best possible insulation job. The smaller motors should be given ample arnish dips and bakes with carefully conrolled viscosity and temperature, and the arger motors should be impregnated by he vacuum-pressure method. Mica inulation is to be used where practicable. The insulation between turns should be ested by a high-frequency testing set in ddition to the conventional high-poential tests between the winding and round.

Insulating materials good for higher emperatures and adverse moisture conitions have been written about widely uring the last year. These are silicone arnishes that are now available in forms milar to the usual varnishes used by the ectrical industry. They have been used ith mica, glass, and asbestos to produce igh-temperature insulations. The improved thermal endurance can be utilized a number of important ways in the degn of electric equipment, such as reducon in size and weight, increased service e for conventional size and weight, and peration in ambient temperatures beond the limits permissible for usual pes of insulation. A few cases of powerant operation may justify trial of silime insulations. One likely application for motors driving induced-draft fans bject to high ambient temperatures.

At the present time quantities of silime products are limited, and their cost high compared to good insulating rnishes. Much is still to be learned about processes of handling and manufacturing that will have to be studied before application to motor equipment becomes general.

MOTOR BEARINGS

Both antifriction and sleeve-type bearings are furnished today. Each type has its advantages and consequently is to be preferred for certain applications. The sleeve bearing has advantages for many general applications. It requires less precision work both in manufacture and maintenance, it inherently is suitable for operation in locations where coal dust and fly ash are present, and it can be operated at higher peripheral speeds. The antifriction bearing may find preference because of smaller space requirements, because of its ability to take thrust loads, or because of the low starting friction inherent to its design. This last point refers particularly to vertical pump motors where heavy thrust is encountered. Prelubricated ball bearings are now used on some small motors. This practice stands to become more popular since field experience with it has been excellent. These bearings have shown unusual freedom from trouble.

Boiler-feed-pump motors generally are connected to the load through flexible couplings. Certain of these couplings have a natural tendency to lock into a fixed running position while transmitting torque, thus preventing the motor and pump from moving axially with respect to each other. Because of shaft expansion which results from high-temperature operation of the pump in handling feed water, heavy thrust can be imposed upon the motor bearings. Failure of these bearings may result, since this thrust is beyond the capacity of simple sleeve-bearing shoulders. Occurrence of such phenomena can be prevented if the "limited end-float" type of coupling and a motor having shaft end play more than the end play of the coupling is specified.

NOISE LEVEL OF MOTORS

Although the noise level in different power plants may vary somewhat, an average figure can be taken to be about 100 decibels in the general vicinity of the turbine and principal auxiliaries. If the noise level of motors is to be kept to a value that will not contribute seriously to powerhouse noise, it is logical to assume that no motors should have a level as high as 100 decibels.

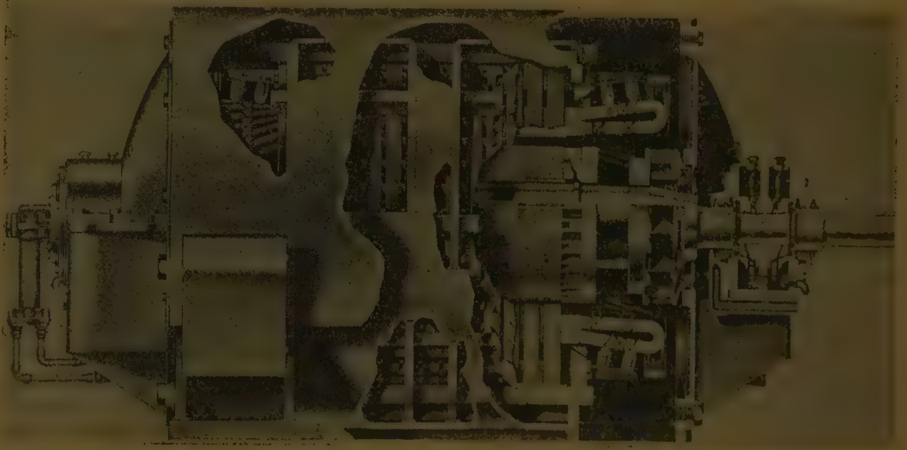
The greatest source of noise in large high-speed motors is windage and the siren effect resulting from jets of air from the rotor vent ducts. A system of axial-flow surface cooling of the rotor is used today that effectively reduces noise level on the very large two-pole motors. Figure 1 illustrates the construction of a large 3,600-rpm motor employing this type of design. Careful selection of slot combinations in the stator and rotor will reduce noise in addition to keeping stray loss and parasitic torques to a minimum.

ADJUSTABLE-SPEED DRIVES

There are several auxiliaries which require adjustable-speed drives. Cranes, coal bridges, and similar material-handling equipment can be driven best by wound-rotor induction motors. The most suitable electric drive for stokers is a series variable-voltage a-c–d-c drive. Boiler-feed pumps are usually driven at constant speed, but when speed adjustment is desired a wound-rotor induction motor with a multipoint drum furnishes adequate control.

Adjustable speed is very desirable for draft-fan drives in order to reduce losses and erosion. Two speeds can be obtained with a pole-changing squirrel-cage motor, and this combination is used frequently with inlet-vane control. Thirteen or 20 speed steps can be obtained with a wound-rotor motor and drum, but this still requires the use of dampers or vanes for continuous control of gas flow. A squirrel-cage motor with a hydraulic coupling gives continuous speed control over about a four-to-one range. The coupling requires cooling water and involves extra bearings. Electric couplings do not require cooling water and give ten-to-one speed range but involve extra bearings,

Figure 1. Cutaway view of 3,600-rpm squirrel-cage induction motor



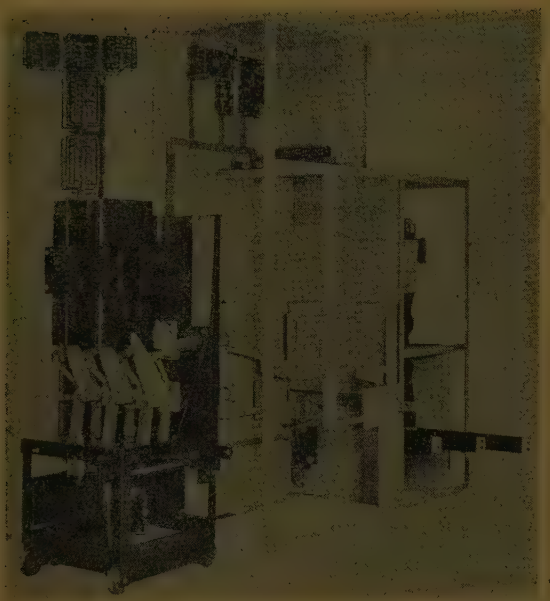
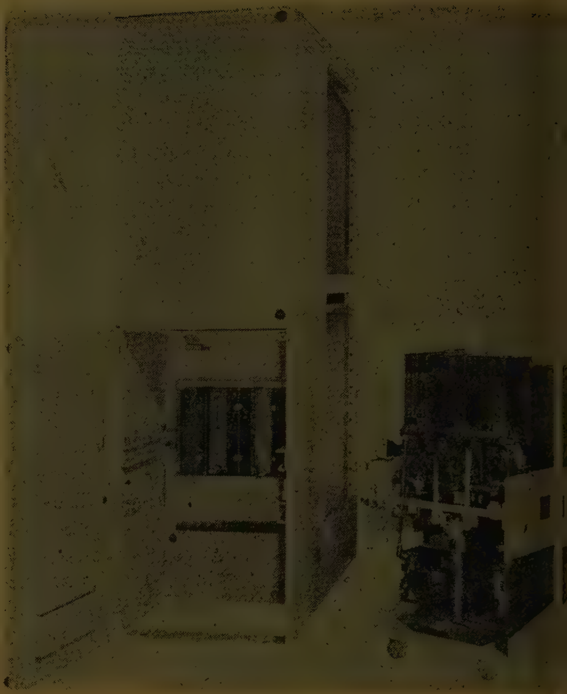


Figure 2 (left). Cell and air breaker for 2,300-volt auxiliary feed (five-kilovolt insulation, 150,000-kva interrupting capacity)

Figure 3 (right). Cell and air breaker for 600 volts or below; breaker shown is 800-ampere unit



an excitation system, and relatively high operating temperatures. A wound-rotor motor with a new type of liquid rheostat for secondary control has been used recently for large draft fans. This combination provides three-to-one speed range. It requires cooling water and an electrolyte, but eliminates extra bearings and rotating apparatus. The liquid rheostat appears to offer an attractive solution for future application.

The cost and efficiency of all of the afore-mentioned drives are comparable, so that the choice of drive should be made primarily to give maximum reliability with the particular operating conditions which will be encountered.

Switchgear and Control

As pointed out in the introduction, reliability is the outstanding qualification for all central-station auxiliary devices, and switchgear is no exception. Safety of operation, ease of installation, ease of maintenance, and so forth, are all important qualifications desirable here, just as in other classes of service, but dependability is paramount. Circuit breakers, contactors, fuses, and assemblies of these component parts all stress this virtue.

METAL-CLAD GEAR

These requirements all are met by up-to-date metal-clad gear in its various forms. The service record of metal-enclosed gear is enviable, and the manufacturers continue to make improvements in the breakers, housings, and other parts. Purchasers of metal-enclosed switchgear can rely on receiving the benefits of all new developments promptly.

All breaker and auxiliary units are arranged for line assembly. They can be throat-connected to liquid-cooled transformers or supplied as part of an integral structure with air-cooled transformers. These unit substation arrangements form a most satisfactory assembly of transforming and distributing apparatus. Switchgear structures, or the switchgear portion of unit substations, can be shipped fully assembled or in groups as large as the purchaser's handling equipment can accommodate. This usually saves considerable installation cost. Factory connections on control wiring are not disturbed, so that tests need not be repeated.

The reliability of metal-clad switchgear is now so well recognized that double-breaker arrangements for load circuits seldom are considered necessary, and even auxiliary or transfer busses are not common. Special circuits may have double feeds, usually from different breaker groups, but duplication of breakers beyond this is not often justified. Where motors may require frequent starting, air breakers in metal-clad gear are well suited for this service. Duplicate power supply or normal and alternate power supply to auxiliary busses with automatic transfer is frequently provided. Relays for this function and for other normal protective functions are mounted on the gear. Relays are preferably of the Flexitest type with removable elements and built-in test switches. Functioning of all such control circuits is tested at the factory.

Switchgear for service voltages below 600 is now usually supplied with drawout breakers which have safety interlocking features similar to those in high-voltage metal-clad switchgear. Increased use and improved manufacturing methods have brought the prices of drawout gear so close to the equivalent enclosed breakers with fixed mountings that the advantages of the removable breaker types easily justify the difference.

Low-voltage metal-clad switchgear is well suited for distributing power in radial, loop, or low-voltage network circuits. Standard units are available whi
contain the limiters and loop-circuit d
connecting switches when the switchge
is part of a network system. The fle
bility with which the various units can
grouped permits arranging the limi
sections and supply circuits where
they best tie in with the plant distributi
system. The network breakers which a
used in network systems may be mount
on the transformers or in a unit of t
switchgear assembly.

Unit substations and power cente
usually with air-cooled transformers, fo
the most modern arrangement for suppl
ing low-voltage power. These assembli
also may include high-voltage gear if t
plant distribution system makes t
arrangement desirable. The transforme
used in network distribution arrang
ments also can be provided as parts
unit substations or power centers. A c
ordinated housing is provided which giv
good uniform over-all appearance a
full accessibility to the transformers a
switchgear.

AIR CIRCUIT BREAKERS

Air circuit breakers virtually have
placed oil for power-plant auxiliary wor
The switchgear for central-station au
iary service is built around air breakers,
practically all ratings up through t
15,000-volt class. Various forms of hig
voltage air breakers have been used sin
1929, and all have given satisfacto
operation. Their reliability has be
definitely proved. The many advanta
of air interruption, particularly for t
service, are well recognized, and cons
quently they are usually included tod
in the purchaser's specifications.

Figure 2 shows a typical five-kilov
removable unit, 150,000-kva interrupti
capacity, equipped with its own whe

or movement outside the steel cell. Units of five-kilovolt insulation class (for ,300-volt auxiliary service) with 60-cycle est of 25 kv and impulse test of 60 kv are supplied in interrupting ratings up to 150,000 kva and in current ratings up to ,000 amperes. These ratings adequately over the larger central-station auxiliary ield.

The low-voltage breakers are available n all ratings to cover the necessary application to central-station auxiliary lrives. Interrupting ratings at 600 volts o as high as 100,000 amperes, and curent ratings as high as 6,000 amperes at his highest interrupting rating. Figure shows an electrically operated 800-mpere removable breaker and cells for 00-volt application.

Low-voltage breakers often are applied n the cascade principle, an arrangement which permits the lower breakers in the eries to open fault currents in excess of heir interrupting rating. This is pernitted on the basis that the main or roup breakers shall be free to open intantaneously to assist the smaller breakers the interruption. It is obvious that such n arrangement does not provide selective ripping and that feeder circuits other than he one in trouble may be deenergized.

The standard duty cycle is not possible ith breakers applied in cascade arrangements, and it is necessary that they be inpected before being reclosed if they have pened currents above their interrupting bility. This is indicated by the breaker ext higher in the cascade having opened assist in clearing the fault.

Because of the limited selectivity which xists in cascaded circuits, the applicaon of this arrangement in central-staon auxiliary service seldom will be found tisfactory. It usually will be necessary make breaker applications in the same ay that they are made with high-voltage rcuits, with all interrupters of adequate pacity for the faults to which they will subjected. This will frequently mean at the cost of breakers for a given inallation will be somewhat more, but it is e only way in which the essential full lective tripping on faults can be obined. On nonessential auxiliaries, casded arrangements might be considered, t in general their application in central-ation auxiliary installations requires reful scrutiny.

Table I. High-Interrupting-Capacity Contactors

Air Break, 600 Volts and Below

ormal Eight-Hour ating, nperes	Interrupting Capacity, Rms Amperes	Maximum Horsepower Ratings	
		220 Volts	440-550 Volts
150	15,000	50	100
300	25,000	100	200
00	40,000	200	400

Currently, consideration is being given to mechanical timing as a means of co-ordinating breakers in series to avoid unnecessary tripping of circuits other than the faulted one, since breakers with means for instantaneous tripping will frequently go out simultaneously under heavy currents. Such co-ordinated timing is not permissible with cascaded breakers. All breakers in a time-co-ordinated scheme must have fully adequate interrupting capacity.

CONTACTORS, STARTERS, AND FUSES

There has been considerable discussion of the use of interrupters other than circuit breakers for protecting load circuits during high fault currents. High-interrupting-capacity contactors at 600 volts and below, of the air-break variety, are now available. Typical ratings are shown in Table I.

The contactors listed in Table I are designed and rated in accordance with National Electrical Manufacturers Association Standards They are furnished with overload relays which provide thermal time-delay characteristics suitable for motor starting and protection in combination with instantaneous trip to take care of short-circuit conditions. The overload relays can be connected for alarm only on essential auxiliaries.

Still another idea has been receiving consideration to provide a motor starter which has a high interrupting capacity at a moderate cost. This is the combination of an oil-immersed contactor with overload relays and a current-limiting power fuse.

These starters may be applied safely to 2,300-volt circuits having a short-circuit capacity of 150,000 kva and to 4,600-volt circuits having a short-circuit capacity of 250,000 kva. They are available with either 100- or 200-ampere contactors. When determining the size of fuse to be used with a motor of a given rating it is necessary to give consideration to the locked-rotor current as well as the full-load current of the motor so that a co-ordinated design will result and the fuses will furnish the necessary short-circuit protection without blowing under ordinary motor starting conditions.

For essential circuits, such as most power-plant auxiliaries, it is believed that interrupting means that can be electrically or manually closed after fault clearing should be used. If an interrupter must be replaced after each short circuit, valuable time is lost that in the extreme case could be disastrous. For a few of the less essential auxiliary circuits, and particularly where short circuits are least likely to occur, the suggested combination starter can provide a very economical installation.

A blanket recommendation of this starter-and-fuse combination is impossible. Each case must stand on its own merits, or be discarded in favor of an air circuit breaker. If rapid restoration of service is of value in any given case, air breakers are dictated. If the available ratings of current-limiting fuses do not completely satisfy the individual requirements, breakers are again the answer. Central-station auxiliary service can be considered as the application requiring the utmost of dependability. As such, engineering and economics require very careful balancing before a decision to use the fuse and starter is reached.

Power-Distribution System

The radial system of power supply, studied carefully in many alternate forms, for decades has provided satisfactory service. It is characteristic of a radial system that the degree of reliability is somewhat proportional to its cost, and a high-performance system or a very skimpy system can be designed, depending upon just how far the engineer will go in sacrificing economics. The author believes that the more popular and time-proved forms of the radial system are well known to central-station operating men and that detailed description has no justified place in this discussion. Figure 4 pictures a typical radial system of supply to auxiliaries.

THE SECONDARY NETWORK FOR POWERHOUSE SUPPLY

The application of a secondary network to auxiliary supply is rather novel, and detailed description, at least of the salient principles of application, will be included. The fact that central-station engineers have had a long-time knowledge of secondary networks for urban distribution has misled many when power-plant application is under consideration. The different forms that an auxiliary system network may take frequently are overlooked, since the engineer pictures only the city network and hence does not consider flexible arrangements applicable particularly to central-station auxiliary supply. Factors pertinent to the selection of a network system for auxiliary supply will be discussed.

The a-c secondary network system has

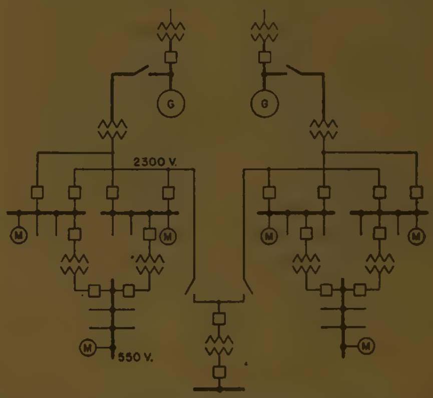


Figure 4. Radial distribution system for central-station auxiliary supply

been used for almost a quarter of a century in the commercial areas of some cities. Today there are 167 cities in the United States whose business districts are served by a-c secondary network systems, and this type of system is now recognized as the standard for serving heavy-load-density areas where the highest grade of service is required. During the last three or four years the advantages of the secondary network system have begun to be recognized in the industrial field, and the system has been installed in about 50 industrial plants. To date, however, the system has been used very little for supplying generating-station auxiliaries. One installation operating at 120/208 volts was made in a small station about eight years ago. Four installations operating at 460 volts have been made during the last four years. The performance of these five a-c secondary networks supplying generating-station auxiliaries has been up to expectations.

Each generating station is a separate problem requiring individual study. The network system for a plant may take the form of a grid, a loop, one or more spot network busses, or a combination of these. It is dangerous to generalize, but detailed studies on 12 stations ranging in size from 10 to 100 megavolt-amperes, indicate several things that are general enough to be of assistance in laying out a network for a particular power station.

First, in more than 50 per cent of the cases, a network grid will be more economical than a secondary loop or spot busses. Small stations are the exception to this rule. Next, a 460-volt grid is cheaper than a 208-volt network, except in very small stations. And also, unless maximum motor size exceeds 600 horsepower, all auxiliaries should be supplied from a 460-volt network. Motors above 600 horsepower should be supplied directly at 2,300 volts, or from a grid or spot networks at this voltage. The same primary windings are used for both the 2,300- and 460-volt networks. If 208 volts should prove economical in a small station, then all motors larger than about 50 or 75 horsepower should be served directly at 2,300 volts, avoiding the third voltage classification. A typical network for auxiliary supply is shown in Figure 5.

Four points afford a reasonable criterion for a distribution system in a power plant: reliability, flexibility, simplicity of operation and ease of maintenance, and economics. The secondary network will be analyzed briefly with respect to these qualifications.

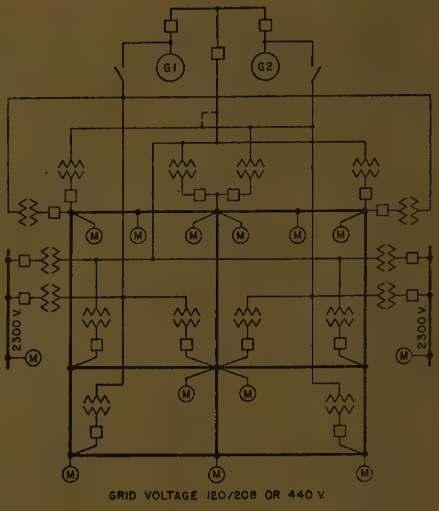


Figure 5. Secondary-network distribution system for central-station auxiliary supply

The secondary network system provides maximum reliability and flexibility. The many years of experience with city network systems indicates that they are substantially as reliable as their source of supply and that they can be readily and economically expanded to take care of load growth. The network system is at least as simple as any type of system which will give somewhere near the same degree of reliability. The fact that any primary feeder together with its associated transformers can be removed from service without any interruption to service facilitates the operation, maintenance, and expansion of the system. Based on a number of careful economic studies, the investment cost of the network system has been found to vary from about 15 per cent more to about 14 per cent less than the cost of a radial system. In about 75 per cent of these studies the network system showed a saving of from 6 to 14 per cent. The result of any cost comparison will depend upon the form of network system used and the form of radial system with which it is compared. If lesser reliability is accepted, a radial system always can be designed that is the cheapest usable scheme. The foregoing figures refer to radial systems currently used and considered adequate.

The matter of short-circuit current on a secondary network is something that must be watched in designing the system; otherwise the fault currents may run considerably higher than on a radial system, and the cost of the load circuit breakers will become excessive. The following are methods of keeping the interrupting duty on the load circuit breakers down.

1. Limit the amount of load on any one secondary grid or loop to a maximum o around 5,000 or 6,000 kva. This usuall means a separate secondary grid or loop fo each 100,000 kva of generating capacity o thereabouts, and less if small generators ar involved and all load is supplied at lo voltage.

2. Where necessary, two or more separat secondary grids or loops can be supplie from the same group of primary feeders The secondary networks preferably shoul be supplied over three or four primary feeders to keep the amount of spare-networ transformer capacity to a small value.

3. The network transformers should hav an inherent impedance of around ten pe cent. This high impedance has the additional advantage of improving load divisio between the transformers supplying a network, and of minimizing circulating current between primary feeders whose voltage may be slightly different in magnitude an phase angle.

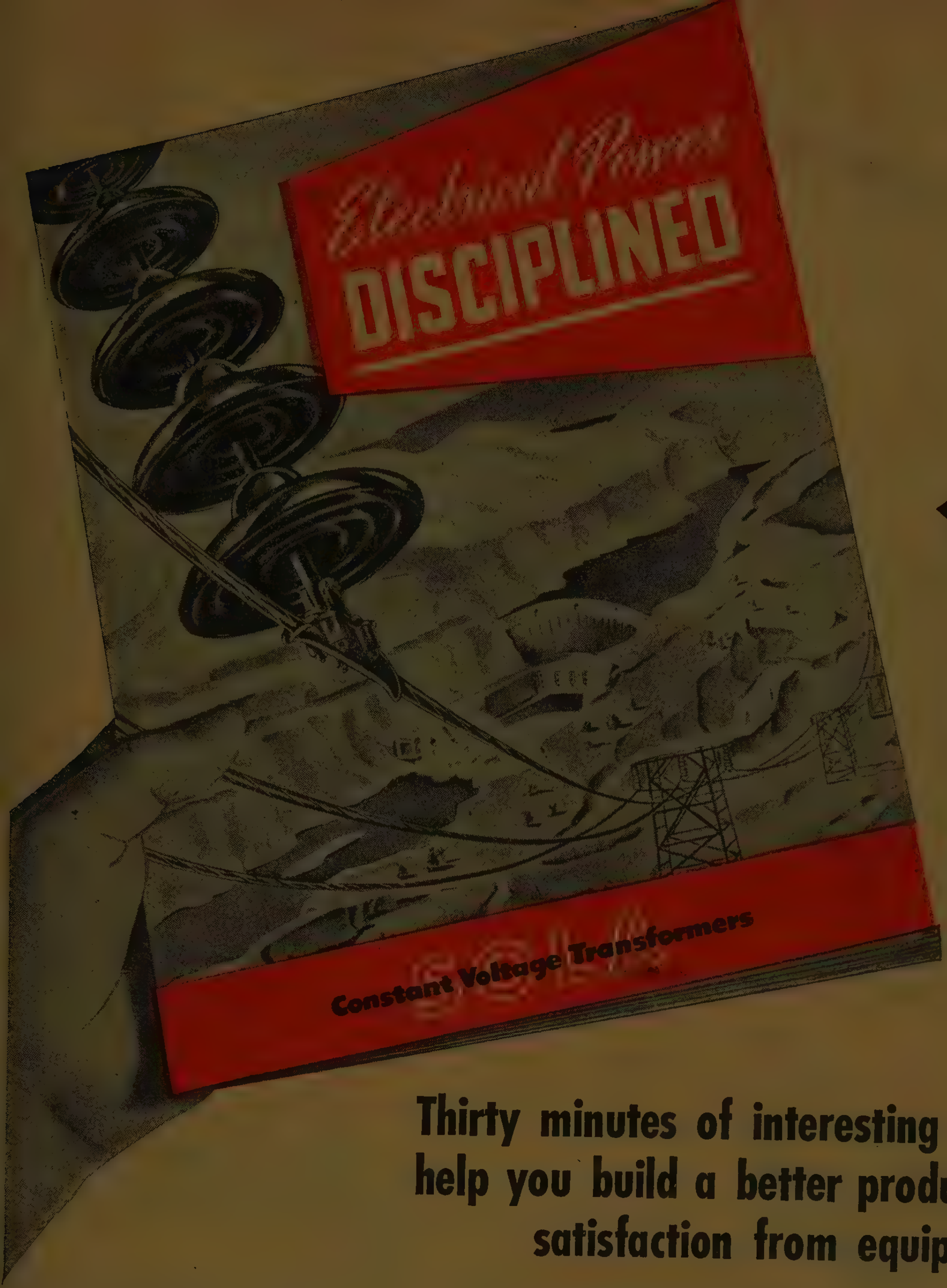
4. If a spot network system is employe instead of a grid or loop form, the amount o load per spot bus should be kept down t 1,500 to 2,500 kva.

If the foregoing points are kept in min as guides when a network system is bein designed for supplying power to generating-station auxiliaries, the advantages o reliability, flexibility, and simplicity o this type of system usually can be obtained at less cost than those forms o radial system which will give somewher near comparable reliability.

As stated, each generating station requires individual study. If the best system for a particular station is to b chosen, such study should be starte early, before physical details of statio design place limiting factors on the application of one type of system or th other. Whether new generation involve a new plant or simply additions to existing plant, of course affects the choice although this fact is not necessarily limiting in itself.

Conclusion

In conclusion the author wishes to repeat that the foregoing paragraphs wer selected as high lights, not as a genera coverage on the subject of auxiliary apparatus and distribution systems. Th selection of items and the discussion o them prove that central-station auxiliar performance will be improved throug better supply-system layout and more reliable individual motors and control. I is sincerely hoped that the discussion ma prove of help to central-station enginee when auxiliary apparatus specification and auxiliary power-supply problems ar under consideration.



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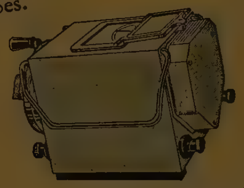

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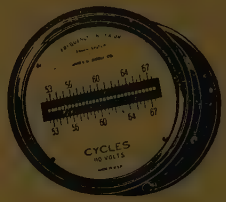

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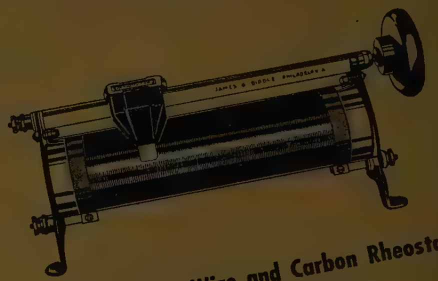

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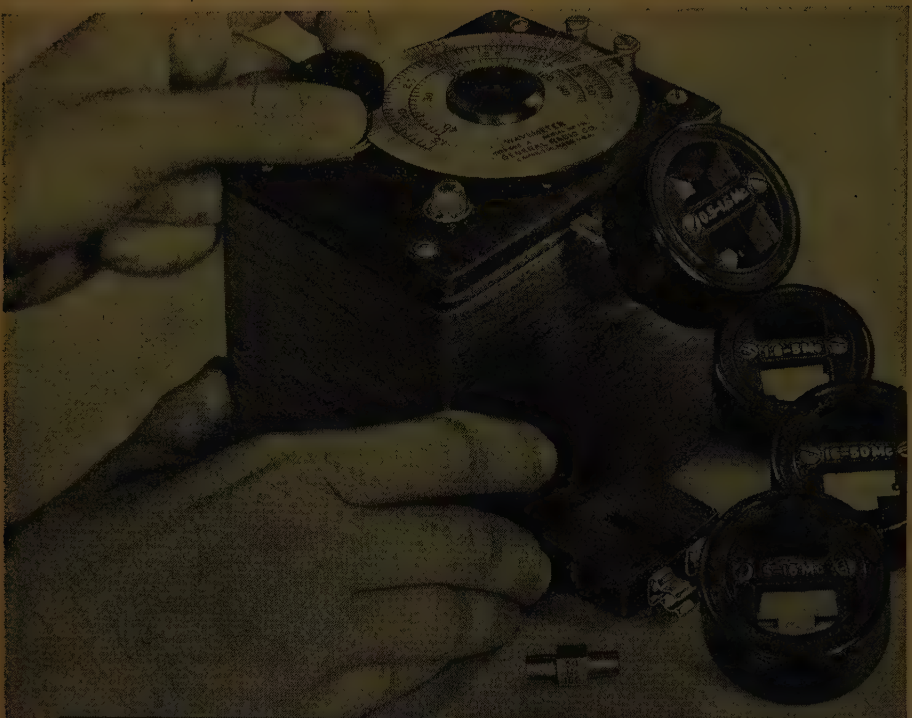

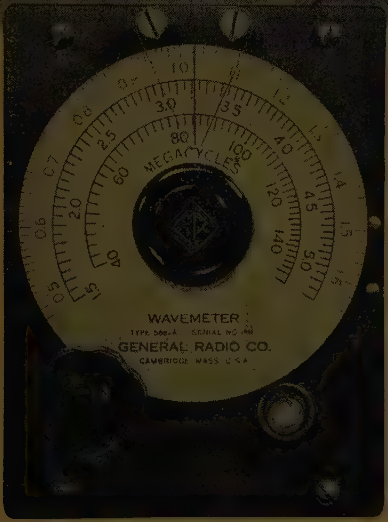

A WAVEMETER with very useful range

NOW AVAILABLE (at the moment from stock) the popular Type 566-A Wavemeter leaves the war effort to return to the civilian laboratory. This meter with its very wide range of 0.5 to 150 Mc (600 to 2 meters) is an extremely handy gadget in any laboratory. Its accuracy is sufficient for a large number of measurements such as determination of coil ranges, oscillator spans, lining up oscillators and transmitters, locating and naming harmonics in either the receiver or the transmitter, and for general experimental work.

All five plug-in coils are stored in a rack on the side of the cabinet. It weighs only three pounds and can be held in the palm of one hand.

Despite its modest price, this wavemeter is built with the same care and is calibrated with the same thoroughness as the most expensive piece of G-R measuring gear.

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SPECIFICATIONS
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FREQUENCY RANGE: 0.5 to 150 Mc

COILS: Five plug-in coils furnished

DIAL: Direct reading in frequency to 2% or better

ACCURACY: 2% for 0.5 to 16 Mc; 3% for 16 to 150 Mc

RESONANCE: Indicator is small incandescent lamp. Two spares supplied

DIMENSIONS: 4¾ x 5⅞ x 5¾ inches, overall

NET WEIGHT: 3 pounds

PRICE: $45.00

GENERAL RADIO COMPANY Cambridge 39, Massachusetts

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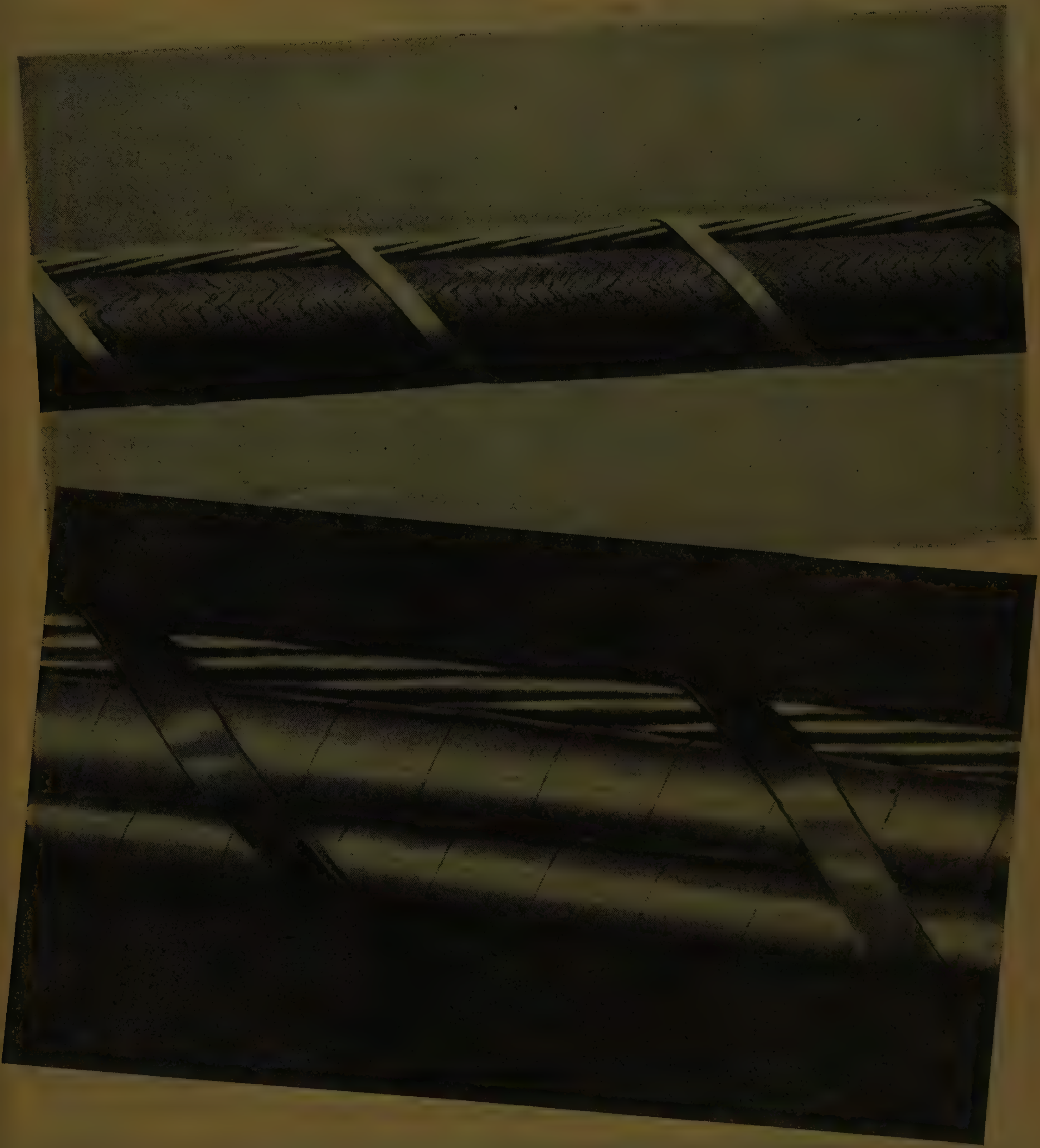

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This is a record of which we are justifiably proud. During our "span of life", we have put forth our every effort to serve the electrical industry faithfully and efficiently in the capacity of insulating varnish specialists.

There has never been, and never will be an insulating problem too difficult for us to do our utmost to solve; or an account or prospective account too "small" for us to service. This policy in itself has brought to us thousands of friends and satisfied users of DOLPH'S Products.

We pledge our continuance of this policy, and with our years of experience and research in the insulating varnish field, we feel fully confident that we can assist you in your own particular insulating problems. Write us; we welcome the opportunity to serve you.

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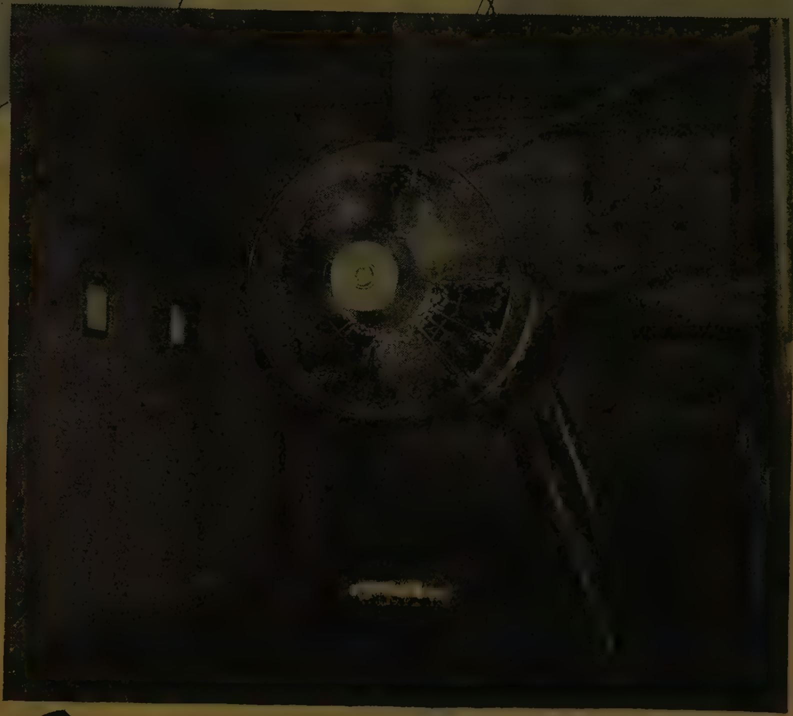
STILL BETTER AIRCRAFT ENGINES BECAUSE OF WRIGHT AERONAUTICAL'S TEST TECHNIQUE BASED ON . . .



A Wright Cyclone engine running on the test stand. Electrical resistance strain gages affixed to various component parts and wired to the remote oscillographic equipment, serve to indicate stress and load under actual operating conditions.

DuMONT *Oscillography*

➧ *Write for literature.*

➧ What are the stress and load on various component parts during *actual operation!* The answer is vital in the design and development of aircraft engines.

Engineers of Wright Aeronautical Corporation use an electrical resistance gage and Wheatstone Bridge, in combination with the DuMont Type 208 Oscillograph, to secure a quick, accurate, explicit answer. They report:

"With this monitoring means, both amplitude and wave form are easily observed. The oscillograph allows immediate observation of sharp changes in amplitude such as occur with relatively undamped resonant phenomena which may be troublesome. Such conditions may thus be recognized quickly and, if necessary, the test procedure adjusted to allow for closer investigation in the critical range.

"Simultaneously, changes in wave form can also be ascertained. Points at which changes in wave form occur may be quickly observed and given closer study. Also, the observer can detect any erratic circuit operation or malfunctioning associated equipment."

This simple technique saves time and money in aircraft engine development. Doubtless other applications of DuMont Oscillography can do a comparable job for you.

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A tight-spot installation is typified by the new 113-foot angle tower at the extreme right. Anchored to its top are the power house double-circuits which here separate into two single circuits; one top-level circuit is diverted back of the old river crossing tower to the substation; the other is dropped some 30 feet and routed under power lines of the river crossing to the substation. The old river crossing tower and adjacent substation were also American Bridge projects.

Typical double-circuit tower carrying transmission lines over the roof of the power house. The unusual flare at the base provides direct connection and anchorage to existing roof trusses under the roof covering.

Double-circuit strain and angle tower, about 120 feet high, located between two existing substations and to clear storage yard conveyor systems.

A special type single-circuit, 57-foot, strain tower provides for an acute angular change in the transmission line from the power house to the substation.

TOWERS...

individually designed and tailored to the job!

DUE to increasing war loads, a recent expansion to generating capacity of an existing electric plant also involved the installation of additional circuit lines for distributing the new power to existing substations and main transmission lines.

The installation of supporting towers for the new circuits, because of limited ground space and overhead clearances imposed by existing structures and a complicated network of power lines, *and the need for quick action*, posed many unusual and difficult engineering and construction problems. All were satisfactorily solved, and with dispatch, by our tower department's technical staff in collaboration with the power company's engineers.

Because of the zigzagging course of the new circuits—around, over and under existing structures and power lines—each tower and its foundation were individually designed and tailored to prevailing site-conditions and to loadings from horizontal and vertical angular changes in alignment.

Pictured here are four of the many tower types designed and fabricated by American Bridge for this particular project. They are indicative of the accumulated experience and ingenuity of our tower engineers, and the fabricating and galvanizing resources of our tower plant.

In emergencies such as this, and when the time comes for postwar construction, both services are yours to draw upon.

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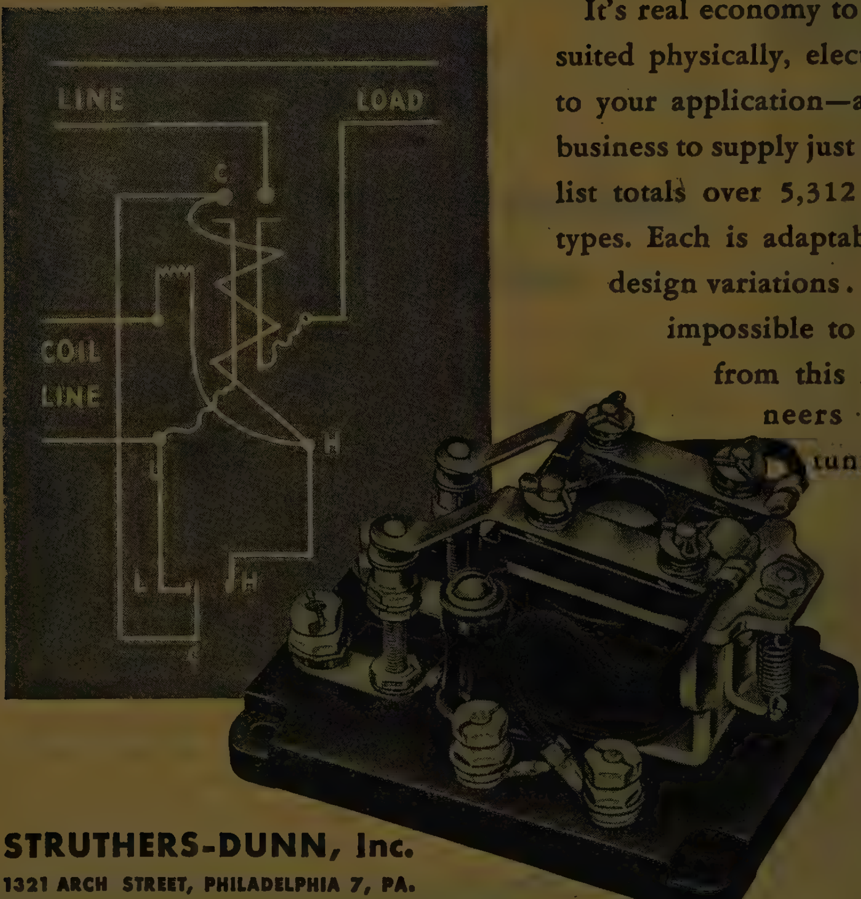
UNITED STATES STEEL

FIT THE RELAY TO THE JOB....

NOT THE JOB TO THE RELAY



It's real economy to use relays that are *exactly* suited physically, electrically, and mechanically to your application—and it is Struthers-Dunn's business to supply just the unit you need. Today's list totals over 5,312 standard Struthers-Dunn types. Each is adaptable to numerous coil and design variations . . . but, should it still prove impossible to match your specification from this list, Struthers-Dunn engineers welcome the opportunity to "tailor" a new relay type to your exact measure.

A typical application of the Struthers-Dunn Type 8BXX50 electrical lock-in relay used with a 3-wire "high-low" temperature control thermostat.

STRUTHERS-DUNN, Inc.
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5,312 RELAY TYPES

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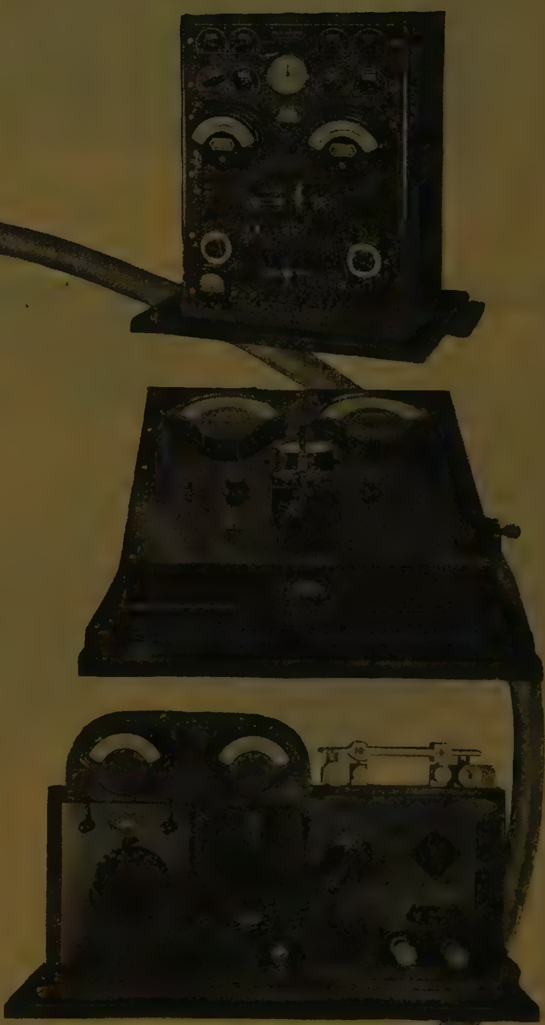

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Literature describing the complete line of WESTON panel and test nstruments is freely offered . . . Weston Electrical Instrument Corporaon, 664 Frelinghuysen Avenue, Newark 5, New Jersey.

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LABORATORY INSTRUMENTS FOR SPEED AND ACCURACY

TAILOR-MADE WAVES FOR RAPID TESTING

The *-hp-* Model 210A Square Wave Generator is designed to amplify and clip the tops of a sine wave, and thus convert it into a wave which has vertical sides and a flat top. When the square wave voltage is applied to the amplifier or network under test, the shape of the output wave immediately shows up any distortion.

In production testing, one or two observations with an *-hp-* Square Wave Generator will accurately check the frequency response. In development work, the *-hp-* Model 210A shows up phase shift and transient effects, both of which are difficult to study by other methods.

In practice, a wave which appears to be perfectly square will contain 30 or more harmonics; and when the amplitude or phase relation of the harmonics is disturbed, the square wave will be distorted. (See Fig. 1.) Thus the application of a square wave to a circuit shows up any irregularities in amplitude or phase transmission of that circuit, not only at the square wave frequency, but also at frequencies far removed from the test point. These characteristics are particularly important in television video amplifier work.

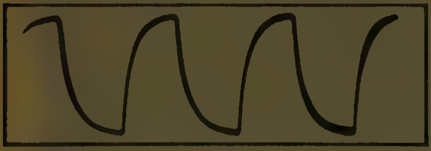

Fig. 1—Shows the square wave distortion caused by poor high frequency response

The output of the generator is square within 1 percent over the frequency range from 20 cps to 10,000 cps; a relatively square wave can be obtained even at 100 kc. The frequency response of the attenuator is sufficiently wide so that the output wave shape is not affected even at the highest frequencies. Once proper criteria have been established, the *-hp-* Model 210A Square Wave Generator is the modern, rapid means of production testing, with the speed and accuracy which are characteristic of all *-hp-* instruments.

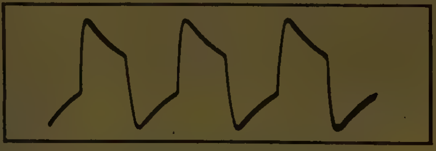

Fig. 2—Low frequency phase distortion serious in television video circuits

Write for complete details on the Model 210A. Ask for *-hp-* Catalog No. 17B, which includes much valuable information on development and measurements.

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Audio Frequency Oscillators　Signal Generators　Vacuum Tube Voltmeters
Noise and Distortion Analyzers　Wave Analyzers　Frequency Meters
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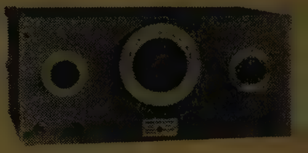
OTHER -hp- INSTRUMENT



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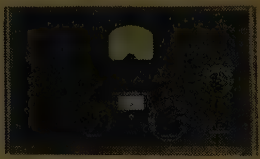

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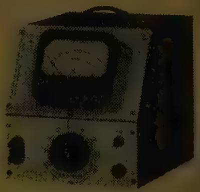
Combines a vacuum-tube voltmeter with a set of fundamental elimination filters for general purpose measurements of total harmonic distortion, noise and voltage level.



VACUUM TUBE VOLTMETER MODEL 400A

Makes accurate voltage measurement from 10 cycles to 1 megacycle, cover nine ranges, (.03 volts to 300 volts) with full scale sensitivity.

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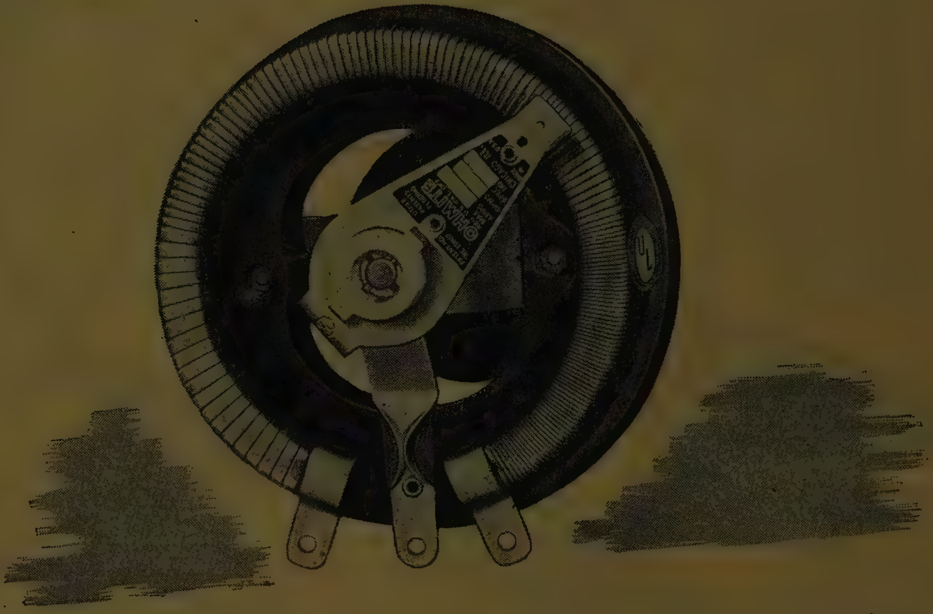
Alcoa can take care of your power conductor requirements for rural and hi-lines. Joints and other accessories for this construction are likewise available.

Aluminum Cable Steel Reinforced has the high strength and dependability required in modern line construction. Alcoa engineers will help you select conductors having suitable characteristics. Write ALUMINUM COMPANY OF AMERICA, 2149 Gulf Bldg., Pittsburgh 19, Pa.

ALCOA A-C-S-R

ALUMINUM CABLE STEEL REINFORCED

Smoother, Closer Control of Generator Voltage with OHMITE Field Rheostats



YOU get exact control *plus* permanently smooth operation . . . in a *compact* unit. The rheostats are tapered or uniformly wire-wound for separately or self excited generators. Continuous variation of resistance in even the smallest sizes saves control-panel space. This saving in space and weight is particularly valuable in portable equipment.

With a series of ten sizes from 25 to 1000 watts available in single or tandem units, there is an Ohmite Field Rheostat suitable for every small or medium size generator. Rheostats designed for special applications. Consult Ohmite engineers on your control problem.

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Foremost Manufacturers of Power Rheostats, Resistors, Tap Switches

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Write on company letterhead for this complete Catalog and Engineering Manual. Gives valuable data on rheostats, resistors, tap switches.

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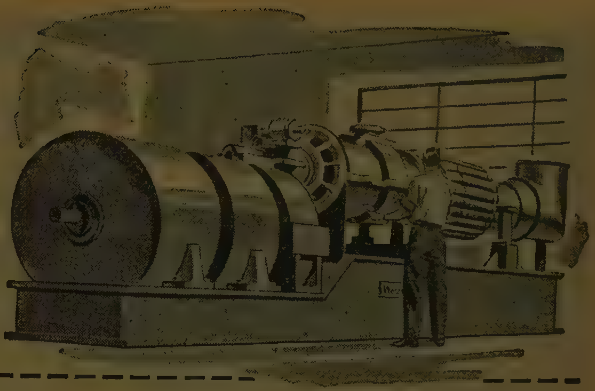
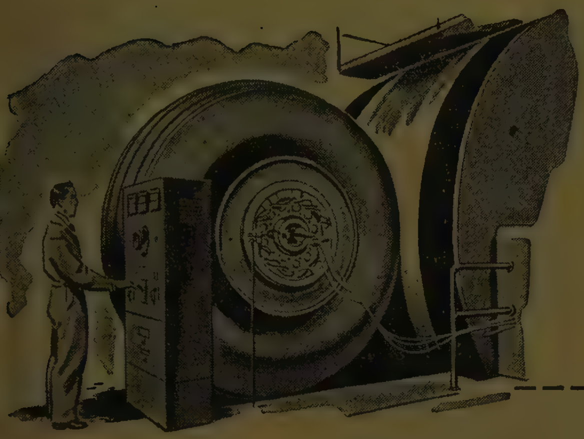
In a laboratory a SCIENTIST experiments with a new gas turbine . . . using heat-resisting alloy blades that are far stronger, at 1100°F., than *ordinary* steel at room temperature.

..the name on the GAS TURBINE is Westinghouse.





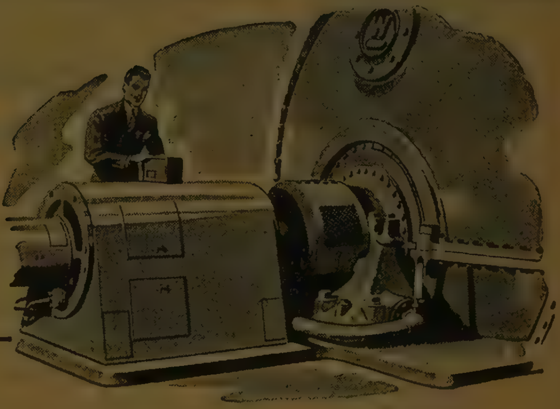
On a special machine a TESTER employs a Rototrol* for smoothly accelerating a large flywheel, used in determining the wear-resisting qualities of tires and brakes—for huge air transports of the future.

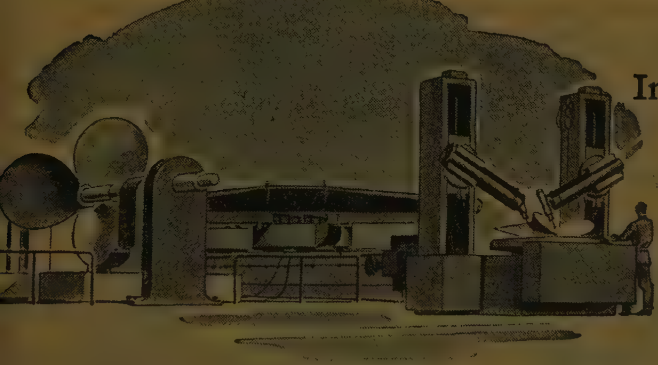
...the name on the ROTOTROL is Westinghouse.

*Registered Trademark

In a power plant an ENGINEER uses a Vibrograph to "take the pulse" of a turbo-generator . . . recording the smallest vibrations as a trace on a film.

..the name on the VIBROGRAPH is Westinghouse.





In a manufacturing plant an OPERATOR uses an electronic control to regulate the movement of milling cutters—for accurately machining irregular contours on giant ship propellers.

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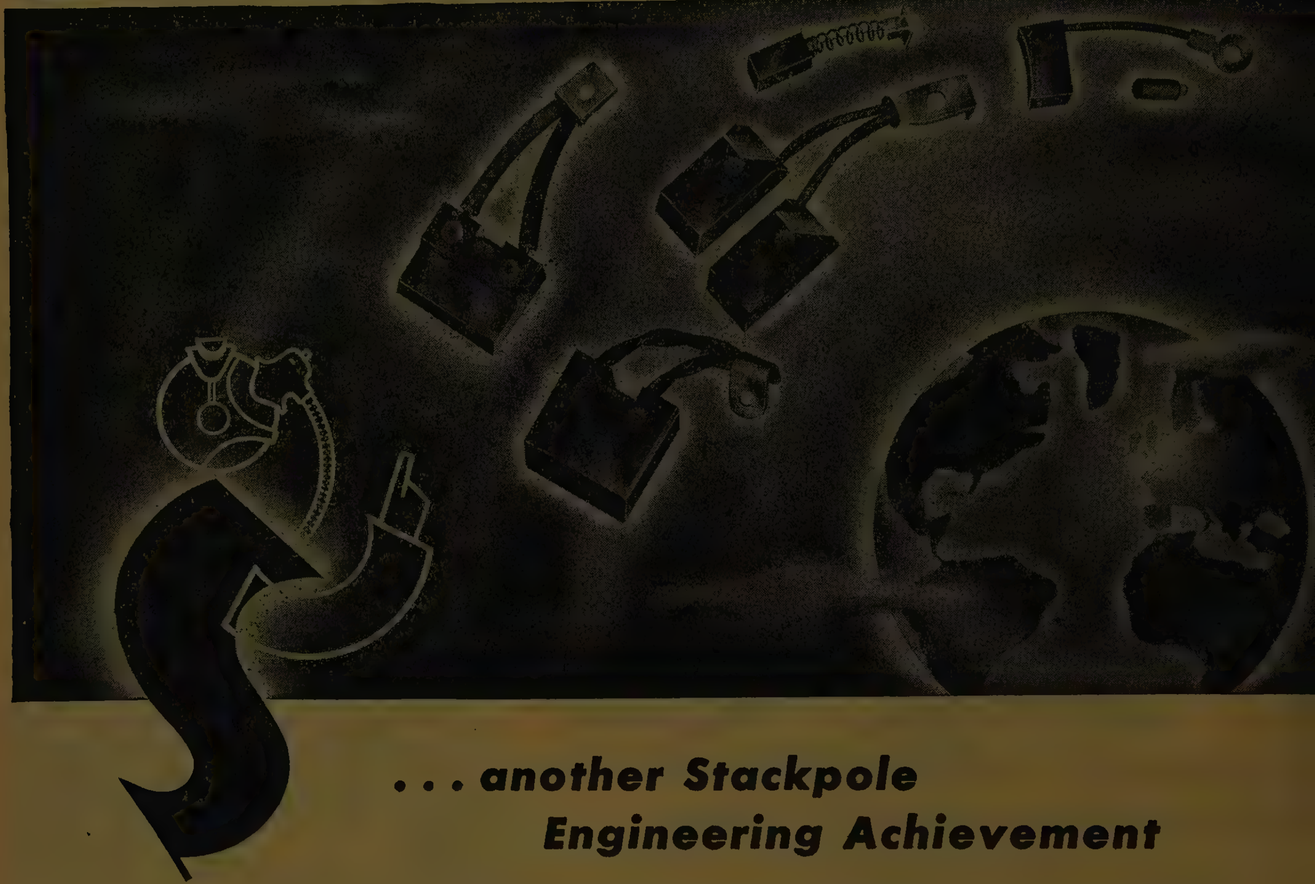

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Brush problems increase tremendously with altitude—and it has remained for Stackpole to overcome these by producing the first really outstanding brushes for high altitude performance. These brushes are treated with a special process developed by the Westinghouse Electric and Manufacturing Co., and have resulted in greatly increased brush life for high-altitude flying.

Such progress is not accidental. It is the natural result of long years of leadership in producing better brushes for all conditions of use.

So rapid have been Stackpole developments in brushes for many kinds of rotating equipment that, whatever your requirements, it pays to check them regularly against the latest types this advanced engineering may have to offer.

(Stackpole Brushes are sold only to manufacturers of original equipment. Replacements can be obtained from the original equipment manufacturers.)

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STACKPOLE *Brushes*

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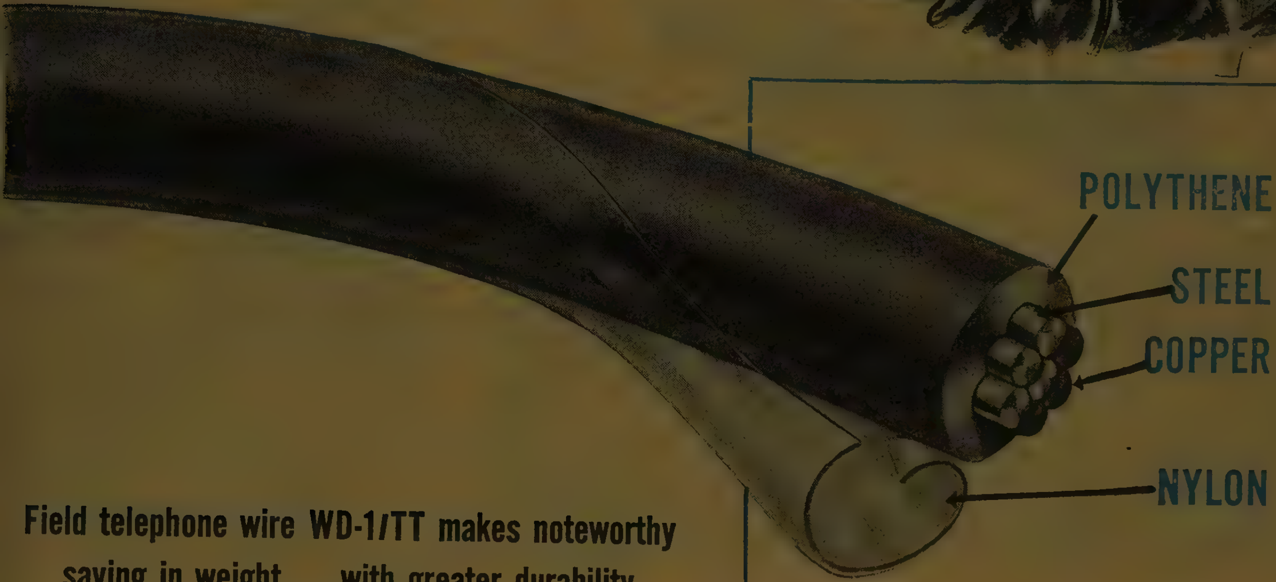
CARBON-GRAPHITE
SILVER-GRAPHITE

FOR ALL TYPES OF
ROTATING EQUIPMENT

ELECTRO-GRAPHITE
COPPER-GRAPHITE
METAL COMPOSITION, ETC.

Insulated with
Du Pont POLYTHENE
Jacketed with
Du Pont NYLON



Section of Field Wire WD-1/TT, magnified approximately 12 times. Wall thickness of polythene insulation; 18 mils. Wall thickness of nylon jacket; 5-6 mils. Overall diameter; 80-82 mils. Average weight per mile: 48 lbs.

Field telephone wire WD-1/TT makes noteworthy saving in weight . . . with greater durability

Here's where *two* Du Pont plastics teamed up to fill a recent rgent need of the Signal Corps. Specifications called for a onsiderably lighter field telephone wire, which would possess he necessary durability as well as a talking range equal to hat of the heavy Army field wire.

Du Pont polythene, because of its outstanding electrical roperties, needs less weight to do a given job of insulation. olythene alone is widely used as insulation on some of the ghtest types of Army wire. But for this field wire it was necesary to protect the insulation against all kinds of weather, and brasion from rocks, trees, sharp corners. That's where the oughness of nylon was called on to help. A wall thickness of nly 5-6 mils of nylon, extruded rapidly over the polythene sulation, gave the necessary protection—and the finished ire weighs a little more than a third as much as the heavier pe of Army field wire—though the talking range is the same.

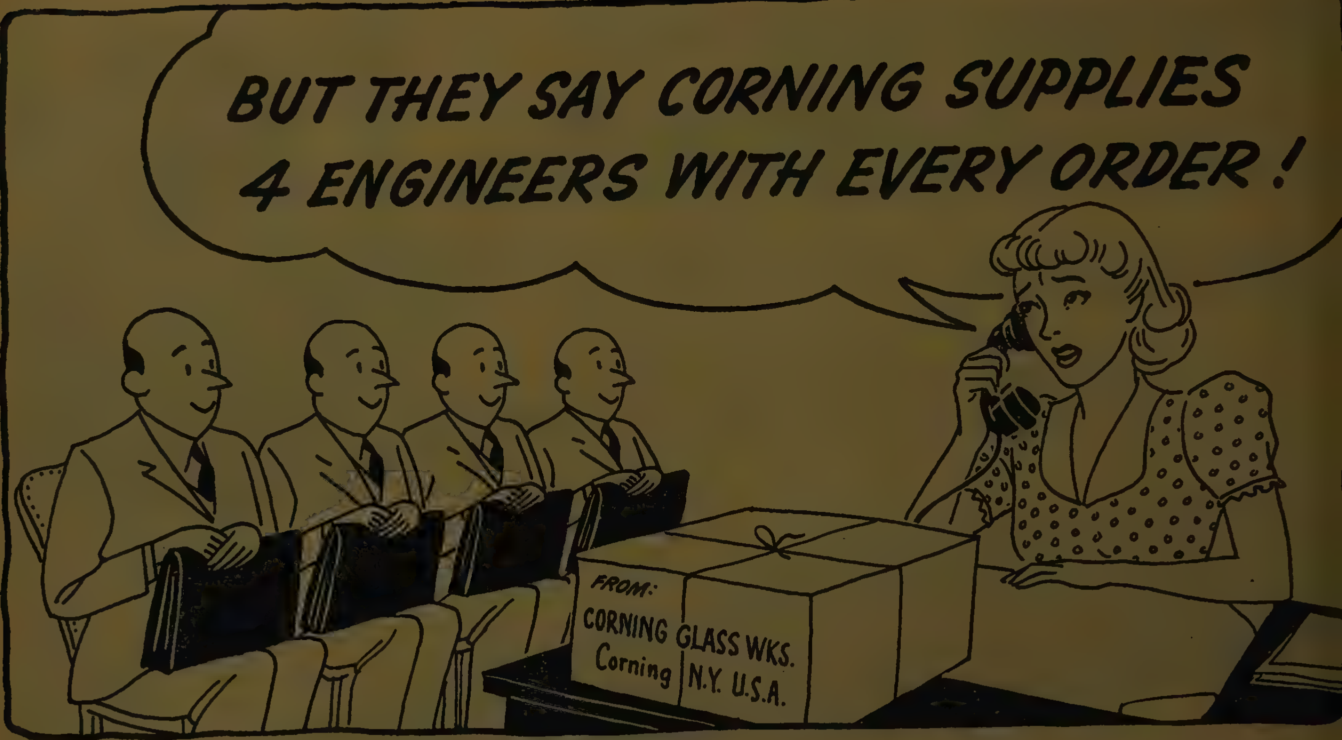
For information on nylon, polythene and other Du Pont astics, write E. I. du Pont de Nemours & Co. (Inc.), Plastics ept., Arlington, N. J.

Share in the Victory—Buy Bonds

FOR PLASTICS . .
CONSULT DU PONT

DU PONT

BETTER THINGS FOR BETTER LIVING
...THROUGH CHEMISTRY


BUT THEY SAY CORNING SUPPLIES 4 ENGINEERS WITH EVERY ORDER!
FROM:
CORNING GLASS WKS.
Corning N.Y. U.S.A.

HOW 4 SPECIAL CORNING SERVICES CAN SAVE YOU LOTS OF GRIEF!

IT'S quite a job getting a new electronic product into production. Materials, methods and prices buzz around your head like a bunch of bees. But you don't have to solve your problems all alone. For Corning has four special engineering services to help you:

1. Sales Engineers—To keep you in touch with latest developments and explain your problems to Corning's technical experts for prompt solution.

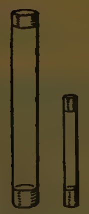
2. Product Engineers—Technical men who translate Corning Research in Glass into practical applications which may solve your particular headaches.

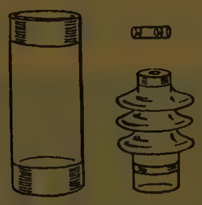
3. Plant Engineers—These men are anxious to see you get the best possible price on your order. They often point out changes in design which reduce costs.

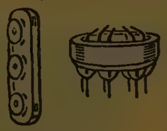
4. Technical Service Engineers — These men get you starte right. They help your people lick the production bugs.

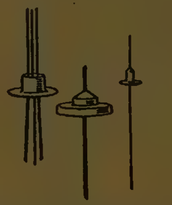
Of course, Corning Electronic Glassware also mean thousands of glass formulae so you can get the righ one for your job. It means Corning's unique metallizin process forming a permanent bond between glass an metal. Tubes, bushings, headers, etc., can be soldere in place to form permanent hermetic seals. It mean an entire plant at Bradford, Pa., devoted exclusivel to the manufacture of electronic specialties quickly in large quantities. To get the fastest service in solvin your pet problem, write, wire or phone Electroni Sales Department, N-12, Technical Products Divisio Corning Glass Works, Corning, New York.

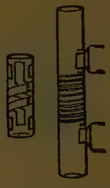
Note—The metallized Tubes and Bushings, Headers and Coil Forms below are all made by the famous Corning Metallizing Process. Can be soldered into place to form true and permanent hermetic seals. Impervious to dust, moisture and corrosion.

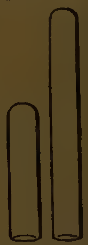

Metallized Tubes for resistors, capacitors, etc. 20 standard sizes ½" x 2" to 1¼" x 10". Mass-produced for immediate shipment.


Metallized Bushings. Tubes in 10 standard sizes, 5/8" x 25/32" to 1" x 4½" in mass production for immediate shipment.


Headers—The best way to get a large number of leads in a small space for assembly in one operation.


Eyelet Terminals—Single or multiple eyelets permit design flexibility. Standard items readily available in quantity.


Coil Forms—Grooved for ordinary frequencies—metallized for high frequencies. In various designs and mountings.


VYCOR Brand cyli ders—very low lo characteristic Stands therm shock up to 900° Can be metallized.

CORNING
means
Research in Glass

Electronic Glassware

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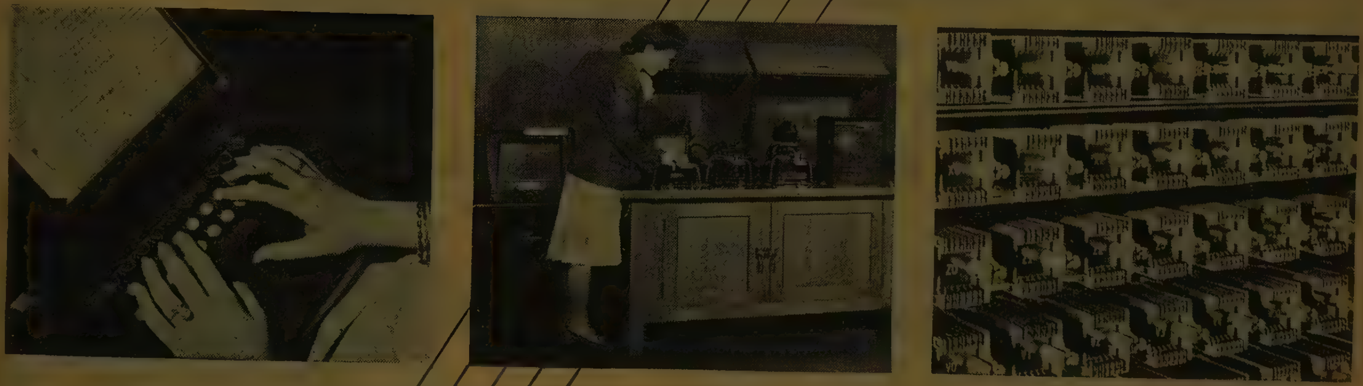
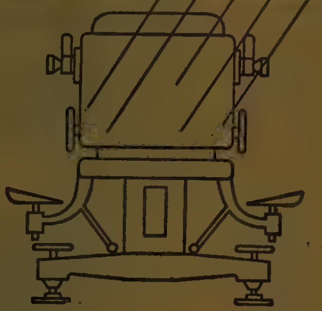
Target practice with Relays and Keys



(Left to right) The operator punches the problem data on tape, which is fed into the computer. The solution emerges in the teletype receiver. Relays which figure out the problem look like your dial telephone system.



In designing the gun-control systems which shot down enemy planes, Army ballistic experts were faced by long hours of mathematical calculations.

So Bell Laboratories developed an electrical relay computer. It solved complicated problems more accurately and swiftly than 40 calculators working in shifts around the clock.

Resembling your dial telephone system, which seeks out and calls a telephone number, this brain-like machine selects and energizes electric circuits to correspond with the numbers fed in. Then it juggles the circuits through scores of combinations corresponding to the successive stages of long calculations. It will even solve triangles and consult mathematical tables. The operator hands it a series of problems with the tips of her fingers — next morning the correct answers are neatly typed. Ballistic experts used this calculator to compute the performance of experimental gun directors and thus to evaluate new designs.

In battle action, Electrical Gun Directors are, of course, instantaneous. Such a director helped to make the port of Antwerp available to our advancing troops by directing the guns which shot down more than 90% of the thousands of buzz bombs.

Every day, your Bell System telephone calls are speeded by calculators which use electric currents to do sums. Even now, lessons learned from the relay computer are being applied to the extension of dialing over toll lines.

BELL TELEPHONE LABORATORIES

EXPLORING AND INVENTING, DEVISING AND PERFECTING FOR CONTINUED IMPROVEMENTS AND ECONOMIES IN TELEPHONE SERVICE

THE CORRECT ANSWER TO YOUR RECTIFIER PROBLEM IS HERE

G-E Builds All Three

Selecting the rectifier best suited for a particular d-c application is not a decision that can be made on a "guess" basis. Construction, basic materials, operating characteristics, weight, size, cost and life expectancy are all factors that should be considered.

G.E. and only G.E. builds the three types of low-voltage rectifiers most generally used—copper-oxide, selenium and Tungar. All three are tops in quality and leaders in their field. To say that one type is better than another is as fatuous as saying a bomber is better than a fighter plane. Each performs best when doing the job for which it was specifically designed.

When blueprints call for rectifiers choose the correct size and type from the G-E line. If you're not sure of what is best for your need let G-E engineers help you. Years of experience qualify them to recommend the rectifier which will give you the most economical, most efficient and most reliable performance. Whether they recommend copper-oxide, selenium or Tungar you can be sure their selection is impartial because G.E. offers all three.

For more information write to Section A1255-32, Appliance and Merchandise Dept., General Electric Co., Bridgeport, Conn.

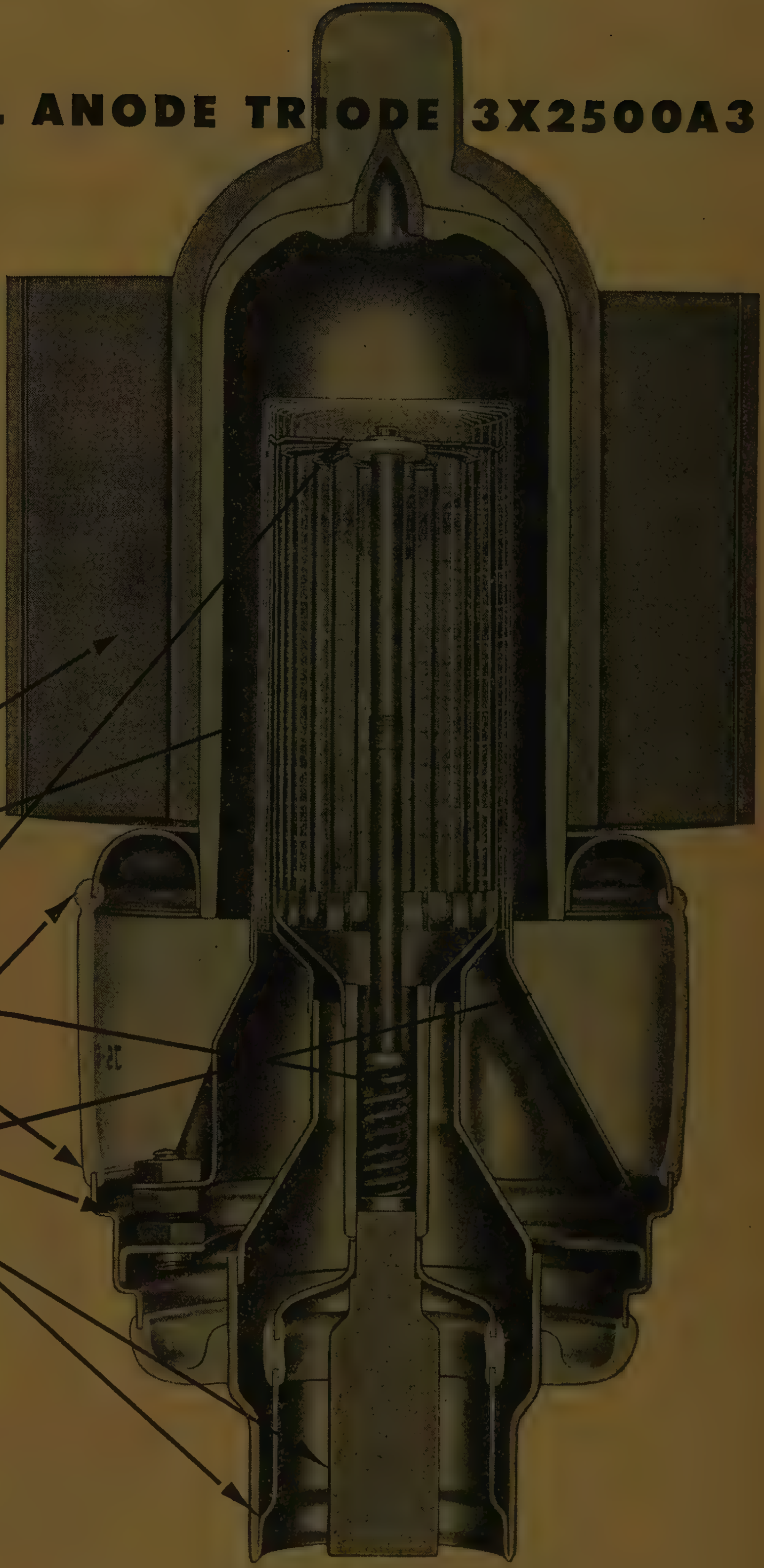
Hear the General Electric radio programs: "Th G-E All Girl Orchestra" Sunday 10 p. m. EST NBC. "The World Today" news every weekda 6:45 p. m. EST, CBS. "The G-E House Party Monday through Friday 4:00 p.m. EST, CBS.

BUY VICTORY BONDS AND KEEP THEM

GENERAL ELECTRIC

Eimac TUBES THE COUNTERSIGN OF DEPENDABILITY IN ANY ELECTRONIC EQUIPMENT

NEW EIMAC EXTERNAL ANODE TRIODE 3X2500A3



Rugged mechanical construction
Outstanding electrical efficiency

In the new 3X2500A3, Eimac engineers have developed a highly efficient external anode triode which, in Class C service, delivers up to 5 KW output at a plate voltage of only 3,500 volts. The mechanical design is radically simple, incorporating a "clean construction" which gives short, low inductance heavy current connections that become an integral part of the external circuits at the higher frequencies.

The external anode, conservatively rated at 2500 watts dissipation, has enclosed fins so as to facilitate the required forced air cooling.

Non-emitting vertical bar grid does not cause anode shadows ordinarily created by heavy supports in the grid structure.

Thoriated tungsten filament. Note unusually large filament area, and close spacing.

Filament alignment is maintained throughout life of the tube by special Eimac tensioning method.

New glass-to-metal seals do not have the RF resistance common to iron alloy seals, nor the mechanical weaknesses of the feather-edged types.

Grid ring terminal mounts a cone grid support which acts as a shield between plate and filament.

A coaxial filament stem structure forms the base of the tube. This makes possible proper connections to the filament lines.

Grid and filament terminal arrangements make it possible to install or remove the 3X2500A3 without the aid of tools.

The new mechanical and electrical features of the Eimac 3X2500A3 external anode triode make it valuable for use on the VHF as well as low frequencies. More complete data and information yours for the asking.

OLLOW THE LEADERS TO

Eimac REG. U.S. PAT. OFF. TUBES

TEL-McCULLOUGH, INC., 1123 San Mateo Ave., San Bruno, Calif.

nts located at: San Bruno, Calif. and Salt Lake City, Utah

TYPE 3X2500A3 — MEDIUM MU TRIODE

ELECTRICAL CHARACTERISTICS

Filament: Thoriated Tungsten	
Voltage	7.5 volts
Current	48 amperes
Amplification Factor (Average) 20	
Direct Interelectrode Capacitances (Average)	
Grid Plate	20 $\mu\mu$fd.
Grid Filament	48 $\mu\mu$fd.
Plate Filament	1.2 $\mu\mu$fd.
Transconductance (i_B=830 ma., E_B=3000 v.) 20,000 μmhos	

NEW EXOTHERM

of arc extinction gives added safet

EXOTHERM ARC QUENCHERS— the reason for low tank pressures Pressure produced by striking arc is confined.

Roller-Smith Exotherm Type Oil Circuit Breaker, 150 TCR-5; 15,000 volts; 1200 amperes; 150,000 kva.

PRINCIPLE

of Low Tank Pressures

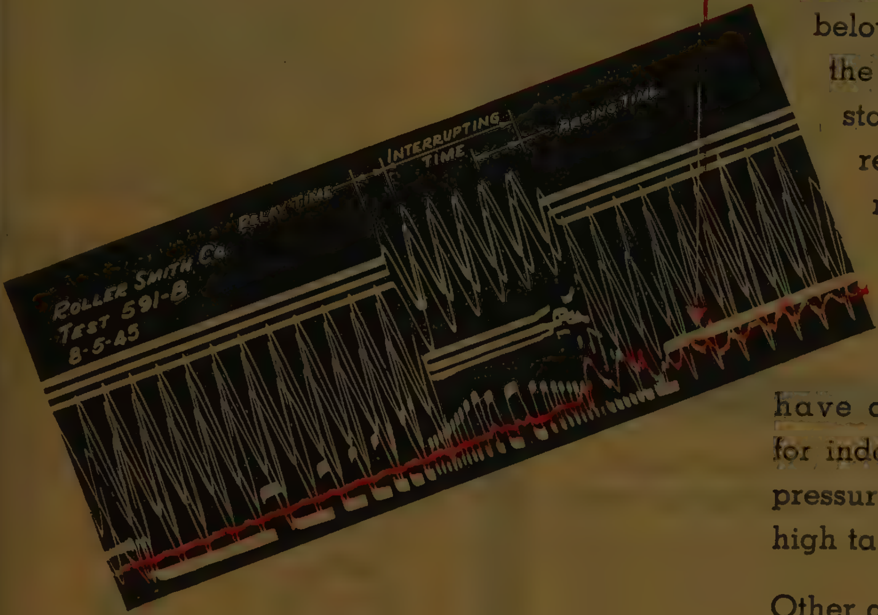
THE Roller-Smith Exotherm principle of arc extinction confines the heated gases in an arc chamber. The arc is extinguished by the combination of internal pressure and lowered gas temperature.

By harnessing the gas pressure within the quencher, the pressure in the tank itself remains at a nominal value well below any danger point. Every test at the Roller-Smith "On the Line" test station includes the tank pressure reading along with voltage, current and time. Average tank pressures do not exceed 40 pounds per square inch.

Exotherm oil circuit breakers have a far greater factor of safety for indoor use as a result of the low pressure than breakers with the usual high tank pressures.

Other advantages of the new Exotherm principle are: 5 cycle operation; reduced contact burning; reduced maintenance and inspection; more consistent arc performance.

For full information on the Exotherm principle applied to Roller-Smith oil circuit breakers, write to dept. EE-6



TANK PRESSURE—Typical test oscillogram of Roller-Smith oil circuit breaker at full rated capacity—note slight fluctuation in tank pressure.

ROLLER SMITH

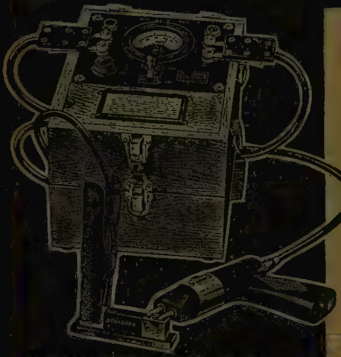
ROLLER SMITH

Oil Circuit Breakers

BETHLEHEM • PENNSYLVANIA

In Canada: Roller-Smith Marsland, Ltd., Kitchener, Ontario

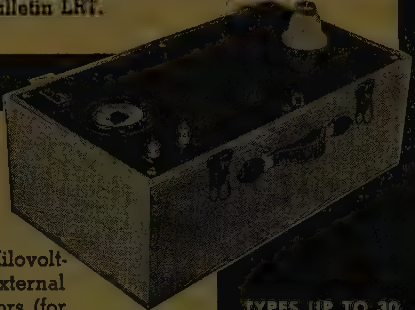
STANDARD AND PRECISION ELECTRICAL INSTRUMENTS • AIRCRAFT INSTRUMENTS • SWITCHGEAR
AIR AND OIL CIRCUIT BREAKERS • ROTARY SWITCHES • RELAYS • PRECISION BALANCES



SHALLCROSS PORTABLE
TYPE 645 FOR FIELD
INSPECTION WORK

STREAMLINING LOW RESISTANCE MEASUREMENTS

Are you required to make hundreds of accurate "Go, No-Go" low resistance measurements? The Shallcross Portable Low-Resistance Test Sets are ideal for making rapid measurements in bond testing, switch and relay contact resistance testing, bar-to-bar commutator readings, etc. Write for Bulletin LRT.



TYPES UP TO 30
KV AVAILABLE
FOR
RAPID DELIVERY!

HIGH-VOLTAGE MEASUREMENT PROBLEMS SOLVED

Shallcross Portable Kilovoltmeters, Kilovoltmeter Multipliers (for use with external meters) and Corona Protected Resistors (for voltages up to 200 KV) comprise a complete line of high-voltage measuring apparatus. Over a period of years Shallcross engineers have pioneered in the field of modern high-voltage measurements. They will welcome an opportunity to put their experience to work on your problem.

TYPE 638-2
COMBINED KELVIN
AND WHEATSTONE
BRIDGE

ONE BRIDGE THAT DOES THE WORK OF TWO

Combining both Kelvin and Wheatstone bridges, this popular Shallcross instrument provides a resistance measurement range from 0.0001 to 11.11 megohms in a single portable instrument. Just the thing for maintenance, production line tests, field investigations, school and laboratory work, etc.

TYPE NO. 845
POPULAR
3-DECADE UNIT

ACCURATE VOLTAGE DIVIDERS

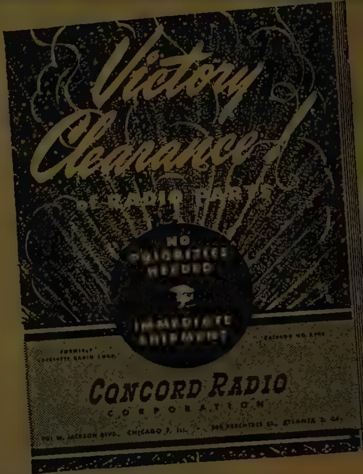
Shallcross Voltage Dividers (Decade Potentiometers) are available in a wide range of total resistance and voltage increments. Many special units regularly produced for special applications. Write for latest data bulletins on any Shallcross instrument type.

Shallcross
MANUFACTURING COMPANY

DEPT. EE-125, COLLINGDALE, PA.

ENGINEERING • DESIGNING • MANUFACTURING

RADIO PART
ELECTRONI
EQUIPMEN



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2½" flange mtg. type. Metal case dull black finish. G.E. 0-200 M.A. C10650. Specially priced $4.95

Output Transforme

Hermetically sealed.
studs, 1, 2, and 3 are
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ind. at 5 V. 1000 cy.: .2
Ratio sec. to pri. 3.0
Size: 3¼ x 2 41/64"
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An Announcement
To Those Who Require
The Best

THIS advertisement is addressed to the manufacturers of electronic equipment whose product demands the best in component parts—who will want the best in transformers, *if* "the best" is offered at a price that will fit the cost specifications of their finished product.

By adapting to peace-time use the major features of the Hermetically-Sealed transformer construction that won war-time leadership, Chicago Transformer is prepared to provide the best in transformers to those who require them. Fully developed basic mounting parts, when utilized on a mass production basis, will provide the element of economy in manufacture.

CHICAGO TRANSFORMER
DIVISION OF ESSEX WIRE CORPORATION
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CHICAGO, 18

MATTHEWS
HOLDFAST LAMP
GUARDS

Better because they remove lamp fire hazards; they stop lamp breakage; they stop unauthorized lamp removal; they are stronger and will last longer.

They are the only lamp guards which are guaranteed for 10 years.

They are approved by Fire Underwriters.

They are made in four sizes, of 14 bwg wire and for brass or weatherproof sockets.

Write for Bulletin 401 which will give you full details including net prices.

W. N. MATTHEWS
CORPORATION
ST. LOUIS, U. S. A.

Permanent Magnets

FOR

LOUD SPEAKERS

From ½ oz. to over 30 lbs. . . . and from start to finish . . . Arnold manufactures better Alnico permanent magnets for loud speakers. They are now available for all civilian applications, and include the proposed Radio Manufacturers Association standard speaker magnets in Alnico V.

The speaker magnets illustrated above are representative, yet are just one type of permanent magnets described in the new 24-page Arnold bulletin, "Permanent Magnets for Industry."

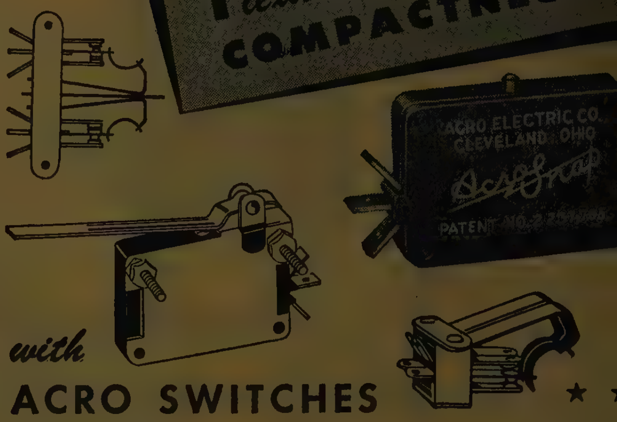
WRITE TODAY on your company letterhead FOR THIS NEW BULLETIN

THE ARNOLD ENGINEERING COMPANY

147 EAST ONTARIO STREET, CHICAGO 11, ILLINOIS

Specialists in the manufacture of ALNICO PERMANENT MAGNETS

★ ★ ★ Flexibility of Design COMPACTNESS


ACRO ELECTRIC CO.
CLEVELAND, OHIO
AcroSnap
PATENT NO.

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ACRO SWITCHES ★

The patented ACRO rolling spring switch lends itself
the designs of your units. Multiple mountings and sma
case shapes—rectangular or curved—available. Ope
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tion pressure from 2 grams (using leaf bracket) to 1¼ lb
Used widely in such applications as valve controls, coi
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IT'S ENGINEERED WITH **Hi-Q** COMPONENTS

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WIRE WOUND RESISTORS

CHOKE COILS

ELECTRICAL REACTANCE CORPORATION

FRANKLINVILLE, N. Y.

PLASTICON* CAPACITORS

Can you use a 10,000 VOLT PLASTICON
⅛ the size of a corresponding paper capacitor?

Because of the 4400 volt per mil breakdown voltage of the Plasticon film dielectric, high voltage Plasticons are smaller, lighter and more economical than paper capacitors.

*Plasticons—plastic film dielectric capacitors

CP

Condenser Products Company
1375 NORTH BRANCH STREET
CHICAGO 22, ILLINOIS

STANDARD SIGNAL GENERATOR Model 80

SPECIFICATIONS:

CARRIER FREQUENCY RANGE: 2 to 400 megacycles.
OUTPUT: 0.1 to 100,000 microvolts. 50 ohms output impedance.
MODULATION: A M 0 to 30% at 400 or 1000 cycles internal.
Jack for external audio modulation.
Video modulation jack for connection of external pulse generator.
POWER SUPPLY: 117 volts, 50-60 cycles.
DIMENSIONS: Width 19", Height 10¾", Depth 9½".
WEIGHT: Approximately 35 lbs. PRICE—$465.00 f.o.b. Boonton.
Suitable connection cables and matching pads can be supplied on order.

MEASUREMENTS CORPORATION
BOONTON NEW JERSEY

TRANSFORMERS FOR POSTWAR ELECTRONIC APPLICATIONS

Designed to meet your specifications, mechanically and electronically.

Open type, plastic impregnated transformers provide adequate weatherproofing for certain applications.

This special Radio Power transformer consists of primary winding, high voltage secondary winding and filament winding.

THE ACME ELECTRIC & MFG. CO.
CUBA, N. Y.

Acme ACME Electric
TRANSFORMERS

How to Get Your Money's Worth in FREQUENCY METERS

Check These Points

Model 33-F, Full-cycle increment, shown indicating frequency of 60 cycles. Black dial for special application.

Here are the facts on J-B-T VIBRATING REED FREQUENCY METERS

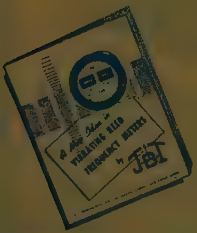
ACCURACY Half-cycle increment, ±0.2%; full-cycle increment, ±0.3%. This accuracy is not affected by normal *temperature* change, *wave form* or *external magnetic* fields.

COMPACTNESS Made in several sizes, most popular of which is the standard 3¼" panel mounting model. Also made to meet C39.2-1944 ASA specifications and Jan-I-6 for mounting and stud size of Electrical Indicating Instruments. No external reactor.

WEIGHT Model 31-F, 3½ inch, 5 reeds, weighs only 0.54 lb; Model 33-F, 3½ inch, 11 reeds, 0.59 lb. Other models are correspondingly *light*.

VOLTAGE VARIATION Will operate on voltages as low as 8 volts. Standard 110-115 volt models will operate satisfactorily over range of 100 to 130 volts. Also made for narrower voltage variation if desired. (Incidentally, current consumption is low. For Model 33-F, for example, ½ watt at 115V.)

RUGGEDNESS No parts to wear out or get out of calibration. All are securely anchored to the base with lock washers at every critical point. The only movement is at the free end of the spring steel reed. J-B-T meters on portable field equipment have established an enviable performance record.

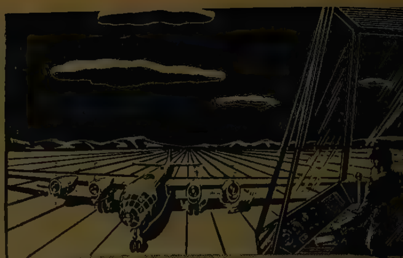


J-B-T Vibrating Reed Frequency Meters are available for frequencies from 12 cycles to 525 cycles with various reed groupings, increments and case sizes. For additional facts on the complete line, send for Bulletins VF-43, VF-43-1A (400 cycle Meters) VF-43-1B (2½" sizes), and VF-43-1C (interesting new applications).

J-B-T

(Manufactured under Triplett Patents and/or Patents Pending)

J-B-T INSTRUMENTS, INC.
459 CHAPEL STREET • NEW HAVEN 8, CONNECTICUT



An invitation to all electrical designers to

TRY SILVER GRAPHALLOY

FOR RELAYS • MOTORS • CONTROLS • ELECTRONIC

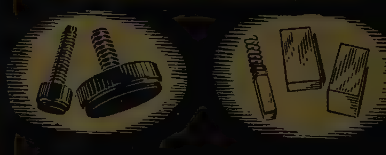
GRAPHALLOY

FOR BRUSHES

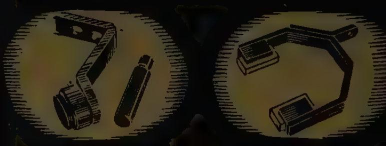
High current density, low contact drop, low electrical noise, and self-lubrication are characteristics of this silver-impregnated molded graphite that may be the answer to your electrical brush problems.

FOR CONTACTS

Low contact resistance and non-welding when breaking surge currents are inherent properties of this unique combination of conductive silver and self-lubricating graphite.



SLIP-RING AND COMMUTATOR



BRUSHES AND CONTACTS

SAMPLES of Silver Graphalloy will be gla
furnished for test on your applications. S
Graphalloy is usually silver plated to permit ea
soldering to leaf springs or holders. Why n
WRITE NOW for your test samples?

ALWAYS SPECIFY
GRAPHALLOY
TRADE MARK REG. U. S. PAT. OFF.

**GRAPHITE METALLIZI
CORPORATION**
1053 NEPPERHAN AVE • YONKERS, NEW Y

Over 300% increase in 3 years

. . . THE SWING IS TO VARNISHED FIBERGLAS* CLOTH FOR ELECTRICAL INSULATION

It's because of the unique combination of characteristics, found only in Fiberglas, so many electrical engineers have standardized on Fiberglas-base Electrical Insulation Materials.

THICKNESSES OF FIBERGLAS CLOTH

Fiberglas Cloths for varnishing are produced in thicknesses from .0015″ to .015″ permitting a wide variety of base cloths for various varnished thicknesses. Thickness of base cloth and finished thickness of varnished cloth should be determined by space-factor, dielectric strength and mechanical requirements of the particular application.

VARNISHED CLOTH AND TAPE

When properly impregnated, Fiberglas-base Insulation material has exceptional dielectric strength and insulation resistance. In addition it has unusually good moisture and temperature resistance and high tensile strength. The strength-to-thinness ratio of varnished Fiberglas Cloth is unmatched.

Varnished Fiberglas Cloth, in thicknesses from .003″ to 030″, is used wherever a high-dielectric material with good mechanical characteristics is required for equipment operating in high temperature, under adverse operating conditions, or where a greater factor of safety is desired. Typical applications are: phase insulation in a-c motors; high-dielectric and high-temperature coil insulation; ground insulation on the slot sections of preformed coils; layer insulation in air-cooled transformers, etc.

PERMITS USE OF THINNER INSULATION

Because of its higher dielectric strength and because it retains its mechanical and electrical characteristics under prolonged exposure to adverse conditions, thinner varnished Fiberglas Cloth or Tape than that required by the use of other materials will give equal or greater insulation.

Varying degrees of flexibility in varnished Fiberglas Cloth, of the same finished thickness and type of varnish, are possible by varying the thickness of the Fiberglas Cloth used as the base —yellow or black varnish treatment.

COMPLETE INFORMATION

If you do not have complete information on all types of Fiberglas-base Electrical Insulation Materials, write for Catalog EL 44-7 today. *Owens-Corning Fiberglas Corporation, 1859 Nicholas Building, Toledo 1, Ohio.*

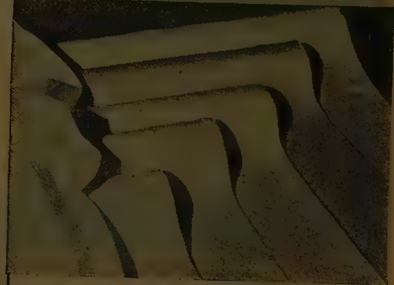
In Canada, Fiberglas Canada Ltd., Oshawa, Ontario.

FIBERGLAS

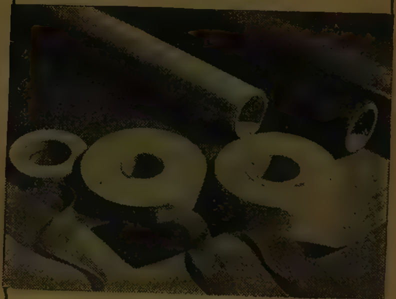
*T. M. Reg. U. S. Pat. Off.

ELECTRICAL INSULATION MATERIALS

There is a FIBERGLAS Insulation Material for every need

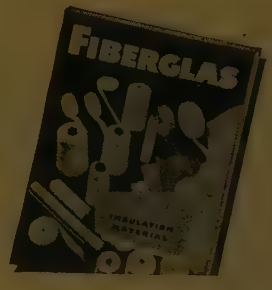


Untreated, continuous filament, Fiberglas Cloth is made in thicknesses from .0015″ to .015″—staple fiber cloth from .012″ to .023″.



Varnished Fiberglas Cloth is available in 36″ widths or cut to any desired tape width.

Treated and untreated Fiberglas Tapes in various widths and thicknesses are used in winding all types of coils.

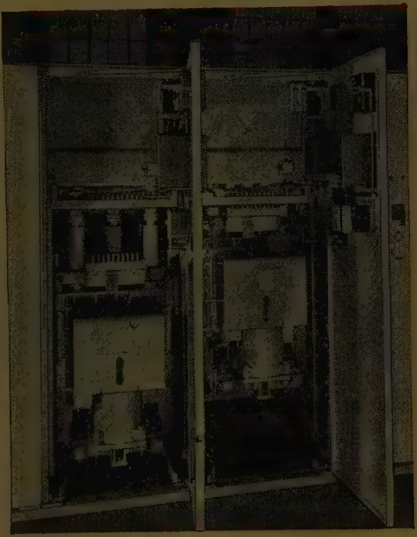


Write for Catalog E 44-7

Each distributor of Fiberglas-base Insulation Materials has his own source of supply, since Owens-Corning Fiberglas Corp. does not process these materials.

See How Easy it Is to

ROUTINE CONTACT INSPECTION IN 3 QUICK STEP
ON OILLESS "RUPTAIR" MAGNETIC AIR BREAKE



1 **LOWER BREAKER.** Photo shows "Ruptair" magnetic air breaker in operating position (at right) and in the lowered disconnect position. The breaker is easily lowered with manually or electrically operated crank-hoist.

2 **WITHDRAW BREAKER.** In disconnect position, the "Ruptair" breaker can be placed on a transfer truck, and pulled out into the aisle. For safety, shutters to isolate disconnect contacts are automatic in operation.

3 **REMOVE SIDE PLATES.** Finally, light-weight in
lating barriers are remov
each barrier being helc
place by two cap-scre
Since the arc chute is above
contacts, it need not be disturl
for contact inspection.

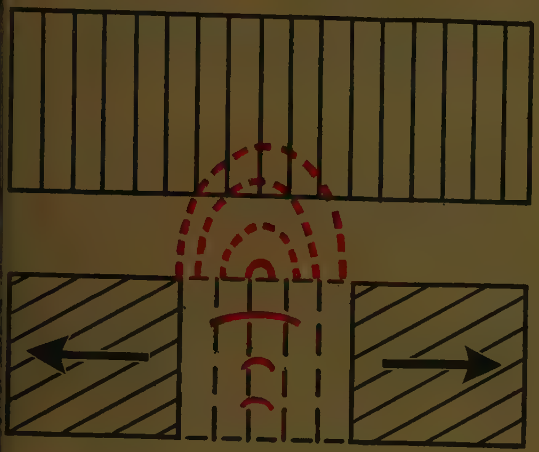
As a MODERATE capacity, oilless circuit interrupter, the "Ruptair" breaker is gaining increased popularity among operators. Elimination of oil means many more repetitive operations without maintenance . . . reduced fire hazard . . . less danger from breaker failure.

In addition to these advantages, and simplest inspection of any magnetic air breaker, the "Ruptair" breaker combines space saving vertical lift construction with horizontal contact separation, shown at top of opposite page. "Ruptair" circuit breakers are built in standard ratings of 5 kv and below, 150,000 kva interrupting capacity and below, for installation in metalclad switchgear, cells, or cubicles. To get the full story, write Allis-Chalmers, Milwaukee 1, Wis.

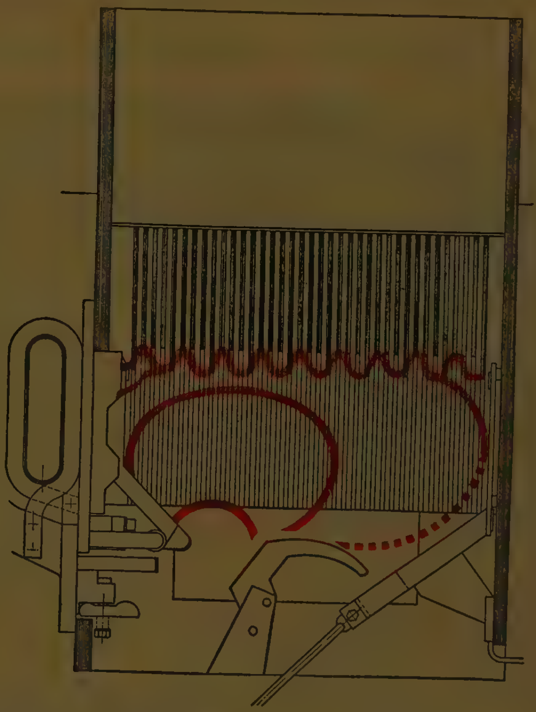
Inspect

A "RUPTAIR" MAGNETIC AIR CIRCUIT BREAKER

WITH HORIZONTAL SEPARATION OF BREAKER CONTACTS, MAINTENANCE AND OPERATION ARE SIMPLIFIED



SINCE THE ARCING contacts on a "Ruptair" magnetic air circuit breaker are separated in the horizontal direction, natural thermal effect assists the arc into the arc chute . . . no complex interrupting structure is required! Position of arc chute *above* contacts makes it possible to inspect contacts without removing the chute. Maximum interrupting effectiveness is achieved in the "Ruptair" circuit breaker, without departing from ideal simplicity of design.



RUPTAIR BREAKER HAS UNIQUE DESIGN

THE RUPTAIR breaker depends for its high efficiency throughout interrupting current range on several design factors . . . coordination of horn-shaped moving contact and arc runner for quick transferral of the arc . . . establishment of a strong magnetic field in the breaker before actual contact separation . . . and special design of the arc chute.

Why Allis-Chalmers offers all 3 types of Circuit Breakers

☞ Ever hear of "special theater modifications"? That's a military term, and refers to the adaptation of aircraft to unusual flying conditions, such as are encountered in Central Alaska, or in torrid tropical climates.

☞ An airplane built for special theater operations ften differs in practically every detail from a plane esigned for duty under normal atmospheric conditions. That is, the Army and Navy have learned that *one* type f plane can't function in *all* theaters of war!

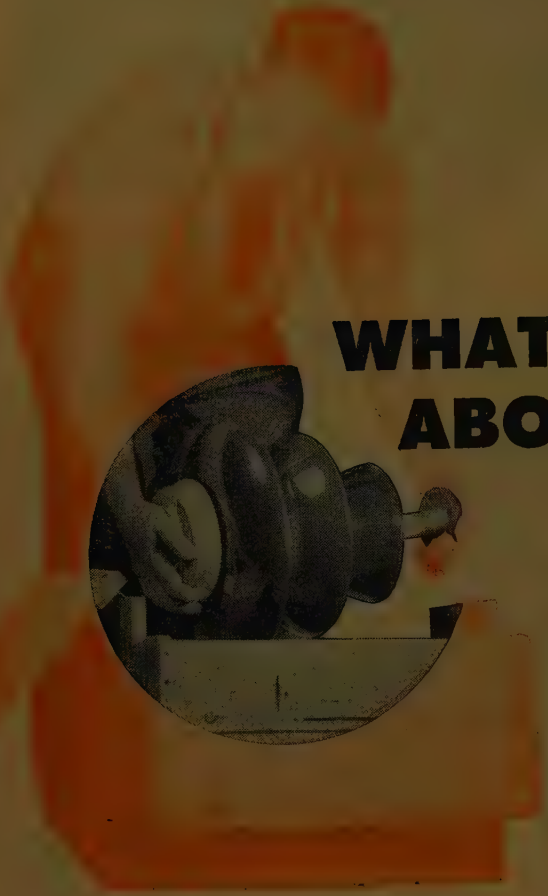
☞ *For the same reasons, Allis-Chalmers engineers know hat one type of circuit breaker won't do the job in all he varied environments encountered in industry.*

☞ Therefore, it is an Allis-Chalmers switchgear policy o offer *all three* basic types of circuit interruptors – oil, magnetic air, and air blast breakers — for unestricted customer choice. Neither by price penalties, or any other means, does Allis-Chalmers seek to coerce buyers into selecting any *one* type of breaker.

☞ Thus the customer has a means of expressing his circuit breaker preference, based on intimate knowledge of local operating conditions. And indirectly, through the medium of free choice, he influences the future trend of switchgear development.

A 1863

ALLIS-CHALMERS
MILWAUKEE



WHAT DO YOU KNOW ABOUT THIS INSULATOR

A NEW station insulator--right out of the crate--represents a host of standards, tests, and inspections, all applying, naturally, to a *new* insulator. But it stops being new the day you put it to work, and gets less new every year thereafter. Important as manufacturing standards may be, they apply only to *new insulators,* and they don't *stay* new!

What do you know about *old* O-B station insulators? You can go back some 30 years of continuous performance, or pick almost any quantity. You can inspect authentic power company records that produce incontestable facts. You will see why most O-B station insulators, today, are bought on repeat orders based upon previous service.

In a new station insulator, you can't reliably predict long future life. Time, alone, furnishes this answer. It is found in long service records--the kind of records O-B has been piling up for better than a quarter of a century. The one thing you want the most--reliability--can't be specified or predicted in tests. It is a inherent quality put there by the man who makes the insulator. Just how much O-B has put into its insula tors is a matter of adequate record.

2612-H

Ohio Brass

MANSFIELD, OHIO

CANADIAN OHIO BRASS COMPANY, LIMITED
NIAGARA FALLS, ONTARIO

TRANSTATS

maintain voltages to a hairline at GULF RESEARCH & DEVELOPMENT COMPANY

Pilot Plant Control Panels, Gulf Research & Development Company, division of Gulf Oil Corporation, Pittsburgh, Penna. Transtat knobs are below ammeters.

TH TRANSTAT A. C. VOLTAGE REGULATORS For 0.3 to 20 KVA

In Gulf's Research Laboratories, unusual precautions are taken to eliminate variations. For example, fifty-six Transtat A. C. Voltage Regulators maintain required voltages to the heating elements inserted between pipe and insulation to compensate for heat lost. The extreme precision and ruggedness of Transtats results from the unique side-commutation. The long, cooler running brush rides on a glass-smooth track, providing arcless, stepless control. Bulletin 51-2 gives construction details. Write for it.

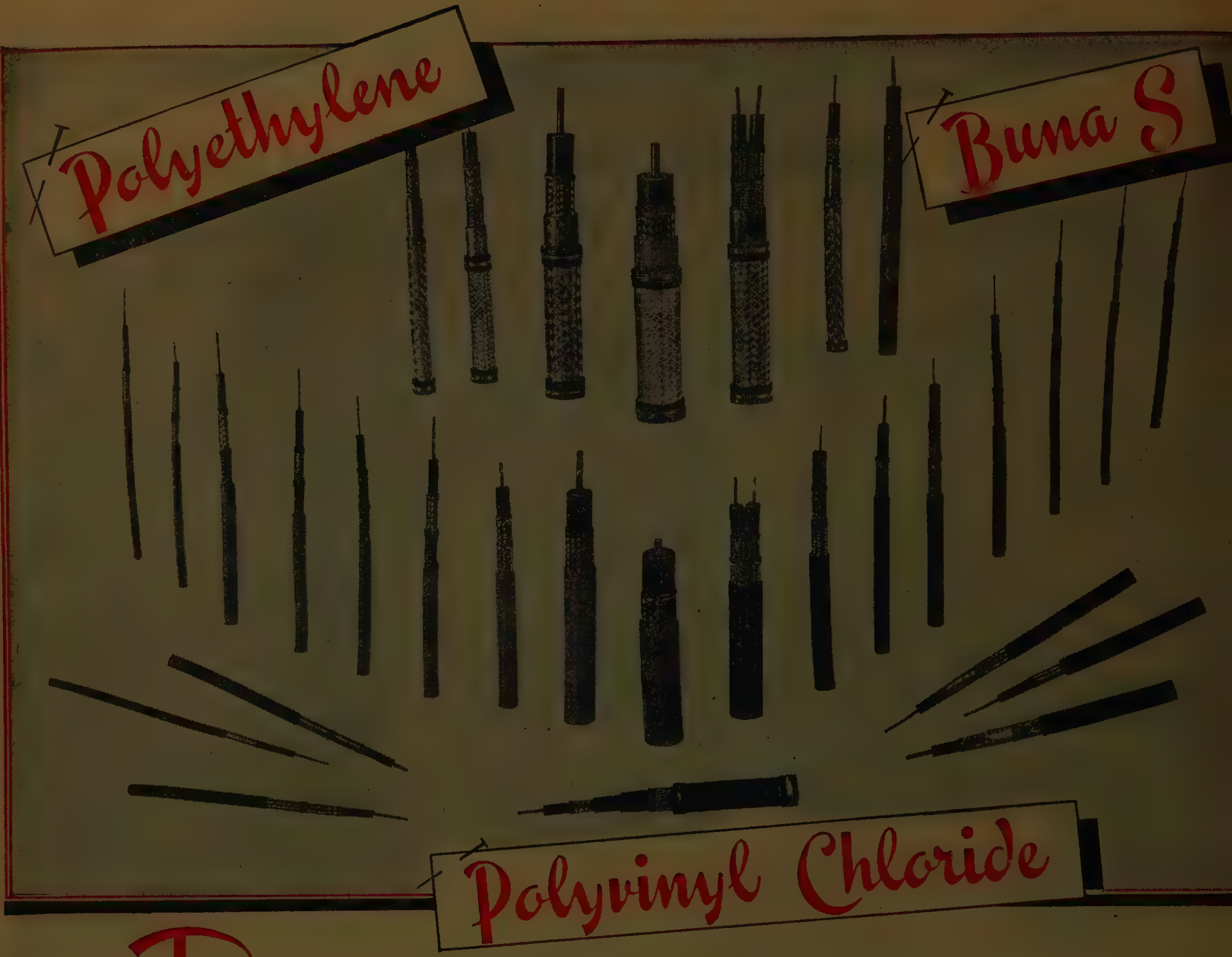
Showing Transtats mounted on rear of panel.

AMERTRAN

REG. U. S. PAT. OFF.
MANUFACTURING SINCE 1901 AT NEWARK, N. J.

AMERICAN TRANSFORMER COMPANY
178 EMMET STREET, NEWARK 5, N. J.

Pioneer Manufacturers of Transformers, Reactors and Rectifiers for Electronics and Power Transmission



These are the insulations used on the Coaxial cables we have made during the war. The cable characteristics vary widely both electrically and physically but their quality is uniformly high.

If you have Coaxial cable requirements perhaps one of these will do what you require. If not, we will be more than happy to design a special cable to meet the specific requirements you have in mind.

REMEMBER THAT THE LATEST ADVANCES IN CABLE DESIGN AND MATERIALS COME FROM SIMPLEX FIRST.

Simplex Wire & Cable Co., 79 Sidney Street, Cambridge 39, Mass.

ARMY E NAVY

M

Simplex

WIRES and CABLES

Sperry FOUND A BETTER WAY

TO HOLD MACHINE PARTS TOGETHER

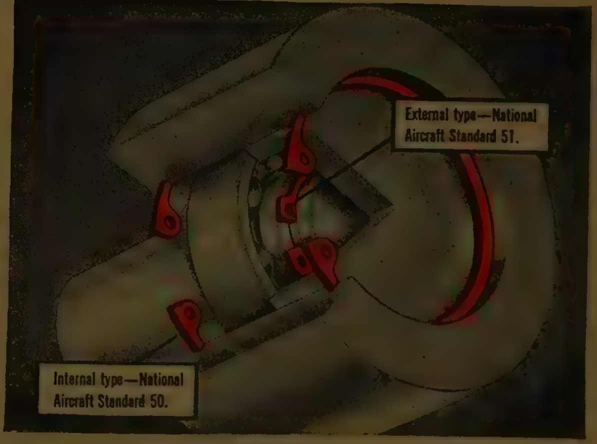
WALDES TRUARC RETAINING RINGS

Before using Waldes Truarc Retaining Rings in Sperry gunsights, Sperry Gyroscope Co. engineers had difficulty with important taper pins working loose while the gunsights were in service. Due to vibration, pins occasionally loosened and fell out affecting the accuracy of the gunsights. Frequent, thorough inspection of taper pins in use didn't entirely solve the problem.

Then they specified Waldes Truarc Rings. Where taper pins had previously worked loose, the rings now held them fast. Assembly of the gunsights became much easier and quicker. Because of this earlier experience, Sperry used Waldes Truarc Rings as original equipment in their K-13 Compensating Gunsights. And Truarc *exceeded* their highest requirement for dependability in action.

Waldes Truarc Retaining Rings are used to save weight, space, cost and man-hours in a wide range of products. For holding and positioning machine parts they offer definite advantages over nuts, shoulders, collars, and pins. They simplify and speed up production. They can be put on and taken off again and again—and still retain the perfect circularity which gives them their unfailing grip. Test them yourself. We'll furnish samples and complete data. Write Dept. F-12.

U. S. PAT. RE—18,144

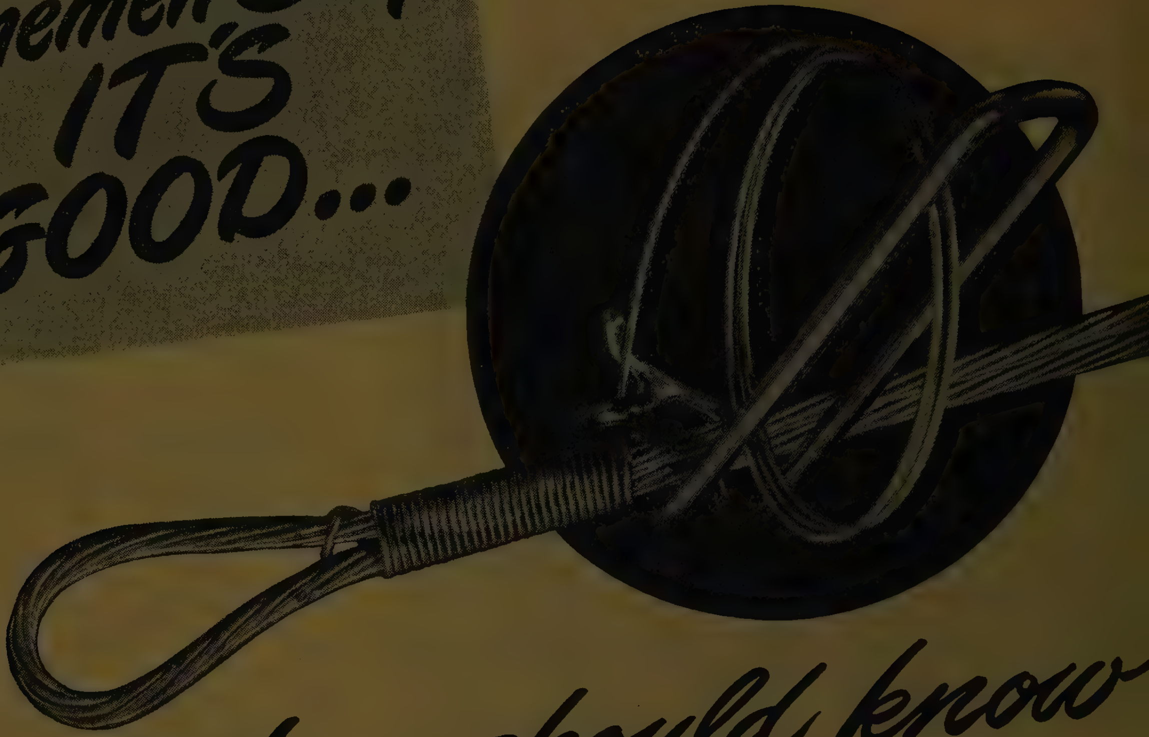

External type—National Aircraft Standard 51.
Internal type—National Aircraft Standard 50.

WALDES TRUARC RETAINING RINGS

TRADE MARK.

WALDES KOHINOOR, INC., LONG ISLAND CITY 1, NEW YORK
CANADIAN REPRESENTATIVES: PRENCO PROGRESS CORP., LTD., 72-74 STAFFORD ST., TORONTO

Linemen Say IT'S GOOD...



and they should know

Bethlehem's serving tool has the look of a "gadget" and a funny one at that. Yet linemen will tell you that it's one of the handiest little devices that ever came along.

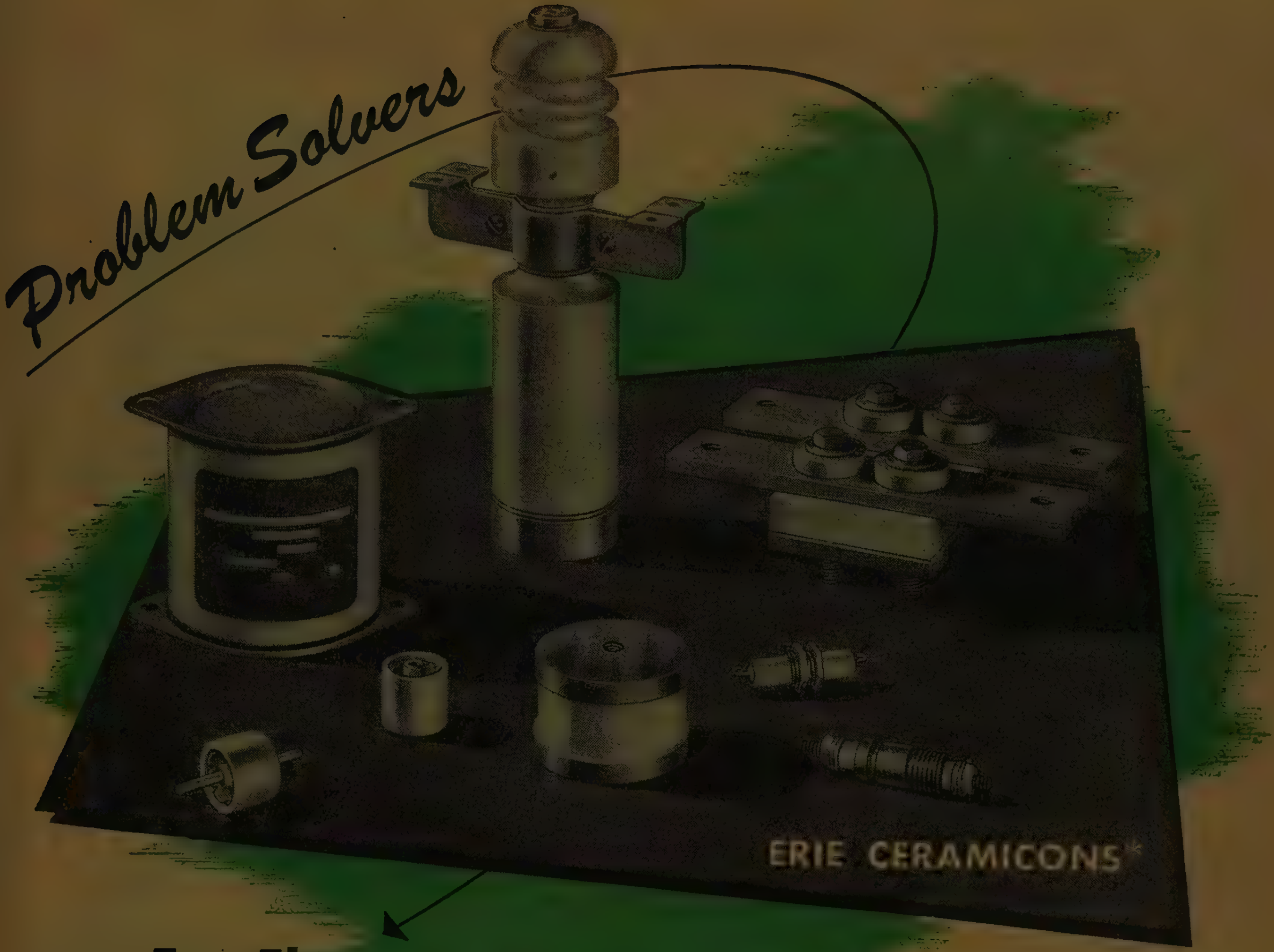
It was designed for making serves on Bethlehem strand by the dead-end coil method. This type of serve consists of a neat, strong wrapping of 11-gage bethanized wire around the dead end. Extensive tests have proved that the serve holds beyond the useful strength of the strand.

Even a novice can learn the technique in a few minutes' time. The lineman simply hooks one end of the wire to the strand, as shown, and engages it between the two fingers of the tool. A simple rotary motion does the rest. It takes about the same length of time as attaching conventional clips, but is less expensive. A 10-foot coil of bethanize dead-end wire costs only a few pennies.

We strongly recommend that you equi your linemen with these tools, which ar intended primarily for use with Bethleher strand. That makes a winning combinatior for Bethlehem strand is flexible, uniform . . always easy to handle . . . and armore against corrosion by a bethanized zinc coa ing of high purity. This coating is so ducti that it is not displaced to any harmf extent by the action of the serving tool.

Write Bethlehem Steel Co., Bethlehem, Pa for further details. We'll gladly supply ther

★ BETHLEHEM STEEL ★



For Electronic Heating and Television Capacitor Applications

WITH the growing importance of electronic heating and television, design engineers can find the solution to many of their capacitor problems in Erie Ceramicons.

These silvered-ceramic condensers offer advantages because of their low inductance at high frequencies and extremely simple construction that eliminates circulating currents tending to reduce power ratings. Design is further simplified by the fact that where required, corona shields are incorporated directly into the ceramic dielectrics.

Erie Resistor has developed a number of High Voltage and High KVA Ceramicons of special design and several standard styles are now in production. Included in the above group is a specially designed 30,000 volt feed-thru Ceramicon; a dual filament by-pass unit having conductors to carry 325 amps. The standard Erie Resistor High Voltage ceramic condensers shown include two styles of double cup Ceramicons; a new double cup unit for television power supply filtering, rated at 500 MMF and 10,000 volts D. C.; two High Voltage feed-thru Ceramicons; and a High KVA, High Voltage, multiple plate condenser comparable in size to mica type CM75 but particularly adapted to use at very high frequencies.

Write for data sheets on standard Erie Ceramicons for television and electronic heating applications. You are invited to make use of our extensive knowledge and background for the development of special Ceramicons for these applications.

*CERAMICON IS THE REGISTERED TRADE NAME OF SILVERED CERAMIC CONDENSERS MADE BY ERIE RESISTOR CORPORATION.

★ BUY VICTORY BONDS ★

Electronics Division

ERIE RESISTOR CORP., ERIE, PA.

LONDON, ENGLAND   TORONTO, CANADA.

USE THIS COMPLETE BRUSH-TESTING LAB

FOR FRACTIONAL HORSEPOWER MOTOR BRUSHES

HERE'S a modern laboratory for testing fractional horsepower motor brushes. It's air conditioned . . . equipped with the latest test facilities . . . staffed by experts . . . backed by over half a century's experience in brush manufacturing.

IT'S AT YOUR SERVICE! WE URGE YOU TO MAKE USE OF IT!

This lab relieves you of making countless time-consuming and costly "elimination tests." Here your motors are tested as you would test them yourself . . . under your own specified load, cycle, and test conditions.

From such tests National Carbon Company, Inc. establishes the proper brush grade for any particular model or type of motor, be it Series Universal, Repulsion-Induction, or for low voltage service. And because these tests are most thoroughly conducted you can be sure that the most suitable grade has been applied.

No delays—tests are made promptly! So send in your motors—make use of this lab!

For more details get in touch with our nearest division office.

A FEW OF THE TESTS RUN

Tests are run to determine optimum performance in respect to:

commutation • commutator condition
quietness of operation
brush life • operating temperature

The word "National" is a registered trade-mark of National Carbon Company, Inc.

NATIONAL CARBON COMPANY, INC.
Unit of Union Carbide and Carbon Corporation
UCC
General Offices: 30 East 42nd Street, New York 17, N. Y.
Division Sales Offices: Atlanta, Chicago, Dallas, Kansas City, New York, Pittsburgh, San Francisco

Federal . . . KNOWS H-F Cable Inside and Out

From inner conductor to outer covering . . . Federal really knows high-frequency transmission lines.

And this knowledge was not easily won. As the pioneer in the field Federal not only developed over 80% of all h-f cable types in use today . . . but developed most of the equipment needed to test them.

Attenuation, high-voltage, dielectric and balance testing equipment, velocity of propagation, braid-resistance and electrical length meters . . . were all Federal-*engineered* to fit specific requirements.

That's why it's logical to turn to the acknowledged leader in the field for the finest in h-f cables, specialty-engineered harnesses and cable assemblies.

Where requirements are critical . . . for transmission lines with special characteristics . . . for custom-built and engineered harnesses and cable assemblies . . . take your high-frequency transmission problems to Federal.

Federal Telephone and Radio Corporation

AN IT&T ASSOCIATE

Newark 1, N. J.

MODEL 2405

Volt•Ohm•Milliammeter

25,000 OHMS PER VOLT D.C.

NEW ENGINEERING • NEW DESIGN • NEW RANGES
30 RANGES

Voltage: 5 D.C. 0-10-50-250-500-1000 at 25000 ohms per volt.
5 A.C. 0-10-50-250-500-1000 at 1000 ohms per volt.
Current: 4 A.C. 0-.5-1-5-10 amp.
6 D.C. 0-50 microamperes — 0-1-10-50-250 milliamperes—0-10 amperes.
4 Resistance 0-4000-40,000 ohms—4-40 megohms.
6 Decibel -10 to +15, +29, +43, +49, +55
Output Condenser in series with A.C. volt ranges.

Model 2400 is similar but has D.C. volts Ranges at 5000 ohms per volt.

Write for complete description

STANDARDS ARE SET BY TRIPLETT

SPECIFICATIONS

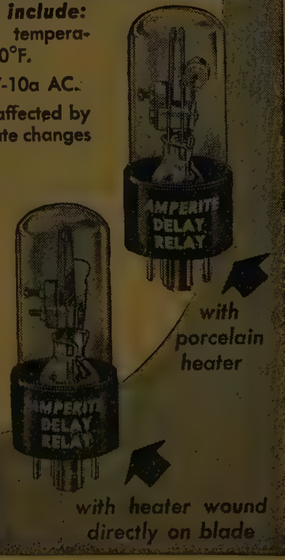
NEW "SQUARE LINE" metal case, attractive tan "hammered" baked-on enamel, brown trim.

■ PLUG-IN RECTIFIER—replacement in case of overloading is as simple as changing radio tube.

■ READABILITY—the most readable of all Volt-Ohm-Milliameter scales—5.6 inches long at top arc.

■ RED•DOT LIFETIME GUARANTEE on 6" instrument protects against defects in workmanship and material.

Triplett
ELECTRICAL INSTRUMENT C
BLUFFTON OHIO.



AMPERITE

THERMOSTATIC METAL TYPE
DELAY RELAYS
PROVIDE DELAYS RANGING FROM 1 TO 120 SECONDS

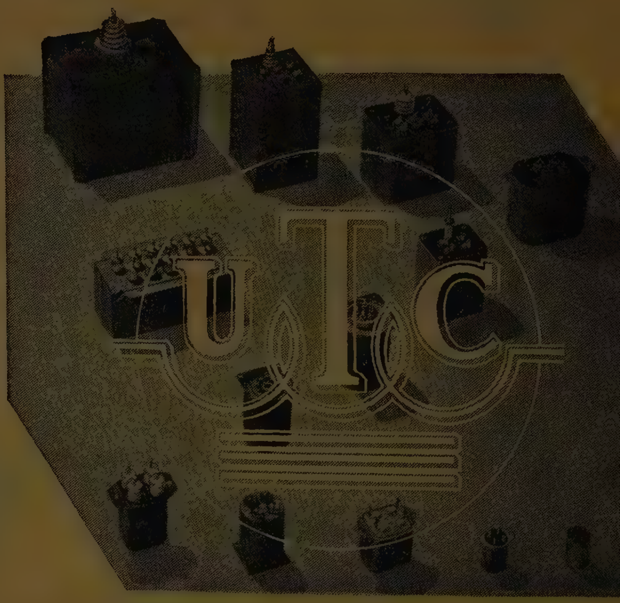
Other important features include:
1. Compensated for ambient temperature changes from —40° to 110°F.
2. Contact ratings up to 115V-10a AC.
3. Hermetically sealed — not affected by altitude, moisture or other climate changes . . . Explosion-proof.
4. Octal radio base permits easy replacement.
5. Compact, light, rugged, inexpensive.
6. Circuits available: SPST Normally Open; SPST Normally Closed.

WHAT'S YOUR PROBLEM? Send for "Special Problem Sheet" . . . and Descriptive Bulletin.

AMPERITE CO.
561 BROADWAY
NEW YORK 12, N. Y.
Canada: Atlas Radio Corp., Ltd.
560 King St. W., Toronto

HERMETIC SEALED TRANSFORMERS

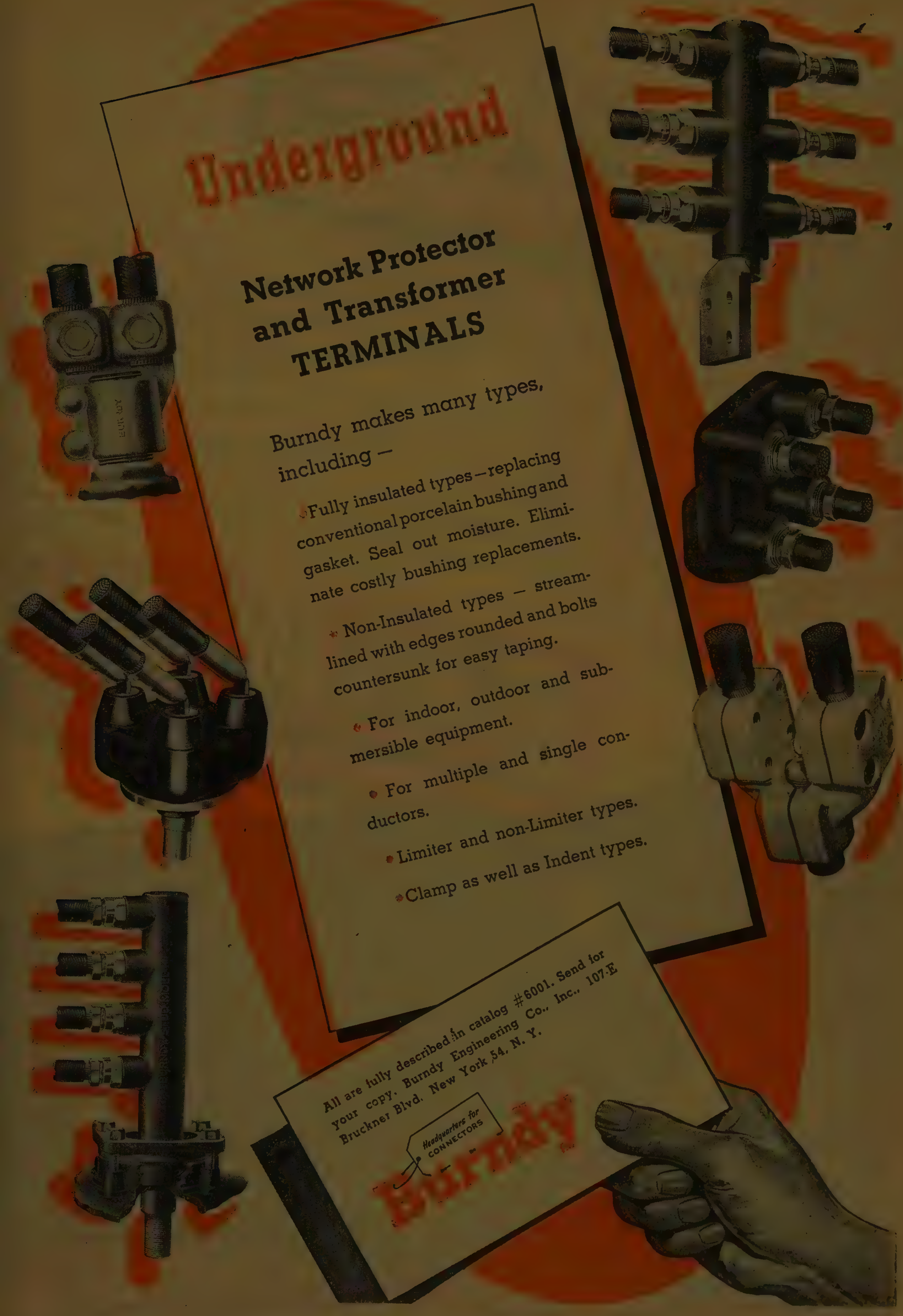
a type for every requirement



May we cooperate with you on design savings for your application...war or post war

ARMY E NAVY
ALL PLANTS

United Transformer Corp.
150 VARICK STREET NEW YORK 13, N. Y.
EXPORT DIVISION: 13 EAST 40th STREET, NEW YORK 16, N. Y., CABLES: ARLAB



Underground

Network Protector and Transformer TERMINALS

Burndy makes many types, including —

- Fully insulated types — replacing conventional porcelain bushing and gasket. Seal out moisture. Eliminate costly bushing replacements.
- Non-Insulated types — streamlined with edges rounded and bolts countersunk for easy taping.
- For indoor, outdoor and submersible equipment.
- For multiple and single conductors.
- Limiter and non-Limiter types.
- Clamp as well as Indent types.

All are fully described in catalog #6001. Send for your copy. Burndy Engineering Co., Inc., 107-E Bruckner Blvd. New York 54, N. Y.

Headquarters for CONNECTORS

Burndy

SELF CONTAINED 2300 VOLT OIL IMMERSED A.C. INDUCTION MOTOR CONTROL

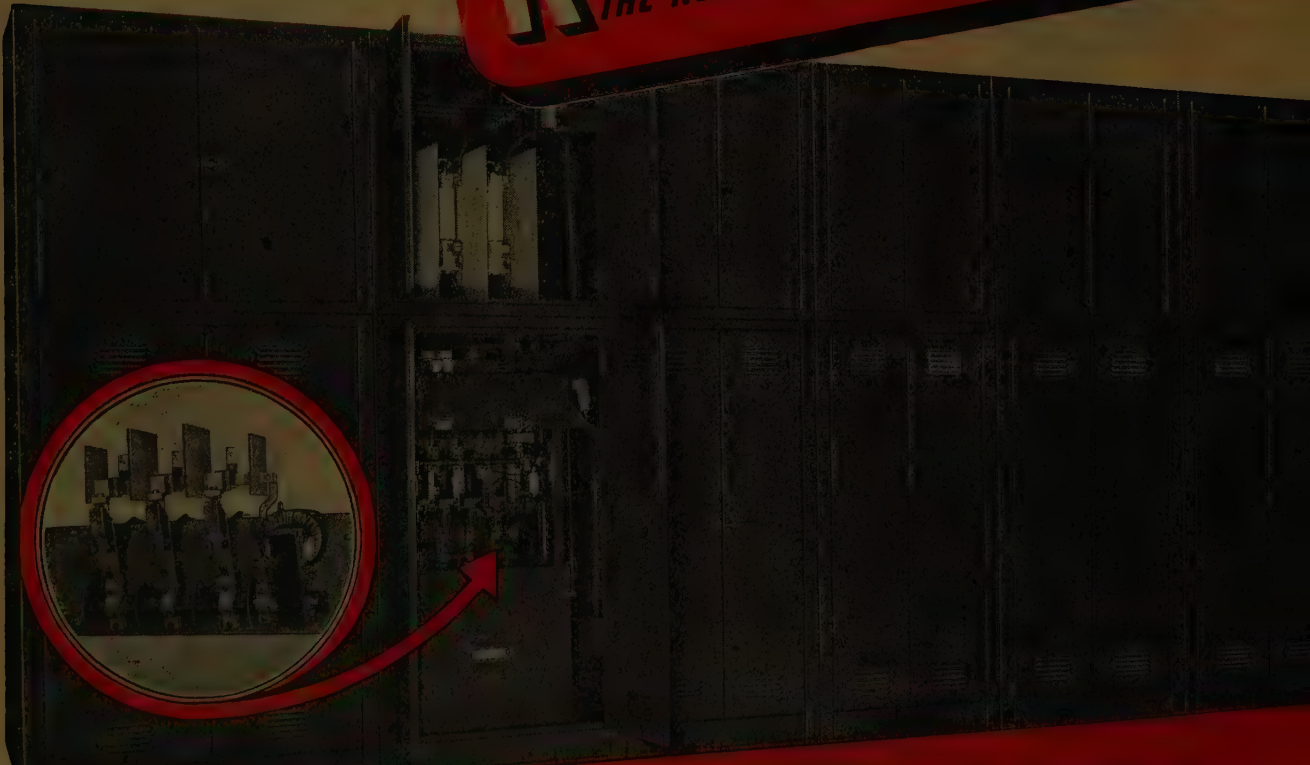
CONSTRUCTION AND OPERATING FEATURES

1. They have an especially designed, quick-acting, heavy-duty contactor.
2. High interrupting disconnect fuses, hook stick operated.
3. Self-contained, seal-off motor terminal compartment.
4. Centralized self-contained low voltage control terminal compartment.
5. Magnetic overload relays with electric reset, instantaneous and inverse time element.
6. Self-contained control potential transformer.
7. Self-contained Tank lowering device.
8. Insulated Bus, supported on porcelain Insulators.

The simplicity, efficiency, and flexibility of this Push Button operat Control have been made possible by Rowan Engineers after thirteen ye of research, and production of this type of equipment. It has be designed for Engineers whose prime thoughts are safety, continu operation, minimum installation and maintenance cost. These start are floor mounted and are arranged for single or group installati They are Push Button operated; of the full voltage or reduced volta type; arranged for indoor, or outdoor service; are of cubicle constructi and are complete within themselves. *Let us have your 2300 V control problems.*

ROWAN CONTROL

THE ROWAN CONTROLLER CO., BALTIMORE, MD.

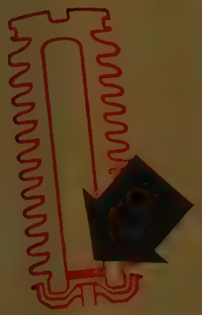


Bank of six control units—Insert in circle shown close-up of quick-acting, heavy-duty contactor.

LINE POST LINES HAVE FEWER INSULATOR OUTAGES FROM FLASHOVER

The Line Post design offers an insulator peculiarly adapted to withstand the shattering forces of power arcover. A rugged post body, it attains leakage distance through the device of many short sturdy petticoats rather than few thin, broad, spreading sections. Repeated flashover usually leaves a unit undamaged. If, under severe flash, a unit is fractured, flashover and leakage distances are not reduced enough to impair the serviceability of the insulator.

LAPP LINE POSTS HAVE A DOUBLE FACTOR OF SAFETY...



1 POSITIVE SEAL

The 15-year service record of Lapp Line Posts is witness to the soundness of Lapp techniques of Post construction. One exclusive Lapp feature is a combination porcelain-and-rubber plug which permanently seals the lower end of the post against moisture.

2 NO HIDDEN CRACKS

Failure of any post-type insulator can usually be attributed to unnoticed internal cracks near the head of the insulator. All Lapp posts are fired with the lower end open, permitting critical inspection of finished porcelain before assembly.

Lapp

LAPP INSULATOR COMPANY, INC., LE ROY, N. Y.

ELECTRONICS **DESIGNER'S**

HERE 3 CAPACITORS DO TWICE THE WORK OF 25

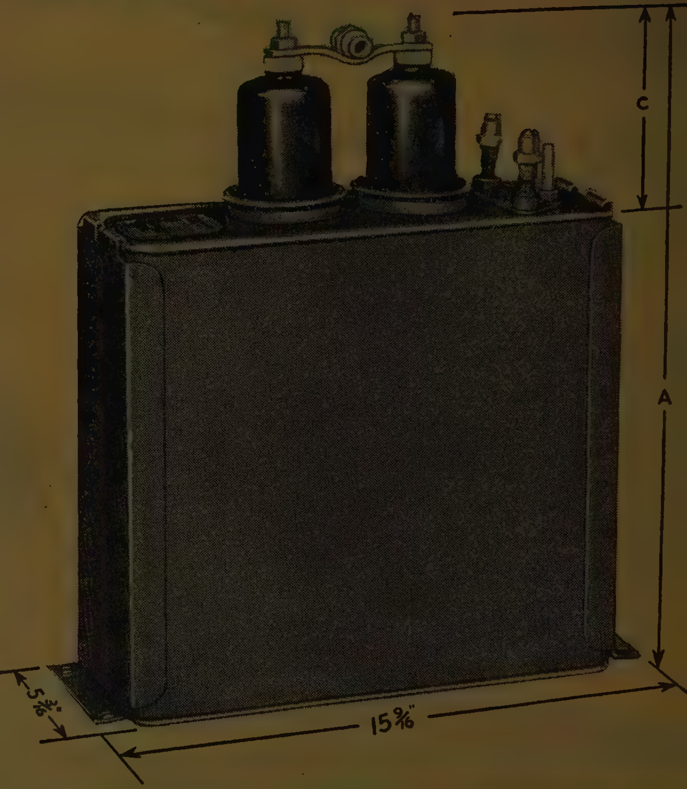
A MANUFACTURER of induction heaters replaced 25 mica capacitors in his resonant (tank) circuit with three General Electric HFP parallel-plate, water-cooled capacitors. He saved nearly half the cost and space, and more than doubled the kva. Result: a more compact, more powerful, and more efficient heater.

Class HFP capacitors, with their two sets of heavy, sheet-aluminum plates, are specifically designed for use in resonant circuits of high-frequency oscillators, such as those employed in electronic heaters. Outstanding features of Class HFP capacitors are compact construction, and ability to operate at high voltages and to carry heavy continuous currents at frequencies from 50 kilocycles up into the megacycles. The special dielectric is a new, stable, synthetic liquid which combines the desirable characteristics of low loss, high dielectric constant, and high dielectric strength.

A coil of copper tubing, for water-cooling the capacitor, is installed inside the case in direct contact with the grounded pair of capacitor plates. Couplings are provided for connection to ⅜-in. copper tubing. The cooling feature permits a compact assembly and high current rating per unit volume.

The cases are of nonmagnetic metal, hermetically sealed, and flexible enough to take care of thermal expansion of the liquid dielectric. Capacitance tolerance is from plus 5 per cent to minus 5 per cent of the rated capacitance at 25 C; Q factor is above 2000 for full load operation at frequencies from 50 kilocycles to one megacycle. Internal inductance is low, which gives resonant frequencies from 3 to 9 megacycles, depending upon the capacitance rating. Write for Bulletin GEA-4365. *Apparatus Dept., General Electric Company, Schenectady 5, N. Y.*

Keep on buying **BONDS**—*and keep all you buy*



CLASS HFP HIGH-FREQUENCY, PARALLEL-PLATE, WATER-COOLED CAPACITORS

Maximum Permissible Rms Working Voltage	Microfarad Rating	Maximum Permissible Rms Working Current in Amp at 540 Kilocycles	Approximate Dimensions in Inches	
			A	C
2000	0.025	170	16 7/32	2 1/2
	0.030	204		
	0.034	230		
3000	0.0165	168	16 7/32	2 1/2
6000	0.0075	153	18 7/16	4 7/16
	0.01	204		
6600	0.009	202	18 7/16	4 7/16
9000	0.0029	88	18 7/16	4 7/16
	0.005	153		
	0.0056	171		

GENERAL ELECTRIC

DIGEST

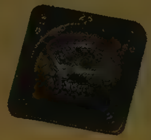
Timely Highlights on G-E Components

LIGHT-WEIGHT TRANSFORMERS *for tight spots*

When size and weight are important and weather resistance isn't, G-E core-and-coil transformers solve a lot of electronic-design problems. Uniform coils, automatically wound over the finest-grade core aminations, are of the same high quality and give the same reliable performance as G-E cased transformers.

Standard core and coil units include 60 types and ratings of plate transformers, 106 filament transformers, 34 plate-and-filament transformers, and 61 reactors. Ratings up to 50 kva (physical size) are wound on standard laminated cores; larger units can be built from special parts. Write for Bulletin GEA-4280.

Small ONLY IN *Size*



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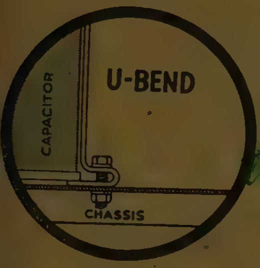

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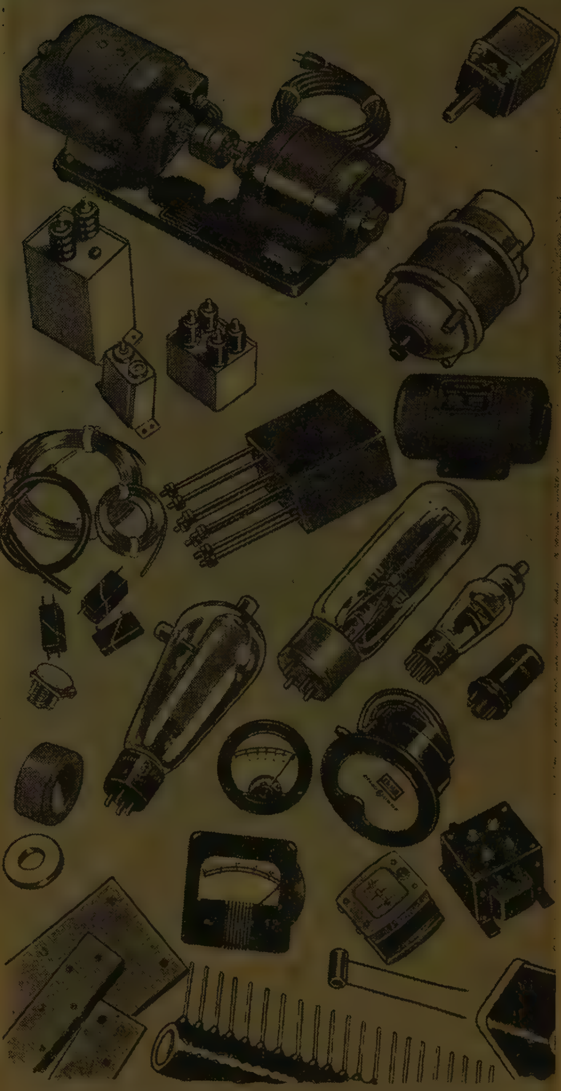

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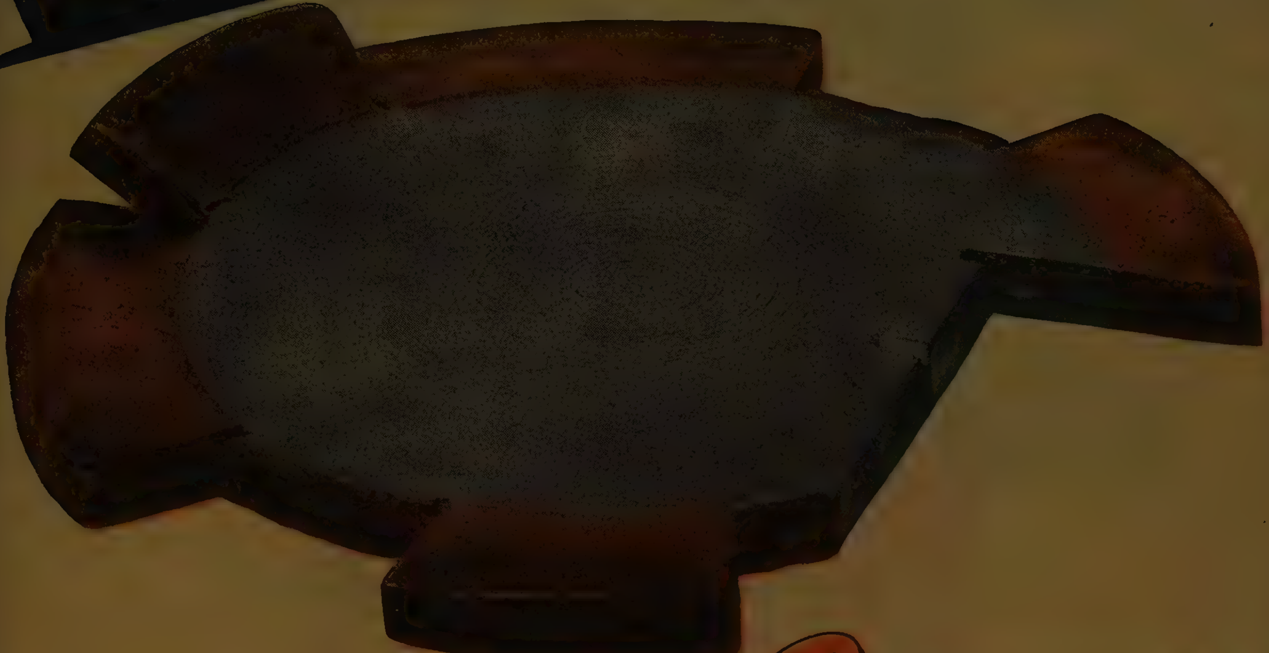

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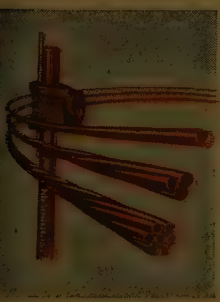

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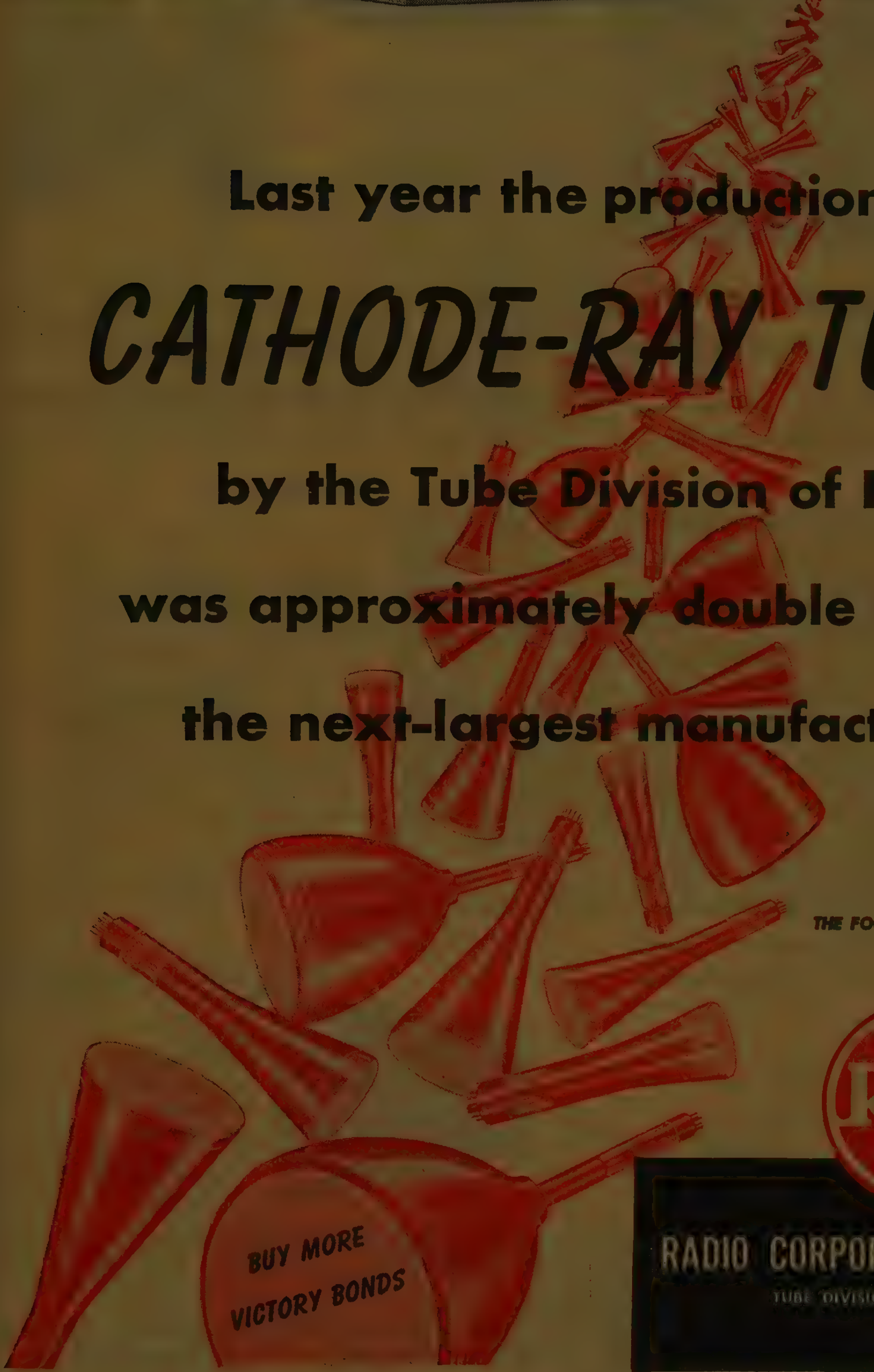

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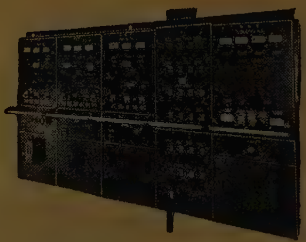

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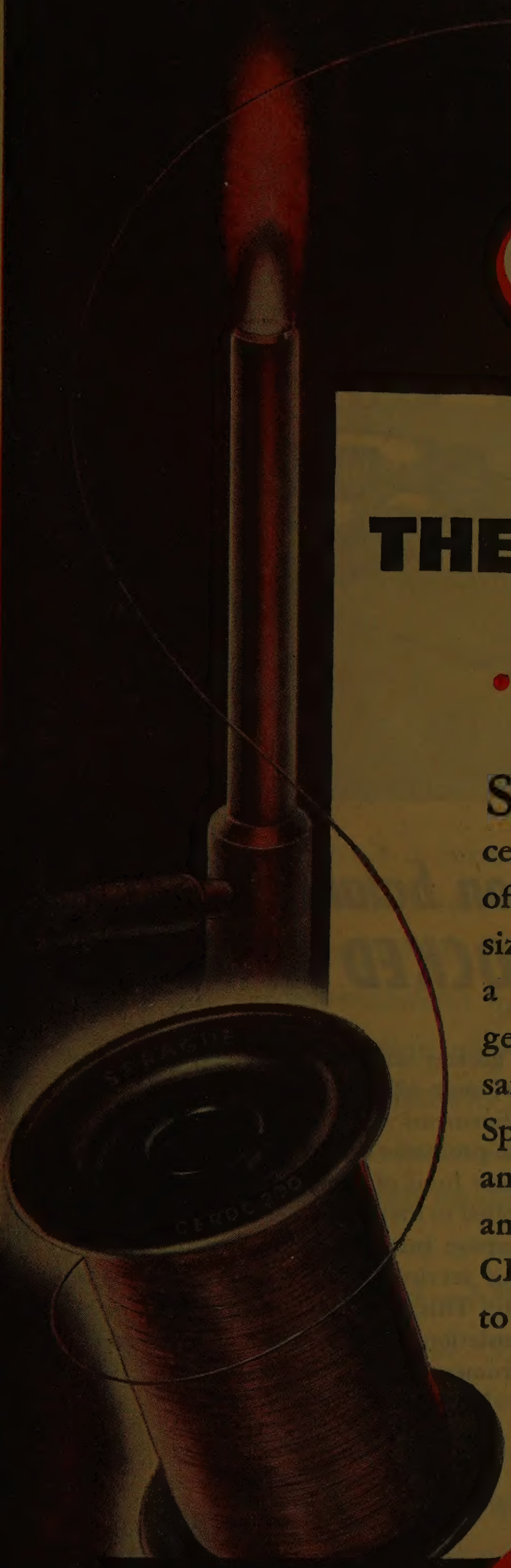

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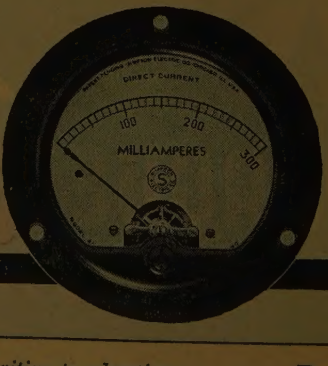

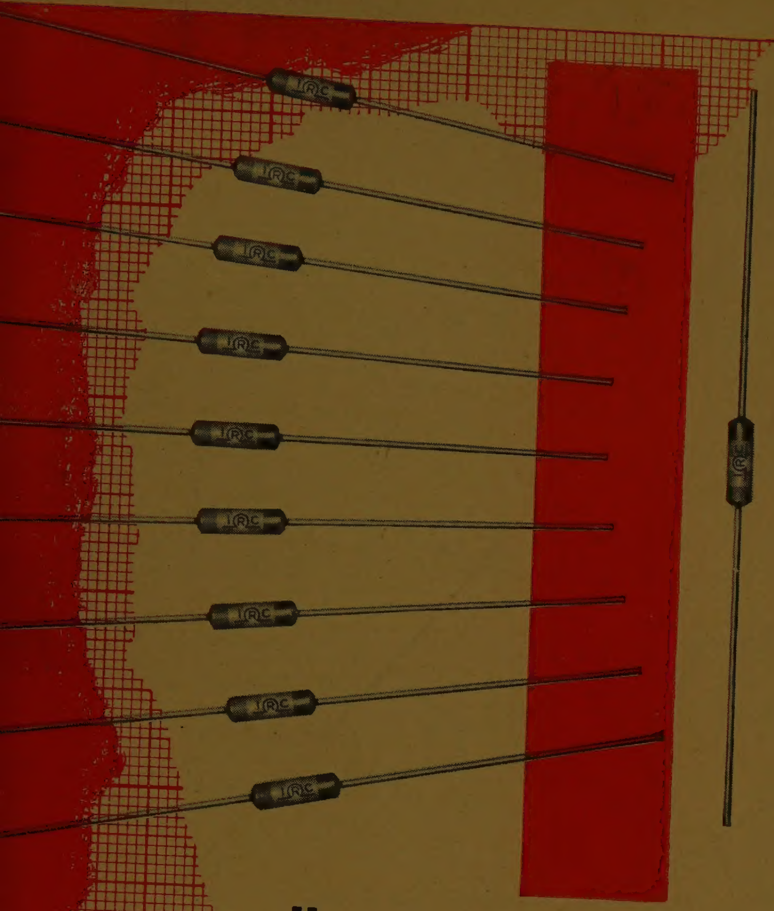

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